Endogenous formation of coalitions in the presence of multilateral environmental externalities

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Abstract

In order to deal effectively with environmental problems, the countries form coalitions or sign agreements, such as international environmental agreements. In this paper we examine the conditions under which asymmetric countries that act rationally are willing to participate in such coalitions. In our analysis the coalition partners share equally the gains from their cooperation. We further assume that the resignation of a single country or of a group of countries from a coalition does not lead the cooperation to a complete breakdown. Furthermore, our analysis allows the existence of multiple noncrossing coalitions. We show that if the benefits from cooperation are high enough, then all the countries form a single coalition. However, the full cooperation may not always prevail, even when the welfare loss is positive.

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1 Introduction

In the last few decades we have witnessed countries forming coalitions and signing agreements to deal with environmental externalities that arise from the economic activity. These problems include

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the climate change or the conservation of the biological diversity and require some form of international coordination, as they cannot be tackled effectively when the countries act separately. The coordination between countries usually takes the form of international environmental agreements (IEAs). Since the international law does not recognize any legal entities with the power to enforce international agreements on sovereign countries, such agreements must provide enough incentives for the countries to join voluntarily.[1] The Kyoto Protocol and the Convention on Biological Diversity are examples of agreements of this type. In this paper we examine the incentives of the countries to sign such agreements as well as their implications on welfare.

By signing the Montreal Protocol on Substances that Deplete the Ozone Layer, its parties intended to protect the ozone layer by phasing out and ultimately eliminating the emissions of substances that deplete it. They also acknowledged that some countries might require a special provision of financial resources in order to develop the capacity to comply with the control measures. For this reason the signatories established a financial mechanism, funded by the developed or industrialized country members of the treaty, that directs aid flows to the developing country members. Further, the disbursement of resources is determined unanimously by a committee with equal representation of the contributing and the benefitting parties.[2].

Here we develop a model that incorporates the main features of the Montreal Protocol. In particular, we examine how asymmetric countries which act rationally may form coalitions, such as IEAs, in the presence of a transfer scheme.[3] We assume that each country has a project with environmental impact and if that project is funded, it generates a local benefit for that country and externalities for the other countries. The externalities can be positive, zero or even negative. China's project to eliminate the production of the ozone depleting substances by the year 2030 is potentially the largest project that funds the financial mechanism of the Montreal Protocol. This project generates positive externalities as, according to the Government of China, it is expected to prevent the emission of more than 4.3 million metric tons of HCFCs or 300,000 tons in terms of its ozone depletion potential.[4] The conversion of tropical forests to commercial land and infrastructure

[1] See Xepapadeas (1997) or Barrett (2005).

[2] According to the protocol, if all efforts at consensus have been exhausted and no agreement reached, decisions shall be adopted by a two-thirds majority vote of the parties present and voting.

[3] A country that acts rationally participates in a coalition when the benefits that it receives are non-negative.

[4] See *Multilateral Fund for the Implementation of the Montreal Protocol* (2013, April 22), "Multilateral Fund approves landmark project for China with ozone and climate benefits - up to US $385 million of funding over the

in Brazil and Indonesia,[5] generates negative externalities as it increases the concentration of carbon dioxide in the atmosphere.[6] A project may have positive externalities for some countries and negative externalities for some other countries. The Inga I and II hydropower dams in the Basin of the Congo River in the Democratic Republic of the Congo have positive externalities for most countries because they generate electricity without air quality impact. However, the dams generate negative externalities for Angola, as they pose a threat to the marine biodiversity in the lower Congo River.[7]

A country that has not signed such agreements decides to undertake its project by comparing the project's local benefit with its cost. Although the externalities of the project occur and affect the other countries, they are not taken into consideration. However, the countries that form coalitions take into consideration the intra-coalition externalities of their projects as well and, as in the Montreal Protocol, they decide unanimously on the projects that they will fund.[8] The undertaken projects may be different in the two cases. Furthermore, they also share the costs of the projects they fund. Empirical evidence supports that very often individuals propose and accept an equal split of the gains (see Ostrom (1998)). We follow the same rule: Each coalition chooses tax rates so as to divide the gains from the cooperation equally among its members.[9] Under this rule, some countries in the coalition might be subsidized. In the example mentioned above, China will receive up to US $385 million of funding until 2030.[10]

The main body of the IEAs literature concludes that when the difference between the non-cooperative and full cooperative benefits is large, the number of signatories of a self-enforcing IEA is small as the countries have high incentives to free ride. This result is verified in a variety of specifications: In static models with symmetric countries (Hoel (1992), Carraro and Siniscalco (1993), Barrett (1994)), asymmetric countries (Barrett (1997), Botteon and Carraro (1998)) or even

next 17 years" [Press release], Retrieved from http://www.multilateralfund.org/

[5]The cause of tropical deforestation is to create farmland for palm tree plantation in Indonesia and for soybean, sugar cane, maize production and pasture in Brazil.

[6]Each acre of tropical forest stores about 180 metric tons of carbon. Worldwide, the tropical forests hold 460-575 billion metric tons of carbon. See Urquhart et al. (1999).

[7]See Norlander, B. (2009, April 20), "Rough waters", *Science World*, Retrieved from http://www.scholastic.com/

[8]In our model unanimity is a result rather than an assumption. See the proof of Corollary 2.

[9]This assumption has been used in other settings as well. See for example Burbidge et al. (1997).

[10]It is also well known the US federal government subsidizes a number of the states and a similar pattern occurs in the EU.

when IEAs are examined as repeated games (Barrett (1994), Finus and Rundshagen (1998)). On the other hand, some authors examine the conditions under which self-financing transfer schemes may increase the number of signatories or even induce the full cooperation of the countries (Chander and Tulkens (1995 and 1997), Petrakis and Xepapadeas (1996), McGinty (2007), Eyckmans and Finus (2009), Colmer (2011)). Among others, Benedick (1998) argues that the increased participation in the Montreal Protocol (197 countries have ratified this treaty) was due to its transfer scheme.

In our approach, we apply the concept of the core. The use of the core theory is not new in the IEAs literature. Mäler (1989) uses the notion of the $\alpha - core$, originally proposed by Aumann (1967), and finds that (a special case of) this concept of the core results in a set of outcomes that is too large.[11] Chander and Tulkens (1995 and 1997) claim that there is no reason why the complementary coalition should behave in the war-like fashion of the $\alpha - core$ and they use the $\gamma - core$ instead.[12] Chander and Tulkens find that for a specific transfer scheme,[13] the full cooperation can prevail and Pareto efficiency can be achieved. In their model, the threat that sustains the grand coalition is the "Nash behavior" of the non-deviating countries.

Nevertheless, the resignation of a single country from an IEA or even of a group of countries from it, does not necessarily lead the cooperation to a complete breakdown, as the γ-game would imply. For instance, neither the failure of the US to ratify the Kyoto Protocol nor the withdrawal of Canada from it led the other parties to renounce the agreement, even though the US accounted for 36% of the controlled emissions (see Dessai (2001)). For this reason, we use the $\delta - core$. We assume that the deviating countries anticipate that those coalitions from which one or more members have left, still stick together.[14] In comparison to the $\gamma - core$, the conditions for the $\delta - core$ are much more restrictive.[15]

In such setup, if the benefits from the full cooperation are large enough, all the countries

[11] Aumann (1967) presumes that when a coalition deviates, the rest of the countries choose strategies that cause the highest damage possible to the deviating coalition. Mäler (1989) assumes that the deviating coalitions can be hurt by up to infinite amounts of pollutants emitted by the countries outside the coalition (hence, individual pollution levels are unbounded from above).

[12] In the $\gamma - game$, the deviating countries expect non deviating members to break-up into singletons and that every singleton adopts the Nash strategy.

[13] This transfer scheme is similar to the ratio equilibrium met in Kaneko (1977) and Mas-Colell and Silvester (1989).

[14] The γ and $\delta - games$ were originally proposed in Hart and Kurz (1983).

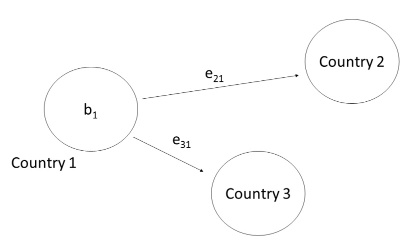
[15] See Appendix C.

participate in a single coalition. No country or group of countries has incentives to deviate from this coalition and free ride, because a number of positive externalities may then not be realized. This result is verified in an economy with n countries. Furthermore, for $n = 3$ we examine the conditions under which, given that the full cooperation of the countries cannot be supported, some countries might form a finer coalition. This occurs if the benefits from the formation of the finer coalition are positive. We show that the only case where the countries do not have incentives to cooperate is when no coalition can generate positive benefits. As mentioned above, we examine the welfare implications of these agreements. We show that it is possible to have welfare losses when countries fail to cooperate.

The rest of the paper is organised as follows: In Section 2 we describe the model. We focus on the coalition formation game in Section 3. In Section 4 we consider the welfare implications. Finally, we conclude in Section 5.

2 Preliminaries

We consider a set N of $1, 2, ..., n$ countries, each populated with citizens of mass 1. Each $i \in N$ has a discrete project that we call it project i, with positive cost c_i. As we want to be as general as possible, we assume that if the project i is funded it may generate a local benefit for country i and externalities of any magnitude for the other countries. We denote $b_i \in \Re$ the local benefit of the project i (b_i may be zero or even negative) and $e_{ji} \in \Re$ the externality of the project i to country j (e_{ji} also might be zero or even negative), $\forall j \in N$, with $e_{ii} = 0$. Information about local benefits, costs and externalities is common knowledge.



The project of the country 1 generates local benefit b_1 for the country 1 and

externalities e_{21}, e_{31} for the countries 2, 3 respectively.

We first examine country i when it is not associated with other countries. We assume that it acts rationally. Country i decides whether it should fund its project or not, by comparing the project's local benefit versus its cost. If it finds that it is profitable to fund it, it obtains the local benefit of project i and covers the cost by using its own resources. Also, country i is full exposed to the (either positive or negative) externalities that reach i and are generated by projects from the other countries or other coalitions of countries. As i does not contribute to their funding, it free rides on these externalities.

On the other hand, when the countries form coalitions, they coordinate on the projects they fund. To do that, they also take into consideration the intra-coalition externalities that these projects generate, i.e. the externalities that reach the members of the coalition from the projects the coalition funds. In such case, a project is funded if its total benefit, that includes its local benefit and the intra-coalition externalities, exceeds its cost. The set of the projects that are funded in the two cases may not be the same. A coalition may not fund a project that an unassociated country would fund and vice versa. Take for example a project i that has a high local benefit but generates negative externalities for the rest of the countries in the coalition. Even though the unassociated country i funds the project, the coalition may not, as it takes into account the negative impact of the project on the other members as well. In both cases, the countries fully free ride on the externalities that are generated from the projects that are funded by the other countries or coalitions.

By choosing to participate in a coalition, the countries also agree to share its surplus and expenses. The surplus is the excess benefit that the members generate by forming the coalition, over the benefit they receive when they are separate. The expenses are the costs of the projects that are funded by the coalition. No other costs are present, e.g. coalition formation costs or costs of entry. The members of the coalition share the expenses by using a predetermined rule that we specify below. Choices about coalition partners are taken simultaneously by all countries and each country can participate in only one coalition.

Formally, a coalition C is a nonempty subset of N, with $|C|$ the number of its members. Every single membered coalition is called a singleton. As it is possible to have many coalitions, we define a coalition structure π to be a partition on N with elements the various coalitions that may be formed,

$$\pi = \{C_1, C_2.., C_h\}.$$

We restrict our attention to coalition structures that are noncrossing and cover N fully. We denote by $\hat{\pi}$ the coalition structure in which the countries are not associated,

$$\hat{\pi} = \{\{1\}, \{2\}, .., \{n\}\}.$$

The coalition structure in which all $1, 2, ..., n$ countries participate in the same coalition, i.e. the grand coalition, is denoted by

$$G = \{1, 2, ..., n\},$$

with $|G| = n$.[16]

As a coalition C chooses which projects to fund in order to maximize the utility of its members, it may not undertake all the available projects. For this reason, we denote $F(\pi, C) \subseteq C$ the set of projects that are funded by C. Also, $F(\pi, N\backslash C) \subseteq N\backslash C$ is the set of projects that are funded by coalitions other than C, and $F(\pi)$ is the set of funded projects by every coalition in π, with $F(\pi) = F(\pi, C) \cup F(\pi, N\backslash C)$. $F(G)$ is the set of the funded projects of the grand coalition.

2.1 Utility Function

Under the coalition structure π, a country i which participates in the coalition C, obtains utility level equal to $u_i(\pi, C, t_i^C)$ as presented below. In the first case, the project i is funded by C while in the second it is not:

$$u_i(\pi, C, t_i^C) = \begin{cases} b_i + \sum\limits_{j \in F(\pi,C)} e_{ij} + \sum\limits_{j \in F(\pi,N\backslash C)} e_{ij} - t_i^C \times A \text{ if } i \in F(\pi, C), \\ \\ \sum\limits_{j \in F(\pi,C)} e_{ij} + \sum\limits_{j \in F(\pi,N\backslash C)} e_{ij} - t_i^C \times A \text{ if } i \notin F(\pi, C), \end{cases} \tag{1}$$

where b_i is the local benefit of project i and occurs only if the project is funded. The country i is fully exposed to both the externalities $\sum\limits_{j \in F(\pi,C)} e_{ij}$ that are generated by the projects the coalition C

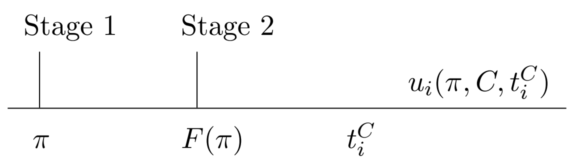
[16] In fact, G is both a coalition and coalition structure.

funds and the externalities $\sum_{j \in F(\pi, N \setminus C)} e_{ij}$ that are generated by the projects that the other countries or coalitions in π fund. We denote A the cost of the projects that are funded by the coalition C, i.e. $A = \sum_{j \in F(\pi, C)} c_j$, and $t_i^C \in \Re$ the tax rate the coalition C charges to country i. The term $t_i^C \times A$ is the contribution that the country i makes to its coalition. A is fully covered as long as $\sum_{i \in C} t_i^C = 1$. In the special case where C is a singleton, $\sum_{j \in F(\pi, C)} e_{ij} = 0$ because i has no co-members. If in such case the project i is funded, the country i covers the cost $A = c_i$ on its own and $t_i^{\{i\}} = 1$.[17]

3 The coalition formation game

3.1 Timing of the game

The game proceeds in the following manner:



In the first stage, the countries choose simultaneously whether to join a coalition or not. As a result we have the formation of the coalition structure π. In the second stage, each coalition decides on which of the projects that are available it should fund, in order to maximize the utility of its members. Each country pays $t_i^C \times A$ to the coalition it belongs. The tax rate t_i^C is based on a predetermined rule which divides the gains from the cooperation equally among the members of the coalition. The projects chosen in the second stage are then funded and utilities are realized. In what follows, we describe in detail the game mentioned above.

3.2 The cost sharing rule

We start our analysis of the game backwards and present the mechanism under which the coalitions determine the tax rates for their members. We assume that the coalitions have already been formed and each coalition has decided on which projects to undertake. Hence, the coalition structure π and the corresponding set of funded projects $F(\pi)$ are known.

[17] But if the country chooses not to fund the project $t_i^{\{i\}} = 0$.

Each coalition chooses the tax rates so as to divide its surplus equally among its members. To this extent, what matters in every coalition is the surplus which is generated by the coalition and not the possible externalities that each member receives. As surplus, we consider the payoff that the members accomplish by forming the coalition, minus the payoff that they receive when they act alone. In the cooperative game theory literature the former payoff level is commonly referred as the worth of the coalition and the latter as the disagreement point. As the externalities of the other coalitions are the same in both cases, they are not taken into consideration when we calculate the surplus.

To formalize our analysis, let a coalition $C \in \pi$ with $C = \{i_1, ..., i_i\}$. We denote the worth of coalition C as

$$w(\pi, C) = \sum_{j \in F(\pi,C)} (b_j + \sum_{i \in C} e_{ij} - c_j) + \sum_{j \in F(\pi, N \setminus C)} \sum_{i \in C} e_{ij}. \tag{2}$$

$w(\pi, C)$ can be seen as the aggregate amount of the payoff that the members of C receive and consists of two terms.

The first term includes the local benefits, costs and externalities that are generated by the projects that are funded by the coalition C. Notice that only the externalities to the members of C enter the coalition's worth, as the term $\sum_{j \in F(\pi,C)} \sum_{i \in C} e_{ij}$ suggests, and not the externalities towards the members of the other coalitions.

The second term includes the externalities from the projects that are funded by the other coalitions and affect the countries in C. As each country is allowed to belong to one coalition only and the members of C do not contribute to the funding of the projects of the other countries, this term corresponds to the externalities C's members free ride.

By $\hat{\pi}_C$, we denote the coalition structure in which all the members of C are singletons while the other countries are still organised in coalitions according to π,

$$\hat{\pi}_C = \{C_1, ..., \underbrace{\{i_1\}, ..., \{i_i\}}_{C}, ...C_h\}.$$

Stated differently, $\hat{\pi}_C$ is a coalition structure in which all the coalitions of π are formed but $i_1, ..., i_i$ fail to agree on the formation of C (or any other subcoalition).[18] We denote $F(\hat{\pi}_C, C) \subseteq C$

[18] The coalition C does not actually dismantle, but we suppose that its members disagree to facilitate our analysis.

the set of projects that are funded by the countries $i_1, ..., i_i$ when they are not associated and $F(\hat{\pi}_C, N\backslash C)$ the set of projects that the other coalitions fund.

We let $d_i(\hat{\pi}_C, C, t_i^{\{i\}})$ to be the utility level of $i \in C$ under the coalition structure $\hat{\pi}_C$. In the first case, the project i is funded by the country i while in the second it is not,

$$d_i(\hat{\pi}_C, C, t_i^{\{i\}}) = \begin{cases} b_i + \sum\limits_{j \in F(\hat{\pi}_C, C)} e_{ij} + \sum\limits_{j \in F(\hat{\pi}_C, N\backslash C)} e_{ij} - c_i \text{ if } i \in F(\hat{\pi}_C, C), \\ \\ \sum\limits_{j \in F(\hat{\pi}_C, C)} e_{ij} + \sum\limits_{j \in F(\hat{\pi}_C, N\backslash C)} e_{ij} \text{ if } i \notin F(\hat{\pi}_C, C). \end{cases} \tag{3}$$

Notice that (3) is a special case of (1) where the countries $i_1, ..., i_i$ are not associated and those that fund their projects face tax rates $t_i^{\{i\}}$ equal to 1. As mentioned before, the utility vector $(d_{i_1}(\hat{\pi}_C, C, t_{i_1}^{\{i_1\}}), ..., d_{i_i}(\hat{\pi}_C, C, t_{i_i}^{\{i_i\}}))$ is usually called the disagreement point. This is different from the threat point used by Nash (1953), in the sense that it is not fixed, but as the term $\sum\limits_{j \in F(\hat{\pi}_C, N\backslash C)} e_{ij}$ suggests, it depends on the coalitions that form the rest of the countries.

We define the surplus of the coalition C as the difference of the worth of C and the aggregate utility that the members of C achieve under $\hat{\pi}_C$:

$$s(C) = w(\pi, C) - \underbrace{\sum_{i \in C} d_i(\hat{\pi}_C, C, t_i^{\{i\}})}_{\text{aggregate utility when the members of } C \text{ are separate}}. \tag{4}$$

From the point of view of the members of C, (4) represents the gains from their cooperation. The fact that all the other countries form the same coalitions in π and $\hat{\pi}_C$ implies that they fund the same projects as well. That is:

$$F(\hat{\pi}_C, N\backslash C) = F(\pi, N\backslash C).$$

Hence, they generate the same externalities for all the members of C:

$$\sum_{j \in F(\hat{\pi}_C, N\backslash C)} e_{ij} = \sum_{j \in F(\pi, N\backslash C)} e_{ij}.$$

We use the latter along with (3) and simplify (4):

$$s(C) = \sum_{j \in F(\pi, C)} (b_j + \sum_{i \in C} e_{ij} - c_j) - \sum_{j \in F(\hat{\pi}_C, C)} (b_j + \sum_{i \in C} e_{ij} - c_j). \tag{5}$$

Thus, the surplus consists solely of the benefits that are generated by the projects that the countries $i_1, ..., i_i$ fund in π over and above $\hat{\pi}_C$.

A coalition C distributes the surplus evenly among its members by adopting the appropriate tax rates. Formally, for every $i \in C$ the tax rate t_i^C is such that:

$$\underbrace{d_i(\hat{\pi}_C, C, t_i^{\{i\}}) + \frac{s(C)}{|C|}}_{\text{Nash Bargaining Solution}} = u_i(\pi, C, t_i^C). \tag{6}$$

This rule implies that the coalition partners set the tax rates in such way to give each member the payoff it obtains when it is alone and then split the surplus equally. Notice that the LHS of (6) corresponds to the payoff level of the Nash Bargaining Solution on the coalition's worth $w(\pi, C)$, given the disagreement point $(d_{i_1}(\hat{\pi}_C, C, t_{i_1}^{\{i_1\}}), ..., d_{i_i}(\hat{\pi}_C, C, t_{i_i}^{\{i_i\}}))$.[19] We adopt this rule because it allows a unique, individually rational and feasible payoff to every coalition member. We can directly get the suitable tax rates after we replace (1) in (6) and solve for t_i^C. Recall that (1) distinguishes between cases where, under π, the project i is funded and where it is not funded. We have that:

$$t_i^C = \begin{cases} \frac{1}{A}[b_i + \sum\limits_{j \in F(\pi,C)} e_{ij} + \sum\limits_{j \in F(\pi,N \setminus C)} e_{ij} - (d_i(\hat{\pi}_C, C, t_i^{\{i\}}) + \frac{s(C)}{|C|})] \text{ if } i \in F(\pi, C), \\ \\ \frac{1}{A}[\sum\limits_{j \in F(\pi,C)} e_{ij} + \sum\limits_{j \in F(\pi,N \setminus C)} e_{ij} - (d_i(\hat{\pi}_C, C, t_i^{\{i\}}) + \frac{s(C)}{|C|})] \text{ if } i \notin F(\pi, C). \end{cases} \tag{7}$$

This mechanism assigns tax rates to the members of the coalition that are determined endogenously and exhibits a series of interesting properties. First, from (6), the payoff that the Nash Bargaining Solution allocates to i is strictly increasing in $d_i(\hat{\pi}_C, C, t_i^{\{i\}})$. For this reason, t_i^C is negatively related with $d_i(\hat{\pi}_C, C, t_i^{\{i\}})$ in (7). The higher the level of utility i obtains in $\hat{\pi}_C$, the lower is its tax rate. The countries with a relatively better position when they act alone, contribute less to the expenses of the coalition, as an incentive to join in it.

On the other hand, expanding the coalition by including countries which cause little or no change in the surplus, increases the tax rates of the existing members. The slightly higher surplus

[19] See Muthoo (1999).

is split among more countries. Every country gets a smaller share and obtains a lower utility level. As a result, the coalition charges higher taxes to the existing members.

The following Lemma introduces the next feature of this mechanism:

Lemma 1 *The rule proposed assigns tax rates that allow the coalition to cover its expenses fully.*

The proof is in Appendix B.

Put differently, if the members of the coalition pay tax rates according to (7), then $\sum_{i \in C} t_i^C = 1$. By Lemma 1, the contributions made to the coalition are sufficient to cover the costs of the projects that are chosen for funding, regardless of the number of projects that the coalition funds. This feature is significant in the second stage of the game, where each coalition decides on which projects to fund. Also, notice that the externalities that are generated by the other coalitions do not affect the tax rates. If we replace $d_i(\hat{\pi}_C, C, t_i^{\{i\}})$ from (3) in (7), these externalities are cancelled out.

Furthermore, the participation of some countries in C may be taxed, while of others might be subsidized. If a member of a coalition has a project with positive and high externalities but with negative net local benefit ($b_i < c_i$), the other members might have incentives to subsidize the participation of this country in the coalition (see the Example 1 below). As we have seen before, this is in accordance with the financial mechanism established by the Montreal Protocol. It is also known that more than half of the states in the U.S. receive more in federal spending than they pay in federal taxes.[20] A similar pattern occurs in the EU. More than half of the EU-27 members are net recipients of the EU funds (see Table 1 in Appendix A and Heinen (2011)).

Whenever in (7) the change of i's payoff due to the formation of C is greater than the share of surplus it receives, i is charged with a positive tax rate and its contribution to C is positive. But, if the opposite occurs, the tax rate is negative and i's participation in the coalition is subsidized. Also, there are cases where the tax rates are identical across all members, e.g. when the members are symmetric.

[20] The Economist online (2011, August 01), "America's fiscal union: The red and the black", *The Economist*, Retrieved from http://www.economist.com/

3.3 Choosing Projects

As the coalition structure π has been formed in the first stage of the game, in the second stage all the countries belong to some coalition. Here, we examine the decision that each coalition makes on the projects it should fund. We find that each coalition funds any project that increases its worth. To make this more obvious, recall that the number of countries in each coalition, in this stage of the game, is fixed. Then, every member of the coalition C is better off if the worth of C increases. In other words, any project with positive net benefit for C should be funded.

3.3.1 Efficient projects

Among the projects that the members of the coalition C may adopt, we focus on the ones that offer to the coalition greater total benefit (local benefit and intra-coalition externalities) than their cost. We call these projects efficient. Formally, a project k is efficient for the coalition C if its local benefit b_k and the sum of its externalities to the members of the coalition exceed its cost,

$$b_k + \sum_{l \in C} e_{lk} > c_k,\ k, l \in C. \tag{8}$$

As mentioned before, in a decentralized economy where each country k stands on its own, each project k is evaluated only for its local benefit b_k and by (8), k is efficient (inefficient) only if $b_k > c_k$ ($b_k \leq c_k$). Notice that even though the externalities of the project k to the other countries occur, they are not taken into account.

On the other hand, when the country k is a member of a coalition, the project k is evaluated not only for its local benefit, but also for the possible externalities it may generate to the other members of the same coalition. For example, if countries k and l form the coalition $C = \{k, l\}$, the coalition takes into account the externalities e_{lk} of the project k to country l in order to decide on whether the project k should be undertaken. In this case, the project k is efficient (inefficient) for C if $b_k + e_{lk} > c_k$ ($b_k + e_{lk} \leq c_k$).[21]

Recall, from Lemma 1, that the coalition covers its expenses fully, regardless of the number of projects it funds. We then have,

[21] The efficiency (inefficiency) of a project k for $\{kl\}$ does not suggest that it is necessarily efficient (inefficient) for another coalition, e.g. $\{km\}$. A project k could be efficient (inefficient) for $\{kl\}$ but inefficient (efficient) for $\{km\}$, as long as $b_k + e_{lk} > c_k$ ($b_k + e_{lk} \leq c_k$) and $b_k + e_{mk} \leq c_k$ ($b_k + e_{mk} > c_k$).

Corollary 1 *The project of the country $i \in C$ is funded if and only if it is efficient for the coalition C.*

Proof. The project of the country $i \in C$ increases the worth of the coalition C in (2) and the surplus in (4) if and only if it is efficient for C. As the number of countries in the coalition is fixed, this project increases the payoff of every member of the coalition in (6). For this reason, the decision to fund such a project is unanimous. ■

From (5) we also have that the surplus of the coalition C is the net benefit generated by the projects funded in π over and above $\hat{\pi}_C$. As the projects that are funded due to the formation of the coalition are efficient for C, its surplus is

$$s(C) \geq 0. \tag{9}$$

If at least one project is funded due to the formation of coalition C, its surplus is positive. But, if no project is funded due to the formation of C, the surplus is zero. We replace $s(C)$ in (9) from (4) and for $C = \{k, l\}$ we have that:

$$\underbrace{w(\pi, C)}_{\text{worth of } C} \geq \underbrace{d_k(\hat{\pi}_C, C, t_k^{\{k\}})}_{\text{worth of } \{k\}} + \underbrace{d_l(\hat{\pi}_C, C, t_l^{\{l\}})}_{\text{worth of } \{l\}}.$$

Since all the other countries form the same coalitions under π and $\hat{\pi}_C$, the above implies that this game is superadditive.[22]

3.4 Coalition Formation

In the first stage of the game we examine how the countries may form coalitions in this setting. We assume that choices about the coalition partners are taken simultaneously by all the countries and each country may participate in only one coalition.

We examine the conditions under which the payoffs of the various coalition structures belong to the $\delta - core$ of the game. The latter occurs when no single country or no group of countries can, by breaking away from a coalition structure, increase the payoffs of its members. We assume that the deviating countries anticipate that those coalitions from which one or more members have left, still stick together. We first consider an economy with $n = 3$ countries and provide some examples.

[22] See Peleg, B. and P. Sudhölter (2007).

In this setup, we examine the conditions under which the payoffs of the grand coalition and of the various fragmented coalition structures are in the $\delta - core$. Then, we generalize the $\delta - core$ of the grand coalition for the case of n countries.

3.4.1 An economy with three countries

For the sake of simplicity, we consider an economy with $N = \{1, 2, 3\}$ countries and $k, l, m \in N$. The possible coalition structures of this economy are:

- The grand coalition $G = \{1, 2, 3\}$.
- The fragmented coalition structures of the type: $\pi = \{C, \{m\}\}$ with $C = \{k, l\}$:

$$\{\{1,2\},\{3\}\}, \quad \{\{1,3\},\{2\}\} \text{ and } \{\{2,3\},\{1\}\}.$$

- The fully fragmented coalition structure

$$\hat{\pi} = \{\{1\},\{2\},\{3\}\}.$$

As stated previously, the payoffs of a coalition structure belong to the $\delta - core$ of the game if no country or group of countries can improve upon their position by deviating. In order to be compatible with the notion of the core, we consider only one deviation at a time, either multilateral or unilateral.

A multilateral deviation from the grand coalition involves at least two countries, say the countries k and l. They break away from the grand coalition to form a finer coalition. This deviation leads to the fragmented coalition structure $\pi = \{\{k,l\},\{m\}\}$. A unilateral deviation from the grand coalition, say by the country m, leads again to the fragmented coalition structure $\pi = \{\{k,l\},\{m\}\}$, as m expects the non-deviating countries k and l to stay together. We do not consider the fully fragmented coalition structure $\hat{\pi}$ as a possible deviation from the grand coalition, because it requires more than one deviations from G to end up in $\hat{\pi}$.

Similarly, a multilateral deviation from a fragmented coalition structure π is a deviation that involves at least two countries. If the countries that deviate are two, we end up with another fragmented coalition structure. For instance, the deviation of the countries k and m from the fragmented coalition structure $\pi = \{\{k,l\},\{m\}\}$ leads to the fragmented coalition structure $\{\{k,m\},\{l\}\}$. If the multilateral deviation involves all the countries it leads to the grand coalition. A unilateral deviation from $\pi = \{\{k,l\},\{m\}\}$, involves either k or l and leads to the fully fragmented coalition structure $\hat{\pi}$.

We examine the conditions under which the payoffs of the various coalition structures are in the $\delta - core$ of the game. We start from the grand coalition:

Proposition 1 *In an economy with $N = \{1,2,3\}$ countries and $k,l,m \in N$, the payoffs of the grand coalition consist the $\delta - core$ of the game (N,u) if and only if for every fragmented coalition structure $\pi = \{C,\{m\}\}$ with $C = \{k,l\}$,*

$$\frac{s(G)}{3} > \frac{s(C)}{2}, \tag{10}$$

and

$$\frac{s(G)}{3} > \sum_{j\in F(\pi,C)} e_{mj} - \sum_{j\in F(\hat{\pi})} e_{mj}. \tag{11}$$

The proof is in Appendix B.

From (10), we have that no group of countries may improve upon their payoffs by leaving G for a finer coalition in π, if the share of the surplus that each deviating country gets in G is higher than the share of the surplus it gets in every two-membered coalition it might be involved. Recall that, from (9), the surplus of any coalition is either positive or zero. For (10) to hold, the surplus of the grand coalition must be positive.[23] This occurs when at least one project is funded due to the formation of the grand coalition. (11) implies that no single country may improve its payoff if it deviates from G to become a singleton, if the share of the surplus that the country receives when it participates in G is greater than the externalities on which this country free rides in π.[24] These are the externalities that generate the projects that are funded under π over and above $\hat{\pi}$.

In order to facilitate the interpretation of Proposition 1, we give an example:

Example 1 *We consider an economy where the projects have the following local benefits, external-*

[23]From (9), we have that $s(G) \geq 0$ and $s(C) \geq 0$. Then, for $\frac{s(G)}{3}$ to be greater than $\frac{s(C)}{2}$ in (10), $s(G)$ must be positive.

[24]Recall that this deviation leads again in π, because in the $\delta - game$ the non-deviating countries stay together.

ities and costs,

		b_i	c_i	e_{1i}	e_{2i}	e_{3i}
	1	1	0.9	0	0.11	0
Country	2	1	1.1	0.11	0	−0.2
	3	−1	1.1	1.06	1.1	0

Notice that the project 1 has local benefit $b_1 = 1$*, cost* $c_1 = 0.9$ *and generates externalities* $e_{21} = 0.11$ *and* $e_{31} = 0$ *for the countries* 2 *and* 3 *respectively. Similarly, we have the parameters for the projects* 2 *and* 3*.*

From (8)*, we have that the projects* 1 *and* 3 *are efficient for the grand coalition, but the project* 2 *is not:*

$$b_1 + e_{21} + e_{31} > c_1 \Rightarrow 1 + 0.11 > 0.9$$

and

$$b_3 + e_{13} + e_{23} > c_3 \Rightarrow -1 + 1.06 + 1.1 > 1.1$$

but

$$b_2 + e_{12} + e_{32} < c_2 \Rightarrow 1 + 0.11 - 0.2 < 1.1.$$

The set of projects that the grand coalition funds is $F(G) = \{1, 3\}$*.*

Although the project 2 *is not efficient for* G *(because of its negative externality to country* 3*), this is not the case when the countries* 1 *and* 2 *form a coalition, as:*

$$b_2 + e_{12} > c_2 \Rightarrow 1 + 0.11 > 1.1.$$

Hence, the set of the funded projects by the coalition $\{1, 2\}$ *is*

$$F(\underbrace{(\{1,2\},\{3\})}_{\pi}, \underbrace{\{1,2\}}_{C}) = \{1, 2\}.$$

In such case, the country 3*, which is not associated with* 1 *and* 2*, funds*

$$F((\{1,2\},\{3\}),\{3\}) = \varnothing.$$

Similarly, the sets of the projects that are funded under the remaining coalition structures are:

$$F((\{1,3\},\{2\}),\{1,3\}) = \{1\}, \quad F((\{1,3\},\{2\}),\{2\}) = \varnothing,$$

$$F((\{2,3\},\{1\}),\{2,3\}) = \varnothing, \quad F((\{2,3\},\{1\}),\{1\}) = \{1\}.$$

When the countries are separate, it is only the project 1 that is efficient as its local benefit exceeds its cost:

$$F(\hat{\pi}, \{1\}) = \{1\}, \quad F(\hat{\pi}, \{2\}) = F(\hat{\pi}, \{3\}) = \varnothing.$$

Below, we calculate the surplus of the grand coalition. Recall that only the projects 1 *and* 3 *are efficient for* G *and that countries* 2 *and* 3 *do not fund their projects when they are separate. From* (4)*, we have that*

$$s(G) = \underbrace{b_1 + b_3 + e_{21} + e_{31} + e_{13} + e_{23} - c_1 - c_3}_{\text{worth of the grand coalition}} - \underbrace{(b_1 + e_{21} + e_{31} - c_1)}_{\text{aggregate utility when the countries are separate}} = 0.06.$$

Similarly, the surpluses that the various coalitions generate are

$$s(\{1,2\}) = 0.01 \text{ and } s(\{1,3\}) = s(\{2,3\}) = 0.$$

Since the surplus of the grand coalition exceeds the surplus of every other coalition by more than $\frac{3}{2}$ *times, the condition* (10) *is satisfied.*

In order to confirm that (11) *holds, we calculate the externalities on which the countries that do not cooperate free ride. These are the externalities that the formation of the two-membered coalitions generate for the non-members.*

The coalition $\{1,2\}$ *funds the project* 2 *over and above the case where the countries* 1 *and* 2 *are unassociated. Then, the country* 3 *receives the externality* $e_{32} = -0.2$ *due to the formation of the coalition* $\{1,2\}$. *The coalition* $\{1,3\}$ *funds no additional projects than the countries* 1 *and* 3 *would fund if they were separate. Hence, the country* 2 *receives no additional externalities due to the formation of* $\{1,3\}$. *The coalition* $\{2,3\}$ *funds no projects and the country* 1 *receives no externalities from it.*

As all the above externalities are lower than the $\frac{1}{3}$ *of the surplus which is generated by the grand coalition, the condition* (11) *is satisfied.*

The payoffs of the grand coalition consist the $\delta-$*core of the game* (N, u). *To verify this, we present the payoffs of the various coalition structures in Table 2 of Appendix A. No multilateral or unilateral deviation from the grand coalition is profitable. According to* (7)*, the tax rates are*

$$(t_1^G, t_2^G, t_3^G) = (0.97, 0.54, -0.51).$$

Notice that even though the project 3 *has a negative local benefit, its externalities to countries* 1 *and* 2 *are so high that these countries substitute the participation of country* 3 *in the grand coalition.*

What is evident from Proposition 1 is that the potential gains from the full cooperation should be sufficiently high in order to support the participation of all the countries in the grand coalition. If not, either a subgroup of the countries can improve their position if they form a finer coalition or a single country is better off when it is not associated with other countries.[25] As we have mentioned before, this result is different from the findings of the main body of the IEAs literature. We also have that:

Corollary 2 *By superadditivity, it is not possible for both* (10) *and* (11) *to hold simultaneously in the reverse direction.*

The proof is in Appendix B.

Hence, the countries do not have enough incentives to form the grand coalition if either (10) or (11) do not hold, or both (10) and (11) are satisfied with equality.

In this setup, we can also examine the conditions under which the payoffs of a fragmented coalition structure $\pi = \{C, \{m\}\}$, where $C = \{k, l\}$, cannot be improved upon:

Proposition 2 *In an economy with* $N = \{1, 2, 3\}$ *countries and* $k, l, m \in N$*, the payoffs of the fragmented coalition structure* $\pi = \{C, \{m\}\}$ *with* $C = \{k, l\}$*, consist the* $\delta - core$ *of the game* (N, u, π) *if and only if*

$$s(C) > s(C') \tag{12}$$

where $C' = \{q, m\}$ *with* $q = k, l$*, given that either* (10) *or* (11) *do not hold.*
Any other fragmented coalition structure for which the surplus of C' *is equal to* $s(C)$ *also has payoffs in the* $\delta - core$*.*

The proof is in Appendix B.

Recall that a multilateral deviation from a fragmented coalition structure π involves either all three countries or two countries. The former deviation leads to the grand coalition and it is not profitable if either (10) or (11) do not hold. The latter leads to another fragmented coalition structure where a member of the coalition $C = \{k, l\}$, say the country $q = k$, leaves C to form with the country m the coalition $C' = \{k, m\}$. The country that does not deviate from C becomes

[25] The appropriate conditions for the case where in the analysis we consider the $\gamma - core$, are in Appendix C.

a singleton. The new fragmented coalition structure is $\pi' = \{\{k, m\}, \{l\}\}$. Since, from (6), each coalition member gets a share of the surplus on top of its disagreement point payoff, this kind of deviation is not profitable if (12) holds. In such case, the country k gets a greater share of surplus if it participates in the coalition C rather than the coalition C'. Futher, a unilateral devition from a fragmented coalition structure π involves a member of the coalition C, say the country k, and leads to the fully fragmented coalition structure $\hat{\pi}$. Such a deviation is not profitable for k if the surplus of C is positive. If (12) is satisfied, then (9) implies that $s(C)$ is positive.[26] This happens if at least one project is funded due to the formation of C. In the special case where in the fragmented coalition structure π' the surplus of C' is equal to $s(C)$, then π' has payoffs in the $\delta - core$ as well, but of the game (N, u, π').

We demonstrate the above in the following example:

Example 2 *We consider the economy of Example 1, but here we make the project of the country* 3 *less attractive. We decrease its local benefit from* -1 *to* -1.05.
The new surplus of the grand coalition is $s(G) = 0.01$ *and it is no longer high enough to satisfy* (10). *The countries* 1 *and* 2 *are better off if they deviate from* G *and form the coalition* $\{1, 2\}$. *The condition* (12) *holds for the coalition* $\{1, 2\}$ *of the fragmented coalition structure* $\{\{1, 2\}, \{3\}\}$. *Every other two-membered coalition generates surplus equal to zero. The payoffs of the fragmented coalition structure* $\{\{1, 2\}, \{3\}\}$ *consist the* $\delta - core$ *of the game* $(N, u, \{\{1, 2\}, \{3\}\})$. *The payoffs of the various coalition structures for this case are in Table 3 of Appendix A. From* (7)*, the tax rates are*

$$(t_1^{\{1,2\}}, t_2^{\{1,2\}}, t_3^{\{3\}}) = (0.5025, 0.4975, 0).$$

Next, we reduce the externality of the project 3 *to the country* 1 *from* 1.06 *to zero and increase the externality of the same project to the country* 2 *from* 1.1 *to* 2.16.
We now have that the surplus of $\{2, 3\}$ *is* 0.01 *and is equal to the surplus of* $\{1, 2\}$. *In this case, the payoffs of the fragmented coalition structure* $\{\{1, 2\}, \{3\}\}$ *consist the* $\delta - core$ *of the game* $(N, u, \{\{1, 2\}, \{3\}\})$ *and the payoffs of the fragmented coalition structure* $\{\{2, 3\}, \{1\}\}$ *consist the* $\delta - core$ *of the game* $(N, u, \{\{2, 3\}, \{1\}\})$. *The payoffs that correspond to the various coalition structures are in Table 4 of Appendix A. The tax rates for* $\{\{1, 2\}, \{3\}\}$ *remain unchanged and*

[26] Since $s(C) > s(C')$ and from (9) we have that $s(C) \geq 0$ and $s(C') \geq 0$, $s(C)$ must be positive.

from (7) *the tax rates for* $\{\{2,3\},\{1\}\}$ *are*

$$(t_1^{\{1\}}, t_2^{\{2,3\}}, t_3^{\{2,3\}}) = (1, 1.959, -0.959).$$

Finally, if the conditions (10), (11) and (12) do not hold, the countries have no incentives to cooperate because no coalition can generate a positive surplus. The utility levels are that of the fully fragmented coalition structure $\hat{\pi}$ for all coalition structures (including the grand coalition). Then, every coalition structure has payoffs in the $\delta - core$.

Example 3 *We consider the economy of the Example 1, but we reduce all externalities to zero. The countries do not expect gains from cooperation, because there is no coalition that generates a positive surplus. See Table 5 of Appendix A for the payoffs under the various coalition structures.*

3.4.2 An economy with n countries

In this section, we consider an economy with $N = \{1, 2, ...n\}$ countries and derive the condition under which the payoffs of grand coalition are in the $\delta - core$. We do not study the $\delta - core$ for the fragmented coalition structures, due to its complexity.

Proposition 3 *In an economy with* $N = \{1, 2, ...n\}$ *countries, the payoffs of the grand coalition consist the* $\delta - core$ *of the game* (N, u) *if and only if* $\forall k \in C$ *and* $\forall C \in N$,

$$\frac{s(G)}{n} > \frac{s(C)}{|C|} + \sum_{j \in F(\hat{\pi}_C, N \backslash C)} e_{kj} - \sum_{j \in F(\hat{\pi}, N \backslash C)} e_{kj}. \tag{13}$$

The proof is in Appendix B.

In a sense, condition (13) can be seen as a combination of (10) and (11) and implies that if the benefits from the cooperation of all the countries are large enough, the grand coalition prevails. According to (13), no group of countries may increase its payoffs by leaving G for a finer coalition C in the fragmented coalition structure $\pi = \{C, N \backslash C\}$, if the share of the surplus that each deviating country gets in G is greater than the share of the surplus it gets in C plus the externalities on which it free rides.[27]

[27] See the Appendix C for a contrast of the $\delta - core$ with the $\gamma - core$ in the case with n countries.

4 Welfare loss

In this section, we examine the loss in the aggregate welfare when the grand coalition does not prevail. We show that the grand coalition may not occur even if the welfare loss is positive.

If all the countries participate in the grand coalition there is no loss in welfare. The project of the country k is evaluated for the externalities that it generates for all the other countries. However, this may not be the case when the countries do not form the grand coalition. The project of the country k in this case is evaluated only for the externalities that it generates to the other countries of the coalition. Even though the externalities that the project k generates for the rest of the countries occur, these are not taken into account. In this case, we may have welfare losses.

4.1 An economy with three countries

Our starting point, for the sake of simplicity, is an economy with three countries. As mentioned before, in the case where all the countries participate in a single coalition there is no loss in the welfare, because all the externalities are internalized. However, when the countries end up in a fragmented coalition structure rather than the grand coalition, we have that:

Proposition 4 *In an economy with $N = \{1, 2, 3\}$ countries and $k, l, m \in N$, if the fragmented coalition structure $\pi = \{C, \{m\}\}$ with $C = \{k, l\}$ prevails, the countries suffer a welfare loss equal to:*

$$W(G) - W(\pi) = s(G) - [s(C) + \sum_{j \in F(\pi, C)} e_{mj} - \sum_{j \in F(\hat{\pi})} e_{mj}].$$

The proof is in Appendix B.

We write the above difference as:

$$W(G) - W(\pi) = 2(\frac{s(G)}{3} - \frac{s(C)}{2}) + [\frac{s(G)}{3} - (\sum_{j \in F(\pi, C)} e_{mj} - \sum_{j \in F(\hat{\pi})} e_{mj})]. \tag{14}$$

Notice that in the RHS of (14), the difference in the parentheses is positive if (10) is satisfied and the difference in the brackets is positive if (11) is satisfied. Then, in case where both (10) and (11) hold, the welfare loss is so high that the countries have incentives to form the grand coalition.

However, we may face cases where the countries do not have enough incentives to cooperate fully (i.e. either (10) or (11) do not hold), even if the aggregate welfare loss is positive. The following example demonstrates such a case:

Example 4 *Recall that for the economy of the Example 2:*

$$F(G) = \{1, 2, 3\},$$

$$F(\underbrace{(\{1,2\},\{3\})}_{\pi}, \underbrace{\{1,2\}}_{C}) = \{1,2\}, \quad F((\{1,2\},\{3\}),\{3\}) = \varnothing,$$

$$F(\hat{\pi}, \{1\}) = \{1\}, \quad F(\hat{\pi}, \{2\}) = F(\hat{\pi}, \{3\}) = \varnothing,$$

$$s(G) = s(\{1,2\}) = 0.01,$$

and

$$\sum_{j \in F((\{1,2\},\{3\}),\{1,2\}))} e_{mj} - \sum_{j \in F(\hat{\pi})} e_{mj} = (e_{31} + e_{32}) - e_{31} = -0.2.$$

Also, from Table 3 of Appendix A we have that the aggregate welfare in the case of full cooperation is $W(G) = 0.22$*, while the aggregate welfare of the fragmented coalition structure* $\pi = \{\{1,2\},\{3\}\}$ *is* $W(\pi) = 0.02$*.*

The position of 3 *improves if* $G = \{1, 2, 3\}$ *prevails instead of* $\pi = \{\{1,2\},\{3\}\}$ *(this implies that* (11) *holds). However, the countries* 1 *and* 2 *are better off without country* 3 *(this implies that* (10) *does not hold). A cooperation with* 3 *would internalize the negative externality that the project* 2 *generates for the country* 3 *and change its efficiency.*

As the countries do not have incentives to participate in a single coalition, the economy suffers an aggregate welfare loss of

$$W(G) - W(\pi) = 0.2.$$

The welfare loss is positive but not high enough to attract the countries 1 *and* 2 *to cooperate with* 3*.*

When the countries do not have incentives to cooperate and we have $\hat{\pi}$, the countries may also suffer a positive welfare loss. In such case the welfare loss is:

Corollary 3 *In an economy with $N = \{1,2,3\}$ countries, if the fully fragmented coalition structure $\hat{\pi} = \{\{1\},\{2\},\{3\}\}$ prevails, the countries suffer a welfare loss equal to:*

$$W(G) - W(\hat{\pi}) = s(G).$$

The proof is in Appendix B.

The example below demonstrates such a case:

Example 5 *We consider an economy where the projects have the following local benefits, externalities and costs,*

		b_i	c_i	e_{1i}	e_{2i}	e_{3i}
	1	1	0.89	0	−0.1	−0.1
Country	2	1	1.1	0	0	−0.2
	3	−1	1.1	0	−0.2	0

Notice that $F(\hat{\pi},\{1\}) = \{1\}$ and the set of the projects that every other coalition funds (including the grand coalition) is a null set. The surplus of the grand coalition is:

$$s(G) = \underbrace{0}_{\text{worth of the grand coalition}} - \underbrace{(b_1 + e_{21} + e_{31} - c_1)}_{\text{aggregate utility when the members are separate}} = 0.09.$$

Since, no projects are funded under G, the aggregate welfare in the case of full cooperation is $W(G) = 0$. But if the countries do not cooperate at all, the aggregate welfare is

$$W(\hat{\pi}) = b_1 + e_{21} + e_{31} - c_1 = -0.09.$$

Here, the country 1 has no incentives to cooperate with the other countries, because in such case the project 1 is not funded. The economy suffers an aggregate welfare loss of

$$W(G) - W(\hat{\pi}) = 0.09.$$

4.2 An economy with n countries

We now generalize for the case where $N = \{1,2,...n\}$. We have that:

Proposition 5 *In an economy with $N = \{1,2,...n\}$ countries and $k \in N$, if the fragmented coalition structure $\pi = \{C, N\backslash C\}$ prevails, the countries suffer a welfare loss equal to:*

$$W(G) - W(\pi) = s(G) - [s(C) + \sum_{k\in C}(\sum_{j\in F(\hat{\pi}_C, N\backslash C)} e_{kj} - \sum_{j\in F(\hat{\pi}, N\backslash C)} e_{kj})]$$

$$- [s(N\backslash C) + \sum_{k \in N\backslash C} (\sum_{j \in F(\hat{\pi}_{N\backslash C}, C)} e_{kj} - \sum_{j \in F(\hat{\pi}, C)} e_{kj})].$$

The proof is in Appendix B.

We write the above as:

$$W(G) - W(\pi) = |C| \times B + |N\backslash C| \times D,$$

where

$$B = \frac{s(G)}{n} - (\frac{s(C)}{|C|} + \sum_{k \in C} (\sum_{j \in F(\hat{\pi}_C, N\backslash C)} e_{kj} - \sum_{j \in F(\hat{\pi}, N\backslash C)} e_{kj}),$$

and

$$D = \frac{s(G)}{n} - (\frac{s(N\backslash C)}{|N\backslash C|} + \sum_{k \in N\backslash C} (\sum_{j \in F(\hat{\pi}_{N\backslash C}, C)} e_{kj} - \sum_{j \in F(\hat{\pi}, C)} e_{kj}).$$

Notice that B is positive if (13) is satisfied for every member of the coalition C. Similarly, D is positive if (13) is satisfied for every member of the coalition $N\backslash C$. Hence, in case where (13) holds for all the countries that participate in the coalition C and the coalition $N\backslash C$, the welfare loss is so high that the countries have incentives to form a single coalition. However, there exist cases where $W(G) - W(\pi)$ is positive but the conditions of Proposition 3 are not satisfied.

5 Conclusion

In this paper we study the endogenous formation of coalitions, such as IEAs, using the $\delta - core$ in an economy that consists of asymmetric countries, when (environmental) externalities are present. The countries are asymmetric because the project that each one of them has generates local benefits and externalities of different magnitudes. Since the countries act rationally, some of them may decide not to associate. On the other hand, other countries may cooperate and form coalitions. Each coalition of countries decides endogenously on which of the available projects it should fund, in order to maximize the utility of its members. The participants of each coalition share equally the gains from their cooperation.

We show that the formation of the grand coalition can be supported, as long as the benefits that it generates are high enough. If this is not the case, we might have more than one coalition. In the special case where the economy consists of $n = 3$ countries, we examine the conditions under which the payoffs not only of the grand coalition, but also of the other coalition structures, are in the $\delta - core$. We examine the welfare losses that the economy may suffer when the grand coalition does not prevail. Finally, in this setup it would be interesting to find a rule that splits the expenses and surplus from the coordination in such a way, that the full cooperation of the countries always prevails.

Appendix A

Table 1: Simple net contribution positions of the EU members,

including traditional own resources (in million euros).

Year / Country	2009	2010	2011	2012	2013
Austria	499.25	805.23	812.86	1086.21	1329.45
Belguim	-967.91	-1361.89	-1870.16	-1725.50	-1918.71
Bulgaria	-589.09	-869.88	-711.91	-1315.55	-1499.31
Croatia	-106.85	-87.65	-113.17	-103.56	-51.72
Cyprus	27.10	6.52	1.21	30.67	-42.24
Czech Republic	-1574.44	-1917.92	-1346.52	-2935.46	-3276.49
Denmark	1162.90	854.79	975.24	1256.75	1464.59
Estonia	-557.93	-665.52	-346.12	-778.55	-761.38
Finland	606.12	392.59	662.18	677.81	662.31
France	-3009.26	-3438.57	-4633.88	-4548.38	-5308.11
Germany	8796.66	11947.39	10994.11	13968.56	16320.00
Greece	-3009.26	-3438.57	-4633.88	-4548.38	-5308.11
Hungary	-2659.73	-2695.01	-4393.51	-3248.75	-4898.74
Ireland	155.61	-671.28	-300.74	-574.07	-143.07
Italy	6045.76	5834.90	6492.09	5586.64	4613.65
Latvia	-494.68	-668.61	-728.66	-950.41	-794.20
Lithuania	-1468.07	-1332.83	-1350.79	-1489.88	-1476.44
Luxembourg	-1166.85	-1293.06	-1255.39	-1253.78	-1276.44
Malta	-7.24	-51.14	-68.78	-72.54	-87.28
Netherlands	1487.50	3467.46	3804.60	3956.64	4288.08
Poland	-6118.98	-8165.16	-10860.22	-11827.56	-11965.02
Portugal	-2087.43	-2530.86	-2980.90	-5023.39	-4369.82
Romania	-1608.90	-1174.33	-1433.54	-1988.45	-4086.28
Slovakia	-480.77	-1257.70	-1091.43	-1544.07	-1226.73
Slovenia	-188.69	-369.15	-445.84	-533.34	-387.97
Sweden	403.49	1596.97	1576.55	2188.09	2550.48
United Kingdom	3864.49	7913.74	7255.18	9243.60	10760.08

Source: Financial Report 2013, European Commission

Table 2: The payoffs of the grand coalition consist the $\delta-core$ of (N,u).

	$\hat{\pi}$	$\{12\},\{3\}$	$\{13\},\{2\}$	$\{23\},\{1\}$	**G**
u_1	0.1	0.105	0.1	0.1	**0.12**
u_2	0.11	0.115	0.11	0.11	**0.13**
u_3	0	-0.2	0	0	**0.02**

Table 3: The payoffs of the fragmented coalition structure {12},{3} consist the $\delta-core$ of $(N,u,\{\{12\},\{3\}\})$.

	$\hat{\pi}$	$\{\mathbf{12}\},\{\mathbf{3}\}$	$\{13\},\{2\}$	$\{23\},\{1\}$	G
u_1	0.1	**0.105**	0.1	0.1	0.103
u_2	0.11	**0.115**	0.11	0.11	0.113
u_3	0	**−0.2**	0	0	0.03

Table 4: The payoffs of the fragmented coalition structures {12},{3} and {2,3},{1} consist the $\delta-core$ of $(N,u,\{\{12\},\{3\}\})$ and $(N,u,\{\{2,3\},\{1\}\})$ respectively.

	$\hat{\pi}$	$\{\mathbf{12}\},\{\mathbf{3}\}$	$\{13\},\{2\}$	$\{\mathbf{23}\},\{\mathbf{1}\}$	G
u_1	0.1	**0.105**	0.1	**0.1**	0.1
u_2	0.11	**0.115**	0.11	**0.115**	0.11
u_3	0	**−0.2**	0	**0.005**	0

Table 5: The case when the countries have no incentives to cooperate.

	$\hat{\pi}$	$\{12\},\{3\}$	$\{13\},\{2\}$	$\{23\},\{1\}$	G
u_1	0.1	0.1	0.1	0.1	0.1
u_2	0.11	0.11	0.11	0.11	0.11
u_3	0	0	0	0	0

Appendix B

Proof of Lemma 1. We sum the tax rates of all the members of C, as given by (7). Recall that C may not fund the projects of all of its participants and for this reason, in (7). We distinguish between the members that have their projects funded in C and the ones that they do not. Below, the term $\sum_{j\in F(\pi,C)} b_j$ is the sum of the local benefits of the projects that are funded by C,

$$\sum_{i\in C} t_i^C = \frac{1}{A}[\sum_{j\in F(\pi,C)} b_j + \sum_{i\in C}(\sum_{j\in F(\pi,C)} e_{ij} + \sum_{j\in F(\pi,N\setminus C)} e_{ij}) - \sum_{i\in C}(d_i(\hat{\pi}_C, C, t_i^{\{i\}}) + \frac{s(C)}{|C|})]$$

$$= \frac{1}{A}[\sum_{j\in F(\pi,C)} b_j + \sum_{i\in C}(\sum_{j\in F(\pi,C)} e_{ij} + \sum_{j\in F(\pi,N\setminus C)} e_{ij}) - \sum_{i\in C} d_i(\hat{\pi}_C, C, t_i^{\{i\}}) - \frac{1}{|C|}\sum_{i\in C} s(C)]$$

$$= \frac{1}{A}[\sum_{j\in F(\pi,C)} b_j + \sum_{i\in C}(\sum_{j\in F(\pi,C)} e_{ij} + \sum_{j\in F(\pi,N\setminus C)} e_{ij}) - \sum_{i\in C} d_i(\hat{\pi}_C, C, t_i^{\{i\}}) - \frac{1}{|C|} \times |C| \times s(C)]$$

$$= \frac{1}{A}[\sum_{j\in F(\pi,C)} b_j + \sum_{i\in C}(\sum_{j\in F(\pi,C)} e_{ij} + \sum_{j\in F(\pi,N\setminus C)} e_{ij}) - \sum_{i\in C} d_i(\hat{\pi}_C, C, t_i^{\{i\}}) - s(C)].$$

If we replace $s(C)$ from (4) we get:

$$\sum_{i\in C} t_i^C = \frac{1}{A}[\sum_{j\in F(\pi,C)} b_j + \sum_{i\in C}(\sum_{j\in F(\pi,C)} e_{ij} + \sum_{j\in F(\pi,N\setminus C)} e_{ij}) - w(\pi, C).$$

If we replace $w(\pi, C)$ from (2) we have:

$$\sum_{i\in C} t_i^C = [\sum_{j\in F(\pi,C)} b_j + \sum_{i\in C}(\sum_{j\in F(\pi,C)} e_{ij} + \sum_{j\in F(\pi,N\setminus C)} e_{ij})$$

$$- \sum_{j \in F(\pi,C)} (b_j + \sum_{i \in C} e_{ij} - c_j) - \sum_{j \in F(\pi,N \setminus C)} \sum_{i \in C} e_{ij}].$$

Since $\sum_i \sum_j e_{ij} = \sum_j \sum_i e_{ij}$ is a property of the summation,

$$\sum_{i \in C} = \frac{\sum_{j \in F(\pi,C)} c_j}{A} = \frac{A}{A} = 1,$$

as $A = \sum_{j \in F(\pi,C)} c_j$. ■

Proof of Proposition 1. In order to prove (10), we consider that a group of countries, say countries k and l, deviate from the grand coalition and form the coalition $C = \{kl\}$ in the coalition structure $\pi = \{\{kl\}, \{m\}\}$. A random member of C, for instance k, has no incentive to deviate from G as long as

$$u_k(G, G, t_k^G) > u_k(\pi, C, t_k^C).$$

We replace $u_k(G, G, t_k^G)$ and $u_k(\pi, C, t_k^C)$ from (6). For k, the disagreement point utility level both in G and π is $d_k(\hat{\pi}, \{k\}, t_k^{\{k\}})$:

$$d_k(\hat{\pi}, \{k\}, t_k^{\{k\}}) + \frac{s(G)}{3} > d_k(\hat{\pi}, \{k\}, t_k^{\{k\}}) + \frac{s(C)}{2} \Leftrightarrow$$

$$\frac{s(G)}{3} > \frac{s(C)}{2}.$$

The same holds for country l.

In order to prove (11), we note that a country m has no incentive to deviate from the grand coalition and be a singleton in π,[28] as long as

$$u_m(G, G, t_m^G) > u_m(\pi, \{m\}, t_m^{\{m\}}).$$

If we replace $u_m(G, G, t_m^G)$ from (6) and $u_m(\pi, \{m\}, t_m^{\{m\}})$ from (1), we have that:

$$d_m(\hat{\pi}, \{m\}, t_m^{\{m\}}) + \frac{s(G)}{3} > \begin{cases} b_m + \sum\limits_{j \in F(\pi,C)} e_{mj} - c_m \text{ if } m \in F(\pi, \{m\}), \\ \sum\limits_{j \in F(\pi,C)} e_{mj} \text{ if } m \notin F(\pi, \{m\}). \end{cases}$$

[28] Because the non-deviating countries k, l still stick together.

Next, we replace $d_m(\hat{\pi}, \{m\}, t_m^{\{m\}})$ from (3). Notice that m is not associated with the other countries under π and $\hat{\pi}$. Thus, if the project m is funded in $\pi = \{\{kl\}, \{m\}\}$, then it is also funded in $\hat{\pi}$:

$$b_m + \sum_{j \in F(\hat{\pi})} e_{mj} - c_m + \frac{s(G)}{3} > b_m + \sum_{j \in F(\pi, C)} e_{mj} - c_m \text{ if } m \in F(\pi, \{m\}),$$

or

$$\sum_{j \in F(\hat{\pi})} e_{mj} + \frac{s(G)}{3} > \sum_{j \in F(\pi, C)} e_{mj} \text{ if } m \notin F(\pi, \{m\}).$$

They both reduce to $\frac{s(G)}{3} > \sum_{j \in F(\pi, C)} e_{mj} - \sum_{j \in F(\hat{\pi})} e_{mj}$. ■

Proof of Corollary 2. Suppose, towards a contradiction, that both (10) and (11) hold simultaneously in the reverse direction. Then, if we multiply (10) times 2 and add (11), we get:

$$s(G) < s(C) + \sum_{j \in F(\pi, C)} e_{mj} - \sum_{j \in F(\hat{\pi})} e_{mj}.$$

If we rewrite $\sum_{j \in F(\pi, C)} e_{mj} - \sum_{j \in F(\hat{\pi})} e_{mj}$ as $u_m(\pi, \{m\}, t_m^{\{m\}}) - d_m(\hat{\pi}, \{m\}, t_m^{\{m\}})$ and replace $s(G)$, $s(C)$ from (4), we have:

$$w(G, G) - \sum_{i \in \{k,l,m\}} d_i(\hat{\pi}, \{i\}, t_i^{\{i\}}) < w(\pi, C) - \sum_{i \in \{k,l\}} d_i(\hat{\pi}, \{i\}, t_i^{\{i\}})$$

$$+ u_m(\pi, \{m\}, t_m^{\{m\}}) - d_m(\hat{\pi}, \{m\}, t_m^{\{m\}}),$$

or

$$\underbrace{w(G, G)}_{\text{worth of G}} < \underbrace{w(\pi, C)}_{\text{worth of \{k,l\}}} + \underbrace{u_m(\pi, \{m\}, t_m^{\{m\}})}_{\text{worth of \{m\}}}.$$

The above is not compatible with the fact that this game is superadditive. ■

Proof of Proposition 2. In order to get (12), we consider a country $q = k$ that participates in the two-membered coalition $C = \{kl\}$ in $\pi = \{\{kl\}, \{m\}\}$. This country has no incentive to break away from C and form with country m the coalition C' in the coalition structure $\pi' = \{\{k, m\}, \{l\}\}$ as long as,

$$u_k(\pi, C, t_k^C) > u_k(\pi', C', t_k^{C'}).$$

We replace $u_k(\pi, C, t_k^C)$ and $u_k(\pi', C', t_k^{C'})$ from (6):

$$d_k(\hat{\pi}, \{k\}, t_k^{\{k\}}) + \frac{s(C)}{2} > d_k(\hat{\pi}, \{k\}, t_k^{\{k\}}) + \frac{s(C')}{2}$$

or $s(C) > s(C')$. The same must hold if $q = l$. The multilateral deviation that involves all the countries from π to the grand coalition is not profitable if either (10) or (11) do not hold. If (10) is not satisfied, the countries k and l are better off in C rather than in G. If (11) is not satisfied, the country m is better off as a singleton rather than a member of the grand coalition.

The country k has no incentive to deviate from C and be a singleton in $\hat{\pi}$ as long as

$$u_k(\pi, C, t_k^C) > u_k(\hat{\pi}, \{k\}, t_k^{\{k\}}).$$

We replace $u_k(\pi, C, t_k^C)$ from (6). Also, as we mentioned earlier, $u_k(\hat{\pi}, \{k\}, t_k^{\{k\}})$ coincides with $d_k(\hat{\pi}, \{k\}, t_k^{\{k\}})$:

$$d_k(\hat{\pi}, \{k\}, t_k^{\{k\}}) + \frac{s(C)}{2} > d_k(\hat{\pi}, \{k\}, t_k^{\{k\}}) \Leftrightarrow$$

$$\frac{s(C)}{2} > 0 \Leftrightarrow s(C) > 0.$$

The latter is implied by (12), since from (5) and Corollary 1 the surplus of any coalition is either positive or zero. The same holds for country l. ■

Proof of Proposition 3. In order to prove (13), we consider that a group of countries leaves the grand coalition and forms the finer coalition C. The countries that deviate expect the non-deviating countries to stay together. The deviation leads to the coalition structure $\pi = \{C, N \backslash C\}$. A random member of C, for instance k, has no incentive to deviate from G as long as

$$u_k(G, G, t_k^G) > u_k(\pi, C, t_k^C).$$

We replace $u_k(G, G, t_k^G)$ and $u_k(\pi, C, t_k^C)$ from (6)

$$d_k(\hat{\pi}, \{k\}, t_k^{\{k\}}) + \frac{s(G)}{n} > d_k(\hat{\pi}_C, \{k\}, t_k^{\{k\}}) + \frac{s(C)}{|C|}.$$

Any efficient project of C in $\hat{\pi}_C$ is also efficient in $\hat{\pi}$ and vice versa. We replace $d_k(\hat{\pi}, \{k\}, t_k^{\{k\}})$ and $d_k(\hat{\pi}_C, \{k\}, t_k^{\{k\}})$ from (3). If $k \in F(\hat{\pi}, C), F(\hat{\pi}_C, C)$ we have

$$b_k + \sum_{j \in F(\hat{\pi},C)} e_{kj} + \sum_{j \in F(\hat{\pi},N \setminus C)} e_{kj} - c_k + \frac{s(G)}{n} > b_k + \sum_{j \in F(\hat{\pi}_C,C)} e_{kj} + \sum_{j \in F(\hat{\pi}_C,N \setminus C)} e_{kj} - c_k + \frac{s(C)}{|C|}.$$

But, if $k \notin F(\hat{\pi}, C), F(\hat{\pi}_C, C)$ we have

$$\sum_{j \in F(\hat{\pi},C)} e_{kj} + \sum_{j \in F(\hat{\pi},N \setminus C)} e_{kj} + \frac{s(G)}{n} > \sum_{j \in F(\hat{\pi}_C,C)} e_{kj} + \sum_{j \in F(\hat{\pi}_C,N \setminus C)} e_{kj} + \frac{s(C)}{|C|}.$$

They both reduce to

$$\sum_{j \in F(\hat{\pi},C)} e_{kj} + \sum_{j \in F(\hat{\pi},N \setminus C)} e_{kj} + \frac{s(G)}{n} > \sum_{j \in F(\hat{\pi}_C,C)} e_{kj} + \sum_{j \in F(\hat{\pi}_C,N \setminus C)} e_{kj} + \frac{s(C)}{|C|}.$$

The members of C are singletons in the coalition structures $\hat{\pi}$ and $\hat{\pi}_C$. They fund the same projects and generate the same externalities for the country k,

$$\sum_{j \in F(\hat{\pi},C)} e_{kj} = \sum_{j \in F(\hat{\pi}_C,C)} e_{kj}.$$

Thus,

$$s(G) > \frac{n}{|C|} s(C) + n\Big(\sum_{j \in F(\hat{\pi}_C,N \setminus C)} e_{kj} - \sum_{j \in F(\hat{\pi},N \setminus C)} e_{kj}\Big).$$

■

Proof of Proposition 4. In case the full cooperation prevails, the aggregate welfare is

$$W(G) = u_k(G, G, t_k^G) + u_l(G, G, t_l^G) + u_m(G, G, t_m^G).$$

If we replace the payoff levels from (6), we have that:

$$W(G) = d_k(\hat{\pi}, \{k\}, t_k^{\{k\}}) + d_l(\hat{\pi}, \{l\}, t_l^{\{l\}}) + d_m(\hat{\pi}, \{m\}, t_m^{\{m\}}) + s(G).$$

However, if a fragmented coalition structure $\pi = \{C, \{m\}\}$ with $C = \{k, l\}$ prevails, the aggregate welfare is

$$W(\pi) = u_k(\pi, C, t_k^C) + u_l(\pi, C, t_l^C) + u_m(\pi, \{m\}, t_m^{\{m\}}). \tag{15}$$

The first two terms in (15) are the aggregate welfare of the countries in the coalition C and the third term is the payoff of the country that is not associated in π. We replace $u_k(\pi, C, t_k^C)$ and $u_l(\pi, C, t_l^C)$ from (6). Note that $u_m(\pi, \{m\}, t_m^{\{m\}})$ is $d_m(\hat{\pi}, \{m\}, t_m^{\{m\}})$ plus the externalities that generate the projects that are funded due to the formation of C : $\sum_{j \in F(\pi,C)} e_{mj} - \sum_{j \in F(\hat{\pi})} e_{mj}$. We thus get that

$$W(\pi) = d_k(\hat{\pi}_C, \{k\}, t_k^{\{k\}}) + \frac{s(C)}{2} + d_l(\hat{\pi}_C, \{l\}, t_l^{\{l\}}) + \frac{s(C)}{2}$$

$$+ d_m(\hat{\pi}, \{m\}, t_m^{\{m\}}) + \sum_{j \in F(\pi,C)} e_{mj} - \sum_{j \in F(\hat{\pi})} e_{mj}.$$

As in $\hat{\pi}_C$ and $\hat{\pi}$ all countries are separate, $d_k(\hat{\pi}_C, \{k\}, t_k^{\{k\}}) = d_k(\hat{\pi}, \{k\}, t_k^{\{k\}})$ and $d_l(\hat{\pi}_C, \{l\}, t_l^{\{l\}}) = d_l(\hat{\pi}, \{l\}, t_l^{\{l\}})$. Then, if the full cooperation of the countries does not prevail, but instead it prevails the fragmented coalition structure π, the countries suffer a welfare loss equal to:

$$W(G) - W(\pi) = s(G) - [s(C) + \sum_{j \in F(\pi,C)} e_{mj} - \sum_{j \in F(\hat{\pi})} e_{mj}].$$

■

Proof of Corollary 3. From (6), the aggregate welfare for G is $\sum_{k \in N} d_k(\hat{\pi}, \{k\}, t_k^{\{k\}}) + s(G)$ and for $\hat{\pi}$ is

$$\sum_{k \in N} u_k(\hat{\pi}, \{k\}, t_k^{\{k\}}) = \sum_{k \in N} d_k(\hat{\pi}, \{k\}, t_k^{\{k\}}).$$

Then, the difference in aggregate welfare under G and $\hat{\pi}$ is $W(G) - W(\hat{\pi}) = s(G)$. ■

Proof of Proposition 5. As mentioned earlier, we consider only one deviation at a time, either multilateral or unilateral. Any deviation from the grand coalition leads to a fragmented coalition structure with two coalitions: $\{C, N \backslash C\}$. The coalition C consists of the countries that deviate from the grand coalition and the coalition $N \backslash C$ consists of the rest of the countries.

In case where the full cooperation prevails, the aggregate welfare is

$$W(G) = \sum_{k \in N} u_k(G, G, t_k^G).$$

We replace $u_k(G, G, t_k^G)$ from (6) and have:

$$W(G) = \sum_{k \in N} d_k(\hat{\pi}, \{k\}, t_k^{\{k\}}) + s(G).$$

However, if the fragmented coalition structure $\pi = \{C, N \backslash C\}$ prevails, the aggregate welfare is

$$W(\pi) = \sum_{k \in C} u_k(\pi, C, t_k^C) + \sum_{k \in N \backslash C} u_k(\pi, N \backslash C, t_k^{N \backslash C}).$$

The first term in the above is the aggregate welfare of the countries in the coalition C and the second the aggregate welfare of the countries in the coalition $N \backslash C$. We replace $u_k(\pi, C, t_k^C)$ and $u_k(\pi, N \backslash C, t_k^{N \backslash C})$ from (6) and get that,

$$W(\pi) = \sum_{k \in C} d_k(\hat{\pi}_C, \{k\}, t_k^{\{k\}}) + s(C) + \sum_{k \in N \backslash C} d_k(\hat{\pi}_{N \backslash C}, \{k\}, t_k^{\{k\}}) + s(N \backslash C).$$

Note that $\forall k \in C$, $d_k(\hat{\pi}_C, \{k\}, t_k^{\{k\}})$ is $d_k(\hat{\pi}, \{k\}, t_k^{\{k\}})$ plus the externalities that generate the projects that are funded due to the formation of $N \backslash C$. Similarly, $\forall k \in N \backslash C$, $d_k(\hat{\pi}_{N \backslash C}, \{k\}, t_k^{\{k\}})$ is $d_k(\hat{\pi}, \{k\}, t_k^{\{k\}})$ plus the externalities that generate the projects that are funded due to the formation of C. We thus have that,

$$W(\pi) = \sum_{k \in C} d_k(\hat{\pi}, \{k\}, t_k^{\{k\}}) + \sum_{k \in C} \Big(\sum_{j \in F(\hat{\pi}_C, N \backslash C)} e_{kj} - \sum_{j \in F(\hat{\pi}, N \backslash C)} e_{kj} \Big) + s(C)$$

$$+ \sum_{k \in N \backslash C} d_k(\hat{\pi}, \{k\}, t_k^{\{k\}}) + \sum_{k \in N \backslash C} \Big(\sum_{j \in F(\hat{\pi}_{N \backslash C}, C)} e_{kj} - \sum_{j \in F(\hat{\pi}, C)} e_{kj} \Big) + s(N \backslash C),$$

or

$$W(\pi) = \sum_{k \in N} d_k(\hat{\pi}, \{k\}, t_k^{\{k\}}) + s(C) + \sum_{k \in C} \Big(\sum_{j \in F(\hat{\pi}_C, N \backslash C)} e_{kj} - \sum_{j \in F(\hat{\pi}, N \backslash C)} e_{kj} \Big)$$

$$+ s(N \backslash C) + \sum_{k \in N \backslash C} \Big(\sum_{j \in F(\hat{\pi}_{N \backslash C}, C)} e_{kj} - \sum_{j \in F(\hat{\pi}, C)} e_{kj} \Big).$$

Then, if the full cooperation of the countries does not prevail, but instead it prevails the fragmented coalition structure π, the countries suffer a welfare loss equal to:

$$W(G) - W(\pi) = s(G) - \Big[s(C) + \sum_{k \in C} \Big(\sum_{j \in F(\hat{\pi}_C, N \backslash C)} e_{kj} - \sum_{j \in F(\hat{\pi}, N \backslash C)} e_{kj} \Big) \Big]$$

$$- [s(N\backslash C) + \sum_{k\in N\backslash C} (\sum_{j\in F(\hat{\pi}_{N\backslash C},C)} e_{kj} - \sum_{j\in F(\hat{\pi},C)} e_{kj})].$$

■

Appendix C

It is noteworthy that in an economy with $n = 3$, the conditions which ensure that either a group of countries or a single country cannot improve upon their payoffs in case the of the $\gamma - core$ are (10) and $s(G) > 0$ respectively. As in Proposition 1, the condition (10) ensures that no multilateral deviations from the grand coalition are profitable. Any unilateral deviation from the grand coalition in the $\gamma - game$ results in a complete breakdown of the grand coalition and the condition $s(G) > 0$ is sufficient for such deviations not to be profitable.[29] As noted earlier, (10) implies that $s(G) > 0$.

In comparison to the $\gamma - core$, the conditions an economy has to satisfy so that its payoffs are in the $\delta - core$ are more restrictive. Consequently, if the payoffs of the grand coalition are in the $\delta - core$ of the game (N, u), they are also in the $\gamma - core$.

The above results are consistent with the general case. The payoffs of the grand coalition are in the $\gamma - core$ of the game (N, u) if $\frac{s(G)}{n} > \frac{s(C)}{|C|}$ and $s(G) > 0$. To verify this, recall that in the $\gamma - game$, every deviation from the grand coalition causes in a complete breakdown of cooperation and results in the fully fragmented coalition structure $\hat{\pi}$. But the term $\sum_{j\in F(\hat{\pi}_C,N\backslash C)} e_{kj} - \sum_{j\in F(\hat{\pi},N\backslash C)} e_{kj}$ in (13) is zero for $\hat{\pi}$. Note that $\frac{s(G)}{n} > \frac{s(C)}{|C|}$ implies that the surplus of the grand coalition is positive.

[29] A unilateral deviation of country k is not profitable as long as

$$u_k(G, G, t_k^G) > u_k(\hat{\pi}, \{k\}, t_k^{\{k\}}).$$

We replace $u_k(G, G)$ from (6), we recall that $u_k(\hat{\pi}, \{k\}, t_k^{\{k\}})$ coincides with $d_k(\hat{\pi}, \{k\}, t_k^{\{k\}})$ and we are left with $s(G) > 0$.

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