A Price Theory of Vertical and Lateral Integration under Productivity Heterogeneity*

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Abstract

We develop a model of organizational choice in a perfectly competitive product market with heterogeneous firms and incomplete contracts. Successful production requires two inputs that are supplied by two different firms. Firms are vertically heterogeneous with respect to their productivity. Each supplier from one side matches endogenously in a stable equilibrium with one supplier from the other side. After they match, and taking the market price as given, they decide whether to integrate or stay as separate units. Each supplier cares about firm profits and private benefits. Organization decisions involve a trade-off between firm profits and private benefits. An important feature of our model is the endogenously determined, through matching, bargaining powers, which as we show have a profound effect on organizational design in a market. We study the interplay between market price, firm productivity and firm boundaries. Integration decisions can be non-monotonic in overall firm productivity. More specifically, it may be the low productivity firms that have stronger incentives to integrate. A higher market price can induce more firms to disintegrate, yielding an industry supply curve that is backward-bending. These results generate new empirical implications.

Keywords: Integration, incomplete contracting, market competition, endogenous matching.

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1 Introduction

There is ample evidence of firm productivity variation within an industry, e.g., Gibbons (2010) and Syverson (2011). Moreover, this variation is correlated with organizational variation, e.g., Gibbons (2010). In this paper, we study the interaction between the product market and firm boundaries and the role of ex-ante firm productivity heterogeneity. We are interested in the following questions. Which firms are more likely to integrate: low or high productivity firms? How does product market competition interact with productivity heterogeneity to shape firms' incentives to integrate? Does integration increase firm productivity, or more productive firms are more likely to integrate? What is the correlation between overall firm productivity in the market and the fraction of integrated firms? Does higher market price always imply higher aggregate output supplied when integration decisions are endogenized?

Our main results are as follows: i) higher firm productivity does not necessarily increase the firm's incentive to integrate; in particular, integration decisions can be *non-monotonic* with respect to firm productivity, where low and high productivity firms integrate but intermediate productivity firms prefer to stay separate, ii) a higher market price does not always lead to more integration; on the contrary, a higher market price can induce more firms to disintegrate, or a lower market price leads to more integration and iii) once we endogenize integration decisions, a higher market price can lower aggregate output (i.e., the "organizationally augmented supply" curve is *backward-bending*). These findings are in contrast to existing literature (see literature review for more details) and generate new empirical implications.

We develop a model in the spirit of the incomplete-contract tradition, Grossman and Hart (1986) and Hart and Moore (1990), with productivity heterogeneity and endogenous matching. Production requires two inputs which are provided by two input suppliers (units) A and B. Each enterprise takes the market price as given and produces output. We can view each unit as a collection of assets run by a manager. The two units can either remain separate or integrate. Each manager's utility depends on firm profits and private benefits. Under nonintegration each manager makes a non contractible decision to maximize his utility. Since managers also care about their private benefits firm profits are not maximized. Under integration managers give decision power to a third party who maximizes firm profit but ignores the managers' private benefits. Neither organizational form is superior: nonintegration results in 'too little' coordination and integration results in 'too much' coordination. Further, we assume that the market consists of a continuum of input suppliers who are endowed with different productivity levels. Each A supplier matches endogenously with one B supplier to form an enterprise. After they match they decide whether to remain as separate units or integrate. We assume that B suppliers have all the bargaining power and make take-it-or-leave-it offers to A suppliers, taking into account the endogenously determined utility of A suppliers.

Higher productivity suppliers are more desirable and hence command a higher equilibrium utility. We find reasonable conditions under which matching is positive assortative: high productivity A suppliers match with high productivity B suppliers, whereas low productivity A suppliers match with low productivity B suppliers.

When the two units are separate they use a contract to govern their relationship. Firm revenue is contractible and the contract stipulates the share of the firm revenues that accrues to each party. The inefficiency under nonintegration is that managers make decisions not only with firm revenues in mind but also with their private benefits. The revenue share is used as an imperfect instrument to gauge manager decisions. As the revenue share a supplier receives increases, the supplier, when making a decision, puts more weight on firm revenue and less weight on private benefits. Unfortunately, this means exactly the opposite for the other supplier, since a higher share for one implies a lower share for the other. Hence, both managers coordinate their non contractible decisions the best when the share is intermediate.[1] This implies that the inefficiency under nonintegration is minimized for intermediate levels of the revenue share. Under integration, on the other hand, a third party maximizes firm revenue, which is shared perfectly between the two managers. There is still an inefficiency though due to the fact that the third party does not care about the private benefits of the managers.

Integration is more likely to be the preferred design the higher the market price is, *holding* the revenue share fixed. A higher price makes the foregone firm revenue under nonintegration more valuable, so the cost of the two firms when they remain separate increases (*market price effect*). However, the revenue share, as we discussed above, also affects integration decisions. Higher productivity A suppliers receive, in a stable equilibrium, a higher share of the revenue. Since intermediate shares are more likely to make nonintegration dominant, we can have a market outcome where the optimal design is *non-monotonic* in firm productivity. In particular, the intermediate productivity enterprises (who are associated with intermediate revenue shares) choose to remain separate, while low and high productivity ones integrate. This *revenue share effect* is a novel effect that emerges from our analysis.

The market price and revenue share effects interact with each other to yield interesting predictions. As the market price increases the equilibrium share decreases (a higher price mitigates the competition among B suppliers for high productivity A suppliers). This has the following implications. As we already know (see the literature review below), a higher market price makes integration more likely. Lower share, on the other hand, as an outcome of a higher price, can have an opposing effect on a firm's decision to integrate. If the share is already high (high productivity

[1] Nevertheless, an intermediate share is not mutually beneficial because each side's utility is an increasing function of its own share of the revenue.

enterprises), then a lower share means a more efficient outcome under nonintegration (because we move closer to intermediate shares which tend to favor nonintegration). Overall, and depending on the underlying productivity distribution, there are instances where a higher market price is associated with less integration (when the revenue share effect dominates the market price effect). Since nonintegrated firms produce less output industry supply can be *backward-bending.*

The model is rich enough and generates new predictions about: i) overall productivity in a market and the incidence of integration, ii) which types of firms (e.g., low or high productivity) are more likely to integrate and iii) the relationship between market price and integration. What turns out to be crucial in determining these predictions is the degree of productivity heterogeneity, how strong the contribution of A supplier productivity is relative to that of B supplier to overall firm productivity and how many high productivity suppliers are relative to low productivity ones.

Each firm (or a subset of firms) in the market exerts an externality on the remaining firms through the equilibrium utilities. This externality affects integration decisions and efficiency in the case of nonintegration. For instance, assume that a subset of firms becomes more productive. This will affect the equilibrium utilities of the managers in the affected firms and their incentives to integrate or not, but it will also affect the equilibrium utilities of all the remaining, unaffected by the productivity shock, firms. To be more precise, a change of utilities at some point in the market (as long as its measure is non-zero) will create a ripple effect that affects the entire market.[2]

Integration increases firm output since coordination between the two suppliers improves, holding productivities fixed (*influence effect*). On the other hand, firm productivities affect the likelihood of integration through endogenous matching (*sorting effect*). The influence effect is monotonic in firm productivity, but the sorting effect is not. Therefore, if higher productivity induces a firm to choose to disintegrate, then higher productivity can lower output. If output is used to infer productivity (as it is usually the case), then one would wrongly conclude that less productive firms are less likely to integrate.

Related literature and contribution

Grossman and Helpman (2002) develop a model of monopolistic competition with free entry to examine the effect of technology and demand on integration decisions.[3] They obtain, non generically though, endogenous heterogeneity in organizational choices from otherwise identical firms. Gibbons, Holden and Powell (2012) obtain generic heterogeneity of ownership by analyzing a rational-expectations equilibrium of price formation and endogenously chosen governance structures. They show that the informativeness of the price mechanism can induce ex-ante homogeneous

[2]Hong, Serfes and Thiele (2013) explore the impact of this ripple effect, caused by entry of new VC firms, in the market for venture capital.

[3]Lafontaine and Slade (2007) review the findings of empirical studies on vertical integration.

firms to choose heterogeneous governance structures. Legros and Newman (2013) also find generically, using a different mechanism than Gibbons, Holden and Powell (2012), that integrated firms can co-exist with nonintegrated firms, when all firms are ex-ante identical. The benefits from integration increase with the market price (our market price effect). For low market prices all firms are nonintegrated and for high market prices all firms are integrated. At one market price firms are indifferent between the two organizational forms and at that point co-existence can emerge. Market price is positively correlated with integration.

Legros and Newman (2013) is the closest paper to our work and our model builds on their model. We differ from them in the following two respects. First, we allow input suppliers to be ex-ante heterogeneous with respect to their productivities and second we endogenize the revenue share and the utilities of the suppliers through matching. These features yield an equilibrium where organizational structure in the market can be non-monotonic in productivity. We highlight the roles of the sources of productivity gains and the endogenously determined bargaining power distribution between the two suppliers. More specifically, higher firm productivity may not be a good indicator of integration decisions. As we show, higher market-wide productivity may be associated with more or less integration depending on whether the gains come mainly from the A or the B suppliers. Finally, higher market price is *not* always associated with more integration and higher output.

Aghion, Griffith and Howitt (2006) provide evidence of a U-shaped relationship between product market competition and vertical integration. Integration is more likely when competition is either soft or intense. In addition to the role of product market competition, we emphasize the significance of competition among different firms for high productivity suppliers in an endogenously matching framework. Our model displays a similar kind of non monotonicity but with respect to firm productivity, holding product market competition constant.

Alfaro et al. (2013) use changes in trade policy (e.g., tariffs) as an exogenous source of price variation. They find empirical evidence that the level of product prices do affect vertical integration decisions. Acemoglu et al. (2010) show that technology intensity affects the likelihood of integration. They find that technology intensity of the downstream producers is positively correlated with integration, while technology intensity of the upstream suppliers is negatively correlated. Our model also disentangles the contributions of the two units to overall firm productivity. Although our notion of productivity contribution and the mechanism we propose are very different from the ones in Acemoglu et al. (2010), our result with respect to the effect of firms' technologies on integration incentives shares some similarities with Acemoglu et al.

The rest of the paper is organized as follows. We present in Section 2 the model. Section 3 contains the analysis and the main results. We offer a discussion and a summary of the main empirical implications in Section 4 and we conclude in Section 5. All proofs are in the Appendix.

2 The Model

The model we develop builds on Hart and Holmström (2010) and Legros and Newman (2013), by introducing productivity heterogeneity. The goal is to derive an industry supply curve that summarizes the relationships among price, quantity and ownership structure in a market where input suppliers (units) have different levels of productivity.

2.1 Technology, preferences and ownership structures

The model features a continuum of input suppliers A (indexed by i) and B (indexed by j). Each supplier is a collection of assets and workers overseen by a manager. There is one consumer good in the market, the production of which requires one A and one B supplier (call it an (i, j) enterprise or firm). This can include lateral as well as vertical relationships. Let $i \in [\underline{i}, \bar{i}]$ with density $f(i)$ and distribution $F(i)$ and $j \in [\underline{j}, \bar{j}]$ with density $g(j)$ and distribution $G(j)$. Input suppliers are vertically heterogeneous with respect to their productivity (or more broadly 'quality'), that is a higher index, either i or j, implies a more productive supplier.[4] We denote by $z(i, j)$ the productivity of the (i, j) supplier-pair (enterprise), with $z(i, j)$ being a continuously differentiable and increasing function of i and j, with complementary supplier productivities $\frac{\partial z^2(i,j)}{\partial i \partial j} \geq 0$, and with $z(i, j)$ greater than a minimum productivity level $\underline{z} > 0$.

Let $m : [\underline{i}, \bar{i}] \to [\underline{j}, \bar{j}]$ be the matching function, with $j = m(i)$ indicating that manager i is paired with manager j to form an enterprise. We assume that the measure of B suppliers is equal to that of A suppliers. This implies $m(\underline{i}) = \underline{j}$. We start by focusing on a given (i, j) pair. Each supplier must make a noncontractible production decision. Denote the decision made by an A supplier by $a \in [0, 1]$ and the decision made by a B supplier by $b \in [0, 1]$. These decisions can be made by the manager of the assets or by someone else. The (i, j) enterprise can either produce output $2q(i, j)$ (success) or nothing (failure). The probability of success is fixed and given by $\frac{1}{2}$ but output in case of success depends on the productivities of the inputs and on the decisions a and b as follows: $q(i, j) = z(i, j) - (a - b)^2$. Thus, expected output is $z(i, j) - (a - b)^2$.[5] If the two managers coordinate their decisions and set $a = b$, then inefficiencies disappear and the enterprise reaches its potential $z(i, j)$. The manager of each supplier is risk neutral and incurs a private (noncontractible)

[4] Bloom and van Reenen (2007), using a survey from medium-sized manufacturing firms from four countries, document that management practices are heterogeneous and affect firm performance. Gibbons (2010) offers a more detailed account of various empirical studies that document persistent performance differences (PPDs).

[5] Legros and Newman (2013) assume that the firm can produce output 1 with probability $1 - (a - b)^2$ and 0 otherwise. Hence, expected output is $1 - (a - b)^2$. Essentially, our formulation allows the expected output to vary with the underlying firm productivities.

cost of the decision made in his unit.[6] The private cost of an A input supplier is $(1-a)^2$ and the private cost of a B supplier is b^2. As it can be seen, there is a disagreement about the direction of the decisions, what is easy for one is hard for the other and vice versa.[7] Also, managers are protected by limited liability, in the sense that incomes must always be nonnegative, and cannot make any side payments (they enter the scene with zero cash endowments). The importance of this assumption is that the division of surplus between the managers will affect the organization structure.

The organization structure can be contractually determined. We assume two different options. Following the property rights literature, each implies a different allocation of decision making power. First, the production units can remain separate firms (nonintegration, N). In this case, managers have full control over their decisions. Second, the two input suppliers can integrate, I, into a single firm, giving control over managerial decisions, a and b, to a third party who always has enough cash to finance the acquisition. The third party is motivated entirely by income and incurs no cost from the a and b decisions. These costs are still borne by the managers.

As argued by Hart and Holmstrom (2010), integration results in an organization where less weight is placed on private benefits than under nonintegration. This, however, is offset by the fact that under integration total profit, rather than individual unit profits, is maximized.

2.2 Contracts

The enterprise's revenue is contractible. We assume that B suppliers have all the bargaining power. Under nonintegration, a contract specifies a revenue share $s(i)$ accruing to i when output is positive (success).[8] The share of j is $1-s(i)$. By limited liability each manager gets a zero revenue when output is zero. Let P be the market price that is taken as given. The expected utilities of the managers as functions of a and b are

$$\begin{aligned} u(i) &= Ps(i)(z(i,j)-(a-b)^2)-(1-a)^2 \\ v(j) &= P(1-s(i))(z(i,j)-(a-b)^2)-b^2. \end{aligned} \tag{1}$$

When the two suppliers integrate, then the third party (call it headquarters HQ) buys the assets A and B for prices π_A and π_B. HQs are supplied perfectly elastically with an opportunity cost normalized to zero.

[6]The private cost can represent, for example, job satisfaction, or a way to capture different beliefs held by managers and workers about the consequences of strategic choices, Hart and Holmstrom (2010).

[7]For example, as discussed in Hart and Holmstrom (2010), the two firms may want to adopt a common standard, as in the Cisco's acquisition of Stratacom. The benefits for the organization from a common standard can be enormous but the private benefits within the firms may decrease because of the change the new standard introduces.

[8]For example, in the airline industry, historically, most contracts between major and regional air carriers were structured as revenue-sharing agreements, Forbes and Lederman (2009).

The overall market equilibrium will determine a, b and the share function $s(i)$. These, in turn, will pin down the equilibrium payoffs $u(i)$ and $v(j)$.

2.3 Product market

The product market is perfectly competitive. Consumers and firms take the market price P as given. We assume that consumers have a quasi-linear utility function, so the solution to the consumer constrained maximization problem is a differentiable demand $D(P)$.

3 Analysis and main results

Equilibrium consists of a stable match in the supplier market and market clearing in the product market. There are two types of coalitions in the supplier market. Under nonintegration, a coalition is supplier i of input A with supplier j of input B with $i = m(j)$. Under integration, a coalition consists of the two suppliers and a HQ. Equilibrium may also involve trivial coalitions consisting of singleton agents. For a given coalition the contract determines the decisions that will be made and the output in case of success. Despite the fact that output is stochastic at the enterprise level, by the law of large numbers industry output is deterministic. When coalitions are formed and contracts are signed, there is a well-defined industry supply $S(P)$. From now on when we will be using i alone we will be referring to the $(i, m(i))$ enterprise.

Definition 1 (Market equilibrium) *An equilibrium consists of a partition of agents into coalitions, a payoff to each agent and a product price P satisfying*

1. *Feasibility: the payoffs to the agents in an equilibrium coalition are feasible given P;*
2. *Stability: no coalition can form and find feasible payoffs for its members that are strictly greater than their equilibrium payoffs;*
3. *Market clearing: the total supply in the industry $S(P)$ is equal to the demand $D(P)$.*

3.1 Choice of organization

First, coalitions are formed, $j = m(i)$ in case of nonintegration and $j = m(i)$ with a HQ in case of integration. Second, contracts are signed and then decisions about a and b are made. Agents correctly anticipate the equilibrium market price P. We analyze each ownership structure separately and in each case we derive the Pareto frontier.

3.1.1 Nonintegration

Managers i and j, with $j = m(i)$, independently choose a and b, both in $[0, 1]$, to maximize their respective utilities as given by (1). The unique Nash equilibrium (same as in Legros and Newman (2013), but with s not being fixed in our model) of this game is

$$a(i) = 1 - s(i)\frac{P}{1+P} \text{ and } b(i) = (1 - s(i))\frac{P}{1+P}. \tag{2}$$

The ideal decision of i is 1, while the ideal decision of j is 0, holding expected output constant. The managers are willing to move away from their ideal decision, and closer to each other as their revenue share and market price increase. Note that the productivity of the (i, j) pair, $z(i, j)$, does not affect the equilibrium decisions. However, it does affect expected output and also the expected payoffs. The expected output in the i enterprise is

$$q^N(i; P) = z(i, j) - \frac{1}{(1+P)^2},$$

which is an increasing function of firm productivity z and market price P.[9]

Let $\mathcal{N} \subseteq [\underline{i}, \bar{i}]$ denote the set of i's, and the associated j's with $j = m(i)$, that do not integrate. Then, the aggregate output among the nonintegrated coalitions is

$$Q^N(P) = \int_{i \in \mathcal{N}} \left(z(i, j) - \frac{1}{(1+P)^2} \right) dF(i). \tag{3}$$

We substitute (2) into (1) to derive the managers' payoffs under nonintegration as a function of individual supplier productivities and market price

$$\begin{aligned} u^N(i; s, P) &\equiv \left(z(i, j) - \frac{1}{(1+P)^2} \right) s(i)P - s(i)^2 \left(\frac{P}{1+P} \right)^2 \\ v^N(j; s, P) &\equiv \left(z(i, j) - \frac{1}{(1+P)^2} \right) (1 - s(i))P - (1 - s(i))^2 \left(\frac{P}{1+P} \right)^2. \end{aligned} \tag{4}$$

Total managerial compensation is

$$u(i)^N + v(j)^N \equiv \left(z(i, j) - \frac{1}{(1+P)^2} \right) P - (s(i)^2 + (1 - s(i))^2) \left(\frac{P}{1+P} \right)^2,$$

and varies from $\left(z(i, j) - \frac{1}{(1+P)^2} \right) P - \left(\frac{P}{1+P} \right)^2$ at $s(i) = 0$ (or $s(i) = 1$) to $\frac{P}{2(1+P)^2}(2z(i, j) + 4z(i, j)P + 2z(i, j)P^2 - 2 - P)$ when $s(i) = \frac{1}{2}$. Nonintegration results in an equilibrium where

[9] The first-best choices maximize the sum of utilities and are given by

$$a = \frac{1+P}{1+2P} \text{ and } b = \frac{P}{1+2P}.$$

Expected output is $z(i, j) - \frac{1}{(1+2P)^2}$. The diminished output under nonintegration reflects the distortionary effect of the imperfect contract. It is similar to the double-marginalization that creates incentives for vertical integration in a world with perfect contracts (Perry (1989)).

managers put low weight on the organizational goal in favor of their private benefits. The Pareto frontier is concave. For any given enterprise, the highest total surplus is achieved when $s(i) = \frac{1}{2}$.

The Pareto frontier is decreasing provided that the share is neither too high nor too low

$$\frac{1 + 2P - z(i,j)(1 + P)^2}{2P} < s(i) < \frac{z(i,j)(1 + P)^2 - 1}{2P}. \tag{5}$$

When the above inequality is not satisfied, an increase in $s(i)$ either increases or decreases the utilities of both suppliers. Observe that the above interval does not collapse to a single point if and only if $z(i,j) > \frac{1}{1+P}$. If $z(i,j) \leq \frac{1}{1+P}$, then $s(i) = \frac{1}{2}$. Finally, if $z(i,j) \geq \frac{1+2P}{(1+P)^2}$ (where $\frac{1+2P}{(1+P)^2} > \frac{1}{1+P}$) then the Pareto frontier is defined for any $s(i) \in [0,1]$. In order to reduce the number of cases we would have to examine we assume that $z(i,j) \geq 1$. This implies that for any $P \geq 0$ the Pareto frontier is defined for any $s(i)$ in the $[0,1]$ interval.

An alternative is integration, but even this form of organization has other incentive problems: the HQ puts too little weight on the managers' private costs.

3.1.2 Integration

The two suppliers integrate and give decision power to a third party called headquarters HQ. The revenue share of HQ is 1. Therefore, it chooses a and b to maximize $(z(i,j) - (a-b)^2)P$. This is maximized at $a = b$. Given this, the managers' private total cost, $(1-a)^2 + b^2$, is minimized when $a = b = \frac{1}{2}$. Although HQ does not care about the managers' private costs we assume it acts so as to minimize total cost. Since HQs compete and have zero opportunity cost, the purchase prices for the assets must total $z(i,j)P$. Expected output in enterprise i is $z(i,j)$ and total managerial welfare under integration is

$$u^I(i) + v^I(i) \equiv z(i,j)P - \frac{1}{2}, \tag{6}$$

which is fully transferable between i and j via the asset prices.

Let $\mathcal{I} \subseteq [\underline{i}, \bar{i}]$ denote the set of i's, and the associated j's with $j = m(i)$, that integrate. Then, the aggregate output among the integrated coalitions is

$$Q^I(P) = \int_{i \in \mathcal{I}} z(i,j) dF(i). \tag{7}$$

An integrated firm produces more output than a nonintegrated firm–$z(i,j)$ versus $z(i,j) - \frac{1}{(1+P)^2}$–holding firm productivity fixed. However, firm productivity affects output both directly and indirectly through integration decisions.

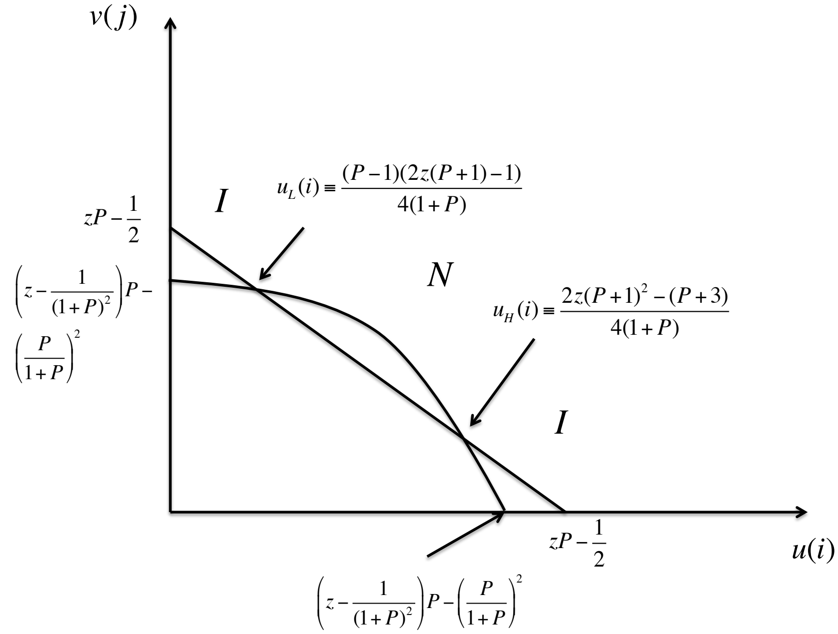

Figure 1: Bargaining frontiers and integration decisions for the (i, j) enterprise

3.1.3 Comparison of ownership structures

When the suppliers integrate they face a trade-off: higher coordination (and hence higher firm profit) but lower private benefits. Figure 1 depicts the Pareto frontiers under nonintegration and under integration. The nonintegration frontier is concave, while the integration frontier is linear. When $P \leq 1$, the Pareto frontier under nonintegration is everywhere above the frontier under integration regardless of the value of $z(i, j)$.[10] Hence, nonintegration is the preferred organizational structure. However, when $P > 1$ the preferred organizational form depends on the reservation utility of the supplier of input A, $u(i)$. The reservation utility depends on the share $s(i)$ which will be determined endogenously. Hence, in the case of $P > 1$, the organizational choice can be *non-monotonic* in productivity. More specifically, for intermediate $u(i)$'s, nonintegration is preferred to integration (see Figure 1). The Proposition below summarizes.[11] (Sometimes, and in order to reduce notational clutter, we write z instead of $z(i, j)$).

Proposition 2 (Organization structure for a given enterprise) *Let*

$$u_L(i) \equiv \frac{(P-1)(2z(P+1)-1)}{4(1+P)} \text{ and } u_H(i) \equiv \frac{2z(P+1)^2 - (P+3)}{4(1+P)}.$$

[10] This is because

$$\left(z(i,j) - \frac{1}{(1+P)^2}\right)P - \left(\frac{P}{1+P}\right)^2 > z(i,j)P - \frac{1}{2}$$

if and only if $P < 1$.

[11] We assume that if an enterprise is indifferent between integrating and remaining nonintegrated it chooses nonintegration.

Nonintegration is chosen if

$$P \leq 1 \text{ or } P > 1 \text{ and } u(i) \in [u_L(i), u_H(i)] \,.$$

Integration is chosen if

$$P > 1 \text{ and either } u(i) < u_L(i) \text{ or } u(i) > u_H(i).$$

Finally, both $u_L(i)$ and $u_H(i)$ are increasing functions of P.

The main idea is that when the two units are separate, and because decisions are non-contractible, utility between the managers can be transferred via the share of the revenue. The share, however, transfers surplus imperfectly since it also affects total surplus. It can be verified that the two utility thresholds $u_L(i)$ and $u_H(i)$ map directly to two share thresholds $s_L(i) \equiv \frac{P-1}{2P}$ and $s_H(i) \equiv \frac{P+1}{2P}$. When the share falls inside the interval $[s_L(i), s_H(i)]$, then nonintegration is optimal. The highest surplus is achieved when coordination among the units is the highest, i.e., when $s(i) = \frac{1}{2}$ which falls into $[s_L(i), s_H(i)]$. On the other hand, when the two units integrate decisions are made by a new boss who maximizes profits but does not care about individual unit private costs. Surplus in this case is transferred perfectly between the two managers, i.e., linear Pareto frontier. Whether integration dominates nonintegration depends on the market price P and the utility $u(i)$ via the share $s(i)$ under nonintegration. A higher market price, as argued by Legros and Newman (2013), makes integration more likely because the foregone output under nonintegration has higher value. We highlight that integration is also more likely for extreme values (low or high) of the utility $u(i)$.[12] At a more intuitive level, nointegration is more likely when neither unit has significant bargaining power, which results in intermediate $u(i)$'s. Given that equilibrium utilities are important determinants (along with market price) of organizational design, the question that arises is: how are they determined in a market equilibrium? The analysis that follows addresses this question, as well as the interplay between equilibrium utilities (and shares), market price and firm boundaries.[13]

3.2 Endogenous matching and evolution of equilibrium utilities and shares

We look for a stable matching between suppliers of input A–indexed by i–and suppliers of input B–indexed by j. Matching is one-to-one, $j = m(i)$. Our goal is to prove that matching is positive

[12]Legros and Newman (2013) also discuss how a change of the outside option u affects integration decisions, but what we add is the co-existence of many u's in a market due to productivity heterogeneity and the endogenous determination of them. In Legros and Newman even when they allow for more than one productivity level, the utilities (which coincide with the outside options) are always fixed and the same across firms.

[13]Note that

$$u_H(i) < \left(z(i,j) - \frac{1}{(1+P)^2} \right) P - \left(\frac{P}{1+P} \right)^2$$

if and only if $z > \frac{3}{2(P+1)}$. If this condition is satisfied then also $u_L(i) > 0$ and the two thresholds are interior. Since we have assumed that $z(i,j) \geq 0$ this condition is always satisfied when $P > 1$.

assortative (PAM), i.e., the stable matching function $m(i)$ is increasing. We begin by assuming a PAM and we determine the matching function $m(i)$. The measure of A suppliers must be equal to the measure of B suppliers for the one-to-one matching equilibrium. Thus, for every i, in order to ensure measure consistency, it must be $F(i) = G(m(i))$. This implies that $m(i) = G^{-1}(F(i))$. Using this consistency condition, we can derive the slope of the matching function $m(i)$

$$\frac{dm(i)}{di} = G^{-1'}(F(i))\, h(i) = \frac{h(i)}{G'(G^{-1}(F(i)))} = \frac{f(i)}{g(m(i))}. \tag{8}$$

The slope of the matching function $m(i)$ is equal to the ratio of the densities of A and B suppliers, $f(i)$ and $g(m(i))$. Thus, the matching function $m(i)$ is the solution to the differential equation (8) with initial condition $m(\underline{i}) = \underline{j}$.

Under integration, the utility of supplier j who matches with supplier i is $v^I(j) = z(i,j)P - \frac{1}{2} - u^I(i)$. Utility in this case is transferred perfectly (TU). PAM is the stable equilibrium since the complementarity condition $\frac{\partial^2 z(i,j)}{\partial i \partial j} \geq 0$ is satisfied (Becker (1973) and Shapley and Shubik (1972)).[14] We need to determine how the utility of i suppliers evolves in equilibrium. The next Lemma states the result.

Lemma 3 (ODE under integration) *When the equilibrium utility $u(i)$ under integration satisfies the following ordinary differential equation (ODE), then each B supplier's expected utility, $v^I(j)$, is strictly concave in i*

$$\frac{du^I(i)}{di} = \frac{\partial z(i, m(i))}{\partial i} P. \tag{9}$$

The above Lemma states that no j supplier finds it profitable to contract with any i supplier other than its assigned in equilibrium i supplier, when the utility of i suppliers evolves according to (9). The marginal utility of i supplier depends on the value of his marginal contribution to overall firm productivity.

Next, we turn to nonintegration, where it is more difficult to prove PAM than under integration. Legros and Newman (2007) have shown that in a NTU environment there are two sufficient conditions for PAM: i) the standard complementarity condition and ii) a higher j implies a (weakly) flatter Pareto frontier. The second condition ensures that it is easier for a more productive input B supplier–who makes offers in our model–to transfer surplus to supplier A, than a less productive B supplier. Our model does not satisfy the second condition. We have verified that the slope of the Pareto frontier does not always become flatter as the productivity of B supplier increases. Our strategy for proving that PAM is an equilibrium is the following. First, we assume PAM and find an

[14] For other matching models in principal-agent settings, see, e.g., Besley and Ghatak (2005), Serfes (2005, 2008) and Dam and Pérez-Castrillo (2006).

$s(i)$, which is increasing in i, so that no j supplier wants to deviate locally from i, where $j = m(i)$. Second, we find a simple condition on $z(i, j)$ under which the expected profit function of every B supplier is strictly concave in i, suggesting that local no deviation implies global no deviation.

The following Lemma characterizes how the equilibrium shares must evolve as a function of the productivity i of supplier A so that no supplier B finds a local deviation profitable (given the matching function $m(i)$). From the shares we can also derive the evolution of the endogenously determined utilities for A suppliers.

Lemma 4 (ODE under nonintegration) *When the equilibrium share $s(i)$ under nonintegration satisfies the following ordinary differential equation (ODE), then B suppliers do not want to deviate locally in i*

$$\frac{ds(i)}{di} = \frac{\frac{\partial z(i,m(i))}{\partial i}(1 - s(i))}{z(i, m(i)) - \frac{1+2P(1-s(i))}{(1+P)^2}}. \tag{10}$$

Our assumptions ensure that the denominator of (10) is positive for all values of $s(i)$ on the Pareto frontier. It is then easy to verify that $\frac{ds(i)}{di} > 0$. Eventhough the outside option of A suppliers is zero, higher productivity A suppliers command a higher than zero utility through a higher share. How fast the revenue share increases with the productivity index i depends positively on i's marginal contribution to firm productivity $\frac{\partial z}{\partial i}$, negatively on the firm's productivity level z and positively on the market price P. This implies that any productivity gain that mainly affects z and to a lesser extent $\frac{\partial z}{\partial i}$–for example j suppliers become more productive–can lead to a decrease of the rate of increase of $s(i)$. We will return to this below, but at this point we would like to draw attention to the differential impact different sources of productivity gains have on the equilibrium revenue share.

Next, we prove that local no deviation implies global no deviation for the nonintegration case.

Lemma 5 *Suppose* $\frac{\partial^2 z}{\partial i \partial j}\left(z - \frac{1+2P}{(1+P)^2}\right) > \frac{\partial z}{\partial i}\frac{\partial z}{\partial j}$ *holds. Then, under nonintegration, where $s(i)$ evolves according to* (10)*, the expected profit function of every B supplier, $v^N(j)$, is strictly concave in i.*

The condition we derive is a strengthening of the complementarity condition from a TU setting. Together with $\frac{ds(i)}{di}$, it guarantees that PAM is an equilibrium, despite the fact that one of the Legros and Newman (2007) conditions is not satisfied. The expected profit function of j supplier when we allow him to optimally choose the organizational form is $v(j) = \max\{v^N(j), v^I(j)\}$. As we have proved, each function inside the max operator is concave. Moreover, by construction, they are both maximized at the same i, where $j = m(i)$. Thus, $v(j)$ must also be concave. It follows that the stable matching in the market, when the organizational form is chosen optimally, is positive assortative.

Proposition 6 (Positive assortative matching) *In a market where the organizational form is chosen optimally and input suppliers on each side are vertically heterogeneous with respect to their productivities, the stable matching between them is positive assortative (if the condition of Lemma 5 is satisfied). This implies that the matching function $m(i)$ is increasing.*

3.3 Equilibrium shares and utilities

We now characterize the solutions to the ODEs, (9) and (10), that govern the evolution of equilibrium utilities and revenue shares. Initial conditions, i.e., the utility given to the lowest productivity A supplier, will always be zero (the outside option of suppliers). This is because B suppliers have all the bargaining power and they only provide higher than zero utility to more productive suppliers. The purpose of this Section is to derive utilities and revenue shares endogenously and analyze how they affect firm boundaries.

The solution to (9), using $u^I(\underline{i}) = 0$, is given by

$$u^I(i) = P \int_{\underline{i}}^{i} \frac{\partial z(\tau, m(\tau))}{\partial \tau} d\tau. \tag{11}$$

Equation (10) is more complicated. According to the Picard-Lindelöf Theorem (e.g., Birkhoff and Rota (1989)) a unique solution exists and is given by (implicitly)

$$s(i) = \int_{\underline{i}}^{i} \frac{ds(\tau)}{d\tau} d\tau, \tag{12}$$

with $s(\underline{i}) = 0$ so that $u^I(\underline{i}) = 0$.[15]

First, we assume that $P \leq 1$. All firms choose to be nonintegrated and revenue shares satisfy (12). The evolution of supplier A's utility can then be derived using $u^N(i)$ (see (4)).

Let's now turn to the $P > 1$ case. Following Proposition 2 firms integrate for low and high $u(i)$'s. Under integration, the lowest u is $u^I(\underline{i}) = 0$, while $u_L(\underline{i}) > 0$. As i increases $u^I(i)$ increases according to (11), that is $\frac{du^I(i)}{di} = P\frac{\partial z}{\partial i}$. The threshold $u_L(i)$ also increases, $\frac{du_L(i)}{di} = \frac{P-1}{2}\left(\frac{\partial z}{\partial i} + \frac{\partial z}{\partial j}\frac{dm}{di}\right)$. If the contribution of j suppliers to total firm productivity, adjusted by the slope of the matching function which reflects the relative scarcity of skills, is not greater than that of i suppliers, $\frac{\partial z}{\partial j}\frac{dm}{di} \leq \frac{\partial z}{\partial i}$, then,

[15] We need $\frac{ds(i)}{di}$ to be Lipschitz continuous in s and continuous in i. Our assumptions ensure that $\frac{ds(i)}{di}$ is continuous in i, because i enters $\frac{ds(i)}{di}$ through $z(i, j)$. The term s enters $\frac{ds(i)}{di}$ linearly in the numerator and denominator. If a differentiable function has a derivative that is bounded everywhere by a real number, then the function is Lipschitz continuous. This clearly holds here. In Section 3.4.1 we offer an example where we solve the ODE numerically.

and given that $P > 1$, $u^I(i)$ is guaranteed to increase faster than $u_L(i)$ (we offer a discussion on this assumption at the end of this Section). Hence, for some unique i, which we denote i_1, $u^I(i_1) = u_L(i_1)$. Beyond i_1 the optimal design will switch to nonintegration. At i_1, by construction, $s(i_1) = \frac{P-1}{2P}$. This is the initial condition for the ODE that governs the evolution of the revenue share, see (10), when nonintegration is the optimal design and $P > 1$. The next threshold, i_2, satisfies

$$\frac{P-1}{2P} + \int_{i_1}^{i_2} \frac{ds(\tau)}{d\tau} d\tau = \frac{P+1}{2P}.$$

When i exceeds i_2 firms are better off switching back to integration. To sum up, for $i \in \mathcal{I}$ firms integrate and for $i \in \mathcal{N}$ they remain separate, where $\mathcal{I} = [\underline{i}, i_1) \cup (i_2, \bar{i}]$ and $\mathcal{N} = [i_1, i_2]$.

When $P > 1$ the form of the organization depends on where the highest productivity supplier $\bar{i}$ lies relative to the two thresholds i_1 and i_2. This depends on how fast the utility $u(i)$ increases which in turn depends on how sensitive $z(i, j)$ is to changes in i, that is how large $\frac{\partial z(i,j)}{\partial i}$ is, the level of productivity $z(i, j)$ and the 'dispersion' of the $[\underline{i}, \bar{i}]$ interval. If marginal productivity of i suppliers is strong (holding total productivity fixed), then the benefit of a j supplier is higher if it contracts with a more productive i supplier. This bids up the revenue shares of i suppliers. However, the opposite happens when the level of productivity increases (holding marginal productivity fixed), since j suppliers now benefit relatively less by matching with a more productive i supplier. This is because j supplier's base profit is higher which lowers their incremental profit they would realize with a more productive supplier. This discussion highlights the role of productivity and that we should pay careful attention to where productivity gains are coming from. If $\bar{i} < i_1$, then all enterprises are integrated. If $i_1 < \bar{i} < i_2$, then low productivity firms are integrated but high productivity firm are nonintegrated. Finally, if $\bar{i} > i_2$ then there is a non-monotonic relationship between firm productivity and organizational forms: low and high productivity firms integrate, while intermediate productivity firm remain nonintegrated.

If $\frac{\partial z}{\partial j}\frac{dm}{di}$ is much higher than $\frac{\partial z}{\partial i}$, then there does not exist an i_1 such that beyond it firms remain nonintegrated. In this case all firms integrate when $P > 1$, exactly as in Legros and Newman (2013) despite the presence of productivity heterogeneity. This can happen either when i suppliers' contribution to total firm productivity, relative to that of j suppliers, is small, and/or when productivity dispersion among i suppliers, relative to that of j suppliers, is small, in which case the slope of the matching function is high. Since this case does not add any new insights, in the rest of the paper we assume that $\frac{\partial z}{\partial j}\frac{dm}{di} \leq \frac{\partial z}{\partial i}$.

3.4 Organizational choices

The next Proposition summarizes the main findings from the previous section.

Proposition 7 (Organization structure in the market) *Depending on the market price P there are two distinct organizational forms in equilibrium:*

1. *Pure nonintegration, $P \leq 1$: All enterprises are nonintegrated.*

2. *Mixed, $P > 1$:*

   - *There can be a co-existence of integrated and nonintegrated enterprises.*
   - *Low productivity enterprises are always integrated.*
   - *The equilibrium organizational form can be non-monotonic in enterprise productivity, where low and high productivity enterprises remain integrated, whereas intermediate productivity enterprises do not integrate.*

The intuition behind the non-monotonic organizational structure is as follows. Productivity heterogeneity and endogenous matching yields a stable equilibrium where higher productivity input A suppliers command a higher utility. Endogenously determined shares, under nonintegration, range from $\frac{P-1}{2P}$ to $\frac{P+1}{2P}$. For shares that are around $\frac{1}{2}$ total profit under nonintegration is maximized, since that share minimizes the inefficiencies associated with nonintegration. Following this logic, for shares that are farther away from $\frac{1}{2}$, either below or above, integration is optimal. If the contribution of the productivity of A suppliers to the enterprise productivity, $\frac{\partial z(i,j)}{\partial i}$, is quite important and A suppliers are sufficiently vertically heterogeneous (which depends on the 'dispersion' of the $[\underline{i}, \bar{i}]$ interval), then B suppliers will compete more aggressively for higher productivity A suppliers and they will bid up their utilities and shares. It is then more likely that we will observe the non-monotonic organizational structure in the market.

Although B suppliers have all the bargaining power in our model, competition among B suppliers for A suppliers determines endogenously each A supplier's threat point in the bargaining problem. Higher productivity A suppliers have more 'bargaining power' in the sense of a higher threat point. Grossman and Helpman (2002) also demonstrate the role of (exogenously given) bargaining power in industry organization. Albeit their model is very different from ours and there is no heterogeneity, outsourcing (which is equivalent to nonintegration) becomes optimal for intermediate values of the bargaining power parameter of the intermediate-good producer.

There are two effects that affect total output for a given enterprise: an *influence effect* and a *sorting effect*. Integration increases firm output, holding firm efficiencies fixed (influence effect). However, integration is endogenously chosen and affected by firm productivities through matching (sorting effect).[16] An interesting result is that the sorting effect is non monotonic in productivity

[16] Sorensen (2007) empirically disentangled these two effects for the VC market. Higher experience VC firms fund higher quality projects but also add more value to the project. He showed that the sorting effect is twice as important

since more productive firms do not always integrate. An implication is that higher firm productivity can lower output if as a result of the higher productivity the firm chooses to disintegrate. For this comparative static we need to assume that the change in a firm's productivity has no effect on market price and the equilibrium share function. This is a reasonable assumption, given that each firm is atomless in our model. Later, we examine the effect of market-wide productivity changes that also affect market equilibrium variables. Hortacsu and Syverson (2007) argue that for the cement and ready-mixed concrete industries the evidence favors the sorting effect (but their paper does not identify these effects, nor does it attempt to empirically estimate them).

3.4.1 Numerical examples

We present two numerical examples where in the first one organizational design is non-monotonic in firm productivity, while in the second one high productivity firms do not integrate. We have used the Maple software to solve the ODE that governs the evolution of the equilibrium revenue share as given by (10) numerically.

We assume that both suppliers, i and j, are distributed in $[0, 4]$ with the same densities. The market price is $P = 2$. Since the densities are the same, the matching function $m(i)$ has slope one (see (8)) and in equilibrium $i = j$. Further, we assume $z(i, j) = 1 + ij$. The interval for the share in which nonintegration dominates is $\left[\frac{P-1}{2P}, \frac{P+1}{2P}\right] = [0.25, 0.75]$. The integration set is $\mathcal{I} = [0, 0.91287) \cup (3.562, 4]$ and the nonintegration set is $\mathcal{N} = [0.91287, 3.562]$. Only firms with intermediate productivity remain nonintegrated. Figure 2 depicts the equilibrium revenue share as a function of supplier i productivity index in the nonintegration region.[17]

Now we modify the above example by assuming that i and j are distributed in $[0, 2]$. In other words, we reduce the dispersion of vertical heterogeneity for both sides. All other assumptions are the same as in the above example. The integration set is $\mathcal{I} = [0, 0.91287)$ and the nonintegration set is $\mathcal{N} = [0.91287, 2]$. High productivity firms do not integrate.

3.5 The organizationally augmented industry supply

The industry supply function is given by

$$S(P) = \begin{cases} Q^N(P), & \text{if } P \leq 1 \\ Q^N(P) + Q^I(P), & \text{for } P > 1, \end{cases}$$

where $Q^N(P)$ and $Q^I(P)$ are given by (3) and (7) respectively. From the analysis in Section 3.3 we know that when $P \leq 1$ the sets of A suppliers that remain nonintegrated and integrated are

as the influence effect. However, in his model, both effects are monotonic in the underlying 'qualities' of the VCs and entrepreneurs.

[17] The conditions of Lemma 5 are satisfied by this example.

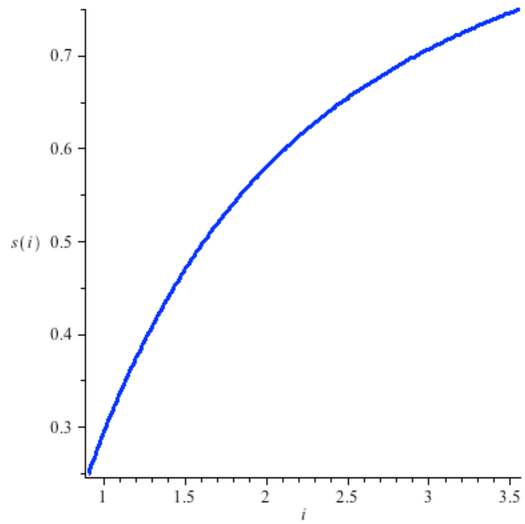

Figure 2: Equilibrium share $s(i)$ for the nonintegrated firms when the market price is $P = 2$. The productivity index i is in $[0.91287, 3.562]$ and $s(i) \in [0.25, 0.75]$. For i's outside this interval, below and above, firms integrate.

$\mathcal{N} = [\underline{i}, \bar{i}]$ and $\mathcal{I} = \emptyset$ respectively. When $P > 1$ the sets are $\mathcal{I} = [\underline{i}, i_1) \cup (i_2, \bar{i}]$ and $\mathcal{N} = [i_1, i_2]$. Therefore, the industry supply can be expressed as follows

$$S(P) = \begin{cases} \int_{\underline{i}}^{\bar{i}} \left(z(i, m(i)) - \frac{1}{(1+P)^2} \right) dF(i), & \text{for } P \leq 1 \\ \int_{\underline{i}}^{i_1} z(i, m(i)) dF(i) + \int_{i_1}^{i_2} \left(z(i, m(i)) - \frac{1}{(1+P)^2} \right) dF(i) + \int_{i_2}^{\bar{i}} z(i, m(i)) dF(i), & \text{for } P > 1. \end{cases}$$

The next task is to characterize $S(P)$. It is clear that when $P \leq 1$, $S(P)$ is an increasing function of P, because the limits of integration do not depend on P. When $P > 1$ an increase in P *also* affects the limits of integration because it affects firms decisions about whether to stay separate or integrate. The sets $\mathcal{N}$ and $\mathcal{I}$ are functions of P through the utilities and the revenue shares. To this end, we need to know how P impacts the equilibrium share. The next Lemma states the result.

Lemma 8 (Effect of market price on equilibrium revenue share) *As the market price increases, the equilibrium share under nonintegration decreases,* $\frac{ds(i)}{dP} < 0$.

The intuition is as follows. Market price and productivity are substitutes, in the sense that supplier j is willing to pay less (in terms of a share offered to i) for a higher, than his assigned in equilibrium, supplier i when the market price increases. In other words, a higher market price mitigates the competition among j suppliers for high productivity i suppliers.

There are two effects of a change in price P on the firms' incentives to choose a specific form of organization: i) *a market price effect* and ii) *an equilibrium revenue share effect*. A higher P makes integration more appealing for similar reasons outlined in Legros and Newman (forthcoming). The foregone output when suppliers are separate is more valuable as P increases, which induces the suppliers to integrate. This is the market price effect. The second effect arises because in our endogenous matching framework equilibrium shares depend (among other things) on market price. As we discussed in Lemma 8, a higher P lowers the equilibrium share for all i and as a result for the marginal i who is indifferent between integrating and remaining as separate units. Recall from Proposition 2 and Figure 1 that the highest surplus under nonintegration is when the share is as close as possible to $\frac{1}{2}$. The high productivity firms that are integrated, i.e., those in $[i_2, \bar{i}]$, offer shares higher than $\frac{1}{2}$. A lower share means a more efficient firm under nonintegration. So, for those firms the two effects work in opposite directions and the net effect depends on which effect dominates. For the low productivity firms, i.e., those in $[\underline{i}, i_1]$, only the price effect is present.

If the revenue share effect is dominant, then it is possible that a higher price leads to fewer integrated firms. Because integrated firms produce more output, it is then possible that higher price lowers the quantity supplied (*backward-bending supply curve*). The numerical examples below illustrate this possibility.

We continue the example we introduced in Section 3.4.1. We assume that both i and j are in the interval $[0, 11]$. First, we assume that $P = 100$ (high price). The set of nonintegrated firms is $\mathcal{N} = [9.9252, 10.1278]$. Now assume $P = 101$. The new set of nonintegrated firms is $\mathcal{N} = [9.97546, 10.17694]$. More low productivity firms integrate, $\frac{di_1}{dP} > 0$, but fewer high productivity firms find integration profitable, $\frac{di_2}{dP} > 0$. Moreover, we have assumed nothing about the densities of the two suppliers, beyond that they are the same. We can now choose densities that put more weight on high productivity firms. The result would be that, overall, fewer firms integrate when the price increases. Since nonintegrated firms produce less output, higher price may imply less output supplied. To illustrate this possibility, let's assume upward sloping densities, that is, $f(i) = i^a$ and $g(j) = j^a$, with $a = 20$. When $P = 100$, 6.1% of the firms in the market are nonintegrated, but when the price is $P = 101$ this fraction increases to 6.7%. The expected aggregate output when $P = 100$ is $S(100) = 3.928414046(10^{22})$, while when the price increases to $P = 101$ expected output decreases to $S(101) = 3.928414030(10^{22})$.

Now assume that $P = 2$ (low price). The nonintegration set is $\mathcal{N} = [0.91287, 3.562]$ (see Section 3.4.1). When $P = 2.4$ the nonintegration set becomes $\mathcal{N} = [1.092757, 3.23]$. More firms–low and high productivity–integrate. Industry supply is upward sloping.

The next Proposition summarizes.

Proposition 9 (Organizationally augmented industry supply) *For low prices, $P \leq 1$, the organizationally augmented industry supply $S(P)$ is increasing in P. A change in market price does* *not change the organizational structure. For $P > 1$, as the price increases more low productivity firms integrate, but it is possible that fewer high productivity firms integrate (especially when the market price is high). Overall, a higher price can induce less integration. In that case, it is possible that $S(P)$ is backward-bending: an increase in P can result in less output supplied.*

To summarize, for the high productivity firms there are two opposing effects (the market price and the revenue share effects) on their incentives to integrate as market price increases. The revenue share effect, even when it does not dominate the market price effect, slows down the rate of integration when price increases. If the market price is high enough (for example, the intensity of competition in the market is low), then it is more likely that the revenue share effect dominates the market price effect and fewer high productivity firms integrate. If on top of that there are more high productivity, relative to low productivity, firms in the market, then higher price increases the fraction of nonintegrated firms. Finally, if this fraction increases significantly, then total output decreases in response to a higher price.

3.6 Market-wide productivity gains and firm boundaries

We examine how firm productivity affects a firm's decision to integrate. Productivity appears in two forms in our model: overall productivity level $z(i,j)$ and marginal productivity of supplier i, $\frac{\partial z(i,j)}{\partial i}$. Suppose first that the productivity level increases for all firms, holding marginal productivity of i suppliers constant. Here, it may very well be the case that it is the marginal productivity of B suppliers that has increased, or the synergies between the two sides. Then, from $u^I(i_1) = u_L(i_1)$, i_1 will increase (because $u_L(i)$ increases, while $u^I(i)$ does not change). That is, more low productivity firms will choose to integrate. From (10) it follows that the revenue share function decreases. The intuition is that a higher productivity level lowers a j supplier's incremental profit from a more productive i supplier, which mitigates competition for i suppliers. The implication is that i_2 increases, i.e., more high productivity firms disintegrate, assuming that $\bar{i} > i_2$ to begin with.[18] Since this result is independent of the distribution of productivities, $F(\cdot)$ and $G(\cdot)$, we can choose distributions that assign more mass to high productivity firms, so that the mass of the firms that disintegrate is higher than the mass of firms that integrate. As a result, higher productivity can lead to less integration overall. Of course, if the mass of more productive firms is not higher than the mass of low productivity firms, higher productivity leads to more integration.

If the marginal productivity of i suppliers increases, holding the overall productivity level fixed, then revenue shares will increase. Competition among j suppliers for productive i suppliers intensifies. This follows directly from (10). Since revenue shares increase, more high productivity firms

[18] If $i_1 < \bar{i} < i_2$, then higher productivity leads to more integration and if $\bar{i} < i_1$ higher productivity has no effect on integration (all firms are integrated and they remain so after the productivity increase).

will integrate. On the other hand, and because $u^I(i)$ increases, but not $u_L(i)$, fewer low productivity firms integrate. This is because the utility of i suppliers increases faster and the threshold for nonintegration is reached faster, i.e., lower i_1.

Intuitively, if productivity gains do not come from the A suppliers, then B suppliers have weaker incentives to compete for high productivity A suppliers. Lower competition implies lower revenue share given to A suppliers. But for the high productivity firms, who already offer a high share, the share reduction moves them closer to the center of the Pareto frontier and the inefficiency under nonintegration is mitigated. Therefore, some high productivity firms find now nonintegration more appealing. We summarize in the Proposition below.

Proposition 10 (Relationship between productivity and integration) *Higher market-wide productivity can induce more or less integration in the market, depending on where the productivity gains are coming from. If higher productivity is not associated with higher marginal productivity for the A suppliers (the ones who receive the offers), then fewer high productivity firms integrate, but more low productivity firms integrate. If, on the other hand, higher productivity comes from stronger marginal productivity of A suppliers, then more high productivity firms integrate, but fewer low productivity firms integrate. Overall, productivity gains can imply less integration.*

The above Proposition highlights the role of productivity gains on firms' decisions to integrate and why we ought to pay close attention to where these gains are coming from.

The impact of a productivity shock that affects a subset of firms will be felt in the entire market. To illustrate the main idea, let's assume that the firms in $[i_1, \tilde{i}]$, with $\tilde{i} < i_2$, experience an increase of the marginal productivity of A suppliers (holding total productivity fixed). From (10) it follows that the revenue shares for the affected firms increase. This will force the unaffected firms, that is those in $[\tilde{i}, i_2]$, to also increase their revenue shares. Efficiency for all the nonintegrated enterprises will be affected, but also, following from Proposition 10, some of the unaffected by the shock firms will switch to integration (those in the neighborhood of i_2).

4 Discussion and empirical implications

The most important results can be summarized as follows:

1. For low market prices all firms are nonintegrated.

2. For high market prices the organizational design can be mixed and non-monotonic with respect to firm productivity.

- Low productivity firms are always integrated.
- If low and high productivity firms are integrated, then it must be that intermediate productivity firms are nonintegrated. Hence, there can be a non-monotonic relationship between productivity and integration.

3. The relationship between market price, integration decisions and expected output is as follows.

   - Market price impacts low and high productivity firms differently. A higher market price always increases the low productivity firms' incentives to integrate, but not always the high productivity firms' incentives.
   - When the price is low (highly competitive markets) a higher market price induces more firms overall to integrate and increases expected output.
   - For a high market price (less competitive markets), however, a price increase can result in fewer high productivity firms choosing to integrate. This can lead to less overall integration and in less output being produced.

4. If a single firm becomes more productive, then its output increases for a given organizational design (influence effect), but its organizational design can also change (sorting effect).

   - Owning to the non monotonic relationship between productivity and integration decisions, higher productivity for a firm can lead to lower output if the firm switches from integration to nonintegration (holding market price fixed).

5. If all firms in a market become more productive, then, there may be less integration in the market and less output (holding market price fixed).

6. Productivity shocks that affect a subset of firms will be felt in the entire market (holding market price fixed).

Result 1 and part of 3 confirm the findings in Legros and Newman (2013), although in our model the change in the fraction of firms that choose integration when price changes is continuous due to the productivity heterogeneity we have assumed. Results 2, 3, 4 and 5 are new. Result 6 echoes the findings in Legros and Newman (2013), where they show that a firm that undergoes a re-organization need not have experienced any change in technology. Our mechanism, however, behind this result is very different.

The non-monotonicity result is more likely to occur when the one side, suppliers B in our model, competes intensively to attract higher productivity suppliers from the other side, A suppliers. This competition bids up the equilibrium share and is responsible for the non-monotonic result (see the intuition after Proposition 7). Such competition is more intense the stronger the contribution of A

suppliers productivity is to the overall productivity of the enterprise (as captured by $\frac{\partial z(i,j)}{\partial i}$) and/or the more vertically heterogeneous the A suppliers are (as captured by the dispersion of the interval $[\underline{i}, \bar{i}]$). Hortacsu and Syverson (2007) show that, in the cement and ready-mix concrete industries, more productive firms are more likely to be integrated, but some high productivity firms remain nonintegrated. Our non-monotonicity result can offer an explanation for this finding. Despite the fact that in our equilibrium low productivity firms integrate, since high productivity firms also integrate, on average integration can be associated with higher productivity (if for example there are more high productivity firms and/or the high productivities are sufficiently high). However, because firms of intermediate productivities do not integrate, we can observe nonintegrated firms that have productivity higher than the average productivity of integrated firms.

Following Proposition 9, a higher market price can lead to more or less integration. If it leads to less integration, then it is possible that the aggregate supply is backward-bending. This generates the following three cases: i) backward-bending supply and less integration as price increases, provided we are on the downward sloping part of the aggregate supply, ii) upward sloping supply and less integration as price increases, starting from a high price level and iii) upward sloping supply and more integration as price increases.

Empirical evidence about the relationship between price and the incidence of integration is mixed: positive correlation (Hastings, 2004; Alfaro et al. 2013), or negative (Hortacsu and Syverson, 2007). We will use the taxonomy outlined above to analyze the empirical evidence from a new angle. If we are on the downward sloping part of the industry supply curve (when the supply curve is backward-bending), a lower price is associated with more integration and higher output (see Figure 3). This is case i) in the taxonomy above. Hence, we can have two markets with identical productivity distributions where due to (say) a weaker demand in one market we observe: i) a lower price, ii) higher output and iii) more integration than the other market (negative correlation between price and integration). This result is consistent with the findings in Hortacsu and Syverson (2007) and our model offers an alternative explanation to the one in Legros and Newman (2013). Legros and Newman generate an outcome with a lower price, higher output and more integration by assuming two levels of productivities (high and low) and then increasing the measure of the more productive firms. This leads to more integration (since in their model more productive firms are more likely to integrate; which would also be the case in our model had we not endogenized the revenue shares) and stronger supply which results in higher output and lower price when we move on a downward sloping demand curve. In contrast, we hold the productivities fixed and we work with a fixed supply curve.

But even if the aggregate supply is not backward-bending, a higher price can be associated with less integration: i) higher price, ii) higher output and iii) less integration (negative correlation

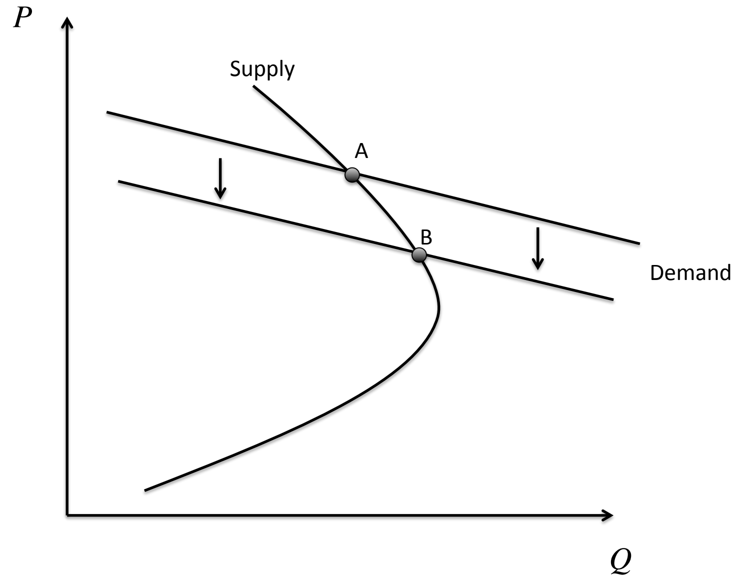

Figure 3: At point B, relative to point A, price is lower, quantity is higher and there are more integrated firms.

between price and integration). This is case ii) in the taxonomy above. In all other cases (case iii) in the taxonomy above), we have: i) higher price, ii) higher output and iii) more integration (positive correlation between price and integration).

We offer a theoretical basis for empirical work that would attempt to estimate to what extent it is the integration that is responsible for higher output, or more productive firms have a different probability of integrating to begin with (the influence and sorting effects).

We can have an equilibrium where only low productivity firms integrate, see also the numerical example of Section 3.4.1. Higher productivity firms choose to remain separate and it may very well be the case that their output is lower, since nonintegration leads to less output.[19] Thus, output can be positively correlated with integration. If, then, output is used as a proxy for productivity, one may wrongly conclude that less productive firms are more likely to remain nonintegrated, when it is exactly the opposite.

5 Concluding remarks

We analyze the effect of productivity heterogeneity in a model with endogenous integration (vertical or lateral) decisions, incomplete contracts and a perfectly competitive product market. Successful production requires two inputs that are provided by two input suppliers A and B. When the two firms are separate, decisions in each firm are made by a manager who cares about firm profits

[19]For this to happen it must be that the productivity increase in i is not very strong so that it does not dominate the reduction in efficiency due to nonintegration.

but also about his private benefits. Under nonintegration, a contract is employed to govern the relationship between the two input suppliers. In particular, the contact specifies a share of the total revenue each manager receives. The higher the share the higher the weight a manager puts on revenue relative to private benefits, yielding a more efficient outcome for the organization. But a higher share for one manager implies a lower share for the other. So, intermediate revenue shares is what maximizes the available surplus. When the two suppliers integrate they transfer the rights to make decisions to a third party, who then maximizes the organization's revenue but completely ignores private benefits. Neither organizational form is always superior. For intermediate revenue shares nonintegration dominates.

Next, we endogenize revenue shares. Input suppliers on each side are vertically heterogeneous with respect to productivity. One input A supplier matches with one input B supplier. We assume B suppliers have all the bargaining power and make take-it-or-leave-it offers taking into account the endogenously determined utility of A suppliers. We show that the stable matching is positive assortative. The endogeneously determined utility of A suppliers is an increasing function of their productivity. This implies that the revenue share under nonintegration is also increasing. Because matching is positive assortative low productivity firms are associated with low supplier A utilities, while for high productivity firms the utilities of A suppliers are also high. Since intermediate shares (and utilities) favor nonintergation, we can obtain an equilibrium where integration is non-monotonic in firm productivities. Low and high productivity firms integrate, but intermediate productivity firms remain separate.

Having established the properties of a stable match and the equilibrium revenue shares and utilities, we then turn to their interaction with the market price and to comparative statics with respect to a single firm productivity change and market-wide productivity changes. A higher market price, say due to a stronger demand, makes integration more profitable holding revenue shares fixed. But it also affects the shares. A higher market price decreases the equilibrium shares. There are two opposing effects on high productivity firms' incentives to integrate. If the share effect dominates the price effect, then a higher market price can induce more firms to disintegrate. Industry supply curve in this case can be backward-bending.

If a single firm becomes more productive then its output will increase holding the form of organization fixed. But a more productive firm may choose to disintegrate owning to the non-monotonic relationship between productivity and integration decisions. Since nonintegrated firms produce less output, higher productivity can reduce output. If all firms in a market become more productive, then we can have an equilibrium response with fewer or more integrated firms. Finally, a productivity shock that affects a subset of firms, will have an effect on all firms in the market. What is important for the direction of the change is where the productivity gains are coming from, the A or the B suppliers (who make the offers).

We now offer a discussion about the plausibility of our main modeling assumptions. First, we assume that neither organization structure is first-best. This is a widely accepted view in the organization literature (e.g., Hart and Holmstrom (2010) and Legros and Newman (2013)). Second, when firms are separate, we are in a NTU environment due to contract incompleteness, while under integration utility is fully transferable. These give rise to a concave Pareto frontier under nonintegration and a linear one under integration. Therefore, nonintegration becomes optimal when bargaining powers are balanced between the two firms (as Figure 1 illustrates). Up to this point, our assumptions are the same as in Legros and Newman (2013). Third, we have assumed that one side (B suppliers) has all the bargaining power. Equilibrium payoffs depend on bargaining powers and on the endogenously determined threat points. The results would not change if we gave all the bargaining power to A suppliers instead. They would also not change much qualitatively if both sides were given some strictly positive (fixed) bargaining power. All these imply that the side who makes the contract offers under nonintegration is willing to offer a higher utility to a higher productivity firm from the other side. This fact, in conjunction with the shape of the Pareto frontiers we mentioned above, suggests that the nonmonotonicity of integration incentives with respect to firm productivity is a natural consequence. These assumptions are responsible for the revenue share effect, which can work in opposite direction from the price effect. Finally, the exogenously given distribution of firm productivities determines the strength of the revenue share effect, which in turn influences the fraction of integrated firms as well as aggregate output, as the market price changes.

A Appendix: Proofs of Lemmas and Propositions

A.1 Proof of Proposition 2

We set $u^N(i)$, from (4), equal to u and solve for s. There are two solutions, we retain the one that is below one. (We omit the expression because it is long). We then substitute s into $v^N(j)$ to obtain a direct relationship (rather than indirectly through the share s) between v and u on the Pareto frontier. We then find the u's such that $v^N(j) = v^I(j)$, where $v^I(j)$ is given by (6). We obtain the two thresholds, $u_L(i)$ and $u_H(i)$, that are reported in Proposition 2. Once we substitute these two thresholds into the solution for s we described above we also derive two thresholds in terms of s: $s_L(i) \equiv \frac{P-1}{2P}$ and $s_H(i) \equiv \frac{P+1}{2P}$.

A.2 Proof of Lemma 3

The equilibrium utility of i supplier must evolve in such a way so that no j supplier, with $j = m(i)$, finds a local deviation in i profitable. We differentiate $v^I(j)$ with respect to i, we then set $j = m(i)$ and solve for $\frac{du^I(i)}{di}$. This yields (9). Given (9) a local no deviation implies global no deviation. This can be seen as follows. First, we compute the second derivative of $v^I(j)$ with respect to i (holding j fixed)

$$\frac{d^2v^I(j)}{di^2} = \frac{\partial^2 z(i,m(i))}{\partial i^2}P - \frac{d^2u^I(i)}{di^2}. \tag{A.13}$$

Second, and using (9), we calculate the second derivative of $u^I(i)$ (where now we allow j to vary with i according to $m(i)$)

$$\frac{d^2u^I(i)}{di^2} = \left(\frac{\partial^2 z(i,m(i))}{\partial i^2} + \frac{\partial^2 z(i,m(i))}{\partial i \partial j}\frac{dm(i)}{di}\right)P.$$

After we substitute the above into (A.13) we obtain

$$\frac{d^2v^I(j)}{di^2} = -\frac{\partial^2 z(i,m(i))}{\partial i \partial j}\frac{dm(i)}{di}P < 0.$$

A.3 Proof of Lemma 4

We assume the enterprise is nonintegrated. Let j be matched with i. We need to determine $s(i)$ so that input supplier j, with $j = m(i)$, has no incentive to match with any other input supplier A in the neighborhood of i. For such a local no deviation we need to set the derivative of the utility of j, $v^N(j)$ as given by (4), with respect to i equal to zero, holding j fixed. This yields

$$\frac{dv^N(j)}{di} = \frac{\partial z(i,j)}{\partial i}P(1-s(i)) - \left(z(i,j) - \frac{1}{(1+P)^2}\right)P\frac{ds(i)}{di} + 2(1-s(i))\left(\frac{P}{1+P}\right)^2\frac{ds(i)}{di} = 0.$$

We solve the above with respect to $\frac{ds(i)}{di}$, then set $j = m(i)$, and simplify to obtain the ODE given by (10).

A.4 Proof of Lemma 5

We take $s(i)$ as given–for simplicity here denoted s– and we differentiate the expected profit function of supplier j, $v^N(j)$, twice with respect to i. We will show that the expected profit function is strictly concave in i. This implies that a zero first order condition is sufficient for a maximum and the matching by construction is positive assortative.

$$\begin{aligned}
&\frac{d^2v^N(j)}{di^2} = \frac{\partial^2 z}{\partial i^2}P(1-s) - \frac{\partial z}{\partial i}\frac{ds}{di}P - \frac{\partial z}{\partial i}\frac{ds}{di}P - \left(z - \frac{1}{(1+P)^2}\right)P\frac{d^2s}{di^2} - 2\left(\frac{ds}{di}\right)^2\left(\frac{P}{1+P}\right)^2 \\
&+2(1-s)\left(\frac{P}{1+P}\right)^2\frac{d^2s}{di^2} = \\
&\frac{\partial^2 z}{\partial i^2}P(1-s) - 2\frac{\partial z}{\partial i}\frac{ds}{di}P - 2\left(\frac{ds}{di}\right)^2\left(\frac{P}{1+P}\right)^2 \\
&-\left(\left(z - \frac{1}{(1+P)^2}\right)P - 2(1-s)\left(\frac{P}{1+P}\right)^2\right)\frac{d^2s}{di^2} = \\
&\frac{\partial^2 z}{\partial i^2}P(1-s) - 2\frac{ds}{di}\left(\frac{\partial z}{\partial i}P + \frac{ds}{di}\left(\frac{P}{1+P}\right)^2\right) - \left(z - \frac{1+2P(1-s)}{(1+P)^2}\right)P\frac{d^2s}{di^2}.
\end{aligned}$$

We substitute $\frac{ds}{di}$, from (10), into the second derivative above and for convenience we let

$$A \equiv z - \frac{1+2P(1-s)}{(1+P)^2} > 0.$$

This yields

$$\frac{d^2v^N(j)}{di^2} = \frac{\partial^2 z}{\partial i^2}P(1-s) - \frac{2(1-s)\left(\frac{\partial z}{\partial i}\right)^2 P\left(1 + \frac{(1-s)P}{(1+P)^2A}\right)}{A} - AP\frac{d^2s}{di^2}. \tag{A.14}$$

Next, we compute $\frac{d^2 s(i)}{di^2}$ by differentiating (10) with respect to i.

$$\frac{d^2 s}{di^2} = \frac{\left(\left(\frac{\partial^2 z}{\partial i^2} + \frac{\partial^2 z}{\partial i \partial j}\frac{dm(i)}{di}\right)(1-s) - \frac{\partial z}{\partial i}\frac{ds}{di}\right)(A) - \frac{\partial z}{\partial i}(1-s)\left(\frac{\partial z}{\partial i} + \frac{\partial z}{\partial j}\frac{dm(i)}{di} + \frac{2P}{(1+P)^2}\frac{ds}{di}\right)}{(A)^2} =$$

using $\dfrac{ds}{di}$ from (10)

$$\frac{\frac{\partial^2 z}{\partial i^2}(1-s)(A) + (1-s)\frac{dm}{di}\left(\frac{\partial^2 z}{\partial i \partial j}A - \frac{\partial z}{\partial i}\frac{\partial z}{\partial j}\right) - 2(1-s)\left(\frac{\partial z}{\partial i}\right)^2 - \frac{2(1-s)^2 P\left(\frac{\partial z}{\partial i}\right)^2}{(1+P)^2 A}}{(A)^2} =$$

$$\frac{\frac{\partial^2 z}{\partial i^2}(1-s)(A) + (1-s)\frac{dm}{di}\left(\frac{\partial^2 z}{\partial i \partial j}A - \frac{\partial z}{\partial i}\frac{\partial z}{\partial j}\right) - 2(1-s)\left(\frac{\partial z}{\partial i}\right)^2\left(1 + \frac{(1-s)P}{(1+P)^2 A}\right)}{(A)^2} =$$

$$\frac{(1-s)\frac{\partial^2 z}{\partial i^2}}{A} + \frac{(1-s)\frac{dm}{di}\left(\frac{\partial^2 z}{\partial i \partial j}A - \frac{\partial z}{\partial i}\frac{\partial z}{\partial j}\right)}{A^2} - \frac{2(1-s)\left(\frac{\partial z}{\partial i}\right)^2\left(1 + \frac{(1-s)P}{(1+P)^2 A}\right)}{A^2}.$$

Now we substitute $\frac{d^2 s(i)}{di^2}$ into (A.14). After simplifications we obtain where

$$\frac{d^2 v(j)}{di^2} = -\frac{(1-s)P\frac{dm}{di}\left(\frac{\partial^2 z}{\partial i \partial j}A - \frac{\partial z}{\partial i}\frac{\partial z}{\partial j}\right)}{A}.$$

Therefore, and given that $\frac{dm}{di} > 0$ due to PAM, $\frac{d^2 v(j)}{di^2} < 0$ if and only if

$$\frac{\partial^2 z}{\partial i \partial j}A > \frac{\partial z}{\partial i}\frac{\partial z}{\partial j}.$$

A sufficient condition is derived when we set $s = 0$ in which case A is the lowest, given the other exogenous parameters. This yields the following sufficient condition

$$\frac{\partial^2 z}{\partial i \partial j}\left(z - \frac{1+2P}{(1+P)^2}\right) > \frac{\partial z}{\partial i}\frac{\partial z}{\partial j}.$$

A.5 Proof of Lemma 8

We would like to determine $\frac{ds(i)}{dP}$ for any i. The revenue share under nonintegration must (implicitly) satisfy

$$s(i) = \frac{P-1}{2P} + \int_{i_1}^{i} \frac{ds(\tau)}{d\tau}d\tau,$$

where the integrand is given by (10).

Using Leibniz's rule we differentiate both sides with respect to P

$$\frac{ds(i)}{dP} = \frac{1}{2P^2} - \frac{ds(i_1)}{di}\frac{di_1}{dP} + \int_{i_1}^{i} \frac{d}{dP}\left\{\frac{ds(\tau)}{d\tau}\right\}d\tau.$$

Next, we determine $\frac{di_1}{dP}$, which appears in the second term on the RHS. By totally differentiating $u^I(i_1) - u_L(i_1) = 0$, see Section 3.3 where this expression appears, which is used to determine i_1, we obtain

$$\frac{di_1}{dP} = \frac{-\int_{\underline{i}}^{i_1} \frac{\partial z(i,m(i))}{\partial i} di + \frac{z(i_1,m(i_1))}{2} - \frac{1}{2(P+1)^2}}{P\frac{\partial z(i_1,m(i_1))}{\partial i} - \frac{P-1}{2}\left(\frac{\partial z(i_1,m(i_1))}{\partial i} + \frac{\partial z(i_1,m(i_1))}{\partial j}\frac{dm(i_1)}{di}\right)} > 0. \tag{A.15}$$

The denominator is positive, since $P > 1$ and we have assumed that i suppliers contribute more to the total firm productivity than j suppliers. The numerator is positive. This can be seen as follows. For the region we are focusing, $u^I(i) \leq \frac{zP}{2} - \frac{1}{4}$, that is supplier i receives less than half of the available surplus under integration, see also Figure 1. Therefore, $u^I(i) = P\int \frac{\partial z}{\partial \tau} d\tau \leq \frac{zP}{2} - \frac{1}{4} \rightarrow \int \frac{\partial z}{\partial \tau} d\tau \leq \frac{z}{2} - \frac{1}{4P}$. Using the last inequality it follows easily that the numerator is always positive. So, more low productivity firms integrate when the market price increases.

Using (10), the fact that at i_1, $s(i_1) = \frac{P-1}{2P}$, and $\frac{di_1}{dP}$ from above, the effect of P on $s(i)$ can be written as

$$\begin{aligned}
\frac{ds(i)}{dP} &= \frac{1}{2P^2} - \left(\frac{\frac{\partial z}{\partial i}\frac{P+1}{2P}}{z - \frac{P+2}{(1+P)^2}}\right)\left(\frac{-\int_{\underline{i}}^{i_1} \frac{\partial z}{\partial i} di + \frac{z}{2} - \frac{1}{2(P+1)^2}}{\frac{\partial z}{\partial i}\frac{P+1}{2} - \frac{\partial z}{\partial j}\frac{dm}{di}\frac{P-1}{2}}\right) + \int_{i_1}^{i} \frac{d}{dP}\left\{\frac{ds(\tau)}{d\tau}\right\} d\tau \\
&= \frac{1}{2P^2} - \left(\frac{1}{P\left(z - \frac{P+2}{(1+P)^2}\right)}\right)\left(\frac{-\int_{\underline{i}}^{i_1} \frac{\partial z}{\partial i} di + \frac{z}{2} - \frac{1}{2(P+1)^2}}{1 - \frac{\frac{\partial z}{\partial j}(P-1)}{\frac{\partial z}{\partial i}(P+1)}}\right) + \int_{i_1}^{i} \frac{d}{dP}\left\{\frac{ds(\tau)}{d\tau}\right\} d\tau. \qquad \text{(A.16)}
\end{aligned}$$

The RHS of the above expression has three terms. Note that the z's in the second term are all evaluated at i_1.

The third term on the RHS can be expressed as follows

$$\begin{aligned}
&\int_{i_1}^{i} \frac{d}{dP}\left\{\frac{ds(\tau)}{d\tau}\right\} d\tau \\
&= \int_{i_1}^{i} \frac{-\frac{\partial z}{\partial \tau}\frac{ds}{dP}\left(z - \frac{1+2P(1-s)}{(1+P)^2}\right) - \left(\frac{\partial z}{\partial \tau}(1-s)\right)\left(\frac{2(P(1-s)+s)}{(1+P)^3} + 2\frac{P}{(1+P)^2}\frac{ds}{dP}\right)}{\left(z - \frac{1+2P(1-s)}{(1+P)^2}\right)^2} d\tau. \qquad \text{(A.17)}
\end{aligned}$$

We would like to show that the sum of the first two terms of (A.16) is always negative. We know that $u^I(i) = P\int_{\underline{i}}^{i} \frac{\partial z}{\partial \tau} d\tau$ and at $i = i_1$, $u^I(i_1) = u_L(i_1)$, where $u_L(i) \equiv \frac{(P-1)(2z(P+1)-1)}{4(1+P)}$ as given in Proposition 2. Therefore, the second term of (A.16) can be expressed as follows

$$\left(\frac{1}{P\left(z - \frac{P+2}{(1+P)^2}\right)}\right)\left(\frac{-\frac{(P-1)(2z(P+1)-1)}{4P(1+P)} + \frac{z}{2} - \frac{1}{2(P+1)^2}}{1 - \frac{\frac{\partial z}{\partial j}(P-1)}{\frac{\partial z}{\partial i}(P+1)}}\right).$$

Moreover, the above term is bounded from below by

$$\frac{-\frac{(P-1)(2z(P+1)-1)}{4P(1+P)}+\frac{z}{2}-\frac{1}{2(P+1)^2}}{P\left(z-\frac{P+2}{(1+P)^2}\right)}.$$

Using the lower bound, the first two terms of (A.16) can be expressed as follows

$$\frac{1}{2P^2}-\frac{-\frac{(P-1)(2z(P+1)-1)}{4P(1+P)}+\frac{z}{2}-\frac{1}{2(P+1)^2}}{P\left(z-\frac{P+2}{(1+P)^2}\right)}=-\frac{3+P^2}{4(z+2zP+zP^2-P-2)P^2}<0$$

for any $P>1$. This implies that the sum of the first two terms of (A.16) is always negative.

It now follows that $\frac{ds(i)}{dP}<0$. Suppose not. If $\frac{ds(i)}{dP}\geq 0$, then the third term of (A.16) is negative (see (A.17)). Given that the sum of the first two terms of (A.16) is also negative, we reach a contradiction.

A.6 Proof of Proposition 9

The effect of P on i_1 is given by (A.15). The threshold i_2 must satisfy

$$\frac{P-1}{2P}+\int_{i_1}^{i_2}\frac{ds(\tau)}{d\tau}d\tau=\frac{P+1}{2P}. \tag{A.18}$$

We differentiate both sides with respect to P

$$\begin{aligned}&\frac{1}{2P^2}-\frac{ds(i_1)}{di}\frac{di_1}{dP}+\int_{i_1}^{i_2}\frac{d}{dP}\left\{\frac{ds(\tau)}{d\tau}\right\}d\tau+\frac{ds(i_2)}{di}\frac{di_2}{dP}=-\frac{1}{2P^2}\Rightarrow\\ &\frac{1}{P^2}-\frac{ds(i_1)}{di}\frac{di_1}{dP}+\int_{i_1}^{i_2}\frac{d}{dP}\left\{\frac{ds(\tau)}{d\tau}\right\}d\tau+\frac{ds(i_2)}{di}\frac{di_2}{dP}=0.\end{aligned} \tag{A.19}$$

The first two terms of (A.19) are very similar to the first two terms of (A.16), except that now the first term is $\frac{1}{P^2}$ instead of $\frac{1}{2P^2}$. But the sign of $\frac{di_2}{dP}$ is in general indeterminate, due to the fact that the sign of the third term is indeterminate.

We have not been able to come up with clean conditions on parameters that would help us sign $\frac{di_2}{dP}$. Nevertheless, we were able to show that $\frac{di_2}{dP}$ can be either positive or negative. The LHS of (A.18) decreases as P increases, holding i_2 fixed (see Lemma 8). The RHS of (A.18) also decreases, but for a large P, the decrease is very small. Hence, to restore the equality, i_2 must increase, $\frac{di_2}{dP}>0$. A numerical example has confirmed this possibility (see Section 3.5). This implies that fewer high productivity firms integrate as the price increases, provided that the price is already high. When P is low, $\frac{di_2}{dP}<0$. This possibility has also been confirmed by a numerical example in Section 3.5.

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