Collective Choice in Dynamic Public Good Provision: Real versus Formal Authority

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Abstract

Who has real authority to decide the scope of a public project? We study a dynamic game in which two heterogeneous agents exert effort over time to bring a public project to completion, and the project scope can be determined at any point in time via collective choice. A larger project scope requires greater cumulative effort, and delivers higher benefits to both agents on completion. We show that for a given project scope, the efficient agent (*i.e.*, the one with the lower effort cost) exerts more effort than the inefficient agent at every stage of the project in the unique Markov Perfect equilibrium. We show that the efficient agent always prefers a smaller project scope than the inefficient agent. Preferences over project scope, however, are *time-inconsistent* - the efficient agent would like to reduce the scope of the project as it progresses, whereas the inefficient agent would like to increase it. We study the equilibrium project scope under commitment and no commitment. With commitment real authority is equivalent to formal authority. With no commitment the efficient agent always has real authority and at completion wants to discontinue the project, whereas the inefficient agent would like to continue the project. We show that in any institution the efficient agent chooses a project scope that is too small, whereas the inefficient agent chooses a project scope that is too large.

1 Introduction

In many economic settings agents must collectively decide the goal or *scope* of a public project. A greater scope reflects a more ambitious project and requires more effort from each agent, but yields greater reward upon completion. In such settings, if agents' preferences over the project scope are aligned, then the natural choice for project scope is the mutually agreed ideal and there will be little debate over the project scope. Yet it is common to find disagreement about when to complete a public project and at what stage. Examples include climate agreements, trade agreements and early stage entrepreneurial ventures. The Kyoto Protocol, for example, has seen several initial signatories exit the agreement, and others refusing to implement parts of the agreement.[1] The General Agreements on Tariffs and Trade (GATT) has been mired in disagreement and stagnation throughout the Doha round, and the debate often pits developed against developing countries. In business ventures, there is often dissent on when a project is ready to be marketed. Central to many of these conflicts is the asymmetry between participants. In this paper we investigate making decisions on the scope of a public project that requires voluntary effort when agents' costs of effort are different. In this setting, we ask who has real authority to make the decision on the project scope.

We begin by establishing the outcome of a voluntary contribution game when agents are heterogeneous with respect to effort costs. It is well-established that free-riding occurs when agents must make voluntary contributions to a public project. We have a good understanding of the outcome when agents are symmetric, and of basic comparative statics (e.g., the effect of changes in effort costs, discount rates, team size, payoffs, etc.). However, relatively little is known about this problem when agents are heterogeneous. We study a model in which agents voluntarily exert costly effort over time to complete a public project, agents differ in the cost of their effort, the reward is shared by all agents on completion, and the reward increases with a larger project scope. The agent with the lower effort cost is the *efficient* agent and the agent with the higher effort cost is *inefficient*. We show that the efficient agent exerts more effort at every stage of the project than the efficient agent, and moreover, gets a lower discounted payoff at every stage of the project. In spite of having lower cost of effort, the magnitude of effort exerted by the efficient agent penalizes the efficient agent to the extent that his discounted payoff is lower.

We study the choice of project scope when it can be selected at any time by collective choice, and consider the implications for real and formal authority. We say that an agent has formal authority if the institution is such that he is appointed as a dictator. We say that an agent as real authority when he has control over the project scope. That is, an agent has real authority over the project scope if at the time the project scope is decided, it is the agent's ideal project scope. We consider two institutions - dictatorship and unanimity. Under each institution we consider that agents either have the ability to write a binding contract on the scope of the project (i.e. can commit to not renegotiate), or do not have this ability. With commitment the project scope

[1] See Nordhaus (2015) for a recent discussion.

is decided at the start of the project. In this case, either agent as dictator achieves his ex-ante ideal project scope, thus real and formal authority are equivalent with commitment. If binding contracts cannot be written, then the project scope is not decided until completion. That is, at every moment the agents decide whether to complete the project immediately, or to keep exerting effort and re-evaluate their decision to complete the project a moment later. The efficient agent as dictator achieves his ideal project scope at completion, and so has real authority. However, when the inefficient agent is a dictator a continuum of equilibria exist. In each equilibrium, at completion the efficient agent wishes to stop the project, but the inefficient agent would prefer to continue. These are also the equilibrium outcomes under unanimity. Thus with no commitment, the efficient agent retains veto power and formal authority is not equivalent to real authority.

We demonstrate that with asymmetric agents under either institution an inefficient project scope is selected. The efficient agent will choose a project scope that is too small and the choice of the inefficient agent will be too large.

This paper joins a large political economy literature studying collective decisions when agents preferences are heterogeneous including the seminal work of Romer and Rosenthal (1979). More recently the literature has turned its attention to the dynamics of collective decision making including papers by Baron (1996), Battaglini and Coate (2008), Strulovici (2010), Bowen, Chen and Eraslan (2014), Diermeier and Fong (2011), Dixit, Grossman and Gul (2000). Other papers, for example, Lizzeri and Persico (2001), have looked at alternative collective choice institutions. Our paper joins this literature by studying the collective choice of agents deciding the scope of a long-term public project, and compares the outcomes under two different institutions - dictatorship and unanimity.

Numerous papers look at the problem of agents providing voluntary contributions to a public good including Admati and Perry (1991), Georgiadis, Lippman and Tang (2014), Georgiadis (2015), Bonatti and Rantakari (2015), Battaglini, Nunnari and Palfrey (2014) to name a few. We contribute to this literature by considering asymmetric agents and the endogenous choice of the terminal state of the public good.

We study agents reaching agreement to make voluntary contributions without the ability to commit. This relates to a large number of papers studying international agreements. Several of these study environmental agreements (for example, Nordhaus, 2015; Battaglini and Harstad, forthcoming), and trade agreements (see, Maggi, 2014).[2] To our knowledge this literature has not examined the dynamic selection of project scope (or goals) in these agreements with asymmetric agents, and identified this is a source of disagreement.

Our interest in real and formal authority relates to the literature studying formal and informal authority including Aghion and Tirole (1997), Callander (2008) and Hirsch and Shotts (2015). These papers focus on the role of information in determining real authority. In our paper the source of real authority is endowed ability to contribute to the public project. The efficient agent who wishes to cease the project before the inefficient agent has the ability to hold up the inefficient

[2] Bagwell and Staiger (2002) discuss the economics of trade agreements in depth. Others look at various aspects of specific trade agreements such as flexibility or forbearance in a non-binding agreement, (see, for example, Beshkar and Bond, 2010; Bowen, 2013).

agent in a sense.

The remainder of the paper is organized as follows. In Section 2 we present the basic model of two agents contributing to a public project and provide some preliminary results for the equilibrium with an exogenous project scope. In Section 4 we endogenize the project scope and examine the outcome under two collective choice institutions - dictatorship and unanimity. In Section 5 we analyze the determinants of real and formal authority under each collective choice institution. In Section 6 we discuss the welfare properties of the collective choice institutions.

2 Model

Two agents $i \in \{1, 2\}$ make voluntary contributions to complete a project. Time is continuous and indexed by $t \in [0, \infty)$. Agents are risk-neutral and discount time at common rate $r > 0$.

At any time t, the status of the project is summarized by the project state q_t, that represents the cumulative joint contribution of the agents at time t. The project starts at initial state $q_0 = 0$. At every time t, each agent i chooses his instantaneous effort level $a_{it} \geq 0$ and incurs flow cost $c_i(a_{it}) = \frac{\gamma_i}{2} a_{it}^2$. The agents' actions influence the state of the project q_t which evolves according to $dq_t = (a_{1t} + a_{2t})\, dt$. Both agents have complete information: at every time each agent observes the current project state and the effort of the the other agent.

The project is completed as soon as the project state hits the completion state, denoted by Q, and referred to as *project scope*. The project yields zero payoff to the agents before its completion. As soon as the project is completed, it yields a payoff $\alpha_i Q$ to agent i, where $\alpha_i \in [0, 1]$ is the share of agent i. If the project is completed at time τ (by convention $\tau = \infty$ if the project is never completed), then agent i gets discounted payoff

$$e^{-r\tau} \alpha_i Q - \int_0^\tau e^{-rt} \frac{\gamma_i}{2} a_{it}^2 dt\,.$$

In the sequel we assume without loss of generality that $\frac{\gamma_1}{\alpha_1} \leq \frac{\gamma_2}{\alpha_2}$. This implies that agent 1's marginal cost of effort relative to his stake in the project is less than agent 2's. That is, agent 1 is *relatively more efficient* than agent 2. If the benefit from the project is the same (that is $\alpha_1 = \alpha_2$) this implies that agent 1 is absolutely more efficient than agent 2.

In the baseline model analyzed in the next section, the project scope is exogenous and common knowledge.[3] In Section 4, we allow the agents to decide on the project scope via collective choice.

3 Preliminaries: exogenous project scope

This section analyzes the benchmark case in which the project scope Q is exogenous and common knowledge. We focus the analysis on Markov perfect equilibria, where the project state is chosen as state variable. A Markov strategy for agent i is a map $a_i(\cdot)$ that gives, at every time t, agent

[3]The benchmark case is similar to Georgiadis (2015) but in our setting agents are allowed to be asymmetric whereas in Georgiadis (2015) agents are symmetric.

i's effort $a_{it} = a_i(q_t)$ as a function of the project state q_t. If a state $q < Q$ is reached where $a_1(q) = a_2(q) = 0$, then the project stays at state q and is never completed.

Consider a Markov strategy profile $\{a_1(\cdot), a_2(\cdot)\}$, and let $J_i(q)$ be the discounted payoff of agent i when the project is at state q. Using standard arguments (Chang, 2004), at equilibrium if J_i is continuously differentiable on $[0, Q]$, J_i satisfies the Hamilton-Jacobi-Bellman (HJB) equation

$$rJ_i(q) = \max_{\hat{a}_i \geq 0} \left\{ -\frac{\gamma_i}{2}\hat{a}_i^2 + (\hat{a}_i + a_j(q)) J_i'(q) \right\}, \tag{1}$$

where j denotes the other agent, and J_i is satisfies the boundary condition $J_i(Q) = \alpha_i Q$.

If $J_i'(q) \geq 0$, the right-hand side is maximized for $\gamma_i \hat{a}_i = J_i'(q)$. That is, at every moment, the agent chooses his effort level such that the marginal cost of effort is equal to the marginal benefit associated with bringing the project closer to completion. Noting that $J_i'(q) < 0$ cannot occur in equilibrium,[4] in any equilibrium with continuously differentiable value functions, J_1 and J_2 satisfy the coupled (implicit) ordinary differential equations:[5]

$$rJ_i(q) = \frac{(J_i'(q))^2}{2\gamma_i} + \frac{1}{\gamma_j} J_i'(q) J_j',(q) \tag{2}$$

for $i \in \{1, 2\}$ and $j \neq i$, subject to

$$J_i(Q) = \alpha_i Q. \tag{3}$$

Any solution to the HJB equations rewritten as equations (2), (3) defines a Markov perfect equilibrium. For any equilibrium in which the project is not completed each agent receives a payoff of zero, so henceforth we focus on equilbria in which the project is completed. The first proposition gives necessary and sufficient conditions such that we have a unique project completing MPE.

Proposition 1. *For any Q, either*

(i) the project is not completed, in which case $J_i(q) = 0$ and $a_i(q) = 0$ for all q and all i, or

(ii) there is a unique project-completing MPE, in which case $J_i(q; Q) > 0$, $J_i'(q; Q) > 0$, $J_i''(q; Q) > 0$ and $a_i'(q; Q) > 0$, for all q and all i.

All proofs are in the Appendix.

It is intuitive that $J_i(\cdot)$ is strictly increasing: each agent is better off the closer the project is to completion. Agents discount time, and they incur the cost of effort at the time effort is exerted. Since they only receive their reward upon completion, they have stronger incentives the closer the project is to completion. An implication of this result is that efforts are strategic complements

[4]First note that at any state, each agent can guarantee himself a payoff of zero by not exerting any effort, so that in equilibrium $J_1, J_2 \geq 0$. If $J_i'(q) < 0$ at project state $q < Q$, then agent i should put no effort at state q. Then agent j also puts zero effort as otherwise $J_i(q) < 0$ which cannot occur in equilibrium. Thus the project at state q stays at state q and is never completed. Thus $J_1(q) = J_2(q) = 0$. If however $J_i'(q) < 0$ then $J_i(q + \epsilon)$ is negative for small enough ϵ, which again cannot occur in equilibrium.

[5]This system of ODE's can be normalized by letting $\tilde{J}_i(q; Q) = \frac{J_i(q;Q)}{\gamma_i}$. This becomes equivalent to a game in which $\gamma_1 = \gamma_2 = 1$, and agent i receives $\frac{Q}{2\gamma_i}$ upon completion of the project.

across time in this model.[6] By raising his effort, an agent brings the project closer to completion, thus inducing the other agent to raise his future efforts.

The next proposition establishes the main properties of the equilibrium with asymmetric agents.

Proposition 2. *In any Markov perfect equilibrium:*

1. *The relatively more efficient agent exerts more effort in every state, i.e.,* $a_1(q) \geq a_2(q)$*, and if* $\frac{\gamma_1}{\alpha_1} < \frac{\gamma_2}{\alpha_2}$ *then* $a_1(q) > a_2(q)$*.*

2. *If* $\alpha_1 = \alpha_2$ *then* $J_1(q) \leq J_2(q)$*. That is, the relatively more efficient agent receives a lower payoff if the project shares are equal.*

It is also straightforward to see that the more efficient agent works harder than the less efficient agent. What is perhaps surprising is the result that the more efficient agent obtains a lower discounted payoff than the less efficient agent. This is because the more efficient agent not only works harder at every moment, but he also incurs a higher cost of effort at every moment.

4 Endogenous project scope

Renee: Add intuition. In this section we endogenize the project scope Q. Project scope is selected by a collective choice institution. We consider two such institutional settings, dictatorship and unanimity with a permanent agenda-setter, both with and without commitment to not renegotiate.

4.1 Agent preferences over project scope

From Proposition 1 when Q is endogenous, agents will choose a project scope that gives each a strictly positive payoff.

To begin we characterize each agent's optimal project size, that is, the Q that maximizes the agent's discounted payoff given the current state q, and assuming both agents follow the MPE characterized in Proposition 1 for the project scope Q. To make the dependence on the project scope explicit, we now let $J_i(q;Q)$ be agent i's value function at project state q when the project scope is Q. Let $Q_i(q)$ denote agent i's ideal project size when the state of the project is q:

$$Q_i(q) = \arg\max_{Q \geq q} \{J_i(q;Q)\}.$$

Proposition 3. *Suppose* $J_i(q;Q)$ *is strictly concave in* Q *for all* i *and* q*. Consider agent* i*'s optimal project size* Q *when both agents choose their strategies based on the project scope* Q*. Then:*

1. *The efficient agent's ideal project size is always smaller than the inefficient agent's.*

2. *The efficient agent's ideal project size is decreasing in the state of the project.*

3. *The inefficient agent's ideal project size is increasing in the state of the project.*

[6] Strategic complementarity has been shown in earlier work with symmetric agents, as in, for example, Kessing (2007).

That is, if $\frac{\alpha_1}{\gamma_1} < \frac{\alpha_2}{\gamma_2}$, *then* $Q_1(q) < Q_2(q)$, $Q_1'(q) < 0$, *and* $Q_2'(q) > 0$ *for all* q; *and if* $\frac{\alpha_1}{\gamma_1} = \frac{\alpha_2}{\gamma_2}$, *then* $Q_1(q) = Q_2(q)$, *and* $Q_1'(q) = Q_2'(q) = 0$ *for all* q.

We know from Proposition 1 that at every state, the more efficient agent works harder and is worse off than the less efficient agent. In other words, the more efficient agent is the one who has to incur the majority of the cost to complete the project, and as a result, he prefers a smaller project than the less efficient agent. If agents are symmetric then they have the same preferences over the project scope.

Note that the agents' preferences over the scope of the project are time-inconsistent when they are asymmetric. That is, the less efficient agent would like to extend the project as it progresses, whereas the more efficient agent would like to shrink it. This time-inconsistency is due to the externality that each agent imposes on the other. Symmetric agents have time consistent preferences over project scope.Figure 1 illustrates Proposition 3 for asymmetric agents.

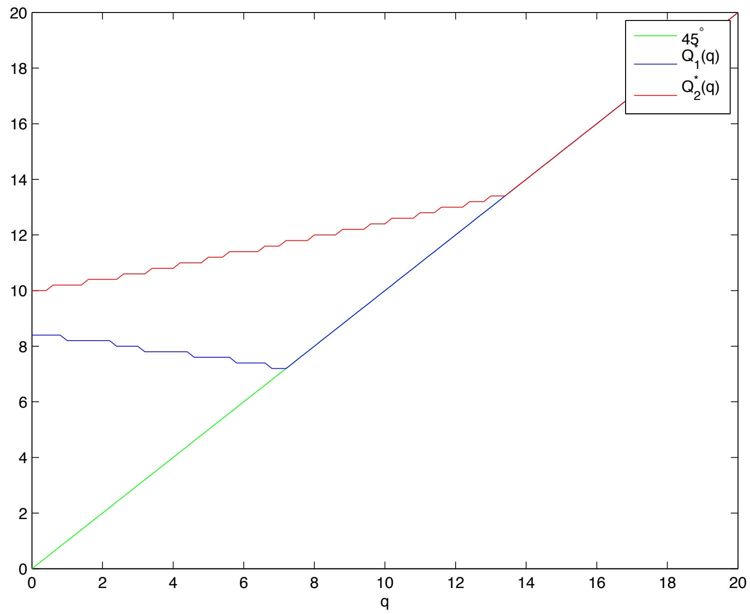

Figure 1: Agent i optimal project size $Q_i(q)$

The following lemma characterizes agent i's optimal project size when he works alone on the project, which will be useful for the subsequent analysis. We use this to characterize the equilibrium with endogenous project scope.

Lemma 1. *Suppose that agent* i *works alone on the project. Then his optimal project scope satisfies*

$$\hat{Q}_i(q) = \frac{1}{4r\,\gamma_i},$$

and it is independent of q. *Moreover,* $\hat{Q}_2(q) < \hat{Q}_1(q) < Q_1(q) < Q_2(q)$ *for all* q.

This lemma asserts that when an agent works in isolation, then his preferences over the scope of the project are time-consistent. Intuitively, this is because there are no externalities imposed on the agent and he solves a dynamic program by backward induction.[7] The second part of this lemma rank-orders the agents' ideal project sizes. If an agent works in isolation, then he cannot rely on the other to carry out part of the project, and therefore the less efficient agent prefers a smaller project scope than the more efficient one. Lastly, it is intuitive that the more efficient agent's ideal project size is larger when he works with the other agent relative to when he works alone, so $\hat{Q}_1(q) < Q_1(q)$ for all q.

4.2 Dictatorship

If agent i is the dictator then a Markov strategy for agent i at time t is a pair $(a_i(q_t), \theta_i(q_t))$. We restrict agent i to choose a project scope that is at least as great as the current state of the project. Let $\mathbb{Q}(q_t) = \{Q \in \mathbb{R}_+ \cup \emptyset : Q \geq q_t \text{ if } Q \in \mathbb{R}_+\}$ be the set of feasible project scopes. Then

$$\theta_i : \mathbb{R}_+ \to \mathbb{Q}(q_t).$$

We will say that if $\theta_i(q_s) \in \emptyset$ for all $s \leq t$, then there is no decision on the project scope at t.

In the case of *commitment:* $Q = \theta_i(q_T)$ where T is the first time at which $\theta_i(q_T) \in \mathbb{R}_+$. In the case of *no commitment:* $Q = \theta_i(q_{\hat{T}})$ where $\hat{T}$ is the first time at which $\theta_i(q_{\hat{T}}) = q_{\hat{T}}$. Denote

$$\overline{Q}_i = \min\{q : q = \theta_i^*(q)\}.$$

The next lemma shows the more efficient agent prefers to stop the project earlier than the less efficient agent.

Lemma 2. *We have the strict inequality $\overline{Q}_1 < \overline{Q}_2$. Moreover:*

$$\sqrt{\overline{Q}_1} = \frac{\sqrt{2r/3}a(\alpha_1/\gamma_1)}{r(\alpha_1/\gamma_1) + \frac{r}{12}[a+3b]^2}$$

and

$$\sqrt{\overline{Q}_2} = \frac{\sqrt{2r/3}a(\alpha_2/\gamma_2)}{r(\alpha_2/\gamma_2) + \frac{r}{12}[a-3b]^2}$$

where a and b are defined as

$$a = \sqrt{\frac{\alpha_1}{\gamma_1} + \frac{\alpha_2}{\gamma_2} + 2\sqrt{\left(\frac{\alpha_1}{\gamma_1}\right)^2 + \left(\frac{\alpha_2}{\gamma_2}\right)^2 - \frac{\alpha_1}{\gamma_1}\cdot\frac{\alpha_2}{\gamma_2}}}$$

and

$$b = \left(\frac{\alpha_1}{\gamma_1} - \frac{\alpha_2}{\gamma_2}\right) \Bigg/ \sqrt{\frac{\alpha_1}{\gamma_1} + \frac{\alpha_2}{\gamma_2} + 2\sqrt{\left(\frac{\alpha_1}{\gamma_1}\right)^2 + \left(\frac{\alpha_2}{\gamma_2}\right)^2 - \frac{\alpha_1}{\gamma_1}\cdot\frac{\alpha_2}{\gamma_2}}}.$$

Renee: Comparative statics?

Proposition 4. *Suppose $J_i(q;Q)$ is strictly concave in Q for all $Q \in [0, \overline{Q}_i]$. With commitment, if agent i is the dictator, then the unique equilibrium project scope is $\theta_i(q_0)$. With no commitment,*

[7] Similarly, if the agents are symmetric, then Georgiadis et al. (2014) shows that they are time-consistent with respect to their preferences over the project size.

if agent 1 is the dictator, then the unique equilibrium project scope is $\overline{Q}_1$, and if agent 2 is the dictator, then any $Q \in \left[\overline{Q}_1, \overline{Q}_2\right]$ can be part of an equilibrium.

It is clear that if an agent has dictatorship rights and can commit to some project scope at the outset of the game, then he will choose his ideal project size. Turning attention to the case in which the agents do not have the ability to commit, suppose first that agent 1 (i.e., the more efficient agent) has dictatorship rights. Note that $\overline{Q}_1 < \overline{Q}_2$ and both agents are better off if the project is completed at $\overline{Q}_1$ rather than any $q < \overline{Q}_1$. Therefore, the unique equilibrium has agent 1 completing the project at $\overline{Q}_1$. Next, suppose that agent 2 is the dictator. Fix any $Q \in \left[\overline{Q}_1, \overline{Q}_2\right]$ and suppose that each agent i plays action profile $a_i\left(q;Q\right)$ if $q < Q$ and exerts 0 effort for all $q \geq Q$. We shall argue that agent 2 finds it optimal to complete the project instantaneously, and this strategy is a best response to agent 1's strategy. Given that agent 1 exerts no effort for all $q \geq Q$, agent 2 would have to make any additional progress single-handedly, and noting that for any $Q > \hat{Q}_2\left(q\right)$, he does not have any incentive to deviate by exerting non-zero effort. Next, given the strategy of agent 2, because $Q \geq \overline{Q}_1$, agent 1 is better off if the project is completed instantaneously, and so he does not have an incentive to deviate from his strategy either. As such, this constitutes an equilibrium. [8]

4.3 Unanimity

For simplicity, under unanimity we assume agent $i \in \{1,2\}$ is the permanent agenda-setter but requires the approval of agent j to determine the project scope. A strategy for agent i at time t is $(a_i(q_t), \theta_i(q_t))$ where

$$\theta_i : \mathbb{R}_+ \to \mathbb{Q}(q_t).$$

A strategy for agent $j \neq i$ at time t is a $(a_j(q_t), Y_j(q_t, Q_j))$ where

$$Y_j : \mathbb{R}_+ \times \mathbb{R}_+ \to \{0,1\},$$

indicates if agent j agrees to agent j's proposal. Let $Q_0 \in \mathbb{R}_+$ be the exogenous status quo project scope in the event of no agreement. Under *commitment* with unanimity $Q = \theta_i(q_T)$ where T is the first time at which $(\theta_i(q_T), Y_j(q_T, \theta_i)) \in \mathbb{R}_+ \times \{1\}$. Under no commitment the status quo is simply the current state of the project, and is thus *endogenous*. That is, with no commitment agents effectively decide at every moment if they wish to complete the project immediately, or continue for one more instant.[9] Under *no commitment* with unanimity, $Q = \theta_i(q_{\hat{T}})$ where $\hat{T}$ is the first time at which $(\theta_i(q_{\hat{T}}), Y_j(q_{\hat{T}}, \theta_i)) \in \{Q : Q = q_{\hat{T}}\} \times \{1\}$.

[8] It is useful to note a simple refinement that eliminates the multiplicity of equilibria when agent 2 has dictatorship rights. Suppose that at every moment, given q_t, he can commit that $Q \geq q_t + \delta$, where $\delta > 0$ (*e.g.*, by making a public announcement in regards to the project scope that is costly to retract). Then the unique equilibrium project scope is $\overline{Q}_2^*$.

[9] Note this endogeneity of the status quo is not due to enforcement by an exogenous institution, as in Bowen et al. (2014), Bowen, Chen, Eraslan and Zapal (2015) and other papers, but rather arises naturally because of a lack of commitment to agree on a project completion state.

With commitment, the equilibrium with an exogenous status quo Q_0 is similar to what is found in Romer and Rosenthal (1979). If $Q_0 \in \left[\overline{Q}_1, \overline{Q}_2\right]$, then the agents will always agree to project scope Q_0. If $Q_0 < \overline{Q}_1$, then they will agree to a project scope closer to agent 1's ideal, and if $Q_0 > \overline{Q}_1$, they will agree to a project scope closer to agent 2's ideal. We focus on the more interesting case of no commitment in the next proposition.

Proposition 5. *Suppose $J_i\left(q;Q\right)$ is strictly concave in Q for all $Q \in \left[0, \overline{Q}_i\right]$. With no commitment, any $Q \in \left[\overline{Q}_1, \overline{Q}_2\right]$ can be part of an equilibrium.*

The intuition for why any project scope in the interval $Q \in \left[\overline{Q}_1, \overline{Q}_2\right]$ is similar to that for the case of agent 2 as the dictator. Suppose $Q \in \left[\overline{Q}_1, \overline{Q}_2\right]$. Fixing an action path for agent i such that $a_i(q) = 0$ for all $q \geq Q$, then agent $j \neq i$ has no incentive to exert effort beyond Q because it is not optimal to continue the project single-handedly. Similarly, it is optimal to exert effort up to Q because the discount payoff is positive with both agents exerting effort.

5 Real and Formal Authority

The equilibria of these games shed light on who has real authority to decide the scope of a public project. We say the institution grants agent i formal authority when agent i is appointed as the dictator. Real authority, however is determined by the agent that has greater control over project scope and can impose its ideal. We define real authority as follows.

Definition 1. Agent i has *real authority* if agent i can control the equilibrium project scope. That is, if project scope Q selected at time t satisfies $Q = Q_i(q_t)$ in any equilibrium.

The following is an immediate corollary to Proposition 4.

Corollary 1. *Under commitment if agent i is the dictator, agent i has real and formal authority. Under no commitment if agent 1 is the dictator, agent 1 has real and formal authority. If agent 2 is the dictator, agent 2 has formal authority but not real authority and agent 1 has real authority.*

When the efficient agent is a dictator he can credibly discontinue the project when it is his ideal because the inefficient agent will not continue the project single-handedly. There is no possibility for coordinating on an alternate project size because the efficient agent gives the signal. However, when the inefficient agent is the dictator, the efficient agent can "hold up" the inefficient agent in a sense, with the coordinated belief that he will stop before $\overline{Q}_2$. The same intuition holds for unanimity with no commitment. The following is an immediate corollary to Proposition 5.

Corollary 2. *Under unanimity and no commitment, neither agent has formal authority, but agent 1 has real authority.*

6 The efficient project scope

We compare the agents' choice of project scope to the project scope that maximizes the sum of discounted payoffs. Denote the project scope that maximizes the agents' total discounted payoff as $Q^*(q)$. This is

$$Q^*(q) = \arg\max \{J_1(q;Q) + J_2(q;Q)\}.$$

The following proposition shows that the efficient project size $Q^*(q)$ lies between the two agents' optimal project scopes for any state of the project.

Proposition 6. *Suppose that $J_i(q;Q)$ is strictly concave in Q for all i and q. Then the project scope that maximizes the agents' total discounted payoff $Q^*(q) \in (Q_1(q), Q_2(q))$.*

Note in general, $Q^*(q)$ will not be independent of q. That is, the "efficient" project scope is also time-inconsistent. We illustrate Proposition 3 in Figure 2 below.

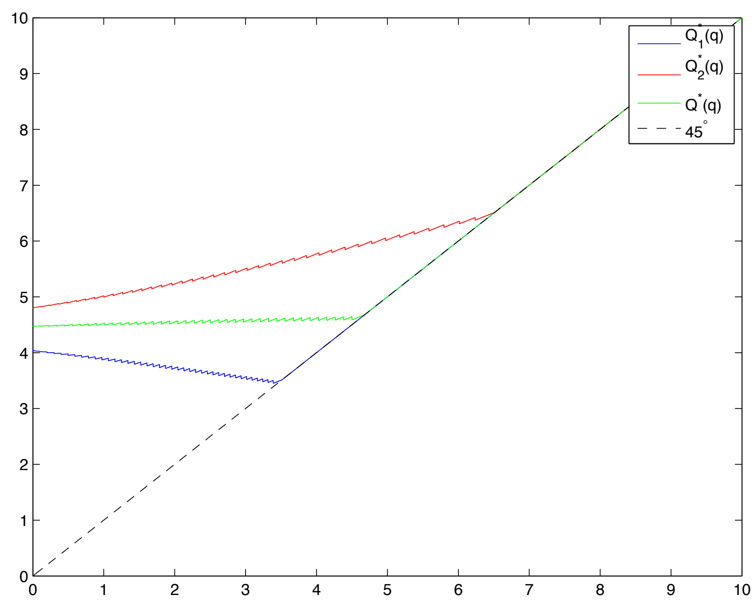

Figure 2: Efficient project scope $Q^*(q)$

Thus the efficient agent's ideal project scope is too low and the inefficient agent's project scope is too large. The efficient agent anticipates working harder than the inefficient agent thus wishes to quit the project sooner than is optimal and vice versa. A natural question is what institutions can induce greater efficiency. We explore this in future work.

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A Appendix

A.1 Proof of Proposition 1

We begin with the following lemma.

Lemma 3. *Given an arbitrary project scope Q, there exists a $\underline{q} < Q$ such that the game defined by (2) subject to the boundary condition (3) for each $i \in \{1, 2\}$ has a unique project-completing MPE if and only if $q_0 \geq \underline{q}$.*

Proof. Sufficiency follows from Cvitanić and Georgiadis (2015). From Cvitanić and Georgiadis (2015) we know given an arbitrary project scope Q, there exists a $\underline{q} < Q$ such that the game defined by (2) subject to the boundary condition (3) for each $i \in \{1, 2\}$ has a unique project-completing MPE on $\left(\underline{q}, Q\right]$. Moreover, $J_i\left(q;Q\right) > 0$, $J_i'\left(q;Q\right) > 0$, and $J_i''\left(q;Q\right) > 0$ for all i and $q > \underline{q}$; i.e., each agent's discounted payoff is strictly positive, strictly increasing, and, if twice differentiable, strictly convex in q. The latter also implies that each agent's effort is strictly increasing in q; i.e., $a_i'\left(q;Q\right) > 0$.

Necessity (George). □

Finally, observe that the system of implicit ODE can be rewritten into an explicit ODE. We will make use of the explicit ODE in the next section.

Lemma 4. *Suppose $Q > \bar{Q}$. Let $\tilde{J}_i = J_i/\gamma_i$. Then $(\tilde{J}_1, \tilde{J}_2)$ is the unique continuously differentiable solution to:*

$$\tilde{J}_1' = \sqrt{\frac{r}{6}}\sqrt{\sqrt{\tilde{J}_1^2 + \tilde{J}_2^2 - \tilde{J}_1\tilde{J}_2} + \tilde{J}_1 + \tilde{J}_2} + \sqrt{\frac{r}{2}}\sqrt{\sqrt{\tilde{J}_1^2 + \tilde{J}_2^2 - \tilde{J}_1\tilde{J}_2} - \tilde{J}_1 + \tilde{J}_2}$$

$$\tilde{J}_2' = \sqrt{\frac{r}{6}}\sqrt{\sqrt{\tilde{J}_1^2 + \tilde{J}_2^2 - \tilde{J}_1\tilde{J}_2} + \tilde{J}_1 + \tilde{J}_2} - \sqrt{\frac{r}{2}}\sqrt{\sqrt{\tilde{J}_1^2 + \tilde{J}_2^2 - \tilde{J}_1\tilde{J}_2} - \tilde{J}_1 + \tilde{J}_2},$$

with terminal condition $\tilde{J}_i(Q) = \frac{\alpha_i}{\gamma_i}Q$ for $i = 1, 2$.

The proof is relegated to the appendix.

Lemma 5. *For any Q such that $\underline{q} \geq q_0$ then $J_i(q_0) = 0$ for all $i \in \{1, 2\}$.*

Proof. (To be added) □

Lemma 6. *$\underline{q} \leq 0$ for two agent problem.*

Proof. Consider the single-agent problem. We can solve this in closed form and show that $\underline{q} \leq 0$. Since agents can only do better in the two-agent problem, it must be that $\underline{q} \leq 0$ for the two-agent problem. □

A.2 Proof of Proposition 2

For convenience, we use the normalization $\tilde{J}_i(q;Q) = \frac{J_i(q;Q)}{\gamma_I}$, then the system of ODE defined by (2) becomes equivalent to

$$r\tilde{J}_i(q;Q) = \frac{1}{2}\left[\tilde{J}_i'(q;Q)\right]^2 + \tilde{J}_i'(q;Q)\,\tilde{J}_j'(q;Q) \tag{4}$$

subject to

$$\tilde{J}_i(Q;Q) = \frac{\alpha_i}{\gamma_i}Q\,, \tag{5}$$

and each agent i's effort function is given by $a_i(q;Q) = \tilde{J}_i'(q;Q)$.

The reader is referred to Cvitanić and Georgiadis (2015) for the proof of parts 1.

To prove part 2, let $\tilde{D}(q;Q) = \tilde{J}_1(q;Q) - \tilde{J}_2(q;Q)$, and note that $\tilde{D}(\cdot;Q)$ is smooth, $\tilde{D}(q;Q) = 0$ for q sufficiently small, and $\tilde{D}(Q;Q) = \left(\frac{\alpha_1}{\gamma_1} - \frac{\alpha_2}{\gamma_2}\right)Q$. Observe that either $\tilde{D}'(q;Q) \geq 0$ for all q, or $\tilde{D}(\cdot;Q)$ is increasing in some intervals and decreasing in some others. Suppose that the latter is the case, in which case there exists some $\bar{q}$ such that $\tilde{D}'(\bar{q};Q) = 0$. Then it follows from (4) that $\tilde{D}(\bar{q};Q) = 0$, which implies that $\tilde{D}(q;Q) \geq 0$ for all q, and $\tilde{D}'(q;Q) > (=)\,0$ if and only if $\tilde{D}(q;Q) > (=)\,0$. Therefore, $\tilde{D}'(q) \geq 0$ for all q, which implies that $a_1(q;Q) \geq a_2(q;Q)$ for all q.

To prove part 3 note first the result for actions follows from the previous paragraph with all weak inequalities replaced with strict inequalities. Let $D(q;Q) = J_1(q;Q) - J_2(q;Q)$, and note that $D(\cdot;Q)$ is smooth, $D(q;Q) = 0$ for q sufficiently small, and $D(Q;Q) = 0$. Observe that either $D(q;Q) = 0$ for all q, or $D(\cdot;Q)$ has an interior extreme point. Suppose that the former is true. Then for all q, we have $D(q;Q) = D'(q;Q) = 0$, which using (*2*) implies that

$$rD(q;Q) = \frac{\left[J_1'(q;Q)\right]^2}{2}\left(\frac{1}{\gamma_2} - \frac{1}{\gamma_1}\right) = 0 \Longrightarrow J_1'(\bar{q};Q) = 0\,.$$

However, this is a contradiction, and so the latter must be true. Then there exists some $\bar{q}$ such that $D'(\bar{q};Q) = 0$. Using (*2*) and the fact that $J_i'(q) \geq 0$ for all q and $J_i'(q) > 0$ for some q, this implies that $D(\bar{q};Q) \leq 0$. Therefore, $D(q;Q) \leq 0$ for all q, which completes the proof.

A.3 Proof of Proposition 3

This proof comprises of 5 steps.

Step 1: *Show that $J_i'(Q;Q)$ is strictly increasing in Q, and so $Q_i(Q)$ is the unique solution to $J_i'(Q_i(Q);Q_i(Q)) = \alpha_i$.*

Consider agent i's optimization problem given state q. We would like to find the smallest q such that $q = \arg\max_Q \{J_i(q;Q)\}$. For such q, it must be the case that $\frac{\partial}{\partial Q}J_i(q;Q)\Big|_{q=Q} = 0$. Note that $J_i(Q;Q) = \alpha_i Q$, and totally differentiating this with respect to Q yields

$$J_i'(Q;Q) = \alpha_i\,. \tag{6}$$

By noting that $J_i(q;Q)$ is strictly concave in Q for all q, it follows that (6) is necessary and sufficient for a maximum. We next show that $J_i'(Q;Q)$ is strictly increasing in Q, so that the solution to (6) is unique.

Lemma 7. *For $i = 1, 2$, $J_i'(Q; Q)$ is strictly increasing with Q.*

Proof. The explicit form of the HJB equations of Lemma 4 yields $J_i'(Q;Q) = J_i'(1;1)\sqrt{Q}$. □

Step 2: *Show that $Q_1(Q) < Q_2(Q)$. [Need to assume that $\alpha_1 = \alpha_2$.]*
Pick Q such that it satisfies $J_1'(Q;Q) = \alpha_1$, and define $D(q;Q) = J_1(q;Q) - J_2(q;Q)$. Note that $D(\cdot;Q)$ is smooth, $D(q;Q) = 0$ for q sufficiently small, and $D(Q;Q) = (\alpha_1 - \alpha_2)Q = 0$. From Proposition 1, we know that $D(q;Q) \leq 0$ for all q, which implies that $J_1(Q;Q) \geq J_2(Q;Q)$. Suppose that $J_1(Q;Q) = J_2(Q;Q)$. Then it follows from (*2*) and the fact that $J_1'(Q;Q) > 0$ that

$$rD(Q;Q) = \frac{[J_1'(Q;Q)]^2}{2}\left(\frac{1}{\gamma_2} - \frac{1}{\gamma_1}\right) < 0\,,$$

which contradicts the fact that $D(Q;Q) = 0$, and so it must be the case that $J_1(Q;Q) > J_2(Q;Q)$. We know that $J_1'(Q;Q) = \alpha_1$, and so $J_2'(Q;Q) < \alpha_2$, which implies that agent 2 finds it optimal to continue work on the project. By a converse argument, one can show that if Q is chosen such that $J_2'(Q;Q) = \alpha_2$, then $J_1'(Q;Q) > \alpha_1$, which implies that agent 1 would like to have completed the project at an earlier state. Since $J_i(q;Q)$ is strictly concave in Q, it follows that $Q_2(Q) > Q_1(Q)$.

Step 3: *Show that for $q \geq Q_1(Q)$, ${Q_2}'(q) \geq 0$.*
To begin, we differentiate $\tilde{J}_i(q;Q)$ in (4) with respect to Q to obtain

$$r\partial_Q \tilde{J}_1(q;Q) = \partial_Q a_1(q;Q)\left[a_1(q;Q) + a_2(q;Q)\right] + a_1(q;Q)\;\partial_Q a_2(q;Q)$$
$$r\partial_Q \tilde{J}_2(q;Q) = \partial_Q a_2(q;Q)\left[a_1(q;Q) + a_2(q;Q)\right] + a_2(q;Q)\;\partial_Q a_1(q;Q)$$

where $\partial_Q \tilde{J}_i(q;Q) = \frac{\partial}{\partial Q}\tilde{J}_i(q;Q)$, $\partial_Q a_i(q;Q) = \partial_Q \tilde{J}_i'(q;Q) = \frac{\partial^2}{\partial Q\,\partial q}\tilde{J}_i(q;Q)$, and $a_i(q;Q) = \tilde{J}_i'(q;Q) = \frac{\partial}{\partial q}\tilde{J}_i(q;Q)$. Re-arranging terms yields

$$\frac{(a_1 + a_2)^2 - a_1 a_2}{r}(\partial_Q a_1) = (a_1 + a_2)\left(\partial_Q \tilde{J}_1\right) - a_1\left(\partial_Q \tilde{J}_2\right) \tag{7}$$
$$\frac{(a_1 - a_2)^2 + a_1 a_2}{r}(\partial_Q a_2) = (a_1 + a_2)\left(\partial_Q \tilde{J}_2\right) - a_2\left(\partial_Q \tilde{J}_1\right)\,, \tag{8}$$

where we drop the arguments q and Q for notational simplicity. Observe that because $a_i, a_j > 0$ in the domain of interest, $(a_1 + a_2)^2 - a_1 a_2 > 0$ and $(a_1 - a_2)^2 + a_1 a_2 > 0$. Let $Q_i(q)$ denote agent i's optimal project size given the current state q. Then for all $q < Q_i(q)$ and for the smallest q such that $q = Q_i(q)$, we have $\frac{\partial}{\partial Q}\tilde{J}_i(q;Q_i(q)) = 0$. Differentiating this with respect to q yields

$$\frac{\partial^2}{\partial Q\,\partial q}\tilde{J}_i(q;Q_i(q)) + \frac{\partial^2}{\partial Q^2}\tilde{J}_i(q;Q_i(q))\,Q_i'(q) = 0 \Longrightarrow Q_i'(q) = -\frac{\partial_Q a_i(q;Q_i(q))}{\partial_Q^2 \tilde{J}_i(q;Q_i(q))}\,.$$

Since $\partial_Q^2 \tilde{J}_i(q;Q) < 0$ (by assumption), it follows that $Q_i'(q) \leq 0$ if and only if $\partial_Q a_i(q;Q) \geq 0$.

Next, fix some $\hat{q} \in (Q_1(Q), Q_2(Q))$. By the strict concavity of $\tilde{J}_i(q;Q)$ in Q, it follows that $\partial_Q \tilde{J}_1(\hat{q}, Q_2(\hat{q})) < 0$ and $\partial_Q \tilde{J}_2(\hat{q}, Q_2(\hat{q})) = 0$; *i.e.*, agent 1 would prefer to have completed the project at a smaller project scope than $Q_2(\hat{q})$, whereas agent 2 finds it optimal to complete the project at $Q_2(\hat{q})$. Using (8) it follows that $\partial_Q a_2(\hat{q}, Q_2(\hat{q})) > 0$, which implies that ${Q_2}'(\hat{q}) > 0$.

Therefore, $Q_2{}'(q) > 0$ for all $q \in (Q_1(Q), Q_2(Q))$ and $Q_2(Q_1(Q)) > Q_1(Q)$, where the last inequality follows from the facts that $\tilde{J}_2(\cdot;Q)$ is strictly concave and so it admits a unique maximum, and that $\tilde{J}_2'(Q_1(Q);Q_1(Q)) < \frac{\alpha_2}{\gamma_2}$, which implies that he prefers to continue work on the project rather than complete it at $Q_1(Q)$.

Step 4: *Show that for $q \leq Q_1(Q)$, $Q_1{}'(q) \leq 0$ and $Q_2{}'(q) \geq 0$.*
Because $Q_2(Q_1(Q)) > Q_1(Q)$ and $Q_i(\cdot)$ is smooth, there exists some $\bar{q} \geq 0$ such that $Q_2(q) > Q_1(q)$ for all $q \in (\bar{q}, Q_1(Q))$. Pick some q in this interval, and note that $\partial_Q \tilde{J}_1(q, Q_2(q)) < 0$ and $\partial_Q \tilde{J}_2(q, Q_2(q)) = 0$, which together with (8) implies that $\partial_Q a_2(q, Q_2(q)) > 0$. Similarly, we have $\partial_Q \tilde{J}_1(q, Q_1(q)) = 0$ and $\partial_Q \tilde{J}_2(q, Q_1(q)) > 0$, which together with (7) implies that $\partial_Q a_1(q, Q_1(q)) < 0$. Therefore, $Q_1{}'(q) < 0$ and $Q_2{}'(q) > 0$ for all $q \in (\bar{q}, Q_1(Q))$.

Next, assume that there exists some q such that $Q_1(q) > Q_2(q)$ for some $q < \bar{q}$. Because $Q_i(q)$ is smooth, by the intermediate value theorem, there exists some $\tilde{q}$ such that $Q_1(\tilde{q}) > Q_2(\tilde{q})$ and at least one of the following statements is true: $Q_1{}'(\tilde{q}) < 0$ or $Q_2{}'(\tilde{q}) > 0$. This implies that for such $\tilde{q}$, we must have $\partial_Q \tilde{J}_1(\tilde{q}, Q_2(\tilde{q})) > 0$, $\partial_Q \tilde{J}_2(\tilde{q}, Q_2(\tilde{q})) = 0$, $\partial_Q \tilde{J}_1(\tilde{q}, Q_1(\tilde{q})) = 0$ and $\partial_Q \tilde{J}_2(\tilde{q}, Q_1(\tilde{q})) < 0$. Then it follows from (7) and (8) that $\partial_Q a_1(\tilde{q}, Q_2(\tilde{q})) > 0$ and $\partial_Q a_2(\tilde{q}, Q_1(\tilde{q})) < 0$. This in turn implies that $Q_1{}'(\tilde{q}) > 0 > Q_2{}'(\tilde{q})$, which is a contradiction. Therefore, it must be the case that $Q_2(q) \geq Q_1(q)$ for all q, and therefore $Q_1{}'(q) \leq 0$ *and* $Q_2{}'(q) \geq 0$ for all q.

Step 5: *Show that there does not exist any q such that $Q_1(q) = Q_2(q)$.*
We now want to show that if there exists some $\bar{q}$ such that such that $Q_1(\bar{q}) = Q_2(\bar{q})$, then it must be the case that $Q_1(q) = Q_2(q)$ for all $q \leq \bar{q}$. Suppose that the converse is true. Then by the intermediate value theorem, there exists some $\tilde{q}$ such that $Q_1(\tilde{q}) < Q_2(\tilde{q})$ and at least one of the following statements is true: $Q_1{}'(\tilde{q}) > 0$ or $Q_2{}'(\tilde{q}) < 0$. This implies that for such $\tilde{q}$, we must have $\partial_Q \tilde{J}_1(\tilde{q}, Q_2(\tilde{q})) < 0$, $\partial_Q \tilde{J}_2(\tilde{q}, Q_2(\tilde{q})) = 0$, $\partial_Q \tilde{J}_1(\tilde{q}, Q_1(\tilde{q})) = 0$ and $\partial_Q \tilde{J}_2(\tilde{q}, Q_1(\tilde{q})) > 0$. Then it follows from (7) and (8) that $\partial_Q a_1(\tilde{q}, Q_2(\tilde{q})) < 0$ and $\partial_Q a_2(\tilde{q}, Q_1(\tilde{q})) > 0$. This in turn implies that $Q_1{}'(\tilde{q}) < 0 < Q_2{}'(\tilde{q})$, which is a contradiction. Therefore, if there exists some $\bar{q}$ such that $Q_1(\bar{q}) = Q_2(\bar{q})$, then $Q_1(q) = Q_2(q)$ and $\partial_Q a_1(q;Q) = \partial_Q a_2(q;Q) = 0$ for all $q \leq \bar{q}$ and $Q = Q_1(q)$.

Lastly, to complete the proof, we rule out the possibility that there exists some q for which $Q_1(q) = Q_2(q)$. Note that each agent's normalized discounted payoff function can be written in integral form as

$$\tilde{J}_i(q_t;Q) = e^{-r[\tau(Q)-t]} \frac{\alpha_i}{\gamma_i} Q - \int_t^{\tau(Q)} e^{-r(s-t)} \frac{a_i^2(q_s;Q)}{2} ds \,.$$

Differentiating this with respect to Q yields the first order condition

$$e^{-r[\tau(Q)-t]} \frac{\alpha_i}{\gamma_i} \left[1 - rQ\tau'(Q)\right] - e^{-r[\tau(Q)-t]} \tau'(Q) \frac{a_i^2(Q;Q)}{2} - \int_t^{\tau(Q)} e^{-r(s-t)} a_i(q_s;Q) \, \partial_Q a_i(q_s;Q) \, ds = 0 \tag{9}$$

Suppose there exists some $\bar{q}$ such that $Q_1(\bar{q}) = Q_2(\bar{q}) = Q^*$. Then we have $Q_1(q) = Q_2(q)$ and $\partial_Q a_1(q; Q^*) = \partial_Q a_2(q; Q^*) = 0$ for all $q \le \bar{q}$. Therefore, fixing some $q \le \bar{q}$ and $Q^* = Q_1(\bar{q})$, it follows from (9) that

$$2\left[1 - rQ^*\tau'(Q^*)\right] = \tau'(Q^*)\frac{\gamma_1}{\alpha_1}a_1^2(Q^*; Q^*) = \tau'(Q^*)\frac{\gamma_2}{\alpha_2}a_2^2(Q^*; Q^*)$$

Observe that $\partial_Q a_1(q; Q^*) = \partial_Q a_2(q; Q^*) = 0$, which implies that $\partial_Q\left[a_1(q; Q^*) + a_2(q; Q^*)\right] = 0$, and hence $\tau'(Q^*) > 0$. Moreover, by assumption, $\frac{\gamma_1}{\alpha_1} < \frac{\gamma_2}{\alpha_2}$, and we shall now show that $\frac{\gamma_1}{\alpha_1}a_1^2(Q^*; Q^*) > \frac{\gamma_2}{\alpha_2}a_2^2(Q^*; Q^*)$. Let $D(q; Q^*) = \sqrt{\frac{\gamma_1}{\alpha_1}}\tilde{J}_1(q; Q^*) - \sqrt{\frac{\gamma_2}{\alpha_2}}\tilde{J}_2(q; Q^*)$, and note that $D(q; Q^*) = 0$ for q sufficiently small, $D(Q^*; Q^*) = \left(\sqrt{\frac{\alpha_1}{\gamma_1}} - \sqrt{\frac{\alpha_2}{\gamma_2}}\right)Q^* > 0$, and $D(\cdot; Q^*)$ is smooth. Therefore, either $D'(q; Q^*) > 0$ for all q, or there exists some z such that $D'(z; Q^*) = 0$. If the former is true, then $D'(Q^*; Q^*) > 0$, and we obtain the desired result. Now suppose that the latter is true. It follows from (4) that

$$rD(z; Q^*) = \frac{\left[\tilde{J}_1'(q; Q^*)\right]^2}{2}\left(\sqrt{\frac{\gamma_1}{\alpha_1}\frac{\alpha_2}{\gamma_2}} - 1\right) < 0\,,$$

which implies that it must be the case that $D'(Q^*; Q^*) > 0$, and hence $\frac{\gamma_1}{\alpha_1}a_1^2(Q^*; Q^*) > \frac{\gamma_2}{\alpha_2}a_2^2(Q^*; Q^*)$. This contradicts the assumption that there exists some q such that $Q_1(q) = Q_2(q)$, and the proof is complete.

A.4 Proof of Proposition 6

Let $S(q; Q) = J_1(q; Q) + J_2(q; Q)$. Because $J_i(q; Q)$ is strictly concave in Q for all i and q, it follows that $S(q; Q)$ is also strictly concave in Q for all q. Therefore, $Q^*(q)$ will satisfy $\frac{\partial}{\partial Q}S(q; Q) = 0$ at $Q = Q^*(q)$ and $\frac{\partial}{\partial Q}S(q; Q)$ is strictly decreasing in Q for all q. We know from Proposition 3 that $Q_1(q) < Q_2(q)$ for all q. We know that (i) $\frac{\partial}{\partial Q}J_1(q; Q) \ge 0$ and $\frac{\partial}{\partial Q}J_2(q; Q) > 0$ and so $\frac{\partial}{\partial Q}S(q; Q) > 0$ for all $q \le Q_1(q)$, and (ii) $\frac{\partial}{\partial Q}J_1(q; Q) < 0$ and $\frac{\partial}{\partial Q}J_2(q; Q) \le 0$ and so $\frac{\partial}{\partial Q}S(q; Q) < 0$ for all $q \ge Q_2(q)$. Because $\frac{\partial}{\partial Q}S(q; Q)$ is strictly decreasing in Q, it follows that $\frac{\partial}{\partial Q}S(q; Q) = 0$ for some $Q \in (Q_1(q), Q_2(q))$.

A.5 Proof of Lemma 1

The following lemma characterizes the MPE for the special case in which the agents are symmetric.

Lemma 8. *Suppose that the team comprises of $n = 2$ symmetric agents (i.e., $\alpha_i = \alpha$ and $\gamma_i = \gamma$ for all i). Then the game defined by (4) subject to the boundary condition (3) for all i has a unique project-completing MPE on $\left(\underline{q}, Q\right]$, where $\underline{q} = Q - \sqrt{\frac{2(2n-1)\alpha Q}{r\gamma}}$. This equilibrium is symmetric,* *each agent's effort strategy and discounted payoff satisfies*

$$\bar{a}_n(q;Q) = \frac{r}{2n-1}\left(q - Q + \sqrt{\frac{2(2n-1)\alpha Q}{r\gamma}}\right)$$

$$\textit{and} \quad \bar{J}_n(q;Q) = \frac{r\gamma}{2(2n-1)}\left(q - Q + \sqrt{\frac{2(2n-1)\alpha Q}{r\gamma}}\right)^2,$$

respectively, while the project is completed at $\tau(Q) = \frac{2n-1}{r\,n}\ln\left[1 - \frac{Q}{C(Q)}\right]$.[10]

The expression for $\hat{Q}_i(q)$ follows by maximizing the agent's discounted payoff function in Corollary 1, where $n = 1$. The inequality $\hat{Q}_2(q) < \hat{Q}_1(q)$ follows from the fact that $\gamma_1 < \gamma_2$.

Next, we show that $\hat{Q}_1(q) < Q_1(Q)$. Fix $Q = Q_1(Q)$, and define $\hat{\Delta}(q) = J_1(q;Q_1) - \bar{J}_1(q;Q_1)$. Note that $J_1'(Q;Q) = \alpha_1$, $\hat{\Delta}(Q) = 0$, $\hat{\Delta}(q) = 0$ for sufficiently small q, and $\hat{\Delta}(\cdot)$ is smooth. Therefore, either $\hat{\Delta}(q) = 0$ for all q, or it has an interior local extreme point. In either case, there exists some $z \in (-\infty, Q)$ such that $\hat{\Delta}'(z) = 0$. Using (2) it follows that

$$r\hat{\Delta}(z) = \frac{J_1'(z;Q)\,J_2'(z;Q)}{\gamma_2}.$$

Because $J_1'(q;Q_1)\,J_2'(q;Q_1) > 0$ for at least some q, it follows that it cannot be the case that $\hat{\Delta}(q) = 0$ for all q. Because $J_1'(q;Q_1)\,J_2'(q;Q_1) \geq 0$, it follows that any extreme point z must satisfy $\hat{\Delta}(z) > 0$, which together with the boundary conditions implies that $\hat{\Delta}(q) \geq 0$ for all q. Therefore, $\hat{\Delta}'(Q) < 0$, which in turn implies that $\bar{J}_1'(Q;Q) > J_1'(Q;Q) = \alpha_1$. By noting that $\bar{J}_1'\left(\hat{Q}_1(q);\hat{Q}_1(q)\right) = \alpha_1$ and $\bar{J}_1'(Q;Q)$ is strictly increasing in Q, it follows that $\hat{Q}_1(q) < Q_1(Q)$.

Since ${Q_1}'(q) < 0$ for all q, it follows that $\hat{Q}_1(q) < Q_1(q)$ for all q, and we know from Theorem that $Q_1(q) < Q_2(q)$ for all q.

A.6 Proof of Proposition 4

(To be added)

A.7 Proof of Proposition 5

(To be added)

[10]To simplify notation, because the equilibrium is symmetric and unique, the subscript i is dropped.

A.8 Proof of Lemma 4

Let $G = \tilde{J}_1 + \tilde{J}_2$ and $F = \tilde{J}_1 - \tilde{J}_2$. The normalized value functions $\tilde{J}_1$ and $\tilde{J}_2$ satisfy the normalized HJB equations

$$r\tilde{J}_1 = \frac{1}{2}(\tilde{J}_1')^2 + \tilde{J}_1'\tilde{J}_2' \tag{10}$$

$$r\tilde{J}_2 = \frac{1}{2}(\tilde{J}_2')^2 + \tilde{J}_1'\tilde{J}_2' \tag{11}$$

with initial conditions $\tilde{J}_i(Q) = (\alpha_i/\gamma_i)Q$ for $i = 1, 2$.

Thus, subtracting (11) from (10),

$$r(\tilde{J}_1 - \tilde{J}_2) - \frac{1}{2}(\tilde{J}_1' + \tilde{J}_2')(\tilde{J}_1' - \tilde{J}_2') = 0,$$

and adding (11) to (10),

$$r(\tilde{J}_1 + \tilde{J}_2) - \frac{1}{2}(\tilde{J}_1' + \tilde{J}_2')^2 = \tilde{J}_1'\tilde{J}_2'.$$

These two equations can be rewritten in terms of F and G,

$$rF - \frac{1}{2}F'G' = 0$$

$$rG - \frac{1}{2}(G')^2 = \frac{1}{4}(G')^2 - \frac{1}{4}(F')^2.$$

The first equation yields $F' = 2rF/G'$ (we have already shown in Proposition 1 that on the interval of interest $G' > 0$) and the second after plugging in the value of F' becomes

$$rG - \frac{1}{2}(G')^2 = \frac{1}{4}(G')^2 - r^2\frac{F^2}{(G')^2},$$

The equation is quadratic in $(G')^2$, and the unique positive root is

$$(G')^2 = \frac{2r}{3}\left(\sqrt{G^2 + 3F^2} + G\right)$$

which yields, as $G' > 0$,

$$G' = \sqrt{\frac{2r}{3}}\sqrt{\sqrt{G^2 + 3F^2} + G}$$

and

$$F' = \frac{2rF}{G'} = \frac{\sqrt{6r}F}{\sqrt{\sqrt{G^2 + 3F^2} + G}}.$$

Besides, since we know that $F > 0$ on the interval of interest, we get

$$F' = \sqrt{2r}\sqrt{\sqrt{G^2 + 3F^2} - G}$$

and

$$G' = \sqrt{\frac{2r}{3}}\sqrt{\sqrt{G^2 + 3F^2} + G},$$

so in terms of $\tilde{J}_1'$ and $\tilde{J}_2'$,

$$\tilde{J}_1' = \sqrt{\frac{r}{6}}\sqrt{\sqrt{\tilde{J}_1^2 + \tilde{J}_2^2 - \tilde{J}_1\tilde{J}_2} + \tilde{J}_1 + \tilde{J}_2} + \sqrt{\frac{r}{2}}\sqrt{\sqrt{\tilde{J}_1^2 + \tilde{J}_2^2 - \tilde{J}_1\tilde{J}_2} - \tilde{J}_1 + \tilde{J}_2}$$

$$\tilde{J}_2' = \sqrt{\frac{r}{6}}\sqrt{\sqrt{\tilde{J}_1^2 + \tilde{J}_2^2 - \tilde{J}_1\tilde{J}_2} + \tilde{J}_1 + \tilde{J}_2} - \sqrt{\frac{r}{2}}\sqrt{\sqrt{\tilde{J}_1^2 + \tilde{J}_2^2 - \tilde{J}_1\tilde{J}_2} - \tilde{J}_1 + \tilde{J}_2}.$$

A.9 Proof of Lemma 2

Let $A_i(Q)$ be the effort agent i put in a project of size Q at the very end of the project. We have already shown that

$$A_1(Q) + A_2(Q) = a\sqrt{2r/3}\sqrt{Q}$$
$$A_1(Q) - A_2(Q) = b\sqrt{6r}\sqrt{Q}.$$

so

$$A_1(Q)^2 = \frac{Qr}{6}\left[a + 3b\right]^2$$
$$A_2(Q)^2 = \frac{Qr}{6}\left[a - 3b\right]^2.$$

For a project of size Q, agent i gets value $\alpha_i Q$ at the completion of the project, when $q = Q$. If the project is instead of size $Q + \Delta Q$ (for small enough ΔQ), and if the current state is $q = Q$, there is a delay ϵ before the project is completed. To the first order in ϵ, the relationship $\Delta Q = (A_1(Q) + A_2(Q))\epsilon$ holds. Thus, to the first order in ϵ, the net discounted value of the project to agent i at state $q = Q$ is

$$\alpha_i(Q + (A_1(Q) + A_2(Q))\epsilon)e^{-r\epsilon} - \frac{\gamma_i}{2}(A_i(Q))^2\epsilon.$$

At a project size $Q = Q_i$, the agent is indifferent between stopping the project now (corresponding to a project size Q_i) and waiting an instant later (corresponding to a project size $Q_i + \Delta Q$ for an infinitesimal ΔQ). So to the first order,

$$\alpha_i Q_i = \alpha_i(Q_i + (A_1(Q_i) + A_2(Q_i))\epsilon)e^{-r\epsilon} - \frac{\gamma_i}{2}(A_i(Q_i))^2\epsilon.$$

So:

$$\alpha_i(A_1(Q_i) + A_2(Q_i)) - r\alpha_i Q_i - \frac{\gamma_i}{2}(A_i(Q_i))^2 = 0.$$

Solving this equation for $i = 1, 2$ yields

$$\sqrt{Q_1} = \frac{\sqrt{2r/3}a(\alpha_1/\gamma_1)}{r(\alpha_1/\gamma_1) + \frac{r}{12}\left[a + 3b\right]^2}$$

and

$$\sqrt{Q_2} = \frac{\sqrt{2r/3}a(\alpha_2/\gamma_2)}{r(\alpha_2/\gamma_2) + \frac{r}{12}\left[a - 3b\right]^2}$$

Note that

$$\frac{\sqrt{Q_1}}{\sqrt{Q_2}} = \frac{12 + \left(\frac{\alpha_2}{\gamma_2}\right)^{-1} [a - 3b]^2}{12 + \left(\frac{\alpha_1}{\gamma_1}\right)^{-1} [a + 3b]^2} = \frac{12b^{-2} + \left(\frac{\alpha_2}{\gamma_2}\right)^{-1} [a/b - 3]^2}{12b^{-2} + \left(\frac{\alpha_1}{\gamma_1}\right)^{-1} [a/b + 3]^2}$$

In particular, $Q_1 < Q_2$ if and only if

$$[a/b + 3] - \left(\frac{\alpha_1}{\gamma_1}\right)^{1/2} \left(\frac{\alpha_2}{\gamma_2}\right)^{-1/2} [a/b - 3] > 0.$$

So defining f as

$$f(x) = 1 + x + 2\sqrt{1 - x + x^2} + 3(x - 1) - \sqrt{x}(1 + x + 2\sqrt{1 - x + x^2} - 3(x - 1))$$

the previous inequality holds if and only if $f((\alpha_1/\gamma_1) \cdot (\alpha_2/\gamma_2)^{-1}) > 0$. After simplification, we have

$$f(x) = 2\left(\sqrt{x} - 1\right)\left(-\sqrt{x^2 - x + 1} + x + 3\sqrt{x} + 1\right)$$

so noting that $\sqrt{x^2 - x + 1} < x$ if $x > 1$, we immediately get that $f > 0$ on $[1, +\infty)$, and so $Q_1 < Q_2$.

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