Doing Business in the Shadows: Informal Firms, Illegal Immigrants and the Government

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Abstract

We develop a search and matching model with two sectors, a formal and an informal, and illegal immigration. The two sectors differ in several aspects, e.g., only firms that operate in the formal sector pay a payroll tax and severance payments; similarly, only workers employed in the formal sector pay an income tax and social security contributions. We find that an increase in illegal immigration raises the size of the shadow economy, at the expense of the formal sector; an immigration amnesty, on the other hand, has the opposite results. We also study the effects of various labor market policies and show that the so-called Todaro paradox may arise. Finally, we calibrate the model and obtain quantitative estimates regarding the effects of these policies for three Southern European economies, namely, Greece, Italy and Spain.

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1 Introduction

The shadow economy is of great importance in most world economies. It significantly affects macroeconomic factors, such as wages and unemployment while tax evasion constitutes a key controversy between politicians. Schneider (2010) finds that the average size of the informal sector of Southern Europe countries[1], during the period 1999-2007, was 25% of official GDP.

There is a rich literature studying the size of the informal sector, the reasons of its existence and how it emerges. For example, Bosh and Pretel (2012) use data from Brazil to calibrate a two- sector search and matching model. They suggest that policies reducing the cost of entry in the formal sector or increase the cost of informality, increase the size of the formal sector. Fugazza et al (2004) also employ a search and matching model and suggest a similar way to deal with the problem; increase incentives to participate in the formal sector, rather than employ deterrence policies. Zenou (2008) suggests a model with search frictions in the formal sector and a competitive informal sector. He finds a clear positive effect on the employment in the formal sector, when a policy of wage subsidy or hiring subsidy is incorporated.

Immigration and its impact on the labor market outcomes has been in the centre of a lively debate among economists. The empirical results on the subject are often ambiguous ; some, such as Card (1990),find little or no effect of immigration on the wage of native workers, whereas others, such as Borjas et al (1997), find a strong negative effect.

The elasticity of substitution between native and immigrants workers is of high importance. Chassamboulli and Peri (2014) construct a search and matching model with skill heterogeneity and different search cost between natives and immigrants. In the case of skilled natives complementing immigrants in production, they find a clear positive effect on their wage and employment. The results are inconclusive in the case of unskilled native workers who compete with immigrants. Chassamboulli and Palivos (2014) present a model with search frictions and skill heterogeneity and analyze the impact of an immigration influx in the US during the period 2000-2009. They conclude that skill-biased immigration increased the total income of native workers, but also had distributional effects.

Illegal immigrants can only be employed in the informal sector. In this sector, firms are

[1] Namely, Spain, Portugal ,Greece and Italy.

unregulated and therefore cannot be directly affected by labor market policies. Although, they might have indirect effects through policies applied in the formal sector, such as unemployment benefits, taxes and severance payments. To our knowledge, only Cuff et al. (2011) have studied illegal immigration in an economy with two sectors. In this context, they analyze optimal policies and find that in equilibrium, wages between undocumented workers and domestic workers are equal, even if they work in a different sector.

In the present paper, we develop a dynamic search and matching model, with two sectors, a formal and an informal one. Workers can be either natives or illegal immigrants. The former have access in both sectors, whereas the latter can only be employed in the informal sector. Native workers trade of the costs and benefits of the two sectors in order to make an optimal decision. If they find a job in the formal sector, they have to pay an income tax but are entitled to unemployment benefits and a severance payment.

Firms also decide optimally the sector in which they want to post a vacancy. Firms operating in the formal sector, are entitled to a subsidy for maintaining a position, but are obliged to pay a payroll tax and face a firing cost which includes a severance payment as well as some administrative costs.

On the other hand, workers and firms in the informal sector do not have to pay taxes or a firing cost, but face the probability to get audited. If that happens the match is terminated and the firm has to pay a penalty. If an illegal immigrant is caught working in the informal sector, he gets deported. Separation rate is higher in the informal sector. Search frictions exist in both sectors and wages in each sector are determined by Nash bargaining between firms and workers. Illegal immigrants have lower outside option, and thus lower bargaining power. The wage of each worker is a combination of his outside option and his productivity in that job.

We assume that there are two intermediate goods produced in the formal and the informal sector respectively. When produced, they are sold in a competitive market to produce the final good, which uses capital and intermediate goods as inputs. Government collects revenue from income and payroll taxes on workers and firms in the formal sector and from fines imposed on audited firms in the informal sector. It uses its revenue to finance unemployment benefits, vacancy subsidies in the formal sector and lump-sum transfers.

We experiment with several government policies, namely, an immigration amnesty, a change in unemployment benefits, a change in firing cost, a change in tax rates, a change in the auditing probability and the penalty rate associated with it and lastly a subsidy to a vacancy in the formal sector. We prove that under certain conditions, a steady-state equilibrium exists and is unique. We study the aforementioned labor market policies in the special case that formal sector intermediate good and the informal sector intermediate good are perfect substitutes and obtain analytical results.

Specifically we find that an illegal immigration influx, will increase the size of the informal sector . This is the exact opposite result of that of an immigration amnesty. Following, we show that an increase in unemployment benefits in the formal sector can possibly increase the unemployment in the same sector, leading us to the Todaro paradox.[2] In this sense, this paper is related to the literature on rural-urban migration, where Todaro (1976) shows that creating urban jobs can increase urban unemployment due to the negative effect of rural migration, being stronger than the positive effect of creating jobs. Our analytical results suggest that an increase or a decrease of a labor policy instrument can increase both employment and unemployment.

The rest of the paper is organized as follows. Section 2 describes the model and proves the existence of a unique equilibrium. Section 3 presents analytical results on some government policy experiments in the special case that the intermediate good from the two sectors are perfect substitutes. Section 4 presents the calibration our model in its general case, using data from 2000-2010 from Spain, Greece and Italy. In Section 7 we conduct simulations of different government policies. Section 8 concludes the paper

2 The Model

We consider an economy with three sectors: two of them, one formal and one informal, produce intermediate inputs Y_F and Y_I, respectively. The third sector is also a formal sector that produces a final consumption good Y. Throughout the paper, we take the final good to be the numeraire.

There is a continuum of workers, who are either natives (N) or illegal immigrants (M) and are indexed by $i \in \{N, M\}$.[3] The mass of native workers is normalized to one, while that of illegal immigrants is also constant and denoted by M. Native workers seek employment in any of two intermediate sectors, whereas illegal immigrants can work only

[2] See, among others, Todaro (1969), Harris and Todaro (1970) and Zenou (2008).

[3] We abstract from legal immigration. Alternatively, one can assume that legal immigrants are lumped together with natives.

in the informal sector.

The mass of jobs in each sector is determined endogenously, as specified below. Time is continuous. All agents are risk neutral and discount the future at a constant rate $r > 0$.

2.1 Production

The final good Y is produced using capital K and a composite input Z. More specifically, we assume that

$$Y = AK^{\alpha}Z^{1-\alpha}, \quad \alpha \in (0,1), \tag{1}$$

where α is the output elasticity with respect to capital, A denotes total factor productivity and Z is an input composed of Y_F and Y_I according to the following expression:

$$Z = [\chi Y_F^{\rho} + (1-\chi)Y_I^{\rho}]^{\frac{1}{\rho}}, \quad \rho \in (0,1). \tag{2}$$

In equation (2), $\chi \in (0,1)$ is a productivity parameter and ρ governs the elasticity of substitution ($= 1/(1-\rho)$) between the formal- and the informal-sector intermediate inputs, Y_F and Y_I, respectively.

The intermediate inputs are Y_F and Y_I are produced using only labor. In particular, the intermediate sector operate the following linear technologies:

$$Y_F = e_{NF}, \quad Y_I = \sigma e_{NI} + (1-\sigma)e_{MI}, \tag{3}$$

where e_{ij} is the number of workers who are employed in sector $j = F, I$ and are of origin $i = N, M$; σ is a labor productivity parameter. Accordingly, a job in the formal intermediate sector can be filled only by a native worker and the outcome from such a pair is one unit of Y_F. By contrast, a job in the informal sector can be filled either by a native or by an immigrant worker and the outcome is $\sigma \in (0,1)$ and $1-\sigma$, respectively, units of Y_I.[4]

2.2 Markets

Each of the two intermediate inputs is sold in a competitive market. Thus, their prices are equal to their marginal products:

[4] Firms in the informal sector do not use or use less public inputs, such as police and fire protection or the judicial system (which enforces property rights), in order to hide their operation. Thus, they are likely to exhibit lower labor productivity.

$$p_F = \chi(1-\alpha) Y Z^{-\rho} Y_F^{\rho-1}, \tag{4}$$

$$p_I = (1-\chi)(1-\alpha) Y Z^{-\rho} Y_I^{\rho-1}. \tag{5}$$

We also assume that there exists a competitive capital market in which firms can buy and sell capital without delay. Since the market is competitive, the marginal product of capital is also equal to its rental price (p_K), which is in turn equal to the interest rate (r) plus its depreciation rate (ξ). Thus,

$$p_K = \alpha \frac{Y}{K} = r + \xi. \tag{6}$$

Finally, in the labor markets, there are frictions that prevent market clearing. More specifically, each firm possesses one vacancy and must decide first whether to open it in the formal (F) or in informal (I) sector. We use the index $j \in \{F, I\}$ to distinguish between the two types of jobs. There is free-entry in both markets. After opening a vacancy, the firm starts seeking for a worker. Similarly, native workers decide first whether to seek employment in the formal or in the informal sector (as mentioned above, illegal immigrants have no such option).

Job seekers and vacant jobs are matched randomly in a pair-wise fashion. The mass of successful job matches in the formal sector is determined by the matching function $M(v_F, u_{NF})$, where v_F is the mass of formal vacancies and u_{NF} denotes the mass of unemployed native workers in the formal sector. Similarly, the mass of matches in the informal sector is given by the matching function $M(v_I, u_{NI} + u_{MI})$, where v_I is the mass of informal vacancies and u_{NI} (u_{MI}) is the mass of unemployed native (immigrant) workers in the informal sector. The matching functions $M(\cdot)$ are assumed to be twice continuously differentiable, strictly increasing and strictly concave with respect to each of their arguments, exhibit constant returns to scale and satisfy the Inada conditions.

We follow the literature and define the labor market tightness in market j, θ_j, as the number of jobs per unemployed worker; that is, in the formal sector $\theta_F = v_F/u_{NF}$ and in the informal sector as $\theta_I = v_I/(u_{NI} + u_{MI})$. The rate at which vacancies in sector j are filled is $q(\theta_j) = M_j/v_j$, $j = F, I$, where $q'(\theta_j) < 0$. On the other hand, the rate at which unemployed workers (native or immigrant) find jobs in each sector is $m(\theta_j) = \theta_j q(\theta_j)$, where $m'(\theta_j) > 0$. Henceforth, to simplify the notation, we write m_j and q_j instead of $m(\theta_j)$ and $q(\theta_j)$, respectively.

2.3 Institutions

There are some fundamental differences between firms and workers that operate in the two intermediate sectors. First, to maintain a vacancy in sector j a firm must pay an advertising cost c_j, $j \in \{F, I\}$. However, only formal jobs are entitled to any subsidy (h) for maintaining a position.

Second, firms that operate in the formal sector pay a payroll tax at a rate t_F and face some firing costs. We consider two components of firing costs: The first component includes various administrative costs captured by the parameter $f > 0$. These costs include the requirement to give the worker advance notice, procedures that the firm must follow if it wants to lay off, legal expenses in case of a trial, etc. The second component of firing costs is a severance payment, i.e., a transfer from the firm to the employee.[5] As it is the case in most countries, we assume that the severance payment is proportional to the wage, that is, it equals γw_{NF}, where w_{NF} is the wage rate of a worker who is employed in the formal sector (F) and is native (she is of origin N). On the other hand, firms that operate in the informal sector pay neither taxes nor firing costs. However, the government monitors the labor market and if a firm is caught operating in the informal sector, then it it is forced to terminate the match and pay a penalty rate η on output.[6] Such an event occurs with a probability (arrival rate) δ. Hence, $\delta\eta$ is the expected penalty rate paid by a firm in the informal sector.

Third, native workers who work in the formal sector pay an income tax (including social security contributions) at a rate t_w. On the other hand, workers in the informal sector do not pay taxes. Nevertheless, informal jobs are less stable for the following two reasons. First, the arrival rate of negative shocks is probably higher, i.e., the separation rate in the informal sector s_I is higher than that in the formal s_F. Second, as mentioned above, firms are audited at a rate δ and if they are caught operating illegally then they have to terminate the match.

Finally, during unemployment, native workers receive a flow of income b_{Nj}, $i = N, M$, $j = F, I$, which captures the opportunity cost of employment, e.g., the payoff from home production, leisure and unemployment benefits. This income is net of any search cost that they incur when looking for a job. On the other hand, illegal immigrants do not

[5] In our model, there are no quits and every termination of employment is a no-fault dismissal.

[6] We assume that η is the penalty rate net of any administrative cost that is necessary to enforce the law.

receive unemployment benefits; nevertheless, they also incur a cost of searching for a job, which is, in general, higher than that faced by natives.[7] We let b_{MI} denote the income of an immigrant in unemployment, which could be negative. Throughout the paper, we assume that the output of match between a vacancy and a worker exceeds the income of the unemployed worker of the same type, i.e., $p_F > b_{NF}$, $\sigma p_I > b_{NI}$, and $(1-\sigma)p_I > b_{MI}$.

2.4 Asset Values

In general, we let Π and V be the values associated with a filled and an unfilled vacancy, and E and U the values associated with an employed and an unemployed worker, respectively. More specifically, let Π_{ij} be the present discounted value associated with a firm in sector j that is matched with a worker of origin i. Then in steady state:

$$r\Pi_{NF} = p_F - (1+t_F)w_{NF} - s_F\left[\Pi_{NF} - V_F + (f + \gamma w_{NF})\right], \tag{7}$$

$$r\Pi_{NI} = (1-\delta\eta)\sigma p_I - w_{NI} - (s_I + \delta)(\Pi_{NI} - V_I), \tag{8}$$

$$r\Pi_{MI} = (1-\delta\eta)(1-\sigma)p_I - w_{MI} - (s_I + \delta)(\Pi_{MI} - V_I), \tag{9}$$

where w_{ij} is the wage rate of a worker who is employed in sector $j = F, I$ and is of origin $i = N, M$ and V_j is the value associated with an unfilled (vacant) position in sector j. As mentioned above, the total firing cost in the formal sector is $f + \gamma w_{NF}$, where $f > 0$ is a fixed amount. Recall that jobs matched with natives in the informal sector have a higher separation rate than jobs matched with natives in the formal sector ($s_I + \delta > s_F$).

The expected income streams accrued to an unfilled vacancy in the sector $j = F, I$ are given by

$$rV_F = -c_F + q_F(\Pi_{NF} - V_F), \tag{10}$$

$$rV_I = -c_I + q_I\left[\phi_{NI}\Pi_{NI} + (1-\phi_{NI})\Pi_{MI} - V_I\right], \tag{11}$$

where ϕ_{NI} represents the probability that a vacancy meets a native worker ($i = N$) in the informal sector ($j = I$). More specifically,

$$\phi_{NI} = \frac{u_{NI}}{u_{NI} + u_{MI}}. \tag{12}$$

[7] Battisti et al. (2014) cite empirical evidence in support of this assumption.

We turn next to values associated with the workers. The expected income streams accrued to employed workers are given by

$$rE_{NF} = \tau + (1 - t_w)w_{NF} - s_F(E_{NF} - U_{NF} - \gamma w_{NF}), \tag{13}$$

$$rE_{NI} = \tau + w_{NI} - (s_I + \delta)(E_{NI} - U_{NI}), \tag{14}$$

$$rE_{MI} = w_{MI} - (s_I + \delta)(E_{MI} - U_{MI}), \tag{15}$$

where τ is a lump-sum transfer provided by the government to its citizens.

Similarly, the values associated with unemployed workers are:

$$rU_{NF} = \tau + b_{NF} + m_F(E_{NF} - U_{NF}), \tag{16}$$

$$rU_{NI} = \tau + b_{NI} + m_I(E_{NI} - U_{NI}), \tag{17}$$

$$rU_{MI} = b_{MI} + m_I(E_{MI} - U_{MI}). \tag{18}$$

We also assume free entry in establishing either type of vacancy. Thus, in equilibrium, the expected payoff of posting a vacancy is equal to zero, that is,

$$V_j = 0, \;\; j = F, I. \tag{19}$$

2.5 Wage Determination

Once a worker meets a firm, they bargain over the wage rate. We assume that they essentially solve a generalized Nash bargaining problem given by[8]

$$\max_{w_{NF}} \left[E_{NF} - U_{NF} - \gamma w_{NF}\right]^{\beta} \left[\Pi_{NF} - V_F + f + \gamma w_{NF}\right]^{(1-\beta)},$$

for the matches in the formal sector and by

$$\max_{w_{iI}} (E_{iI} - U_{iI})^{\beta} (\Pi_{iI} - V_j)^{(1-\beta)},$$

[8] We assume that wages are constantly renegotiated at no cost. Hence, the relevant wage for an unemployed worker who contacts a firm in the formal sector for the first time, and hence is not entitled to a severance payment, is the same wage as the one for an already employed worker. This is so, because the unemployed worker will immediately renegotiate the wage once a contract is signed.

for the matches in the informal sector, where $\beta \in (0,1)$ represents the worker's bargaining strength. The solution to each of these two problems gives, respectively,

$$(1-\beta)\left[E_{NF} - U_{NF} - \gamma w_{NF}\right] = \beta\left[\Pi_{NF} - V_F + f + \gamma w_{NF}\right], \tag{20}$$

$$(1-\beta)(E_{iI} - U_{iI}) = \beta(\Pi_{iI} - V_j). \tag{21}$$

The total surplus generated by a match in the formal and the informal sector is $S_{NF} = \Pi_{NF} - (V_F - f) + E_{NF} - U_{NF}$ and $S_{iI} = \Pi_{iI} - V_I + E_{iI} - U_{iI}$, $i = N, M$, respectively. In each case, workers and firms get a share β and $1-\beta$, respectively, of this surplus. Notice that the severance payment γw_{NF}, being a pure transfer from the firm to the worker, drops out of the definition of the surplus S_{NF}. By using the above asset value equations, we can derive the expressions for the wage rates.

Substituting for $E_{ij} - U_{ij}$ and Π_{ij}, using equations (7-8) and (13-18), in equations (20) and (21), and noting that $V_j = 0$ (equation 19), we find

$$w_{NF} = \frac{\beta\left[r + s_F + m_F\right](p_F + rf) + (1-\beta)(r + s_F)b_{NF}}{\Phi_F}, \tag{22}$$

$$w_{NI} = \frac{\beta(1-\delta\eta)\left[r + s_I + \delta + m_I\right]\sigma p_I + (1-\beta)(r + s_I + \delta)b_{NI}}{\Phi_I}, \tag{23}$$

$$w_{MI} = \frac{\beta(1-\delta\eta)\left[r + s_I + \delta + m_I\right](1-\sigma)p_I + (1-\beta)(r + s_I + \delta)b_{MI}}{\Phi_I} \tag{24}$$

where $\Phi_F = \beta\Gamma m_F + \{\Delta - \gamma\left[r + (1-\beta)m_F\right]\}(r + s_F)$, $\Gamma = 1 + t_F - r\gamma$, $\Delta = 1 - (1-\beta)t_w + \beta t_F$, and $\Phi_I = r + s_I + \delta + \beta m_I$. In each case, the worker's wage when employed in a particular job is basically a combination of his outside option and his productivity in that job.

Comparing the wages across sectors, we see that if productivity is higher in the formal sector ($p_F > \sigma p_I$), then the wage of a native worker is higher than that of her counterpart in the informal sector. The same effect results from the presence of a firing cost in the formal sector and a higher separation (auditing) rate in the informal sector. On the other hand, the income tax raises the (net) wage in the informal sector, as does the payroll tax. As regards, the wage of natives and immigrants in the informal sector, the former is higher for two reasons: higher productivity (if we assume that $\sigma > 1/2$) and higher unemployment income (lower search cost) ($b_{NI} > b_{MI}$), i.e., higher outside (of employment) option for the natives.

2.6 Steady-State Composition of the Labor Force

The following definitions apply regarding the different sub-groups in the labor force:

$$\begin{aligned} u_{NF} + e_{NF} &= \lambda, \\ u_{NI} + e_{NI} &= 1 - \lambda, \\ u_{MI} + e_{MI} &= M, \end{aligned}$$

where $\lambda \in (0,1)$ and $1-\lambda$ represent the share of native workers in the formal and informal sector, respectively, M denotes the mass of illegal immigrants, and e_{ij} is the number of workers who are employed in sector $j = F, I$ and are of origin $i = N, M$. The share λ is determined endogenously below. Moreover, in steady state, where the flows in and out of unemployment for each sub-group are equal to each other, we have

$$\begin{aligned} u_{NF} &= \frac{s_F}{s_F + m_F}\lambda, \quad e_{NF} = Y_F = \frac{m_F}{s_F + m_F}\lambda, \\ u_{NI} &= \frac{s_I + \delta}{s_I + \delta + m_I}(1-\lambda), \quad e_{NI} = Y_{NI} = \frac{m_I}{s_I + \delta + m_I}(1-\lambda), \\ u_{MI} &= \frac{s_I + \delta}{s_I + \delta + m_I}M, \quad e_{MI} = Y_{MI} = \frac{m_I}{s_I + \delta + m_I}M. \end{aligned} \tag{25}$$

Next, we can write the expression regarding the probability that a firm finds a native worker in the informal sector as

$$\phi_{NI} = \frac{u_{NI}}{u_{NI} + u_{MI}} = \frac{1-\lambda}{1-\lambda+M}. \tag{26}$$

2.7 Steady-State Equilibrium

As mentioned above, native workers must decide in advance whether to search in the formal or in the informal sector. In making their decision, they compare the values of being in the two sectors. In equilibrium, native workers are indifferent between entering the formal or the informal sector. Therefore, the arbitrage condition is given by

$$U_{NF} = U_{NI}.$$

Using equations (7), (16) and (20) to solve for U_{NF} and equations (8), (17) and (21) to solve for U_{NI}, this equality can be written as:

$$\frac{\beta m_F (1 - t_w + s_F\gamma)(p_F + rf) + (\Delta - r\gamma)(r + s_F) b_{NF}}{\Phi_F} = \frac{\beta m_I (1 - \delta\eta)\sigma p_N + (r + s_I + \delta) b_{NI}}{\Phi_I}, \tag{27}$$

where it may be recalled that $\Phi_F = \beta\Gamma m_F + \{\Delta - \gamma[r + (1-\beta)m_F]\}(r + s_F)$, $\Gamma = 1 + t_F - r\gamma$, $\Delta = 1 - (1-\beta)t_w + \beta t_F$, and $\Phi_I = r + s_I + \delta + \beta m_I$.

Definition. A steady-state equilibrium is a set $\{\theta_j^*, e_{ij}^*, u_{ij}^*, w_{ij}^*, \lambda^*\}$, where $i \in \{N, M\}$ and $j \in \{F, I\}$, such that

1. The intermediate input markets clear (equations 4 and 5);
2. The capital market clears (equation 6);
3. The free-entry condition for vacancies of each sector j is satisfied (equations 19);
4. The Nash bargaining optimality condition for each sector j and origin i holds (equations 20 and 21);
5. The numbers of employed and unemployed workers for each sector j and origin i remain constant (equations 25);
6. The equilibrium mobility condition is satisfied (equation 27).

Using the free-entry conditions (equations 19), we derive the following two equations

$$\frac{c_F}{q_F} = \frac{\Theta_F(p_F + rf) - (1-\beta)\Psi b_{NF}}{\Phi_F} - f, \tag{28}$$

$$\frac{c_I}{q_I} = (1-\beta)\frac{\phi_{NI}[(1-\delta\eta)\sigma p_I - b_{NI}] + (1-\phi_{NI})[(1-\delta\eta)(1-\sigma)p_I - b_{MI}]}{\Phi_I}, \tag{29}$$

where $\Theta_F = (1-\beta)(1-t_w) - \gamma(r + \beta s_F + m_F)$, $\Psi = 1 + t_F + s_F\gamma$, and ϕ_{NI} is defined in equation (26).

Substituting the steady-state values of Y_F, Y_{NI} and Y_{MI} into the price equations of p_F, p_N and p_M gives

$$p_F = \chi(1-\alpha)A^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{r+\xi}\right)^{\frac{\alpha}{1-\alpha}}[\chi + (1-\chi)\psi^{\rho}]^{\frac{1-\rho}{\rho}},$$

$$p_I = (1-\chi)(1-\alpha)A^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{r+\xi}\right)^{\frac{\alpha}{1-\alpha}}[\chi\psi^{-\rho} + (1-\chi)]^{\frac{1-\rho}{\rho}},$$

where ψ is defined as the ratio of the two intermediate inputs Y_I and Y_F

$$\psi \equiv \frac{Y_I}{Y_F} = \frac{m_I(s_F + m_F)[\sigma(1-\lambda) + (1-\sigma)M]}{m_F(s_I + \delta + m_I)\lambda}.$$

Next, plugging the expressions for p_F, and p_I into equations (27), (28), and (29) forms a system of three equations that describes the behavior of the three variables $\{\theta_F, \theta_I, \lambda\}$. Having determined θ_F^*, θ_I^* and λ^*, we can obtain the equilibrium values for all the other variables by substituting in the appropriate equations.

3 Analytical Results

The model we have outlined above is too complex to study analytically and we have to rely on numerical analysis. Nevertheless, before doing that, we derive analytical results for a simplified version, namely, when $\rho = 1$. This condition implies that the two intermediate inputs Y_F and Y_I are perfect substitutes and their price, given in general by equations (4) and (5), become constant. Therefore, the effects of any government policy operating through the prices of the intermediate inputs are shut down.

The condition $\rho = 1$ also implies that the steady-state equilibrium system becomes recursive. Specifically, first, by using equation (28), we obtain a unique θ_F^*. Second, combining equation (27) and θ_F^* gives a unique θ_I^*. Finally, plugging θ_I^* into equation (29) yields a unique λ^*.

Proposition 1 (Existence and Uniqueness). Under certain parameter restrictions, confined in the Appendix, a steady-state equilibrium exists and is unique.

Figure 1 demonstrates the way the equilibrium is calculated. Panel (a) shows the equilibrium in the formal sector. The left-hand side of equation (28), which represents the average cost of a vacancy in the formal sector, is upward sloping. On the other hand, under the assumptions listed in the Appendix (see the Proof of Proposition 1), the right-hand side, which represents the expected value of a filled position in the formal sector, is downward sloping. The intersection of the two lines gives the equilibrium value θ_F^*. Next, under the assumptions listed in the Appendix (see again the Proof of Proposition 1), equation (27), which ensures that an unemployed native worker exploits every arbitrage opportunity between the two sectors, is depicted by an upward-sloping curve (see panel b). Given θ_F^*, determined in panel (a), this curve gives the equilibrium value of θ_I^*. Finally, we assume that

$$(1 - \delta\eta)[\sigma p_N - (1 - \sigma)p_M] < b_{NI} - b_{MI}, \qquad \text{(Assumption 1)}$$

i.e., that the expected difference in productivities between a native and an immigrant is lower than the difference in their outside options.[9] Under (Assumption 1) then, equation (29), which represents free entry in the informal sector, is depicted by an upward-sloping curve (see panel c). Given θ_I^*, determined in panel (b), this curve gives the equilibrium value of λ^*.

Next, we analyze various government policies, namely, an immigration amnesty, a change in unemployment benefits, b_{NF}, a change in firing cost parameters, γ and f, a change in tax rates, t_w and t_F, a change in the auditing probability, δ, and the penalty rate associated with it, η, and finally, a subsidy to a vacancy in the formal sector, i.e., an increase in h (decrease in c_F).

3.1 Size and Composition of Immigration

Consider first the impact of an increase in the number of illegal immigrants M. Equations (27) and (28) indicate that θ_F and θ_I remain the same. Using equation (29), we see that λ decreases in M, implying that illegal immigration induces more native workers to participate in the informal sector. The intuition behind this result is that the increase in illegal immigration increases the value of a filled position in the informal sector. This spurs job entry with a concomitant increase in market tightness θ_I. These better job prospects (lower unemployment and higher wage) induce native workers to move from the formal to the informal sector until the previous values of the market tightness, the wage and the unemployment rate are restored. In the end, the mass of native workers employed in the informal sector, $1 - \lambda$, is higher. Also, the numbers of employed and unemployed native workers in the formal sector decrease whereas the numbers of employed and unemployed native and immigrants workers in the informal sector increase. Similarly, the numbers of filled and unfilled vacancies in the former sector decrease and in the latter increase. In other words, an increase in illegal immigration raises the size of the informal sector. Finally, in terms of Figure 1, an increase in M leaves all the curves in panels (a) and (b) unchanged and shifts up the MM curve in panel (c).

Exactly the opposite are the effects of an immigration amnesty, namely, the case where the government grants a legal status to a fraction $\mu < 1$ of illegal immigrants. Consequently, as far as the labor market is concerned, the mass of natives increases to

[9] Obviously, this assumption holds if $p_N = p_M$. Recall also that b_{MI} is small and possibly negative, which makes (Assumption 1) even more likely to hold.

$1+\mu M$ and that of illegal immigrants decreases to $(1-\mu)M$. Following similar steps with those for the analysis of an increase in M, we can see that an international amnesty decreases the size of the informal sector but leaves wages and unemployment rates the same.

3.2 Unemployment Benefits

Next we consider an increase in the value of leisure for formal workers, b_{NF}, which is mostly associated with an increase in unemployment benefits. Using equation (28), we see that an increase in b_{NF} lowers the value of a filled position in the formal sector, which results in firm exit, a decrease in the tightness θ_F and hence an increase in the unemployment rate u_{NF}. As regards the wage rate w_{NF}, there are two countervailing effects: on the one hand, the higher value of unemployment benefits raises workers' outside option and hence increases their wage, but on the other, the lower probability of finding a match, lowers their bargaining position and hence decreases their wage. Similarly, the effect on the value of being unemployed in the formal sector (left-hand side of equation (27)), is ambiguous: on the one hand, the increase in b_{NF} raises it, but on the other, the decrease in θ_F lowers it. Hence, the sign of the movement of workers from the formal to the informal sector is ambiguous, as are the effects on market tightness θ_I, the unemployment rates u_{iI} and the wage rates w_{iI}. Thus, it is even possible that we get the seemingly paradoxical results whereby an increase in unemployment benefits in the formal sector raises the unemployment rate and lowers the wage in that sector, that is, hurts formal workers, whereas it drives the opposite results in the other sector (benefits workers in the informal sector).

Figure 2 illustrates the effects of an increase in unemployment benefits b_{NF}. As shown in panel (a), the value of a filled position decreases. However, the value of being unemployed in the formal sector and hence the change in market tightness θ_I that is necessary to restore the equality $U_{NF} = U_{NI}$ can be on either direction. Figure 2 illustrates both cases. The initial equilibrium is $(\theta_F^*, \theta_I^*, \lambda^*)$, while the final equilibrium can be either $(\theta_F^{'}, \theta_I^{'}, \lambda^{'})$ or $(\theta_F^{''}, \theta_I^{''}, \lambda^{''})$.

3.3 Firing Cost

We analyze first the impact of an increase in f. Under the conditions for existence and uniqueness of steady-state equilibrium, we have $r\Theta_F < \Phi_F$ (the proof is presented in the

Appendix). Using equation (28), we see that an increase in f lowers the value of a filled position in the formal sector, which results in firm exit, a decrease in the tightness θ_F and hence an increase in the unemployment rate u_{NF}. As regards the wage rate w_{NF}, there are two countervailing effects: on the one hand, the higher value of administrative costs raises workers' wage, but on the other, the lower probability of finding a match, lowers their bargaining position and hence decreases their wage. Similarly, the effect on the value of unemployment in the formal sector (left-hand side of equation (27)), is ambiguous: on the one hand, the increase in f raises it, but on the other, the decrease in θ_F lowers it. Hence, the sign of the movement of workers from the formal to the informal sector is ambiguous, as are the effects on market tightness θ_I, the unemployment rates u_{iI} and the wage rates w_{iI}. Thus, it is even possible that we get some results whereby an increase in f in the formal sector raises the unemployment rate and lowers the wage in that sector, that is, hurts formal workers, whereas it drives the opposite results in the other sector (benefits workers in the informal sector).

Next consider the effect of an increase in γ. We have

$$\frac{d\Phi_F}{d\gamma} < 0, \quad \frac{d\Theta_F}{d\gamma} < 0, \quad \text{and} \quad \frac{d\Psi}{d\gamma} > 0.$$

How γ changes θ_F depending on the two forces that work on the right-hand side of equation (28). Similarly, γ affects the left-hand side of equation (27) directly and through θ_F, it's not clear how the left-hand side is affected by γ. Hence, the sign of the movement of workers from the formal to the informal sector is ambiguous, as are the effects on market tightness θ_I, the unemployment rates u_{iI} and the wage rates w_{iI}.

3.4 Tax Rates

We analyze the effects of an a change in t_F and in t_w. Using equation (28), we have

$$\frac{d\Phi_F}{dt_F} > 0, \quad \text{and} \quad \frac{d\Psi}{dt_F} > 0.$$

Using equation (28), we see that an increase in t_F lowers the value of a filled position in the formal sector, which results in firm exit, a decrease in the tightness θ_F and hence an increase in the unemployment rate u_{NF}. The lower probability of finding a match lowers workers' bargaining position and hence decreases their wage. The effect on the value of unemployment in the formal sector (left-hand side of equation (27)), is ambiguous: on the one hand, the increase in t_F raises it through Δ, but on the other, the decrease in θ_F

lowers it. Hence, the sign of the movement of workers from the formal to the informal sector is ambiguous, as are the effects on market tightness θ_I, the unemployment rates u_{iI} and the wage rates w_{iI}. Thus, it is even possible that we get some results whereby an increase in f in the formal sector raises the unemployment rate and lowers the wage in that sector, that is, hurts formal workers, whereas it drives the opposite results in the other sector (benefits workers in the informal sector).

Next we consider a change in t_w. Using equation (28), we have

$$\frac{d\Phi_F}{dt_w} < 0, \quad \text{and} \quad \frac{d\Theta_F}{dt_w} < 0.$$

How t_w changes θ_F depending on the two forces that work on the right-hand side of equation (28). It seems that we can show an increase in t_w lowers θ_F (see the proof in Appendix). Similarly, t_w affects the left-hand side of equation (27) directly and through θ_F, both channels work in the same direction to lower the left-hand side. Therefore, the right-hand side will reduce by lowering θ_I. As a result, the unemployment rates u_{iI} rise and the wage rates w_{iI} reduce. λ will reduce.

3.5 Penalty Rate and Auditing Probability

Consider next an increase in the penalty rate η. Obviously, the equilibrium in the formal sector remains unchanged (see equation 28). On the contrary, the increase in the penalty rate, will lower the wages in the informal sector as well as the value of being unemployed in that sector. Naturally, this will result in a movement from the informal to the formal sector (increase in λ), which will raise the contact rate θ_I. Thus, the unemployment rate in the informal sector decreases, as does it its overall size. On the other hand, the net effect on the wages in that sector, w_{NI} and w_{MI}, is ambiguous: the expected output generated from a match $(1 - \delta\eta)y_N$ is lower, but the probability of forming a match is higher. We illustrate these effects in Figure 3. The equilibrium in panel (a) remains unchanged. On the other hand, an increase in η lowers the value of being unemployed in the informal sector and raises the AA curve in panel (b) up. The change in η lowers also the value of a filled vacancy in the informal sector, which shifts the curve MM in panel (c) to the right.

The effect of an increase in the auditing probability, δ, acts in two ways: a) in manner similar to the penalty rate, it lowers the expected output from a match $(1-\delta\eta)y_N$ and b) it increases the separation rate between workers and firms in the informal sector. Both of these effects work in the same direction (they lower the value of being unemployed and the value of a filled vacancy in the informal sector) and hence the results are as analyzed above (see also Figure 3).

3.6 Vacancy Subsidy

The last type of government policy that we consider is a subsidy for the maintenance of a vacancy in the formal sector, i.e., a decrease in c_F. If the subsidy is at the rate $h \in (0, 1)$, then c_F in equation (28) changes to $(1-h)c_F$. Obviously, such a policy will lower the cost of a vacancy in the formal sector (in terms of Figure 1, it will rotate the curve depicting the average cost of a vacancy to the right). This will spur firm entry, which, in turn, will initiate a number of changes: first, it will increase market tightness θ_F, which means that it will become more likely that an unemployed worker meets a vacancy. This will increase the value of being unemployed in the formal sector and consequently induce a movement of native workers towards the formal sector (increase in λ). In the end, the unemployment rates in both sectors, u_{NF} and u_{NI}, decrease, wages w_{NF} and w_{iN} go up, and the size of the formal sector increases at the expense of the informal sector.

4 Quantitative Analysis

4.1 The Role of the Government

The government collects revenue from an income tax (t_w) and a payroll tax (t_F) on workers and firms, respectively, in the formal sector, and from a fine (η) imposed on firms caught operating in the informal sector. The firing cost f, on the other hand, is a pure waste and hence it does generate any revenue.

The government uses its revenue to finance unemployment benefits, vacancy subsidies in the formal sector (at the rate h) and lump-sum transfers. Accordingly, the budget constraint is given by

$$(t_F + t_w)w_{NF}e_{NF} + \delta\eta \sum_{i=N,M} p_i e_{iI} = \sum_{j=F,I} b_{Nj}u_{Nj} + hc_F v_F + \tau, \tag{30}$$

where e_{ij} is the mass of workers who are employed in sector $j = F, I$ and are of origin $i = N, M$. The left-hand side of equation (30) gives the total government revenue, whereas the right-hand side corresponds to government spending. When we examine the effects of illegal immigration, we will treat b_{ij} as exogenous and let τ, t_F and t_w adjust to balance the government budget.

TO BE COMPLETED

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A Appendix

A.1 Restrictions on Parameter Values

To ensure that all types of workers are employed, all surpluses must be positive. Given the Nash sharing rule this requires that $\Pi_{NF} + f + \gamma w_{NF}$, Π_{NI} and Π_{MI} are all positive. For $\Pi_{NI} > 0$, it is necessary and sufficient to assume that $(1 - \delta\eta)\sigma p_I > b_{NI}$; similarly, $\Pi_{MI} > 0$, iff $(1 - \delta\eta)(1 - \sigma)p_I > b_{MI}$. Finally, a necessary and sufficient condition for the employability of native workers in the formal sector, i.e., $\Pi_{NF} + f + \gamma w_{NF} > 0$, is

$$\sigma p_F + rf > \frac{1 + t_F - r\gamma}{(1 - t_w - r\gamma - \gamma m_F)} b_{NF}. \tag{A.1}$$

All the above assumptions imply that the (expected) net output from a match between a worker and a firm in each sector exceeds the worker's outside option. Moreover, the assumption that $(1-\delta\eta)\sigma p_I > b_{NI}$ guarantees that $\Pi_{NI} > V_I = 0$. Thus, an informal firm that meets a native worker will form an employment relation with her; that is, it will not decide to turn her down and keep searching for an illegal immigrant.

A.2 Proofs

Proof of Proposition 1: Notice first that the left-hand side (LHS) of equation (28) is increasing and approaches zero (infinity) as $\theta_F \to 0$ (∞). Also, for the equation to be meaningful, its right-hand side (RHS) must be positive, or $\Phi_F > 0$ (which requires $\beta\Gamma > (1 - \beta)\gamma(r + s_F)$ and $\Delta > r\gamma$) and

$$\frac{\Theta_F(p_F + rf) - (1 - \beta)\Psi b_{NF}}{\Phi_F} - f > 0,$$

where $\Phi_F = \beta\Gamma m_F + [\Delta - r\gamma - (1 - \beta)\gamma m_F](r + s_F)$, $\Gamma = 1 + t_F - r\gamma$, $\Delta = 1 - (1 - \beta)t_w + \beta t_F$, $\Theta_F = (1 - \beta)(1 - t_w) - \gamma(r + \beta s_F + m_F)$ and $\Psi = 1 + t_F + s_F\gamma$.
Moreover,

$$\frac{d\Theta_F}{d\theta_F} = -\gamma m_F' < 0 \quad \text{and} \quad \frac{d\Phi_F}{d\theta_F} = [\beta\Gamma - (1 - \beta)\gamma(r + s_F)]m_F' > 0.$$

Finally, $\lim_{\theta_F \to 0} \Theta_F = (1 - \beta)(1 - t_w) - \gamma(r + \beta s_F) > 0$ (by assumption), $\lim_{\theta_F \to \infty} \Theta_F = -\infty$, $\lim_{\theta_F \to 0} \Phi_F = (\Delta - r\gamma)(r + s_F) > 0$, and $\lim_{\theta_F \to \infty} \Phi_F = \infty$.
Assuming that $[(1 - \beta)(1 - t_w) - \gamma(r + \beta s_F)](p_F + rf) > (1-\beta)\Psi b_{NF} + (\Delta - r\gamma)(r + s_F)f$, the RHS of (28) becomes positive as $\theta_F \to 0$; applying L'Hôpital's rule, we see that it is negative as $\theta_F \to \infty$. The determination of θ_F is then depicted in Figure 1, panel (a).

Next, define

$$\Upsilon = \frac{\beta m_I(1-\delta\eta)\sigma p_I + (r+s_I+\delta)b_{NI}}{\Phi_I} - \varrho_F.$$

where

$$\varrho_F = \frac{\beta m_F(1-t_w+s_F\gamma)(p_F+rf) + (\Delta - r\gamma)(r+s_F)b_{NF}}{\Phi_F}.$$

Note that for a given value of θ_F, we have

$$\frac{d\Upsilon}{d\theta_I} = \frac{\beta\left[(1-\delta\eta)\sigma p_I - b_{NI}\right](r+s_I+\delta)m_I'}{\Phi_I^2} > 0.$$

Similarly, for a given value of θ_I, we have

$$\frac{d\Upsilon}{d\theta_F} = -\frac{\{\beta(1-t_w+s_F\gamma)(p_F+rf) - [\beta\Gamma - (1-\beta)\gamma(r+s_F)]b_{NF}\}(\Delta - r\gamma)(r+s_F)m_F'}{\Phi_F^2}.$$

Under the necessary and sufficient condition for the employability of native workers in the informal sector, $d\Upsilon/d\theta_F$ becomes negative. We then have

$$\frac{d\theta_I}{d\theta_F} = -\frac{d\Upsilon/d\theta_F}{d\Upsilon/d\theta_I} > 0.$$

See also panel (b) in Figure 1, which depicts equation (27). Given θ_F^*, determined in panel (a), this curve gives the unique equilibrium value of θ_I^*. Finally, we substitute the value of θ_I^* into equation (29) to obtain a unique value of λ^*; see panel (c) in Figure 1. ■

Effects on wages: The effects on w_{NF}, w_{NI}, and w_{MI} follow immediately upon differentiation of equations (22), (23) and (24). Moreover, differentiating equation (22) yields

$$\begin{aligned}\frac{dw_{NF}}{d\theta_F} &= \frac{r+s_F}{\Phi_F^2}\{\beta(p_F+rf)\left[\Delta - r\gamma - \beta\Gamma + (1-\beta)\gamma(r+s_F)\right] \\ &\quad -(1-\beta)\left[\beta\Gamma - (1-\beta)\gamma(r+s_F)\right]b_{NF}\}\, m_F',\end{aligned}$$

which becomes positive when the following condition holds.

$$p_F + rf > \frac{(1-\beta)\left[\beta\Gamma - (1-\beta)\gamma(r+s_F)\right]}{\beta\left\{\Delta - r\gamma - \left[\beta\Gamma - (1-\beta)\gamma(r+s_F)\right]\right\}} b_{NF}.$$

Differentiating equation (23) and (24) yields

$$\frac{dw_{NI}}{d\theta_I} = \frac{\beta(1-\beta)(r+s_I+\delta)m_I'}{\Phi_I^2}\left[(1-\delta\eta)\sigma p_I - b_{NI}\right],$$

$$\frac{dw_{MI}}{d\theta_I} = \frac{\beta(1-\beta)(r+s_I+\delta)m_I'}{\Phi_I^2}\left[(1-\delta\eta)(1-\sigma)p_I - b_{MI}\right],$$

which are both positive under the conditions $(1-\delta\eta)\sigma p_I > b_{NI}$ and $(1-\delta\eta)(1-\sigma)p_I > b_{MI}$.

■

Condition for the paradoxical result: Next, we derive the condition under which the seemingly paradoxical results whereby an increase in unemployment benefits in the formal sector b_{NF} raises the unemployment rate and lowers the wage in the formal sector, whereas it drives the opposite results in the informal sector.

Differentiating the LHS of equation (27) with respect to b_{NF} gives rise to

$$\begin{aligned}\frac{dLHS}{db_{NF}} &= \frac{\{\beta(1-t_w+s_F\gamma)(y_F+rf)-[\beta\Gamma-(1-\beta)\gamma(r+s_F)]b_{NF}\}(\Delta-r\gamma)(r+s_F)m'_F}{\Phi_F^2}\frac{d\theta_F}{db_{NF}} \\ &\quad +\frac{(r+s_F)(\Delta-r\gamma)}{\Phi_F}.\end{aligned}$$

If $dLHS/db_{NF} > 0$ which requires

$$\frac{\{\beta(1-t_w+s_F\gamma)(y_F+rf)-[\beta\Gamma-(1-\beta)\gamma(r+s_F)]b_{NF}\}m'_F}{\Phi_F}\frac{d\theta_F}{db_{NF}} > -1,$$

then we have the seemingly paradoxical results as the RHS of equation (28) increases in θ_I. ■

Proof of $r\Theta_F > \Phi_F$: From the proof of existence and uniqueness of steady-state equilibrium, we have

$$\frac{d\Theta_F}{d\theta_F} = -\gamma m'_F < 0 \quad \text{and} \quad \frac{d\Phi_F}{d\theta_F} = [\beta\Gamma-(1-\beta)\gamma(r+s_F)]m'_F > 0,$$

which implies that $r\Theta_F - \Phi_F$ decreases in θ_F. When $\theta_F \to 0$, we have

$$\begin{aligned}\lim_{\theta_F\to 0}(r\Theta_F-\Phi_F) &= r\left[(1-\beta)(1-t_w)-\gamma(r+\beta s_F)\right]-(\Delta-r\gamma)(r+s_F) \\ &= -r\beta(1+\gamma s_F+t_F)-(\Delta-r\gamma)s_F < 0.\end{aligned}$$

Therefore, we have $r\Theta_F < \Phi_F$ for any values of θ_F. ■

The effect of t_w on θ_F: When $\theta_F \to 0$, the right-hand side of equation (28) becomes

$$\lim_{\theta_F\to 0} RHS = \frac{[(1-\beta)(1-t_w)-\gamma(r+\beta s_F)](p_F+rf)-(1-\beta)\Psi b_{NF}}{(\Delta-r\gamma)(r+s_F)} - f,$$

where $\Delta = 1-(1-\beta)t_w+\beta t_F$.

We then differentiate the $\lim\limits_{\theta_F\to 0} RHS$ with respect to t_w.

$$\frac{d\lim\limits_{\theta_F\to 0} RHS}{dt_w} = -\frac{\beta\Psi(p_F+rf)+(1-\beta)\Psi b_{NF}}{(\Delta-r\gamma)^2(r+s_F)} < 0.$$

So the intercept for the curve for the value of a filled position in the formal sector in Panel (a) becomes lower in t_w. It shifts inwards. ■

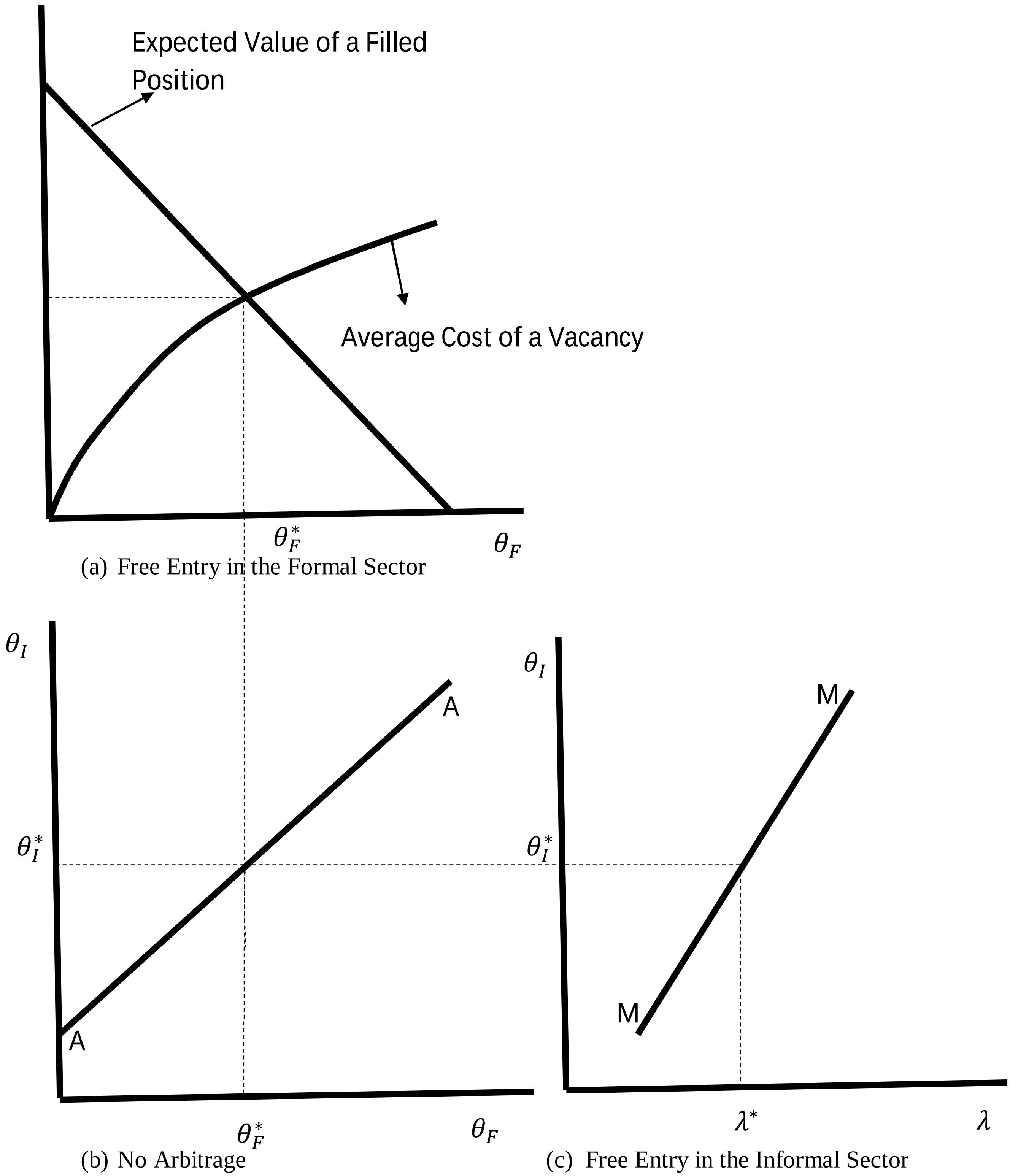
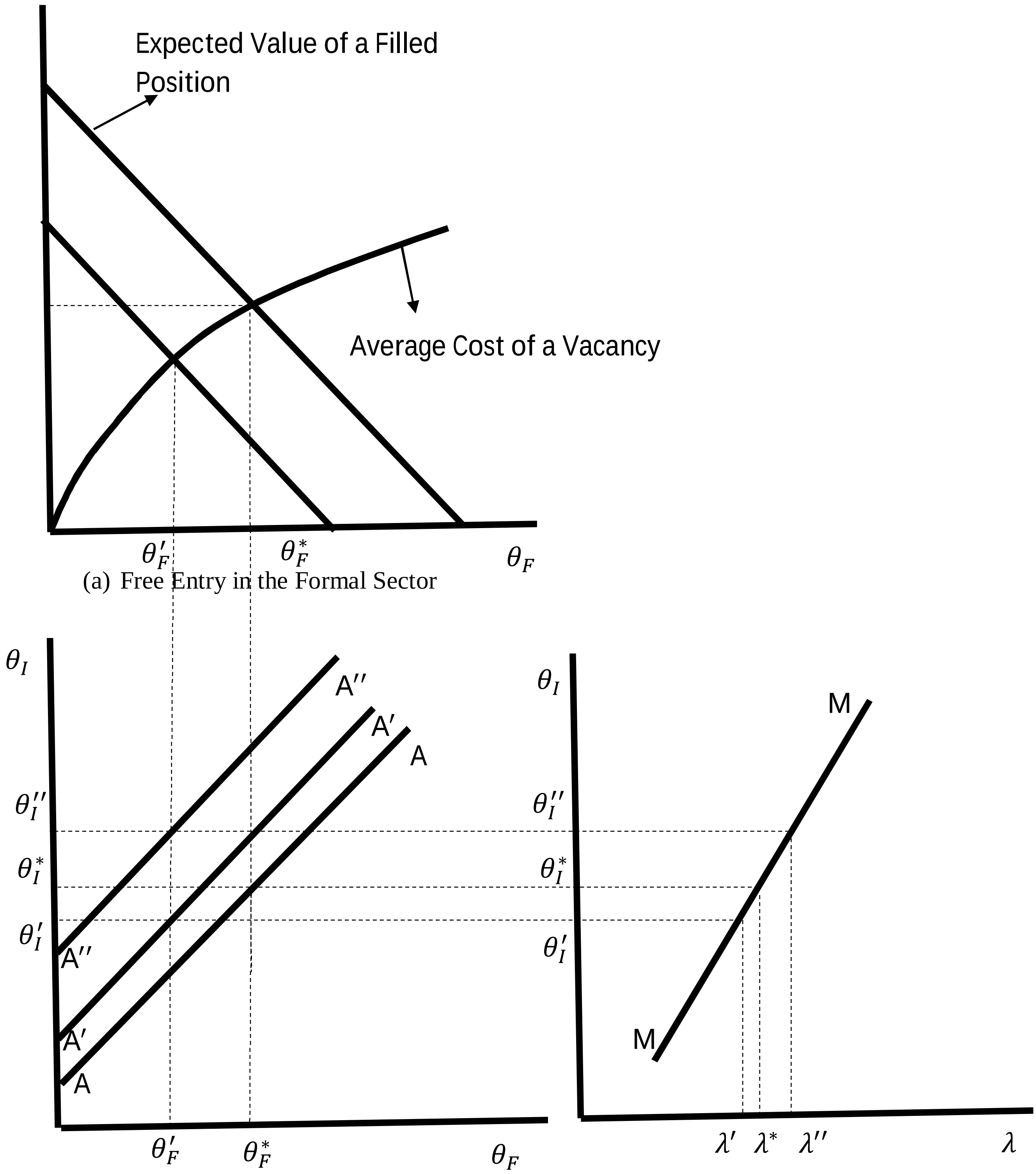


Figure 1. Calculation of the Equilibrium

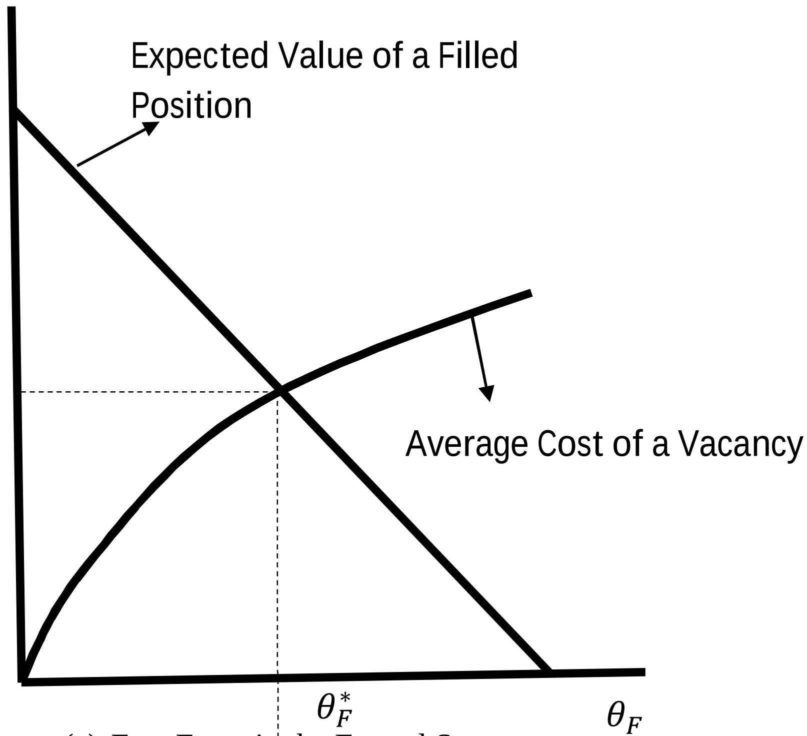


(a) Free Entry in the Formal Sector

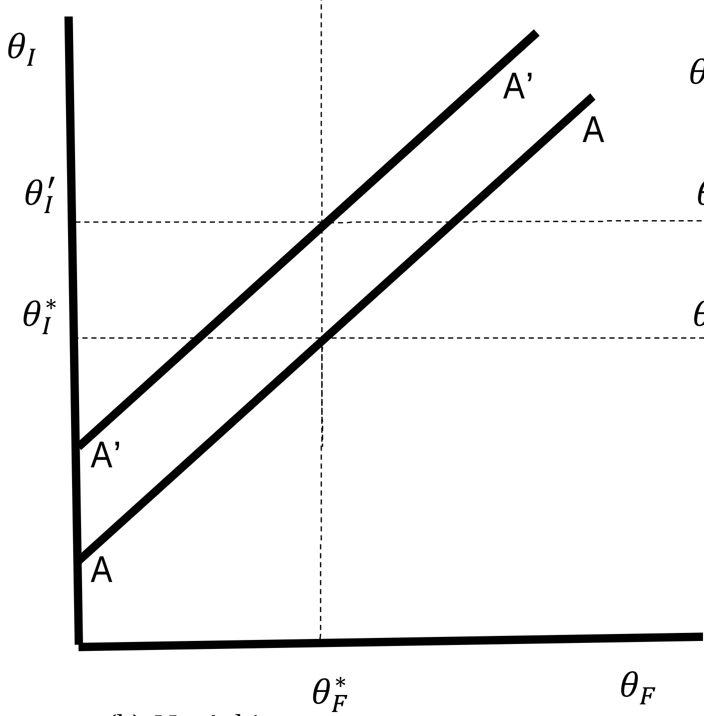
(b) No Arbitrage

(c) Free Entry in the Informal Sector

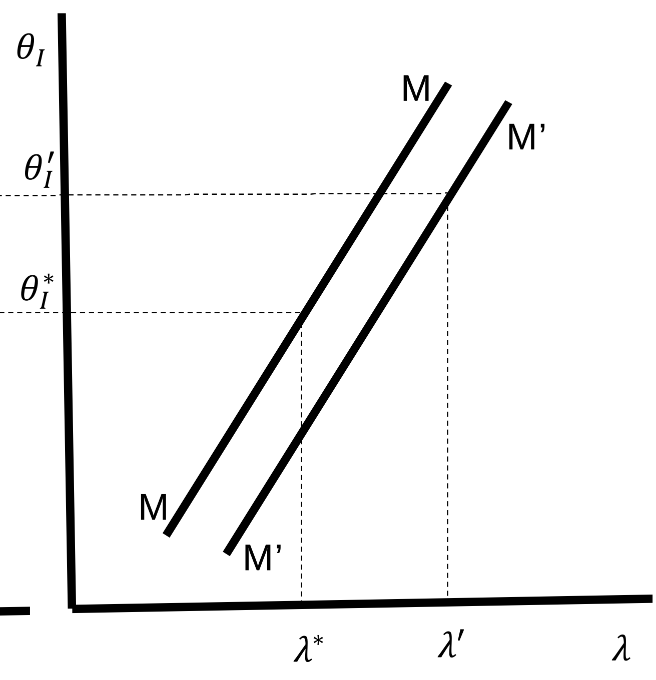
Figure 2. The Effects of an Increase in Unemployment Benefits in the Formal Sector



(a) Free Entry in the Formal Sector



(b) No Arbitrage



(c) Free Entry in the Informal Sector

Figure 3. The Effects of an Increase in the Penalty Rate η