Upward Pricing Pressure formulations with logit demand and endogenous partial acquisitions

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Abstract

The aim of this paper is to derive the formula of Gross Upward Pricing Pressure Index (*GUPPI*), used on duopoly markets with differentiated products, when we allow for unilateral equity stakes (expressed as a function of victim's market share) to be endogenously determined. The results show that the unilateral effects of partial acquisitions, as they are measured by *GUPPI* when the percentage of equity stakes of the acquirer in the target firm is considered endogenous, may be higher than in the case where the said percentage is exogenously determined.

JEL classifications: G3, L13, L16

Keywords: Differentiated Product Markets; *GUPPI*; Logit Demand; Endogenous Partial Acquisitions.

1 Introduction

Salop and Moresi (2009) were the first who developed a modified version of Upward Pricing Pressure (UPP) methodology to market definition called Gross Upward Pricing Pressure Index ($GUPPI$).[1] According to this in markets with differentiated products $GUPPI$ 'measures only the upward pricing component before netting out the downward pricing pressure

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[1] The two methodologies are based on Bertrand competition with differentiated products. The *UPP* methodology measures the merger induced unilateral effects net of any potential efficiencies emerged from the merger. Following Farrell and Shapiro (2010) the *UPP* in pre-merger values on product 1 is defined as $UPP_1 = DR_{12} \times l_2 \times \frac{p_2}{p_1} - E_1(1 - l_1)$, where DR_{12} is the diversion ratio from product 1 to product 2

from efficiencies' (Moresi, 2010). Mathematically, suppose the merger between products 1 and 2. The $GUPPI$ for product 1 is defined as $GUPPI = DR_{12} \times l_2 \times (p_2/p_1)$.[2] Merger causes gross upward pricing pressure if $GUPPI > 0$.

In this paper we derive the formula of $GUPPI$ in duopoly markets with unilateral partial acquisitions and rough information regarding the structure of products' demand. For this reason we use a logit demand function (Anderson and de Palma, 1990)[3] and we endogenise the amount of acquired equity stake with respect to the market share of the victim firm. The rationale behind this follows directly from Willig (1991) who, inter alia, states that 'the bigger 2's share, the more the merger will drive up the price of 1, and obversely'.

Despite the rich body of literature concerning unilateral effects of partial acquisitions, none of the existing studies has used logit demand function in order to calculate $GUPPI$.[4] Besides, to the best of our knowledge equity holdings have never been used endogenously with respect to victim firm's market share. Hence, the novelty of this paper is to provide an alternative index for measuring unilateral effects of partial acquisitions, consistent with traditional horizontal merger analysis.

The remaining of the paper is organized in the following way: Section 2 presents the basic model set up and the results. Lastly, section 3 provides some policy implications and section 4 concludes.

2 The model

In the pre-acquisition stage, each firm i ($i = 1, 2$) chooses its price p_i to maximize its profits $(p_i - c_i^0)Q^i(p_i, p_{-i})$ where c_i^0 denotes the marginal cost of firm i, and Q^i and p_{-i} are the demand function and the price of the ith competitor, respectively. By assuming a logit demand function, demand for good i will be

$$Q^i(p_i, p_{-i}) = \frac{e^{-\gamma p_i}}{\sum_{j=1}^{2} e^{-\gamma p_j}} \tag{1}$$

where $\gamma \in (0, 1)$ is a positive constant denoting the rate of substitution between the

(Shapiro, 1996; Hausman et al. 2011), $l_2 \equiv \frac{p_2 - c_2}{p_2}$ is the variable profit margin of product 2 as a fraction of revenue, $\frac{p_2}{p_1}$ is the price of product 2 relative to price of product 1, E_1 denotes the merger-induced variable cost savings for Product 1 and $l_1 \equiv \frac{p_1 - c_1}{p_1}$ is the variable profit margin of product 1 as a fraction of revenue. In the symmetric case the UPP for each merger product is $UPP = DR \times \frac{l}{1-l} - E$. The merger causes upward pricing pressure if $UPP_1 > 0$ (or $UPP > 0$).

[2] If we assume $p_2 = p_1$ then $GUPPI = DR_{12} \times l_2$ (Salop and Moresi, 2009).

[3] Logit demand is based on Luce's Choice Axiom. See Luce (1959) and Willig (1991).

[4] Willig (2011) calculates UPP and $GUPPI$ using a general demand function under Bertrand competition with differentiated products.

products (the lower the γ, the greater the differentiation between products).

The equilibrium prices p_i^0 in the pre-acquisition stage are given by the solution to the following system of equations:

$$-(p_1^0 - c_1^0)\gamma Q^1(p_1^0, p_2^0)Q^2(p_1^0, p_2^0) + Q^1(p_1^0, p_2^0) = 0 \tag{2}$$

$$-(p_2^0 - c_2^0)\gamma Q^1(p_1^0, p_2^0)Q^2(p_1^0, p_2^0) + Q^2(p_1^0, p_2^0) = 0 \tag{3}$$

In the post-acquisition stage where m percent of firm 2 is acquired by firm 1, the profits of firm 1 are given by

$$\Pi_1^m = (p_1 - c_1^0)Q^1(p_1, p_2) + m(p_2 - c_2^0)Q^2(p_1, p_2) \tag{4}$$

In (4), we assume that the marginal costs do not change in the post-acquisition stage. In contrast to the existing literature about $GUPPI$ calculation, m is determined endogenously in our analysis. More specifically m is assumed to be a function of target firm's market share, i.e. $m = m(Q^2/(Q^1 + Q^2)) = m(p_1, p_2)$. Hence, the post-acquisition change in the profits of firm 1 with respect to a change in p_1 is given by

$$\begin{aligned}
\frac{\partial \Pi_1^m}{\partial p_1} &= -(p_1 - c_1^0)\gamma Q^1(p_1, p_2)Q^2(p_1, p_2) + Q^1(p_1, p_2) \\
&\quad + m'(p_1, p_2)\gamma Q^1(p_1, p_2)Q^2(p_1, p_2)(p_2 - c_2^0)Q^2(p_1, p_2) \\
&\quad + m(p_1, p_2)(p_2 - c_2^0)\gamma Q^1(p_1, p_2)Q^2(p_1, p_2)
\end{aligned} \tag{5}$$

where $m' > 0$ is the derivative of m with respect to $Q^2/(Q^1 + Q^2)$ with $m'' < 0$.[5] Evaluating (5) at the pre-acquisition price levels (Willig, 2011), we get

$$\begin{aligned}
\left.\frac{\partial \Pi_1^m}{\partial p_1}\right|_{p_i = p_i^0} &= -(p_1^0 - c_1^0)\gamma Q^1(p_1^0, p_2^0)Q^2(p_1^0, p_2^0) + Q^1(p_1^0, p_2^0) \\
&\quad + m'(p_1^0, p_2^0)\gamma Q^1(p_1^0, p_2^0)Q^2(p_1^0, p_2^0)(p_2^0 - c_2^0)Q^2(p_1^0, p_2^0) \\
&\quad + m(p_1^0, p_2^0)(p_2^0 - c_2^0)\gamma Q^1(p_1^0, p_2^0)Q^2(p_1^0, p_2^0)
\end{aligned} \tag{6}$$

From (2) and by rearranging, we get that the condition for UPP is

[5] Since we focus on partial rather than full acquisitions, we assume here that m increases as the market share of the victim firm increases but at a decreasing rate.

$$m(p_1^0, p_2^0) \times \frac{(p_2^0 - c_2^0)}{p_1^0} \times [1 + \epsilon_m^0] > 0 \tag{7}$$

where ϵ_m^0 is the elasticity of m with respect to the target firm's market share evaluated at the pre-acquisition prices.[6]

Proposition 1 *If the percentage of equity stakes of the acquirer in the target firm is endogenously determined by the market share of the target firm and demand is approximated by the logit specification in (1), then the GUPPI is given by*

$$GUPPI^{end} = m(p_1^0, p_2^0) \times \frac{(p_2^0 - c_2^0)}{p_1^0} \times [1 + \epsilon_m^0] \tag{8}$$

Proposition 2 *The value for the $GUPPI$ with exogenous percentage of partial equity stakes, $\tilde{m}$, as per Willig (2011) and the logit demand function in (1) is*

$$GUPPI_{\log it}^{ex} = \tilde{m} \times \frac{(p_2^0 - c_2^0)}{p_1^0} \tag{9}$$

Combining Propositions 1 and 2, we get Proposition 3

Proposition 3 *If the demand is approximated by the logit specification in (1), then the $GUPPI$ is downward biased when the percentage of equity stakes of the acquirer in the target firm is assumed to be exogenous. The degree of biasness is captured by ϵ_m^0.*

Proof. *Proposition 3 comes straightforwardly from Propositions 1 and 2. If we pick a value for $\tilde{m}$ which is equal to $m(p_1^0, p_2^0)$, then it can be easily shown that $GUPPI^{end} - GUPPI_{\log it}^{ex} = GUPPI_{\log it}^{ex} \times \epsilon_m^0 > 0$.*[7] ■

According to Proposition 3, there is a degree of biasness between Willig's model of $GUPPI$ and our specification. Specifically, we argue that when one out of the two interrelated hypotheses in our specification is violated (i.e. logit demand assumption is satisfied but m continues to be exogenous) then Willig's $GUPPI$ exhibits a downward biasness. The level of this biasness is measured by the elasticity of m with the respect to target firm's pre-acquisition market share. More specifically, the higher the elasticity, the higher the downward biasness of Willig's $GUPPI$.

The analogous expression for (9) if we assume linear demand function of the form $Q^i = \frac{a - p_i - \gamma(a - p_j)}{1 - \gamma^2}$ (where $a > 0$)[8] is given by

[6] Note that in (7) the "diversion" ratio from product 1 to product 2 (Shapiro, 1996; Hausman et al. 2011) is equal to 1. In our model market shares rather than demand elasticities play crucial role in determining $GUPPI$.

[7] Dividing both sides by $GUPPI_{\log it}^{ex}$, we get that the percentage change in $GUPPI$s is equal to ϵ_m^0.

[8] See Singh and Vives (1984) and Alipranti et al. (2014).

$$GUPPI_{linear}^{ex} = \tilde{m} \times \frac{(\tilde{p}_2^0 - c_2^0)}{\tilde{p}_1^0} \times \gamma \tag{10}$$

where $\tilde{p}_i^0$ (for $i = 1, 2$) denotes the equilibrium pre-acquisition price of firm i under the aforementioned linear demand function and γ is the diversion ratio (the diversion ratio coincides with the rate of substitution between the products).[9]

It can be further shown that the downward bias of the $GUPPI$ with an exogenous m is evident for any functional form of the demand function.[10]

3 Policy implications

In this paper we derive an alternative index for measuring unilateral effects of partial acquisitions. We focus on markets with differentiated products and we show that with endogenous acquired equity stakes (expressed as a function of victim's market share) and logit demand the unilateral effects of partial acquisitions, as they are measured by $GUPPI$, may be higher than in the case where the minority shareholdings are not endogenously determined.

Our $GUPPI$ specification is consistent with traditional horizontal merger analysis which is mainly based on market shares in order to assess the effects of partial acquisitions on consumer welfare.

4 Conclusion

The scope of this paper is to develop a formula of $GUPPI$ in duopoly markets with unilateral partial acquisitions and rough information about the products' demand structure. For this reason we use a logit demand function and we endogenise the amount of acquired equity stake with respect to the market share of the victim firm.

The results show that if the percentage of equity stakes of the acquirer in the target firm is considered exogenous, then the $GUPPI$ is downward biased. In other words, the unilateral effects of partial acquisitions may be lower with exogenously rather than with endogenously determined minority shareholdings.

[9] For a more detailed discussion about the $GUPPI$ under linear demand functions see Hausman et al. (2011).

[10] For instance, the $GUPPI$ formula in Willig (2011) with endogenous m and general functional form of demand function is given by $m(\hat{p}_1^0, \hat{p}_2^0) \times \frac{(\hat{p}_2^0 - c_2^0) DR_{12}}{\hat{p}_1^0} \times [1 + \frac{\epsilon_m^0 Q^2(\hat{p}_1^0, \hat{p}_2^0)}{Q^1(\hat{p}_1^0, \hat{p}_2^0) + Q^2(\hat{p}_1^0, \hat{p}_2^0)} \times (\frac{Q^1(\hat{p}_1^0, \hat{p}_2^0)}{Q^2(\hat{p}_1^0, \hat{p}_2^0)} + \frac{1}{DR_{12}})]$ (where $\hat{p}_1^0$, $\hat{p}_2^0$ are the pre-acquisition prices under general demand function). The downward bias is given by the term $\frac{\epsilon_m^0 Q^2(\hat{p}_1^0, \hat{p}_2^0)}{Q^1(\hat{p}_1^0, \hat{p}_2^0) + Q^2(\hat{p}_1^0, \hat{p}_2^0)} \times (\frac{Q^1(\hat{p}_1^0, \hat{p}_2^0)}{Q^2(\hat{p}_1^0, \hat{p}_2^0)} + \frac{1}{DR_{12}}) > 0$.

We may derive different results if we assume cost asymmetries or/and possible efficiencies emerged from the acquisitions. Besides, bilateral equity stakes between firms may also play a critical role in our specification. Therefore, further research may be based on these considerations.

References

[1] Alipranti, M., Milliou, C., and Petrakis, E. (2014). Price vs. quantity competition in a vertically related market. *Economics Letters* 124(1), 122-126.

[2] Anderson, P. S., and de Palma, A. (1990). The logit as a model of product differentiation: Further results and extensions. Discussion Paper No 913. Northwestern University.

[3] Farrell, J., and Shapiro, C. (2010). Antitrust evaluation of horizontal mergers: An economic alternative to market definition. *The B.E. Journal of Theoretical Economics (Policies and Perspectives)* 10(1), 1-39.

[4] Hausman, J., Moresi, S., and Rainey, R. (2011). Unilateral effects of mergers with general linear demand. *Economics Letters* 111(2), 119-121.

[5] Luce, R. D. (1959). Individual Choice Behavior: Theoretical Analysis. John Wiley and Sons.

[6] Moresi, S. (2010). The use of upward price pressure indices in merger analysis. *Antitrust Source* 9(3), 1-12.

[7] Salop, C. S., and Moresi, S. (2009). Updating the merger guidelines: Comments (available at http://www.ftc.gov/os/comments/horizontalmergerguides/545095-00032.pdf).

[8] Shapiro, C., (1996). Mergers with differentiated products. *Antitrust* 10(2), 23-30.

[9] Singh, N., and Vives, X. (1984). Price and quantity competition in a differentiated duopoly. *RAND Journal of Economics* 15, 546-554.

[10] Willig, D. R. (1991). Merger analysis, industrial organization theory, and merger guidelines. *Brookings Papers on Economic Activity: Microeconomics*, 281-332.

[11] Willig, D. R. (2011). Unilateral competitive effects of mergers: Upward pricing pressure, product quality, and other extensions. *Review of Industrial Organization* 39(1-2), 19-38.