The role of Securities with discontinuous payoff function in the Credit Market

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Abstract

In this paper we consider firm financing under ex-ante asymmetric information (hidden types or adverse selection). Firms can use both the amount on the investment and they type of the security issued to credibly convey information to the uninformed side of the market (financiers). We show that the introduction of a security with a discontinuous payoff function (barrier option) leads to a fully revealing equilibrium, where the investment level is optimal (first best).

KEYWORDS: Adverse selection, optimal investment, security design

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1 Introduction

After the seminal work of Modigliani & Miller (1958) there is a debate in the literature trying to give answer about the optimal firm's capital structure. Later innovations regarding informational asymmetries shed extra light to this direction. These asymmetries can be either some hidden types (adverse selection) or some hidden actions (moral hazard). Adverse selection problem can arise when the financiers are uninformed about the characteristics of an entrepreneur, who needs to raise money from the capital market to invest in some projects. What are these characteristics? It can be a personal characteristic like in Jaffee & Russel (1976), or it can be a parameter of an earnings distribution as in Stiglitz & Weiss (1981). However, no matter what is the exact informational friction, it may cause inefficient credit market equilibria and it will affect the capital structure of the firm.

Most existing papers on financing under ex-ante asymmetric information exogenously impose that the security issued for raising the required amount of funds is debt. Take for example the important paper of Stiglitz & Weiss (1981). Assuming "competitive equilibria"[1] and that only debt is issued they show that a credit rationing equilibrium is possible. Credit rationing is a credit market failure that happens when some entrepreneurs are denied credit even though they are willing to pay more interest in order to get the loan they want. This definition is different than that of Jaffee & Russel (1976) where all the entrepreneurs get a loan but are restricted compare to the social efficient level. Stiglitz & Weiss considering mean preserving spreads for the distribution of earnings show that a credit rationing equilibrium is possible when the interest rate, which makes market clear, is higher than the one that maximizes the banks' profit. This happens because an increase of the interest rate results in more dangerous loans and as a consequence the expected profit of the bank decreases. They also examine the role of collateral as a screening device for the entrepreneurs. Assuming risk averse entrepreneurs they show that increasing the collateral would also make the loans riskier and would decrease the banks' profit. Wette (1983) extends the problem to risk neutral entrepreneurs and proves that

[1] "Competitive equilibria" in the sense that financiers compete, and choose a price that maximize their profit.

the inefficiency not only remains but also becomes more severe.

Nevertheless, these models have a crucial assumption. They do not allow interest rate and collateral to be used simultaneously for screening the entrepreneurs and sorting them into risk classes. They examine always either collateral or interest rate separately, while keeping fixed the other. As a result, another strand of the literature starts to use collateral as a screening device and shows that the first-best level of investment can be implemented only if the collateral is limitless and can be used without any cost. Otherwise, there is underinvestment. For example, Bester (1985) demonstrates the simultaneous use of interest rate and collateral can provide a self-mechanism that will allow the screening of the entrepreneurs and no credit rationing will exist in the equilibrium. Specifically, high risk borrowers choose high interest rate with low collateral while low risk borrowers choose lower rate with higher collateral. However, he presumes entrepreneurs face no limit to the amount of collateral they can offer. Bester (1987) drops this assumption and concludes that perfect sorting can happen only when collateral is not constrained by the initial wealth. If this is the case and collateral is constrained by the initial wealth then a credit rationing equilibrium is again possible.

Besanko & Thakor (1987b) change the assumption of mean preserving spreads and assume first order stochastic dominance for the distribution of earnings. They consider two market structures, monopoly and perfect competition. Under monopoly there are two plausible outcomes; either there is a pooling equilibrium where both entrepreneurs issue debt at the high-risk entrepreneur interest rate (when his social surplus is sufficiently high) or the high risk entrepreneurs are denied credit. Under perfect competition the result is similar with the previous literature. On the one hand, when wealth is relatively low, collateral cannot be used effectively to screen the entrepreneurs and the credit rationing is probable, where the low-risk entrepreneurs are the ones that may be denied credit. On the other hand, when collateral is adequate the results are same as Bester (1985).

Myers & Majluf (1984) show that when the firms are unable to credibly convey their private information to the investors, the resulting adverse selection problem may lead the firms to forego profitable investment opportunities when they use only equity financing, resulting in significant welfare loss. Notwithstanding, the main reason of the underinvestment. that arises in

Myer and Majluf work, is that they use contracts with securities whose expected payoff is lower than the raised money, according to Brennan & Kraus (1987). Brennan & Kraus provide the conditions under which the problem of adverse selection can be costlessly resolved with revealing equilibria. In cases that the distribution of returns is ranked by first order stochastic dominance, equity issue with debt retirement is the equilibrium contract. If they assume mean preserving spreads then the optimal contract contains convertible debt or junior bond or packages of bonds and warrants. In a similar context but with different initial capital structure Constantinides & Grundy (1989) indicate that the optimal contract consists of convertible debt plus share repurchase when investment is fixed, and only debt with varying investment, when investment is not fixed. Both papers, reject the conventional wisdom of using either equity or debt as the only signaling device. In order to achieve revealing equilibrium a contract with at least two signalling factors is required.

In another paper Besanko & Thakor (1987a), maintaining the assumption of first order stochastic dominance for the distribution of earnings and that of constrained collateral, conclude that low success entrepreneurs borrow small amount of money and repay debt with risk free rate because their wealth is enough to make the debt riskless. On the contrary, good entrepreneurs invest more compare to the first best level while they pay higher interest than risk-free rate since their wealth is not enough. Thus, the higher is their wealth the closer to their first best level of investment they get (negative correlation between wealth and level of investment). We notice here that adverse selection problem provokes overinvestment and no credit is denied. De Meza & Webb (1987) get the same result of overinvestment but they don't use the collateral as screening device and the economy is in a pooling equilibrium (no self mechanism is available). Their result is quite important and precisely contrary to the prevailing view that if credit market fails, the direction of the bias is that investment falls short of the socially optimal level.

Martin (2009) gets also some results that contradict the previous literature. First of all, in his economy there may be either separating or pooling equilibria. Pooling equilibrium arises when wealth is relatively low, while separating equilibrium when wealth is higher and can be used more effectively as collateral to screen the entrepreneurs. Which equilibrium emerges finally depends on the profit of the good entrepreneur. Secondly, he shows that, even when aggregate investment is constrained due to the presence of adverse selection, it need not be monotonically increasing in entrepreneurial wealth[2]. He demonstrates that in pooling equilibrium when the ratio of good to bad entrepreneurs is high enough, both of them will get loans of higher size than those of the separating equilibrium. While the entrepreneur's wealth increases the equilibrium will switch the equilibrium from pooling to separating and this will result in a discontinuous fall of investment, which contradicts the conventional wisdom of monotonically increasing investment with entrepreneur's wealth. However, regardless of the equilibrium configuration the level of aggregate investment is sub-optimal.

In this paper, we consider firm financing under ex-ante asymmetric information (hidden types or adverse selection). More specifically, in our environment, there are two groups of agents: entrepreneurs and financiers. Each entrepreneur has a positive net present value project but not initial wealth. To be undertaken, each project requires some investment. Hence, in order to invest in their projects, the entrepreneurs have to raise the required amount of funds from the financiers (market) by issuing some securities. Each project's return depends both on the amount of funds invested and its inherent quality (type). The project type is private information of the entrepreneur. The financiers can use two instruments to screen projects: the amount of the investment and the payoff function of the securities issued.

Firms do not have any assets in place which could be used as collateral but they are allowed to issue any type of security they wish (including non-monotonic securities). We show that there always exists a fully revealing separating equilibrium (without any use of collateral) where the level of investment is optimal (first-best). The security issued by the high quality entrepreneurs in this separating equilibrium is a barrier option. This option has a discontinuous payoff because it can be exercised only if the payoff of the underlying asset (project) is below or above a certain threshold which is determined endogenously.

The paper proceeds as follows. In section 2 the model is presented and the assumptions discussed. Section 3 solves the benchmark case under full information, and adverse selection as well when the only financing tools are equity and debt. In section 4 we discuss the use of barrier options as a

[2]On the other hand in Besanko and Thakor (1987a) the aggregate investment is initially higher than the first best level.

tool of sorting entrepreneurs in risk classes and restoring social efficiency. Finally, in section 5 we conclude with the final remarks.

2 The environment

Assume the following simple one-period model of financing that involves adverse selection. There are two dates $t \in \{0, 1\}$, and one homogeneous good which can be used either for consumption or investment purposes. There is a continuum financiers and entrepreneurs that consume only at date 1. All agents are risk neutral and consume only at date 1. Entrepreneurs have an indivisible project but no initial wealth (we can change it later). More importantly, they are endowed with a decreasing returns to scale technology for transforming consumption goods time 0 into consumption goods time 1. The fact that this technology can be operated solely by them, though, means that it is potentially subject to informational frictions. Financiers have a vast initial endowment which are willing to lend inelastically at zero interest rate.

There are two groups of entrepreneurs: Good (G) or Bad (B) depending on their technology, with respective proportions in the population κ and $1-\kappa$, $0 < \kappa < 1$. At time 0 entrepreneur invests I_i for his project (with $i = G, B$), and he is the only one that knows the type of the project (**Adverse Selection**). Since he has no initial wealth, he needs to raise I_i from the capital market. There are two states of nature: Success and Failure. If successful (unsuccessful) an entrepreneur of type i who invests I_i obtains a gross return of $X_i = \alpha_i f(I)$ (zero[3]), where $\alpha_G < \alpha_B$ and $p_G \alpha_G > p_B \alpha_B$, implying second order stochastic dominance and specifically mean reversing spreads for the bad type.

It is assumed that $f(\cdot)$ is increasing, concave and satisfies Inada conditions. Also, there are no taxes, no bankruptcy or financial distress costs and the returns X_i are observable and verifiable by both agents at time 1. Finally, there is no conflict of interest between managers and entrepreneurs. In fact, firms are run by entrepreneurs.

[3] Without loss of generality, we set the return in the bad state of nature equal to zero in order to make the calculations easier.

The Game

The financiers and the entrepreneurs play the following two-stage signaling game:

- **Stage 1:** The entrepreneur approaches the financier and offers a combination of securities in exchange of I_i.
- **Stage 2**: The financier either accepts or rejects the contract the entrepreneur proposes.

We only consider pure-strategy perfect Bayesian equilibria.

3 Benchmark: Firms can only issue Debt and Equity

3.1 Full information problem

In order to understand what happens under full information we will presume that the only available security is debt. However, any security could be used and get the same result. Entrepreneurs and financiers sign a contract of the form (I, R, D) where I is the amount raised from the market and invested, R is the interest factor on the amount raised, and D is the face value of the the debt, $D = RI$. In the good state of nature, entrepreneurs pay the amount borrowed, while in the bad state of nature they default and the banks take the residual value of the project. This implies that the expected utility of the entrepreneur given limited liability is:

$$U_i(I, R) = \max(p_i[a_i f(I_i) - R_i I_i], 0) \tag{1}$$

We follow the adverse selection literature in making the following assumptions regarding bank competition. The first is a condition of no cross-subsidization, by which banks are not allowed to offer contracts that lose money in expectation. The second assumption is that entrepreneurs can apply to at most one of the contracts offered. This assumption implies entrepreneurs borrow only from one bank, implicitly assuming that banks can monitor contract applications made by entrepreneurs[4]. It is also considered that each bank gets the same share of total deposits and, if they design the same contract, they get the same share and composition of loan applications. Given these assumptions, the financier's expected profit of accepting an application to a contract (I, R) from a type-i entrepreneur is given by:

$$\Pi_i(R_i, I_i) = p_i \min[a_i f(I_i), R_i I_i] - I_i \tag{2}$$

In the absence of asymmetric information the equilibrium is the following (proof is trivial and is omitted). The equilibrium is $\{(I_{B^*}, R_{B^*}), (I_{G^*}, R_{G^*})\}$, and satisfies the succeeding conditions:

$$\left.\begin{aligned} f'(I_{i^*}) &= \frac{1}{\alpha_i p_i}, \\ R_{i^*} &= \frac{1}{p_i}, \\ D_i &= \frac{I_i}{p_i}, \end{aligned}\right\} \qquad \text{for } i \in \{G, B\} \tag{3}$$

Thus, under full information, good entrepreneurs invest more than bad ones and financiers break even in both contracts[5]. Moreover, we observe that the interest rate is independent of the raised amount of money. Specifically, good type borrows at R_G that is lower than the interest rate of bad type.[6]

3.2 Pure Adverse selection

Let's consider now what happens when the financier cannot distinguish the type of the entrepreneur. We begin by restricting financing instruments to debt and outside equity. Debt claims are zero coupon-bonds and are senior to equity. A contract $A = (\lambda, R, D, I)$ provides the entrepreneur with the required amount of funds I_i, in return for a combination of debt with interest factor R_i and face value $D_i = R_i J_i$[7], and a proportion of equity

[4]The assumption is similar to that of Rothschilld & Stiglitz (1976) applied in the insurance markets.

[5]The same result we get if the security instead of debt is equity, or any combination of these two securities or any other security in general.

[6]Interest rate depends only on the probability of success, while the fraction of equity shares depends on the realizations, which depend on total investment.

[7]When only debt is issued then $J_i = I_i$.

of the project λ, $0 \leq \lambda \leq 1$, $R \geq 0$.

Given risk neutrality and limited liability, the entrepreneur seek to maximize:

$$\begin{aligned} U_i(I_i, R_i, D_i, \lambda_i) &= p_i \max[(1-\lambda)(a_i f(I_i) - R_i J_i), 0] \\ &= p_i \max[(1-\lambda)(a_i f(I_i) - D_i), 0] \end{aligned} \tag{4}$$

where U is the expected utility of an entrepreneur of type i when choosing the contract $A_i = (\lambda_i, R_i, I_i)$.

The financiers also seek to maximize their expected profit. The expected profit Π_F of a financier offering a contract (λ, R, I), given limited liability, is:

$$\Pi_i(I_i, R_i, D_i, \lambda_i) = p_i\{\max[\lambda(a_i f(I_i) - D_i), 0] + \min[a_i f(I_i), D_i] - I_i \tag{5}$$

Lemma 1 *Let U_i denote the family of indifference curves of type i, and u_i denote a member of this family. In the (λ, D) space for $0 \leq \lambda < 1$ and $0 \leq D < a_i f(I)$,*

a) u_i are downward sloping and concave.

b) At any (λ, D) pair, u_B is flatter than u_G and so the indifference curves of G- and B-type cross only once.

Proof. For any $0 \leq \lambda < 1$ and $0 \leq D < a_i f(I)$, equation 4 becomes:

$U_i(I_i, R_i, D_i, \lambda_i) = p_i(1-\lambda)(a_i f(I_i) - D_i), \quad i = G, B$

By total differentiation, we can get the slope of the indifference curve:

$(\frac{d\lambda}{dD})_{u_i=\bar{u}} = -\frac{1-\lambda}{a_i f(I) - D} < 0$

Taking into consideration that $u_i = \bar{u}$ implicitly defines λ as a function of D, we obtain:

$(\frac{d^2\lambda}{dD^2})_{u_i=\bar{u}} = -\frac{1-\lambda}{a_i f(I) - D} + \frac{d\lambda/dD}{a_i f(I) - D} = -\frac{2(1-\lambda)}{(a_i f(I) - D)^2} < 0$

Consequently, the indifference curves of both types are downward sloping and concave. Moreover, by assumption we have that $a_G < a_B$, at any pair (λ, D) pair, u_G is steeper than u_B and as a result they cross only once. ■

When the investment is funded by 100% debt then we notice that the interest rate does not depend on the amount borrowed. Specifically, from the zero profit condition of the financier the interest factor of debt is $R_i = 1/p_i$. This means that no matter what is the investment level or the amount raised with debt the interest rate would not change. Only the face value of the debt would change. On the other hand, if the investment is funded by 100% equity, then the fraction of the firm's shares that the financier will obtain is depended on the amount invested. Again from the zero profit condition of the financier, the required fraction of shares is $\lambda_i = I/p_i a_i f(I)$

Lemma 2 *In the (λ, D) space and for a specific level of investment:*

a) The zero profit lines (ZP) are downward sloping and strictly concave. Also at any (λ, D) pair, ZP_B is flatter than ZP_G.

b) ZP_G lies entirely below ZP_B and the two lines never intersect.

c) u_i and ZP_i never cross each other, $i = G, B$.

Proof. a) The equations for the zero profit lines ZP_i, PZP (pooling zero profit line) are respectively the following:

$$p_i\{\lambda(a_i f(I) - D) + D\} = I$$

$$\kappa p_G[\lambda(a_G f(I_P) - D) + D\} + (1-\kappa)p_i[\lambda(a_B f(I) - D) + D\} = I$$

By total differentiation, we obtain their slopes in the (λ, D) space, which are:

$$(\frac{d\lambda}{dD})_{ZP_i} = -\frac{1-\lambda}{a_i f(I) - D} < 0$$

$$(\frac{d\lambda}{dD})_{PZP} = -\frac{1-\lambda}{\kappa a_G(f(I)-D)+(1-\kappa)a_B(f(I)-D)} < 0.$$

b) For the same level of investment we can calculate the corner values of u_i:

- when $\lambda = 0$, then $D_G = \frac{I}{p_G} < \frac{I}{p_B} = D_B$,
- when $D = 0$, then $\lambda_G = \frac{I}{p_G a_G f(I)} < \frac{I}{p_B a_B f(I)} = \lambda_B$.

Thus in the (λ, D) and always for the same level of investment, the ZP_G lies below the ZP_B. Moreover it is obvious that at any given pair (λ, D),

$\left|(\frac{d\lambda}{dD})_{ZP_G}\right| \geq \left|(\frac{d\lambda}{dD})_{PZP}\right| \geq \left|(\frac{d\lambda}{dD})_{ZP_B}\right|$

c) By Lemmas 1 and 2 we have:

$(\frac{d\lambda}{dD})_{u_i=\bar{u}} = (\frac{d\lambda}{dD})_{ZP_i} = -\frac{1-\lambda}{a_i f(I)-D} < 0.$

Thus, u_i and ZP_i never intersect. ∎

Figure 1 shows the zero profit lines of the financiers when offering a contract (λ, D, I)[8] to the entrepreneurs and both of them invest the same amount. The zero profit line of the good type lines entirely under the the zero profit line of the bad type. In this case the bad entrepreneur has always the incentive to mimic the good one and invest more while paying less interest rate or giving smaller fraction equity. As a result, separating equilibrium is impossible and we consider only the pooling equilibrium, which is the line PZP on the graph. Let's see what is the new optimal investment for the good type when both types are pooled.

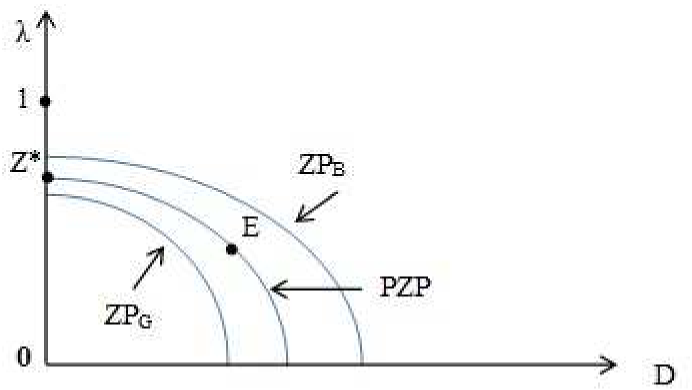

Figure 1: Zero Profit lines of the financiers for the 2-types.

Proposition 1 *There exists a unique "reasonable"* [9] *pooling equilibrium where both types issue only equity. The good type issues underpriced equity while the bad type overpriced.*

[8]We don't include R_i anymore since it is incorporated in the face value of debt D, and is independent of the borrowed amount.

[9]An equilibrium is "reasonable" in the sense that it satisfies the "intuitive criterion" of Cho & Kreps (1987)

Proof. Due to competition among financiers, any equilibrium contract implies zero profit and as a result any pooling equilibrium must lie on the pooling zero-profit curve (PZP). Consider, for example, contract E. The G-type can credibly reveal his type by offering a contract involving less debt and more equity (between the two indifference curves through point E). Given contract E, a financier observing the deviant contract will reasonably infer that it has been proposed by the G-type and so is profitable. But if contract B is taken only by the B-type it becomes loss-making and so no rational financier would accept it. Therefore, this pooling equilibrium collapses. The same argument applies to all points along PZP (except the equilibrium point Z^*). Given Z^*, if a financier observes a deviant contract between ZP_G and ZP_B, he will reasonably infer that it is proposed by the B-type. Thus, it would be loss-making and it will be rejected by the financier. ■

Intuitively, because the G-type project has both higher expected return and higher success probability, in absolute terms, both debt and equity issued by the G-type are more valuable than those of the B-type and so the B-type will always mimic. However, in relative terms, debt is more valuable for the good type and equity for the bad one. Thus, whenever some (mispriced) debt is issued the G-type can reveal his type by issuing more equity and less debt. By doing so, he can reduce the mispricing of his securities. Nevertheless he reduces the mispricing for the pooling level of investment that is different than his optimal investment with full information. We can calculate the new level of investment as follows.

Under 100% equity financing the financier's profit is equal to

$$\Pi_F(\lambda, I) = p_i \max[a_i f(I_i), 0] - \bar{I} \tag{6}$$

Solving this equation, we can find the equilibrium percentage of equity:

$$\bar{\lambda} = \frac{\bar{I}}{\kappa p_G \alpha_G f(\bar{I}) + (1-\kappa) p_B a_B f(\bar{I})} \tag{7}$$

Afterwards, we replace the equilibrium percentage of equity (7) inside the good type's utility (4) and calculate the optimal investment for the pooling equilibrium.

$$f'(\bar{I}) = \frac{1}{\kappa p_G \alpha_G + (1-\kappa) p_B a_G} \tag{8}$$

If we compare the full information equilibrium with the adverse selection equilibrium we can make two observations. First of all, the good type issues mispriced securities and his investment is lower than his first best choice. In particular, he gives a higher fraction of the project's return to the financier under equity financing (underpriced security). On the other hand the bad type invests more in comparison with his first best choice and he gives a lower fraction of the project's return to the financier (overpriced equity). This implies that the good type subsidizes the bad type in the equilibrium. Nevertheless, since there are no assets in place he will always issue mispriced securities and invest[10].

4 Introduction of barrier options

Under adverse selection the first best equilibrium (FB) is not feasible anymore. The B-type has always incentive to mimic the G-type. Thus, we need to introduce a contract that will prevent B-type from mimicking, restore the efficiency in the market and give us FB equilibrium if it is feasible. This contract has to satisfy both the incentive compatibility and the participation constraints of the two types. Suppose the entrepreneurs can also use a new security, the barrier option, except debt and equity. The exercise of the barrier option conditions to the value of the underlying asset. If it is exercised, its holder receives a pre-specified fraction of the firm's (project's) shares η.

For simplicity, and without loss of generality, consider a barrier option which can be exercised only if the gross return of the underlying project is higher than X_G and its strike price is zero. Then the expected payoff of the entrepreneur offering a contract involving the barrier option and debt[11], taken into consideration limited liability, is given by:

$$U_i(D_i, I_i, \eta_i) = \begin{cases} p_i[a_i f(I) - D], & \text{if} \quad X_i \leq X_G \\ (1-\eta)p_i[a_i f(I) - D], & \text{if} \quad X_i > X_G \end{cases} \tag{9}$$

[10] Unlike Mayers & Majluf (1984) where the firms have assets in place. When the mispricing of the securities is larger than the net present value of the new projects, then the good type will not invest and only bad entrepreneurs will invest.

[11] The entrepreneur borrows all the amount I_i that is need to invest and repays $R_i I_i$ to the financier. The option works as punishment and exercised only when $X_i > X_G$.

The expected profit, Π_i, of a financier is given by:

$$Pi_i(D_i, I_i, \eta_i) = \begin{cases} p_i D_i - I_i, & \text{if} \quad X_i \leq X_G \\ a_i f(I) - D_i) + D_i] - I_i, & \text{if} \quad X_i > X_G \end{cases} \tag{10}$$

The incentive compatibility and participation constraints for the two types are the following respectively:

$$\begin{aligned} U_G(I_G, D_G, \eta) &\geq U_G(I_B, D_B, \eta) \\ U_B(I_B, D_B, \eta) &\geq U_B(I_G, D_G, \eta) \\ U_B &\geq 0 \\ U_G &\geq 0 \end{aligned}$$

Proposition 2 *There is a pooling equilibrium where both G-type and B-type invest the same I_G, issuing debt with face value $D_{G^*} = \frac{I_{G^*}}{p_G}$ and a barrier option with strike price zero which can be exercised only if $X_i > X_G$ and if it is exercised its holder obtains an $\bar{\eta}$ fraction of the project's shares. Both types invest I_{G^*}.*

There also exists a continuum of separating equilibriums where i) G-type issues debt with face value $D_{G^*} = \frac{I_{G^*}}{p_G}$ to invest I_{G^*}, including a barrier option with strike price zero that can be exercised only if $X_i > X_G$ and if it is exercised its holder obtains an $\eta > \bar{\eta}$ fraction of the project's shares, and ii) B-type invests as in his first best choice I_{B^*}, issuing any combination of debt and equity along the corresponding zero profit line ZP_B of the full information problem.[12]

Proof. The G-type's barrier option is never exercised for the specific level of investment (I_{G^*}), and as a result his ZP is $D_{G^*} = \frac{I_{G^*}}{p_G}$. Hence the corresponding expected return for the G-type and the expected profit of the financier are the equations on the upper brunch of equations 9 and 10. The G-type will invest as in full information problem and repay his debt in the good state of nature. What is needed now is to structure the barrier option

[12]We need to be careful here in order don't get confused by the figure 2. The figure shows the zero profit lines of the financiers when both invest I_{G^*}. However, on the graph we can see only the pooling equilibrium. The separating equilibria are not shown on this figure because the B-type invests at his first best level of investment and the zero profit line cannot be depicted on the same space with that of the G-type.

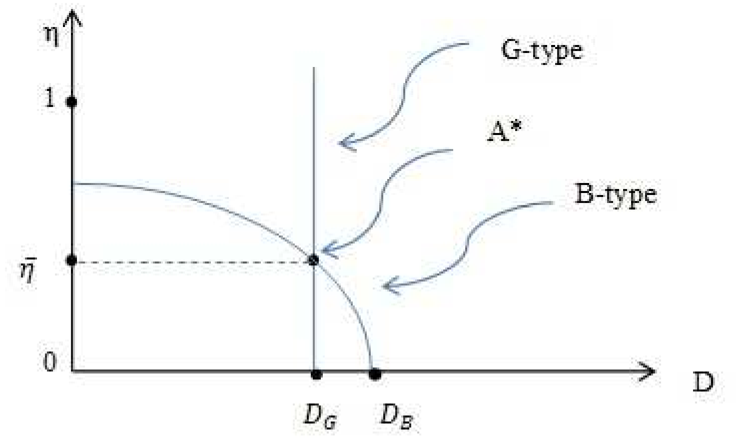

Figure 2: Zero Profit lines of the financiers for the 2-types when the level of investment is I_{G^*}.

in a way that will prevent the B-type from mimicking the good one. We will accomplish this, by introducing an η, (the fraction of the project's shares that the financier will take if he exercises the option), that makes him indifferent between mimicking and revealing the truth. As a consequence, his payoff should be at most equal to his full information payoff or,

$$\begin{aligned} U_B(I_{G^*}, D_{G^*}, \eta) &\leq U_B(I_{B^*}, D_{B^*}) \Rightarrow \\ p_B(1-\eta)[\alpha_B f(I_{G^*}) - D_{G^*}] &\leq p_B[\alpha_B f(I_{B^*}) - D_{B^*}] \Rightarrow \\ \eta &\geq 1 - \frac{\alpha_B f(I_{B^*}) - D_{B^*}}{\alpha_B f(I_{G^*}) - D_{G^*}} = \bar{\eta} \end{aligned}$$

Therefore, for this $\bar{\eta}$ bad type is indifferent between mimicking or revealing the truth.

Intuitively, suppose that the G-type offers a contract that includes debt with face value $D_{G^*} = \frac{I_{G^*}}{p_G}$ and a barrier option that can be exercised if $X_i < X_G$ and the holder obtains a fraction of $\eta > \bar{\eta}$ of the project's shares. If B-type mimics the good counterpart the financier will make a strictly positive profit, since this contract lies above the ZP_B. Hence, B-type will issue mispriced securities if he mimics. As a result, he will avoid mimicking and he will reveal his type by offering any contract that lies on his zero profit line that corresponds to total level of investment I_{B^*}. Moreover we need to add here that the B-type if he deviates from A^* he would never choose any contract in the right of A^* since he can always do better by choosing his first best investment and move to other ZP_B. For example, consider the case in the right of A^* that he issues only debt. The face value will be $D_B = I_G/p_B$. His utility will become

$$U_B = p_B(a_B f(I_G) - D_B) < p_B(a_B f(I_{B^*}) - D_{B^*}) = U_{B^*}$$

since I_G doesn't maximize his utility (he prefers to issue less by equation 3). Accordingly, suppose that B-type issues only debt with face value $D_{B^*} = \frac{I_{B^*}}{p_B}$ as in full information case. G-type has no incentive to mimic since if he does so, he will sell non fairly prices securities and he will raise lower I, than his optimal choice The financier will also make strictly positive profit since $D_{B^*} > \frac{I_{B^*}}{p_G}$. However, this means that the G-type not only doesn't invest as much as he wants, but also also with a higher cost. Consequently, he would never mimic the B-type and both types' incentive compatibility constraints are satisfied and a fully separating equilibrium arises. ■

5 Conclusion

The main results of this paper already are presented in the introduction. Hence, instead of conclusion we will make a small synopsis of what we did and provide some ideas for future work and improvements in this area. In this paper we examined a credit market plagued with adverse selection. Financiers cannot distinguish the types of projects/entrepreneurs and as a result inefficient equilibria arise. We propose that the introduction of a barrier option can help the financier to distinguish the two types and get a fully revealing equilibrium where the entrepreneurs invest like their first best and at fair terms.

An interesting idea for an extension of the current work would be to increase the possible types of entrepreneurs or drop the assumption of fully verifiable result. Specifically assume that entrepreneurs can hide some profit and either overstate or understate the profit. Financiers are only able to know only the interval that the real profit is or alternatively they could verify the reported profit with some cost (costly state verification).

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