Mitigation and Climate Engineering under Deep Uncertainty*

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Abstract

We study an ambiguity model of climate policy design in terms of emissions and solar radiation management (SRM) under deep uncertainty. Uncertainty is modelled in terms of the stock of dynamics for solar radiation management. We determine the impact of uncertainty on mitigation and SRM activities, concentration of GHGs, and global temperature and we show that the presence of SRM provides an incentive for relatively more emissions. SRM is found to be decreasing in the degree of uncertainty, hence for high uncertainty SRM and mitigation become substitutes. The analysis suggests that concerns about model misspecification induce conservative behavior and this behavior leads to low geoengineering effort and emissions both at the cooperative and the noncooperative solution. The results suggest that for low uncertainty levels countries will prefer to use geoengineering intensively and avoid mitigation, in order to generate private benefits through GHGs emissions.

Keywords: Climate change, mitigation, solar radiation management, cooperation, Knightian uncertainty, robust control, feedback Nash equilibrium.

JEL Classification: Q53, Q54.

1 Introduction

Anthropogenic greenhouse gas emissions have increased since the pre-industrial era, driven largely by economic and population growth, and are now higher than ever. This has led to atmospheric concentrations of carbon dioxide, methane

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and nitrous oxide that are unprecedented in at least the last $800,000$ years. Their effects, together with those of other anthropogenic drivers, have been detected throughout the climate system and are extremely likely to have been the dominant cause of the observed warming since the mid-$20th$ century. Warming of the climate system is unequivocal, and since the $1950s$, many of the observed changes are unprecedented over decades to millennia. The atmosphere and ocean have warmed, the amounts of snow and ice have diminished, and sea level has risen (IPCC, 2014: Summary for Policymakers).

At the same time, slow progress in international climate negotiations has given rise to skepticism about the prospect of global cooperative action on climate change. Given the scope of the coordination challenge the quest for a more comprehensive international climate treaty with binding targets continues. The slow progress in climate change mitigation policies aimed at reducing greenhouse gas emissions has led to the discussion about alternative policy options in order to cope with the impacts from climate change. In particular geoengineering has been proposed as a means to reduce surface greenhouse-induced warming and help avert discontinuous state transitions in the Earth system and dangerous climate change. Geoengineering refers to the deliberate intervention in the planetary environment of a nature and scale intended to counteract anthropogenic climate change and its impacts (Secretariat of the Convention of Biological Diversity 2012). Solar Radiation Management (SRM) which involves reflecting more of the Sun's rays away from the planet back into space proposes a method which involves putting sulphur aerosols into the high reaches of the atmosphere (Keith 2000, Ricke et al. 2008, Shepherd 2009, Secretariat of the Convention of Biological Diversity 2012). This mimics what occasionally occurs in nature when a powerful volcano erupts. For example, the Mount Pinatubo eruption in 1991 injected huge volumes of sulphur into the stratosphere. The particles produced in subsequent reactions cooled the planet by about 0.5^oC over the next two years by reflecting sunlight back out to space (e.g., Randel et al. 1995, Robock 2000, Barrett 2008).

One common feature of geoengineering is that it tends to be speculative, in the sense that no large scale experiments have been conducted in order to assess the full potential to counteract global warming. It is believed that we do not really know the consequences of solar radiation management on the earth's climate. Moreover once SRM is applied it might be difficult to reverse outcomes (Robock 2008, Barrett 2008). Thus geoengineering itself is highly uncertain, apart from a general kind of uncertainty associated with the future costs and benefits of SRM, there is the specific uncertainty associated with the evolution of the natural system. This uncertainty could arise from sources such as major gaps in knowledge, limited modelling capacity and lack of theories to anticipate thresholds, and emergence of surprises and unexpected consequences. Frank Knight (1921) introduced a concept of uncertainty in order to represent a situation where there is ignorance, or not enough information to assign probabilities to events. Knightian uncertainty is contrasted to risk (measurable or probabilistic uncertainty) where probabilities can be assigned to events and are summarized by a subjective probability measure or a single Bayesian prior. The concept of Knightian uncertainty or ambiguity can be modeled by associating it with the case of a decision maker who is trying to make good choices when he has concerns about possible misspecifications of the correct model and wants to incorporate these concerns into the decision-making rules (e.g., Hansen and Sargent, 2001; Hansen et al., 2006). The misspecification concerns emerge because the regulator cannot assign probabilities to events and this means that the regulator distrusts his model and wants good decisions over a cloud of models that surrounds the regulator's benchmark model. The cloud of models is obtained by disturbing a benchmark model by introducing a misspecification error, so that the admissible disturbances reflect the set of possible probability measures that the decision maker is willing to consider. The more ambiguous the regulator is, the larger is the cloud of approximate models that he is willing to consider.

In this paper ambiguity is modeled in terms of a robust control problem. The standard expected utility maximizing model could be derived as a special case of the robust control model when the regulator has no concerns about model misspecification and completely trusts the benchmark model. Our paper expands the standard linear-quadratic model of pollution control, studied by Dockner and VanLong (1993) among many others, to allow for misspecification of stock dynamics of geoengineering. We study a simple dynamic game of climate change policy design in terms of emissions and geoengineering efforts. The model we develop consists of a traditional economic benefit function along with a climate module based on a simplified energy balance climate model ($EBCM$) (e.g., North, 1975 a,b; North, 1981, Coakley 1979). $EBCMs$ are based on the idea of global radiative heat balance. In radiative equilibrium the rate at which solar radiation is absorbed matches the rate at which infrared radiation is emitted. The purpose of SRM as a policy instrument is to reduce global average temperature by controlling the incoming solar radiation.

We consider a world consisting of two countries with production activities that generate GHG emissions and we formulate the problem in terms of linear-quadratic (LQ) differential game. This $GHGs$ emissions generate private benefits for each country. The stock of $GHGs$ blocks outgoing radiation and causes temperature to increase. Geoengineering in the form of SRM blocks incoming radiation which is expected to cause a drop in temperature. This drop does not, at least in the way that our model is developed, depend on the accumulated $GHGs$. We analyze the problem, as it is usual in this type of problems, in the context of cooperative and noncooperative solutions. In the cooperative case there is coordination between the two countries for the implementation of geoengineering and the level of emissions in order to maximize the joint, or global, welfare. In the noncooperative case, each government chooses SRM and emissions policies noncooperatively. The noncooperative solution is analyzed in terms of feedback Nash equilibrium ($FBNE$) strategies. We first derive the optimal paths and the steady-state levels of GHG emissions, SRM, global average temperature and $GHGs$ accumulation under the risk scenario, corresponding to cooperation and feedback Nash strategies. This scenario in practice serves as a useful benchmark against which outcomes corresponding to the case of model misspecification may be compared. Adopting the Hansen–Sargent framework, we introduce Knightian uncertainty into the basic model and study the effect of model misspecification on optimal geoengineering efforts and mitigation. We focus on uncertainty surrounding geoengineering and in particular its accumulation dynamics. Specifically, uncertainty is introduced in the underlying diffusion process, reflecting concerns about our benchmark probabilistic model. We seek to characterize cooperative and noncooperative mitigation (or equivalently $GHGs$ emission) and SRM strategies in the framework of a robust control problem.

Our results suggest that concerns about model misspecification formulated in the context of robust control induce conservative behavior in the sense of reducing emissions both at the cooperative and the noncooperative solution relative to the pure risk case. If the decision maker has high concerns about model misspecification he will adopt a more restrained policy and he will choose mitigation over geoengineering. Our main policy-relevant finding is that comparisons of the welfare functions for different levels of uncertainty in case of cooperation between the two countries indicate that the more the regulator is concerned about model misspecification, the more costly is the design of robust control policies. However in $FBNE$ this result seems to reverse due to the extensive use of geoengineering and robustness seems to be more costly under cooperation relative to the $FBNE$.

We also show that the optimal geoengineering effort is decreasing in uncertainty. Thus for high levels of geoengineering incentives to mitigate weaken and when uncertainty increases, geoengineering and mitigation become substitutes. In our simulations it seems that when we implement asymmetry in uncertainty between the two countries, then the country with the higher level of uncertainty allows low levels of emissions and geoengineering. Finally, from the numerical results it is clear that as the uncertainty decreases, countries will prefer the intensive use of geoengineering in order to reduce mitigation efforts and this leads to high emissions and $GHGs$ accumulation.

The rest of the paper is organized as follows. Section 2 develops the model with an economic and a climate module. In section 3 we determine cooperative and noncooperative solutions under risk (the benchmark case). In section 4 we determine cooperative and noncooperative solutions in terms of $FBNE$ under model misspecification. In section 5 we use numerical simulations to compare the solutions under ambiguity. Section 6 concludes.

2 Robust control

2.1 Model misspecification and damage control geoengineering

The basic model and functional forms are adopted from Dockner and van Long (1993) as presented in Manoussi, Xepapadeas (2014) . The world consists of two countries, or two groups of countries, indexed by $i = 1, 2$. We develop our model along the lines of the standard linear quadratic model of international pollution control. Output is a function of emissions $F(E_i)$, where $F(\cdot)$ is strictly concave with $F(0) = 0$. Individual country utility, or private utility, without environmental externalities is given by $U(F(E_i(t))) - C(\zeta_i(t))$ where $C(\zeta_i)$ is strictly increasing and convex function of the private cost of geoengineering or SRM activity $\zeta_i(t)$. Utility is given by the quadratic function

$$U(F(E_i(t))) = A_1 E_i(t) - \frac{1}{2} A_2 E_i^2(t) \tag{1}$$

where A_1, A_2 are parameters indicating the intercept and the slope of the private marginal benefits from emissions which are defined as $A_1 - A_2 E_i(t)$. Thus A_1 can be regarded as reflecting the level effect on marginal benefits, while A_2 as reflecting the strength of diminishing returns.

We use North ($1975a, b$; 1981; $North\ et\ al.$1979) as basis for our exposition in order to describe climate by a simplified "homogeneous-earth" EBCM[1] and we follow Manoussi, Xepapadeas (2014) for the derivation of the equation for the global temperature

$$T = \frac{-A + Sq(1-\alpha) - \phi \sum_{i=1}^{2} \zeta_i + \eta (G - G_0)}{B} \tag{2}$$

where A, B are constants used to relate outgoing infrared radiation with the corresponding surface temperature, S is the mean annual distribution of radiation, q is the solar constant that includes all types of solar radiation, not just the visible light, α is the average albedo of the planet, the function $\varphi(\zeta) = \frac{\phi}{B} \sum_{i=1}^{2} \zeta_i$ is the reduction in solar radiation due to aggregate geoengineering $\sum_{i=1}^{2} \zeta_i$, $\phi > 0$ is the sensitivity of incoming radiation to geoengineering in reducing the average global temperature,[2] η is a measure of climate's sensitivity and G, G_0 denote the $GHGs$, where G is the current accumulation of $GHGs$ and G_0 is the preindustrial $GHGs$ accumulation.

Emissions contribute to the stock of $GHGs$ denoted by $G(t)$ at time t. The evolution of $GHGs$ emitted by both countries is described by the linear differential equation:

$$\dot{G}(t) = E_1(t) + E_2(t) - mG, \quad G(0) = G_0 \tag{3}$$

where $0 < m < 1$ is the natural decay rate of $GHGs$.

We assume a simple quadratic cost function for the private cost of geoengineering in each country,

$$C(\zeta_i(t)) = \frac{1}{2} \delta \zeta_i^2(t), \ \delta > 0 \tag{4}$$

[1] A homogeneous-earth model is a "zero-dimensional" model since it does not contain spatial dimensions but only the temporal dimension.

[2] SRM can be regarded as increasing the global albedo, since it blocks incoming radiation. We use a sensitivity function which is linear in aggregate SRM instead of a nonlinear function in order to simplify the exposition.

We also assume two types of damage functions related to climate change which affect private utility. The first one reflects damages from the increase in the average global surface temperature because of $GHGs$ emissions. This damage function is represented as usual by a convex, quadratic in our case, function,

$$\Omega_T(T) = \frac{1}{2}c_T T^2, \Omega_T(0) = 0, (\Omega_T(T))' > 0, (\Omega_T(T))'' > 0 \tag{5}$$

where $c_T T$ is the marginal damage cost from a temperature increase for each country.

The second is the social damage function associated with SRM effects, such as for example ocean acidification, increased acid depositions, change in precipitation patterns etc.[3] The global damages from geoengineering will be

$$\begin{aligned} \mathbb{D}_\zeta(\mathcal{Z},\zeta) &= c_\zeta \mathcal{Z}(t) \\ \text{thus, } \mathbb{D}_\zeta(\mathbf{0}) &= 0, (\Omega_\zeta(\boldsymbol{\zeta}))' > 0, (\Omega_\zeta(\boldsymbol{\zeta}))'' > 0 \end{aligned} \tag{6}$$

Risk is introduced to the standard model so that the stock of the geoengineering $\mathcal{Z}(t)$ accumulates according to the diffusion process :

$$d\mathcal{Z}(t) = \left(\beta \sum_{i=1}^{2} \zeta_i + \gamma \mathcal{Z}(t)\right) dt + \sigma \mathcal{Z}(t)\, dW(t) \tag{7}$$

where $\beta > 0, \gamma < 0$ [4] are model parameters and $W(t)$ is a Brownian motion on an underlying probability space $(\Omega, \mathcal{F}, \mathcal{G})$.

If there were no fear of model misspecification solving the symmetric problem without uncertainty[5] would be sufficient. As this is not the case, model misspecification can be reflected by a family of stochastic perturbations to the Brownian motion so that the probabilistic structure implied by stochastic differential equation (7) is distorted and the probability measure $\mathcal{G}$ is replaced by another $\mathcal{Q}$. The perturbed model is obtained by performing a change of measure and replacing $W(t)$ in (7) by

$$\hat{W}(t) + \int_0^t v(s)\, ds \tag{8}$$

[3] As mentioned in the introduction, the use of geoengineering methods could intensify ocean acidification. Although the natural absorption of CO_2 by the world's oceans helps mitigate the climatic effects of anthropogenic emissions of CO_2, it is believed that since geoengineering will cause an increase in GHG emissions, the resulting decrease in pH will have negative consequences, primarily for oceanic calcifying organisms, and there will be an impact on marine environments. For a discussion of damage functions related to climate change see Weitzman (2010).

[4] Calibration of model parameters based on Manoussi, Xepapadeas (2014). When there is no uncertainty (i.e., when $\theta \to \infty$), the calibration results in an optimally controlled steady-state carbon stock of approximately $965GtC$ ($453ppm\ CO_2$) that, according to prevailing climate science, is consistent with a 2^oC warming stabilization target. Following the previous assumption we calibrated the parameters β, γ, σ and we defined their values as: $\beta = 0.07, \gamma = -0.09$ and $\sigma = 0.5$.

[5] For the symmetric problem without uncertainty under noncooperative (feedback) strategies see Manoussi, Xepapadeas (2014).

where $\left\{ \hat{W}(t) : t \geqslant 0 \right\}$ is a Brownian motion and $\{v\,(t) : t \geqslant 0\}$ is a measurable drift distortion such that $v\,(t) = v\,(\mathcal{Z}\,(s) : s \leq t)$. Thus, changes to the distribution of $W(t)$ are parameterized as drift distortions to a fixed Brownian motion $\left\{ \hat{W}(t) : t \geqslant 0 \right\}$. The measurable process v could correspond to any number of misspecified or omitted dynamic effects such as : (a) a miscalculation of geoengineering damages, (b) a miscalculation of the decay rate of geoengineering, and (c) an ignorance of more complex dynamic structure involving irreversibility, feedback or hysteresis effects. The distortions will be zero when $v \equiv 0$ and the two measures $\mathcal{G}$ and $\mathcal{Q}$ coincide. Geoengineering dynamics under model misspecification are given by :

$$d\mathcal{Z}\,(t) = \left(\beta \sum_{i=1}^{2} \zeta_i + \gamma \mathcal{Z}\,(t) + \sigma v\,(t) \right) dt + \sigma \mathcal{Z}\,(t)\, d\hat{W}\,(t) \tag{9}$$

where $\{v\,(t) : t \geqslant 0\}$ is a measurable drift distortion which can be interpreted as a misspecification error of the geoengineering dynamics which is expressed in terms of deviations from the benchmark case. The benchmark case is defined for $v\,(t) = 0$.

As discussed in Hansen and Sargent (2001), the discrepancy between the two measures $\mathcal{G}$ and $\mathcal{Q}$ is measured through their relative entropy :

$$\mathcal{R}\,(\mathcal{Q}) = \int_0^\infty e^{-\rho t} \frac{1}{2} \mathbf{E}_{\mathcal{Q}}\,(v\,(t))^2\, dt \tag{10}$$

where $\mathbf{E}$ denotes the expectation operator. To allow for the notion that even when the model is misspecified the benchmark model remains a "good" approximation, the misspecification error is constrained so that we only consider distorted probability measures $\mathcal{Q}$ such that

$$\mathcal{R}\,(\mathcal{Q}) = \int_0^\infty e^{-\rho t} \frac{1}{2} \mathbf{E}_{\mathcal{Q}}\,(v\,(t))^2\, dt \leq \xi < \infty \tag{11}$$

where $e^{-\rho t}$ is the appropriate discount factor. By modifying the value of in (11) the decision maker can control the degree of model misspecification he is willing to consider. In particular, if the decision maker can use physical principles and statistical analysis to formulate bounds on the relative entropy of plausible probabilistic deviations from his benchmark model, these bounds can be used to calibrate the parameter ξ.

The robust control problem emerging when the decision maker is concerned about model misspecification takes the form

$$W = \max_{E_i, \zeta_i} \min_{v_i} \int_0^\infty e^{-\rho t} \left[U\,(F\,(E_i)) - C\,(\zeta_i) - \Omega\,(T) - \mathbb{D}_\zeta\,(\mathcal{Z}, \zeta) + \frac{1}{2} \theta v_i^2 \right] dt \ ,\ i = 1, 2$$

$$\text{subject to } (2)\ , (3)\ ,\ (9)\,,\ G\,(0) = G_0$$

In our problem, the minimizing agent, Nature, chooses v. The parameter $\theta > 0$ restrains the minimizing choice of the $v(t)$ function and it can be regarded as the level of uncertainty in our model. The lower value of θ is a so-called breakdown point beyond which it is fruitless to seek more robustness. When $\theta \to \infty$ then there are no concerns about model misspecification.

3 The Benchmark Model

As a reference scenario we look at the outcome of the game in a world without uncertainty. We will determine the steady state level of emissions, geoengineering, $GHGs$ accumulation and average global temperature under cooperation and noncooperation between the two countries.

3.1 The cooperative solution under risk

The problem of the social planner is to maximize the joint welfare of both countries by choosing paths for emissions $E_i(t)$ and geoengineering $\zeta_i(t)$ subject to the constraint of the accumulation of $GHGs$, the constraint of the average global temperature and the stochastic differential equation of the stock of geoengineering.

To solve for the cooperative game we formulate the LQ optimal-control problem.

$$W = \max_{E_i,\zeta_i} \int_0^\infty e^{-\rho_i t} \left\{ \sum_{i=1}^{2} \left[U\left(F\left(E_i\right)\right) - C\left(\zeta_i\right) - \Omega\left(T\right) - \mathbb{D}_\zeta\left(\mathcal{Z},\zeta\right) \right] \right\} dt$$

$$s.t \ \ (2)\ \ ,(3)\ \ ,(7)\,,\ G\left(0\right) = G_0$$

Given the LQ structure of the problem, a quadratic value function

$$V^0\left(G,\mathcal{Z}\right) = \varepsilon_0^0 + \gamma_1^0 G + \gamma_2^0 G^2 + \delta_1^0 \mathcal{Z} + \delta_2^0 \mathcal{Z}^2 + \varepsilon^0 G\mathcal{Z} \tag{12}$$

where

$$V_G^0 = \gamma_1^0 + 2\gamma_2^0 G + \varepsilon^0 \mathcal{Z}\ ,\ V_{\mathcal{Z}}^0 = \delta_1^0 + 2\delta_2^0 \mathcal{Z} + \varepsilon^0 G,\ V_{\mathcal{Z}\mathcal{Z}}^0 = 2\delta_2^0$$

The $Hamilton-Jacobi-Bellman$ (HJB) equation for this problem, where $V_G^0, V_{\mathcal{Z}}^0$ and $V_{\mathcal{Z}\mathcal{Z}}^0$ denote the first and second derivatives of the value function respectively, is :

$$\rho V^0\left(G,\mathcal{Z}\right) = \max_{E_i,\zeta_i} \left\{ \sum_{i=1}^{2} \left[A_1 E_i - \frac{1}{2} A_2 E_i^2 - \frac{1}{2}\delta\zeta_i^2 - \frac{1}{2} c_T \cdot \right.\right.$$

$$\left. \cdot \left(\frac{-A + Sq\left(1-\alpha\right) - \phi\sum_{i=1}^{2}\zeta_i + \eta\left(G - G_0\right)}{B} \right)^2 - c_\zeta \mathcal{Z}\left(t\right) \right] +$$

$$\left. + V_G^0\left(E_1 + E_2 - mG\right) + V_{\mathcal{Z}}^0\left(\beta\sum_{i=1}^{2}\zeta_i + \gamma\mathcal{Z}\right) + \frac{1}{2}\left(\sigma\mathcal{Z}\right)^2 V_{\mathcal{Z}\mathcal{Z}}^0 \right\}$$

Optimality implies

$$E_i^* = \frac{A_1 + V_G^0}{A_2}, \quad E_i^* = E_j^*,\ i, j = 1, 2 \tag{13}$$

$$\zeta_i^* = \frac{B^2 \beta V_{\mathcal{Z}}^0 + 2\phi c_T \left(-A + Sq\left(1-\alpha\right) + \eta\left(G - G_0\right)\right)}{B^2\delta + 4\phi^2 c_T},\ \zeta_i^* = \zeta_j^* \tag{14}$$

where E_i^*, ζ_i^* are the optimal cooperative emissions and geoengineering efforts for each country in a feedback form. The symmetric-cooperative solution under risk determines the levels of long-run $GHGs$ stock and of the average global temperature, through the optimal policy for emissions and geoengineering.

Using the numerical values for the parameters we can define the steady-state level of emissions, geoengineering, $GHGs$ stock and temperature in the symmetric-cooperative game as

$$\begin{aligned} E_i^* &= 4.02057 \\ \zeta_i^* &= 4.6765 \\ G^* &= 968.813 \\ T^* &= 17.1975. \end{aligned}$$

3.2 The feedback Nash solution under risk

In this section we analyze the noncooperative game in a world without uncertainty and characterize its equilibrium outcome. We assume that each country follows feedback strategies regarding the level of emissions and geoengineering efforts. Feedback strategies are associated with the concept of feedback Nash equilibrium which is a strong time-consistent noncooperative equilibrium solution. The $FBNE$ for the linear quadratic climate change game can be obtained as solution the dynamic programming representation of the non-cooperative dynamic game.

$$W = \max_{E_i,\zeta_i} \int_0^\infty e^{-\rho_i t} \left\{\left[U\left(F\left(E_i\right)\right) - C\left(\zeta_i\right) - \Omega\left(T\right) - \mathbb{D}_\zeta\left(\mathcal{Z},\zeta\right)\right]\right\} dt$$

$$s.t\ \ (2)\ ,(3)\ ,(7)\,,\ G\left(0\right) = G_0, i = 1, 2$$

The value function for each country is

$$V_i^0\left(G,\mathcal{Z}\right) = \varepsilon_{0i}^0 + \gamma_{1i}^0 G + \gamma_{2i}^0 G^2 + \delta_{1i}^0 \mathcal{Z} + \delta_{2i}^0 \mathcal{Z}^2 + \varepsilon_i^0 G\mathcal{Z} \tag{15}$$

and the corresponding HJB for each country is

$$\begin{aligned} \rho V_i^0\left(G,\mathcal{Z}\right) = \max_{E_i,\zeta_i} & \left\{ A_1 E_i - \frac{1}{2}A_2 E_i^2 - \frac{1}{2}\delta\zeta_i^2 - \frac{1}{2}c_T\left(\frac{-A + Sq\left(1-\alpha\right) - \phi\sum_{i=1}^2 \zeta_i + \eta\left(G-G_0\right)}{B}\right)^2 - c_\zeta \mathcal{Z} \right. \\ & \left. + V_{iG}^0\left(E_1 + E_2 - mG\right) + V_{i\mathcal{Z}}^0\left(\beta\sum_{i=1}^2 \zeta_i + \gamma\mathcal{Z}\right) + \frac{1}{2}\left(\sigma\mathcal{Z}\right)^2 V_{i\mathcal{Z}\mathcal{Z}}^0 \right\} \end{aligned}$$

For the full solution of the problem the parameters of the value function are obtained as usual by substituting the optimal controls into the HJB equation and then equating coefficients of the same power. Optimality implies

$$E_i^* = \frac{A_1 + V_{iG}^0}{A_2} \ , \ \ E_i^* = E_j^*, \ i,j = 1,2 \tag{16}$$

$$\zeta_i^* = \frac{B^2 \beta V_{i\mathcal{Z}}^0 + \phi c_T \left(-A + Sq\left(1-\alpha\right) + \eta\left(G - G_0\right) - \phi\zeta_j\right)}{B^2\delta + \phi^2 c_T}, \ \zeta_i^* = \zeta_j^* \tag{17}$$

where E_i^*, ζ_i^* are the optimal noncooperative emissions and geoengineering efforts for each country in a feedback form. It is clear that both emissions and geoengineering efforts are in a linear feedback form and depend on the current stock of $GHGs$. The slope of the emission feedback rule is negative, while the slope of the geoengineering feedback rule is positive. This means that one country expects the other country to reduce emissions and to increase geoengineering efforts when the stock of $GHGs$ increases. The symmetric-noncooperative solution under risk determines the levels of steady state long-run $GHGs$ stock and of the average global temperature, through the optimal policy for emissions and geoengineering.

Using the numerical values for the parameters, the steady-state level of emissions, SRM, $GHGs$ stock and average global temperature in the $FBNE$ under risk are shown below, with the percentage increase relative to the cooperative solution in parentheses:

$$\begin{aligned} E_i^* &= 10.108 \ (151\%) \\ \zeta_i^* &= 69.0842 \ (1377\%) \\ G^* &= 2435.66 \ (151\%) \\ T^* &= 32.5372 \ (89.2\%) \end{aligned}$$

It is interesting to note that at the $FBNE$ steady-state emissions increase by 151% and geoengineering increases by 1377%. Thus the presence of geoengineering provides an incentive for relatively more emissions. This is to be expected since more emissions are in principle desirable because benefits will increase, while the cost of increased emissions, in terms of global warming, is counterbalanced by SRM. This results in the increase in the steady state $GHGs$ in the $FBNE$.

4 Robust control and Model Misspecification

4.1 The cooperative solution under ambiguity

When concerns about misspecification of the pollution dynamics exist, then the cooperative solution can be obtained as the solution of the following multiplier extremization problem, which has already been defined above, as:

$$W = \max_{E_i, \zeta_i} \min_{v_i} \int_0^\infty e^{-\rho t} \left\{ \sum_{i=1}^{2} \left[U\left(F\left(E_i\right)\right) - C\left(\zeta_i\right) - \Omega\left(T\right) - \mathbb{D}_\zeta\left(\mathcal{Z}, \zeta\right) + \frac{1}{2}\theta v^2 \right] \right\} dt \ , \ i = 1,2$$

$$\text{subject to } (2)\ ,(3)\ ,(9)\,,\ G(0)=G_0$$

Given the LQ structure of the problem, a quadratic value function

$$V(G,\mathcal{Z})=\varepsilon_0+\gamma_1 G+\gamma_2 G^2+\delta_1\mathcal{Z}+\delta_2\mathcal{Z}^2+\varepsilon G\mathcal{Z} \tag{18}$$

where

$$V_G=\gamma_1+2\gamma_2 G+\varepsilon\mathcal{Z}\ ,\ V_{\mathcal{Z}}=\delta_1+2\delta_2\mathcal{Z}+\varepsilon G,\ V_{\mathcal{Z}\mathcal{Z}}=2\delta_2$$

The $Hamilton-Jacobi-Bellman$ (HJB) equation for this problem, where $V_G, V_{\mathcal{Z}}$ and $V_{\mathcal{Z}\mathcal{Z}}$ denote the first and second derivatives of the value function respectively, is :

$$\rho V(G,\mathcal{Z})=\max_{E_i,\zeta_i}\min_{v_i}\left\{\sum_{i=1}^{2}\left[A_1E_i-\frac{1}{2}A_2E_i^2-\frac{1}{2}\delta\zeta_i^2-\frac{1}{2}c_TT^2-c_\zeta\mathcal{Z}(t)+\frac{1}{2}\theta v^2\right]+\right.$$

$$\left.+V_G(E_1+E_2-mG)+V_{\mathcal{Z}}\left(\beta\sum_{i=1}^{2}\zeta_i+\gamma\mathcal{Z}+\sigma v\right)+\frac{1}{2}(\sigma\mathcal{Z})^2V_{\mathcal{Z}\mathcal{Z}}\right\} \tag{19}$$

Minimizing first with respect to v, we obtain

$$v^*=-\frac{\sigma V_{\mathcal{Z}}}{\theta} \tag{20}$$

so that $Eq.\,(19)$ becomes

$$\rho V(G,\mathcal{Z})=\max_{E_i,\zeta_i}\left\{\sum_{i=1}^{2}\left[A_1E_i-\frac{1}{2}A_2E_i^2-\frac{1}{2}\delta\zeta_i^2-\frac{1}{2}c_TT^2-c_\zeta\mathcal{Z}(t)+\frac{1}{2}\theta(v^*)^2\right]+\right.$$

$$\left.+V_G(E_1+E_2-mG)+V_{\mathcal{Z}}\left(\beta\sum_{i=1}^{2}\zeta_i+\gamma\mathcal{Z}+\sigma v^*\right)+\frac{1}{2}(\sigma\mathcal{Z})^2V_{\mathcal{Z}\mathcal{Z}}\right\}$$

Maxmin optimal emissions and geoengineering satisfy

$$E_i^* = \frac{A_1+V_G}{A_2}\ ,\ \ E_i^*=E_j^*,\ i,j=1,2 \tag{21}$$

$$\zeta_i^* = \frac{B^2\beta V_{\mathcal{Z}}+2\phi c_T(-A+Sq(1-\alpha)+\eta(G-G_0))}{B^2\delta+4\phi^2c_T},\ \zeta_i^*=\zeta_j^* \tag{22}$$

while the worst-case misspecification v is given by

$$v^*=-\frac{\sigma V_{\mathcal{Z}}}{\theta}=-\frac{\sigma(\delta_1(\theta)+2\delta_2(\theta)\mathcal{Z}+\varepsilon(\theta)G)}{\theta} \tag{23}$$

4.2 The feedback Nash solution under ambiguity

The robust $FBNE$ can be obtained as the solution of the extremization multiplier problem

$$W = \max_{E_i,\zeta_i} \min_{v_i} \int_0^\infty e^{-\rho t} \left[U\left(F\left(E_i\right)\right) - C\left(\zeta_i\right) - \Omega\left(T\right) - \mathbb{D}_\zeta\left(\mathcal{Z},\zeta\right) + \frac{1}{2}\theta v_i^2 \right] dt \ , \ i = 1,2$$

$$\text{subject to } (2) \ , (3) \ , (9) \, , \ G(0) = G_0$$

where v_i is the misspecification error for country i when countries follow time stationary linear feedback strategies. Assuming again a quadratic value function $V_i\left(G,\mathcal{Z}\right) = \varepsilon_{0i} + \gamma_{1i}G + \gamma_{2i}G^2 + \delta_{1i}\mathcal{Z} + \delta_{2i}\mathcal{Z}^2 + \varepsilon_i G\mathcal{Z}$, the HJB for each country is

$$\begin{aligned} \rho V_i\left(G,\mathcal{Z}\right) \quad = \quad & \max_{E_i,\zeta_i} \min_{v_i} \left\{ A_1 E_i - \frac{1}{2}A_2 E_i^2 - \frac{1}{2}\delta\zeta_i^2 - \frac{1}{2}c_T T^2 - c_\zeta \mathcal{Z} + \frac{1}{2}\theta v_i^2 \right. \\ & + V_{iG}\left(E_1 + E_2 - mG\right) + V_{i\mathcal{Z}}\left(\beta\sum_{i=1}^{2}\zeta_i + \gamma\mathcal{Z} + \sigma v_i\right) \\ & \left. + \frac{1}{2}\left(\sigma\mathcal{Z}\right)^2 V_{i\mathcal{Z}\mathcal{Z}} \right\} \end{aligned} \tag{24}$$

Optimality implies

$$v_i^* = -\frac{\sigma V_{i\mathcal{Z}}}{\theta} = -\frac{\sigma\left(\delta_{1i}\left(\theta\right) + 2\delta_{2i}\left(\theta\right)\mathcal{Z} + \varepsilon\left(\theta\right)G\right)}{\theta} \tag{25}$$

$$E_i^* = \frac{A_1 + V_{iG}}{A_2} \ , \ \ E_i^* = E_j^*, \ i,j = 1,2 \tag{26}$$

$$\zeta_i^* = \frac{B^2\beta V_{i\mathcal{Z}} + \phi c_T\left(-A + Sq\left(1-\alpha\right) + \eta\left(G - G_0\right) - \phi\zeta_j\right)}{B^2\delta + \phi^2 c_T}, \ \zeta_i^* = \zeta_j^* \tag{27}$$

The HJB equation (24) implies that the parameters of the value function and the optimal feedback strategy for each country depend on the parameter θ. Thus (24) can be used to determine a robust $FBNE$ which is the $FBNE$ under conditions of ambiguity. As $\theta \to \infty$ the robust $FBNE$ tends to the $FBNE$ under conditions of risk.

5 Numerical Results

In this section we perform a numerical exercise that provides some context for the theoretical results. To make the analysis concrete, we focus on a climate-change application of our model and calibrate the relevant parameters according to Manoussi, Xepapadeas (2014). Comparison of the results obtained from the numerical example suggest that concerns about model misspecification generate a more conservative behavior from the policy maker. Furthermore, reduced emissions and geoengineering under robust control lead to lower expected steady-state $GHGs$ accumulation.

5.1 Cooperation under uncertainty

The model with no misspecification concerns can be regarded as a special case of the robust control model when $\theta \to \infty$. Thus when θ becomes large, the results regarding optimal emissions, optimal geoengineering, global temperature and the expected steady state $GHGs$ accumulation should converge to the results obtained for the benchmark model.

From $Table$ 1 it is clear that emissions and geoengineering are decreasing in uncertainty. Concerns about model misspecification formulated in the context of robust control induce conservative behavior in the sense of reducing emissions both at the cooperative and the noncooperative solution relative to the pure risk case. If the decision maker has high concerns about model misspecification he will adopt a more restrained policy. He will choose mitigation over geoengineering and, as we can see from $Table$ 1, this decision will lead to low levels of emissions, stock of $GHGs$ and a lower global temperature compare to those of the benchmark model. The worst-case model misspecification corresponding to low values of θ and high values of misspecification error (v) implies a negative geoengineering level and low emissions.

The welfare function, which the social planner wants to maximize, is:

$$W_C = \max_{E_i,\zeta_i} \int_0^\infty e^{-\rho t} \left\{ \sum_{i=1}^{2} \left[U\left(F\left(E_i^*\right)\right) - C\left(\zeta_i^*\right) - \Omega\left(T^*\right) - \mathbb{D}_\zeta\left(\mathcal{Z}^*, \zeta^*\right) + \frac{1}{2}\theta\left(v^*\right)^2 \right] \right\} dt \ , \ i = 1,2 \tag{28}$$

where all the values are the corresponding steady state values of the problem.

The term $\frac{1}{2}\theta\left(v^*\right)^2$ in (28) is a fictitious term, which wants to capture the misspecification error of the problem. Thus the presence of this term greatly affects the welfare results as we can see in $Table$ 1. In cooperation each agent (country) wants to maximize the welfare function

$$W_C^{v=0} = \max_{E_i,\zeta_i} \int_0^\infty e^{-\rho t} \left\{ \sum_{i=1}^{2} \left[U\left(F\left(E_i^*\right)\right) - C\left(\zeta_i^*\right) - \Omega\left(T^*\right) - \mathbb{D}_\zeta\left(\mathcal{Z}^*, \zeta^*\right) \right] \right\} dt \ , \ i = 1,2 \tag{29}$$

where $v \equiv 0$ and all the values are the corresponding steady state values of the problem.

$Table$ 1 shows the values of the welfare function at different levels of θ in two cases. In the first case (W_C) social planner maximizes the joint welfare by taking into account the fictitious term $\frac{1}{2}\theta\left(v^*\right)^2$ of the misspecification error. In the second case $\left(W_C^{v=0}\right)$ social planner maximizes the joint welfare with the steady state values of the initial problem but without the fictitious term of the misspecification error.

As misspecification concerns increase, the changes in the welfare function, as θ is reduced, can be interpreted as the cost of robustness or the cost of being more precautionary in order to avoid potentially severe damages. In $W_C^{v=0}$ (absence of the fictitious term) the welfare is reduced as θ decreases indicating that as concerns about model misspecification increase, the robust control of the system becomes more costly as the system loses welfare. Therefore robust control under concerns about model misspecification becomes more costly. On the other hand the presence of the fictitious term $\frac{1}{2}\theta\left(v^{*}\right)^{2}$ in the welfare function seems to reverse the welfare result (W_C) and as uncertainty increases the the robust control of the system becomes less costly as the system gains welfare.

As misspecification concerns increase, the reduction in geoengineering and a more intensive use of mitigation (which is clear by the reduction in emissions), can be interpreted as the cost of being more precautionary. An increase in misspecification concerns makes the regulator cautious. It is possible that policy maker will adopt a policy with low emissions and this will lead to reduced expected $GHGs$ accumulation and global temperature.

Finally $Figure$ 1 presents the time path of expected welfare functions for $\theta = \{100, 10, 5, 2.5, 1, 0.5\}$ in both cases. As misspecification concerns are reduced, that is θ increases, the steady state welfare level increases in $W_C^{v=0}$ and decreases in W_C. An increase in misspecification concerns (decrease of θ), because the regulator increases the size of the 'maximum' misspecification to be incorporated into the decision rule, will reduce the robust welfare policy function and will lead to reduced expected $GHGs$ accumulation. As θ increases, ambiguity goes down and misspecification concerns decrease, robust emissions and geoengineering increase and as a result steady state stock increases too and eventually converges to the benchmark value.

Table 1 : Cooperation under uncertainty

$\boldsymbol{\theta}$	$\mathbf{G}$	$\mathbf{E}_i$	ζ_i	$\mathbf{T}$	$\mathbf{v}$	$\mathbf{W_C}$	$\mathbf{W_C^{v=0}}$
1000	968.787	4.02047	4.65764	17.1974	0.0311	-159881	-159892
100	968.525	4.01938	4.48199	17.1962	0.2531	-159786	-159893
10	965.826	4.00818	2.69665	17.1836	2.53935	-158822	-159896
5	962.709	3.99524	0.655355	17.1689	5.09765	-157737	-159902
2.5	956.058	3.96764	-3.62367	17.1367	10.274	-155522	-159934
1	931.714	3.86662	-18.2818	17.0097	26.3337	-148504	-160250
0.5	866.387	3.59551	-50.7249	16.604	55.1689	-135928	-162769

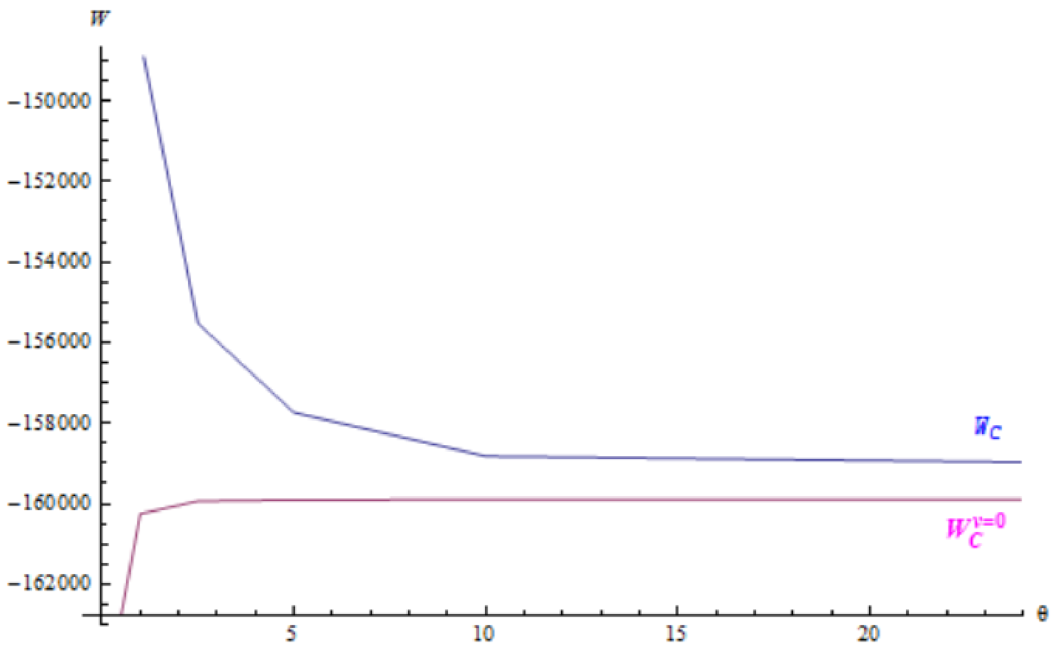


Fig1 Optimal welfare for different θ

5.2 Feedback under uncertainty

As $\theta \to \infty$ the robust $FBNE$ tends to the $FBNE$ under conditions of risk. $Table$ 2 refers to the non-cooperative solution under uncertainty. For all the chosen values of θ emissions, geoengineering, stock of $GHGs$ and global temperature are lower than the corresponding steady state levels of the benchmark model under risk. For low degree of uncertainty ($\theta \geqslant 10$) emissions, stock of $GHGs$ and global temperature do not seem to be very sensitive to changes of θ.

The optimal geoengineering effort is decreasing in uncertainty. Thus for high levels of geoengineering and since mitigation is more costly, incentives to mitigate weaken to the extent that mitigation is actually reduced and emissions as the stock of $GHGs$ are increased. In this case we observe that when uncertainty decreases, geoengineering and mitigation become substitutes.

$Tables$ 1 *and* 2 imply that emissions, geoengineering, the stock of $GHGs$ and the global temperature will always be above the cooperative levels for the same degree of uncertainty. This intuition is expected if we consider the fact that when the countries are not forced to cooperate they will choose to substitute mitigation with geoengineering, which is cheaper, and this decision will increase the level of emissions, which afterwards will increase the stock of $GHGs$ and consequently the level of global temperature.

The welfare function, which each agent (country) wants to maximize, is:

$$W_{FB} = \max_{E_i,\zeta_i} \int_0^\infty e^{-\rho t} \left[U\left(F\left(E_i^*\right)\right) - C\left(\zeta_i^*\right) - \Omega\left(T^*\right) - \mathbb{D}_\zeta\left(\mathcal{Z}^*, \zeta^*\right) + \frac{1}{2}\theta\left(v_i^*\right)^2 \right] dt \ , \ i = 1, 2 \tag{30}$$

where all the values are the corresponding steady state values of the problem. For the determination of the welfare in (30) we include the fictitious term for the misspecification error of the problem. In non-cooperation the presence of this term does not significantly affect the welfare results as we can see in *Table* 2. If we do not include the term for the misspecification error, then each agent (country) wants to maximize the welfare function

$$W_{FB}^{v=0} = \max_{E_i,\zeta_i} \int_0^\infty e^{-\rho t} \left[U\left(F\left(E_i^*\right)\right) - C\left(\zeta_i^*\right) - \Omega\left(T^*\right) - \mathbb{D}_\zeta\left(\mathcal{Z}^*, \zeta^*\right) \right] dt \ , \ i = 1, 2 \tag{31}$$

where $v \equiv 0$ and all the values are the corresponding steady state values of the problem.

The last two columns of *Table* 2 shows the values of the welfare functions at different levels of θ. The changes in the welfare function as θ reduces can be interpreted as the cost of each country being robust when no cooperation is taking place. The level of welfare increases as θ decreases indicating that as concerns about model misspecification increase the robust control of the system in each country becomes less costly in contrast to the cooperative solution. Thus we can claim that the presence of geoengineering as an alternative policy has the ability to reduce the welfare losses as concerns for model misspecification increase.

Figure 2 presents the time path of the expected robust welfare functions $W_{FB}, W_{FB}^{v=0}$ for $\theta = \{100, 10, 5, 2.5, 1, 0.5\}$. As individual misspecification concerns increase, that is θ is reduced, the expected steady-state welfare level increases too.

Table 2 : Feedback under uncertainty

$\boldsymbol{\theta}$	$\mathbf{G}$	$\mathbf{E}_i$	$\boldsymbol{\zeta}_i$	$\mathbf{T}$	$\mathbf{v}_i$	$\mathbf{W_{FB}}$	$\mathbf{W_{FB}^{v=0}}$
1000	2435.64	10.1079	69.0479	32.5373	0.0202	-133092	-133098
100	2435.27	10.1064	68.7103	32.5364	0.2026	-133039	-133090
10	2431.49	10.0907	65.2702	32.5276	2.045	-132479	-132997
5	2427	10.0721	61.2991	32.5161	4.13374	-131830	-132886
2.5	2417.13	10.0311	52.8541	32.4881	8.45217	-130439	-132644
1	2376.32	9.86175	22.6575	32.3278	22.7368	-125152	-131537
0.5	2126.73	8.82592	-49.6231	30.293	50.7092	-103655	-120635
0.44	1119.85	4.64737	-49.5588	19.3481	39.1249	-70275.3	-84594.2

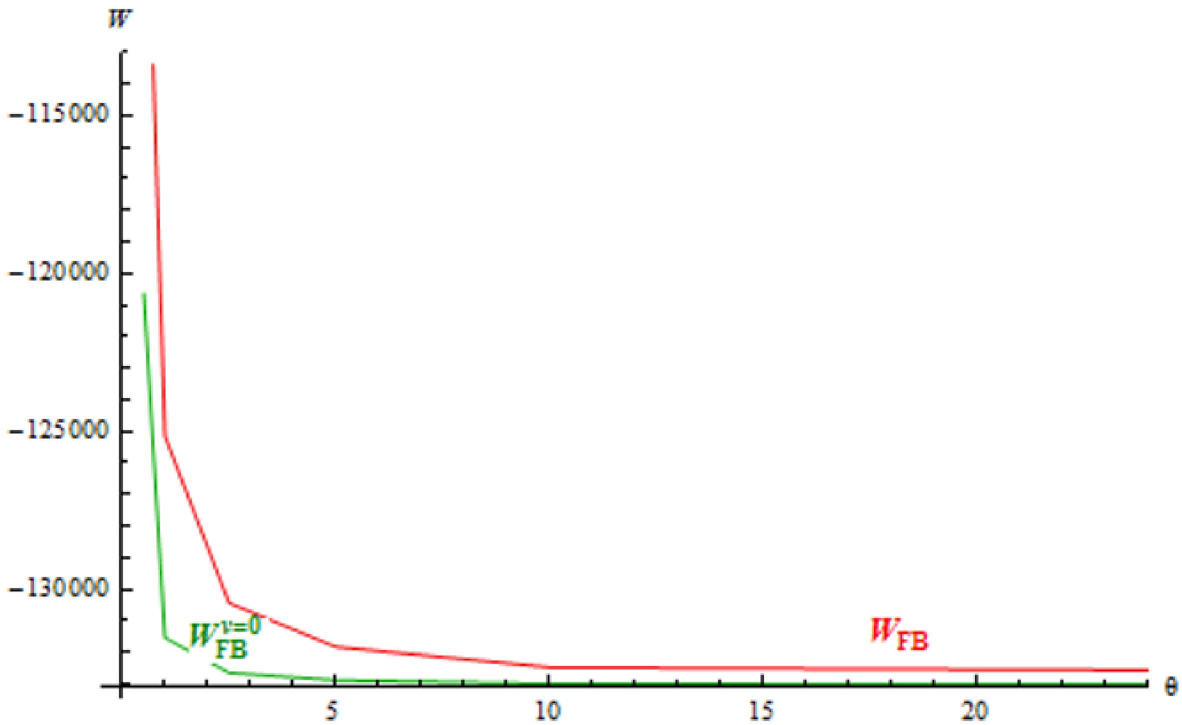


Fig2 Optimal welfare for different θ

Table 3 presents the percentage difference of the welfare between cooperation and feedback Nash for different levels of θ. If we include the fictitious term for the misspecification error of the problem, then as uncertainty increases the difference between cooperation and feedback Nash in terms of welfare decreases. Thus in this case uncertainty can be considered as a substitute of cooperation between the two countries. The opposite result occurs when we assume that $v \equiv 0$ in the computation of the welfare. In this case as uncertainty increases we observe a moderate increase of the percentage difference between cooperation and feedback Nash in terms of welfare.

Table 3 : Welfare percentage change

$\boldsymbol{\theta}$	$\frac{\mathbf{W_C - W_{FB}}}{\mathbf{W_{FB}}}\%$	$\frac{\mathbf{W_C^{v=0} - W_{FB}^{v=0}}}{\mathbf{W_{FB}^{v=0}}}\%$
1000	20.1%	20.1%
100	20.1%	20.1%
10	19.9%	20.2%
5	19.7%	20.3%
2.5	19.2%	20.6%
1	18.7%	20.8%

5.3 Feedback under asymmetric uncertainty

We assume that the level of uncertainty in each country is different. The decision maker in *country* 1 has more concerns about possible misspecifications of the correct model than the one of *country* 2, both policy makers want to incorporate these concerns into the decision-making rules. The results are shown in $Table$ 4.

Comparisons of emissions and geoengineering for different levels of θ indicate that the more the regulator is concerned about model misspecification, the more costly is the design of robust control policies. The country with high θ, that is less concerns about model misspecification, allows high levels of emissions and geoengineering. On the other hand *country* 1 with the high uncertainty level, will be more conservative than *country* 2 and she will try to maintain emissions and geoengineering in low levels. From $Table$ 4 it is clear that the greater the difference in uncertainty between the two countries, the greater will be the difference in emissions and in geoengineering.

We observe that both countries increase emissions[6], but most importantly *country* 2, which experiences lower level of uncertainty, increases geoengineering a lot more than the corresponding reduction in country one, which experiences the higher level of uncertainty. The net increase in emissions increases the steady-state stock of $GHGs$. In general increased emissions and SRM lead to an increase of the steady-state stock of $GHGs$, while there is a corresponding moderate increase in global average temperature.

The welfare functions, which each agent (country) wants to maximize, are:

$$W_{FB}^{1} = \max_{E_1,\zeta_1} \int_0^{\infty} e^{-\rho t} \left[U\left(F\left(E_1^*\right)\right) - C\left(\zeta_1^*\right) - \Omega\left(T^*\right) - D_{\zeta}\left(Z^*, \zeta_1^*\right) + \frac{1}{2}\theta_1 \left(v_1^*\right)^2 \right] dt \quad (32)$$

$$W_{FB}^{2} = \max_{E_2,\zeta_2} \int_0^{\infty} e^{-\rho t} \left[U\left(F\left(E_2^*\right)\right) - C\left(\zeta_2^*\right) - \Omega\left(T^*\right) - D_{\zeta}\left(\mathcal{Z}^*, \zeta_2^*\right) + \frac{1}{2}\theta_2 \left(v_2^*\right)^2 \right] dt \quad (33)$$

where all the values are the corresponding steady state values of the problem. For the determination of the welfare in (32),(33) we include the fictitious term for the misspecification error of the problem. If we do not include the term for the misspecification error, then each agent (country) wants to maximize the welfare function

$$W_{FB}^{1,v=0} = \max_{E_1,\zeta_1} \int_0^{\infty} e^{-\rho t} \left[U\left(F\left(E_1^*\right)\right) - C\left(\zeta_1^*\right) - \Omega\left(T^*\right) - D_{\zeta}\left(\mathcal{Z}^*, \zeta_1^*\right) \right] dt \quad (34)$$

$$W_{FB}^{2,v=0} = \max_{E_2,\zeta_2} \int_0^{\infty} e^{-\rho t} \left[U\left(F\left(E_2^*\right)\right) - C\left(\zeta_2^*\right) - \Omega\left(T^*\right) - D_{\zeta}\left(\mathcal{Z}^*, \zeta_2^*\right) \right] dt \quad (35)$$

where $v \equiv 0$ and all the values are the corresponding steady state values of the problem.

The second part of $Table$ 4 shows the values of the welfare functions of each country at different levels of θ. The country which experiences lower level of

[6] This result holds for all the cases except from the first case where $\theta_1 = 1$ and $\theta_2 = 1000$. In this case we observe a moderate reduction in country's 1 emissions.

uncertainty (*country* 2), that is less concerns about model misspecification, has higher level of welfare. Thus in *country* 1, which is more conservative than *country* 2 and maintains low levels of geoengineering, the robust control of the system becomes more costly as the system loses welfare.

Table 4 : Feedback under asymmetric uncertainty

$\boldsymbol{\theta_1}$	$\boldsymbol{\theta_2}$	$\mathbf{E_1}$	$\mathbf{E_2}$	$\boldsymbol{\zeta_1}$	$\boldsymbol{\zeta_2}$	**G**	**T**
1	1000	−0.2042	20.555	−626.123	586.874	2451.91	33.5462
1	800	6.2079	12.7607	−580.847	543.279	2285.38	31.7281
5	500	9.70398	10.0804	−38.8354	54.8802	2383.66	32.5449
50	200	9.90503	9.92922	7.07328	13.361	2389.67	32.5896

$\boldsymbol{\theta_1}$	$\boldsymbol{\theta_2}$	$\mathbf{v_1}$	$\mathbf{v_2}$	$\mathbf{W_1}$	$\mathbf{W_2}$	$\mathbf{W_1^{v=0}}$	$\mathbf{W_2^{v=0}}$
1	1000	60.7156	−0.0086	−470399	−294097	−124497	−124498
1	800	56.7771	−0.0093	−383805	−291637	−190975	−144456
5	500	5.28242	0.042	−286711	−283100	−133968	−132005
50	200	0.476439	0.117	−284578	−284357	−133059	−132937

6 Conclusions

The present paper has assessed the interplay between geoengineering and mitigation under Knightian uncertainty by adopting the robust control framework of Hansen and Sargent (2001). We introduced uncertainty in our model through geoengineering and in particular its accumulation dynamics. We wanted to study the effect of the geoengineering uncertainty in the use of geoengineering and the level of emissions. If we consider geoengineering as a comparable cheap and effective alternative to traditional mitigation, then this possibility will affect the choice of the optimal climate change policies.

To provide a benchmark for comparisons we first obtain a solution of the model under risk and then compare it with the solution under ambiguity in terms of individual emissions, individual *SRM* activities, the steady-state stock of *GHGs* and the average global temperature. When $\theta \to \infty$ the results regarding optimal emissions, optimal geoengineering, global temperature and the expected steady state *GHGs* accumulation converge to the results obtained for the benchmark model. Our main finding is that concerns about model misspecification induce conservative behavior and this behavior leads to low geoengineering effort and emissions both at the cooperative and the noncooperative solution relative to the pure risk case.

On the other hand the presence of geoengineering provides an incentive for relatively more emissions. Moreover geoengineering is always decreasing in uncertainty but optimal mitigation is not. Thus for high geoengineering and since mitigation is more costly, incentives to mitigate weaken and emissions increasing. We observe that when uncertainty increases, geoengineering and mitigation become substitutes.

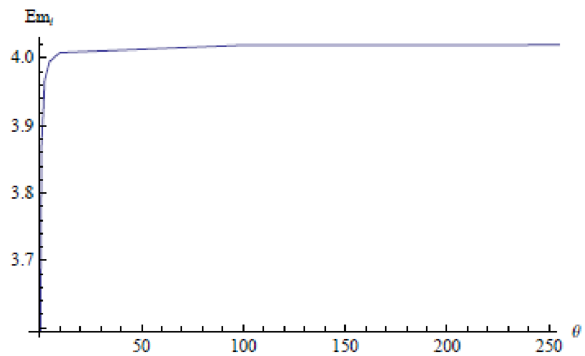
In the noncooperative case under ambiguity, for low degree of uncertainty $(\theta \geqslant 10)$, emissions, stock of $GHGs$ and global temperature do not seem to be very sensitive to changes of uncertainty. If we assume a different level of uncertainty in each country, then less concerns about model misspecification, allows high levels of emissions and geoengineering. Also it is clear that the greater the difference in uncertainty between the two countries, the greater will be the difference in emissions and in geoengineering.

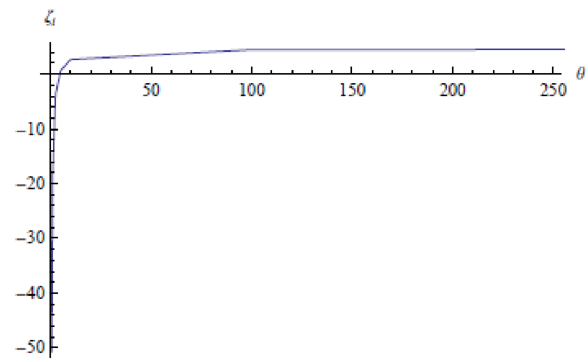
Comparison of the results obtained from the numerical example suggest that concerns about model misspecification formulated in the context of robust control induce conservative behavior in the sense of reducing emissions both at the cooperative and the noncooperative solution relative to the pure risk case. Furthermore, reduced emissions under robust control lead to lower expected steady-state $GHGs$ accumulation. Comparisons of the welfare functions for different levels of uncertainty in case of cooperation between the two countries indicate that the more the regulator is concerned about model misspecification, the more costly is the design of robust control policies, if we ignore the fictitious term for the misspecification error of the problem. However in $FBNE$ this result seems to reverse and in contrast to the cooperative solution as concerns about model misspecification increase the robust control of the system in each country becomes less costly. Thus robustness seems to be more costly under cooperation relative to the $FBNE$. The results suggest that the use of geoengineering can substitute the absence of a binding environmental commitment among countries and has the ability to moderate the welfare losses as concerns for model misspecification increase in the case of non-cooperation among countries.

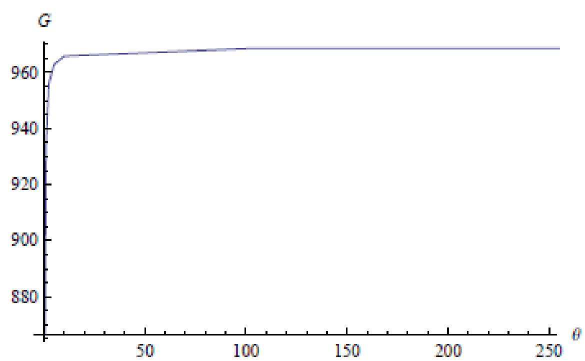
The analysis was kept at the linear quadratic level in order to make clear certain key issues. Possible extensions could be more extensive simulations in order to better trace relative precautionary costs, and introducing adaptation, in addition to mitigation and SRM, as alternative policy options against climate change.

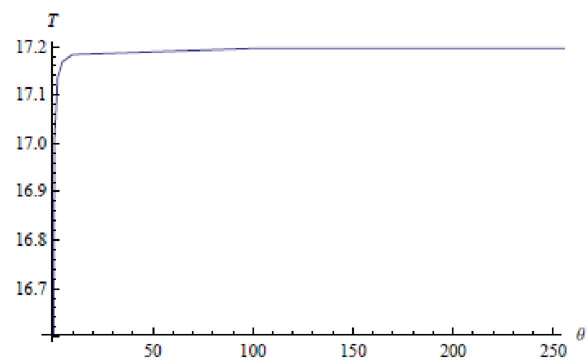
Appendix

A.1 Cooperation under uncertainty

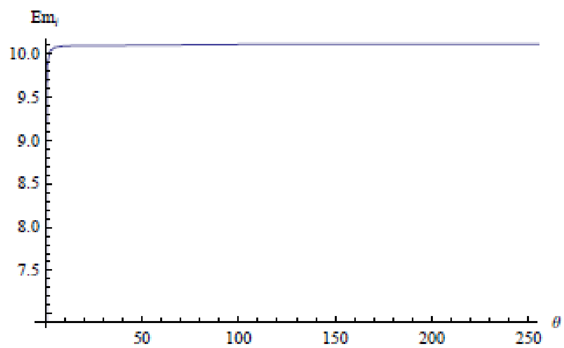


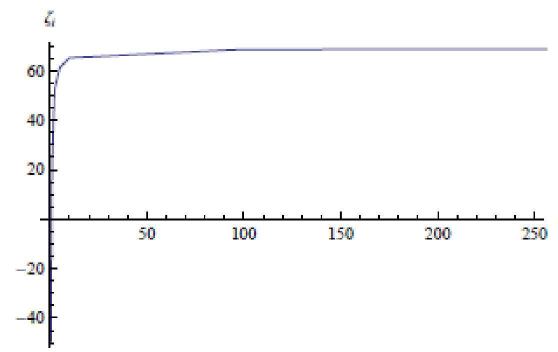


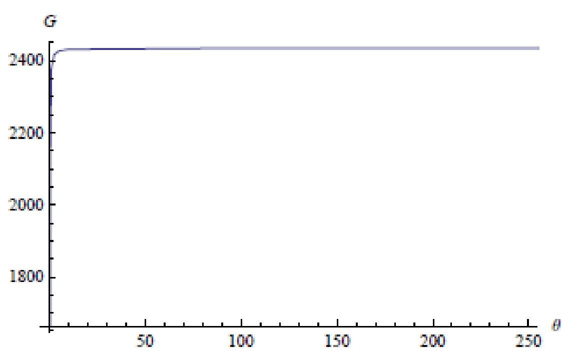


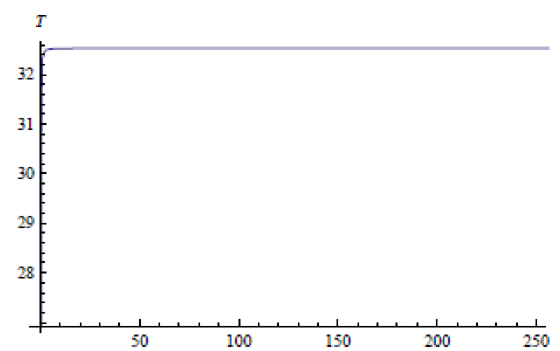


A.2 Feedback under uncertainty

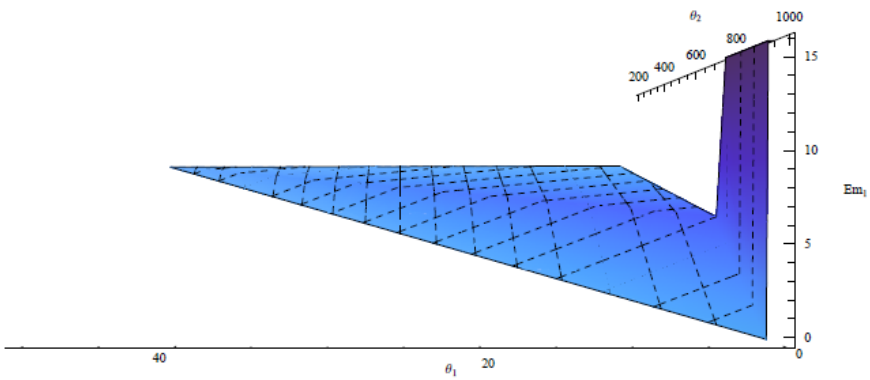


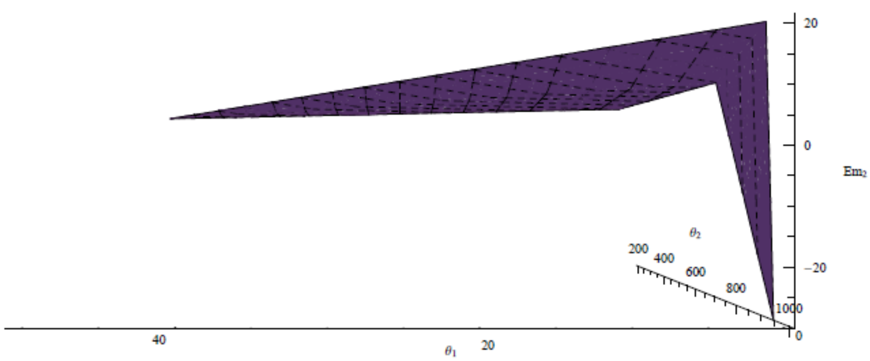






A.3 Feedback under asymmetric uncertainty





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