Partial Default*

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September 1, 2012

VERY PRELIMINARY AND INCOMPLETE

Abstract

In this paper we produce a theory of partial default applicable to sovereign debt. The theory uses Markovian equilibria and the notion that circulating unpaid coupons of any given country courtail its productive capabilities. As a consequence no issues of equilibrium selection appear in the analysis. The theory allows for renegotiation of the debt which occurs in particular dire circumstances. This theory, in contrast with the ones in the literature, is consistent with the main facts of international debt crisis, which we document.

*The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

1 Introduction

Individuals in the U.S. and Canada often file for bankruptcy when they hold debt and in a dire economic situation. This is a legal process that allows these people to have their debts condoned and have a fresh start (Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007), Livshits, MacGee, and Tertilt (2007)). Nations when facing both debt and harsh economic conditions do not have access to this legal possibility and the outcomes of these situations, often called debt crisis, are very different. For example, Benjamin and Wright (2009) document that often defaults are partial, haircuts are minor, and countries exit default with more debt than prior to default. Yet, the theories that we have (Eaton and Gersovitz (1981), Atkeson (1991), Arellano (2008) to cite a few) assume that the debt is either repaid in full or completely defaulted and the country goes over an extended period of being in a pariah status. Here we propose a model of partial default where the resolution of defaults can range from simply paying the debt in arrears with minor haircuts to situations where renegotiations are lengthy and haricuts are large.

The properties of sovereign default and restructuring episodes from a sample of 90 defaults since 1980 can be summarized by:

1. A typical default starts with a partial default on some bond where its interest payment is missed. Then the country might continue to default on other bonds. Sometimes it burst into a wide spread default as in the case of Argentina 2001, and sometimes it remains contained to a subset of bonds, as in the case of Russia 1998. Even in the case of Argentina where they defaulted on all external bonds, they might had continued to repay other loans (to banks or to the IMF)

2. About 1/4 of the defaults since 1980 have been restructured with small haircuts (less than 10%, and often even 0 %)). In some examples, like Venezuela 2005, they missed 1 or 2 interest payments and just they made up for them later. The average haircut however is 38%. Renegotiations have high direct costs, with estimates ranging from 0.5 - 3 % of the value of the loan. They are also likely to have high indirect costs by creating the shadow of

lawsuits to trading partners by holders of unpaid coupons.

3. After renegotiations countries leave with higher debt/GDP ratios than they had when they defaulted (about 25% more). This is clearly due to the combination of the accumulation of the interest of the unpaid debt, whatever changes GDP has, and the haircuts due to renegotiation.

In this paper we develop a Markovian theory of debt without commitment that accounts for these properties. The theory uses explicitly the observation that while there is no international legal system to enforce contracts, there is enough of a legal system to difficult the economic activity of countries with outstanding unpaid debt until either the debt is repaid (including the additional interest accumulated during the partial default episode) or there is an agreement between the debtor and the holders of unpaid debt. We model this difficulty as a loss of output where the size of the loss is an increasing function of the unpaid debt. In our model, missing debt payments acts as an alternative form of borrowing, that in some circumstance provides more advantageous terms. Unpaid debt does not disappear, it accumulates at an institutionally determined rate $\overline{R}$. Depending on the circumstances of the borrower, the debt is sometimes paid in full while other times a negotiation over repayment that entails costs to both parties ensues and the debt is renegotiated in such a way as it is no longer delinquent.

Our model consists of one large agent that borrows and many small lenders. There is limited commitment on the part of the borrower. Lending will always exist as long as is profitable like in Krueger and Uhlig (2006). Default is partial in the sense that only some of the debt is unpaid. This is because the larger the unpaid debt the larger the output loss inflicted by the unpaid debt, and by the fact that in any case the debt does no disappear. The debt is eventually repaid when circumstances get better. The borrower could possibly stay in a position of never paying the debt as the output loss has a limit. It is this possibility what gives it renegotiation power, power that is exercised in particular dire circumstances. In our model there is no real need to keep track of the seniority of the debt, each period the borrower just decides how much to repay and how much to default on out of the of the total debt due in the period which clearly includes the debt plus interest that was unpaid in previous period

Unlike the rest of the sovereign debt literature that sustains borrowing through trigger strategies of some kind, we are only interested in Markovian arrangements, this is, arrangements that are the limit of finite economies and therefore, we only look at states that are payoff relevant. We can do this because we model explicitly the international legal system which has enough power, if not to guarantee repayment, at least to, reduce the output of the defaulting borrowers. We think that this assumption resembles more closely the international order in place than an assumption where there is absolutely no international legal system.

Our paper is related to the literature that studies sovereign default and renegotiation. The work of Yue (2010)) and D'Erasmo (2008) extend the model of complete default by Eaton and Gersovitz (1981) and Arellano (2008) to allow for renegotiation of debt. During renegotiation the borrower and the creditors bargain over the debt haircut. They show that haircuts are decreasing in the level of debt outstanding and that allowing for renegotiation improves the model's ability to match the frequency of defaults and debt levels in emerging markets. Benjamin and Wright (2009) study inefficient delays in debt renegotiation that arises because the borrower's lack of commitment to repay debt affects the payoffs during renegotiations making it worthwhile to wait until default risk is low enough. In contrast to these works, in our model we allow for partial default of debt the resolution of defaults go through costly renegotiations only when the size of the defaulted debt is large enough.

Our work is also related to the literature on private defaultable debt. As in the literature of sovereign debt, the majority of the work has focused on full defaults and private bankrupcty (See for example, Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007) and Livshits, MacGee, and Tertilt (2007)). Recently, however, analyzing partial defaults in gaining more attention because defaults outside formal bankruptcy procedures is substantial, as documented by Dawsey and Ausubel (2004). In the work of Mateos-Planas and Seccia (2007) for example, households default partially on their debts plays and such outcome gives rise to incomplete consumption insurance in an environment with a complete set of securities.

We start describing in some detail the nature of actual sovereign debt episodes in Section 2. We pose the model first without renegotiation in Section 3 and we extend it to consider the renegotiation branches in Section 4. Section 5 characterizes equilibria while Section 6 compares its properties with those of the data.

2 The empirical properties of sovereign default

To be written.

3 The model without renegotiation

We consider a dynamic model of debt with partial default. A small open economy has a stochastic endowment stream. The economy borrows one period discount bonds and can partially default on its debt payments at any time. The defaulted debt becomes delinquent and accumulates over time at rate $\overline{R}$. Default entails a cost that is increasing in the amount of the delinquent debt. The economy can also pay off its delinquent debt at any time.

We use capital letters to denote the fact that the borrower is large, say a sovereign country, while the lenders may be small.

3.1 The borrower

The small open economy - henceforth the borrower - receives utility from consumption, C and discounts time at rate β. The borrower receives an endowment $\widehat{y}$ which is drawn from a distribution Γ_z. The index of the distribution z is itself a discrete Markov process with transition probabilities $\pi(z'|z)$.

Each period the borrower has total debt obligations A, and chooses how much of the debt to default on, D, and how much to borrow in the form of new loans B. Defaulted debt grows at rate $\overline{R}$ and new loans carry a price $q(z, B, D)$ that compensates for the possible losses due to default. The choices of D and B imply that the next period total debt obligations are $A' = \overline{R}D + B$.

Having delinquent debt $D > 0$ carries a direct cost on the endowment $\widehat{y}'$ that is increasing in the level of delinquent debt and is given by the function $\Psi(\overline{R}D)$.

The state vector of the borrower consists of three variables, (z, A, y). The Markovian structure of the shocks require us to keep track of z, the index of the distribution of the endowment. A is the total debt due. Finally, the effective income of the borrower is $y = \widehat{y}\Psi(\overline{R}D)$ and it incorporates today's idiosyncratic shock $\widehat{y}$, and the difficulties of having a delinquent position $\Psi(\overline{R}D)$. The recursive problem of the borrower whose state is $\{z, A, y\}$ is

$$V(z, A, y) = \max_{c,B,D} u(c) + \beta \; E\{V(z', A', y') \mid z\} \tag{1}$$

$$\text{s.t.} \qquad c = y - (A - D) + q(z, B, D)\; B, \tag{2}$$

$$A' = \overline{R}D + B, \tag{3}$$

$$y' = \widehat{y}'\; \Psi(\overline{R}D), \tag{4}$$

$$0 \le D \le A. \tag{5}$$

Here $q(z, B, D)$, the price of the new loan B, is an equilibrium object to be determined. We assume that Ψ is decreasing, $\Psi(0) = 1$, $\lim_{D\to 0}, \Psi(D) < 1$, and $\lim_{D\to\infty} \Psi(D) > 0$.

This problem gives the optimal debt and default functions $B(z, A, y)$ and $D(z, A, y)$ as well as the consumption function $c(z, A, y)$.

3.2 The lenders

There are many identical lenders who lend to the borrower. They discount time at rate $1/R$. While the aggregate state of the arrangement is $\{z, A, y\}$, the individual state of any measure zero lender is $\{a\}$ which is its holdings of debt. Each lender takes as given the borrowers decision rules $B(z, A, y)$ and $D(z, A, y)$.

The value function of a lender is given by $\Omega(z, A, y, a)$ as follows

$$\Omega(z, A, y, a) \;=\; a\left(1 - \frac{D(z, A, y)}{A}\right) + \frac{1}{R}\, E\left\{\, \Omega(z', A', y', a') \mid z\right\}, \qquad \text{with} \tag{6}$$

$$A' \;=\; \overline{R}D(z, A, y) + B(z, A, y), \tag{7}$$

$$y' \;=\; \widehat{y}\Psi(\overline{R}D(z, A, y)), \tag{8}$$

$$a' \;=\; \frac{\overline{R}D(z, A, y)}{A}\; a. \tag{9}$$

We can establish that $\Omega(z, A, y, a)$ is linear in a and solve it explicitly by guess-and-verify. Specifically, it takes on the form:

$$\Omega(z, A, y, a) = a\; H(z, A, y), \tag{10}$$

where the aggregate-state-dependent coefficient $H(.)$ solves

$$H(z, A, y) = \left(1 - \frac{D(z, A, y)}{A}\right) + \frac{\overline{R}\; D(z, A, y)}{R\; A}\;\; E\left\{H(y', A', z') \mid z\right\}. \tag{11}$$

with A' and y' determined as above.

The expression $H(z, A, y)$ is the value to a claim of one unit of debt a. This value is easy to interpret. When the lender owns a claim to one unit of a, it gets this period the portion $\left(1 - \frac{D(z,A,y)}{A}\right)$ of the face value. The portion $\frac{D(z,A,y)}{A}$ of the face value is defaulted on and becomes delinquent debt. Delinquent debt accumulates at predetermined rate $\overline{R}$ and becomes part of tomorrow's total debt. The expected value for every unit of debt tomorrow is precisely $E\,[H(y', A', z')|z\,]$.

Competition among lenders imply that every new loan to the borrower makes zero profits in expected value. This means that the bond price function is

$$q(z, B, D) \;=\; \frac{1}{R}\;\; E\left\{H(\overline{R}D + B, \widehat{y}'\Psi(\overline{R}D), z') \mid z\right\}. \tag{12}$$

Using the definition for $H(z, A, y)$, the bond price function solves the functional equation

$$q(z, B, D) = \frac{1}{R} \; E\left\{ \left(1 - \frac{D(z', A', y')}{A'}\right) + \frac{D(z', A', y')}{A'} \, \overline{R} \; q'(z', B', D') \mid z \right\}, \tag{13}$$

where $A' = D\overline{R} + B$, and $y' = \widehat{y}'\Psi(\overline{R}D)$.

3.3 Equilibrium

Definition 1. *A Markov Perfect Equilibrium without renegotiation is a set of decision rules for* $\{B, D, c\}$*, value for the lenders* Ω*, and the bond price function* q *such that the decision rules solve problem (1), the value for the lenders satisfies (10) and the equilibrium price satisfies (25).*

3.4 Characterization of Equilibrium

First order conditions We now analyze the first order conditions for the borrower. We assume that the bond price function $q(z, B, D)$ and the value function $v(z, A, y)$ are differentiable.

We first substitute equations (2), (3), and (4) into the objective function. The first order conditions with respect to B and D are

$$\begin{aligned}
u_c(c)[q(z, B, D) + q_B(z, B, D)B] + \beta E V_A(\overline{R}D + B, \widehat{y}'\Psi(\overline{R}D), z') &= 0, \\
u_c(c)[1 + q_D(z, B, D)B] + \lambda_0 - \lambda_A + & \\
\beta \overline{R} E\left\{ V_A(\overline{R}D + B, \widehat{y}'\Psi(\overline{R}D), z') + \widehat{y}'\Psi'(\overline{R}D) V_y(\overline{R}D + B, \widehat{y}'\Psi(\overline{R}D), z') \right\} &= 0,
\end{aligned}$$

where λ_0 and λ_B and the multipliers on the constraints $D \geq 0$ and $A - D \geq 0$ respectively.

The envelope conditions are:

$$\begin{aligned}
V_A(z, A, y) &= -u_c(c) + \lambda_A, \\
V_y(z, A, y) &= u_c(c).
\end{aligned}$$

Combining these conditions with the first order conditions we get the following Euler equations

$$u_c(c)[q(z,B,D) + q_B(z,B,D)B] = \beta E[u_c(c') - \widehat{\lambda}'_A], \tag{14}$$

$$u_c(c)[1/\overline{R} + q_D(z,B,D)B/\overline{R}] + \widehat{\lambda}_0 - \widehat{\lambda}_A = \beta E[u_c(c')(1 - \widehat{y}\Psi'(\overline{R}D)) - \widehat{\lambda}'_A]. \tag{15}$$

where $\widehat{\lambda}_j = \lambda_j/\overline{R}$.

We can combine the two Euler equations and get the following portfolio equation

$$u_c(c)[q(z,B,D) - 1/\overline{R} + B[q_B(B,D) - q_D(B,D)/\overline{R}]] \;\; = \;\; \widehat{\lambda}_0 - \widehat{\lambda}_A + \beta E u_c(c')\, \widehat{y}\Psi'(\overline{R}D).$$

The $RHS < 0$ for any $D > 0$ because here $\Psi'(D) < 0$ and $\widehat{\lambda}_0 = 0$ and $\widehat{\lambda}_A \geq 0$. Hence, if the $LHS > 0$ then the solution requires that $D = 0$. The interpretation is the following. Because, issuing bad debt carries a direct cost tomorrow with lower endowment $\beta E u_c(c')\, \widehat{y}\Psi'(\overline{R}D) < 0$ a necessary condition for issuing bad debt today is that the marginal benefit today from issuing bad debt is larger than for issuing good debt, $LHS < 0$.

Now let's work out the an expression for $[q_B(z,B,D) - q_D(z,B,D)/\overline{R}]$ with the bond price equation derived above. First, define

$$T(z',A',y') \;\; = \;\; \left[\left(1 - \frac{D(z',A',y')}{A'}\right) + \frac{D(z',A',y')}{A'}\,\overline{R}\; q'(B(z',A',y'), D(z',A',y'), z')\right].$$

Hence the bond price function can be written as

$$q(z,B,D) = \frac{1}{R} E\;\, T(z',A',y'),$$

where $A' = B + D\overline{R}$, and $y' = \widehat{y'}\Psi(\overline{R}D)$.

The derivatives of the bond price function are

$$
\begin{aligned}
q_B(z,B,D) &= \frac{1}{R} E\, \frac{dT(z',A',y')}{dA'}. \\
q_D(z,B,D) &= \frac{\overline{R}}{R} E\, \left(\frac{dT(z',A',y')}{dA'} + \frac{dT(z',A',y')}{dy'} \widehat{y}'\Psi'(\overline{R}D) \right),
\end{aligned}
$$

which implies

$$
q_D(z,B,D) = q_B(B,D,z)\overline{R} + \frac{\overline{R}}{R} E\, \left(\frac{dT(z',A',y')}{dy'} \widehat{y}'\Psi'(\overline{R}D) \right).
$$

Plugging these derivatives back into the portfolio equation we get

$$
u_c(c)\left(q(z,B,D) - 1/\overline{R} - \frac{B}{R}\, E\left(\frac{dT(z',A',y')}{dy'} \widehat{y}'\Psi'(\overline{R}D) \right) \right) = \widehat{\lambda}_0 - \widehat{\lambda}_A + \beta E u_c(c')\widehat{y}\Psi'(\overline{R}D).
$$

This equation shows that when $D = 0$, the $LHS > 0$ if $q(z,B,D) - 1/\overline{R} > 0$. Moreover, if $q(z,0,0) > 1/\overline{R}$ then the borrower would always choose *small* loans with regular debt: $A' = B$ and $D = 0$ when $A' < \widehat{A}$.

Also the equation illustrates that having delinquent debt is very expensive. The cost of having delinquent debt is not only the cost of low output tomorrow (encoded in the RHS), but also the cost from having a tighter bond price due to the fact the low output tomorrow reduces the probability to repay tomorrow (encoded in the $\widehat{y}'\Psi'(\overline{R}D)dT/dy'$ term of the LHS). However this extra cost is reduced when B is small, and can be eliminated if $B = 0$.

Domain of States For the same reasons as in the Aiyagari (1994) model, if $\beta < R^{-1}$, the borrower will not hold assets below a (negative) value $\underline{A}$. Similarly, for sufficiently large accumulated delinquent debt, the borrower will not recover and will go in a perpetual state without ever paying back. Denote such value by $\overline{A}$. Without loss of generality we can bound the set of possible asset states with these values $\mathcal{A} = [\underline{A}, \overline{A}]$. The borrower's endowment $\widehat{y}$ is drawn from a distribution Γ_z, whose index z is itself a Markov process. Both random variables $\widehat{y}$ and z are assumed to have compact support taking values in $\mathcal{Y} \in [\underline{y}, \overline{y}]$ and $\mathcal{Z} \in [\underline{z}, \overline{z}]$. All this guarantees that the state space $\mathcal{Z} \times \mathcal{Y} \times \mathcal{A}$ is compact.

Some properties of the equilibrium are that

1. The borrower will not default, $D > 0$ if $\overline{R} > \frac{1}{q(z,D,B)}$. This is clearly the case for any real cost of default.

2. The borrower may default, $D > 0$ on some of the debt when it is cheaper (including the loss of future output) than borrowing normally. Because the loss of output is expected, it is not a sufficient condition.

3. There is a sufficiently large amount of debt $\overline{A}$, such that the borrower will just default forever. It is clear that a value such that $\overline{R}\hat{y}^{max}$ is such a value. It is an absorbing state.

To prevent that the economy goes into this absorbing state, we now turn to explicitly model renegotiation, a costly situation where debtors agree to reduce their debt.

4 Model with Renegotiation

In this section we extend the model to allow for renegotiation of debt. For the borrower and the lender, renegotiation is a costly but sometimes rewarding process by which the debt owed to the lender is reduced. The cost of the renegotiation is κ which is incurred by the borrower and we pose it (for now) as a loss of utility during the period of renegotiation.

4.1 The borrower

In the problem with renegotiation, the borrower chooses each period whether to renegotiate or not. Therefore, the borrower solves

$$V(z, A, y) = max\left\{V^n(z, A, y), V^r(z, A, y)\right\}, \tag{16}$$

where $V^n(z, A, y)$ is the value of not renegotiating and $V^r(z, A, y)$ is the value of renegotiating.

If the borrower doesn't renegotiate the debt, then it solves a similar problem to (1) as follows

$$V^n(z, A, y) = \max_{D,B,c} u(c) + \beta \; E\{V(z', A', y')|z\} \tag{17}$$

$$\text{s.t.} \quad c = y - (A - D) + q(z, B, D)B, \tag{18}$$

$$A' = \overline{R}D + B, \tag{19}$$

$$y' = \widehat{y}'\Psi(\overline{R}D), \tag{20}$$

$$0 \le D \le A. \tag{21}$$

When the borrower chooses to renegotiate the debt, the lender and borrower exchange the debt due this period A for two objects: a current payment and a continuation amount of debt A'. The current payment specifies the fraction ϕ of the debt due A that the borrower pays the lender in the renegotiation period.[1] Moreover, the new debt A' consists only of new loans, which implies that delinquent debt is zero, $D = 0$.[2] Successful renegotiation implies that there is no loss of current output. The value of renegotiating the debt is then

$$V^r(z, A, y) = u(y - \phi A) - \kappa + \beta E\{V(A'_r, y'_r, z')\}. \tag{22}$$

The values for ϕ and A'_r are determined during renegotiation through bargaining. This problem is described below.

4.2 Lenders

With renegotiation the payoff for lenders has an extra branch relative to the problem in (10), which incorporate the nodes where renegotiation takes place. The value function of a lender who owns a

[1] It is conceivable that the payment goes in the other direction and the lenders extend even more credit.

[2] So far we are assuming that all lenders agree.

strip of debt with size a when the aggregate state is (z, A, y) is given by $\widetilde{\Omega}(z, A, y, a)$ as follows

$$\Omega(z, A, y, a) = \begin{cases} a\left(1 - \frac{D(z,A,y)}{A}\right) & + \frac{1}{R}\, E\left\{\Omega(a', A', y', z')|z\ \right\} \quad \text{if no renegotiation,} \\ \phi a & + \frac{1}{R}\, E\left\{\ \Omega(a'_r, A'_r, y'_r, z')|z\ ]\right\} \quad \text{if renegotiation,} \end{cases} \tag{23}$$

with

$$A' = \overline{R}D(z, A, y) + B(z, A, y),$$

$$y' = \widehat{y}\Psi(\overline{R}D(z, A, y)),$$

$$a' = \frac{a}{A}\,\overline{R}\,D(z, A, y),$$

$$A'_r = A'_r(z, A, y),$$

$$y'_r = \widehat{y},$$

$$a'_r = \frac{a}{A}\,A'_r(z, A, y).$$

In this problem $\Omega(z, A, y, a)$ is also linear in a and it takes on the form:

$$\Omega(z, A, y, a) = a \begin{cases} H(z, A, y) & \text{if no renegotiation,} \\ J(z, A, y) & \text{if renegotiation,} \end{cases} \tag{24}$$

where the aggregate-state-dependent coefficients $H(.)$ and $J(.)$ now solve:

$$\begin{aligned} H(z, A, y) &= \left(1 - \frac{D(A, ya, z)}{A}\right) + \\ &\qquad \frac{\overline{R}}{R}\,\frac{D(z, A, y)}{A}\, E\left\{[1 - g(z', A', y')]H(y', A', z') + g(z', A', y')J(z', A', y') \mid z\right\}, \\ J(z, A, y) &= \phi(z, A, y) + \frac{1}{R}\, E\left\{[1 - g(A'_r, y'_r, z')]H(y'_r, A'_r, z') + g(A'_r, y'_r, z')J(A'_r, y'_r, z') \mid z\right\}. \end{aligned}$$

with A', y', A'_r, and y'_r are determined as above, and where function g takes the value of 1 if there is renegotiation and of 0 if there is not.

Competition among lenders imply that every new loan to the borrower makes zero profits in expected value. This means that the bond price function is

$$q(z, B, D) = \frac{1}{R} E\{[1 - g(z', A', y')]H(z', A', y') + g(z', A', y')J(z', A', y')\}. \tag{25}$$

Although there are many identical lenders it is instructive to characterize the aggregate value of lenders with some abuse of notation by writing

$$\Omega(z, A, y) = A\{[1 - g(z, A, y)]H(z, A, y) + g(z, A, y)J(z, A, y)\}.$$

4.3 Renegotiation

During renegotiation the borrower and the representative lender bargain over the fraction ϕ of the debt due A that the borrower pays the lender in the renegotiation period and the new debt A'_r issued. The bargainer on account of the lenders uses function Ω. In case of breakdown of the agreement, they revert to the nonnegotiated outcome but the cost of renegotiation has already been endured.

To set up the renegotiation the problem, recall that in the problem without renegotiation $v(z, A, y)$ is the value for the borrower and $\Phi(z, A, y)$ is the value for representative lender. These are the values that the agents get in case of a breakup. The renegotiation problem is

$$\begin{aligned} \max_{\phi, A'_r} \; & [u(y - \phi A) + \beta E\{V(A'_r, y'_r, z')\} - V^n(z, A, y)]^{\mu} \\ & \Bigg[\phi a + \frac{1}{R} E\{\,\Omega(a'_r, A'_r, y'_r, z')|z\} \\ & \quad - a\left(1 - \frac{\widehat{D}(z, A, y)}{A}\right) - \frac{1}{R} E\left\{\Omega(z', \widehat{A}'(z, A, y), y')|z\right\}\Bigg]^{1-\mu} \end{aligned} \tag{26}$$

where be μ is the bargaining weight of the borrower and $y_r = \widehat{y}$ because during renegotiation $D = 0$. More importantly, functions $\widehat{D}()$ and $\widehat{A}'()$ are the choices that the borrower would have made if the negotiations were to break down which solve problem (17) (for those states where the borrower chooses the renegotiate). These problems give equilibrium functions $\phi(z, A, y)$ and $A_r(z, A, y)$.

4.4 Equilibrium with Renegotiation

Equilibrium with renegotiation is essentially the same as before as we have just added in each period another branch of the tree. It involves all the objects in the previous section as these new ones, but the conditions are essentially unchanged.

Definition 2. *A Markov Perfect Equilibrium with renegotiation is a set of decision rules for* $\{B, D, c, g\}$, *value for the lenders* Ω, *a bond price function* q, *and the renegotiation decisions for* $\{\phi, A_r\}$ *such that the decision rules solve problem (1), the value for the lenders satisfy (24) the equilibrium price satisfies (25), and renegotiation outcome satisfies (26) .*

4.5 Characterization of Equilibrium with Renegotiation

The actual bargaining is over the amount transferred this period and the amount of new debt issued. Renegotiation will typically imply a positive primary surplus in the current period, although extreme negative fundamental conditions (a very low z) may imply even a rescue (a transfer in the current period). In general, we expect that the level of regular debt will increase.

5 Characterization of Equilibria: Results and Examples

To be written.

6 A comparison with the empirical properties of sovereign default

To be written.

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