Corruption in All-Pay Auctions

Katerina-Chara Papioti

April 2015

Abstract

As discussed in recent bibliography, auctions performed by an intermediary between the seller of the good and buyers can be penetrable by corruption. Furthermore, corruption can enter auctions in different forms. In the context of All-Pay Auctions, used to model lobbying, labour-market tournaments and contests, we compare the effects of pure pecuniary corruption and favouritism on bidding behaviour and the auctioneer's expected revenue. We provide conditions under which favouritism makes bidders more or less aggressive than in the benchmark model without corruption, and prove that bidders are always more aggressive when faced with a non favouritist corrupt auctioneer. In both cases, the revenue maximizing auctioneer deprives his collaborator of all "surplus of corruption". Finally, we study the auctioneer's choice of corruption type, and find that his expected revenue is not necessarily monotonic in the probability that he choses one type of corruption or the other.

1 Motivation and Literature

As mechanisms that involve an intermediary, auctions have been repeatedly used in the literature for the study of corruption, since in the presence of an intermediary, no mechanism is immune to it. In our study of corruption, we chose the sealed bid All-Pay Auction as our main setting; as far as we know there is no literature studying corruption in this setting. The basic strand of literature focuses on auctions as procurement mechanisms and consequently uses the First Price Auction. Nevertheless, we deem it important to study the effects of corruption in All-Pay Auctions for two reasons: they represent economic procedures-such as lobbying, labour market tournaments, contests- that are not immune to corruption, and due to their modeling particularity can offer insights that First Price Auctions cannot.

Examples of corruption in labour market tournaments can be found in several empirical papers; in most of them, corruption takes the form of favouritism, or nepotism. For example Combes et al.(2008) using data from

1984 to 2003 find that network connections are more important than actual merits as determinants of success at the "concours d'agregation en sciences economiques", the centralized hiring procedure of economics professors in France. Although both publication records and professional network were found to be statistically significant determinants of success in the french economics departments' job market, the network effect was greater. On nepotism, Kramarz et al.(2007), using a Swedish population-wide employer-employee data set, show that it is very common for highschool graduates to work in the same plants as their parents (and especially their fathers); they also show that firms hire more graduating workers when children of their existing employees graduate, even if they have lower school grades, offering them more job stability and higher initial wages.

Although corruption and lobbying have always been faced in the literature as two separate economic phenomena, most notably argued as being substitutes (see for example Campos and Giovannoni 2008), recent scandals of corrupt lobbyists imply that lobbying is indeed not immune to corruption itself. In 2005, Jack Abramoff, a then top lobbyist for Greenberg Traurig law firm, was accused, and later convicted for offering free meals up to 150.000 US dollars, plus 65.000 on his personal tab at his restaurant "Signatures" to republican congressmen he was lobbying to in 2002-2003, while no present over 100 US dollars per year is allowed by any lobbyist to any politician. According to New York Times (July 6th 2005), "...In the restaurant' s early months, a customer list noted who could dine for free...handwritten notes next to 18 names - lawyers, lobbyists and eight current or former lawmakers...". Abramoff, who was later caught overcharging the Native American Casino Association, clients of his, on lobbying costs, up to 80 million, is not the only corrupt lobbyist to be discovered by justice. In 2013, British Energy Policy MP Tim Yeo was accused by the Sunday Times of "tutoring" businesses how to lobby the government at 7.000 pounds per day.

An interesting application on the All-Pay Auction can be found in Baye, Kovenock and de Vries(1993), who find that politicians seeking to maximize political rents, are better off excluding the lobbyists with the highest valuations.

When studying corruption in auctions, recent literature focuses on specific types of corruption. For example Arozamena and Weinschelbaum (2009) model favouritism, while Menezes and Monteiro (2006) build a First Price auction where the auctioneer colludes with the highest bidder, after observing bids.

In our paper, we not only consider these two types of corrupt auctioneers separately, but also attempt a comparison between the two. We find that the corrupt auctioneer does not necessarily prefer to be one type or the other with probability 1, and depending on the dustribution of valuations might chose both with positive probability.

When the auctioneer approaches the highest bidder after observing bids

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(a situation which henceforth we will call posterior corruption), players bid more aggressively than in the benchmark model without corruption[1].We find then that the revenue maximizing auctioneer robs his collaborating bidder of all "surplus" of corruption.

An interesting result of our paper that is particular to All-Pay auctions is that, contrary to Second Price Auctions that are immune to corruption, in the merely theoretical concept of a Second Price All-Pay Auction this is not the case. This is due to the fact that in a Second Price All-Pay Auction players do not have a dominant strategy to follow, thus corruption alters bidding behaviour and bidders become more aggressive than without corruption.

Finally, we discuss the possibility that, within the setting of posterior corruption, the second highest bidder is approached by the auctioneer. We find that in an auction with only two participants an equilibrium does not exist however, against our original intuition, for $N > 2$ an equilibrium can exist. This type of corruption where the auctioneer choses to collaborate with a bidder after observing bids, has been studied by Menezes and Monteiro (2006), Lengwiler and Wolfstetter (2010) and Celentani and Ganuza (2002) among others. While Menezes and Monteiro investigate how corruption affects the outcome of a first-price auction (bidding behavior, efficiency and the sellers expected revenue) and argue that the auctioneer approaches only the winner to offer the possibility of a reduction in his bid in exchange for a bribe, Lengwiler and Wolfstetter (2010) argue that the assumption that the auctioneer would ask for a bribe from the winner of the auction is not the only choice, as it might be profitable to ask for a bribe from the second highest bidder, which would in turn create inefficiencies. Previous work on the possibility of inefficient outcomes was done by Burguet and Che (2004), who study competitive procurement administered by a corrupt agent who is willing to manipulate his evaluation of contract proposals in exchange for bribes. They find that if the agent is corrupt and has large manipulation power, bribery makes it costly for the efficient firm to secure a sure win, so in equilibrium it loses the contract with positive probability.

The second type of corruption we focus on is favouritism. Unlike Arozamena and Weinschelbaum (2009) that study favouritism in First Price Auctions, we built an All-Pay auction and find that in the presence of a favourite bidder, other bidders become more or less aggressive given some conditions on the curvature of the distribution of valuations; and as in the posterior corruption case, the revenue maximizing auctioneer will deprive his "favourite" of all "surplus" of corruption. One of the first papers to explore favouritism is Laffont and Tirole (1991), who study favouritism in multidimensional auctions; they find that it might appear when the auctioneer assesses product

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[1]Analogous conditions for the First Price Auction can be found in Menezes and Monteiro (2006)

quality.

The remainder of the paper is organized as follows. Section 2 describes the benchmark All-Pay Auction model without corruption. In Section 3 we study bidding behaviour and the auctioneer's maximization problem in Posterior corruption and Favouritism. Section 4 is devoted to the auctioneer's choice of corrupt environment. We conclude in Section 5. Proofs can be found in Appendices A through D.

2   Benchmark Model: The All-Pay Auction

Our setting is one where the seller of an indivisible object wishes to sell it to one of N buyers. The mechanism through which the winner is chosen, is a First Price All-Pay Auction, where buyers place one-dimensional price bids; in the setting of the all pay auction, the highest bidding agent wins the object, and all agents pay their respective bids. The seller assigns the auction procedure to an agent of his, henceforth called the auctioneer, which for simplicity, and without loss of generality is assumed to gain a flat wage, which we normalize to 0.

We consider the incomplete information setting, where N risk neutral (payoff maximizing) players place bids for the indivisible object; each player i's valuation of the object v_i is private information, however it is common knowledge that valuations are iid distributed over $[\underline{v}, \overline{v}]$ with common distribution F. We assume that F has full support and positive and continuous density f.

As a first step, we discuss the agents' bidding behaviour without corruption. Under no corruption, the highest bidder wins the object, however all bidders have to pay their respective bids; in case of a tie, we assume that one of the tying bidders is randomly assigned the object. Thus, the payoffs for player i are:

$$
\pi_i = \left\{
\begin{array}{c}
v_i - b_i \text{ , if } b_i > b_j \ \forall j \neq i \\
-b_i \text{ , if } b_i < b_j \text{ for some } j \neq i \\
\frac{v_i}{\#\{m:b_m=b_i\}} - b_i \text{ , if } b_i = b_m > b_j \forall i, m \neq j \text{ and } \forall i \neq m
\end{array}
\right\}
$$

We focus only in the symmetric equilibrium and strategies β increasing in valuations. We assume that $\beta(\underline{v}) = 0$, since otherwise the bidder with the lowest valuation would make losses with probability 1[2]. Assume all players but player i bid given the increasing strategy $\beta()$. Then, bidder i wins by placing bid b, if $b \geq \beta(y_1) \forall j \neq i \Rightarrow y_1 \leq \beta^{-1}(b)$, where y_1 is the highest of the remaining $N - 1$ valuations, i.e. the highest order statistic of the $N - 1$

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[2]The condition can also be derived using the property of Revenue Equivalence

remaining values. Thus, i maximizes his expected payoff (we drop subscript i for simplicity) :

$$\max_b \Pi = vF^{(N-1)}(\beta^{-1}(b)) - b$$

In a symmetric equilibrium $b = \beta(v)$, and thus:

$$\frac{v(N-1)F(v)^{(N-2)}f(v)}{\beta'(v)} = 1 \Rightarrow v(N-1)F(v)^{(N-2)}f(v) = \beta'(v)$$

Keeping in mind that $\beta(\underline{v}) = 0$, the increasing symmetric bidding function is given by:

$$\beta(v) = vF^{(N-1)}(v) - \int_0^v F^{(N-1)}(y)\,dy$$

The resulting symmetric bidding function is indeed increasing, with $\beta(\underline{v}) = 0$ and $\beta(v) < v$. Notice that because in an All-Pay Auction every player forfeits their bid, bids are smaller than in an analogous First Price Auction.

3 The Model: Posterior Corruption and Favouritism

Always using an All-Pay Auction as our basic setting, we now look at two different types of corrupt auctioneers. First, we will consider the situation where the corrupt auctioneer **after** observing the bids approaches the highest bidder and proposes a bribe scheme, to be described later on. Second, we will study an environment where the corrupt auctioneer has a favourite bidder to whom he wishes to allocate the good **before** observing bids[3]. Henceforth we will refer to the first situation as Posterior Corruption and to the second situation as Favouritism.

3.1 Posterior Corruption

After observing bids, the auctioneer approaches the highest bidder, and proposes to lower his bid to the second highest bid, thus securing him a win and lowering his payment. In return, he asks for a bribe, proportional to the difference between the highest and second highest bid; This "surplus of corruption" is divided between the auctioneer and the winner by the sharing parameter $\alpha_{pc} \in [0,1]$, which for now we consider exogenous. Then, the maximization problem of bidder i bidding b when all other bidders are playing given a symmetric increasing bidding function of their valuation $\beta()$ is:

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[3]For a similar application in the First Price Auction see for example Arozamena and Weinschelbaum (2009)

$$\max_b \Pi = F(\beta^{-1}(b))^{N-1}[v - \mathrm{E}[\beta(y_1) \mid \beta(y_1) \leq b] -$$
$$\alpha(b - \mathrm{E}[\beta(y_1) \mid \beta(y_1) \leq b]]) - b[1 - F(\beta^{-1}(b))^{N-1}]$$

As in the benchmark case, bidder i will win if his bid is higher than the highest of everyone else's bid, i.e. if $b \geq \beta(y_1) \Rightarrow \beta^{-1}(b) \geq y_1$, a manipulation we are able to do since by assumption $\beta()$ is continuous and increasing and thus invertible. Furthermore, y_1 is the highest order statistic of $N-1$ valuations distributed according to $F()$, and so its distribution is $F()^{N-1}$. Now with probability $F(\beta^{-1}(b))^{N-1}$ bidder i wins and has to pay the second highest bid to the seller, as well as a proportion of the "corruption surplus" to the corrupt auctioneer. In case he loses, he has to pay his own bid b. Since player i cannot know the highest of $N-1$ bids, he calculates its expectation in case of a win:

$$\mathrm{E}[\beta(y_1) \mid \beta(y_1) \leq b] = \frac{1}{F(\beta^{-1}(b))^{N-1}} \int\limits_{\underline{v}}^{\beta^{-1}(b)} \beta(y_1) f(y_1)(N-1) F(y_1)^{N-2} \, dy_1$$

The resulting first order condition after imposing symmetry is:

$$\beta'_{pc}(v) = \frac{v f(v)(N-1) F(v)^{N-2}}{1 - (1-\alpha) F(v)^{N-1}}$$

Integrating with initial condition $\beta(\underline{v}) = 0$ yields the increasing symmetric bidding function:

$$\beta_{pc}(v) = \int\limits_{\underline{v}}^{v} \frac{y_1 f(y_1)(N-1) F(y_1)^{N-2}}{1 - (1-\alpha) F(y_1)^{N-1}} \, dy_1$$

Setting $\varphi'(y_1) = \frac{f(y_1)(N-1)F(y_1)^{N-2}}{1-(1-\alpha)F(y_1)^{N-1}}$ we get:

$$\beta_{pc}(v) = v \frac{Log[1 - (1-\alpha) F(v)^{N-1}]}{\alpha - 1} - \int\limits_{\underline{v}}^{v} \frac{Log[1 - (1-\alpha) F(y_1)^{N-1}]}{\alpha - 1}$$

Proposition 1 *The limit of $\beta_{pc}(v)$ as $\alpha -> 1$ is equal to $\beta(v)$, the increasing symmetric equilibrium bidding function in the benchmark no corruption case. Also, $\beta^{PostC}(v)$ at $\alpha = 0$ is equal to $\beta_{sp}(v)$, the increasing symmetric equilibrium bidding function in a Second Price All Pay auction, where the highest bidder wins the object and pays the second highest bid, and all other bidders forfeit their bids.*

The proof of Proposition 1 is provided in Appendix A1. Notice that since $\alpha \leq 1$, and $Log(x)$ is an increasing function of x, then $\frac{\partial \beta_{pc}(v)}{\partial \alpha} < 0 \rightarrow \beta_{pc}(v)$ decreasing in α. Thus, with a corrupt auctioneer bidders become uniformly more aggressive than without corruption, i.e. $\beta_{pc}(v) \geq \beta(v)$; also, the less they have to bribe the auctioneer, the higher their bids become.

After having obtained the bidding function for players, we turn to the auctioneer's point of view. Up until now we considered the sharing parameter α_{pc} exogenous. However, the auctioneer will chose the sharing parameter that maximizes his expected revenue.

Proposition 2 *The auctioneer will set $\alpha_{pc}^* = 1$, leaving his collaborator indifferent between accepting or rejecting the corrupt agreement.*

The formal proof of Proposition 2 can be found in Appendix A2. The auctioneer's expected revenue is nothing but the sum of the ex-ante expected payments of players towards him. As we saw before, bids are decreasing in α_{pc}; however the difference between the two highest bids is weakly increasing in α_{pc}. Thus, the auctioneer's expected revenue[4]

$$ER_{pc} = \alpha N \int_{\underline{v}}^{\overline{v}} \frac{v(1 - F(v))g(v)G(v)}{1 - (1 - \alpha)G(v)} \, dv$$

is strictly increasing in α_{pc} and as such, maximized at $\alpha_{pc}^* = 1$

3.1.1 Posterior Corruption in Second Price All-Pay Auctions

As has been discussed in several papers, the Second Price Auction is not affected by corruption. The underlying reason is that in this type of auction, players have a dominant strategy-bidding their own valuation. However, this is not the case in the context of a Second Price All-Pay Auction, where the highest bidder wins the object and pays the second highest bid, and all other players forfeit their bids. The maximization problem of an agent in a Second Price All-Pay Auction with a corrupt auctioneer is[5]:

$$\max_b \Pi = F(\beta^{-1}(b))^{N-1}[v - (1 - \alpha) \operatorname{E}[\beta(y_2) \mid \beta(y_1) \leq b] - \alpha \operatorname{E}[\beta(y_1) \mid \beta(y_1) \leq b]) - b[1 - F(\beta^{-1}(b))^{N-1}]$$

Taking the first order condition and imposing symmetry yields the following differential equation:

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[4]Where we substituted $g() = (N-1)f()F()^{N-2}$ and $G() = F()^{N-1}$

[6]In this setting the corruption agreement between the auctioneer is the same as before, the only difference being that the winner of the object that would normally pay the second highest bid, will now pay the third highest bid.

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$$\beta'_{spc}(v) - \beta(v)\left(\frac{f(v)(N-1)(1-\alpha)}{1-F(v)^{N-1}}\right)(F(v)^{N-2} - (N-2)f(v)F(v)^{N-3}) = \frac{v(N-1)F(v)^{N-2}f(v)}{1-F(v)^{N-1}}$$

Without corruption, the analogous differential equation resulting from the maximization problem of agents is[6]:

$$\beta'_{sp}(v) = \frac{v(N-1)F(v)^{N-2}f(v)}{1-F(v)^{N-1}}$$

Notice that the right hand sides of the two equations are equal. Thus, $\beta_{sp}(v) \neq \beta_{spc}(v)$ unless $\alpha = 1$.

Proposition 3 *Contrary to the Second Price Auction where bidders place bids, the highest bidder wins and pays the second highest bid where corruption does not affect the symmetric bidding equilibrium (see for example Menezes and Monteiro (2006)), in the Second Price All-Pay Auction, corruption alters players' symmetric bidding behaviour, making them uniformly more aggressive.*

3.1.2 Posterior Corruption: Second Highest Bidder Approached by the Auctioneer

In the previous parts of the paper, we looked only at situations where the highest bidder is approached by the corrupt auctioneer. Now we consider the situation where the auctioneer, after observing bids, approaches the second highest bidder; his proposal is to match his bid to the highest bid, thus securing him a win, in exchange for a proportional bribe. The bidder will accept only if his valuation is high enough to be able to afford the good and the bribe.

Proposition 4 *In the All-Pay auction where the corrupt auctioneer after observing bids approaches the second highest bidder and proposes to match his bid to the highest bid, thus securing him a win in exchange for a proportional bribe, for arbitrarily small sharing parameter α and $N = 2$, an increasing symmetric equilibrium does not exist. For $\alpha > 0$ and general N an increasing symmetric equilibrium can exist.*

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[7]The proof can be found in Appendix A2

In order to prove that an increasing symmetric equilibrium does not exist in All-Pay auctions where the corrupt auctioneer approaches the second highest bidder, with $N = 2$ and $\alpha = 0$, we will look at the bidding decision of player 1 with valuation $v_1 = \overline{v}$. Suppose that player 2 places a bid according to some increasing function of his valuation $\beta(v_2)$. The expected payoff for player 1 with strategy $b = \beta(\overline{v})$ is:

$$\Pi(\overline{v}, \beta(\overline{v})) = Prob(\beta(\overline{v}) \leq v_2)(-\beta(\overline{v})) + (1 - Prob(\beta(\overline{v}) \leq v_2))(\overline{v} - \beta(\overline{v}))$$

The player with the highest valuation knows that by bidding according to $\beta()$ his bid will be the highest. Given that the auctioneer will approach the second bidder, player 1 knows that he will be the eventual winner only if the second bidder's valuation is not high enough. Thus his probability of winning is not equal to 1. If instead player 1 chooses to bid $b = 0$, his expected payoff is:

$$\Pi(\overline{v}, 0) = \overline{v} - \mathrm{E}[\beta(v_2) \mid v_2 \leq \overline{v}] = \overline{v} - \mathrm{E}[\beta(v_2]$$

Given that $\beta()$ is increasing, notice that $\mathrm{E}[\beta(v_2] \leq \beta[\overline{v}]$, and thus $\Pi(\overline{v}, 0) \geq \Pi(\overline{v}, \beta(\overline{v}))$. Then, for the player with the highest valuation it is not best response to play according to the increasing bidding function used by the second player; we can conclude that an increasing symmetric equilibrium does not exist.

The intuition behind this result is that in such an environment players are torn between wishing to have the highest or second highest bid, as in both cases they win with positive probability. The player with the highest valuation knows that by placing $\beta(\overline{v})$ he looses the object with positive probability; in a setting with only 2 players however, he can chose $b = 0$ and win with probability 1, as he will have placed the second highest bid, and can afford to pay any bid $b \in [\underline{v}, \overline{v}]$. With $N > 2$, choosing any bid $b < \beta(\overline{v})$ would not guarantee him a win with probability 1, and thus an increasing symmetric $\beta(v)$ might exist.

3.2 Favouritism

Now we will consider an All-Pay Auction where the auctioneer has a "favourite" bidder[7] to whom he wishes to allocate the good, before observing bids. Before the auction starts, the auctioneer and his favourite reach an agreement that the former will reveal all bids to the later, and allow him to bid afterwards. In case the favourite's bid is a winning bid, the auctioneer asks for a compensation proportional to the difference between the favourite's valuation and his bid, with sharing parameter $\alpha_f \in [0, 1]$. Since in an All-Pay

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[7]An example of favouritism in the literature is Arozamena and Weinschelbaum, who study the welfare effects of favouritism in a First Price Auction

Auction all bidders forfeit their bids, the favourite bidder will then bid:

$$b_f = \begin{cases} 0 & \text{if } v_f < \bar{b} \\ \bar{b} & \text{otherwise} \end{cases}$$

, where $\bar{b}$ is the highest of $N-1$ bids. In case his valuation is high enough to afford paying the highest of $N-1$ bids for the good, he will do so in order to win, whereas if his valuation is not high enough he will bid 0 in order to minimize his costs.

We assume that the rest $N-1$ bidders are aware of the presence of a favourite, and we look at their bidding behaviour. The maximization problem of non-corrupt bidder i bidding b if all other non corrupt bidders play according to an increasing function of their valuations, is:

$$\max_{b} \Pi = vF(\beta^{-1}(b))^{N-2}F(b) - b$$

In this setting player i knows that he is bidding against $N-2$ honest and 1 corrupt player. Thus he knows that in order to win, his bid needs to be higher than $N-2$ bids and the favourite's valuation. The resulting differential equation, after imposing symmetry, is:

$$\beta'_f(v) = \frac{vf(v)(N-2)F(v)^{N-3}F(\beta(v))}{1 - vF(v)^{N-2}f(\beta(v))}$$

The above differential equation cannot be analytically solved unless we assume a specific distribution of valuations. Nevertheless, although we cannot guarantee uniqueness, in Appendix B we prove the existence and optimality of a symmetric increasing bidding function solving the differential equation, under the condition that $f(v)$ decreasing in v. Given the existence of an increasing symmetric $\beta_f(v)$, we are interested in comparing bidding behaviour in favouritism and the benchmark model without corruption.

Proposition 5 *If $(\beta_f^{-1}(b))' > \frac{f(\beta_f(v))}{f(v)}$ then bidders bid uniformly less aggressively than in the benchmark no corruption case. Analogously, if $(\beta_f^{-1}(b))' < \frac{f(\beta_f(v))}{f(v)}$ then bidders bid uniformly more aggressively than in the benchmark no corruption case.*

More (less) aggression in our context means that $\beta_f(v) > (<)\beta(v)$, i.e. player i with valuation v will place a higher (lower) bid in the presence of a favouritist auctioneer than in an honest environment. An alternative view is that player i will place the same bid in both settings, only if his valuation is lower (higher) in favouritism than without corruption, i.e. if $\beta_f(v_1) = \beta(v_2)$ then $v_1 < (>)v_2$. The proof of Proposition 5 can be found in Appendix

C. The intuition behind this result lies in the fact that non-corrupt bidders are driven by two different forces. On one hand, they know that with their valuation there are cases where they would have won in the benchmark no corruption case, but now lose for sure; this effect drives them to bid less aggressively. However, they also know that if the valuation of the corrupt bidder is not high enough given their own bid, they can win. This effect drives bidders to bid more aggressively. Ultimately it is the curvature of $F()$ that defines whether or not players place uniformly higher or lower bids than in the benchmark model.

After having obtained the conditions for equilibrium and the properties of the bidding function, we turn to the auctioneer's point of view. As said before, if the favourite's valuation is high enough, he will set $\bar{b}$ and win the object. Now we know that $\bar{b} = \beta_f(y_1)$, the bid of the player with the highest among $N-1$ valuations. In this case the auctioneer and his favourite share the "surplus" of corruption which is the difference between v_f and $\beta_f(v)$, with sharing parameter $\alpha_f \in [0,1]$ Since the auctioneer does not know his favourite's valuation, his expected revenue will be equal to the ex-ante payment of his favourite bidder towards him.

Proposition 6 *The auctioneer will set $\alpha_f^* = 1$*

The favourite's expected payment to the auctioneer is:

$$p(v) = \alpha_f Prob(v > \beta_f(y_1))(v - \mathrm{E}\,\beta_f(y_1)) \mid \beta_f(y_1) < v)$$

The ex-ante expected payment is:

$$\mathrm{E}(p(V)) = \alpha_f [\int_{\underline{v}}^{\overline{v}} [vF_{Y1}(\beta_f(v^{-1})) - \int_{\underline{v}}^{\beta_f^{-1}(v)} \beta_f(y_1)(F_{Y1}(y_1))'\,dy_1]f(v)\,dv] = ER_f$$

Since $\beta_f(v)$ is constant in α_f then ER_f is strictly increasing in α_f and the auctioneer's revenue maximizing α_f^* is equal to 1.

3.3 The Corrupt Auctioneer

Throughout the paper we have assumed that there exist two types of corrupt auctioneers: one that strictly prefers posterior corruption, and one that strictly prefers favouritism. In this section we drop this assumption and allow the auctioneer to chose which type he prefers to be. We assume that bidders know that they are facing an auctioneer that prefers posterior corruption with probability λ, and favouritism with probability $1 - \lambda$.

We assume that the auctioneer's choice is binding, and check how he will chose λ in order to maximize his expected revenue. We will solve the game backwards, first solving the bidder's maximization problem, taking into consideration our previous result that $\alpha_{pc}^* = \alpha_f^* = 1$.

For conveniece we will consider $N = 2$ without loss of generality, as our qualitative results hold for any N. As before, players place their respective bids. Then, with probability λ the highest bidder wins (and pays all the surplus of corruption to the auctioneer), and with probability $1 - \lambda$ the auctioneer asks his favourite bidder to rebid, after revealing his rival's bid. In order to be able to obtain a symmetric bidding function, we will assume that before placing bids, bidders do not know whether with probability $1 - \lambda$ they will be the favourite or not. Thus, they assign probability $\frac{1}{2}$ to both events. Player i knows that with probability $\frac{1}{2}(1 - \lambda)$ he will be able to rebid after observing his rival's bid, and thus will set:

$$b_f = \begin{cases} 0 & \text{if } v_i \text{ smaller than his rival's bid} \\ \text{rival's bid} & \text{otherwise} \end{cases}$$

As before, if v_i is bigger than his rival's bid, the auctioneer will take away all the surplus of corruption from him. With probability λ player i is no longer the favourite, and thus will chose $b = argmax \, \text{E} \, \Pi(v)$. Then, the maximization problem of player i is:

$$\max_b \Pi(v,b) = \lambda[vProb(b > \beta(y)) - b] + \frac{1}{2}(1 - \lambda)[vProb(b > y) - b]$$

Arranging the probabilities, the maximization problem becomes:

$$\max_b \Pi(v,b) = \lambda[vF(\beta^{-1}(b) > y) - b] + \frac{1}{2}(1 - \lambda)[vF(b) - b]$$

The symmetric first order condition yields the differential equation:

$$\beta'(v_h) = \frac{2\lambda v_h f(v_h)}{1 + \lambda - (1 - \lambda)v_h f(\beta(v_h))}$$

As a convex combination of the differential equations discussed before, this has an increasing solution. However notice that now $f()$ need not be strictly decreasing. Now, the auctioneer's expected revenue is:

$$ER(\lambda) = \lambda ER_{pc}(\lambda) + (1 - \lambda)ER_f(\lambda)$$

, where $ER_{pc}(\lambda) = 2\int_{\underline{v}}^{\overline{v}}[1 - F(y)]F(y)\frac{\partial \beta(y,\lambda)}{\partial y}\,dy$ and $ER_f(\lambda) = \int_{\underline{v}}^{\overline{v}}[1 - F(\beta(y,\lambda))]F(y)\frac{\partial \beta(y,\lambda)}{\partial y}\,dy$
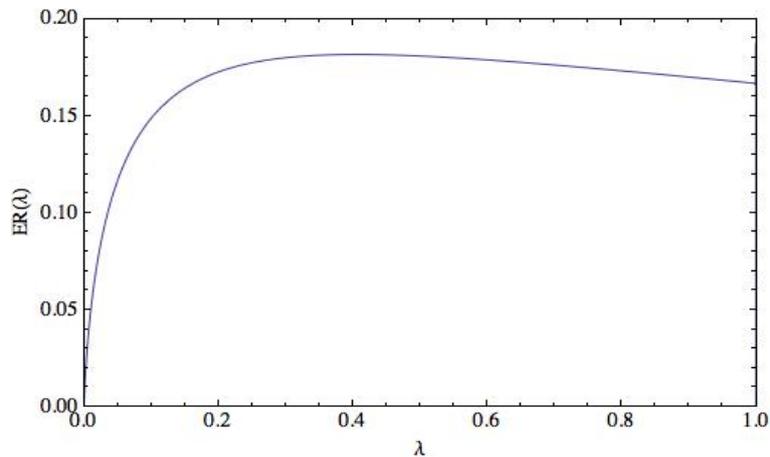
Thus:

Figure 1: Auctioneer's Expected Revenue as a function of λ $(v_i \sim U[0,1])$

$$ER(\lambda) = \int\limits_{\underline{v}}^{\overline{v}} F(y) \frac{\partial \beta(y,\lambda)}{\partial y} [2\lambda(1 - F(y)) + (1 - \lambda)(1 - F(\beta(y,\lambda)))] \, dy$$

Proposition 7 *The auctioneer's expected revenue is not necessarily monotonic in λ. The curvature of the expected revenue depends on the curvature of the bidding function and the pdf of valuations. For example, with uniform distribution on [0,1], the expected revenue is a concave function of λ and maximized at interior λ.*

The proof of Proposition 7 can be found in Appendix D. In Figure 1 we show $ER(\lambda)$ for $v_i \sim Uniform[0,1]$; $ER(\lambda)$ is concave and maximized at interior λ.

4   Conclusions and Future Work

Throughout the paper we tried to explore symmetric equilibrium bidding behaviour and the auctioneer's choices, in the setting of an All-Pay Auction where corruption can occur. We considered two different types of corruption schemes: one where the corrupt auctioneer colludes with the winner of the auction, and one where the corrupt auctioneer has a preferred bidder, whom he wishes to allocate the good to. We found conditions under which players bid more or less aggressively in favouritism that in the benchmark no corruption model, and showed that in posterior corruption bidders are always more aggressive.

Furthermore we argued that a Second Price All-Pay Auction is penetrable to corruption as, contrary to a Second Price Auction, the game is not dominance solvable. Coming to the auctioneer's choices, we found that in both

corruption settings he will deprive his colluding bidders of any "surplus of corruption". Finally, we saw that the auctioneer's revenue is not necessarily monotonic in the probability that he choses one type of corruption or the other.

What we did not talk about is the welfare effects of corruption both on the bidders and the seller; an interesting extension of our paper could give monitoring power to the seller, and potential punishments to the auctioneer if caught "cheating", which might lead to less/no corruption.

Appendix A1

In order to prove that $\lim_{\alpha\to 1}\beta_{pc}(v)=\beta(v)$, we only need to apply L'Hopital's rule. Since:

$$\lim_{\alpha\to 1} Log[1-(1-\alpha)F(v)^{N-1}] = \lim_{\alpha\to 1}\alpha - 1 = 0$$

, and $\lim_{\alpha\to 1}\dfrac{\frac{\partial Log[1-(1-\alpha)F(v)^{N-1}]}{\partial\alpha}}{\frac{\partial\alpha-1}{\partial\alpha}} = F(v)^{N-1}$, then:

$$\lim_{\alpha\to 1}\beta_{pc}(v) = vF(v)^{N-1} - \int_{\underline{v}}^{v}F(y_1)^{N-1}\,dy_1 = \beta(v)$$

In order to conclude the proof of Proposition 1, we need to show that $\beta_{pc}(v,\alpha=0)=\beta_{sp}$. Thus, we need to solve for the increasing symmetric equilibrium bidding function for the Second Price All Pay auction, where the highest bidder wins the object and pays the second highest bid, and all other bidders pay their respective bids. Revenue Equivalence implies:

$$[1-F(v)^{N-1}]\beta_{sp}(v)+\int_{\underline{v}}^{v}\beta_{sp}(y_1)f(y_1)(N-1)F(y_1)^{N-2}\,dy_1 = \int_{\underline{v}}^{v}y_1f(y_1)(N-1)F(y_1)^{N-2}\,dy_1$$

Differentiating with respect to v, we get:

$$\beta'_{sp}(v) = \frac{vf(v)(N-1)F(v)^{N-2}}{1-F(v)^{N-1}} \Rightarrow \beta_{sp}(v) = \int_{\underline{v}}^{v}\frac{y_1f(y_1)(N-1)F(y_1)^{N-2}}{1-F(y_1)^{N-1}}\,dy_1$$

Setting $\varphi'(y_1) = \frac{f(y_1)(N-1)F(y_1)^{N-2}}{1-(1-\alpha)F(y_1)^{N-1}}$:

$$\beta_{sp}(v) = v[-Log(1 - F(v)^{N-1})] - \int \underline{v}^v - Log(1 - F(y_1)^{N-1}) \, dy_1$$

, and thus: $\beta_{pc}(v, \alpha = 0) = \beta_{sp}(v)$, which concludes our proof.

Appendix A2

In this Appendix we prove that the auctioneer's expected revenue is strictly increasing in α_{pc}. We will begin our proof by computing player i's expected payment to the auctioneer. The expected payment is :

$$p(v) = \alpha Prob(win)(\beta(v) - \mathrm{E}[\beta(y_1) \mid y_1 \le v]) \Rightarrow$$

$$p(v) = \alpha[G(v) \int\limits_{\underline{v}}^{v} \frac{y_1 g(y_1)}{1 - (1 - \alpha)G(y_1)} \, dy_1 - \int\limits_{\underline{v}}^{v} \int\limits_{\underline{v}}^{y_1} \frac{yg(y)}{1 - (1 - \alpha)G(y)} \, dyg(y_1) \, dy_1]$$

, where we substituted $g() = (N - 1)f()F()^{N-2}$ and $G() = F()^{N-1}$. With a change of variables we get:

$$p(v) = \alpha \int\limits_{\underline{v}}^{v} \frac{y_1 g(y_1)G(y_1)}{1 - (1 - \alpha)G(y_1)} \, dy_1$$

Now, we can calculate the ex-ante expected payment to the auctioneer, ie player i's expected payment to the auctioneer before knowing his valuation v_i:

$$\mathrm{E}(p(V)) = \alpha \int\limits_{\underline{v}}^{\overline{v}} \int\limits_{\underline{v}}^{v} \frac{y_1(1 - F(y_1))g(y_1)G(y_1)}{1 - (1 - \alpha)G(y_1)} \, dy_1 f(v) \, dv$$

Again, with a change of variables, we have:

$$\mathrm{E}(p(V)) = \alpha \int\limits_{\underline{v}}^{\overline{v}} \frac{v(1 - F(v))g(v)G(v)}{1 - (1 - \alpha)G(v)} \, dv$$

The expected revenue of the auctioneer, is nothing but the sum of ex-ante expected payments of bidders. Thus $ER_{pc} = N \, \mathrm{E}(p(V))$. The auctioneer wishes to maximize his expected revenue with respect to α. The derivative of ER_{pc} with respect to α is:

$$\frac{\partial ER_{pc}}{\partial \alpha} = N \int_{\underline{v}}^{\overline{v}} \frac{v(1 - F(v))g(v)G(v)}{1 - (1 - \alpha)G(v)} \frac{1 - G(v)}{1 - (1 - \alpha)G(v)} \, dv > 0$$

Since $\frac{\partial ER_{pc}}{\partial \alpha} > 0, \forall \alpha \in [0, 1]$, the auctioneer maximizes his expected revenue by setting $\alpha_{pc}^* = 1$. Then as we already saw, bidders will play according to the increasing symmetric function without corruption.

Appendix B

To prove existence we follow Li and Tan(2000) .We wish to show that an increasing solution to the differential equation

$$\beta_f'(v) = \frac{v f(v)(N - 2)F(v)^{N-3}F(\beta(v))}{1 - v F(v)^{N-2}f(\beta(v))}$$

exists. With $N = 2$, the FOC becomes $f(b) = \frac{1}{v}$ which has an increasing solution, as long as $f()$ is decreasing in its argument.

For $N > 2$, we are looking for an increasing solution $\beta_f(v)$ such that $v \in [\underline{v}, \overline{v}]$, $\beta_f(v) \in [0, \overline{v}]$ and $\beta_f(v) \leq v \forall v$, with the initial condition $\beta_f(\underline{v}) = 0$. Thus, our domain is $D = \{v \in [\underline{v}, \overline{v}], b \in [0, \overline{v}] : b \leq v\}$. However, in this domain, both the numerator and the denominator of our differential equation can be 0, so we need to restrict our domain of existence of a solution, in order to be able to apply existence theorems.

The numerator is zero for $b = \underline{v}$, thus we need to exclude such b in order for the numerator to be strictly positive. Also, the denominator can be 0 if $1 - v F(v)^{N-2}f(\beta_f(v)) = 0$. Let $k(b, v) = 1 - v F(v)^{N-2}f(b)$, and let $\tilde{b}$ be such that $k(\tilde{b}, v) = 0$. Since $f(b)$ decreasing, then $k(b, v)$ increasing in b, and thus for any $b > b$, $n(b, v) > 0$.

Our new domain is:

$$D' = \{b \in (0, \overline{v}], v \in (\underline{v}, \overline{v}], \hat{b} < b \leq \overline{v}, \hat{b} = \begin{cases} \tilde{b} & \text{if } k(\tilde{b}) = 0 \text{ for } \tilde{b} \in (0, \overline{v}) \\ 0 & \text{otherwise} \end{cases}$$

On the new domain the standard existence theorems apply. Then for any $(b_0, v_0) \in D'$ there exists a unique solution $\beta_{f0}(v)$ in a neighbourhood of v_0 such that $\beta_{f0}(v_0) = b_0$ and that solution is continuous in v_0. The solution can be extended to the left and right and then be defined in a larger interval. Let (V_0, V_1) be the biggest open interval where the solution exists. Then, by the extensibility theorem, when v approaches V_1, $(v, \beta_{f0}(v))$ approaches a point on the boundary of D'.

We wish to show that $\lim_{v \to V_1} \beta_{f0}(v) \neq \tilde{b}$. Suppose that $\lim_{v \to V_1} \beta_{f0}(v) = \tilde{b}$. Then $k(V_1, \lim_{v \to V_1} \beta_{f0}(v)) = 0$ and thus $\beta_{f0}'(v)$ can be made arbitrarily

large. Since there exists V_1 such that as v approaches V_1, $\beta_f(v)$ approaches b, then for any $\epsilon > 0$, there exist $\delta > 0$ such that if $V_1 - v < \delta$, $\mid \beta_f(v) - b \mid < \epsilon$. Since for sufficiently ϵ, $\beta'_f(v)$ is arbitrarily large, if v moves closer to V_1 by a given distance, $\mid \beta_f(v) - b \mid < \epsilon$ does not hold. Thus, $\lim_{v \to V_1} \beta_{f0}(v) = \overline{v}$. Then we have a strictly increasing solution that takes values on the interval $[v_0, \overline{v}]$ and we need to extend it again such that it takes values on the whole interval of valuations $[\underline{v}, \overline{v}]$.

Now we need to prove that the second order condition for optimality holds. In this part of the proof we follow Arozamena and Weinschelbaum (2001). The expected payoff of bidder i is:

$$\Pi(b) = vF^{(N-2)}(\beta_f^{-1}(b))F(b) - b$$

Then:

$$\Pi'(b) = -1 + v[g(\beta_f^{-1}(b))\frac{1}{\beta'_f(\beta_f^{-1}(b))}F(b) + G(\beta_f^{-1}(b))f(b)]\mathbf{(a)}$$

Also recall that the First Order Condition under symmetry is:

$$1 = v[g(v)\frac{1}{\beta'_f(v)}F(\beta_f(v)) + G(v)f(\beta_f(v))]\mathbf{(b)}$$

Plugging $\mathbf{(b)}$ into $\mathbf{(a)}$ yields:

$$\pi'_b = v[g(\beta_f^{-1}(b))\frac{1}{\beta'_f(\beta_f^{-1}(b))}F(b) + G(\beta_f^{-1}(b))f(b) - g(v)\frac{1}{\beta'_f(v)}F(\beta_f(v)) + G(v)f(\beta_f(v))]$$

Since all the above functions are increasing, when $\beta_f^{-1}(b) < v \Rightarrow \pi'_b < 0$, and when $\beta_f^{-1}(b) > v \Rightarrow \pi'_b > 0$. Thus, it is optimal for bidder i to chose $b = \beta_f(v)$ such that $\beta_f^{-1}(b) = v$, as there $\pi'_b = 0$, which concludes our proof.

Appendix C

In order to prove that under some conditions, $\beta(v) > (<)\beta_f(v)$, we will follow Arozamena and Weinschelbaum (2009), and prove that if for some $\tilde{b} \in [0, \overline{v}]$, $\beta^{-1}(\tilde{b}) = \beta_f^{-1}(\tilde{b})$ then $(\beta^{-1}(\tilde{b}))' > (<)(\beta_f^{-1}(\tilde{b}))'$ if $(\beta_f^{-1})' < (>)\frac{f(\tilde{b})}{f(\beta_f^{-1}(\tilde{b}))}$. For convenience we will set $\beta_f^{-1}(b) = q_f$ and $\beta^{-1}(b) = q$. Note here that since $\beta(v)$ and $\beta_f(v)$ do not appear in the differential equations without corruption and favouritism, a particularity of the All-Pay Auction, in order to be able to compare them, for the purpose of this proof we will use the equivalent differential equations:

$$q = \frac{1}{(N-1)F(q)^{N-2}q'}$$

and

$$q_f = \frac{1}{(N-2)F(q_f)^{N-3}f(q_f)F(b)q_f' + F(q_f)^{(N-2)}f(b)}$$

Suppose that for some $\tilde{b}$ we have $q(\tilde{b}) = q_f(\tilde{b})$. Then, from the above we have:

$$q' = \frac{(N-2)F(b)}{(N-1)F(q_f)}q_f' + \frac{f(b)}{(N-1)f(q_f)}$$

We can see that if $\frac{f(b)}{f(q_f)q_f'} > \frac{(N-1)F(q_f)-(N-2)F(b)}{F(q_f)}$ then $q' > q_f'$. However notice that since $\frac{(N-1)F(q_f)-(N-2)F(b)}{F(q_f)} > 1$ always, then $\frac{f(b)}{f(q_f)q_f'} > \frac{(N-1)F(q_f)-(N-2)F(b)}{F(q_f)}$ implies that $\frac{f(b)}{f(q_f)q_f'} > 1$. Thus, we get that $q' > (<)q_f'$ iff $q_f' < (>)\frac{f(b)}{f(q_f)}$, i.e. players bid more (less) aggressively in favouritism than in the case of no corruption.

In order to conclude our proof, we need to show that such $\tilde{b}$ does not exist, ie if $q_f' < (>)\frac{f(b)}{f(q_f)}$, then $q > (<)q_f \forall b \in [0, \overline{v}]$. We will focus on the case where $q_f' < \frac{f(b)}{f(q_f)}$, as the proof for the opposite case is analogous.

Suppose towards a contradiction that for some $b_0 \in [0, \tilde{b}]$, $q(b_0) \leq q_f(b_0)$. This implies that $\frac{F(q_f(b_0))}{F(q(b_0))} > 1$. Then it should be true that $\frac{\partial \frac{F(q_f(b_0))}{F(q(b_0))}}{\partial b_0} > 0$. However:

$$\frac{\partial \frac{F(q_f(b_0))}{F(q(b_0))}}{\partial b_0} = \frac{q_f'f(q_f)F(q) - q'F(q_f)f(q)}{F(q(b_0))^2} < 0$$

, since $q_f' < \frac{f(b)}{f(q_f)}$, $f()$ is decreasing, $F()$ is increasing, and $q(b_0) < q_f(b_0)$, which is a contradiction.

Appendix D

The derivative of the auctioneer's expected revenue with respect to λ is:

$$\frac{\partial ER(\lambda)}{\partial \lambda} =$$

$$\overbrace{2\int\limits_{\underline{v}}^{\overline{v}}[1-F(y)]F(y)[\underbrace{\frac{\partial \beta(y,\lambda)}{\partial y}}_{\textbf{a'}}+\lambda\underbrace{\frac{\partial^2 \beta(y,\lambda)}{\partial y \partial \lambda}}_{\textbf{a''}}]\,dy}^{\textbf{a}} - \overbrace{\int\limits_{\underline{v}}^{\overline{v}}[1-F(\beta(y,\lambda))]F(y)\frac{\partial \beta(y,\lambda)}{\partial y}\,dy}^{\textbf{b}} +$$

$$\underbrace{(1-\lambda)\int\limits_{\underline{v}}^{\overline{v}}F(y)[(1-F(\beta(y,\lambda)))\overbrace{\frac{\partial^2 \beta(y,\lambda)}{\partial y \partial \lambda}}^{\textbf{c'}} - \overbrace{\frac{\partial F(\beta(y,\lambda))}{\partial \lambda}}^{\textbf{c''}}\frac{\partial \beta(y,\lambda)}{\partial y}]\,dy}_{\textbf{c}}$$

Notice that **b** is strictly positive. Also notice that **a** is also strictly positive as long as **a''**> 0. If instead **a''**< 0, then the sign of **a** is uncertain, and depends on the relative magnitudes of **a'** and **a''**. As for **c**, notice that its sign is also uncertain, and depends on the signs of **c'** and **c''**, and their relative magnitudes. Thus $ER(\lambda)$ might be increasing, decreasing or concave, depending on the curvature of $F(v)$, which concludes our proof.

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