The role of taxation in a Ramsey model with environmental externalities

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Abstract

We examine the long-run properties of a dynamic general equilibrium model for a Ramsey economy with endogenous labor and an environmental negative externality. We find that the existence of the environmental externality gives a non-zero capital tax in the long run, which is in contrast to the result of Chamley (1986) and Judd (1985). Moreover, the endogeneity of labor creates an additional channel of substitution between consumption and labor, besides the channel of substitution between consumption today and tomorrow. This characteristic of our model improves its performance in terms of stylized facts of the economy compared to the equivalent model where labor is exogenous.

1 Introduction

Considering the role of environmental quality as a public good in dynamic macroeconomic models is gaining a lot of ground over the last two decades. The premise is that governments levy an environmental tax to, *inter alia*, prevent environmental degradation. Increasing environmental awareness places further pressure on governments to reconsider their environmental policies. The Fukushima nuclear disaster in 2011 strengthened considerably the share of people opposing the use of nuclear power (BBC, 2011). Germany decided to shut down all nuclear plants by 2022, despite the obvious impact of this decision on output and employment, especially given the ongoing economic turmoil in the European Union. Such decisions place inevitably the role of environemntal awareness in a central position within the labor decisions of households and fiscal decisions of governments.

The purpose of this paper is to study the impact of environmental externalities within a basic neoclassical growth model with labor and capital taxes. Our setup augments the seminal models of Chamley (1986) and Judd (1985), henceforth Chamley-Judd, by adding an environmental externality to an economy in which both labor and capital are determined endogenously.[1] The representative economy consists of a large number of identical infinitely-lived private agents, whose utility depends on private consumption, labor and the stock of environmental quality. The agents consume, save and produce a single good. The production of this good yields environmental pollution and this worsens environmental quality, which is assumed to be a public good. In other words, private agents do not internalize the effects of their actions on environmental quality. The decentralized equilibrium is inefficient and policy intervention is justified.

This paper has two novelties. First, to the best of our knowledge, there is no other study that examines the existence of an environmental externality in a model similar to that of Chamley-Judd. We find that the capital tax is uniquely determined in equilibrium by the

[1] The Chamley-Judd result states that in a steady state there should be no wedge between the intertemporal rate of substitution and the marginal rate of transformation, i.e. the optimal tax on capital is zero.

environmental parameters of our model, namely pollution technology and abatement policy. More specifically, in the case where the pollution technology is zero, the capital tax is also zero and our result is identical to the Chamley-Judd result. In contrast, in the presence of a negative environmental externality, the tax on capital is positive and the Chamley-Judd result does not hold. We could say that in our model with an environmental externality, we obtain a second-order Chamley-Judd result, where the capital tax is always positive but constant and responds only to changes in environmental parameters.

Second, we endogenize labor supply. In this way, individuals face two types of trade-offs, one between consumption and investment and another between consumption and leisure. We introduce this additional channel of substitution in a dynamic general equilibrium (DGE) model with an environmental externality. This is mostly observed in the real business cycle (RBC) literature, where labor is endogenous, and this creates an additional choice for intratemporal substitution (see e.g., Kydland and Prescott, 1982, Plosser, 1989). By endogenizing the labor/leisure decision, increases in the public good maximize a individual's utility, and therefore agents are willing to pay more relative to the case where labor is exogenous. When the provision of the public good is reduced, the endogeneity of labor softens the reaction of the society and less compensation is required to keep utility at a higher level. Thus, our model becomes more realistic, it is consistent with recent trends in environmental awareness, and agrees with the stylized facts describing the relationship between environmental quality (measured by concentration of sulfur dioxide) and awareness (measured by indices constructed on the basis of information from the World Values Survey).

Both novelties relate to an important literature on macro-dynamic models. The existence of a non-zero limiting capital tax is in line with the literature studying non-environmental models. This literature shows that the optimal factor taxation may involve positive tax rates on both capital and labor incomes. For example, Correia (1996) shows that when there exists an extra factor that cannot be taxed, the capital tax is non-zero. The same result comes from Stiglitz (1987), where there are limitations on the government's information and on the government's ability to impose taxes. Jones, Manuelli and Rossi (1997) study a dynamic model with human capital and show that if the tax code is not sufficiently rich and there are pure profits from accumulating human capital, the optimal tax rate on income from physical capital does not go to zero. Finally, Acemoglu, Golosov and Tsyvinski (2011) show that when taxes and public good provision are decided by a self-interested politician, who cannot commit to policies, the long-run capital taxation is no longer equal to zero if the politician is less patient than the citizens. Evidently, in all of these papers, a non-zero capital tax arises due to constraints on the government to impose taxes. In our model these constraints are not needed; the mere existence of an environmental externality yields this result.

Our work is also related to the growth and environmental literature. In their recent work, Angelopoulos, Economides and Philippopoulos (2010) use a micro-founded dynamic stochastic general equilibrium (DSGE) model with exogenous labor to rank different environmental policy instruments under uncertainty. Xepapadeas (2005) and Economides and Philippopoulos (2008) study the case where the government imposes taxes on polluting activities and then uses the revenues to finance abatement activities. Bovenberg and Smulders (1995) explore the link between environmental quality and economic growth in an endogenous growth model that incorporates pollution-augmenting technological change. A common characteristic of these papers is that the utility function is independent from the labor/leisure decision.

In fact, many endogenous growth models with fiscal policy treat labor supply as inelastic. This treatment limits certain aspects of fiscal policy (Turnovsky, 2000). More precisely, De Hek (2006), studies an endogenous growth model with physical capital and suggests that the flexibility of the labor supply may induce agents to spend more or less time on leisure activities, depending on the relative sizes of the substitution and income effects. Even more importantly, Flores and Graves (2008) argue that exogeneity of labor generally results in undervaluation of utility due to increases in the provision of a public good. Phrased differently, if the labor supply is exogenously fixed, the Le Chatelier-Samuelson principle holds. Intuitively, this follows from the fact that an increase in the cost of the public good will result in a higher marginal valuation of ordinary private goods, as their quantities are reduced to pay for the public good, and this in turn will result in a higher marginal cost of leisure.

The rest of the paper is organized as follows. In the next section we describe our model. In Section 3 we solve for the decentralized competitive equilibrium, check for its stability and compare our model with the equivalent model with endogenous labor using impulse responces and stylized facts. In Section 4 we solve for the Ramsey equilibrium and check for its stability. Moreover we compare our result with the Chamley-Judd result, offer some numerical examples and illustrate the dynamic responses to permanent shocks in some of the parameters of interest in our model. Section 5 concludes.

2 Description of the economy

In this section we describe our basic framework, placing particular emphasis on the fact that labor decisions are endogenous in the individual's preferences and their weight in the utility function is proportional to the weight placed on consumption and environmental quality. Subsequently, we describe the decisions of firms the laws of motion of natural resources, the resources constraint and we close the model with the government budget constraint.

2.1 Households

We assume that the population size is constant and equal to one. The representative infinitely-lived household maximizes the intertemporal utility

$$\sum_{t=0}^{\infty} \beta^t U(c_t, lt, Q_t), \tag{1}$$

where c is the private consumption, l is leisure, Q is the stock of environmental quality and $\beta \in (0,1)$ is the time discount rate. Without loss of generality, the utility function has the form:

$$U(c_t, l_t, Q_t) = \frac{[(c_t)^{\mu_1}(l_t)^{\mu_2}(Q_t)^{1-\mu_1-\mu_2}]^{1-\sigma}}{1-\sigma}, \tag{2}$$

where $\mu_1, \mu_2, 1-\mu_1-\mu_2 \in (0,1)$ are preference parameters that assign weights to consumption, leisure and environmental quality, respectively, and $\sigma \geq 0$ is a measure of risk aversion. The household is endowed with one unit of time that can be used for leisure l_t or labor n_t, thus $n_t + l_t = 1$. Each household can save in the form of capital k_t, receiving a rate of return r_t. Households also supply inelastically one unit of labor services and receive labor income $w_t n_t$. Further, they receive dividends π_t. Each household has to pay a portion of its income to the government in the form of linear taxes. τ_t^k is the tax on capital income and τ_t^l is the tax on labor income. The flow budget constraint of the household is

$$k_{t+1} - (1-\delta^k)k_t + c_t = y_t = (1-\tau_t^l)w_t n_t + (1-\tau_t^k)r_t k_t + \pi_t, \tag{3}$$

where k_{t+1} is the end-of-period capital stock, k_t is the beginning-of-period capital stock and $\delta^k \in$[0,1] is the rate of capital depreciation.

From all the above it follows that the household's problem is to

$$\max_{\{c_t, l_t, k_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \frac{[(c_t)^{\mu_1}(1-n_t)^{\mu_2}(Q_t)^{1-\mu_1-\mu_2}]^{1-\sigma}}{1-\sigma}$$

$$s.t. k_{t+1} - (1-\delta^k)k_t + c_t = (1-\tau_t^l)w_t n_t + (1-\tau_t^k)r_t k_t + \pi_t,$$

taking w_t, r_t and the policy as given. The FOCs for this problem are

$$U_{c_t} = \lambda_t, \tag{4}$$

$$\frac{c_t}{1-n_t} = \frac{\mu_1}{\mu_2}(1-\tau_t^l)w_t, \tag{5}$$

$$U_{c_t} = \beta U_{c_{t+1}}[(1-\tau_{t+1}^k)r_{t+1} + 1 - \delta^k]. \tag{6}$$

The last equation is the Euler equation for capital. It tells us that along an optimal path, the marginal utility from consumption at any point in time is equal to the opportunity cost of consumption.

2.2 Firms

The production function of the representative firm is a neoclassical function with constant returns to scale of the form

$$y_t = Ak_t^a n_t^{1-a} = f(k_t, n_t), \tag{7}$$

where $a \in (0,1)$ is the output elasticity of private capital and $1-a \in (0,1)$ is the private elasticity of labor. A is total factor productivity or the index of production technology, which is assumed to be constant. In each period, the representative firm takes w_t and r_t as given and uses capital and labor services from households. The objective of the firm is to

$$\max_{\{c_t, l_t, k_{t+1}\}_{t=0}^{\infty}} \pi_t = y_t - w_t n_t - r_t k_t. \tag{8}$$

The FOCs for this problem are

$$r_t = a\frac{y_t}{k_t}, \tag{9}$$

$$w_t = (1-a)\frac{y_t}{n_t}, \tag{10}$$

so that $\pi = 0$.

2.3 Laws of motion of natural resources

The evolution of the stock of environmental quality is given by

$$Q_{t+1} = (1-\delta^q)\bar{Q} + \delta^q Q_t - p_t + \nu g_t, \tag{11}$$

where $\bar{Q} \geq 0$ is the environmental quality without pollution, p_t is the current pollution flow, $\delta^q \in$[0,1] is the degree of environmental persistence, g_t is public spending on abatement activities and $\nu \geq 0$ shows how public abatement spending is transformed into units of renewable resources. The flow of pollution is caused by the production of output and is given by

$$p_t = \varphi A k_t^a n_t^{1-a}, \tag{12}$$

where ϕ is an index of pollution technology or it represents the emissions per unit of output.

2.4 Government budget constraint

The government collects revenues from the taxes on labor and capital. On the expenditure side, it spends on abatement policy g_t. Assuming a balanced budget, we have

$$g_t = A k_t^a n_t^{1-a}[a\tau_t^k + (1-a)\tau_t^l]. \tag{13}$$

2.5 Resource constraint (technology)

Output can be consumed by households, used to increase the capital stock and/or used by the government. The resource constraint therefore is

$$c_t + g_t + k_{t+1} = y_t + (1-\delta^k)k_t. \tag{14}$$

3 Decentralized competitive equilibrium

We solve the problem described in Section 2 for a DCE in which (i) households maximize welfare, (ii) firms maximize profits, (iii) all constraints are satisfied and (iv) all markets clear. The DCE of the above economy is given by the following equations:

$$\frac{c_t}{1-n_t} = \frac{\mu_1}{\mu_2}(1-\tau_t^l)w_t, \tag{15}$$

$$U_{c_t} = \beta U_{c_{t+1}}[(1-\tau_{t+1}^k)r_{t+1} + 1 - \delta^k], \tag{16}$$

$$Q_{t+1} = (1-\delta^q)\bar{Q} + \delta^q Q_t - \varphi A k_t^a n_t^{1-a} + \nu g_t, \tag{17}$$

$$g_t = A k_t^a n_t^{1-a}[a\tau_t^k + (1-a)\tau_t^l], \tag{18}$$

$$c_t + k_{t+1} = A k_t^a n_t^{1-a} - g_t + (1-\delta^k)k_t. \tag{19}$$

The DCE holds for given initial conditions for the stock variables k_0 and Q_0, the FOCs of the representative firm's problem, the exogenous variables A and φ, for given policy (which is summarized by the tax rates τ^l, τ^k) and provided that $r_t = aAk_t^{a-1}n_t^{1-a}$, $w_t = (1-a)Ak_t^a n_t^{-a}$. Therefore, we have a system of five equations in $\{c_t, n_{t,} k_{t+1}, Q_{t+1}, g_t\}_{t=0}^{\infty}$. We can obtain the long-run DCE if we simply drop the time subscripts.

3.1 Steady state

To find the steady state we solve the above system for c^*, n^*, Q^*, k^*, g^*, where the asterisk denotes the steady state value of each variable. Therefore, we have that

$$c^* = \frac{\mu_1}{\mu_2}(1-\tau^l)(1-a)AX^{\frac{a}{a-1}}(1-X^{\frac{1}{1-a}}k^*), \tag{20}$$

$$n^* = X^{\frac{1}{1-a}}k^*, \tag{21}$$

$$Q^* = \bar{Q} - k^*\frac{AX}{(1-\delta^q)}[\varphi - \nu a\tau^k - \nu(1-a)\tau^l], \tag{22}$$

$$g^* = AXk^*[a\tau^k + (1-a)\tau^l], \tag{23}$$

where

$$k^* = \frac{\frac{\mu_1}{\mu_2}(1-a)AX^{\frac{a}{a-1}}(1-\tau^l)}{\delta^k - [\frac{\mu_1}{\mu_2}(1-a) + a(1-\tau^k) + (1-a)(1-\tau^l)]AX} \tag{24}$$

and

$$X = \frac{(1 - \beta + \beta\delta^k)}{a\beta A(1 - \tau^k)}. \tag{25}$$

The steady state values show that an increase in the capital tax τ^k causes c^* and k^* to fall, but n^*, $Q^*$and g^* to rise. Agents substitute consumption with labor and the tax increase helps the government to spend more in abatement policy, which improves environmental quality. We obtain almost the same results by an increase in τ^l, with the only difference that the amount of labor in equilibrium falls. Moreover, a change in the environmental parameters φ and ν, affects only the environmental quality. Specifically, a deterioration in pollution technology or an increase in increases in government's spending for abatement improve environmental quality. An increase in the ratio $\frac{\mu_1}{\mu_2}$ (weight on consumption) or an increase in β, increases consumption, labor and capital in the steady state. Although government spending for abatement decreases, the environmental quality rises. Therefore we obtain exactly the same results either the weight to consumption increases (weight to consumption and/or weight to environmental quality falls), or agents become more patient and care more about future.

3.2 Linearization

By substituting Eq. (18) in the rest of the equations of the DCE, the DCE becomes:

$$c_t + k_{t+1} = Ak_t^a n_t^{1-a} - Ak_t^a n_t^{1-a}[a\tau_t^k + (1-a)\tau_t^l] + (1 - \delta^k)k_t, \tag{26}$$

$$\frac{c_t}{1 - n_t} = \frac{\mu_1}{\mu_2}(1 - \tau_t^l)(1 - a)Ak_t^a n_t^{-a}, \tag{27}$$

$$U_{c_t} = \beta U_{c_{t+1}}[(1 - \tau_{t+1}^k)aAk_{t+1}^{a-1}n_{t+1}^{1-a} + 1 - \delta^k], \tag{28}$$

$$Q_{t+1} = (1 - \delta^q)\bar{Q} + \delta^q Q_t - \varphi Ak_t^a n_t^{1-a} + \nu Ak_t^a n_t^{1-a}[a\tau_t^k + (1-a)\tau_t^l]. \tag{29}$$

We linearize the system of Eqs. (26)-(29) around the steady state, using Taylor's Theorem. We asume that the exogenous tax rates, τ^l and τ^k, take the values from the respective

Ramsey optimization problem. After replacing the rest of the parameters with the values used in the literature (e.g., Angelopoulos, Economides and Philippopoulos, 2010), we find that the model is stable (for the proof, see Appendix A).

3.3 Impulse responses and stylized facts

To see how the endogeneity of labor affects the equilibrium results, we compare our DCE with the DCE of an equivalent model with exogenous labor. We illustrate the dynamic responses of both these economies to permanent unitary increases in some of the parameters of interest in the models. To do so, we set the output to be equal in both models. Capital is a state variable and, thus, we initialize it at t = 0 based on what is assumed in the model with endogenous labor. Initially, all variables are at their steady-state levels.

Figure 1 shows how the economy with endogenous labor reacts to a 1% increase in the weight on environmental quality. We observe that when both μ_1 and μ_2 decrease output, consumption, labor and capital also decrease. As discussed above, there is one channel of substitution towards investment and against consumption and another channel towards labor and against consumption. The environmental quality improves permanently. Households internalize the fact that the output is harmful for the environment, so they produce and consume less. In this way the taxes paid to the government are reduced and less funds are needed for abatement policy. Through this mechanism welfare also improves permanently. Notably, in the endogenous labor model preferences of individuals for a better environment are positively correlated with variables such as emissions, which characterize the environmental quality. This is a realistic finding, and is in line with the stylized facts that we examine below in this section. Similar results hold for the measure of risk aversion σ (see Appendix B).

Figure 2 shows how the economy with exogenous labor reacts to a 1% increase in the weight on environmental quality. The results now are quite different. Output and capital increase. Consumption decreases and, even though the government spending used in abatement policy increases, pollution increases by even more. This leads to a degradation of environmental quality. Agents can now only substitute current consumption with future consumption but obviously this is not adequate to improve environmental quality. The outcome is an increase in welfare, despite the fact that environmental quality deteriorates. This is obviously in contrast with the finding of a positive relationship between environmental quality and welfare presented in Figure 1.

Figure 3 shows how the economy with endogenous labor reacts to a 1% increase in the pollution parameter. An increase in the pollution parameter means that agents use a more polluting technology in the production of the output, therefore they care less for the environmental quality. We observe that, with the exception of consumption, all variables decrease. Agents substitute labor and investment with current consumption. The environmental quality is depleted permanently. Households do not internalize the fact that the output is harmful for the environment and, therefore, welfare decreases.

Finally, Figure 4 shows the response to a 1% increase in the pollution parameter φ of the fundamentals of the economy with exogenous labor. Again, the findings are quite different from those reported in Figure 3. Agents substitute future consumption with current consumption and therefore current consumption increases. They produce more polluting output and consume more. In this way they pay higher taxes to the government, which are used in abatement policy. The polluting production technology has a stronger effect than the abatement policy and environmental quality and welfare decrease. An increase in the abatement parameter ν has exactly the opposite results (see Appendix B).

We can see in both cases that the change in the environmental quality affects welfare and this effect is greater in the model with endogenous labor than in the case where labor is exogenous. Therefore, the Le Satelier Samuelson principle holds and exogeneity of labor results in undervaluation for increases in the environmental quality (public good).

The importance of endogenizing labor is further supported by the stylized facts relating environmental awareness and environmental quality. Figures 5-7 report this bivariate relationship using data from the World Values Survey and Yale's Environmental Performance Index. We measure environmental quality with the concentration of sulfur dioxide (SO_2). This is the measure most commonly employed in the related empirically literature for a number of reasons. First, air quality is one of the most important indicators of environmental quality and SO_2 is one of the "Criteria Air Contaminants" used by the United States Environmental Protection Agency, the World Bank, the OECD, and other authorities to describe air quality. Second, SO_2 is a major air pollutant and has significant impacts on human health, ecosystems, and the economy. Third, SO_2 emissions can be controlled by altering the techniques of production. Fourth, there are good data are available for a large number of countries and over long time periods. Finally, Bernauer and Koubi (2009) show that most forms of air pollution (such as SO_2, CO_2, N_2O and NO_x) behave quite similarly across countries, and, thus, SO_2 captures general trends in overall air pollution.

As a measure of environmental awareness we use the relative information from the World Values Survey. More specifically, we build three indices by aggregating the responses to the following questions:

- Income provision: would give part of my income for the environment, with variable/question code b001, for the years 2005-2007
- Environmental action 1: attend meeting, signed petition, with variable/question code b014, for the years 1994-1999
- Environmental action 2: contributed to environmental organization, with variable/question code b015, for the years 1994-1999

An increase in the index constructed from the first question implies that people are less willing to give part of their income for the environment, reflecting a decreasing environmental awareness. From Figure 5 we note that the relationship between this aspect of environmental awareness and SO_2 is positive. The slope from this simple regression with robust standard errors is 24.4 and statistically significant at the 5% level. This means that as environmental awareness increases, the emissions of SO_2 decrease, and therefore environmental quality increases. Further, Figures 6 and 7 show that the relationship between SO_2 and the two indices measuring environmental action is negative. This means that as environmental action increases, the emissions of SO_2 decrease, therefore, environmental quality increases. These results are not affected by the presence of the few outliers shown on the Figures. Evidently, the stylized facts are in line with the model where labor is endogenous. This provides further evidence that endogenizing labor is very important mainly because increases in the public good maximize utility, and therefore agents are willing to pay more relative to the case where labor is exogenous. In other words, treating labor as fixed results in measurement errors.

4 The Ramsey problem with an environmental externality

In this section we place our DCE framework within the framework of Ramsey taxation. Ramsey taxation provides a compelling argument against taxing capital income in the long run in dynamic macroeconomic models. Here we show how this result is transformed when we consider a negative environmental externality. Following Chamley (1986), we replace r_t and w_t with net factor prices $\widetilde{r}_t$ and $\widetilde{w}_t$, where

$$\widetilde{r}_t = (1 - \tau_t^k) r_t, \tag{30}$$

$$\widetilde{w}_t = (1 - \tau_t^l) w_t. \tag{31}$$

In this way, the four instruments $\tau_t^k, \tau_t^l, r_t, w_t$ reduce to two. Thus, the DCE is given by

$$\frac{c_t}{1 - n_t} = \frac{\mu_1}{\mu_2} \widetilde{w}_t, \tag{32}$$

$$U_{c_t} = \beta U_{c_{t+1}} (\widetilde{r}_{t+1} + 1 - \delta^k), \tag{33}$$

$$Q_{t+1} = (1 - \delta^q)\bar{Q} + \delta^q Q_t - \varphi A k_t^a n_t^{1-a} + \nu g_t, \tag{34}$$

$$g_t = A k_t^a n_t^{1-a} - \widetilde{w}_t n_t + \widetilde{r}_t k_t, \tag{35}$$

$$c_t + k_{t+1} - (1 - \delta^k) k_t + g_t = A k_t^a n_t^{1-a}. \tag{36}$$

The FOCs of the above problem are:

$$U_{c_t} = \frac{1}{1-n_t}\lambda_t + \chi_t - \partial U_{c_t}/\partial c_t[\psi_{t-1}(\widetilde{r}_t + 1 - \delta) - \psi_t], \tag{37}$$

$$U_{n_t} = \frac{c_t}{(1-n_t)^2}\lambda_t - (1-a)Ak_t^a n_t^{1-a}(\xi_t - \zeta_t\varphi + \chi_t) \tag{38}$$

$$+\xi_t\widetilde{w}_t + \partial(U_{c_t}/\mu_1)/\partial n_t[\psi_t - \psi_{t-1}(\widetilde{r}_t + 1 - \delta)],$$

$$\chi_t = \beta[\chi_{t+1}(f_k + 1 - \delta^k) + \xi_{t+1}(f_k - \widetilde{r}_{t+1}) - \zeta_{t+1}\varphi f_k], \tag{39}$$

$$U_{Q_t}[\psi_t(\widetilde{r}_{t+1} + 1 - \delta^k) - \psi_{t+1}] = \frac{\zeta_t}{\beta} - U_{Q_{t+1}} - \zeta_{t+1}\delta^q, \tag{40}$$

$$\xi_t k_t = \psi_{t-1}U_{c_t}, \tag{41}$$

$$\lambda_t\frac{\mu_1}{\mu_2} = \xi_t n_t, \tag{42}$$

$$\nu\zeta_t = \xi_t + \chi_t, \tag{43}$$

$$\frac{\mu_1}{\mu_2}\widetilde{w}_t = \frac{c_t}{(1-n_t)}, \tag{44}$$

$$U_{c_t} = \beta U_{c_{t+1}}[\widetilde{r}_{t+1} + 1 - \delta^k], \tag{45}$$

$$Q_{t+1} = (1-\delta^q)\bar{Q} + \delta^q Q_t - \varphi Ak_t^a n_t^{1-a} + \nu g_t, \tag{46}$$

$$Ak_t^a n_t^{1-a} - \widetilde{w}_t n_t + \widetilde{r}_t k_t = g_t, \tag{47}$$

$$c_t + k_{t+1} = Ak_t^a n_t^{1-a} + (1-\delta^k)k_t - g_t. \tag{48}$$

Eq. (39) tells us that a marginal increase of capital investment in period t increases the amount of available goods in period $t+1$ by $(f_k + 1 - \delta)$ with social marginal value χ_{t+1}. Moreover, tax revenues increase by $(f_k - \widetilde{r}_{t+1})$, which enables the government to reduce its debt on other taxes by the same amount. This increase has a social marginal value equal to ξ_{t+1}, which is interpreted as the extra burden imposed to the society due to the existence of distortionary taxation. β is the discount factor in period $t+1$ and χ_t is the social marginal value of investment good in period t. Therefore, χ_t and ξ_t are positive for all t. Finally, we can see that the increase of capital investment worsens environmental quality by φf_k with social marginal value ζ_{t+1}.

We obtain the long-run conditions by dropping the time subscripts. To simplify the FOCs we set in the utility function $U(c_t, l_t, Q_t)$, $\sigma = 1$, which then limits to

$$U(c_t, l_t, Q_t) = \mu_1 \ln(c_t) + \mu_2 \ln(l_t) + (1 - \mu_1 - \mu_2) \ln(Q_t). \tag{49}$$

As we did with the DCE, we linearize the system of Eqs. (37)-(48) around the steady state, using Taylor's Theorem. We use the same values for the parameters and we find that the model is stable (for details see Appendix D). Once again there is a unique equilibrium and the economy converges to this through a saddle path.

4.1 The Chamley-Judd approach to the Ramsey problem

Eq. (39) in the long run reduces to

$$\beta[(r - \widetilde{r})\xi + (r + 1 - \delta)\chi - r\varphi\zeta] = \chi. \tag{50}$$

From Eq. (45) in the long run it holds that $(1 - \delta) = \frac{1}{\beta} - \widetilde{r}$. By replacing this result into (49) and rearranging we have

$$(r - \widetilde{r})(\chi + \xi) - r\phi\zeta = 0. \tag{51}$$

We now consider two cases, where $\varphi = 0$ and $\varphi \neq 0$. In this case the environmental externality is zero, and Eq. (50) becomes

$$\tau^k(\chi + \xi) = 0. \tag{52}$$

The marginal social value of goods χ is strictly positive and the marginal social value of reducing government taxes ξ is nonnegative, therefore r must be equal to $\widetilde{r}$, so that τ^k is equal to zero. This is the result of the papers by Chamley (1986) and Judd (1985).

We can see this result using a simple numerical example. In Table 1 we provide the parameter values and in Column I of Table 2 the results. The values used for the parameters are as in most dynamic general equilibrium studies (e.g., Angelopoulos, Economides and Philippopoulos, 2010). The findings show that $\tau^k = 0$ and the discounted welfare for $t = 100$ is

$$\begin{aligned} U^*(c,n,Q) &= \frac{(1-\beta^t)}{(1-\beta)} U(c,n,Q) = \frac{(1-\beta^{100})}{(1-\beta)} \frac{(c^{\mu_1}(1-n)^{\mu_2} Q^{(1-\mu_1-\mu_2)})^{(1-\sigma)}}{(1-\sigma)} \\ &= -37.28275513. \end{aligned}$$

In the case where $\varphi \neq 0$ the first term of Eq. (50) is exactly the same with the Chamley-Judd result. The second term of Eq. (50) appears because of the positive environmental externality. By substituting $\widetilde{r}$ with $r(1-\tau^k)$ and by using Eq. (43) we have that

$$\tau^k = \frac{\varphi}{\nu}. \tag{53}$$

It must hold that $\tau^k < 1 \Leftrightarrow \frac{\varphi}{\nu} < 1$, or $\varphi < \nu$.

In Column II of Table 2 we provide the results from the numerical example where φ is positive and equal to 0.5. The values of the parameters are as before. Evidently, τ^k is positive and discounted welfare in this case for $t = 100$ is given by

$$\begin{aligned} U^*(c,n,Q) &= \frac{(1-\beta^t)}{(1-\beta)} U(c,n,Q) = \frac{(1-\beta^{100})}{(1-\beta)} \frac{(c^{\mu_1}(1-n)^{\mu_2} Q^{(1-\mu_1-\mu_2)})^{(1-\sigma)}}{(1-\sigma)} \\ &= -39.49321353. \end{aligned}$$

The presence of the environmental externality worsens environmental quality. Taxes increase and this leads to a lower level of utility, compared to the case where the environmental externality is equal to zero. Thus, in our model with an environmental externality, we obtain a second-order Chamley-Judd result, where the capital tax is always positive and responds only to changes in environmental parameters.

4.2 Impulse response functions for the Ramsey economy

In this section we illustrate the dynamic response of the Ramsey economy to permanent unitary increases in some of the parameters of interest in our model. Here we study the responses due to a 1% increase in the weight to environmental quality and a 1% increase in the pollution parameter φ. Responses to abatement technology ν, rate of risk aversion σ and time preference rate β are given in the Appendix D.

Figure 8 shows how the economy responds to a 1% increase in the weight to environmental quality. We observe that in the long-run output, consumption, labor and capital decrease. There is a channel of substitution running from consumption to investment and another running from consumption to labor. The second channel of substitution stems from the fact that labor is endogenous. The environmental quality is much higher compared to the initial equilibrium. Households realize that the output is harmful for the environment, so they produce and consume less. The labor tax increases in the long run and this increases the amount of money used in abatement policy by the government. The capital tax does not change in the long run, as its value is determined only by environmental parameters. Finally welfare is higher compared to the initial equilibrium. Once again this finding is in line with the stylized facts presented in Section 3.3.

Similarly, a 1% increase in the pollution parameter φ has a negative effect on the environmental quality (Figure 9). An increase in the pollution parameter implies that agents use a more polluting technology for the production of the output production, therefore placing less weight on environmental quality. Households substitute current consumption for labor; however, the decrease in current consumption is small because agents prefer to consume more today than in the future. Given this mechanism, capital decreases and both capital and labor taxes increase in the long-run. Agents produce less output and, even though they pay more taxes to the government and the government spending used for abatement policy increases, the increase in the pollution production prevails. Environmental quality is depleted and welfare decreases.

5 Conclusions

This paper builds on the literature of taxation of capital and labor to study a dynamic general equilibrium model with an environmental externality. Our model without an environmental externality gives the result of a zero optimal capital tax, which is in line with the seminal contributions of Chamley (1985) and Judd (1986). However, the inclusion of the environmental externality alters this result and yields in equilibrium a positive and constant capital tax that responds only to changes in the environmental parameters.

An important element of our framework is that labor decisions of households are endogenous. This assumption is consistent with the increasing environmental awareness, linking employment and output decisions with environmental quality. Endogenizing the supply of labor also produces non-trivial changes in the results. In particular, the model without endogenous labor predicts that an increase in the weight on environmental quality actually by households actually leads to environmental degradation. Using simple indicators of environmental quality and awareness, we show that this finding contrasts the stylized facts. In turn, the same model with endogenous labor predicts an improvement in environmental quality following a positive shock on the weight placed on environmental quality by households, which is a result in line with the stylized facts.

Interesting extensions to this model are the inclusion of uncertainty, imperfect competition and/ or a different production technology. As this paper has covered considerable ground, we leave these ideas for future research.

Appendix A: The Linearization of the DCE

Eq. (26) becomes

$$f(c_t, k_{t+1}, k_t, n_t) = \hat{c}_t + \hat{k}_{t+1} + f_a \hat{n}_t + f_b \hat{k}_t = 0 \tag{A1}$$

where for any variable x of the system it holds that $\hat{x}_t = x_t - x^*$, with x^* being the steady state value of the variable and

$$f_a = f_{n_t}(\cdot) = [-A(1-a)(k^*)^a (n^*)^{-a}[1 - a\tau^k - (1-a)\tau^l]] \tag{A2}$$

$$f_b = f_{k_t}(\cdot) = [-[aA(k^*)^{a-1}(n^*)^{1-a}[1 - a\tau^k - (1-a)\tau^l] + (1-\delta^k)]] \tag{A3}$$

Eq. (27) becomes

$$g(c_t, k_t, n_t) = \mu_2 \hat{c}_t + g_a \hat{n}_t + g_b \hat{k}_t = 0 \tag{A4}$$

where

$$g_a = g_{n_t}(\cdot) = [\mu_1 a A(k^*)^a (n^*)^{-a-1} + \mu_1 (1-a) A(k^*)^a (n^*)^{-a}](1-\tau^l)(1-a) \tag{A5}$$

$$g_b = g_{k_t}(\cdot) = [-\mu_1 (1-n^*)(1-a) a A(k^*)^{a-1}(n^*)^{-a}(1-\tau^l)] \tag{A6}$$

Eq. (28) becomes

$$h(c_{t+1}, n_{t+1}, Q_{t+1}, k_{t+1}, c_t, n_t, Q_t) = h_a \hat{c}_{t+1} + h_b \hat{n}_{t+1} + h_c \hat{Q}_{t+1} \tag{A7}$$
$$+ h_d \hat{k}_{t+1} + h_e \hat{c}_t + h_f \hat{n}_t + h_g \hat{Q}_t = 0$$

where

$$h_a = h_{c_{t+1}}(\cdot) = -[\beta[\mu_1(1-\sigma)-1](c^*)^{\mu_1(1-\sigma)-2}[(1-n^*)^{\mu_2}(Q^*)^{1-\mu_1-\mu_2}]^{1-\sigma}] \tag{A8}$$

$$[(1-\tau^k)aA(k^*)^{a-1}(n^*)^{1-a}+1-\delta^k]$$

$$h_b = h_{n_{t+1}}(\cdot) = [\beta\mu_2(1-\sigma)(c^*)^{\mu_1(1-\sigma)-1}(1-n^*)^{\mu_2(1-\sigma)-1}(Q^*)^{(1-\mu_1-\mu_2)(1-\sigma)} \tag{A9}$$

$$[(1-\tau^k)aA(k^*)^{a-1}(n^*)^{1-a}+1-\delta^k]-\beta(1-a)(c^*)^{\mu_1(1-\sigma)-1}$$

$$[(1-n^*)^{\mu_2}(Q^*)^{1-\mu_1-\mu_2}]^{1-\sigma}[(1-\tau^k)aA(k^*)^{a-1}(n^*)^{-a}]$$

$$h_c = h_{Q_{t+1}}(\cdot) = -[\beta(1-\mu_1-\mu_2)(1-\sigma)(c^*)^{\mu_1(1-\sigma)-1} \tag{A10}$$

$$(1-n^*)^{\mu_2(1-\sigma)}(Q^*)^{(1-\mu_1-\mu_2)(1-\sigma)-1}][(1-\tau^k)aA(k^*)^{a-1}(n^*)^{1-a}+1-\delta^k]$$

$$h_d = h_{k_{t+1}}(\cdot) = [-\beta(c^*)^{\mu_1(1-\sigma)-1}(1-n^*)^{\mu_2(1-\sigma)}(Q^*)^{(1-\mu_1-\mu_2)(1-\sigma)} \tag{A11}$$

$$(1-\tau^k)a(a-1)A(k^*)^{a-2}(n^*)^{1-a}]$$

$$h_e = h_{c_t}(\cdot) = [[\mu_1(1-\sigma)-1](c^*)^{\mu_1(1-\sigma)-2}[(1-n^*)^{\mu_2}(Q^*)^{1-\mu_1-\mu_2}]^{1-\sigma} \tag{A12}$$

$$h_f = h_{n_t}(\cdot) = -[\mu_2(1-\sigma)(c^*)^{\mu_1(1-\sigma)-1}(1-n^*)^{\mu_2(1-\sigma)-1}(Q^*)^{(1-\mu_1-\mu_2)(1-\sigma)}] \tag{A13}$$

$$h_g = h_{Q_t}(\cdot) = [(1-\mu_1-\mu_2)(1-\sigma)(c^*)^{\mu_1(1-\sigma)-1}(1-n^*)^{\mu_2(1-\sigma)}(Q^*)^{(1-\mu_1-\mu_2)(1-\sigma)-1}] \tag{A14}$$

Finally Eq. (29) becomes

$$m(Q_{t+1}, n_t, Q_t, k_t) = \delta^q\hat{Q}_t + m_a\hat{n}_t + m_b\hat{k}_t - \hat{Q}_{t+1} = 0 \tag{A15}$$

where

$$m_a = m_{n_t}(\cdot) = -A(1-a)(k^*)^a(n^*)^{-a}[v[a\tau^k+(1-a)\tau^l]-\phi] \tag{A16}$$

$$m_b = m_{k_t}(\cdot) = -aA(k^*)^{a-1}(n^*)^{1-a}[v[a\tau^k+(1-a)\tau^l]-\phi] \tag{A17}$$

The 4 by 4 system in matrix notation is

$$\begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -h_a & -h_b & -h_d & -h_c \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{c}_{t+1} \\ \hat{n}_{t+1} \\ \hat{k}_{t+1} \\ \hat{Q}_{t+1} \end{bmatrix} = \begin{bmatrix} f_a & 1 & f_b & 0 \\ g_a & \mu_2 & g_b & 0 \\ h_f & h_e & 0 & h_g \\ m_a & 0 & m_b & \delta^q \end{bmatrix} \begin{bmatrix} \hat{c}_t \\ \hat{n}_t \\ \hat{k}_t \\ \hat{Q}_t \end{bmatrix} \Longleftrightarrow A\hat{X}_{t+1} = B\hat{X}_t$$

One way to check the stability of equilibrium is with the approach of Blanchard and Kahn. We observe that the second equation is a static equation. To obtain a real Jacobian we substitute this equation into the other three equations of the system. Thus, the system becomes

$$\begin{bmatrix} 0 & 1 & 0 \\ h_1 & h_2 & h_c \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{c}_{t+1} \\ \hat{k}_{t+1} \\ \hat{Q}_{t+1} \end{bmatrix} = \begin{bmatrix} f_1 & f_2 & 0 \\ h_3 & h_4 & -h_g \\ m_1 & m_2 & \delta^q \end{bmatrix} \begin{bmatrix} \hat{c}_t \\ \hat{k}_t \\ \hat{Q}_t \end{bmatrix} \Leftrightarrow E\hat{X}_{t+1} = F\hat{X}_t \Leftrightarrow \hat{X}_{t+1} = FE^{-1}\hat{X}_t \Leftrightarrow \hat{X}_{t+1} = C\hat{X}_t$$

Using the parameter values in the paper of Angelopoulos, Economides and Philippopoulos (2010), we find that there are two eigenvalues with module smaller than 1 for the backward looking variables $\hat{k}_t$ and $\hat{Q}_t$, and one eigenvalue with module larger than 1 for the forward looking variable $\hat{c}_t$. When we solve the 4 by 4 system using Dynare we find that the eigenvalue of n_t, which is a forward looking variable too, has module larger than 1. The Blanchard-Kahn conditions are satisfied and the model is stable. The steady state of the system is a saddle path, therefore it has a unique equilibrium.

Given that in the initial 4 by 4 system the matrix A is singular, we can also check its stability using the approach of Klein. We first recover the generalized Schur decomposition of (A, B). We get the matrices of complex numbers Q and Z, such that $S = QAZ$ and $T = QBZ$ are upper triangular, and $QQ' = ZZ' = I$. Then the dynamics equation can be rewritten as

$$AZZ'X_{t+1} = BZZ'X_t. \tag{A18}$$

Let us define $\varpi_t = Z'X_t$ to get

$$AZ\varpi_{t+1} = BZ\varpi_t \tag{A19}$$

and premultiply both sides by Q

$$QAZ\varpi_{t+1} = QBZ\varpi_t, \tag{A20}$$

which is equal to

$$S\varpi_{t+1} = T\varpi_t. \tag{A21}$$

$\frac{T_{ii}}{S_{ii}}$ are the Generalized eigenvalues of the system. We find that we have 2 stable eigenvalues with modulus below unity, which are associated with the variables k_t and Q_t, and 2 unstable eigenvalues with modulus greater than unity, which are associated with the variables c_t and n_t. Therefore, the model is stable, the steady state of the system is a saddle path and it has a unique equilibrium.

Appendix B: Other impulse responses for the DCE

Figure 10 shows how the economy with endogenous labor responds to a 1% increase in the abatement technology. An increase in ν implies that public spending in abatement activities increases, and therefore environmental quality is more important for the society. We observe that, with the exception of consumption, all variables increase. Agents substitute current consumption with labor and investment. The environmental quality and welfare increase permanently.

Figure 11 shows the response to a 1% increase in the abatement technology of the economy with exogenous labor. The results are exactly the opposite compared to the case where φ increases. Agents substitute current consumption with future consumption and therefore current consumption decreases. They produce less polluting output and consume less. In this way they pay lower taxes to the government and less expenditure is used for abatement policy. Environmental quality and welfare increase.

Figure 12 shows how the economy with endogenous labor responds to a 1% increase in the coefficient of relative risk aversion, σ. We observe that when σ increases output, consumption, labor and capital decrease. As discussed above, there is one channel of substitution towards investment and against consumption and another channel towards labor and against consumption. The environmental quality improves permanently because households become less risky. They internalize the fact that the output is harmful for the environment, so they produce and consume less. In this way less taxes are paid to the government and less funds are needed for abatement policy. Through this mechanism welfare also improves permanently. Again, in the endogenous labor model preferences of individuals for a better environment are positively correlated with variables such as emissions, which characterize the environmental quality. This is a realistic finding, and is in line with the stylized facts of the economy presented in Figures 5-7.

The effects of a 1% increase in the coefficient of relative risk aversion in model with exogenous labor can be seen in Figure 13. The results now are quite different. Output and capital increase. Consumption decreases and, even though the government spending used in abatement policy increases, pollution increases by even more. This leads to a degradation of environmental quality. Agents can now only substitute current consumption with future consumption but obviously this is not adequate to improve environmental quality. The outcome is an increase in welfare, despite the fact that environmental quality deteriorates. This is obviously in contrast with the finding of a positive relationship between environmental quality and welfare presented in Figure 12.

Figure 14 shows how the economy with endogenous labor reacts to a 1% increase in the time discount factor. We observe that when β increases all variables increase in the long run. As discussed above, there is one channel of substitution towards investment and against consumption and another channel towards labor and against consumption. The environmental quality improves permanently just because households evaluate their future higher. They care more about the environment and pay more taxes to the government that are used for abatement policy. Through this mechanism welfare also improves permanently. Again, in the endogenous labor model preferences of individuals for a better environment are positively correlated with variables such as emissions, which characterize the environmental quality. This is a realistic finding, and is in line with the stylized facts of the economy.

Finally, Figure 15 shoes us how the economy with exogenous labor reacts in a 1% increase in the time discount factor β. The results are similar to the case where labor is endogenous. All variables increase with an exception on capital, which decreases. There is only one channel of substitution between current consumption with future consumption. Agents produce more and pay more taxes to the government that are used in abatement policy. Environmental quality and welfare increase.

Appendix C: Linearization of the Ramsey model

For

$$U(c_t, l_t, Q_t) = \mu_1 \ln(c_t) + \mu_2 \ln(l_t) + (1 - \mu_1 - \mu_2) \ln(Q_t) \tag{C1}$$

the FOCs of the Ramsey problem become:

$$\mu_1 c_t - \frac{c_t^2\lambda_t}{1-n_t} + \beta\psi_t\frac{\mu_1}{c_t^2} - c_t^2\chi_t - \psi_{t-1}\frac{\mu_1}{c_t^2}[(1-\tau_t^k)Ak_t^a n_t^{1-a} + 1 - \delta^k] = 0 \tag{C2}$$

$$(1-a)Ak_t^a n_t^{-a}[\chi_t - \zeta_t\phi + \xi_t[a\tau_t^k + (1-a)\tau_t^l]] \tag{C3}$$

$$+\psi_{t-1}\frac{\mu_1}{c_t}(1-\tau_t^l)a(1-a)Ak_t^{a-1}n_t^{-a}$$

$$-\frac{c_t\lambda_t}{(1-n_t)^2} - \lambda_t(1-\tau_t^l)\frac{\mu_1}{\mu_2}a(1-a)Ak_t^a n_t^{-1-a} = 0$$

$$\psi_t\frac{\mu_1}{c_{t+1}}(1-\tau_{t+1}^k)a(a-1)Ak_{t+1}^{a-2}n_{t+1}^{1-a} - \frac{\chi_t}{\beta} + \lambda_{t+1}(1-\tau_{t+1}^l)\frac{\mu_1}{\mu_2}a(1-a)Ak_{t+1}^{a-1}n_{t+1}^{-a} \tag{C4}$$

$$+aAk_{t+1}^{a-1}n_{t+1}^{1-a}(\chi_{t+1} - \zeta_{t+1}\phi + \xi_{t+1}[a\tau_{t+1}^k + (1-a)\tau_{t+1}^l]) + \chi_{t+1}(1-\delta^k) = 0$$

$$-\zeta_t Q_{t+1} + \beta(\frac{1-\mu_1-\mu_2}{Q_{t+1}} + \zeta_{t+1}\delta^q) = 0 \tag{C5}$$

$$\nu\zeta_t - \chi_t - \xi_t = 0 \tag{C6}$$

$$\xi_t k_t - \psi_{t-1}\frac{\mu_1}{c_t} = 0 \tag{C7}$$

$$\xi_t n_t - \lambda_t\frac{\mu_1}{\mu_2} = 0 \tag{C8}$$

$$\frac{\mu_1}{\mu_2}(1-a)Ak_t^a n_t^{-a}(1-\tau_t^l) - \frac{c_t}{1-n_t} = 0 \tag{C9}$$

$$\beta\frac{\mu_1}{c_{t+1}}[(1-\tau_{t+1}^k)aAk_{t+1}^{a-1}n_{t+1}^{1-a} + 1 - \delta^k] = \frac{\mu_1}{c_t} \tag{C10}$$

$$Q_{t+1} = (1-\delta^q)\bar{Q} + \delta^q Q_t - \varphi Ak_t^a n_t^{1-a} + \nu g_t \tag{C11}$$

$$g_t = Ak_t^a n_t^{1-a}[a\tau_t^k + (1-a)\tau_t^l] \tag{C12}$$

$$c_t + k_{t+1} = Ak_t^a n_t^{1-a} - g_t + (1-\delta^k)k_t \tag{C13}$$

From the Eqs. $(C2)-(C12)$ we can eliminate g_t, ζ_t, ψ_t and ψ_{t-1} by substituting $(C11)$, $(C5)$ and $(C6)$ at time t and $t+1$ so that Eqs. $(C2)-(C12)$ be written as:

$$\mu_1 c_t - \frac{c_t^2\lambda_t}{1-n_t} + \beta\xi_{t+1}k_{t+1}c_{t+1} - c_t^2\chi_t - \xi_t k_t c_t[(1-\tau_t^k)Ak_t^a n_t^{1-a} + 1 - \delta^k] = 0 \tag{C15}$$

$$(1-a)Ak_t^a n_t^{-a}[\chi_t(1-\frac{\varphi}{\nu}) + \xi_t[a\tau_t^k + (1-a)\tau_t^l - \frac{\varphi}{\nu}]] - \frac{\mu_2}{1-n_t} + \xi_t(1-\tau_t^l)a(1-a)Ak_t^a n_t^{-a} \tag{C16}$$

$$-\frac{c_t\lambda_t}{(1-n_t)^2} - \lambda_t(1-\tau_t^l)\frac{\mu_1}{\mu_2}a(1-a)Ak_t^a n_t^{-1-a} = 0$$

$$\xi_{t+1}k_{t+1}(1-\tau_{t+1}^k)a(a-1)Ak_{t+1}^{a-2}n_{t+1}^{1-a} - \frac{\chi_t}{\beta} + \lambda_{t+1}(1-\tau_{t+1}^l)\frac{\mu_1}{\mu_2}a(1-a)Ak_{t+1}^{a-1}n_{t+1}^{-a} \tag{C17}$$

$$+aAk_{t+1}^{a-1}n_{t+1}^{1-a}[\chi_{t+1}(1-\frac{\varphi}{\nu}) + \xi_{t+1}[a\tau_{t+1}^k + (1-a)\tau_{t+1}^l - \frac{\varphi}{\nu}]] + \chi_{t+1}(1-\delta^k) = 0$$

$$-(\chi_t + \xi_t)\frac{1}{\nu}Q_{t+1} + \beta\frac{1-\mu_1-\mu_2}{Q_{t+1}} + \frac{\beta\delta^q}{\nu}(\chi_{t+1} + \xi_{t+1}) = 0 \tag{C18}$$

$$\xi_t n_t - \lambda_t\frac{\mu_1}{\mu_2} = 0 \tag{C19}$$

$$\mu_2 c_t - \mu_1(1-n_t)(1-a)Ak_t^a n_t^{-a}(1-\tau_t^l) = 0 \tag{C20}$$

$$\beta\mu_1 c_t[(1-\tau_{t+1}^k)aAk_{t+1}^{a-1}n_{t+1}^{1-a} + 1 - \delta^k] = \mu_1 c_{t+1} \tag{C21}$$

$$(1-\delta^q)\bar{Q} + \delta^q Q_t - \varphi Ak_t^a n_t^{1-a} + \nu Ak_t^a n_t^{1-a}[a\tau_t^k + (1-a)\tau_t^l] - Q_{t+1} = 0 \tag{C22}$$

$$c_t + k_{t+1} - Ak_t^a n_t^{1-a}[1 - a\tau_t^k - (1-a)\tau_t^l] - (1-\delta^k)k_t = 0 \tag{C23}$$

In this way we have a system with nine equations in $\{c_t, n_t, k_{t+1}, Q_{t+1}, \tau_t^k, \tau_t^l, \lambda_t, \chi_t, \xi_t\}_{t=0}^{\infty}$. We want to linearize Eqs. $(C15) - (C23)$ around the steady state in order to analyze its behavior. By using Taylor's Theorem we expand the functions of the system around the steady state.

$(C15)$ becomes

$$f_1(c_t, n_t, \lambda_t, \xi_{t+1}, k_{t+1}, c_{t+1}, \chi_t, \xi_t, k_t, \tau_t^k) = f_{1a}\hat{c}_t + f_{1b}\hat{n}_t + f_{1c}\hat{\lambda}_t + f_{1d}\hat{\xi}_{t+1} + f_{1e}\hat{k}_{t+1} \tag{C24}$$

$$+f_{1f}\hat{c}_{t+1} + f_{1g}\hat{\chi}_t + f_{1h}\hat{\xi}_t + f_{1i}\hat{k}_t + f_{1j}\hat{\tau}_t^k$$

where

$$f_{1a} = f_{1c_t}(c^*, n^*, \lambda^*, \xi^*, k^*, c^*, \chi^*, \xi^*, k^*, \tau^{k*}) = [\mu_1 - \frac{2c^*\lambda^*}{1-n^*} - 2c^*\chi^* - \xi^* k^*[(1-\tau^{k*}) \tag{C25}$$

$$A(k^*)^a(n^*)^{1-a} + 1 - \delta^k]]$$

$$f_{1b} = f_{1n_t}(c^*, n^*, \lambda^*, \xi^*, k^*, c^*, \chi^*, \xi^*, k^*, \tau^{k*}) = [-[\frac{(c^*)^2\lambda^*}{(1-n^*)^2} \tag{C26}$$

$$+\xi^* k^* c^*(1-\tau^{k*})(1-a)A(k^*)^a(n^*)^{-a}]]$$

$$f_{1c} = f_{1\lambda_t}(\cdot) = [-\frac{(c^*)^2}{1-n^*}] \tag{C27}$$

$$f_{1d} = f_{1\xi_{t+1}}(\cdot) = \beta k^* c^* \tag{C28}$$

$$f_{1e} = f_{1k_{t+1}}(\cdot) = \beta \xi^* c^* \tag{C29}$$

$$f_{1f} = f_{1c_{t+1}}(\cdot) = \beta \xi^* k^* \tag{C30}$$

$$f_{1g} = f_{1\chi_t}(\cdot) = [-(c^*)^2] \tag{C31}$$

$$f_{1h} = f_{1\xi_t}(\cdot) = [-[k^* c^*[(1-\tau^{k*})A(k^*)^a(n^*)^{1-a} + 1 - \delta^k]]] \tag{C32}$$

$$f_{1i} = f_{1k_t}(\cdot) = [-\xi^* c^*[(1-\tau^{k*})A(k^*)^a(n^*)^{1-a} + 1 - \delta^k] \tag{C33}$$

$$-\xi^* c^*(1-\tau^{k*})Aa(k^*)^a(n^*)^{1-a}]$$

$$f_{1j} = f_{1\tau_t^k}(\cdot) = \xi^* c^*(1-\tau^{k*})A(k^*)^{a+1}(n^*)^{1-a} \tag{C34}$$

$(C16)$ becomes

$$f_2(k_t, n_t, \chi_t, \xi_t, \tau_t^k, \tau_t^l, c_t, \lambda_t) = f_{2a}\hat{k}_t + f_{2b}\hat{n}_t + f_{2c}\hat{\chi}_t + f_{2d}\hat{\xi}_t + f_{2e}\hat{\tau}_t^k + f_{2f}\hat{\tau}_t^l + f_{2g}\hat{c}_t + f_{2h}\hat{\lambda}_t \tag{C35}$$

where

$$f_{2a} = f_{2k_t}(\cdot) = [(1-a)aA(k^*)^{a-1}(n^*)^{-a}[\chi^*(1-\frac{\varphi}{\nu}) \tag{C36}$$

$$+\xi^*[a\tau^{k*} + (1-a)\tau^{l*} - \frac{\varphi}{\nu}]] + \xi^*(1-\tau^{l*})a^2(1-a)A(k^*)^{a-1}(n^*)^{-a}$$

$$-\lambda^*(1-\tau^{l*})\frac{\mu_1}{\mu_2}a^2(1-a)A(k^*)^{a-1}(n^*)^{-1-a}]$$

$$f_{2b} = f_{2n_t}(\cdot) = [-a(1-a)A(k^*)^a(n^*)^{-a-1}[\chi^*(1-\frac{\varphi}{\nu}) \tag{C37}$$

$$+\xi^*[a\tau^{k*} + (1-a)\tau^{l*} - \frac{\varphi}{\nu}]] - \frac{\mu_2}{(1-n^*)^2} - \xi^*(1-\tau^{l*})a^2(1-a)A(k^*)^a(n^*)^{-a-1}$$

$$-\frac{2c^*\lambda^*(1-n^*)}{(1-n^*)^4} + \lambda^*(1-\tau^{l*})\frac{\mu_1}{\mu_2}a(1-a)(a+1)A(k^*)^a(n^*)^{-a-2}]$$

$$f_{2c} = f_{2\chi_t}(\cdot) = (1-a)A(k^*)^a(n^*)^{-a}(1-\frac{\varphi}{\nu}) \tag{C38}$$

$$f_{2d} = f_{2\xi_t}(\cdot) = [(1-a)A(k^*)^a(n^*)^{-a} \tag{C39}$$

$$[a\tau^{k*} + (1-a)\tau^{l*} - \frac{\varphi}{\nu}]] + (1-\tau^{l*})a(1-a)A(k^*)^a(n^*)^{-a}] \tag{C40}$$

$$f_{2e} = f_{2\tau^k_t}(\cdot) = (1-a)A(k^*)^a(n^*)^{-a}\xi^*a \tag{C41}$$

$$f_{2f} = f_{2\tau^l_t}(\cdot) = [(1-a)^2A(k^*)^a(n^*)^{-a}\xi^* - \xi^*a(1-a)A(k^*)^a(n^*)^{-a} \tag{C42}$$

$$+\lambda^*\frac{\mu_1}{\mu_2}a(1-a)A(k^*)^a(n^*)^{-a-1}]$$

$$f_{2g} = f_{2c_t}(\cdot) = [-\frac{\lambda^*}{(1-n^*)^2}] \tag{C43}$$

$$f_{2h} = f_{2\lambda_t}(\cdot) = [-\frac{c^*}{(1-n^*)^2} - (1-\tau^{l*})\frac{\mu_1}{\mu_2}a(1-a)A(k^*)^a(n^*)^{-a-1}] \tag{C44}$$

$(C17)$ becomes

$$f_3(\xi_{t+1}, k_{t+1}, \tau^k_{t+1}, n_{t+1}, \chi_t, \lambda_{t+1}, \tau^l_{t+1}, \chi_{t+1}) = f_{3a}\hat{\xi}_{t+1} + f_{3b}\hat{k}_{t+1} + f_{3c}\tau^k_{t+1} + f_{3d}\hat{n}_{t+1} \tag{C45}$$

$$+f_{3e}\hat{\chi}_t + f_{3f}\hat{\lambda}_{t+1} + f_{3g}\hat{\tau}^l_{t+1} + f_{3h}\hat{\chi}_{t+1}$$

where

$$f_{3a} = f_{3\xi_{t+1}}(\cdot) = [(1-\tau^{k*})a(a-1)A(k^*)^{a-1}(n^*)^{1-a} \tag{C46}$$
$$+aA(k^*)^{a-1}(n^*)^{1-a}[a\tau^{k*} + (1-a)\tau^{l*} - \frac{\varphi}{\nu}]]$$

$$f_{3b} = f_{3k_{t+1}}(\cdot) = [\xi^*(1-\tau^{k*})a(a-1)^2A(k^*)^{a-2}(n^*)^{1-a} \tag{C47}$$
$$+\lambda^*(1-\tau^{l*})\frac{\mu_1}{\mu_2}a(1-a)^2A(k^*)^{a-2}(n^*)^{-a}$$
$$+a(a-1)A(k^*)^{a-2}(n^*)^{1-a}[\chi^*(1-\frac{\varphi}{\nu}) + \xi^*[a\tau^{k*} + (1-a)\tau^{l*} - \frac{\varphi}{\nu}]]]$$

$$f_{3c} = f_{3\tau^k_{t+1}}(\cdot) = [a^2A(k^*)^{a-1}(n^*)^{1-a}\xi^* - \xi^*a(a-1)A(k^*)^{a-1}(n^*)^{1-a}] \tag{C48}$$

$$f_{3d} = f_{3n_{t+1}}(\cdot) = [-\xi^*(1-\tau^{k*})a(a-1)^2A(k^*)^{a-1}(n^*)^{-a} \tag{C49}$$
$$-\lambda^*(1-\tau^{l*})\frac{\mu_1}{\mu_2}a^2(1-a)A(k^*)^{a-1}(n^*)^{-1-a}$$
$$+a(1-a)A(k^*)^{a-1}(n^*)^{-a}[\chi^*(1-\frac{\varphi}{\nu}) + \xi^*[a\tau^{k*} + (1-a)\tau^{l*} - \frac{\varphi}{\nu}]]]$$

$$f_{3e} = f_{3\chi_t}(\cdot) = [-\frac{1}{\beta}] \tag{C50}$$

$$f_{3f} = f_{3\lambda_{t+1}}(\cdot) = (1-\tau^{l*})\frac{\mu_1}{\mu_2}a(1-a)A(k^*)^{a-1}(n^*)^{-a} \tag{C51}$$

$$f_{3g} = f_{3\tau^l_{t+1}}(\cdot) = [a(1-a)A(k^*)^{a-1}(n^*)^{1-a}\xi^* - \lambda^*\frac{\mu_1}{\mu_2}aA(k^*)^{a-1}(n^*)^{-a}] \tag{C52}$$

$$f_{3h} = f_{3\chi_{t+1}}(\cdot) = [aA(k^*)^{a-1}(n^*)^{1-a}(1-\frac{\varphi}{\nu}) + 1 - \delta^k] \tag{C53}$$

$(C18)$ becomes

$$f_4(\chi_t, \xi_t, Q_{t+1}, \chi_{t+1}, \xi_{t+1}) = f_{4a}\hat{\chi}_t + f_{4b}\hat{\xi}_t + f_{4c}\hat{Q}_{t+1} + f_{4d}\hat{\chi}_{t+1} + f_{4e}\hat{\xi}_{t+1} \tag{C54}$$

where

$$f_{4a} = f_{4\chi_t}(\cdot) = -\frac{Q^*}{\nu} \tag{C55}$$

$$f_{4b} = f_{4\xi_t}(\cdot) = -\frac{Q^*}{\nu} \tag{C56}$$

$$f_{4c} = f_{4Q_{t+1}}(\cdot) = [(\chi^* + \xi^*)\frac{1}{\nu} - \frac{\beta(1-\mu_1-\mu_2)}{(Q^*)^2}] \tag{C57}$$

$$f_{4d} = f_{4\chi_{t+1}}(\cdot) = \frac{\beta\delta^q}{\nu} \tag{C58}$$

$$f_{4e} = f_{4\xi_{t+1}}(\cdot) = \frac{\beta\delta^q}{\nu} \tag{C59}$$

$(C19)$ becomes

$$f_5(\xi_t, n_t, \lambda_t) = f_{5a}\hat{\xi}_t + f_{5b}\hat{n}_t + f_{5c}\hat{\lambda}_t \tag{C60}$$

where

$$f_{5a} = f_{5\xi_t}(\cdot) = n^* \tag{C61}$$

$$f_{5b} = f_{5n_t}(\cdot) = \xi^* \tag{C62}$$

$$f_{5c} = f_{5\lambda_t}(\cdot) = [-\frac{\mu_1}{\mu_2}] \tag{C63}$$

$(C20)$ becomes

$$f_6(c_t, n_t, k_t, \tau_t^l) = \mu_2\hat{c}_t + f_{6a}\hat{n}_t + f_{6b}\hat{k}_t + f_{6c}\hat{\tau}_t^l \tag{C64}$$

where

$$f_{6a} = f_{6n_t}(\cdot) = [\mu_1(1-a)A(k^*)^a(n^*)^{-a}(1-\tau^{l*}) \tag{C65}$$

$$+a\mu_1(1-n^*)(1-a)A(k^*)^a(n^*)^{-a-1}(1-\tau^{l*})]$$

$$f_{6b} = f_{6k_t}(\cdot) = [-\mu_1(1-n^*)(1-a)aA(k^*)^{a-1}(n^*)^{-a}(1-\tau^{l*})] \tag{C66}$$

$$f_{6c} = f_{6\tau_t^l}(\cdot) = \mu_1(1-n^*)(1-a)A(k^*)^a(n^*)^{-a} \tag{C67}$$

$(C21)$ becomes

$$f_7(c_t, \tau_{t+1}^k, k_{t+1}, n_{t+1}, c_{t+1}) = f_{7a}\hat{c}_t + f_{7b}\tau_{t+1}^k + f_{7c}\hat{k}_{t+1} + f_{7d}\hat{n}_{t+1} + [-\mu_1]\hat{c}_{t+1} \tag{C68}$$

where

$$f_{7a} = f_{7c_t}(\cdot) = \beta\mu_1[(1-\tau^{k*})aA(k^*)^{a-1}(n^*)^{1-a} + 1 - \delta^k] \tag{C69}$$

$$f_{7b} = f_{7\tau_{t+1}^k}(\cdot) = [-\beta\mu_1 c^* aA(k^*)^{a-1}(n^*)^{1-a}] \tag{C70}$$

$$f_{7c} = f_{7k_{t+1}}(\cdot) = \beta\mu_1 c^*(1-\tau^{k*})a(a-1)A(k^*)^{a-2}(n^*)^{1-a} \tag{C71}$$

$$f_{7d} = f_{7n_{t+1}}(\cdot) = \beta\mu_1 c^*(1-\tau^{k*})a(1-a)A(k^*)^{a-1}(n^*)^{-a} \tag{C72}$$

$(C22)$ becomes

$$f_8(Q_t, k_t, n_t, \tau_t^k, \tau_t^l, Q_{t+1}) = \delta^q\hat{Q}_t + f_{8a}\hat{k}_t + f_{8b}\hat{n}_t + f_{8c}\hat{\tau}_t^k + f_{8d}\hat{\tau}_t^l - \hat{Q}_{t+1} \tag{C73}$$

where

$$f_{8a} = f_{8k_t}(\cdot) = [-\varphi a A(k^*)^{a-1}(n^*)^{1-a} + \nu A a(k^*)^{a-1}(n^*)^{1-a}[a\tau^{k*} + (1-a)\tau^{l*}]] \tag{C74}$$

$$f_{8b} = f_{8n_t}(\cdot) = [-\varphi(1-a)A(k^*)^{a}(n^*)^{-a} + \nu(1-a)A(k^*)^{a}(n^*)^{-a} \tag{C75}$$

$$[a\tau^{k*} + (1-a)\tau^{l*}]]$$

$$f_{8c} = f_{8\tau_t^k}(\cdot) = \nu A(k^*)^{a}(n^*)^{1-a}a \tag{C76}$$

$$f_{8d} = f_{8\tau_t^l}(\cdot) = \nu A(k^*)^{a}(n^*)^{1-a}(1-a) \tag{C77}$$

$(C23)$ becomes

$$f_9(c_t, k_{t+1}, k_t, n_t, \tau_t^k, \tau_t^l) = \hat{c}_t + \hat{k}_{t+1} + f_{9a}\hat{k}_t + f_{9b}\hat{n}_t + f_{9c}\hat{\tau}_t^k + f_{9d}\hat{\tau}_t^l \tag{C78}$$

where

$$f_{9a} = f_{9k_t}(\cdot) = [-[aA(k^*)^{a-1}(n^*)^{1-a}[1 - a\tau^{k*} - (1-a)\tau^{l*}] + (1-\delta^k)]] \tag{C79}$$

$$f_{9b} = f_{9n_t}(\cdot) = [-A(1-a)(k^*)^{a}(n^*)^{-a}[1 - a\tau^{k*} - (1-a)\tau^{l*}]] \tag{C80}$$

$$f_{9c} = f_{9\tau_t^k}(\cdot) = A(k^*)^{a}(n^*)^{1-a}a \tag{C81}$$

$$f_{9d} = f_{9\tau_t^l}(\cdot) = A(k^*)^{a}(n^*)^{1-a}(1-a) \tag{C82}$$

We observe that the three of the equations in the system are static. In order to obtain a real Jacobian we substitute these equations into the other six equations of the system. More specifically we solve f_2, f_5 and f_6 with respect to $\hat{\tau}_t^k$, $\hat{n}_t$ and $\hat{\tau}_t^l$ respectively:

$$\begin{bmatrix} f_2(k_t, n_t, \chi_t, \xi_t, \tau_t^k, \tau_t^l, c_t, \lambda_t) = 0 \\ f_5(\xi_t, n_t, \lambda_t) = 0 \\ f_6(c_t, n_t, k_t, \tau_t^l) = 0 \end{bmatrix}$$

$$\Leftrightarrow \begin{bmatrix} \hat{\tau}_t^k = \hat{k}_t(\frac{f_{2f}}{f_{2a}} - \frac{f_{2a}}{f2e}) + \hat{\xi}_t(\frac{f_{2b}}{f_{2a}}\frac{n^*}{\xi^*} - \frac{f_{2d}}{f_{2a}} - \frac{f_{2f}}{f2e}f_{6a}\frac{n^*}{\xi^*}) + \hat{\lambda}_t(\frac{f_{2b}}{f_{2a}}\frac{f_{5a}}{\xi^*} - \frac{f_{2f}}{f_{2a}}f_{6a}\frac{f_{5a}}{\xi^*} - \frac{f_{2h}}{f_{2a}}) \\ +\hat{\chi}_t(-\frac{f_{2c}}{f_{2a}}) + \hat{c}_t(\frac{f_{2f}}{f_{2a}}\mu_2 - \frac{f_{2g}}{f_{2a}}) \\ \hat{n}_t = [-\frac{n^*}{\xi^*}]\hat{\xi}_t + [-\frac{f_{5a}}{\xi^*}]\hat{\lambda}_t \\ \hat{\tau}_t^l = -\mu_2\hat{c}_t - f_{6b}\hat{k}_t + f_{6a}\frac{n^*}{\xi^*}\hat{\xi}_t + f_{6a}\frac{f_{5a}}{\xi^*}\hat{\lambda}_t \end{bmatrix}$$

Therefore, the system becomes

$$\begin{bmatrix} f_{1f} & f_{1e} & 0 & 0 & 0 & f_{1d} \\ f_{35} & f_{32} & 0 & f_{33} & f_{34} & f_{31} \\ 0 & 0 & f_{4c} & 0 & f_{4d} & f_{4e} \\ f_{75} & f_{71} & 0 & f_{73} & f_{74} & f_{72} \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{c}_{t+1} \\ \hat{k}_{t+1} \\ \hat{Q}_{t+1} \\ \hat{\lambda}_{t+1} \\ \hat{\chi}_{t+1} \\ \hat{\xi}_{t+1} \end{bmatrix} = \begin{bmatrix} f_{11} & -f_{15} & 0 & -f_{13} & -f_{14} & -f_{12} \\ 0 & 0 & 0 & 0 & -f_{3e} & 0 \\ 0 & 0 & 0 & 0 & -f_{4e} & -f_{4b} \\ -f_{7a} & 0 & 0 & 0 & 0 & 0 \\ f_{85} & f_{83} & \delta^q & f_{82} & f_{84} & f_{81} \\ f_{91} & f_{92} & 0 & f_{94} & f_{95} & f_{93} \end{bmatrix} \begin{bmatrix} \hat{c}_t \\ \hat{k}_t \\ \hat{Q}_t \\ \hat{\lambda}_t \\ \hat{\chi}_t \\ \hat{\xi}_t \end{bmatrix}$$

$$\Leftrightarrow D\hat{X}_{t+1} = E\hat{X}_t \Leftrightarrow \hat{X}_{t+1} = ED^{-1}\hat{X}_t \Leftrightarrow \hat{X}_{t+1} = F\hat{X}_t$$

By using the parameter values in the paper of Angelopoulos et al. (2010), we find that the three eigenvalues of F have absolute value smaller than one, while the other three have absolute value larger than one. The Blanchard-Kahn conditions are satisfied and the model is stable. The steady state of the system is a saddle path, therefore it has a unique equilibrium.

Appendix D: Other impulse responces for the Ramsey model

In Figure 16 we see how the economy responds in a 1% increase in the abatement policy. We can see that an increase in ν has exactly the opposite results by an increase in φ. An increase in the abatement parameter means that public spending in abatement activities increases, therefore environmental quality is more important for the society. They substitute labor with current consumption and current consumption with investment, therefore capital increases. Agents produce more output, but the increase in the abatement parameter reduces taxes and government spending used for abatement policy decreases too. Finally the increase of public spending for abatement counterbalances the increase of pollution. Environmental quality increases and welfare increases too.

In Figure 17 we see how the economy responds in a 1% increase in the coefficient of relative risk aversion. We observe that in the long-run all the variables remain unaltered. Households become less risky, and since σ has an immediate effect only in welfare, in the long-run welfare is increased.

In Figure 18 we see how the economy responds in a 1% increase in the rate of time preference. We observe that the economy moves to a better steady state. There is one channel of substitution towards investment and against consumption and a second channel of substitution towards labor and against consumption, just because labor in our model is endogenous. The environmental quality finally is much higher than in the initial equilibrium. Households evaluate their future higher, so they substitute consumption with labour and investment. At the same time they care more about the environment, they pay more in abatement policy and this increases environmental quality. In the new equilibrium welfare is higher.

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Table 1

Parameter values for the numerical example

Parameter	Description	Value
α	Capital share in production	0.33
δ^k	Capital depreciation rate	0.1
σ	Curvature parameter in utility function	2
β	Time discount factor	0.97
μ_1	Consumption weight in utility function	0.2
μ_2	Leisure weight in the utility function	0.6
$\bar{Q}$	Environmental quality without pollution	1
δ^q	Persistence of environmental quality	0.9
A	Long-run total factor productivity	1
φ	Long-run pollution technology	0
ν	Transformation of spending into units of nature	5

Table 2

Long-run values when

Variable name	I	II
c	0.156	0.135
n	0.217	0.217
Q	5.964	5.168
k	0.861	0.737
τ^k	0.000	0.100
τ^l	0.434	0.483
g	0.099	0.116
λ	0.575	0.681
ψ	0.101	0.084
ζ	0.295	0.361
ξ	0.885	1.046
χ	0.589	0.757

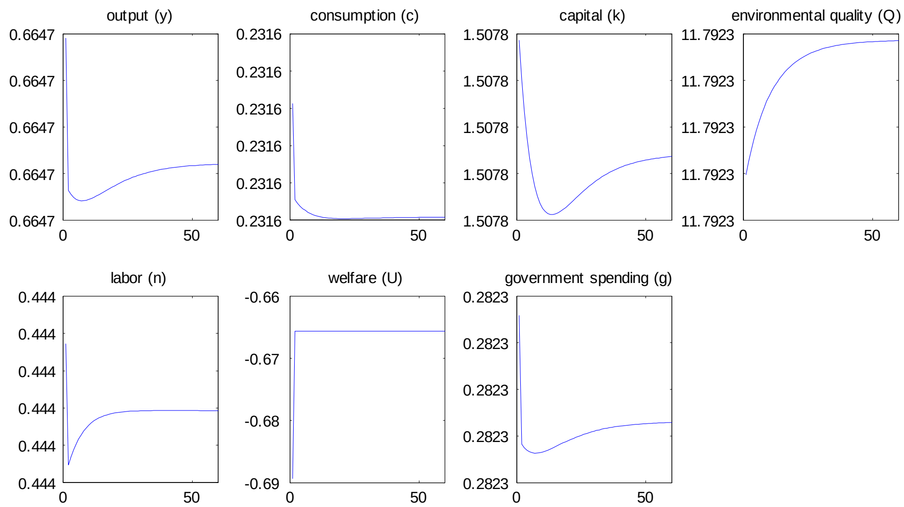

Figure 1: Model with endogenous labor, 1% increase in the weight on environmental quality

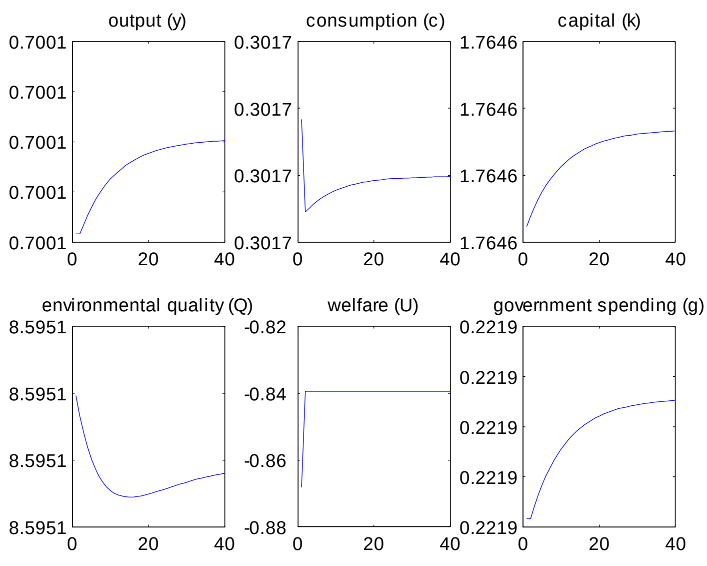

Figure 2: Exogenous labor model, 1% increase in the weight to environmental quality

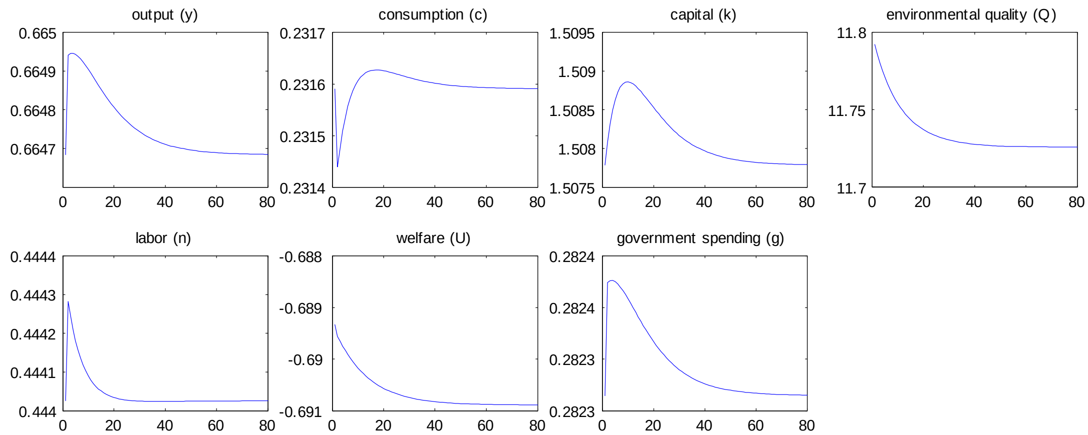

Figure 3: Endogenous labor model, 1% increase in the pollution parameter

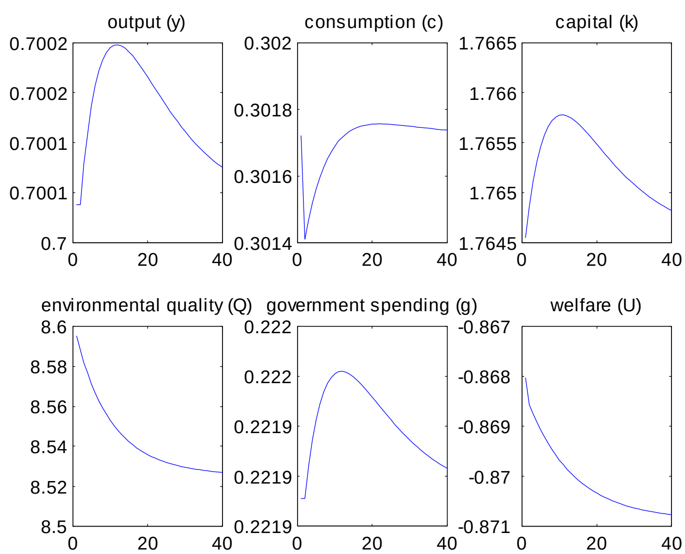

Figure 4: Exogenous labor model: 1% increase in the pollution parameter

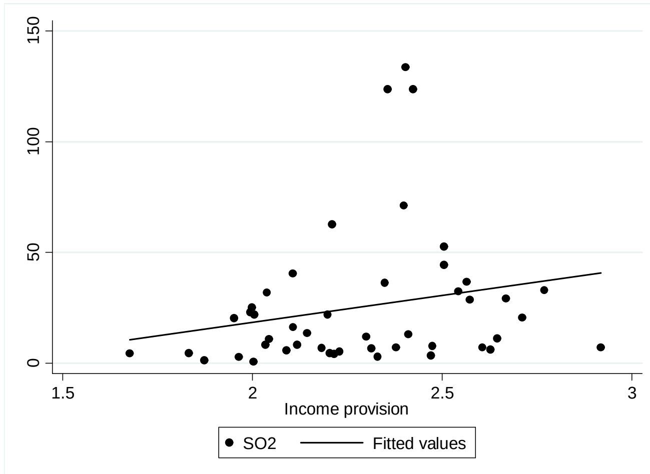

Figure 5: Income provision

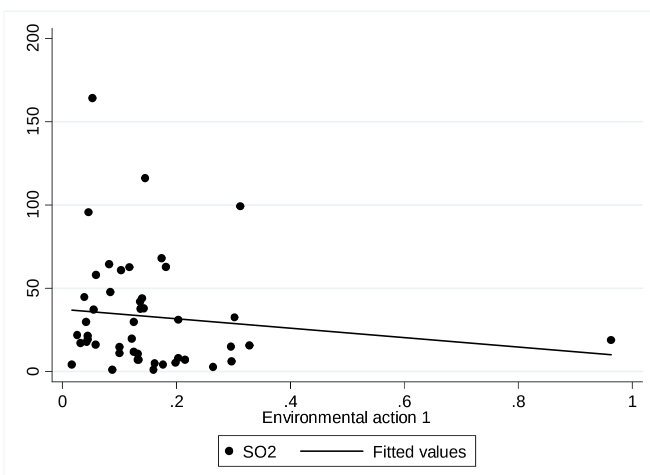

Figure 6: Environmental action 1

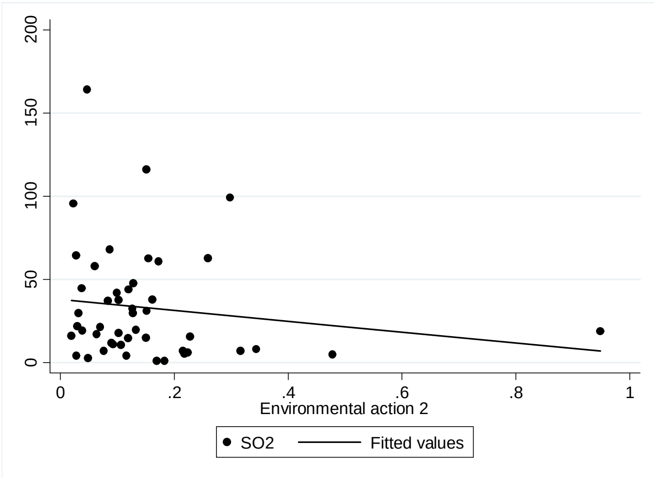

Figure 7: Environmental action 2

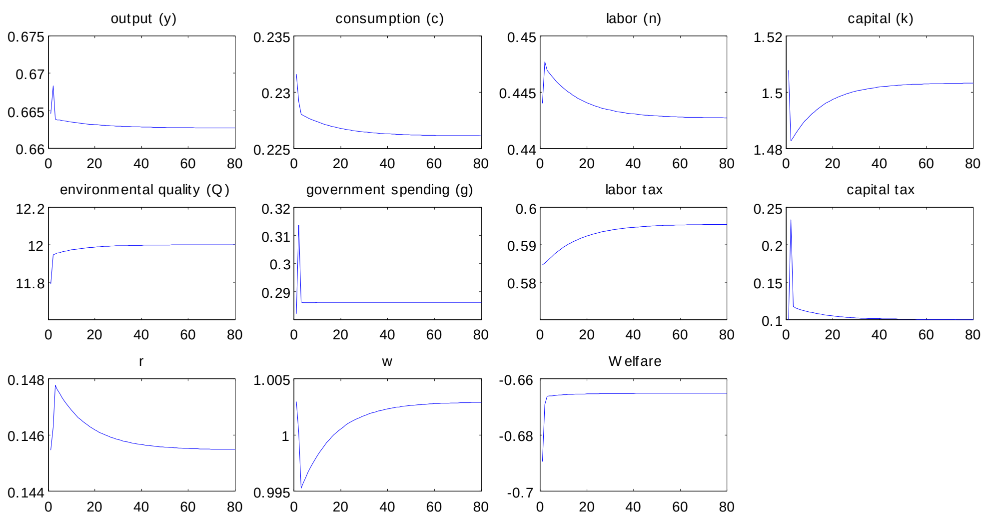

Figure 8: Response of Ramsey economy to 1% increase in the weight of environmental quality

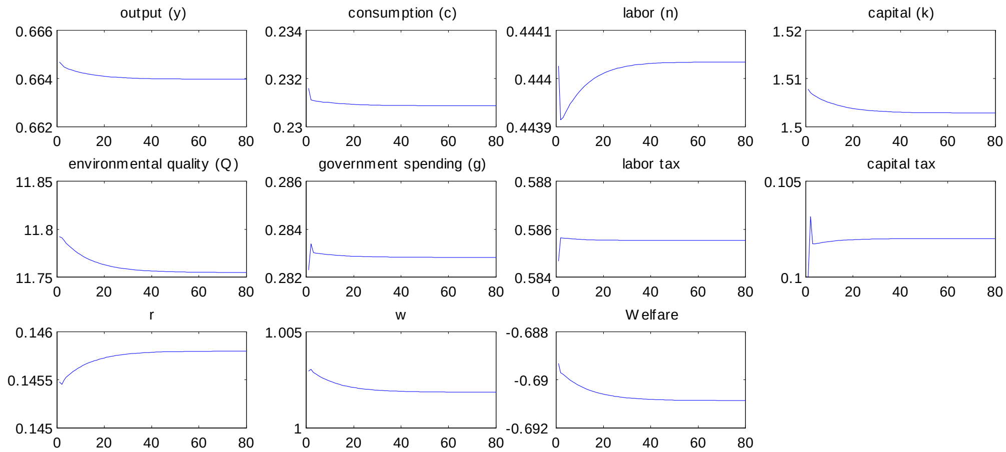

Figure 9: Response of Ramsey economy to 1% increase in the pollution parameter

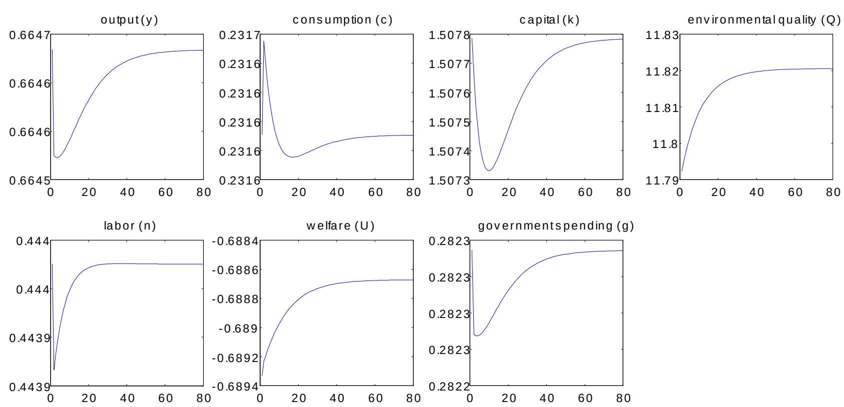

Figure 10: Endogenous labor model, 1% increase in the abatement technology

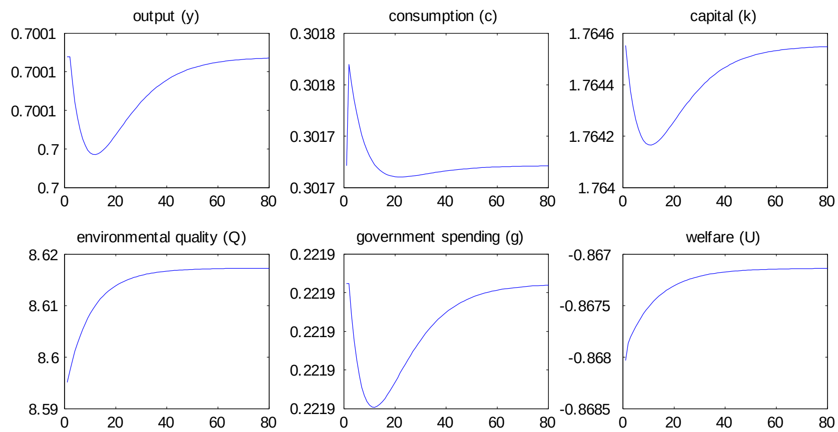

Figure 11: Exogenous labor model: 1% increase in the abatement policy

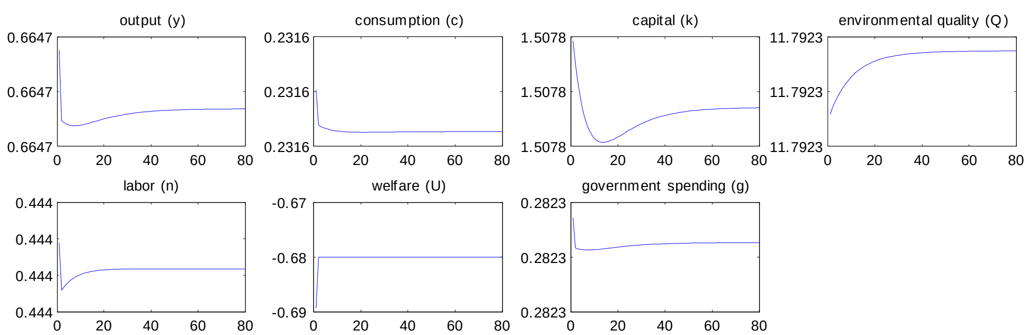

Figure 12: Endogenous labor model, 1% increase in the coefficient of relative risk aversion

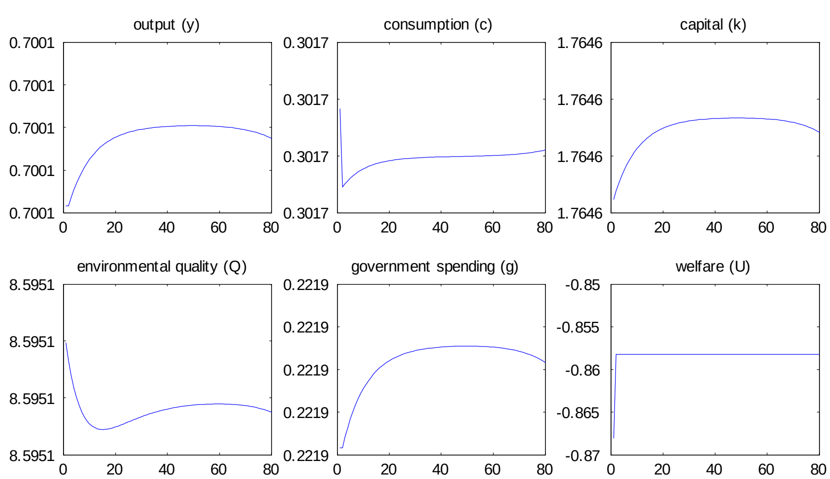

Figure 13: Exogenous labor model, 1% increase in the coefficient of relative risk aversion

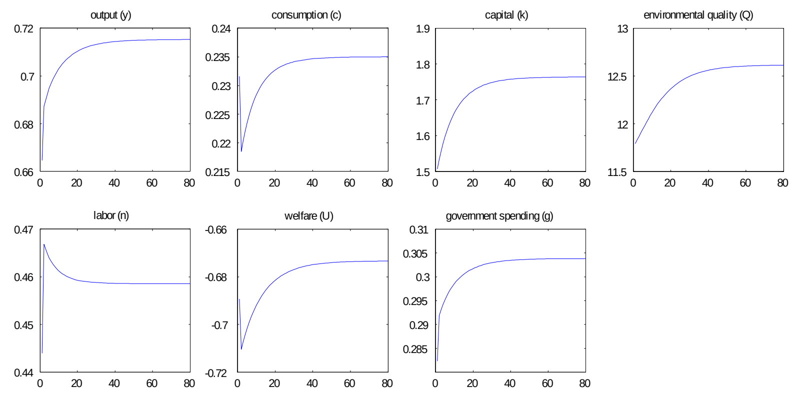

Figure 14: Endogenous labor model: 1% increase in the time discount rate

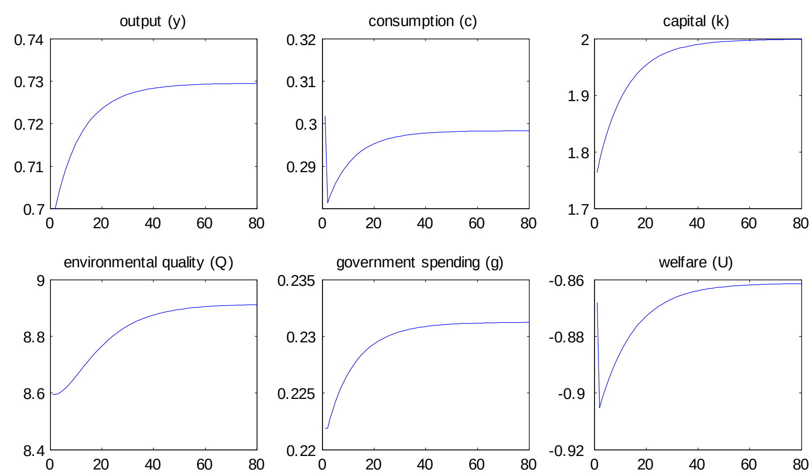

Figure 15: Exogenous labor model: 1% increase in the time discount rate

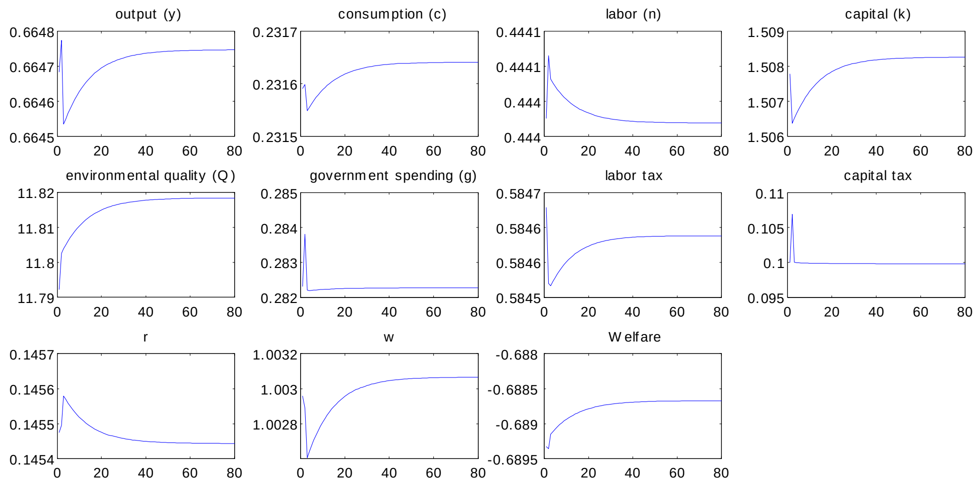

Figure 16: Response of Ramsey economy to 1% increase in the abatement policy

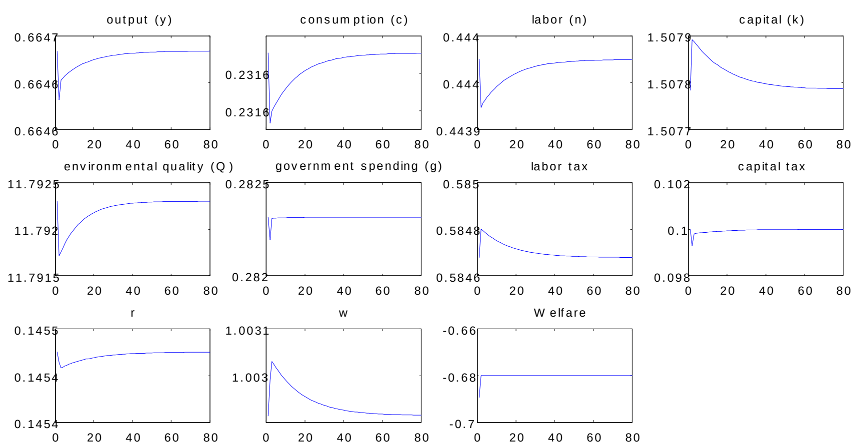

Figure 17: Response of Ramsey economy to 1% increase in the coefficient of relative risk aversion

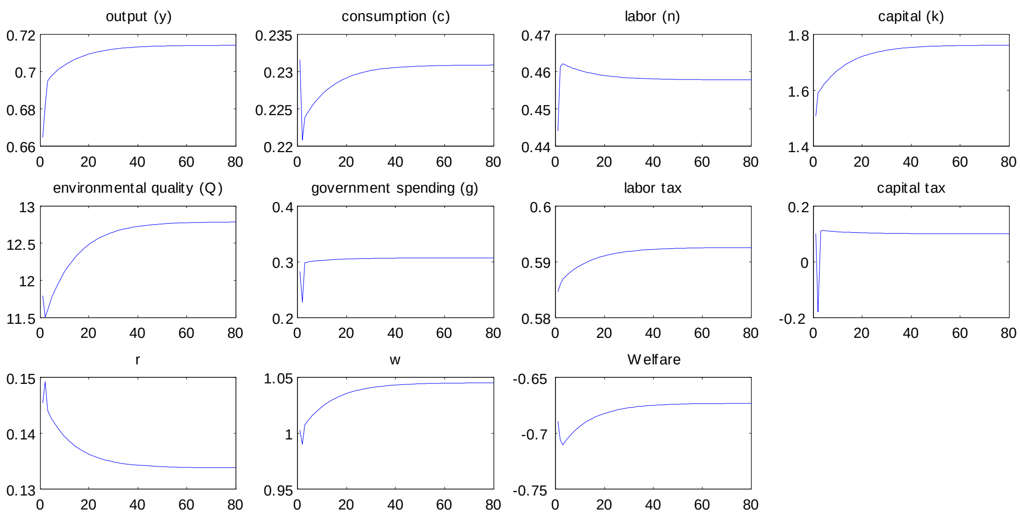

Figure 18: Response of Ramsey economy to 1% increase in the time discount rate