Time Substitution and the EU Stability and Growth Pact

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May 25, 2012

(First draft)

This paper addresses a single DEA (Data Envelopment Analysis) topic, time substitution. The time substitution topic is the study of how to allocate resources in time, with the two options (i) determine when to apply inputs and (ii) how long, i.e., in how many periods should inputs be used.

These options allow the decision maker to determine when to start production and for how long to produce. The topic was initiated by Shephard and Färe (1980) in connection with ship building, refined by Färe and Grosskopf (1996), and finally given a DEA formulation by Färe, Grosskopf, and Margaritis (2010). Their results are presented and generalized here.

After an introductory theoretical presentation we devote the rest of the paper to an empirical application examining the Pact for Stability and Growth adopted by the countries comprising the single currency euro zone. Particular attention will be given to the potential implications of this analysis for Greece's adjustment under the EU/ECB/IMF fiscal austerity plan.

1. Theoretical Underpinning

The idea of time substitution was discussed by Shephard and Färe (1980) and deals with the idea "when should inputs be used?" A producer may wait and at the end work with high intensity or go about life evenly. The problem of time substitution has been discussed by Färe and Grosskopf (1996) and Färe, Grosskopf, and Margaritis (2010). Here we build on their theoretical work. In the next section we apply the theory to two problems: when should inputs be used and government purchases made to produce GDP given a government debt constraint, and, when should inputs be used and government purchases made in order to minimize the buildup of government debt given a target level of real GDP growth?

To set the stage let

$$T = \{(x, y): \quad x \text{ can produce } y\}$$

with $x = (x_1, ..., x_N)$ a vector of inputs and y a scalar output. Define the production function as

$$F(x) = \max\{y: \quad (x, y) \in T\}.$$

By the assumptions on T, this function exists and it is a representation of the technology, i.e.,

$$(x, y) \in T \text{ if and only if } F(x) \geq y.$$

Time substitution models the various ways inputs can be applied to production. To illustrate this consider the following time line.

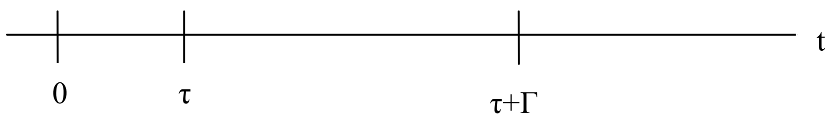

Figure : Timeline of input use

Suppose further than inputs are non-negative vectors that can be allocated over time. We also assume that inputs are essential for production, i.e.,

$$F(0) = 0,$$

No input, no output, i.e., no free lunch. Suppose the total amount of inputs is used in the time interval $[\tau, \tau + \Gamma]$, so that

$$\begin{aligned} F(x^t) &= 0, \quad 0 \le t < \tau \\ F(x^t) &> 0, \quad \tau \le t \le \tau + \Gamma \\ F(x^t) &= 0, \quad \tau + \Gamma < t. \end{aligned}$$

Thus production is positive only on the "support of inputs", i.e., when inputs are applied to production. Now there are two ways to change the input use;

(i) Change τ

(ii) Change Γ.

In the first case if τ is increased input use is delayed, i.e., the initial use is postponed, keeping the same length of the production period Γ. In the second case the length of the production period is changed. A smaller Γ yields higher production intensity while a larger Γ implies more "leisurely" production.

The time substitution problem is to find the "best" τ and Γ given some objective function. Data are frequently reported in discrete time rather than continuously[1], hence we think of the timeline as discrete intervals, so that x^t means the amount of inputs applied in period t; a week, a year, etc.

Our first problem is to find the best τ and Γ when total production, here a scale, is maximized, i.e.,

$$\max \quad \sum_{t=0}^{\infty} F^t(x^t), \quad x^\tau + ... + x^{\tau+\Gamma} \le \bar{x},$$

[1] As scanner data becomes more frequently available continuous time production can be analyzed.

where $\bar{x}$ is a finite amount of inputs and τ and Γ are fixed times. Let $F^t(x^t)$ be continuous and $F^t(x^t)=0$ for $t \notin [\tau, \tau+\Gamma]$. Then a solution exists to the maximization problem which we denote by $F(\tau,\Gamma,\bar{x})$ and our time substitution problem is to find

$$\max_{\tau,\Gamma} F(\tau,\Gamma,\bar{x}).$$

To provide intuition about the time substitution model we follow Färe, Grosskopf, and Margaritis (2010) and look at its solution under two economic scenarios: (i) returns to scale and (ii) technical change.

In case one, suppose there are two periods τ and Γ and that the single input $\bar{x}$ is allocated over these periods; i.e.,

$$x^{\tau} + x^{\tau+\Gamma} \leq \bar{x}.$$

We assume that there is no technical change and that the production function is

$$F(x) = x^{\alpha}.$$

If $\alpha > 1$ we have increasing returns to scale, and constant or decreasing returns to scale if $\alpha = 1$ or $\alpha < 1$. Our maximization problem is

$$\max \quad (x^t)^{\alpha} + (x^{\tau+\Gamma})^{\alpha} \quad s.t. \quad x^{\tau} + x^{\tau+\Gamma} \leq \bar{x}.$$

The solutions to this problem are

$$\begin{array}{lll}
\alpha > 1 & (i) & x^{\tau} = 0 \text{ and } x^{\tau+\Gamma} = \bar{x} \\
 & (ii) & x^{\tau} = \bar{x} \text{ and } x^{\tau+\Gamma} = 0 \\
\alpha = 1 & (iii) & x^{\tau} + x^{\tau+\Gamma} = \bar{x} \\
\alpha < 1 & (iv) & x^{\tau} = x^{\tau+\Gamma} = \bar{x}.
\end{array}$$

In words, under increasing returns to scale, use all input at one time, but the time does not matter; the decision maker should take advantage of increasing returns to scale. When $\alpha = 1$ (CRS) the decision maker is indifferent as to when to use input and finally, under decreasing returns to scale inputs are split equally between the two periods. Thus, in all three cases the solution is independent of τ and Γ.

Our second example is the case of technical change under constant returns to scale. Again assume there are two periods τ and Γ. We assume the production function is of the form

$$F^t(x^t) = A(t) \cdot x^t,$$

where $A(t)$ is the technical change factor. Our problem is now

$$\max \quad A(\tau) \cdot x^\tau + A(\tau + \Gamma) \cdot x^{\tau+\Gamma} \quad s.t. \quad x^\tau + x^{\tau+\Gamma} \leq \overline{x}.$$

If there is technical progress, i.e.,

$$t' > t \quad \Rightarrow \quad A(t') > A(t),$$

then by increasing the length of the production period from Γ to Γ', we have

$$A(\tau) \cdot 0 + A(\tau + \Gamma) \cdot \overline{x} < A(\tau) \cdot 0 + A(\tau + \Gamma') \cdot \overline{x},$$

and the solution tells us to extend the support from $[\tau, \tau + \Gamma]$ to $[\tau, \tau + \Gamma']$.

In the case of technical regress, i.e.,

$$t' > t \quad \Rightarrow \quad A(t') < A(t),$$

It follows that the production should be done early, i.e.,

$$A(\tau) \cdot \overline{x}, \text{ where } x^\tau = \overline{x} \text{ and } x^{\tau+\Gamma} = 0.$$

Similarly, if there is technical progress it pays to delay the production start by increasing τ and if there is technical regress it pays to begin production earlier by decreasing τ. We conclude that technical change impacts the choice of τ and Γ, but returns to scale does not.

Given that we have finite τ and Γ, the DEA solution of the time substitution problem is obtained by solving

$$
\begin{aligned}
\max \quad & y^{\tau} + \ldots + y^{\tau+\Gamma} \\
s.t. \quad & y^{\tau} \leq \sum_{k=1}^{k} z_k^{\tau} y_k^{\tau} \\
& \sum_{k=1}^{K} z_k^{\tau} x_k^{\tau} \leq x_n^{\tau}, \quad n = 1,\ldots,N \\
& z_k^{\tau} \geq 0, \quad k = 1,\ldots,K \\
& \ \vdots \\
& y^{\tau+\Gamma} \leq \sum_{k=1}^{K} z_k^{\tau+\Gamma} y_k^{\tau+\Gamma} \\
& \sum_{k=1}^{K} z_k^{\tau+\Gamma} x_k^{\tau+\Gamma} \leq x_n^{\tau+\Gamma}, \quad n = 1,\ldots,N \\
& z_k^{\tau+\Gamma} \geq 0, \quad k = 1,\ldots,K \\
& x_n^{\tau} + \ldots + x_n^{\tau+\Gamma} \leq \bar{x}, \quad n = 1,\ldots,N,
\end{aligned}
$$

where the last inequality sum the allocation of the inputs over the production period. For each DMU, $\bar{x}_{nk'}$ is given and the problem is solved for each τ and Γ within the date periods. Thus, eg. t=0,1,2, we solve for

τ	Γ
0	0,1,2
1	1,2
2	0

For instance, when $\tau = 0$ and $\Gamma = 0$ production begins and ends in period t=0. When $\tau = 0$ and $\Gamma = 1$, production begins in t=0 and ends in t=1, and $\tau = 0$ and $\Gamma = 2$ production begins in t=0 and ends in t=2. The maximum for these solutions gives the optimal τ and Γ for the particular DMU.

A generalization of the above problem is to identify inputs that are fixed in time and inputs that can be allocated over times, e.g. labor may be allocated over time, while capital is not. In this case we reformulate the inputs constraints (here just for τ, the same holds for the other periods)

$$\sum_{k=1}^{K} z_k^{\tau} x_{kn}^{\tau} \leq x_{k'n}^{\tau}, \quad n = 1,...,N^1$$

$$\sum_{k=1}^{K} z_k^{\tau} x_{kn}^{\tau} \leq \overline{x}_{k'n}, \quad n = N^1,...,N.$$

The summing up constraints then apply to just the $n = N^1,...,N$ inputs.

Assume next that $y \in R_+^M$ and that we know the output prices $p \in R_+^M$. In this case there is a simple generalization of the time substitution problem to multi-outputs. Note that revenue equals

$$p_1 y_1 + ... + p_M y_M \cdot$$

Hence by studying the revenue maximization problem, the following DEA time substitution problem holds (we assume each DMU faces the same prices, which can be generalized).

$$\begin{aligned}
\max \quad & \sum_{m=1}^{M} p_m^{\tau} y_m^{\tau} + ... + \sum_{m=1}^{M} p_m^{\tau+\Gamma} y_m^{\tau+\Gamma} \\
s.t. \quad & y_m^{\tau} \leq \sum_{k=1}^{K} z_k^{\tau} y_{km}^{\tau}, \quad m = 1,...,M \\
& \sum_{k=1}^{K} z_k^{\tau} x_{kn}^{\tau} \leq x_n^{\tau}, \quad n = 1,...,N \\
& z_k^{\tau} \geq 0, \quad k = 1,...,K \\
& \vdots \\
& y_m^{\tau+\Gamma} \leq \sum_{k=1}^{K} z_k^{\tau+\Gamma} y_{km}^{\tau+\Gamma}, \quad m = 1,...,M \\
& \sum_{k=1}^{K} z_k^{\tau+\Gamma} x_{kn}^{\tau+\Gamma} \leq x_n^{\tau+\Gamma}, \quad n = 1,...,N \\
& z_k^{\tau+\Gamma} \geq 0, \quad k = 1,...,K \\
& x_n^{\tau} + ... + x_n^{\tau+\Gamma} \leq \overline{x}_n, \quad n = 1,...,N
\end{aligned}$$

Note that prices may differ between periods which is indicated by p_m^{τ} and $p_m^{\tau+\Gamma}$, m=1,…,M.

2. Empirical Application-Reassessing the EU Stability and Growth Pact

Monetary union among sovereign countries helps facilitate trade by reducing transaction costs of currency exchange. When countries have different levels of sovereign default risk, an agreement among those countries to form a monetary union can lend a deficit bias to the area as interest rate differences between those countries disappear (Schuknecht 2002). A deficit bias can also occur when an ageing population defers payment for retirement and health care to future generations, when fiscal policy is captured by special interests, or when lower level governments think they will receive transfers from a central government in the event of insolvency. As deficits accumulate and a 90% debt to GDP threshold is passed, Reinhart and Rogoff (2010) found that economic growth tends to slow, although Panizza and Presbitero (2012) attributed the finding to correlation, rather than causation. To alleviate the bias toward deficits, member countries can agree to a fiscal constitution requiring, for instance, balanced government budgets except in emergencies and/or maximum allowable ratios of government debt to GDP. However, rules can be time inconsistent, since there may be instances when it is sub-optimal to abide by previous commitments.

The 1992 Maastricht Treaty proposed a monetary union among European countries that met certain fiscal criteria. From 1992 to 1998 the fiscal deficits of the European countries considering monetary union improved from an average of 5% of GDP to only 2% of GDP. However, only Ireland, Luxembourg, and Finland met the balanced budget requirement. Although France and Germany were producing near potential output in 1998 they continued to run deficits in excess of 2% of GDP (Schuknecht, Moutot, Rother, and Stark 2011). In 1998, eleven European countries formed the initial euro currency zone and the euro came into existence. Subsequently, Greece was admitted into the euro zone in 2001, followed by Slovenia (2007), Malta and Cyprus (2008), Slovakia (2009), and Estonia (2011) bringing the number of euro zone countries to seventeen. As part of the 1992 Maastricht Treaty member countries agree to a "no-bailout" rule prohibiting the European Central Bank from directly acquiring the sovereign debt of member countries and holding harmless member countries for the debts of another country.

To formalize the budget criteria under the Maastricht Treaty the 1998 Pact for Stability and Growth required that the budgets of member countries be balanced or in surplus during normal times, so that automatic fiscal stabilizers could be allowed to operate. Allowable exemptions include countries which have experienced 2% declines in GDP. Those countries having excessive deficits would be required to contribute 0.2% of GDP to a non-interest bearing deposit at the European Central Bank that would convert to a fine that could rise to as much of 0.5% of GDP given agreement by a majority of countries (Schuknecht et al. 2011).

One of the main rationales behind formal fiscal rules, such as the Pact for Stability and Growth, is that the deficit bias that arises from the common pool problem among member countries could create contagion effects if default in one country spills over to other countries. To test for contagion effects Eichengreen and Wyplosz (1998) examine US states and find little evidence of interstate contagion effects. In addition, they test the null hypothesis of no interest rate spillovers between European countries using Granger causality tests. They conclude that European countries borrow on a global capital market with only small interest rate spillovers between countries. Furthermore, Eichengreen and Wyplosz (1998) simulate the effects of Stability Pact rules for OECD countries and find that the gap between potential and GDP increases as the budget surplus increases. Since the Stability Pact rules weaken sovereign fiscal authority, the rules undermine citizen support for market reforms, especially policies that make labor markets more flexible and open.

Wyplosz (2012) defines fiscal discipline as occurring when the long-run debt to GDP ratio is stationary. Using and augmented Dickey-Fuller test and the KPSS stationary test he finds evidence of a non-stationary debt to GDP ratio for 15 out of 19 OECD countries during 1960-2011.

It is important for fiscal rules to be flexible enough to accommodate unforeseen emergencies such as real estate or stock market bubbles or a financial crisis. However, the temptation to amend a fiscal rule becomes greatest when it is needed most: at the point the rule becomes binding (Wyplosz 2012).

The success of Ulysses in hearing the sirens sing was only possible because his faithful crew followed his orders. Will today's European Monetary Union odyssey become a Greek tragedy because fiscal rules were not followed? Or, as austerity opponents argue, were the rules the source of instability contributing to the on-going debt crisis in countries such as Greece?

In this paper we simulate the effects of the Pact for Stability and Growth by examining the lost output that might result from budget rules that constrain the ratio of debt to GDP to be less than some arbitrary target. Rather than require that budgets meet the target in each and every year we impose a budget target over a longer period. Our method follows Färe, Grosskopf, and Margaritis (2010) and Färe, Grosskopf, Margaritis, and Weber (2012) by allowing inputs to be substituted across time and governments to choose when to spend in order to maximize the sum of real GDP over time given the fiscal budget constraint. We also simulate how resources might be substituted across time in order to minimize government debt given a real GDP growth constraint.

Method

We consider two time substitution problems. In both problems the amount of labor is fixed in each year set at the actual amount of labor used. The amount of capital is fixed over the entire period but can be reallocated between periods. In the first problem budget deficits over the period 1995-2011 are balanced, but deficits in a given period are allowed if they are offset by a surplus in a different period. In the second problem we simulate a setting where countries attempt to minimize the sum of the budget deficits during the period subject to technological constraints and a policy goal of achieving 2% annual growth in real GDP throughout the period. In both problems the amount of labor and amount of capital are fixed over the entire period but labor and capital can be reallocated between periods.

We assume there are *k*=1,…,*K* countries producing in *t*=1,…,*T* periods. Each country is observed to produce y_k^t real GDP using L_k^t units of labor and K_k^t units of capital. Real GDP consists of the sum of real consumption expenditures, real investment expenditures, real government spending, and real next exports. The sum of consumption, investment, and net exports is represented by CIX_k^t and government spending equals G_k^t so that $y_k^t = CIX_k^t + G_k^t$. Government tax revenues, R_k^t, equal the product of the average tax rate, tr_k^t, and real GDP, y_k^t: $R_k^t = tr_k^t \times y_k^t$. The total amount of capital used during the period is represented by $\overline{K}_k = \sum_{t=1}^{T} K_k^t$ and the total labor used during the period is $\overline{L}_k = \sum_{t=1}^{T} L_k^t$ Transfer payments are represented by TP_k^t and the budget deficit is $deficit_k^t = G_k^t + TP_k^t - R_k^t$. The problem of time substitution involves choosing a period to begin production, τ and a length of time to engage in production, Γ, so as to maximize production or minimize government debt.

For country *o*, the first problem takes the form:

$$\max_{z,G,K,\tau,T} \sum_{t=\tau}^{\tau+\Gamma} y^t = CIX_o^t + G^t \quad \text{subject to}$$
$$L^t \geq \sum_{k=1}^{K} z_k^t L_k^t \quad t=\tau,...,\tau+\Gamma, \quad \sum_{t=\tau}^{\tau+\Gamma} L^t \leq \bar{L}_o$$
$$K^t \geq \sum_{k=1}^{K} z_k^t K_k^t \quad t=\tau,...,\tau+\Gamma, \quad \sum_{t=\tau}^{\tau+\Gamma} K^t \leq \bar{K}_o$$
$$CIX_o^t \leq \sum_{k=1}^{K} z_k^t CIX_k^t \quad t=\tau,...,\tau+\Gamma, \tag{1}$$
$$G^t \leq \sum_{k=1}^{K} z_k^t G_k^t \quad t=\tau,...,\tau+\Gamma,$$
$$R^t = tr_o^t \times (CIX_o^t + G^t), \quad t=\tau,...,\tau+\Gamma,$$
$$\sum_{t=\tau}^{\tau+T} (G^t + TP_o^t - R^t) \leq DEBT_{\text{target}},$$
$$z_k^t \geq 0, \quad t=\tau,...,\tau+T, \quad k=1,...,K, \quad G^t \geq 0, \quad K^t \geq 0, \quad t=\tau,...,\tau+\Gamma.$$

In (1) the choice variables are the intensity variables, z_k^t, when to begin production, τ, the length of period to engage in production, Γ, the amount of labor and capital to be used in each period, L^t and K^t, and the amount of government spending, G^t. The optimal values determine government revenues, R^t. The $DEBT_{\text{target}}$ is a pre-determined value for the total accumulation of debt (deficits) over the period. A debt (change) target equal to zero would require government to run a balanced budget during the entire period but would allow a deficit in one year to be offset by a surplus in another year. Alternatively, the debt (deficit) target could be set as a percent of GDP, such as a 2% target.

Given the difficulties of countries in the EU to obtain budget balance we consider an alternative problem. Here we simulate an objective whereby countries minimize the amount of debt they accumulate throughout the period subject to a constraint that requires a minimum amount of average annual real GDP growth. Real GDP in period t=1 for country o is represented by y_o^1. Given the seventeen year period 1995-2011, let the sum of real GDP that satisfies the growth target be $\bar{y}_o = y_o^1 \times \sum_{t=0}^{16}(1+r)^t$, where r is the target for average annual real GDP growth.

The second problem we consider takes the form:

$$
\begin{aligned}
&\min_{z,G,K} \quad \sum_{t=\tau}^{\tau+\Gamma}(G^t + TP_o^t - R^t) \text{ subject to} \\
&L^t \geq \sum_{k=1}^{K} z_k^t L_k^t \quad t=\tau,...,\tau+\Gamma, \quad \sum_{t=\tau}^{\tau+\Gamma} L^t \leq \bar{L}_o \\
&K^t \geq \sum_{k=1}^{K} z_k^t K_k^t \quad t=\tau,...,\tau+\Gamma, \quad \sum_{t=\tau}^{\tau+\Gamma} K^t \leq \bar{K}_o, \\
&CIX_o^t \leq \sum_{k=1}^{K} z_k^t CIX_k^t \quad t=\tau,...,\tau+\Gamma, \\
&G^t \leq \sum_{k=1}^{K} z_k^t G_k^t \quad t=\tau,...,\tau+\Gamma, \\
&R^t = tr_o^t \times (CIX_o^t + G^t), \quad t=\tau,...,\tau+\Gamma, \\
&\sum_{t=\tau}^{\tau+\Gamma}(CIX_o^t + G^t) \geq \bar{y}_o, \\
&z_k^t \geq 0, \quad t=\tau,...,\tau+\Gamma, \quad k=1,...,K, \quad G^t \geq 0, \quad K^t \geq 0, \quad t=\tau,...,\tau+\Gamma.
\end{aligned} \tag{2}
$$

The choice variables for (2) are the same as problem (1): the intensity variables, z_k^t, when to begin production, τ , the length of period to engage in production, Γ, the amount of capital to be used in each period, L^t and K^t, and the amount of government spending, G^t. However, in (2), countries are allowed to run deficits as long as the sum of real GDP throughout the period meets the target: $\sum_{t=\tau}^{\tau+\Gamma}(CIX_o^t + G^t) \geq \bar{y}_o$.

To estimate the time substitution model we employ panel data on 23 countries for the period 1995 to 2011. Included are the seventeen euro zone countries plus Denmark, Japan, Sweden, Switzerland, the UK, and US. The six non-euro zone countries are included since they help form the reference technology or because they are part of the European community. Table 1 reports descriptive statistics for the variables comprising problems (1) and (2) for the panel and for the seventeen euro zone countries and six non-euro zone countries. The average deficit to real GDP ratio equals 0.27 for the entire panel, with an average of 0.28 among the euro zone countries and 0.22 for the six non-euro zone countries. Finland had the smallest deficit to GDP ratio (largest surplus) of -0.068 in 2000 and in fact, ran a surplus from 1998 to 2008. Ireland had the largest deficit to GDP ratio of 0.313 in 2010, but actually managed a surplus in eleven out of the seventeen years. Nine countries (Austria, France, Greece, Italy, Japan, Malta, Portugal, Slovakia, and Slovenia) ran deficits in every year 1995 to 2011. Luxembourg ran a deficit in only four years, followed by Finland with six years of deficits, and then Ireland, Estonia, and Switzerland with seven years of deficits.

In Table 2 (see also Fig. 1) we report the results from solving problem (1) with a balanced budget and with debt (deficit) targets equal to 0%, 1%, 2%, and 3% of accumulated real GDP. As part of the solution to (1), the period to begin production was $\tau = 1995$ and the length of production was $\Gamma = 17$ years for all countries. To estimate the effects of the deficit constraint on real GDP we estimated (1) with and without the constraint $\sum_{t=\tau}^{\tau+T}(G^t + TP_o^t - R^t) \leq DEBT_{\text{target}}$.

The lost output due to the deficit constraint was calculated as

$$\text{Lost Output} = \left(\sum_{t=\tau}^{\tau+T} y_t^*(\text{No Debt}_{\text{target}}) \Big/ \sum_{t=1995}^{2011} y_t^* \ (\text{with Debt}_{\text{target}}) \right) - 1.$$

Luxembourg and Denmark experience no lost output given all four debt targets. For the 2% and 3% targets Finland would have experienced no lost output and for the 3% target Switzerland would have experienced no lost output. Such a result was expected since each of those countries had average deficit to real GDP ratios less than 0 during the period indicating that on average, the countries ran a surplus. Estonia also ran a cumulative surplus during the period but would have experienced some lost output. This result occurred because Estonia was inefficient and problem (1) maximizes potential real GDP=CIX+G by choosing G. For Estonia, the optimal level of government purchases (G) with no debt constraint was greater than the optimal level of government purchases with the debt constraint and the actual level of government purchases, which resulted in the lost potential output. Among the euro zone countries the effect of the alternative debt constraints was greatest for Greece, followed by Spain. These countries would have suffered 45% and 31% in lost output had they balanced their budgets during the period and even under a 3% debt target the two countries would have experienced 36.5% and 24.8% in lost output.

Among the non euro zone countries Japan would have experienced 23.3% in lost output had they satisfied the 0% debt constraint with lost output falling to 16.3% for the 3% debt target. For the 3% debt target, Sweden, the UK and the US would have experienced approximately 11% in lost output. For all countries (except those with 0% lost output) as the debt constraint is relaxed the lost output due to the constraint declines. For instance, relaxing the debt target from 2% to 3% resulted in an output gain for Greece equal to $(.391-.365)\times 100\% = 2.6\%$.

Given the high cost in terms of lost output that most countries would incur from balancing their budgets we also estimated problem (2) to determine the minimum potential debt that might be incurred if the countries produced efficiently and achieved a given growth rate in real GDP. Table 3 lists the actual average annual growth rate of real GDP for each of the countries.[2] During the 1995-2011 Ireland had the maximum average growth rate at 7% followed Estonia, Slovakia, the UK and the US. Slovenia experienced a negative growth rate and other

[2] We calculate the average annual growth rate using formula for the future value of an annuity flow: $FV = A\times\left(\frac{(1+r)^t}{r}\right)$ where FV equals cumulative real GDP for the seventeen year period, A=real GDP in 1995, and t=17.

slow growing countries included Japan and Germany which both experienced less than 1% average annual growth in real GDP.

To obtain a solution to problem (2) we have to specify a target level of output $(\bar{y})$ such that cumulative real GDP throughout the period is greater than the target: $\sum_{t=\tau}^{\tau+\Gamma}(CIX_o + G) \geq \bar{y}_o$. In our first simulation we assume each country has a target for cumulative real GDP equal to its actual cumulative real GDP during the seventeen year period 1995-2011. Given this country specific target problem (2) chooses government purchases (G) so as to minimize the cumulative debt. The optimal time to begin production (τ) is in 1995 and production takes place in all periods. Table 3 (see also Fig. 2) reports the actual debt accumulated during the 1995-2011 period and the simulated debt given the real GDP growth target. Luxembourg would have experienced a surplus equal to 24.7% of 2011 real GDP and this surplus equaled its actual surplus. For the other countries the cumulative debt is smaller (or cumulative surplus is larger) when resources are optimally allocated across time. Greece, with the largest actual debt to GDP ratio, also had the largest simulated debt to GDP ratio, with both actual and simulated debt exceeding 100% of real GDP.

Figure 1

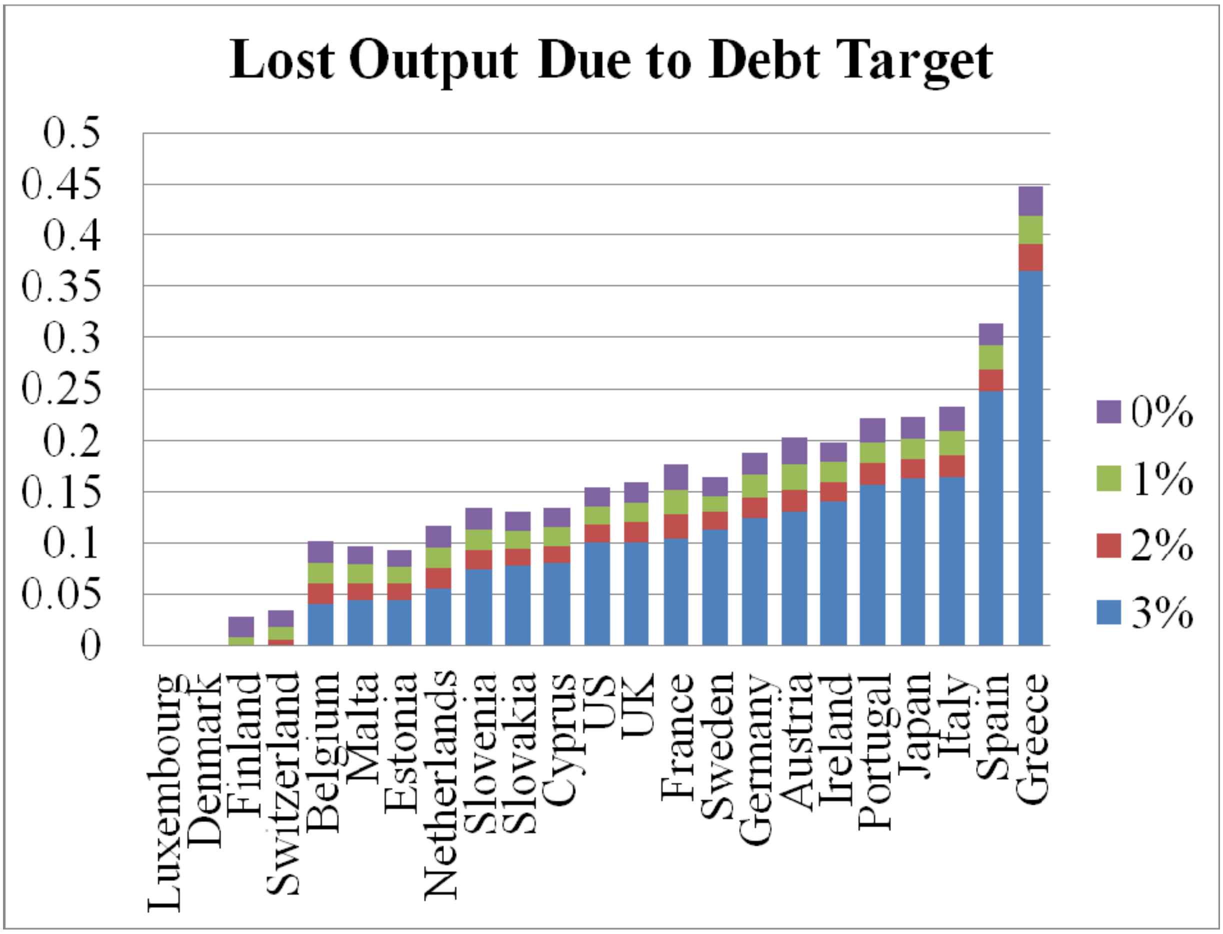
Lost Output Due to Debt Target
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0%
1%
2%
3%
Luxembourg
Denmark
Finland
Switzerland
Belgium
Malta
Estonia
Netherlands
Slovenia
Slovakia
Cyprus
US
UK
France
Sweden
Germany
Austria
Ireland
Portugal
Japan
Italy
Spain
Greece

Figure 2

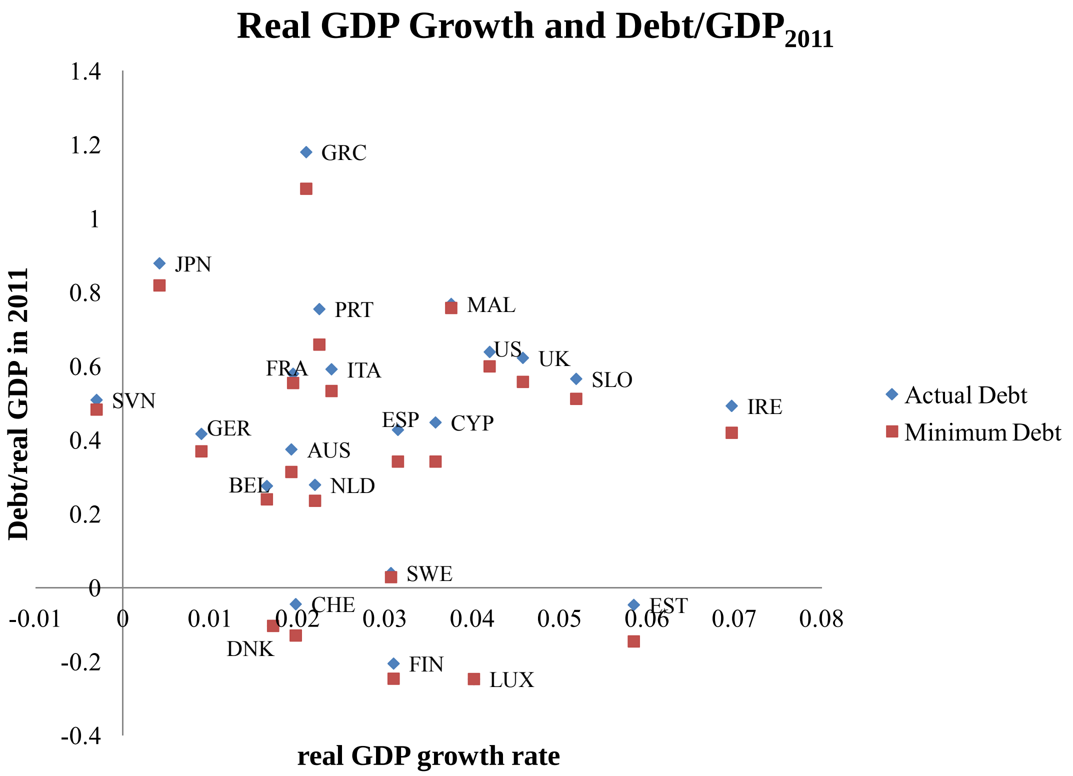
Real GDP Growth and Debt/GDP2011
Debt/real GDP in 2011
real GDP growth rate
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.01
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
GRC
JPN
PRT
MAL
US
UK
FRA
ITA
SLO
SVN
IRE
GER
ESP
CYP
AUS
BEL
NLD
SWE
CHE
EST
DNK
FIN
LUX
Actual Debt
Minimum Debt

Table 1. Descriptive Statistics, 23 countries 1995-2011

Variable	N	Mean	Std. Dev.	Minimum	Maximum
Labor=L	391	16650.2	30473.0	141.8	148295.0
Capital=K	391	2762.4	4991.8	6.0	26326.4
Y=CIX+G	391	952.1	1912.7	3.0	11173.2
CIX	391	780.2	1605.7	2.5	9521.7
Government expenditures=G+TP	391	397.2	721.7	1.2	3907.9
government purchases=G	391	171.9	310.3	0.6	1662.0
Transfer payments=TP	391	225.2	412.5	0.6	2256.4
Tax revenue=R	391	361.1	646.4	1.1	3847.4
Average tax rate=tr	391	0.425	0.071	0.305	0.597
Deficit/Y	391	0.027	0.039	-0.068	0.313
	17 Euro countries				
Labor=L	289	8222.7	11015.4	141.8	41028.4
Capital=K	289	1376.2	1956.4	6.0	7180.8
Y=CIX+G	289	412.6	597.5	3.0	2310.7
CIX	289	328.1	476.2	2.5	1865.6
Government expenditures=G+TP	289	199.9	294.4	1.2	1075.4
Government purchases=G	289	84.6	122.7	0.6	450.0
Transfer payments=TP	289	115.4	172.5	0.6	691.6
Tax revenue=R	289	187.3	275.5	1.1	1025.2
Average tax rate=tr	289	0.428	0.056	0.314	0.565
Deficit/Y	289	0.028	0.039	-0.068	0.313
	6 Non-Euro Countries (Denmark, Japan, Sweden, Switzerland, UK, US)				
Labor=L	110	40527.9	49605.4	2643.0	148295.0
Capital=K	110	6690.2	8014.1	392.9	26326.4
Y=CIX+G	110	2480.6	3149.0	153.4	11173.2
CIX	110	2061.1	2658.4	114.7	9521.7
Government expenditures=G+TP	110	956.0	1156.3	84.7	3907.9
Government purchases=G	110	419.5	495.1	27.9	1662.0
Transfer payments=TP	110	536.5	663.1	48.3	2256.4
Tax revenue=R	110	853.8	1032.3	77.9	3847.4
Average tax rate=tr	110	0.420	0.103	0.305	0.597
Deficit/Y	110	0.022	0.039	-0.050	0.115

Source: European Commission. Economic and Financial Affairs. Annual Macroeconomic Database. http://ec.europa.eu/economy_finance/db_indicators/ameco/zipped_en.htm

Table 2. Lost output due to the deficit target

Country	Lost output given 0% deficit target	Lost output given 1% deficit target	Lost output given 2% deficit target	Lost output given 3% deficit target
Austria	0.202	0.176	0.152	0.130
Belgium	0.102	0.080	0.060	0.040
Cyprus	0.135	0.115	0.097	0.080
Estonia	0.093	0.076	0.060	0.045
Finland	0.027	0.008	0.000	0.000
France	0.176	0.151	0.127	0.104
Germany	0.188	0.166	0.144	0.124
Greece	0.447	0.419	0.391	0.365
Ireland	0.198	0.178	0.159	0.141
Italy	0.233	0.209	0.186	0.164
Japan	0.222	0.201	0.182	0.163
Luxembourg	0.000	0.000	0.000	0.000
Malta	0.097	0.078	0.060	0.043
Netherlands	0.116	0.095	0.075	0.056
Portugal	0.221	0.198	0.177	0.156
Slovakia	0.130	0.111	0.094	0.077
Slovenia	0.134	0.113	0.093	0.074
Spain	0.315	0.292	0.270	0.248
Denmark	0.000	0.000	0.000	0.000
Switzerland	0.034	0.018	0.005	0.000
Sweden	0.164	0.146	0.129	0.112
UK	0.159	0.139	0.120	0.101
US	0.154	0.136	0.118	0.101

Lost output $= \left(\sum_{t=\tau}^{\tau+T} y_t^* (\text{No Debt}_{\text{target}}) \Big/ \sum_{t=1995}^{2011} y_t^* \ (\text{given Debt}_{\text{target}}) \right) - 1$ under constant returns to scale.

Table 3. Actual Debt and Simulated Debt given a growth Target and Time Substitution

Country	Actual average annual growth rate of Real GDP	Actual debt Accumulated (1995-2011) as a % of 2011 GDP	Simulated debt accumulated (1995-2011) given real GDP growth target
Austria	0.0193	0.375	0.314
Belgium	0.0165	0.276	0.240
Cyprus	0.0358	0.448	0.342
Estonia	0.0585	-0.046	-0.145
Finland	0.0310	-0.205	-0.246
France	0.0195	0.580	0.555
Germany	0.0090	0.417	0.370
Greece	0.0210	1.180	1.081
Ireland	0.0697	0.493	0.420
Italy	0.0239	0.592	0.533
Japan	0.0042	0.879	0.819
Luxembourg	0.0402	-0.247	-0.247
Malta	0.0376	0.769	0.758
Netherlands	0.0220	0.279	0.236
Portugal	0.0225	0.755	0.659
Slovakia	0.0519	0.566	0.512
Slovenia	-0.0030	0.509	0.483
Spain	0.0315	0.428	0.342
Denmark	0.0172	-0.103	-0.103
Switzerland	0.0198	-0.044	-0.129
Sweden	0.0307	0.040	0.029
UK	0.0458	0.623	0.558
US	0.0420	0.639	0.600

Miscellaneous Table-Shephard Distance Function Estimates CRS and VRS (No deficit constraint)

Country	$D_o(\overline{x, y \mid CRS})$	$D_o(\overline{x, y \mid VRS})$
Austria	0.637	0.832
Belgium	0.822	0.947
Cyprus	0.806	0.822
Estonia	0.753	0.766
Finland	0.858	0.914
France	0.774	0.994
Germany	0.706	0.892
Greece	0.499	0.582
Ireland	0.602	0.879
Italy	0.649	0.827
Japan	0.739	0.976
Luxembourg	1.000	1.000
Malta	0.989	1.000
Netherlands	0.780	0.935
Portugal	0.668	0.781
Slovakia	0.843	0.878
Slovenia	0.826	0.860
Spain	0.517	0.620
Denmark	1.000	1.000
Switzerland	0.894	0.989
Sweden	0.699	0.988
UK	0.769	0.931
US	0.833	1.000

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