

Vacancies in Housing and Labor Markets¹

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Abstract

The housing and business cycles are clearly tied together, and this fact has recently attracted attention by the literature. We contribute to this literature by analyzing the housing and labor markets by means of a DMP model that highlights interdependence of vacancies in housing and labor markets. We estimate the model using US data at the MSA level on housing vacancies from the US Census Bureau's Housing Vacancy Survey (HVS) starting in 1986, data on job vacancies from the Conference Board's Help-Wanted Index starting in 1951, and from the BLS's Job Opening and Labor Turnover Survey (JOLTS) since December 2000. We estimate a Beveridge curve for labor markets, while allowing for spillovers from the housing market, as well as a novel Beveridge curve for housing markets. We then specify VAR models for housing and job vacancies and estimate, based on the VAR results, impulse response functions to study how shocks to either the housing or labor markets will propagate themselves in the other market. The results show that shocks to the owner and rental vacancies have negative and significant impacts on job vacancies.

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1 Introduction

The housing and business cycles are clearly tied together and this has become even more apparent since the Great Recession of 2007–2009. Figure 1 shows the national growth rate for real house prices and GDP and the unemployment rate. The house price index tracks quite well with both the real GDP growth rate and the unemployment rate; the correlation is 0.5 and -0.5, respectively. This is an indication that the national business and housing cycles are closely synchronized. In fact, Leamer (2007) has claimed that housing *IS* the business cycle. He shows that, at the national level, residential investment is a much better predictor of recessions than aggregate business activity (though this same relationship is not as strong at the MSA level [see Ghent and Owyang (2010)]).

Recent work has looked at the impact of homeownership on the labor market. One notable example is the so called “Oswald hypothesis” (Oswald 1977a, b; Coulson and Fisher 2009) that asserts that homeownership and unemployment rates are positively related given the relatively higher moving costs for homeowners. However, as Beugnot *et al.* (2014) show theoretically, even if the unemployment and homeownership rates are positively correlated, individuals would be better off in economies where homeownership is promoted, and the costs from higher homeownership rates, if any, are principally associated with mobility costs, which are higher for homeowners. A second way that such a link has been established is through the literature that estimates the quality of life based on hedonic models of wages and house prices. We propose a new approach based on the link between housing and labor market vacancies.

Vacancy rates in both the housing and labor markets emerge naturally from search models. The vacancy rate is a long-established concept in labor markets, that is central to the study of frictions in those markets as established under the DMP framework. Once created job openings may remain unfilled until suitable workers are found. Vacancies of dwelling units in housing markets are also rigorously grounded in search models (Wheaton 1990; Ngai and Sheedy 2013) and studies of housing market adjustment through search have been long-standing (Ioannides 1975; Genovese and Han 2012). Housing units may remain

unoccupied until buyers or renters may be found.

It is natural to think of joint search models for the labor and housing markets. Housing search is often associated with as well as prompted by job change. Specifically, using data from the PSID for 1991-1993, Ioannides and Kan (1996) report that for 1974-1983, the proportion of moves combined with job changes was 6% for household heads, while the proportion of job changes was 15% per year, and that for residential moves was 15.6%. Thus, more than 40% of the movers also changed jobs, which implies a substantial correlation between moving and job change. Furthermore, nearly two-thirds of movers did so to rent, and one-third to own. These facts agree with data from the CPS for 2004: Ioannides and Zanella (2008), Table 1, report that 17% of moves occur for work-related reasons and 52.7% for housing- and neighborhood- related reasons.

One additional feature of the housing market is that there are both rental and home-ownership vacancy rates. These are linked via housing tenure choice, that is via households' choice of renting versus owner-occupancy. We investigate both vacancy rates in our analysis of labor and housing market vacancies.

Vacancies in housing markets are not as well understood from an empirical standpoint as those in labor markets. In part, this has to do with the fact that so far no housing market counterpart to the Beveridge curve has been proposed. The latter is a firmly established tool in the study of labor markets that tracks the joint movement of unemployment and job vacancy rates over the business cycle. Whereas the Beveridge curve is simply an “accounting relationship” that links vacancy and unemployment rates in steady state situations, shifts in the job creation condition allows us to track the movement of the economy during the business cycle as the points where the Beveridge curve and the job creation condition intersect; see Figures 2 and 3 in Pissarides (2011). Furthermore, when the economy undergoes restructuring, this is reflected on shifts of Beveridge curve, which is in fact evident from Figure 4 below.

Our theoretical model gives rise to an empirical model that includes the Beveridge Curve and its counterpart in the housing market. To do so, we develop a concept for the housing market that intuitively corresponds to unemployment in the labor market, that of the

unfulfilled demand on the part of renters for owner occupied housing. We find that the cyclical movement along this curve is in the opposite direction as that on the labor market Beveridge curve. We believe that we are the first to develop a housing market counterpart of the Beveridge curve. Our theoretical model of housing and labor market vacancies explicitly captures the interdependence of the two markets via a mechanism that we elaborate on further below. Empirically, vacancies in the housing market can shift the labor market Beveridge curve and vice versa.

We take our theoretical model to the data that are obtained from: the US Census Bureau's Housing Vacancy Survey (HVS), the national version of the American Housing Survey (NAHS), the Help-Wanted Index from the Conference Board, and BLS's Opening and Labor Turnover Survey (JOLTS). We estimate models at the national level as well as for 37 US Core-Based Statistical Areas (CBSAs). Annual national-level data on housing vacancies are available starting in 1950 and MSA-level data are measured back as far as 1986. Monthly data on job vacancies begin in 1951. Following Barnichon (2010), we combine the early print index with the recent online index to construct a consistent index of job openings for 1951-2014 at both the national and CBSA level.

Our estimates at the national level show that housing market vacancies do shift the labor market Beveridge Curve. Furthermore, we estimate the "unfulfilled homeownership rate" from a housing tenure choice equation for multiple waves of the NAHS. We define a renter household to be an "unfulfilled owner" if its predicted probability of owning is greater than or equal to 0.5. We then use this prediction to estimate the housing market version of Beveridge Curve. Our results are consistent with the theoretical prediction that the cyclical movement along this curve is opposite to that of the labor market Beveridge curve.

Finally, using the data at the CBSA level, we develop a VAR model of housing and labor market vacancies and estimate impulse response functions to study how shocks to either the housing or labor markets will propagate themselves in the other market. The results show that shocks to the owner and rental vacancies have negative and significant impacts on job vacancies. This is consistent with the thinking that during the Great Recession of 2007-2009 the downturn in the housing market led to the subsequent decline in the labor market and

the real economy.

Pissarides (2013) wonders whether the housing market crash that was associated with the Great Recession might be an appealing explanation for it. Since homeowners are known to be less mobile than renters, the extraordinary expansion of home ownership in recent years might have contributed to the decline in residential mobility. He argues, however, that there is little evidence of “house-lock effect”, namely that falling house prices and the negative equity in many houses are factors behind the fall in mobility. Pissarides argues nonetheless that composition effects due to the shift to more home ownership could still be significant. He speculates that if a secular decline in mobility, whatever its origin might be, is to persist, we should expect future recessions in the US to definitely impact the labor Beveridge curve.

The remainder of the paper is organized as follows. Section 2 provides a review of the recent literature that employs search models in the empirical study of housing markets. Sections 3 and 4 develop the theoretical model that captures the interdependence of the housing and labor markets with frictions. Section 5 describes the data and Section 6 presents the results.

2 Literature Review

There have been a number of papers in the literature that employ search models in the empirical study of housing markets, though very few of them examine both the housing and labor market by means of the full complement of ideas proposed here. Both Head and Lloyd-Ellis (2012; 2014) and Rupert and Wasmer (2012) develop models of joint housing-labor search, which are complementary to one another. Head and Lloyd-Ellis (2012) focus on frictions in the housing market and the role of housing markets in generating frictions between labor markets. They do not, however, allow for frictions originating in the labor market, which is assumed to be Walrasian. Our paper extends the model of Head and Lloyd-Ellis and is discussed in great detail below. A notable recent study is Limnios (2014), who explores whether frictions in the rental housing market can help explain frictions in the labor market.

Rupert and Wasmer (2012) develop a theory of the relationship between unemployment and housing market frictions that focuses on the trade-off between commuting time and location decisions within a single labor market. Jobs break up with an exogenous probability, and new job offers arrive at a rate that is specified in the usual way for labor markets with frictions as a function of labor market tightness, the ratio of vacancies to unemployment. Since the only housing amenity that the model considers is commuting distance, which also indexes wage offers, the job acceptance decision is solely in terms of the maximum acceptable distance [*ibid.*, Eq. (6)]. Workers reject job offers that are too far from their current location. Since labor income net of commuting depends on commuting distance, the value of employment is conditional on commuting distance.

Rupert and Wasmer show that a higher arrival rate of housing opportunities makes workers less choosy about jobs. That is, with more opportunities, workers are prepared to live further away, because they know that opportunities to move closer are more frequent. Also, a higher arrival rate of housing opportunities increases the job vacancy rate and thus the rate of job creation, because of the higher rate of job acceptance by workers. As a result, improved opportunities to move decrease unemployment because of workers' being more likely to accept offers and firms' increasing vacancies and thus labor market tightness.

In a variation of the model, workers receive demographic ("family") shocks, which change the valuation of the current dwelling and allows workers to sample from the existing stock of job vacancies, as opposed to just new vacancies when their jobs break up. Job separations now reflect the possibility that workers may not find an acceptable offer and thus vacancy, and the distribution of commuting distances occupied by workers is suitably adjusted [*ibid.*, Eq. (20)]. In equilibrium, the distribution of workers' commuting distances is a linear combination of the distribution function of new vacancies, weighted by the rate at which new job opportunities arrive, of the distribution function of all vacancies, weighted by the rate at which demographic shocks arrive, and of the distribution of job offers over commuting distances, conditional on their coming from acceptable commuting distances, weighted by the rate of total separations. It is thus clear that labor turnover and frictions, including demographic shocks, have profound effects on individuals' location choices. With job and

housing vacancy searches being jointly indexed by commuting distance, the housing search process is subsumed into the job search. The housing market is not modelled, however, and therefore the impact of location decisions on housing prices is not reflected in the model. The spatial distribution of new and existing vacancies play the role of housing supply, but demand is not rationed by housing price.

Unlike Rupert and Wasmer, Head and Lloyd-Ellis (2012) do distinguish between ownership and renting, a significant distinction in their paper and in the economy. Their Bellman equations are defined separately for employed and unemployed renters and owners and are conditional on two different city types. All those four types possibly coexist in each city. The economy consists of many identical cities of two types, with the types differing in terms of the average productivity of labor. The housing market is intermediated by real estate firms. The value of vacant homes differs across the two city types. The Bellman equations for dwelling units as assets reflect the capital gains associated with offers by either employed or unemployed workers, adjusted by the rate at which offers are made. New housing units may be constructed, with unit costs that increase with city population to reflect congestion.

Head and Lloyd-Ellis rely on the steady-state equilibrium values of the Bellman equations to establish that the rent differential across the two city types is determined by unemployed renters who are assumed to move costlessly between cities, even if they do not receive a job offer. In contrast, the differential in the value of houses is determined by the marginal (unemployed) home-owner who must first receive an outside offer and then incur the (endogenous) liquidity cost of selling their house. This result suggests that anchoring the opportunity cost of owning housing calculations on rent differentials must account for basic characteristics of labor turnover across different city types.

A key friction modelled by Head and Lloyd-Ellis pertains to the illiquidity of housing for home ownership. Because homeowners accept job offers from other cities at a lower rate than do renters, a link is generated between homeownership and unemployment both at the city level and in the aggregate. Their calibration of the model in order to match aggregate US statistics on mobility, housing, and labor flows predicts that the effect of homeownership on aggregate unemployment is small. When unemployment is high, however, changes in the

rate of homeownership can have economically significant effects.

In a sequence of papers, Ngai and Sheedy focus on the frictions associated with buying and selling homes. Ngai and Sheedy (2015) emphasize the dynamic impact of the fact that the majority of housing purchase transactions involve households moving from one house to another, whereby they put their existing homes on the market and plan to buy new homes. This is motivated by households' desire to improve match quality, and consequently their decisions produce a cleansing effect on the quality distribution. Moving may be triggered by an event, like a demographic shock to a household that causes a reassessment of its housing demand. Ngai and Sheedy (2013) emphasizes sellers' decisions, namely when to put a house up for sale and when to agree to a sale. Ngai and Sheedy do not take a position on the correlation between residential moves and job changes.

To the best of our knowledge, ours is the first paper to introduce a Beveridge curve for housing markets in a manner that is consistent with the original definition for the labor market. Peterson (2009) introduces a Beveridge curve for housing markets based on a relationship between the vacancy rate for housing and the rate of household formation, which he intends as a "long-run supply" relationship. Peterson argues that the rate of capital formation being a decreasing function of the housing vacancy rates can be explained by the following: one, the marginal cost of a new house is decreasing in the growth rate of the housing stock; and two, the probability of selling a new house is decreasing in the vacancy rate. Whereas the former assumption is counterintuitive, in view of urban congestion, the latter does agree with intuition.

3 Theoretical Model

We adopt the dynamic model of the housing market with frictions, due to Head and Lloyd-Ellis (2012). The model allows us to examine housing vacancies in the ownership market while also accounting for owning and renting. We extend their model to study frictions in the labor market by using the same Bellman equations for individuals to structure labor market bargaining as the ones they introduce for housing decisions. This leads naturally to

interactions between frictional housing and labor markets. We also propose an extension to allow for frictions in the rental housing market. The remainder of the presentation of the theoretical model is structured as follows. First we introduce the preference structure, which we employ in studying both the housing and labor markets with frictions. This is followed by the development of the Bellman equations for owners and renters, the specification of supply of dwelling units, separately for the owner-occupied and rental stock, and the determination of the value of vacant housing, which reflects critically the illiquidity of housing. The Bellman equations are solved after we have determined the relative numbers of agents in different states, that is employed and unemployed owners and renters. Finally, we extend the model to allow for taste shocks leading to turnover in the ownership market and for frictions in the rental market (neither of which are allowed by Head and Lloyd-Ellis 2012). We develop a concept to express the counterpart of unemployment in the housing market which allows us to introduce a Beveridge curve for housing markets. Next we extend the model to study the labor market with frictions, where we start with the model of Pissarides (1985), as presented in Pissarides (2000). We use the solutions for the Bellman equations obtained earlier to characterize the determination of wages and job vacancy rates. The solutions establish the presence of spillovers from the housing market to the labor market through the wage curve. For most of the analysis, we examine the model at a steady state equilibrium.

3.1 Preferences

Let W^j, U^j , denote expected lifetime utility, conditional on being employed, unemployed, for a renter, homeowner, $j = R, H$ respectively, which is expressed in real terms, and under the assumption of unrestricted borrowing or lending at a fixed rate of interest, ρ . These are generated by utility flow, denoted by π^j , and defined in terms of preference over consumption c , labor supply, l , and housing consumption, z , in a typical period:

$$\pi^j(c, l, z) = c^j - l + z^j, j = R, H. \quad (1)$$

We assume that a person is either employed, supplying one unit of labor, in area i earning w_i , or unemployed, receiving $b_i < w_i$. We suppress the location-specific subscript i , unless it

is strictly necessary for clarity.

We allow for the definition of utility flow (1) to account for housing consumption to depend on housing unit costs that differ for renters and owners and to depend on local conditions. Let: non-housing consumption be the numeraire, with its price set equal to 1, and let κ be rent per unit of rental housing. Ignoring commuting costs, the quantity of housing consumed by renters in a particular area is given by rent expenditure divided by κ . Let p_h be the annual user cost of owner-occupied housing. The latter is defined in standard fashion [Poterba (1986); Henderson and Ioannides (1987)], given a housing price index, p , as the annualized user cost of housing per unit of housing value. A dwelling unit of value V generates an annualized user cost of $p_h V$.² The respective quantity of housing consumed, that is, housing services, is given by $\frac{p_h V}{p}$. Suppose that there are no taxes, nor maintenance, depreciation, and appreciation, and an individual borrows at the real rate of interest ρ to finance living in a house of value V . She would thus incur annual housing costs equal to the opportunity cost of housing of value V , ρV . Equivalently, since housing is durable, services from an actual housing stock V emanate at the real rate of interest, ρ , and are given by $\rho \frac{V}{p}$. More generally, the user cost of housing reflects the implications of the tax treatment of housing as well as its durability.³

²This definition maintains consistency between the rental and the ownership sectors. However, this could be modified so as to be based on transactions values instead of the vacant unit value. Also property tax rates, maintenance and depreciation rates as well as housing price appreciation rates may be area-specific.

³Following Poterba (1986) and Henderson and Ioannides (1987) the user cost of housing reflects mortgage payments at a rate of interest ι , times the portion of the value of a dwelling unit that is financed, $1 - \text{equ}$, and adjusted for the tax deductibility of mortgage interest associated with the portion of the value of owner-occupied housing that is leveraged, by multiplying by $1 - \text{tax}$, the marginal US income tax rate, $1 - \tau$. Property taxes, denoted by rate τ_p here, are also deductible for US income tax purposes. In addition, allowing for maintenance and depreciation, at rates maint and depr , respectively, and deducting the rate of expected housing price appreciation, appr^e , yield the annual user cost of housing as:

$$p_h = [(1 - \tau)(\iota(1 - \text{equ}) + \tau_p) + \text{depr} + \text{maint} - \text{appr}^e].$$

This definition maintains consistency between the rental and the ownership sectors. However, this could be modified so as to be based on transactions values instead of the vacant unit value. Also property taxes rates, maintenance and depreciation rates as well as housing price appreciation rates may be area-specific. In the

Under the assumption of perfect capital markets, with individuals' being able to borrow against their expected future income and to save at rate ρ , the Bellman equations for $W^j, U^j, j = H(\text{omeowner}), R(\text{enter})$, may be defined as follows. Spending in a particular period is equal to the wage (or unemployment compensation) rate plus dissaving. This plus spending on non-housing consumption equals the wage rate (or unemployment compensation, as appropriate) plus dissaving. Following Head et al. (2014), Eq. (3), we let the flow of utility be linear in non-housing and housing consumption. For a homeowner, ignoring the disutility of work and allowing for institutional considerations entering through the definition of p_h , utility per period may be written as:

$$\pi^H(w) = w - \rho V + \text{dissaving} + \frac{p_h V}{p}, \quad (2)$$

where $-\rho V$ denotes the opportunity cost (dissaving) associated with holding (durable) housing stock of value V . For a renter:

$$\pi^R(w) = w - \text{rent expenditure} + \text{dissaving} + \frac{\text{rent expenditure}}{\kappa}. \quad (3)$$

For an unemployed individual, b takes the place of w in both sides of the above expressions.

We see the implications of this formulation, in the simplest possible case at the steady state, with renters and owners maintaining their housing tenure status for ever. Let δ denote the job destruction rate and μ the job finding rate (which will be specified in section 4 below as a function of labor market tightness). The Bellman equations for the conditional value functions are, first for owners:

$$\rho W^H = \pi^H(w) + \delta[U^H - W^H]; \quad (4)$$

$$\rho U^H = \pi^H(b) + \mu[W^H - U^H]; \quad (5)$$

and correspondingly for renters:

$$\rho W^R = \pi^R(w) + \delta[U^R - W^R]; \quad (6)$$

$$\rho U^R = \pi^R(b) + \mu[W^R - U^R]; \quad (7)$$

absence of taxation, maintenance and capital gains, and with $\iota = \rho$, the above definition yields $p_h = \rho$ as a special case of the user cost concept.

where the flow utilities are specified in (2)-(3) above, except that the term dissaving is of course dropped when we integrate from the flow to the stock (the value functions). From now on, we will use $\pi^j, j = H, R$ without the term dissaving.⁴

The associated steady-state unemployment rate is given by: $\frac{\delta}{\delta+\mu}$. The job destruction rate is typically assumed to be exogenous. It may vary across MSAs because of differences in their industrial compositions. The job finding rate is typically specified in terms of the job matching process and labor market tightness, to which we come further below. It can reflect individual characteristics, which is relevant at the empirical stage. Housing spells of homeowners are initially assumed to last for ever, if job market events and housing tenure events are independent. At such a steady state, we could assume that housing units for renters and owners are perfect substitutes.

3.2 Frictions in Housing Markets

Both housing and labor markets are subject to frictions. The individual (or household, the two terms will be used interchangeably) is subject to the risk of job loss: jobs break up at a Poisson rate δ , and the unemployed individual finds a job at a rate μ , per unit of time. Regarding the housing market with frictions, we account for matching of dwelling units and individuals via search, which leads to the determination of vacancy rates for ownership and rental housing markets. Suppose first that rental units may be found instantaneously and thus costlessly, while units for owner-occupancy involve a matching process, frictions. Consequently, the values of vacant units as assets may differ from the transaction prices at which they change owners.

Specifically, let γ denote the rate at which new dwelling units sold by construction firms match with prospective homeowners. Head and Lloyd-Ellis specify γ as the product of the rate at which prospective homeowners match dwelling units, λ , times the ratio of prospective

⁴In contrast to Head and Lloyd-Ellis, our definition of $\pi^H(w)$ in (2) above makes it dependent on V , in general, which is endogenous. We will ignore this endogeneity from now on, when we derive the equilibrium value of V below. However, V does not enter the expressions for the $\pi^j, j = H, R$ under the assumptions that $p = 1$, and $p_h = \rho$.

homeowners to vacant units on the market, ϕ :

$$\gamma = \lambda \times \phi. \tag{8}$$

Clearly, ϕ and thus γ may vary across areas, and we may introduce a subscript i , when it is necessary for clarity. This definition may be generalized by specifying a matching function in the standard Pissarides fashion, but now between individuals and vacant dwelling units.⁵ It may also be generalized to account for the time it takes owner-occupied houses to be transferred from one household to another, when turnover in owner-occupied units is allowed; see section 3.6 below. The fact that matching in housing markets involves frictions makes housing to some extent illiquid; its value when vacant depends on the speed with which a buyer can be found for dwelling units on the market. The model highlights this effect. We specify further below ϕ , the ratio of buyers to vacant units in the market. In section 3.5.1 below, we allow for frictions in the rental housing market as well.

3.3 Housing Supply

There are N individuals who are in one of four different states, employed and unemployed homeowners and employed and unemployed renters. Their respective numbers are N^{WH} , N^{UH} , N^{WR} , N^{UR} , and sum up to the total population:

$$N^{WR} + N^{UR} + N^{WH} + N^{UH} = N. \tag{9}$$

⁵Let $I_{b,t}$, $I_{s,t}$, denote the stock of buyers searching for houses and the stock of sellers searching for buyers, respectively. Let the matching process be specified in the standard fashion in terms of the Poisson rate of contacts generated, denoted by $\Gamma_t = \Gamma(I_{b,t}, I_{s,t})$. So, in general, the rate of arrivals of contacts to the typical dwelling unit in MSA i is: $\gamma = \frac{1}{I_{s,t}} \Gamma(I_{b,t}, I_{s,t})$, which under the assumption of constant returns to scale, this may be written as:

$$\gamma = \Gamma(\phi, 1) = \Gamma\left(\frac{I_{b,t}}{I_{s,t}}, 1\right).$$

This differs from the Head and Lloyd-Ellis assumption, (8) above, only because of the nonlinearity, but is consistent with the assumptions typically made about matching models. Parameter λ is subsumed in this formulation.

It is more convenient to work with the relative numbers of agents. By using lower case n 's, i.e. $n^{WH} = \frac{N^{WH}}{N}$, (9) becomes:

$$n^{WH} + n^{UH} + n^{WR} + n^{UR} = 1. \quad (10)$$

The population is assumed to grow at a rate ν , we need to allow for the construction of new dwelling units. Following Glaeser *et al.* (2014), and Head and Lloyd-Ellis, *op. cit.*, the assumption of free entry in the housing construction-real estate business implies a supply equation for rental housing units: the present value of rents equals the asset value of their construction costs, that is:

$$\frac{\kappa}{\rho} = c_0 + c^R \frac{R}{N},$$

where c_0 denotes fixed construction cost, and $c^R \frac{R}{N}$ variable costs that depend linearly on the rental housing stock, R , relative to population, N . We assume initially that the entire stock of rental units are occupied as soon as they are produced, that is, the rental housing market is not subject to frictions. Since all rental units are occupied, $R = N^{WR} + N^{UR}$, the above equation may be rewritten instead in terms of n^{WR} and n^{UR} :

$$\frac{\kappa}{\rho} = c_0 + c^R (n^{WR} + n^{UR}). \quad (11)$$

Housing is assumed to live for ever, and the rental price κ may be assumed to include maintenance costs. Under the assumption of free entry, we need not worry about the owners of rental housing stock.

Regarding units for owner-occupancy, their value when vacant must compensate their producers:

$$V = c_0 + c^H \frac{H}{N}, \quad (12)$$

where $c^H \frac{H}{N}$ denotes variable costs, that depend linearly on the housing stock for ownership, H , relative to the population, in order to express the cost of land due to congestion. We could also allow for costly conversion of dwelling units from one mode of tenure to another.

We note that the supply equations (11)-(12) link “prices,” that is rents and values of vacant units, to their respective stocks relative to the numbers of individuals. Next, we relate supply equations to demand conditions.

3.4 The Value of Vacant Housing

For asset equilibrium, the value of a vacant dwelling in a particular city reflects the fact that it may be purchased by either employed renters or unemployed renters. The return per unit of time to holding an asset of value V is equal to the probability per unit of time that it may be sold either to an employed renter, at price P^W , or an unemployed renter, at price P^U , whichever of the two bids is higher:

$$\rho V = \gamma \mathcal{E} \max_j \{P^j - V\}, j = W, U. \quad (13)$$

The expectation on the rhs of (13), the arbitrage equation for V , may be written out by recognizing that a unit will either be purchased by an employed renter if $P^W > P^U$, an event that occurs with probability equal to the proportion of those employed among all renters, $\alpha = \frac{N^{WR}}{R}$; correspondingly, the event $P^W < P^U$ occurs with probability $1 - \alpha = \frac{N^{UR}}{R}$.

Consistent with the literature of markets with frictions, a seller and a buyer of a dwelling unit engage in Nash bargaining and split the surplus from the transaction, with σ of V going to the seller and $1 - \sigma$ of $W^H - W^R$ to the buyer. So, the prices paid by employed and unemployed households satisfy:

$$P^W = \sigma V + (1 - \sigma) [W^H - W^R]; P^U = \sigma V + (1 - \sigma) [U^H - U^R]. \quad (14)$$

Thus, we may solve for V from (13) and (14):

$$V = \frac{(1 - \sigma)\gamma}{\rho + (1 - \sigma)\gamma} [\alpha[W^H - W^R] + (1 - \alpha)[U^H - U^R]]. \quad (15)$$

Recall that we assume here that once renters purchase dwelling units and become homeowners they remain so forever. Their conditional value functions are given by (4-5) above. Renters, on the other hand, are faced with opportunities, at a rate γ , to purchase dwelling units and become homeowners. Thus, the respective Bellman equations, the counterparts of (6-7), for the conditional value functions become:

$$\rho W^R = \pi^R(w) + \delta[U^R - W^R] + \gamma [W^H - P^W - W^R]; \quad (16)$$

$$\rho U^R = \pi^R(b) + \mu[W^R - U^R] + \gamma [U^H - P^U - U^R]. \quad (17)$$

We may modify the model to allow for interdependence between employment and housing tenure mode transitions, but for the moment, such transitions are assumed to be independent. However, conditions in the housing market have a profound effect on the conditional value functions.

Next, we use (15) in order to express the transaction prices, P^W, P^R , in terms of the conditional value functions, W^H, W^R , and U^H, U^R . We substitute back into the Bellman equations, (4-5) for owners, and (16-17) for renters, solve for the conditional value functions, namely for W^H, W^R, U^H, U^R , as functions of the real wage rate and unemployment compensation, on one hand, and of labor market and housing market tightness, on the other. Labor market tightness enters the job finding rate for owners and renters, as we discuss in more detail in section 4 below.

Housing market tightness enters via the rate at which prospective home-owners make contacts with housing construction firms, defined in (8) above, $\gamma = \lambda\phi$. Recall that λ is a matching parameter and ϕ is the ratio of the number of potential buyers (employed and unemployed renters) to that of vacant units, the latter being equal to the stock of dwelling units for owner-occupancy that is not owned. If H denotes the total housing stock for owner occupancy, rewriting (8) we have for the unit matching rate:

$$\gamma = \lambda\phi = \lambda \frac{N^{WR} + N^{UR}}{H - N^{WH} - N^{UH}}. \quad (18)$$

3.5 Solution for Housing Market

Given wages and other magnitudes including the value of vacant units for sale from (15), which are determined from the labor markets with frictions model, we may solve the model. The transaction prices for owner-occupied units, P^W, P^R , may be expressed in terms of the four conditional value functions, W^H, W^R, U^H, U^R . There are seven unknowns, namely:

- the total stock for owner-occupancy, H ;
- the rental stock, R ;
- the rent, κ ;

- and the four numbers of agents in different states, $N^{WR}, N^{UR}, N^{WH}, N^{UH}$.

By solving equ. (4), (5), (16) and (17) for the conditional value functions, we can express the value of a vacant home, V , in terms of the four unknown numbers of agents in different states, $N^{WR}, N^{UR}, N^{WH}, N^{UH}$, the unknown housing rental price, κ , and the wage rate.

It is more convenient to think of the model in a steady state, with the number of individuals growing at an exogenous rate ν . Along the steady state, all absolute quantities grow at the same rate leaving relative numbers of agents constant. This leads to four flow relationships in terms of relative numbers of agents: First, all N individuals distribute themselves over the four states, so that their respective relative numbers sum up to 1: (10) holds.

Second, the change in the number of employed renters in a given city i , $\frac{dN^{WR}}{dt}$ equals the number of unemployed renters who become employed, μN^{UR} , minus the measure of employed renters whose jobs are destroyed, δN^{WR} , and minus those who cease to be renters because they become owners, λN^{WR} . That is:

$$\frac{dN^{WR}}{dt} = \mu N^{UR} - (\delta + \lambda) N^{WR}.$$

Dividing by N and imposing the condition that for a steady state, $\frac{dN^{WR}}{dt} = \nu N^{WR}$, and rewriting, yields:

$$(\nu + \delta + \lambda)n^{WR} - \mu n^{UR} = 0. \quad (19)$$

Third, working in like manner, the change in the relative number of unemployed homeowners, $\frac{dN^{UH}}{dt} = \nu N^{UH}$, is equal to minus those unemployed homeowners who find jobs, μn^{UH} , plus the number of those employed homeowners who lose their jobs, plus the number of unemployed renters who become homeowners, λn^{UR} . Rewriting, we have:

$$(\mu + \nu)n^{UH} - \delta n^{WH} - \lambda n^{UR} = 0. \quad (20)$$

Fourth, the increase in the number of employed homeowners, νn^{WH} , is equal to the number of unemployed homeowners getting jobs, μn^{UH} , plus the number of employed renters who become homeowners, λn^{WR} , minus those employed homeowners who become unemployed. Rewriting, we have:

$$\nu n^{WH} + \delta n^{WH} - \lambda n^{WR} - \mu n^{UH} = 0. \quad (21)$$

By rewriting the above equations in matrix form, we have:

$$\begin{bmatrix} \delta + \lambda + \nu & -\mu & 0 & 0 \\ 0 & -\lambda & -\delta & \mu + \nu \\ -\lambda & 0 & \delta + \nu & -\mu \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} n^{WR} \\ n^{UR} \\ n^{WH} \\ n^{UH} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}. \quad (22)$$

The matrix on the l.h.s. of (22) depends on the parameters $(\delta, \lambda, \mu, \nu)$ only. This yields the solution:

$$n^{WR} = \frac{\mu\nu}{(\lambda + \nu)(\lambda + \nu + \delta + \mu)}, n^{UR} = \frac{\nu(\lambda + \nu + \delta)}{(\lambda + \nu)(\lambda + \nu + \delta + \mu)}; \quad (23)$$

$$n^{WH} = \frac{\lambda\mu(\lambda + \nu + \delta) + \lambda\mu(\mu + \nu)}{(\lambda + \nu)(\delta + \mu + \nu)(\lambda + \nu + \delta + \mu)}, n^{UH} = \frac{\lambda\delta(\lambda + \nu + \delta + \mu) + \lambda\nu(\delta + \lambda + \nu)}{(\lambda + \nu)(\delta + \mu + \nu)(\lambda + \nu + \delta + \mu)}. \quad (24)$$

With these results, the share of employed renters $\alpha = \frac{n^{WR}}{n^{WR} + n^{UR}}$, and the unit matching rate are given by:

$$\alpha = \frac{\mu}{\lambda + \nu + \delta + \mu}; \quad (25)$$

$$\gamma = \lambda \frac{\frac{\nu}{\lambda + \nu}}{h - \frac{\lambda}{\lambda + \nu}}, \quad (26)$$

where $h = \frac{H}{N}$, is the ratio of the stock for owner-occupancy to the population. In equilibrium, the denominator above must be positive. The model implies an equilibrium homeownership rate,

$$\text{hr} = n^{WH} + n^{UH} = \frac{\lambda}{\lambda + \nu}. \quad (27)$$

The equilibrium rental rate is thus equal to $\frac{\nu}{\lambda + \nu}$.

We note that these rates depend critically on the rate of growth of the population. In contrast, the equilibrium unemployment rate does not depend on population growth. Clearly, in order to arrive at a more general model, we need to specify churning within the housing market. We go some way further in this direction below in sections 3.5.1 and 3.6. In section 3.5.1, we also relax the assumption that all renters seek to become homeowners, which boosts the equilibrium rental rate.

The conditional value functions for homeowners may be obtained by solving (4–5). Thus, we have:

$$W^H = \frac{1}{\rho(\delta + \mu + \rho)} \left[(\rho + \mu)\pi^H(w^H) + \delta\pi^H(b) \right]. \quad (28)$$

$$U^H = \frac{1}{\rho(\delta + \mu + \rho)} \left[\mu\pi^H(w^H) + (\delta + \rho)\pi^H(b) \right]. \quad (29)$$

These expressions allow for the possibility that bargaining between firms and workers may lead to wage rates that are different between renters and owners, w^H, w^R , respectively, from now on.

We can solve for the conditional value functions for renters, (16-17), after we have expressed the transaction prices for vacant units in terms of the conditional value functions. Recall (14), which gives give transaction prices via Nash bargaining. From (14) and (15), we have instead of (16), (17), respectively:

$$\begin{aligned} \rho W^R &= \pi^R(w) + \delta[U^R - W^R] \\ + \gamma_i \frac{\sigma\rho + \sigma(1-\sigma)\gamma_i(1-\alpha_1)}{\rho + (1-\sigma)\gamma_i} [W^H - W^R] - \gamma_i \frac{\sigma(1-\sigma)\gamma_i(1-\alpha_1)}{\rho + (1-\sigma)\gamma_i} [U^H - U^R]; \end{aligned} \quad (30)$$

$$\begin{aligned} \rho U^R &= \pi^R(b) + \mu[W^R - U^R] \\ + \gamma_i \frac{\sigma\rho + \sigma(1-\sigma)\gamma_i\alpha_1}{\rho + (1-\sigma)\gamma_i} [U^H - U^R] - \gamma_i \frac{\sigma(1-\sigma)\gamma_i\alpha_1}{\rho + (1-\sigma)\gamma_i} [W^H - W^R]. \end{aligned} \quad (31)$$

These equations in concise matrix notation become:

$$\begin{aligned} & \begin{bmatrix} \rho + \delta + \sigma\gamma_i \frac{\rho + (1-\sigma)\gamma_i(1-\alpha_i)}{\rho + (1-\sigma)\gamma_i} & - \left(\delta + \frac{\sigma(1-\sigma)\gamma_i^2(1-\alpha_i)}{\rho + (1-\sigma)\gamma_i} \right) \\ - \left(\mu + \frac{\sigma(1-\sigma)\gamma_i^2\alpha_i}{\rho + (1-\sigma)\gamma_i} \right) & \rho + \mu + \sigma\gamma_i \frac{\rho + (1-\sigma)\gamma_i\alpha_i}{\rho + (1-\sigma)\gamma_i} \end{bmatrix} \begin{bmatrix} W^R \\ U^R \end{bmatrix} \\ &= \begin{bmatrix} \pi^R(w^R) + \frac{\gamma_i\sigma\rho W^H}{\rho + (1-\sigma)\gamma_i} + \frac{\gamma_i^2\sigma(1-\sigma)(1-\alpha_i)}{\rho + (1-\sigma)\gamma_i} [W^H - U^H] \\ \pi^R(b) + \frac{\gamma_i\sigma\rho U^H}{\rho + (1-\sigma)\gamma_i} - \frac{\gamma_i^2\sigma(1-\sigma)\alpha_i}{\rho + (1-\sigma)\gamma_i} [W^H - U^H] \end{bmatrix} \end{aligned} \quad (32)$$

While a full solution of (32) is straightforward and would allow us to compute agents' expected lifetime utility for the purpose of welfare analysis, it is helpful to solve for $W^R - U^R$, which is a crucial input to the bargaining problem between firms and unemployed renters.

$$W^R(w^R; b) - U^R(w^R; b) = \frac{1}{D} \frac{\rho(\rho + \gamma)}{\rho + (1-\sigma)\gamma} \left(w^R - b + \gamma\sigma\rho [W^H(w^H; b) - U^H(w^H; b)] \right), \quad (33)$$

where D denotes the determinant of the matrix on the LHS of (32). Naturally, parameters and all variables reflecting housing market conditions through α and γ that enter the matrix in (32).

To recapitulate, the conditional value functions (W^H, U^H, W^R, U^R) have been solved in terms of the wage rates, w^H, w^R , the unit matching rate, γ , which in view of (26) depends on h , and the labor market tightness that enters via the employment rate, μ . From (11) and in view of (23), the rent κ is determined as a function of the share of renters $n^{WR} + n^{UR} = \frac{\nu}{\lambda + \nu}$, and thus is determined and is exogenous. Finally, from (15), V may be expressed, via the conditional value functions, in terms of the wage rates, w^H, w^R , labor market tightness (via the job finding rate μ) and the unit matching rate $\gamma = \lambda \frac{\frac{\nu}{\lambda + \nu}}{h - \frac{\nu}{\lambda + \nu}}$. These derivations when used in (12) yield an equation for the relative stock of owner-occupied units, h . Finally, the equilibrium is fully determined once the wage rates are set. We turn to this next, which requires looking at the labor market with frictions. In contrast, Head and Lloyd-Ellis assume frictionless labor markets.

3.5.1 Allowing for Frictions in Rental Markets

Underlying the model so far is an assumption that the rental housing market is frictionless. However, the concept of vacancies does apply to rental units, and in fact data on rental market vacancies are also available. The purpose of this extension of the Head and Huw-Ellis model is introduce rental market vacancies, which helps structure the use of such data in our estimation. To the variables denoting the relative numbers of individuals in different labor market states, $(n^{WR}, n^{UR}, n^{WH}, n^{UH})$, and the housing stocks for owner-occupancy and renting, H and R , we need to add the stocks of vacant units, v^H and v^R , respectively. We extend the model of frictions by also allowing for mismatch of owners, that is, given their circumstances some owners would rather be renting, as well as of renters, that is, given their circumstances some renters would rather be owning. Both types of mismatch express financial and mobility frictions. Let the numbers of mismatched individuals be $N_{u,own}$ and $N_{u,rent}$, respectively — unfulfilled owners and renters, respectively. Let the respective probability of mismatch for renters and owners be denoted by m^{sm}^R and m^{sm}^H ,

respectively:

$$\text{msm}^R = \frac{N_{u,rent}}{N^{WR} + N^{UR}}, \quad \text{msm}^H = \frac{N_{u,own}}{N^{WH} + N^{UH}}, \quad (34)$$

With the introduction of mismatch of renters, msm^R , but not of owners, $\text{msm}^H = 0$, the first three equations in (22) continue hold with the modification that instead of λ , the rate at which prospective buyers find dwelling units, we now have $\lambda(1 - \text{msm}^R)$.

The supply equation for the rental housing stock satisfies

$$\frac{R}{N} = n^{WR} + n^{UR} + \frac{v^R}{R} \frac{R}{N}, \quad (35)$$

which now accounts for vacancies of the rental stock. We rewrite (11) in terms of the expected value of a vacant rental unit, V^R ,

$$V^R = c_0 + c^R \left(n^{WR} + n^{UR} + \frac{v^R}{R} \frac{R}{N} \right). \quad (36)$$

Allowing for owner mismatch in the form of unfulfilled renters, msm^H , requires a greater modification of the model. That is, there are now transitions of owners, unemployed and employed, into renters. This needs to be accounted for in the Bellman equations and in equ. (??22) that determine the equilibrium distribution of agents across states. The number of employed and of unemployed renters must account for inflow of employed and unemployed owners who are mismatched. The algebra is straightforward but tedious. Specifically, it involves two steps. First, in equ. (19)–(21) λ is replaced by $\lambda(1 - \text{msm}^R)$. Second, the term $\lambda(1 - \text{msm}^H)n^{WH}$ is added in the rhs of (19), the term $\lambda - (1 - \text{msm}^H)n^{UH}$ is added in the rhs of (20), and the term $-\lambda(1 - \text{msm}^H)n^{WH}$ is added in the rhs of (21). The resulting counterpart of equ. (??22) is more complicated but straightforward to solve. The extended model has the advantage that owning is no longer an absorbing state and a nonzero probability of renting is possible even if $\nu = 0$, thus removing a drawback of the previous model.

These definitions allow us to complete the determination of the values of rental housing units, V^R, V_r^R . At equilibrium, vacant and occupied rental units must earn the market return, that is:

$$\rho V^R = \text{maint} + \lambda \frac{(1 - \text{msm}^H)(N^{WH} + N^{UH})}{R - N^{WR} - N^{UR}} [V_r^R - V^R];$$

$$\rho V_r^R = \kappa + \lambda \frac{(1 - \text{msm}^R)(N^{WR} + N^{UR})}{H - N^{WH} - N^{UH}} [V^R - V_r^R].$$

By solving these two equations in terms of (V^R, V_r^R) and plugging back into (36) we obtain a second equation for $(\frac{v^R}{R}, \frac{R}{N})$. Since $\text{msm}^H, \text{msm}^R, \text{maint}$ are given, the solutions for $(n^{WR}, n^{UR}, n^{WH}, n^{UH})$ obtained above allow us to solve explicitly for the vacancy rate in rental housing markets, $(\frac{v^R}{R})$.

3.5.2 Housing Beveridge Curves

The Beveridge Curve for labor markets is a well-established and a widely researched concept. The intuitive similarities between housing and labor markets allows us to exploit analogies in order to obtain a Beveridge Curve for housing. Analogous to vacancies in labor markets, which is unsatisfied demand by firms for workers, there correspond prospective buyers and prospective renters in housing markets, which is unsatisfied demand by individuals for housing. Analogous to unemployed individuals, which is unsatisfied demand for employment by individuals, there are unsatisfied renters who wish to own, and owners who wish to rent. There are prevented from doing so by frictions. Our development of Beveridge Curves for housing markets is adapted to the institutional features of housing markets, where there are owners and renters and conforms to the notion of Beveridge curve as an accounting relationship at the steady state.

We work first with the owner occupancy market and express the vacancy rate in the housing market for owner-occupancy,

$$\frac{v^H}{H} = \frac{H - N^{WH} - N^{UH}}{H} = 1 - \frac{1}{h} (n^{WH} + n^{UH}) \quad (37)$$

in terms of a concept that serves as the unemployment counterpart in housing markets. Now we allow for mismatch among renters giving rise to unsatisfied ownership demand, the solutions for n^{WH} and n^{UH} depend on $\lambda(1 - \text{msm}^R)$ instead of just λ and thus on the incidence of mismatch. Working with the previous solution, recalling (27), and assuming that $\text{msm}^H = 0$, we have that the equilibrium homeownership rate is:

$$n^{WH} + n^{UH} = \frac{\lambda(1 - \text{msm}^R)}{\lambda(1 - \text{msm}^R) + \nu}. \quad (38)$$

The equilibrium homeownership rate decreases with the probability of mismatch. That is, an increase, due to mismatch of renters, in the number of individuals searching to buy homes reduces the homeownership rate.⁶ It thus follows that the vacancy rate in housing market for owner-occupancy $\frac{v^H}{H}$ is increasing in the rate of mismatch, a concept at the heart of the interaction between the rental and ownership markets.

In developing a Beveridge curve for the owner occupancy market, we propose the concept of the unfulfilled homeownership rate as the counterpart of the unemployment rate. This is defined as

$$\text{uhr} = \frac{N_{u,rent}}{N_{u,rent} + N_{own}}, \quad (39)$$

where $N_{u,rent}$ is defined as the number of renters whose imputed probability of owning is greater than 50%, and N_{own} all self-reported owners. Solving yields:

$$N_{u,rent} = \frac{\text{uhr}}{1 - \text{uhr}}(N^{WH} + N^{UH}).$$

In view of (34) above, along with the expressions for the rental and home ownership rates, we have:

$$\frac{\text{msm}^R}{1 - \text{msm}^R} = \frac{\lambda}{\nu} \frac{\text{uhr}}{1 - \text{uhr}}.$$

It thus follows that the unfulfilled homeownership rate is monotone increasing in the mismatch rate, which in turn implies that the vacancy rate in the owner-occupancy market is increasing in the unfulfilled homeownership rate. Finally, by expressing uhr in terms of msm^R , substituting for $n^{WH} + n^{UH}$ from (38) in (37), we obtain an equation for the Beveridge curve in the owner-occupancy market:

$$\frac{v^H}{H} = 1 - \frac{1}{h} \frac{\lambda(1 - \text{uhr})}{\lambda + \nu(1 - \text{uhr})}. \quad (40)$$

In this expression, λ is exogenous but the owner-occupied housing stock, H , which enters $h = \frac{H}{N}$, is endogenous. It is determined by using (15) to solve out for V in (12). It is

⁶In view of the generalization of the matching model in footnote 3 above, the rate at which buyers contact dwelling units, λ , may be written in terms of the matching function $\Gamma(.,.)$, and the ratio of potential buyers to vacant units, ϕ . That is:

$$\lambda = \Gamma(1, \phi^{-1}).$$

thus a function of the conditional value functions $(W^H, U^U; W^R, U^R)$, which are solved for completely at equilibrium. Therefore, even though the vacancy rate is increasing in the unfulfilled homeownership rate, the relationship is shifted by the cyclical variation in h .

Regarding the rental market, by solving for $\frac{v^R}{R}$ from (35) in (??), we have for the rental vacancy rate:

$$\frac{v^R}{R} = 1 - \frac{1}{R/N}(n^{WR} + n^{UR}). \quad (41)$$

Next we propose the concept of the unfulfilled rental rate as the counterpart of the unemployment rate in rental markets. This is defined as

$$\text{urr} = \frac{N_{u,own}}{N_{u,own} + N_{rent}}, \quad (42)$$

where N_{u-own} is defined as the number of owners whose imputed probability of renting is greater than 50%, and N_{rent} all self-reported renters. Solving yields:

$$N_{u,own} = \frac{\text{urr}}{1 - \text{urr}}(N^{WR} + N^{UR}).$$

In view of (34) and by using this in (??) along with the expressions for the rental and homeownership rates, we have:

$$\text{msm}^H = \frac{\text{urr}}{1 - \text{urr}} \frac{N^{WR} + N^{UR}}{N^{WH} + N^{UH}}.$$

It thus follows that the unfulfilled rental rate is monotone increasing in the owner mismatch rate, urr , and proportional to the relative share of renters to owners. By using the expressions for $\text{msm}^R, \text{msm}^H$ in terms of urr, uhr , respectively, we may obtain an expression for the Beveridge Curve for rental housing markets. That is:

$$\frac{v^R}{R} = 1 - \frac{1}{R/N} \frac{1 - \text{urr}}{\text{urr}} \text{msm}^H (N^{WH} + N^{UH}). \quad (43)$$

Since renting and owning are interdependent, in the most general case, it is not surprising that the vacancy rates are also interdependent. As with the vacancy rate in the owner-occupancy market, the rental vacancy rate depends on $\frac{R}{N}$, the rental housing stock per person, which is endogenous and varies procyclically, thus shifting the rental Beveridge curve. Once we have solved for $(n^{WH}, n^{UH}, n^{WR}, n^{UR})$ from the counterpart of (22) in the general case with mismatch that we discuss in section 3.5.1, we can adapt (43), the equation

for the Beveridge Curve for rental housing markets, accordingly. However, unlike its owner-occupancy counterpart, it will depend on urr as well. Correspondingly, the Beveridge Curve for owner-occupancy markets (40) will have to be modified accordingly.

3.6 Residential Moving and Intra-city Turnover

In the model so far, owners stay for ever in the houses they buy. Yet, it is common experience that owners move, as their circumstances change, by selling their homes to buy other homes or to rent. We generate turnover by homeowners by adopting a modeling trick due to Wheaton (1990), also used by Head and Lloyd-Ellis and Ngai and Sheedy, namely that homeowners suffer a taste, or demographic shock. That is, we assume with probability equal to m_{sm} ⁷ home-owners experience housing taste shocks. Upon experiencing a shock, the service flow a home-owner receives from their current home falls permanently to $\pi^H - \epsilon$, *cet. par.*, while the service flow potentially available to them from other houses remains equal to π^H . Analytically, this is analogous to the exogenous job destruction rate that so simplifies the DMP model. This creates a mismatch, and all such *mismatched* owners immediately become potential buyers, search for a new house and match with vacant houses via the same technology as renters. Once they find a new house, they immediately sell their old house to a real estate firm at the market price for vacant units. The real estate firm sells the new house at a price that reflects the usual bargaining outcome:

$$P_i^{WH} = (1 - \sigma) [W^H - \tilde{W}^H] + \sigma V_i, P_i^{UH} = (1 - \sigma) [U^H - \tilde{U}^H] + \sigma V_i,$$

where \tilde{W}^H, \tilde{U}^H denote the conditional value functions for mismatched owners. We need to express this new set of possibilities by rewriting the Bellman equations for home-owners as follows:

$$\begin{aligned} \rho W^H &= \pi^H(w^H) + \delta(1 - m_{sm})[U^H - W^H] + m_{sm} \delta_m [W^H - \tilde{W}^H]; \\ \rho U^H &= \pi^H(b) + \mu(1 - m_{sm})[W^H - U^H] + m_{sm} \delta_m [U^H - \tilde{U}^H], \end{aligned}$$

and for the mismatched owners, the conditional value functions satisfy:

$$\rho \tilde{W}^H = \pi^H(w^H) - \epsilon + \delta[\tilde{U}^H - \tilde{W}^H] + \gamma_i [W^H - P_i^{WH} + V_i - \tilde{W}^H];$$

⁷Note, there is no superscript, to distinguish from m_{sm}^H, m_{sm}^R .

$$\rho \tilde{U}^H = \pi^R(b) - \epsilon + \mu[\tilde{W}^H - \tilde{U}^H] + \gamma_i [W^H - P_i^{UH} + V_i - \tilde{U}^H].$$

To these new set of possibilities, there correspond steady state unemployment and ownership rates.

4 The Labor Market with Frictions

The model so far has taken as given the wage rate and the employment rate. The treatment that follows completes the analysis by employing the same preference structure to examine symmetrically the labor market under frictions. Since housing market magnitudes enter the analysis, it follows that housing market outcomes show up as determinants of wages and the unemployment rate. That is, we embed the above model of individuals into a DMP model, by following the pared down approach of Pissarides (1985), as presented in Pissarides (2000), the canonical equilibrium model of search unemployment.

4.1 Labor market flows

Consider a labor market in a steady state with a fixed number of labor force participants, L who are either employed or unemployed. Time is continuous and agents have infinite time horizons. Recalling the basic details, jobs are destroyed at the exogenous rate δ ; all employed workers thus lose their jobs and enter unemployment at the same rate. Unemployed workers enter employment at the rate μ which is endogenously determined, as we see shortly below. Frictions in the labor market are modeled by a matching function of the form

$$M = \mathcal{M}(uL, vL), \tag{44}$$

where uL is the number of unemployed workers, a stock, and vL the number of job vacancies, also a stock. The matching function is taken as increasing in both arguments, concave and exhibiting constant returns to scale.

Unemployed workers find jobs at the rate

$$\mu = \frac{\mathcal{M}(uL, vL)}{uL} = \mu(\theta),$$

where $\theta \equiv \frac{v}{u}$ is labor market tightness. It follows that firms fill vacancies at the rate

$$q = \frac{\mathcal{M}(uL, vL)}{vL} = \mathcal{M}\left(\left(\frac{v}{u}\right)^{-1}, 1\right) = q\left(\frac{v}{u}\right) = q(\theta). \quad (45)$$

It follows⁸ that:

$$\mu'(\theta) > 0, \quad q'(\theta) < 0.$$

The intuition is straightforward: the tighter the labor market, the easier it is for workers to find a job, and the more difficult for firms to fill a vacancy. A steady state in the labor market requires that the unemployment rate is unchanging over time. This occurs when the inflow from employment into unemployment, $\delta(1-u)L$, equals the outflow from unemployment to

⁸By definition:

$$\mu(\theta) = \theta q(\theta).$$

Differentiating with respect to θ we have:

$$\mu'(\theta) = q(\theta) + \theta q'(\theta).$$

From the definition of q the second term above becomes:

$$\theta q'(\theta) = -\theta \frac{\partial \mathcal{M}}{\partial (uL)} \left(\frac{v}{u}\right)^{-2} = -\frac{\partial \mathcal{M}}{\partial (uL)} \left(\frac{v}{u}\right)^{-1} < 0.$$

Greater labor market tightness reduces the rate at which firms fill their vacancies. Therefore,

$$\mu'(\theta) = \mathcal{M}(\theta^{-1}, 1) - \theta^{-1} \frac{\partial \mathcal{M}}{\partial (uL)} > 0.$$

The inequality follows from the concavity of $\mathcal{M}(\cdot, \cdot)$ by a simple geometric argument, provided that $\lim_{ul \rightarrow 0} \mathcal{M}(uL, vL) \rightarrow \infty$.

employment, $\mu(\theta)uL$. The steady-state unemployment rate⁹ is thus given as:

$$u = \frac{\delta}{\delta + \mu(\theta)}. \quad (47)$$

Since $\mu(\theta)$ is increasing in its argument, this equation also implies a negative relationship, at the *steady state*, between unemployment and vacancies known as the Beveridge curve, typically depicted on an unemployment rate – vacancy rate space, (u, v) space. In an important sense, this is a mechanical accounting relationship, the consequences of flow balance, and it is this feature that we sought to emulate in section 3.5.2 above in defining Beveridge curves for housing markets.

A deterioration of matching efficiency, i.e., a decline in job finding given a certain level of tightness, involves an outward shift of the Beveridge curve in the (u, v) space. An increase in the job destruction rate, possibly induced by faster sectoral reallocation of jobs, is also associated with an outward shift of the Beveridge curve. The Beveridge curve, computed using U.S. monthly data on unemployment and vacancies, is regularly reported by BLS and is based on its Job Openings and Labor Turnover Survey (JOLTS) [www.bls.gov/ljt]. The movements in the unemployment rate, u , measured here as unemployment divided by the labor force, and v , the job openings (vacancy) rate, measured here as openings divided by employment plus openings. Monthly observations are used to track the business cycle. See Figures 4 and 5 below.

During the Great Recession, a marked outward shift in the Beveridge curve has been ob-

⁹The full dynamic equation for the unemployment rate readily follows: At any point in time, $(1 - u)L$ people are employed. Of these, per unit of time, $(1 - u)L\delta dt$ people lose their jobs and enter unemployment. Again, during time dt , $uL\mu(\theta)$ are finding jobs, thus reducing the ranks of the unemployed. Consequently,

$$du = (1 - u)L\delta dt - uL\mu(\theta) dt.$$

Rewriting this as a differential equation we have:

$$\dot{u} = (1 - u)L\delta - uL\mu(\theta). \quad (46)$$

The Beveridge curve follows if we impose the condition that the unemployment rate remains constant, equilibrium unemployment $\dot{u} = 0$.

served. Earlier recessions were also associated with such shifts, though not as pronounced.¹⁰ The reasons for this shift are not yet fully understood. However, it is clear that the curve is turning around, exactly as predicted by Pissarides' theory. We come to that shortly below. This feature of the observed Beveridge curve has consequences for the housing market, and it is one of the aims of the present paper to explore it fully.

4.2 The Behavior of Workers at Bargaining

Our benchmark model assumes exogenous search effort. Workers' decisions can influence unemployment only through their impact on wage setting. Workers care about their expected present values of incomes and recognize that these values depend on labor market transition rates as well as wages while employed and unemployment benefits while unemployed.

We work with the value functions developed before. Recall $U^j, j = H, R$ denote the expected present value of income of an unemployed worker and $W^j, j = H, R$, the corresponding present value of an employed worker. Equ. (28-29) for owners, and Equ. (32), for renters, are solved for the quantities, namely for W^H, W^R, U^H, U^R , as functions of the real wage rate, unemployment compensation, and labor market and housing market tightness. The solutions for the conditional value functions enter the bargaining model. Here we are faced with a modeling choice, namely whether or not to make wage determination conditional on whether the individual is a renter or an owner. That is, in modeling bargaining between prospective workers and employers whether we assume that the benefit from employment for the worker is either $W^H - U^H$ or $W^R - U^R$. If we assume that bargaining is conditional of housing tenure status, then the resulting labor market tightness will also be similarly conditional. The spirit of bargaining theory in the context of search models suggests that the tenure status ought to affect bargaining, because the utility of the threat point does. Of course, there is no reason that otherwise equivalent labor should be discriminated in employment decisions, unless of course they have different reservation wages. If we were to posit that the benefit from employment, for the purpose of bargaining, WS, is the expected

¹⁰For recent discussions of shifts in the Beveridge curve for labor markets, see Elsby *et al.* (2014) and Diamond and Sahin (2014).

surplus, it is expressed as follows:

$$WS = \frac{n^{UR}}{n^{UR} + n^{UH}} [W^R - U^R] + \frac{n^{UH}}{n^{UR} + n^{UH}} [W^H - U^H]. \quad (48)$$

Still, the logic of the bargaining model requires that the transactions price be dependent on the buyer's being employed or unemployed, as expressed by (14) above, so in wage bargaining the same logic requires that we treat firms' bargaining with workers separately for homeowners and renters.¹¹

4.3 The Behavior of Firms at Bargaining

Jobs are created by firms that decide to open new positions. Job creation involves some costs and firms care about the expected present value of profits, net of hiring costs. The unit value of a firm's output is p_g . Assume, as is standard in this literature, that firms are small, in the sense that each firm has only one job that is either vacant or occupied by a worker. There is a flow cost, $p_g c$, associated with a vacancy, per unit of time. Note it is defined in terms of the value of the output. Let V_u denote that expected present value of having a vacancy unfilled and V_f the corresponding value of having a job occupied, a vacancy filled, by a worker. A job vacancy is an asset from which the firm expects to make money. A job vacancy is filled at the rate $q(\theta)$, whereas an occupied job is destroyed at the rate δ . The value functions associated with a vacancy and a filled job satisfy, respectively, the following equations:

$$\rho V_u = -p_g c + q(\theta)(V_f - V_u). \quad (49)$$

$$\rho V_f = p_g - w + \delta(V_u - V_f), \quad (50)$$

where p_g denotes the price of the good produced, which for consistency with the earlier part of the paper can be set equal to 1, as the good is the numeraire. The l.h.s. of (49) is the opportunity cost per unit of time of a vacancy. Its r.h.s. is the expected return, when costs are incurred per unit of time, $p_g c$, plus the expected capital gain from a job vacancy's getting

¹¹A way to get around this awkward step would be to assume competitive search, as modeled by Diaz and (2013) and Moen (1997).

filled, $q(\theta)(V_f - V_u)$. Similarly, the l.h.s. of (50) is the opportunity cost per unit of time of a filled vacancy. Its r.h.s. is the expected return, which consist of output minus the wage rate, profit per unit of time, $p_g - w$, plus the expected capital gain from a job's becoming vacant, $\delta(V_u - V_f)$.

Hiring by firms is done indirectly by firms' opening vacancies. Firms open vacancies as long as it is profitable to do so. As firms open up vacancies, the value of a vacancy decreases. At the free entry equilibrium, $V_u = 0$.

So, solving (49) in terms of V_u and setting $V_u = 0$, yields an equation for V_f :

$$V_f = \frac{p_g c}{q(\theta)}. \quad (51)$$

Solving (50) for V_f and setting $V_u = 0$, yields a second equation for V_f , $V_f = \frac{p_g - w}{\rho + \delta}$. From these two equations for V_f we have:

$$w = p_g - (\rho + \delta) \frac{p_g c}{q(\theta)}. \quad (52)$$

Here, p_g is the value of the marginal product of labor. Once filled, each job produces a unit of output per unit of time. It is equal to the wage rate plus the capitalized value of the firm's hiring cost. A vacancy once created is expected to last for $q(\theta)^{-1}$ periods of time, generating costs $\frac{pc}{q(\theta)}$. Each vacancy is created with probability δ per unit of time and the hiring cost incurs an interest cost at a rate ρ . So, So, the capitalized value of the firm's hiring cost is given by $(\rho + \delta) \frac{p_g c}{q(\theta)}$. From this relationship, w as a function of θ in (θ, w) space is downwardsloping.

Equ. (52) is referred to as the *job creation* condition. It plays the role of the demand for labor in the standard model of a labor market without frictions, where the quantity of labor is represented by labor market tightness, $\theta = \frac{v}{u}$, the ratio of the vacancy rate to the unemployment rate. Note that in equilibrium, from Equ. (52), that given p_g, w , the incentives to create vacancies are reduced by a higher real interest rate, a higher job destruction rate and a higher vacancy cost. Vacancy creation is encouraged by improved matching efficiency that exogenously increases the rate at which the firm meets job searchers.

4.4 Wage bargaining

Since the labor market is characterized by frictions and bilateral meetings, the standard wage determination mechanism is not appropriate. The main approach that has been used by the markets with frictions literature assumes that there is bargaining between the employer and the worker. So suppose that wages are set through individual worker-firm bargaining. The logic of our model requires that we distinguish between homeowners and renters in their bargaining with employers.¹²

4.4.1 Homeowners' bargaining and labor market equilibrium

The expected capital gain for an unemployed homeowner from becoming employed is equal to $W^H - U^H$. A firm, on the other hand, gives up $V_u = 0$, in order to gain V_f . Following generalized Nash bargaining, the wage rate is determined so as to split the total surplus,

$$\text{Total Surplus}^H = W^H - U^H + V_f - V_u, \quad (53)$$

in order to

$$\max_{w^H} : (W^H - U^H)^{1-\sigma_L} (V_f - V_u)^{\sigma_L}, \quad (54)$$

where $1 - \sigma_L$ is a measure of the worker's relative bargaining power. With free entry of vacancies, $V_u = 0$, and thus: $V_f = \frac{p-w}{\rho+\delta}$. Note that the threat points in the Nash bargaining are taken to be what the worker and the firm would receive upon separation from each other. As Hall and Milgrom (2008) note, the job-seeker then returns to the market and the employer waits for another applicant. A consequence is that the bargained wage is a weighted average of the applicant's productivity on the job and the value of unemployment. That latter value, in turn, depends in large part on the wages offered by other jobs.¹³

¹²Our approach to both housing and labor markets is based on the original formulation of labor markets with frictions due to Pissarides (1985). It can be extended by means of competitive search models, along the lines of Diaz and Jerez (2013), which is applied to housing markets, and Moen (1997), who aims at job market applications.

¹³Some researchers have made alternative assumptions about the threat points. Hall and Milgrom (2008) assume that the threat point is to delay and postpone bargaining and agreement instead of threatening to

The first-order condition for the maximization of the total surplus is:

$$W^H - U^H = (1 - \sigma_L) [W^H - U^H + V_f],$$

which yields $\sigma_L(W^H - U^H) = (1 - \sigma_L)V_f = (1 - \sigma_L)\frac{p-w^H}{\rho+\delta}$. Following Pissarides (2000), we do not work with the reduced forms for W^H and U^H , to express $W^H - U^H$ and instead follow the logic of the Nash equilibrium and solve for W^H from (4), substitute into the l.h.s. of the above equation and solve for w^H to get:

$$w^H = (1 - \sigma_L)p_g + \sigma_L\rho U^H.$$

Next, we solve for U^H from (5) using the alternative expression for $W^H - U^H$, obtained from $\sigma_L(W^H - U^H) = (1 - \sigma_L)V_f = (1 - \sigma_L)\frac{p_g c}{q(\theta)}$. Solving for ρU^H we have: $\rho U^H = b + \theta p_g c^{\frac{1-\sigma_L}{\sigma_L}}$. This yields:

$$w^H = \sigma_L b + (1 - \sigma_L)p_g(1 + c\theta^H), \quad (55)$$

the wage curve for owners. Not surprisingly, it does not depend on housing market conditions: once individuals become homeowners, they stay as homeowners. Of course, this would change if we were to modify the model and allow for turnover for homeowners as well, as in Section (3.6) above.

The two equations, the wage curve and the job creation condition, (55) and (56), the latter being the same as (52) but expressed in terms of (w^H, θ^H) for clarity,

$$w^H = p_g - (\rho + \delta)\frac{p_g c}{q(\theta^H)}, \quad (56)$$

play the role of the labor supply and demand curves, respectively. Solving them jointly determines the wage rate and labor market tightness in the labor market for homeowners, (θ^H, w^H) . It might sound odd that labor market conditions depend on housing tenure status,

walk out of the deal, as Pissarides does. “The bargainers have a joint surplus, arising from search friction, that glues them together.” Hall and Milgrom (2008). They assume that the threats are to extend bargaining rather than to terminate it. The result is to loosen the tight connection between wages and outside conditions of the Mortensen–Pissarides model. When the labor market is hit with productivity shocks, the Hall–Milgrom bargaining model delivers greater variation in employer surplus, employer recruiting efforts, and employment than does the Nash bargaining model.

but the logic of the model is that bargaining is adapted to all parties' specific circumstances. One way to rationalize this is to consider that owners would likely have smaller separation rate δ than renters. Thus, firms are prepared to invest more in employment relationships with owners.

4.4.2 Renters' bargaining and labor market equilibrium

Working in like manner, we formulate the bargaining problem for renters in order to obtain the wage curve for renters. Because in the model renting is a transitional state, as individuals look forward to becoming owners, the wage curve reflects conditions both for renters and owners. The bargaining model is defined as maximizing $\max_{w^R} : (W^R - U^R)^{1-\sigma_L} (V_f - V_u)^{\sigma_L}$, subject to a total surplus condition, like (53), yields:

$$\sigma_L(W^R - U^R) = (1 - \sigma_L)V_f = (1 - \sigma_L)\frac{p_g - w^R}{\rho + \delta}. \quad (57)$$

Following Pissarides (2000), we do not work with the reduced forms for W^R, U^R to write for $W^R - U^R$ from (33). Instead we follow the logic of Nash equilibrium and solve for W^R from (30):

$$\begin{aligned} \left[\rho + \delta + \gamma \frac{\sigma\rho + \sigma(1-\sigma)\gamma(1-\alpha_1)}{\rho + (1-\sigma)\gamma} \right] W^R &= \pi^R(w) + \left[\delta + \gamma \frac{\sigma(1-\sigma)\gamma_i(1-\alpha_1)}{\rho + (1-\sigma)\gamma_i} \right] U^R \\ &+ \gamma \frac{\sigma\rho + \sigma(1-\sigma)\gamma_i(1-\alpha_1)}{\rho + (1-\sigma)\gamma} W^H - \gamma \frac{\sigma(1-\sigma)\gamma(1-\alpha_1)}{\rho + (1-\sigma)\gamma} U^H. \end{aligned}$$

By substituting into (57), we solve the resulting equation for w^R , in terms of (W^H, U^H) and U^R . Next, we solve from (31) for U^R , in terms of (W^H, U^H) and by using the alternative version of the maximization condition for the surplus, $\sigma_L(W^R - U^R) = (1 - \sigma_L)\frac{pc}{q(\theta)}$. Finally, we use the solutions (28-29) in the resulting expression for the wage curve.

A short cut (that uses the reduced form solution and is not fully accurate) yields the following wage curve for renters:

$$w^R = b + \frac{\sigma_L}{1 - \sigma_L} p_g \left[(\rho + \delta + \gamma\sigma) \frac{c}{q(\theta^R)} + c\theta^R \right] - \frac{\gamma\rho}{\delta + \rho + \mu(\theta^H)} (w^H - b). \quad (58)$$

Since $q(\theta^R)$ is decreasing in θ^R , the wage curve, which plays the role of the supply curve here, is increasing in θ^R . Equation (58) and the job creation condition for renters,

$$w^R = p_g - (\rho + \delta) \frac{p_g^C}{q(\theta^R)}, \quad (59)$$

when solved as a simultaneous system determine (θ^R, w^R) labor market tightness and the wage rate for renters.

Not surprisingly, the wage curve for renters does depend on housing market magnitudes: renters become homeowners at the first opportunity, and thereafter stay as homeowners. Forward-looking agents anticipate this prospect. We note that the result implies a spill-over effect from the housing market. An increase in θ^H in labor market tightness in the labor market for homeowners increases the employment rate for homeowners and shifts upwards the wage curve for renters. A decrease in housing market tightness, γ , associated for example with an increase in housing per person, shifts downwards the wage curve in this labor market. This causes, *cet. par.*, a decrease in labor market tightness θ^R , which increases unemployment and decreases vacancies. This, in turn, shifts the Beveridge curve upwards, exactly as it was observed during the downturn associated with the Great Recession of 2007-2009 in the US. Therein lies the power of the Beveridge curve tool: it allows us to track structural shifts in the overall economy as well as at sectoral or regional levels.

Before we move on with the empirics, some remarks are in order. One, at first sight, it might be puzzling that wage rates for owners and renters are different, since individuals are identical with respect to their productivity. Yet differences in wages between renters and owners follows directly from the model. The economic interpretation rests on the fact that firms anticipate different turnover patterns between renters and owners, due to different mobility costs. There are, of course, numerous ways in which the model can be extended, in addition to developing fully the case of turnover by owners and its implications for wage determination, unemployment and labor and housing vacancy rates. A particularly interesting feature that is worth exploring is to allow for correlation between residential moves and job changes. As discussed in section 2 above, more than one-third of moves are also associated with job change.

5 Data

Annual data at the national level on homeownership and rental vacancies is available from the Census Bureau starting in 1950. Data on housing vacancies at the MSA level come from the Census Bureau’s Housing Vacancy Survey (HVS). The HVS is a regular part of the Current Population Survey (CPS). Units that are found to be vacant or were otherwise not interviewed are included in the HVS.¹⁴ These data are available from 1986-present on an annual basis for the largest 75 MSAs (though there are less than 75 MSAs in the early years). These data are somewhat problematic since MSA definitions change over time.

Data on monthly job vacancies starting in 1951 come from the Help-Wanted Index for the fifty largest MSAs; these are an aggregate of ads carried by the press that is provided by the Conference Board.¹⁵ However, it is known that since the mid- to late-1990s, this “print”-based measure of vacancy posting has become increasingly unrepresentative as advertising over the internet has become more prevalent. Figure 2 plots the National print Help-Wanted Index starting in 1977 (note that it coincides with the composite index until 1994). One can see the drop off around 2000. Barnichon (2010) builds a vacancy posting index that captures the behavior of total “print” and “online”-help-wanted advertising, by combining the print with the online Help-Wanted Index published by the Conference Board since 2005. Figure

¹⁴The definition of a vacant housing unit as given by the Census Bureau is “A housing unit is vacant if no one is living in it at the time of the interview, unless its occupants are only temporarily absent. In addition, a vacant unit may be one which is entirely occupied by persons who have a usual residence elsewhere. New units not yet occupied are classified as vacant housing units if construction has reached a point where all exterior windows and doors are installed and final usable floors are in place. Vacant units are excluded if they are exposed to the elements, that is, if the roof, walls, windows, or doors no longer protect the interior from the elements, or if there is positive evidence (such as a sign on the house or block) that the unit is to be demolished or is condemned. Also excluded are quarters being used entirely for nonresidential purposes, such as a store or an office, or quarters used for the storage of business supplies or inventory, machinery, or agricultural products. Vacant sleeping rooms in lodging houses, transient accommodations, barracks, and other quarters not defined as housing units are not included in the statistics in this report. A vacant unit for rent consists of ”units offered for rent and those offered both for rent and sale.”

¹⁵Pissarides (1986) for Britain, and Blanchard and Diamond (1989) for the US have used the Help-Wanted Index in studying labor market adjustment.

2 includes our version of the combined index. It closely replicates Barnichon's index which goes through 2009 and extends it through June 2014.

Figure 3 plots the job vacancy data (the composite Help-Wanted Index divided by the size of the labor force) along with ownership and rental vacancy rates for 1956-2014. The correlations are given below the figure. There is a reasonably strong negative correlation between job vacancies and both rental and homeownership vacancy rates. This is explained by the observations that there tend to be more job vacancies when the labor market is hot, as workers can be more selective, and fewer rental or home ownership vacancies, when the housing market is hot as renters are motivated to enter the ownership market (though there is more churning). We believe this fact has not been noticed before. A potential causal mechanism is that as vacancies increase in the housing market, job vacancies decrease since this opens up more residential locations and allows workers to make better job matches (given the joint residential location and job matching decision process).

We now have the same job vacancy data at the MSA level for 40 MSAs, We have applied a similar procedure to create a combined Help-Wanted Index (HWI) for each of the MSAs.¹⁶ Details about the construction of our MSA-level combined HWI are given in the Appendix. Summary statistics for the composite HWI for 1986 - present are given in Table 1.

Additional data on monthly job vacancies starting in December 2000 are available from the Bureau of Labor Statistics in the Job Openings and Labor Turnover Survey (JOLTS). These data are only provided at the level of the four Census regions for total nonfarm employment as well as aggregated by a number of industrial categories.

We use the National version of the American Housing Survey (NAHS) to estimate renters unfulfilled desire to be home owners. The NAHS is an unbalanced panel of more than 50,000 housing units that are interviewed every two years and contains detailed information

¹⁶We have data for 51 MSAs for the HW online index (HWOL) and 49 MSAs for the print index. But Austin, Buffalo, Honolulu, Las Vegas, Orlando, Portland, Providence, San Jose, Tucson, and Virginia Beach are included in the online data but not in the print data whereas Albany, Allentown, Dayton, Knoxville, Omaha, Syracuse, Toledo, and Tulsa are included in the print data but not in the online data. Also, Houston is missing the print index for 1996.9 to 2003.7 so it is excluded.

on dwelling units and their occupants through time, including the current owner’s evaluation of the unit’s market value. For now, we use the NAHS for survey years 1997-2011. Summary statistics for all the variables used in this calculation are given in Table 1.

6 Empirics

6.1 Beveridge Curve Regressions: labor market

Recall that the Beveridge curve plots job vacancies versus the unemployment rate. Movements along the Beveridge curve indicate positions in the business cycle: higher unemployment and lower vacancies in periods of recession, and lower unemployment and higher vacancies in periods of expansion. Shifts in the Beveridge curve can arise from a variety of reasons: changes in efficiency of the job matching process, skill mismatch, changes in the labor force participation rates, and others, such as economic and policy uncertainty. See Pissarides (2011) for details on shifts in the US and UK Beveridge curves.

Figure 4 plots the National Beveridge Curve for 1951-2014. The job vacancy rate, v_{jobs} , is the composite help wanted index divided by the labor force. The data are split into five episodes that are determined by apparent shifts in the Beveridge curve. Figure 5 plots the Beveridge curve for the most recent episode using the monthly JOLTS data. The curve begins in the upper left corner in January 2001 in a strong period of low unemployment and a high job vacancy rate. It then moves south east and ends up in the southeast corner at the end of 2009 in a weak period of high unemployment and low job vacancies. There appears to be an outward shift in the Beveridge curve over the next six months followed by a north-west movement to a stronger economic position in 2014.

Our theory establishes spillovers between the wage curves for owners and renters, where the latter depends on conditions in the housing market. This is evident in the last term on the r.h.s. of Eq. (58), via γ , the rate at which new dwelling units sold by construction firms are matched with potential buyers, and $\mu(\theta^H)$, the employment rate in the labor market for homeowners. A larger γ causes a downward shift of the wage curve for renters, thus

increasing the respective labor market tightness and causing a downward movement along the corresponding Beveridge curve. The prediction for the full effect requires that we solve jointly the two wage-job creation curve systems for the labor market for renters and owners.

Denote the unemployment rate in MSA i and time t as $\text{unempl}_{i,t}$, the job vacancy rate as $\text{vjobs}_{i,t}$, the ownership vacancy rate as $\text{vown}_{i,t}$, and the rental vacancy rate as $\text{vrent}_{i,t}$. Then the augmented Beveridge curve is specified as

$$\text{vjobs}_{i,t} = \alpha_0 + \alpha_1 \text{unempl}_{i,t}^{-1} + \alpha_2 \text{vown}_{i,t} + \alpha_3 \text{vrent}_{i,t} + \epsilon_{i,t}. \quad (60)$$

We estimate this model using the national composite help wanted index for 1951-2014 where the jobs vacancy rate is this index divided by the size of the labor force. The results for 1951-2014 are given in Table 2. Column (1) of Table 2 is for the regression of the jobs vacancy rate on the inverse of the unemployment rate. The ownership and rental vacancy rates are then added to produce the augmented Beveridge curve. We also include a flag for the 1951-1955 period since the housing vacancy rates are only observed starting in 1956. The results are given in Column (2) of Table 2. This significantly increases the fit of the regression and increases the (magnitude) of the elasticity of the jobs vacancy rate with respect to the unemployment rate to -0.50 (see Column 2). Both housing vacancy rates are significant. Surprisingly the corresponding coefficient estimates are opposite in sign. The coefficient for the rental vacancy rate has a smaller p -value and an elasticity near -1 . This is in line with the negative relationship between labor and housing market vacancy rates that we hypothesized earlier. We next add in indicators of the five episodes as given in Figure 4. These are all significant and slightly increase (the magnitude) of the unemployment rate elasticity. Now only the rental vacancy rate is significant and its elasticity has shrunk (in magnitude) to -0.40 . See columns 3 and 4 of Table 2.

6.2 Beveridge Curve Regressions: housing market

Our theory aims at a symmetric treatment of the labor and housing markets over and above the presence of spillovers. In particular, we would like to develop a housing market Beveridge curve. Whereas the vacancy rate concept applies equally well to the housing

market, hitherto there is no obvious counterpart of unemployment in the housing market. In the labor market, the unemployment rate measures the extent of the unfulfilled desire of labor market participants to work. We posit a counterpart concept in terms of an unfulfilled desire on the part of renters to become homeowners. Individuals are prevented from owning homes due to the inability to get a mortgage because of down payment constraints, poor credit, or because of discrimination in the mortgage credit market.

We proceed by estimating a housing tenure choice equation and then use these results to predict the probability of owning. We define renters with a probability of homeownership greater than or equal to 0.5 as unfulfilled owners. The number of this group is denoted as $N_{u,rent}$ and the number of owners as N_{own} . The unfulfilled homeownership rate is defined in (42) above.

To obtain uhr , we estimate the following equation for the propensity for household head i in MSA m , and year t to be a homeowner:

$$\begin{aligned} own_{i,m,t}^* &= \alpha_0 + \alpha_1 \frac{\text{index}_{mt}^{\text{value}}}{\text{index}_{mt}^{\text{rent}}} \\ &+ \alpha_2 \text{income}_{imt}^p + \alpha_2 \text{income}_{imt}^T + \mathbf{X}_{imt} \alpha_4 + \epsilon_{it}, \end{aligned} \quad (61)$$

where the discrete indicator $own_{i,m,t}$ is defined as $own_{i,m,t} = 1$, if

$$\epsilon_{it} \geq - \left(\alpha_0 + \alpha_1 \frac{\text{index}_{imt}^{\text{value}}}{\text{index}_{imt}^{\text{rent}}} + \alpha_2 \text{income}_{imt}^p + \alpha_2 \text{income}_{imt}^T + \mathbf{X}_{imt} \alpha_4 \right), \quad (62)$$

and renting otherwise:

$$own_{i,m,t} = 0, \text{ otherwise}$$

where $\text{index}_{imt}^{\text{value}}$, $\text{index}_{imt}^{\text{rent}}$ are rental and house value indices (to be explained below). The value to rent ratio is included in the housing tenure equation to capture the relative cost of owning versus renting. The variables income_{imt}^p , income_{imt}^T are permanent and transitory annual household income. Due to mortgage market imperfections, they have different impacts. In the absence of suitable data, income_{imt}^p proxies for wealth. The vector $\mathbf{X}_{i,m,t}$ includes socioeconomic characteristics, like individual education, gender, race, age, and household size.

We generate the auxiliary variables $\text{index}_{mt}^{\text{rent}}$, $\text{index}_{mt}^{\text{value}}$ from the following hedonic equations, for renters and owners, respectively:

$$\ln(\text{rent}_{imt}) = \alpha_{0,m} + \alpha_1 \mathbf{Y}_{1,i,m,t} + \epsilon_{1,i,t}, i = \text{renter}, \quad (63)$$

where rent_{imt} is reported monthly rent paid, and

$$\ln(\text{price}_{imt}) = \beta_{0,m} + \beta_1 \mathbf{Y}_{1,i,m,t} + \beta_2 \mathbf{Y}_{2,i,m,t} + \epsilon_{2,i,t}, i = \text{owner}, \quad (64)$$

where price_{imt} is the respondent's estimate of the property's market price, $\mathbf{Y}_{1,i,m,t}$ denotes a vector of dwelling unit characteristics, and $\mathbf{Y}_{2,i,m,t}$ property tax and lot size. The intercepts of the above hedonic equations vary by MSA, m . Then the rent and value indices are calculated as follows:

$$\text{index}_{mt}^{\text{value}} = 100 \times \exp[\beta_{0,m}]; \quad (65)$$

$$\text{index}_{mt}^{\text{rent}} = 100 \times \exp[\alpha_{0,m}] \quad (66)$$

The predicted values of the permanent and transitory components of household incomes, income_{imt}^p , income_{imt}^T are obtained as the predicted value and residual, respectively, from the following equation:

$$\ln(\text{income}_{imt}) = \gamma_{0,m} + \gamma_1 \mathbf{Z}_{i,m,t} + \epsilon_{2,m,t} \quad (67)$$

where income_{imt} denotes reported household income and $\mathbf{Z}_{i,m,t}$ denotes a vector that includes functions of education, age, race, and gender.

In order to calculate our unfulfilled homeownership rate variable, uhr_t , defined in (42) above, we estimate the tenure choice equation by probit, where the probability of owning is given by $\Phi(\mathcal{X}_{i,m,t}\hat{\alpha})$, where $\Phi(\cdot)$ denotes the cumulative normal distribution, and the vector $\mathcal{X}_{i,m,t}$ includes all characteristics used so far. We define renter i , $\text{own}_{i,m,t} = 0$, as an unfulfilled renter,

$$\text{u-rent}_{i,m,t} = 1, \text{ if } \Phi(\mathcal{X}_{i,m,t}\hat{\alpha}) \geq 0.5; \text{ and } \text{own}_{i,m,t} = 0. \quad (68)$$

It follows that the unfulfilled homeownership rate is given by: $\text{uhr}_t = 100 \times \frac{N_{u-\text{rent}}}{N_{u-\text{rent}} + N_{\text{own},t}}$, where $N_{u-\text{rent},t} = \sum_i \text{u-rent}_{i,m,t}$, and $N_{\text{own},t}$ all self-reported owners.

Working in like manner, we define the unfulfilled rental rate as: $urr_t = 100 \times \frac{N_{u,own}}{N_{u,own,t} + N_{rent,t}}$, where $N_{u,own,t} = \sum_i u_{,own_{i,m,t}}$, and $N_{rent,t}$ all self-reported owners. Note that the estimation of equ. (68) is sufficient, in view of the binary nature of the tenure choice here. That is, $u_{,own_{i,m,t}} = 1 - u_{,rent_{i,m,t}}$.

We estimate the share of unfulfilled homeowners and renters using the NAHS. The mean of the number of unfulfilled homeowners, those who wish to be renters, as a share the number of all homeowners over the sample period is equal to 13%. Correspondingly, the mean of the number of unfulfilled renters, those who wish to be owners, as a share the number of all renters over the sample period is equal to 44%. These estimates are of course sensitive to the choice of the 50% cutoff point above. The US housing Beveridge curves are plotted in Figure 6 with the rental and owner vacancy rates on the left and right axis, respectively. Note that both curves follow very similar patterns, and they are roughly distorted logistic. Each starts in the southeast corner. This is a sign of a hot market when vacancy rates are low and there is an increased renter willingness to get into the ownership market, increasing the unfulfilled ownership vacancy rate. There is then movement to the northwest corner when the housing market is at its low point in 2009-2011. The housing Beveridge curve then moves southeast in 2011, reflecting improvement in the market. We believe that this is the first time that Beveridge curves for the housing market have been drawn.¹⁷

Next, we specify and estimate the augmented housing Beveridge curves for the ownership and rental markets

$$vx_{i,t} = \beta_0 + \beta_1 uxr_{i,t}^{-1} + \beta_2 vjobs_{i,t} + \varepsilon_{i,t}, \quad (69)$$

where $x = h(own), r(rent)$. Results for the standard Beveridge curve are given in Columns (1) and (3) of Table 3. Of course, this is really only illustrative at this point since we only have

¹⁷Peterson (2009) defines a long-run ‘‘Beveridge Curve’’ in the housing market as the rate of household formation as a decreasing function of the vacancy rate for housing, which he finds to be true for the owner-occupied market, the rental market, and the total market for housing irrespective of ownership status. He sees this as a long-run supply condition that he explains by assuming that, one, the cost to produce new housing is decreasing in the growth rate of the housing stock, and two, the likelihood of selling a new house is decreasing in the vacancy rate. The first condition clashes with a long-held stylized fact of urban congestion; the second one is, however, consistent with the search model.

8 observations. The coefficient estimates for $\text{uhr}_{i,t}^{-1}$ are positive and significant in both cases but the elasticities seem unusually large (approximately -6.0 and -3.0 for the ownership and rental models, respectively). The augmented housing Beveridge curve results are given in Columns (2) and (4). The coefficient estimate for $\text{vjobs}_{i,t}$ is negative in both cases but not significant.

6.3 VAR Models and Impulse Response Functions for Owner, Rental, and Job Vacancy Rates

We next specify and estimate VAR models of job and housing ownership and rental vacancy rates using the data at the CBSA level. The purpose is to establish the interrelationship between the two markets and then to calculate how shocks in one market propagate themselves in the other market using an impulse response function. These models are extensions of the augmented labor and housing market Beveridge curves that include lags of the explanatory and dependent variables, the CBSA house price index as an additional control variable, and CBSA fixed effects. We have data for 37 CBSAs for 1991–2012.

First, we check for unit roots in each of the time series (using `xtunitroot` in Stata). All variables, including the unemployment rate (and its inverse) and the owner house price index are found to have a unit root. So we run the VAR regressions in first differences. Next, we test for Granger Causality. These three regressions for owner, rental, and job vacancy rates include two lags of these variables, fixed effects, and time dummies. Whether we run these tests in levels or in first differences, the only (Granger) causality runs from owner and renter vacancy rates to job vacancy rates.

The VAR equations are reduced form (there are no contemporaneous variables included as explanatory variables). That is:

$$\begin{aligned} \Delta \text{vx}_{i,t} = & \alpha_{0,x} + \sum_{j=1,2} \alpha_{1,j,x} \Delta \text{vown}_{i,t-j} + \sum_{j=1,2} \alpha_{2,j,x} \Delta \text{vrent}_{i,t-j} \\ & + \sum_{j=1,2} \alpha_{3,j,x} \Delta \text{vjobs}_{i,t-j} + \sum_{j=1,2} \alpha_{4,j,x} \Delta \mathbf{X}_{i,t-j} + u_{t,x} + v_{i,x} + \varepsilon_{it,x}, \end{aligned} \quad (70)$$

where $vx = \text{own, rent, job vacancy rates}$, that is, o, r, j , $\mathbf{X}_{i,t-j}$ is a two-vector containing the inverse of the unemployment rate and the house price index. First, we estimate these three equations (70) and get coefficient estimates with two lags included. The results are given in Table 4 below.

After estimating these three equations, we want to look at responses to shocks to $vjobs$, $vown$, and $vrent$. We do so by adding a one standard deviation increase in (values given in Table X) and following the changes in $\Delta vx_{i,t}$, $x = o, r, j$ over time. This produces three sets of impulse response functions (IRFs); with shocks to the first-differences in owner, rental, and job vacancy rates. Note that this means that the ordering of the variables is not necessary. IRFs for the three cases are given below; Figure 7. These are cumulative in the levels of $vown$, $vrent$, $vjobs$. These results reinforce the Granger causality findings. The responses to the owner and rental vacancy rates due to a shock to job vacancies are small and not significantly different from zero. The responses to the owner vacancy rates due to a shock to rental vacancies are small and not significantly different from zero (and vice versa). But the shocks to the owner and rental vacancies do have negative and significant impacts on job vacancies. In the case of the shock to owner vacancies, there is a long-term negative and significant impact of about -0.04 in the job vacancies variable. In the case of the shock to rental vacancies, there is a negative and significant impact for the first few periods but the long-term impact of about -0.15 is not significant. The RMSE from the VAR equation for job vacancies is 0.27 so the ratio of the long-term impact from the shock to the owner vacancy rate is 0.15, a reasonably large impact.

These results reinforce the thinking on the Great Recession where it was the downturn in the housing market that resulted in the subsequent decline in the real economy.

7 Conclusions

This paper explores theoretically and empirically the potential interdependence between the housing and business cycles by analyzing the housing and labor markets by means of a DMP-type model. The model gives rise naturally to vacancy rates in housing and labor

markets. We estimate the model using data at the MSA level on housing vacancies from the US Census Bureau's Housing Vacancy Survey (HVS) starting in 1986 and data on job vacancies from the Help-Wanted Index that starts in 1951 and from the Job Opening and Labor Turnover Survey (JOLTS) since December 2000. We also estimate a Beveridge curve for labor markets, while allowing for spillovers from the housing market, which are predicted by our model. We develop a novel Beveridge curve for the housing market, which is based on a housing market counterpart for the concept of unemployment that we propose, namely that of the unfulfilled homeownership rate. Our estimated housing Beveridge curves for the rental and ownership markets confirm our theoretical predictions. Our estimates accounts for interdependence between those markets, and so do a number of VAR-type models for housing and job vacancies. The results from the VAR models can be used to study how shocks to either the housing or labor markets will propagate themselves in the other market. This can help explain the strong relationship between the housing and labor markets during the Great Recession of 2007-2009.

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9 Figures

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2. National Help Wanted Index, 1977:1-2014:6: Conference Board Print Index. Composite Index, JOLTS National (rescaled)
3. Annual Rental, Homeowner and Job Vacancy Rates: 1956-2014
4. U.S. Beveridge Curve, Help Wanted Index: 1951-2014.
5. US Beveridge Curve, Labor Markets, JOLTS Data: 2000:12–2014:3
6. U.S. Housing Beveridge Curve: 1997-2011.
7. Impulse Response Functions

Figure 1

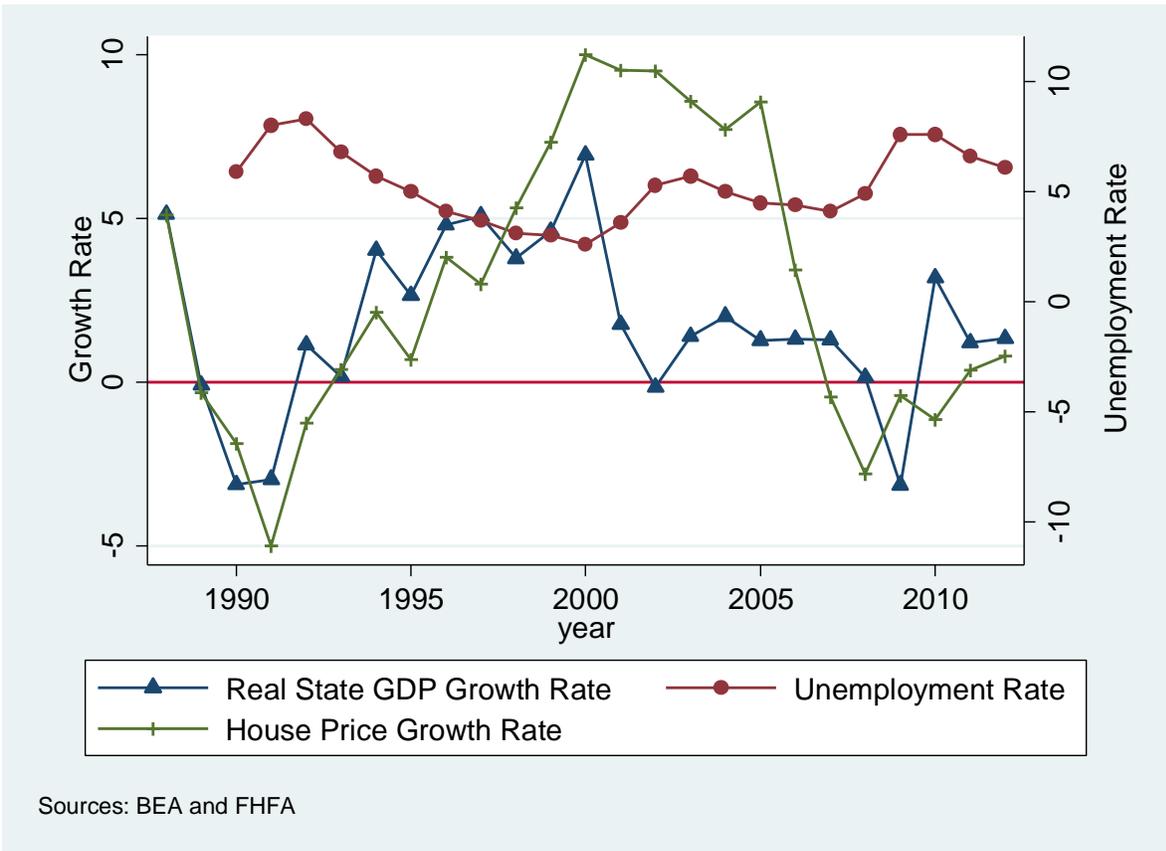


Figure 2

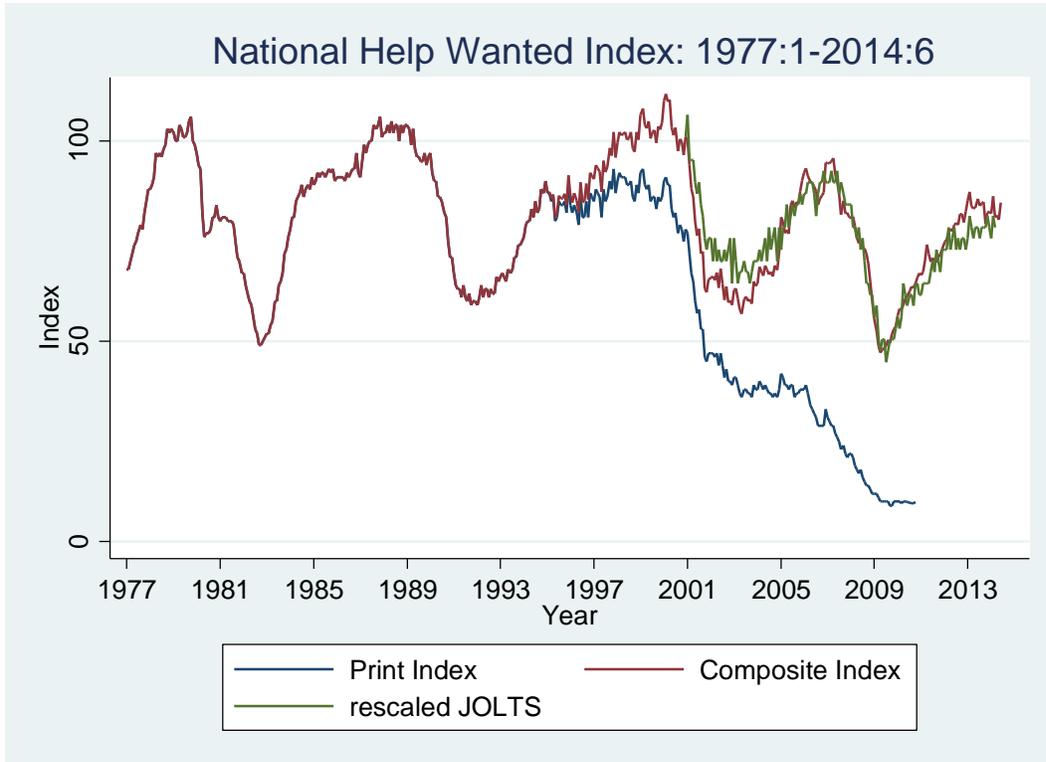
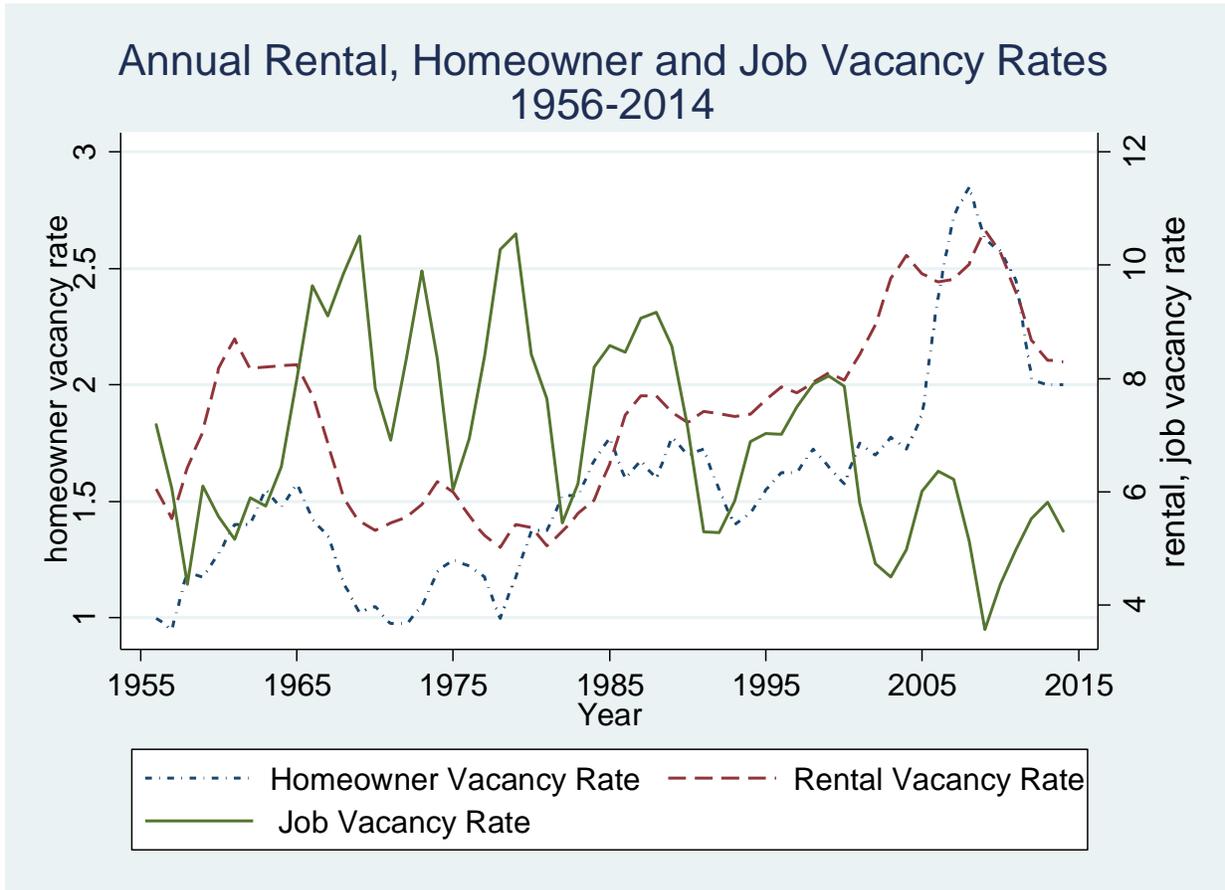


Figure 3



Correlations

	Rental Vacancy	Home Vacancy	Job Vacancy
Home vacancy	0.805		
Job Vacancy	-0.591	-0.495	
Unemployment Rate	0.003	0.294	-0.468

Figure 4

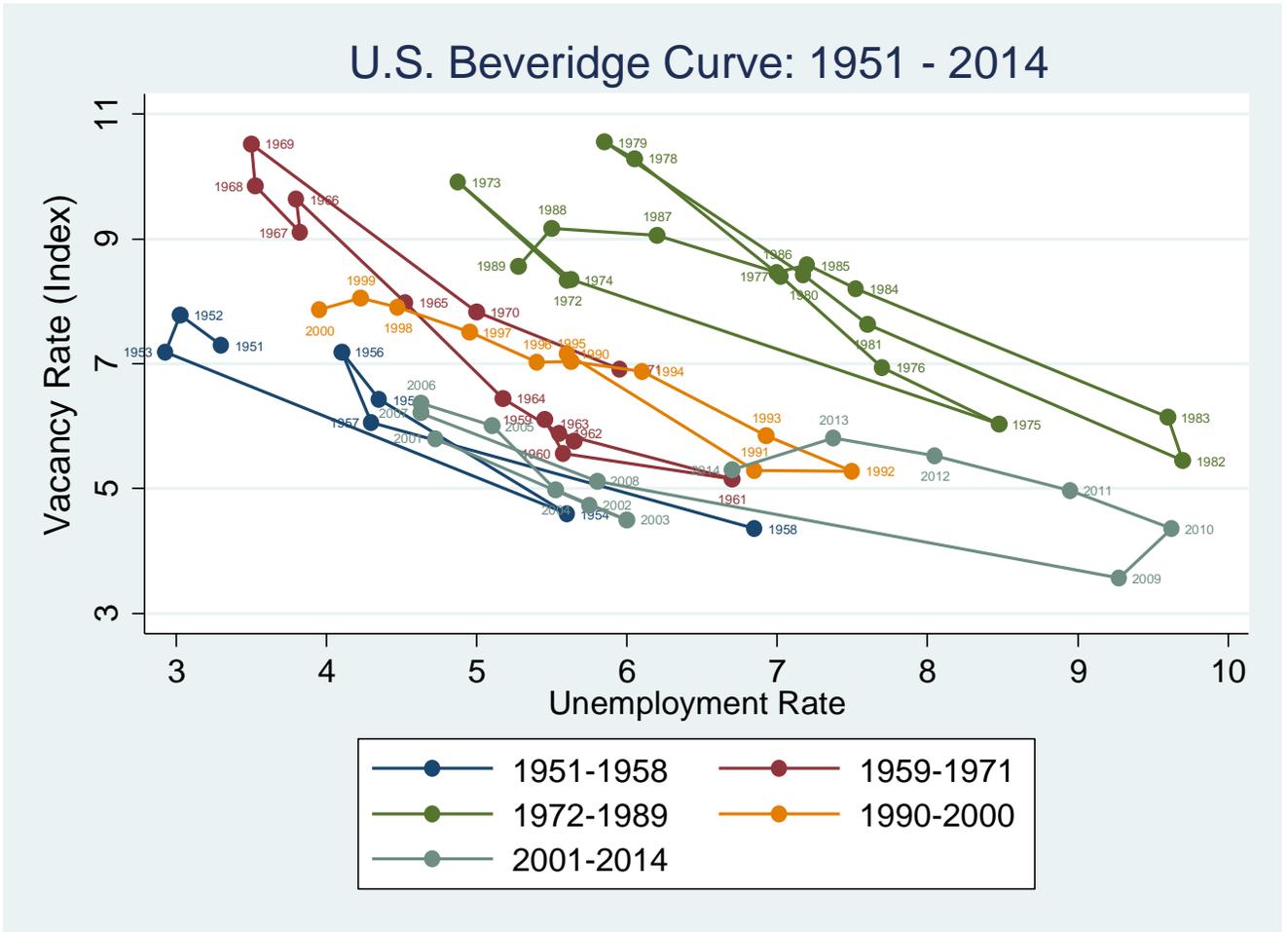


Figure 5

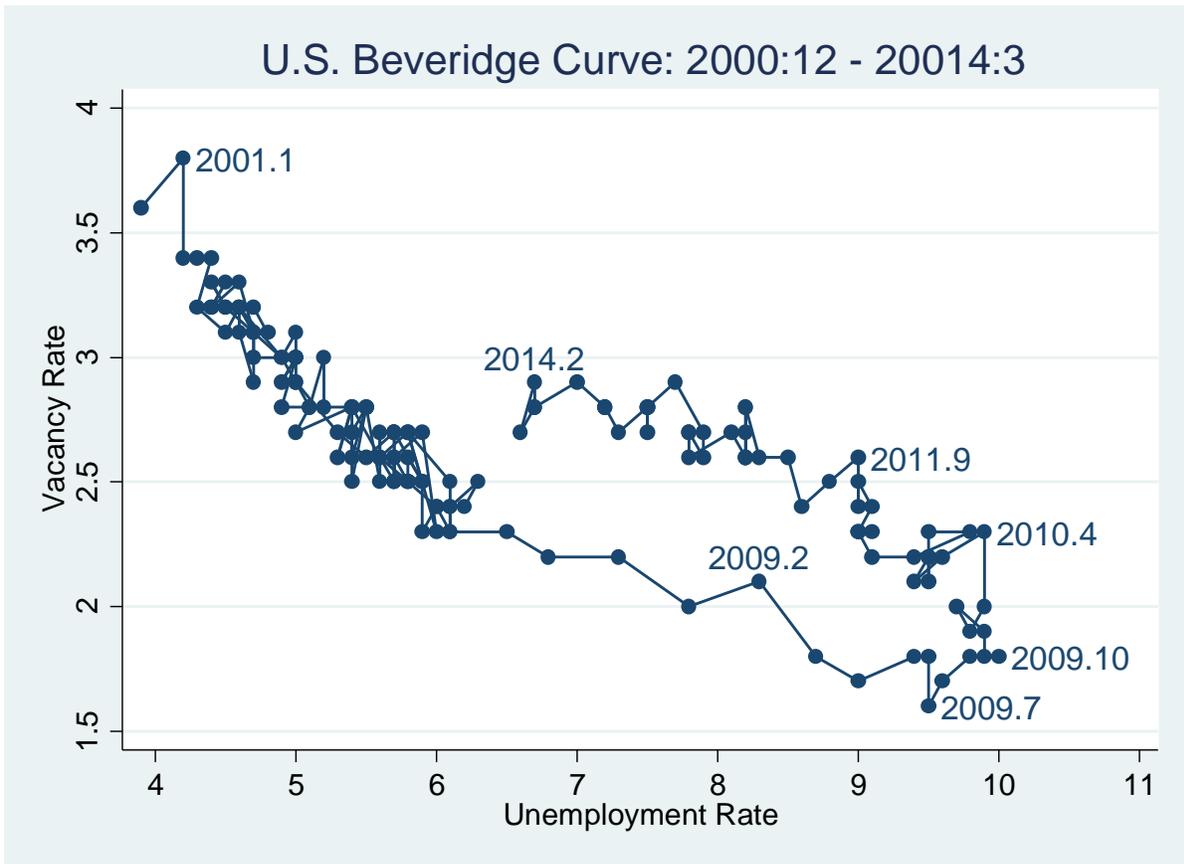


Figure 6

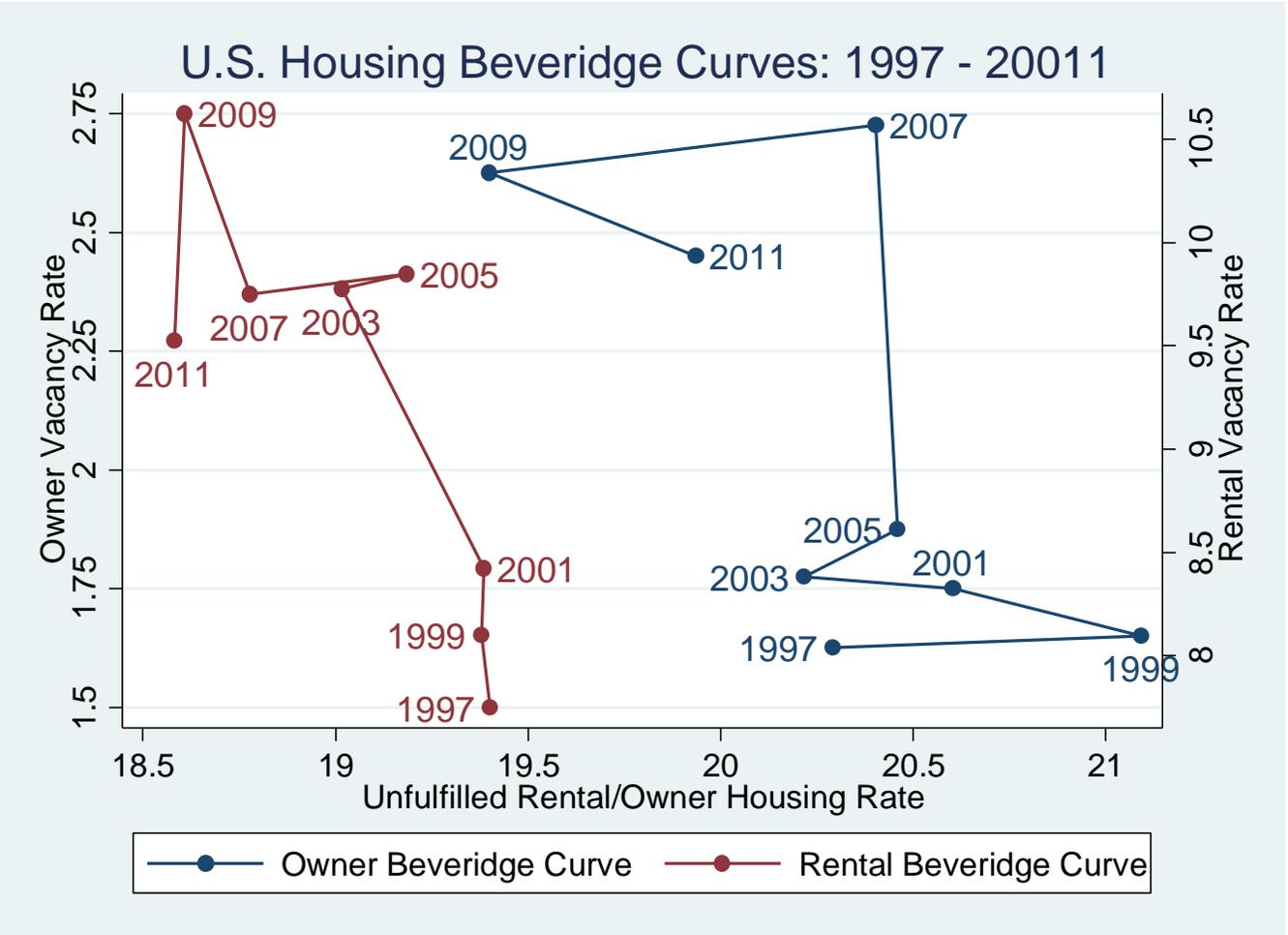
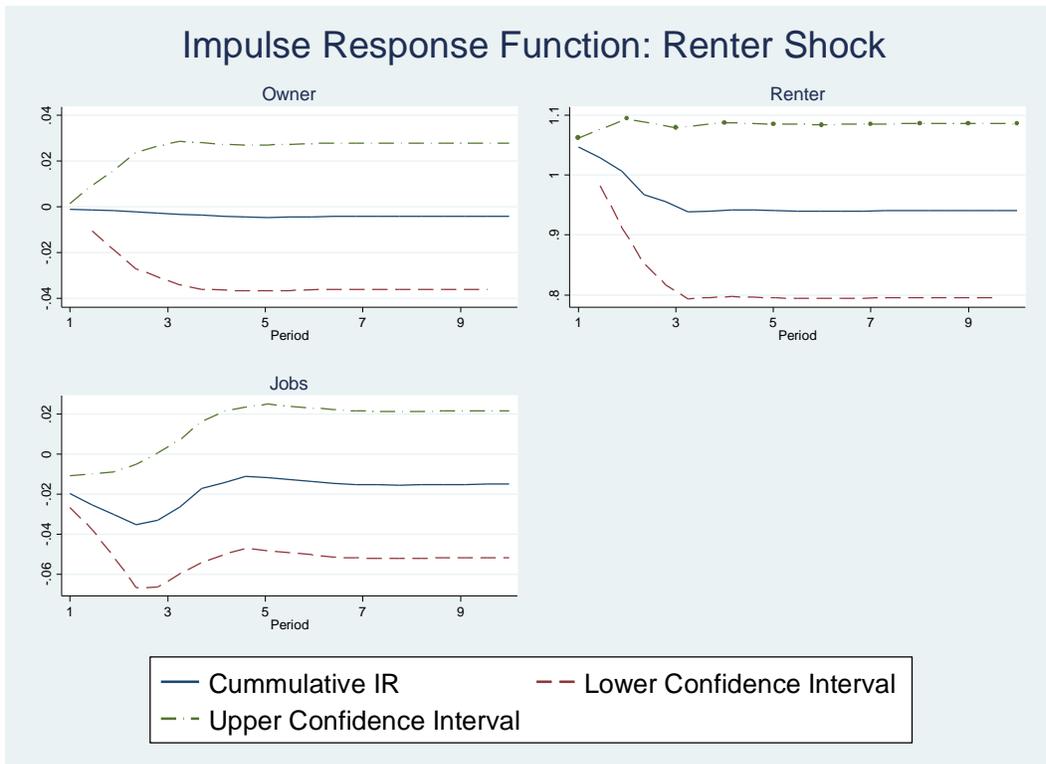
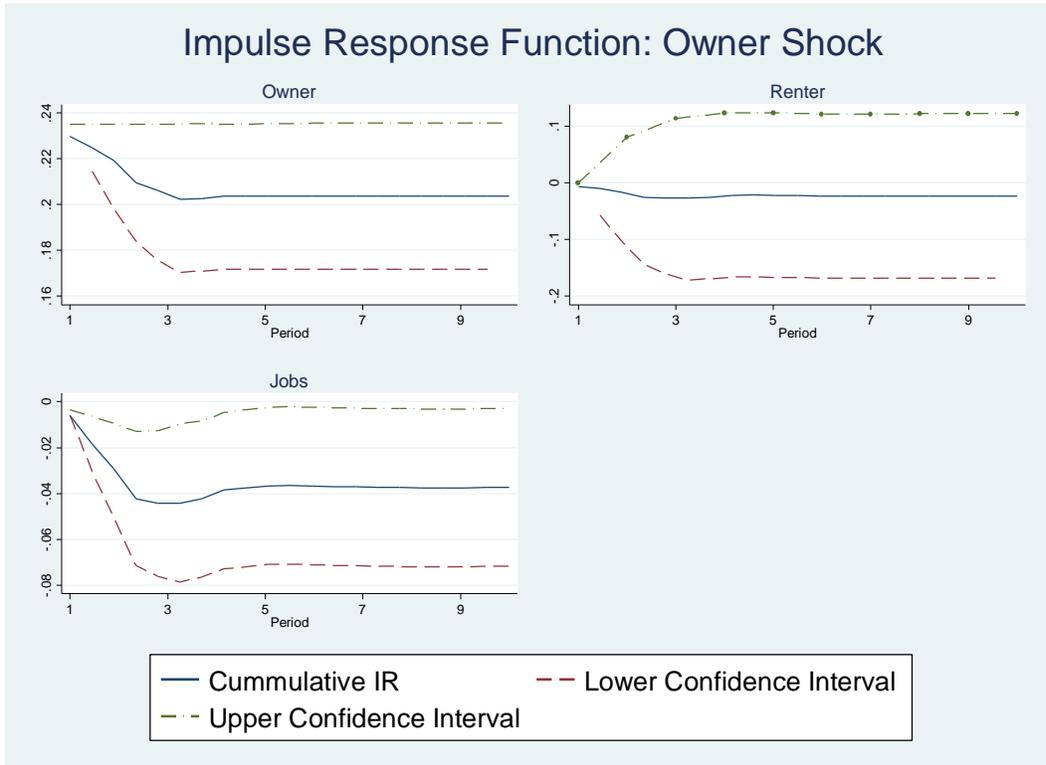
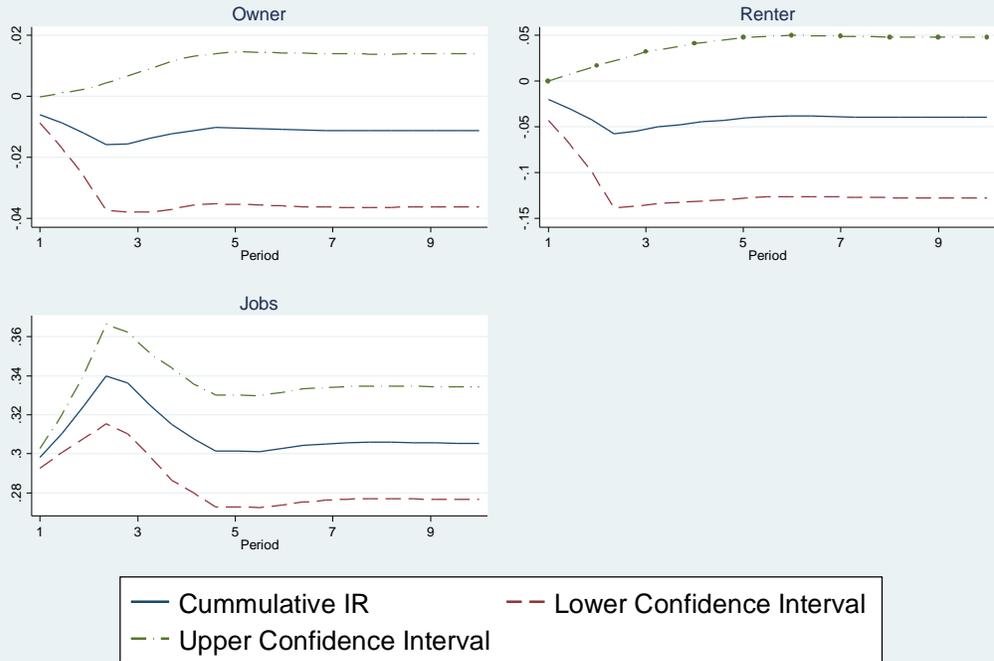


Figure 7: Impulse Response Functions



Impulse Response Function: Job Shock



10 Tables

1. Summary statistics
2. Beveridge Curve 1950 –2014: Dependent Variable is Job Vacancy Rate
3. Housing Beveridge Curve Results: 1997-2011
4. Fixed Effects Results for Homeowner, Rental, and Job Vacancy Rates: CBSA Level

Table 1: Summary Statistics

Variable	Nobs	Mean	Std. Dev.	Minimum	Maximum
HVS Sample					
Single Family House Price Index	1423	144.44	48.26	67.31	333.53
Single Family Housing Permits (in hundreds)	1423	85.76	90.35	0.52	615.58
Owner Occupied Vacancy Rate (HVS)	1492	1.67	1.00	0.10	6.30
Rental Vacancy Rate (HVS)	1492	8.56	4.21	2.50	21.10
Employment (1,000s)	1351	1006.96	1174.33	16.79	7737.40
Fair Market Rent	1423	715.03	231.19	370.16	1791.65
Population (1,000s)	1423	2439.26	2847.47	105.18	19069.80
Per Capita Income (1,000s)	1150	34.31	8.65	15.59	80.14
Age Adjusted Ownership Rate	1423	56.32	6.97	37.75	78.30
Unemployment Rate	1421	5.62	2.21	1.56	15.87
Unemployment Compensation (millions)	1423	396.46	793.45	2.31	11456.67
Wages (1,000s)	1150	39.08	9.75	19.73	94.75
ACS Sample					
Single Family House Price Index	2465	181.32	37.07	105.03	362.87
Single Family Housing Permits (in hundreds)	2465	20.19	45.64	0.10	615.58
Owner Occupied Vacancy Rate	2465	2.34	1.31	0.10	11.90
Employment (1,000s)	2112	283.90	653.85	14.79	7737.40
Fair Market Rent	2465	726.51	191.21	356.17	1730.00
Population (1,000s)	2465	716.26	1596.95	70.26	19069.80
Per Capita Income (1,000s)	2458	35.29	6.79	17.29	80.14
Age Adjusted Ownership Rate	2465	61.05	6.22	38.90	78.30
Unemployment Rate	2463	6.82	2.99	2.07	29.67
Unemployment Compensation (millions)	2458	180.48	567.83	0.59	11456.67
Wages (1,000s)	2458	38.91	6.81	24.44	94.75
Wages – Construction (1,000s)	2443	34.81	8.79	10.48	67.48
Monthly Composite Help Wanted Index					
HWI	13680	98.40	67.27	11.35	628.13
JOLTS Data					
Jobs Vacancy Rate – National	160	2.65	0.41	1.6	3.8
Jobs Vacancy Rate – Northeast	160	2.52	0.36	1.7	4
Jobs Vacancy Rate – Midwest	160	2.39	0.39	1.4	3.8
Jobs Vacancy Rate – South	160	2.82	0.49	1.7	3.9
Jobs Vacancy Rate – West	160	2.77	0.5	1.6	4.3

Table 2: Beveridge Curve Results: 1950-2014
Dependent Variable is Job Vacancy Rate

Variables	(1)	(2)	(3)	(4)
Unempl Rate ⁻¹	12.864*** (3.630)	20.388*** (3.193)	24.618*** (2.332)	25.900*** (2.316)
Owner Vacancy Rate		1.221** (0.587)		0.566 (0.459)
Rental Vacancy Rate		-0.921*** (0.162)		-0.379*** (0.140)
1 if 1959-1971			2.056*** (0.363)	1.789*** (0.489)
1 if 1972-1989			4.352*** (0.406)	3.673*** (0.493)
1 if 1990-2000			2.131*** (0.393)	1.891*** (0.525)
1 if 2001-2014			1.060** (0.403)	1.214* (0.650)
1 if 1951 to 1959		-7.269*** (0.914)		-2.686*** (0.873)
Constant	4.590*** (0.696)	8.282*** (0.889)	0.188 (0.647)	2.173** (0.952)
Elasticities				
Unemployment Rate	-0.31	-0.50	-0.60	-0.63
Owner Vacancy Rate		0.28		0.13
Rental Vacancy Rate		-0.98		-0.40
Observations	64	64	64	64
R-squared	0.168	0.610	0.805	0.836
Standard errors in parentheses				
*** p<0.01, ** p<0.05, * p<0.1				

Table 3: Housing Beveridge Curve Results: 1997-2011

Variables	Dependent Variable			
	Owner Vacancy Rate		Rental Vacancy Rate	
	(1)	(2)	(3)	(4)
Unfulfilled Ownership	241.03*	215.07		
	(120.11)	(233.87)		
Unfulfilled Rental			840.90*	434.63
			(251.41)	(321.23)
Job Vacancy Index		-0.03		-0.36
		(0.19)		(0.21)
Constant	-9.82	-8.39	-34.95	-11.52
	(5.92)	(12.46)	(13.21)	(17.79)
	Elasticities			
Unfulfilled Ownership	-5.76	-5.14		
Unfulfilled Rental			-4.49	-2.32
Job Vacancy Index		-0.9		-0.27
Observations	8	8	8	8
R-squared	0.402	0.404	0.651	0.7081
Standard errors in parentheses				
*** p<0.01, ** p<0.05, * p<0.1				

Table 4 - VAR Regression Results

	Dependent Variable: Vacancy Rates (if First-Differences)		
	Ownership	Rental	Jobs
<u>Vacancies in First Diff</u>			
Ownership _{t-1}	-0.077 (0.048)	-0.090 (0.219)	-0.167 (0.055)**
Ownership _{t-2}	-0.077 (0.048)	-0.093 (0.220)	0.002 (0.056)
Rental _{t-1}	-0.002 (0.010)	-0.066 (0.048)	-0.049 (0.012)**
Rental _{t-2}	-0.004 (0.011)	-0.069 (0.048)	0.034 (0.012)**
Jobs _{t-1}	-0.067 (0.033)*	-0.236 (0.149)	0.323 (0.038)**
Jobs _{t-2}	0.026 (0.031)	0.103 (0.143)	-0.210 (0.036)**
<u>Other Variables in First-Diff</u>			
Unpl _{t-1} ¹	-0.661 (0.459)	-2.080 (2.091)	0.351 (0.528)
Unpl _{t-2} ¹	0.019 (0.443)	0.140 (2.019)	-0.244 (0.510)
House Price Index _{t-1}	-0.002 (0.001)	-0.012 (0.006)*	-0.006 (0.002)**
House Price Index _{t-2}	0.000 (0.001)	0.001 (0.006)	0.004 (0.001)**
_cons	0.002 (0.044)	0.026 (0.198)	0.053 (0.050)
$\hat{\sigma}_\varepsilon$	0.2332	1.0622	0.2685
R^2	0.16	0.37	0.41
N	740	740	740

Standard Errors are in Parentheses. Regressions include time dummies and CBSA fixed effects

* $p < 0.05$; ** $p < 0.01$

11 Appendix: Generating the Composite Help Wanted Index

The method we use to construct the full National help wanted index up through 2014 is similar to Barnichon (2010) but not as complicated. It consists of the following 4 steps.

Step 1. 1951-1994: online help-wanted (HWOL) index, O_t does not exist, $O_t = 0$. As in Barnichon, we use the HWI print index through 1994; $H_t = P_t$ where H_t and P_t are the composite and print Help-Wanted advertising indices, respectively.

Step 2. 1995-2005:5: $O_t > 0$, but not observed. Step 2 is also the same as in Barnichon. To get the composite index, we inflate the print index by the estimated print share: $H_t = P_t/\hat{s}_t^p$. But the procedure we use to estimate is the simpler version in Barnichon. That is, we fit a quartic polynomial to P_t over 1951-2010:10 (the last month that we have the National HWI print index), and estimate as the ratio of the polynomial's value at time t to the polynomial's value in 1994:12. Figure A1 below reproduces Figure 2 in Barnichon. One can see that the print share based on the polynomial trend fits the Sp-JOLTs print share very well. What is key here is that, unlike in Barnichon, the Sp-JOLTs print share DOES exhibit a constant rate of decline and does NOT appear to follow an S -curve. Hence, we use the polynomial trend in the above calculation to estimate and not the more complicated method used by Barnichon.

Step 3. 2005:6-2010:10: both O_t and P_t are observed. Same as in Barnichon: H_t is constructed from

$$\frac{dH_t}{H_{t-1}} = s_{t-1}^p \frac{dH_t}{P_{t-1}} + (1 - s_{t-1}^p) \frac{dO_t}{O_{t-1}}$$

where O_t is the online help-wanted advertising index.

What we have from the online data is the total number of ads (seasonally adjusted and not seasonally adjusted) and the total number of new ads (seasonally adjusted and not seasonally adjusted). We use the seasonally adjusted total number of ads to construct O_t .

Step 4. 2010:11-2014:6: Only O_t is observed. We construct H_t from $d \ln H_t = d \ln O_t$. That is, we assume that $s_t^p = 0$ starting in 2010:11 (the estimated value from the polynomial trend is 0.008). The composite index, the print index and the rescaled JOLTS index are plotted on Figure 5.

We can use the same procedure at the MSA level. The one complication is that the last date that the print index is observed varies across MSAs and can be less than 2010.10; the earliest date for this is June 2005. Between this date and 2010:11 (call this Step 3.1), we use the inflated value of O_t to construct H_t from $d \ln H_t = \ln \frac{O_t}{1-s_t^p} - \ln \frac{O_{t-1}}{1-s_{t-1}^p}$.