The Relationship Between Competition and Innovation: How Important are Firms' Financial Constraints?

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Abstract

This paper studies the relationship between product market competition and innovation. It illustrates that apart from the well-studied impact of product market competition on the incentives of firms to innovate, it also affects their ability to have access to the necessary funds in order to innovate. Due to the fact that innovation projects are by definition risky and external financing subject to moral hazard concerns, lenders may be reluctant to finance projects in which the involved firms are not considered credible borrowers. I develop a step-by-step innovation model and show how product market competition may restrict the ability of firms to be credible borrowers. This effect can become the main driving force for $R\&D$ activities if firms are financial constrained. The relationship between competition and innovation is inverted-U shaped but the peak of the curve moves to lower levels of competition as financial constraints become more intense. Proposed market reforms that target to increase competition should be accompanied with the appropriate credit reforms that secure the access of firms to sufficient funds for their $R\&D$ investments. The theoretical study is complemented by empirical tests (in progress) using a unique data set created by Banque de France.

Keywords: Innovation, $R\&D$, Competition, Financial constraints.
JEL-codes: G15, G31, L13, O31.

1 Introduction

The exact relationship between product market competition and innovation has traditionally drawn the attention of the academic community not only because of its numerous important implications to economic efficiency and growth, but also due to the discrepancies among endogenous growth models, agency models and empirical evidence on whether competition reinforces or discourages innovation. To my knowledge, the existing literature has focused on the impact of competition on the incentives of firms for innovation (which can be defined as the difference in the profits a firm earns when it innovates and when it is not). This "incentive-effect" of competition can be either positive or negative. On the one hand, firms that operate in a competitive market have incentives to innovate to escape from competition and enjoy higher market shares (the "escape-competition" effect following Aghion et. al., 2005, which is a slightly modified version of the "replacement effect" of Arrow, 1962). On the other hand, firms that enjoy monopoly rents have higher incentives to innovate to protect their market position and discourage entry by potential competitors (the Schumpeterian effect, based on the notion of "creative destruction" introduced by Schumpeter, 1942). The most of theoretical contributions have focused on the interaction of these two opposing forces of the "incentive-effect" under different market structures and characteristics of innovations. As a result, even today, it is not clear whether product market competition increases or discourages investments in $R\&D$.

This paper adopts an alternative path to approach this relationship. It illustrates that apart from the "incentive-effect", product market competition has a second effect on innovation, the "ability-effect"[1]. This effect is based on the fact that competition affects both the internal funds that are available for $R\&D$ investments and the ability of firms to have access to external financing in order to innovate. Therefore, it is worthwhile to investigate the interplay between the "incentive-effect" and the "ability-effect" by separating firms in the ones that are financially constrained and the others that do not face financial constraints when they decide whether they will invest in $R\&D$. The main research questions of this study are the following:

- What is the relationship between product market competition and innovation if we take into consideration the firms' financial constraints?
- What are the policy implications of this relationship for proposed reforms that aim to increase competition in order to reinforce investments in $R\&D$?

In the European Community Innovation Surveys the most frequently cited obstacles to innovation by firms are related to high cost of innovation, lack of financing, and economic risk (see Jaumotte and Pain , 2005a, 2005b). Large firms are more innovative than small firms by a factor of three to four when innovativeness is measured by $R\&D$ spending. A smaller proportion of large firms also reports lack of financing among the most significant obstacles to innovation. Access to external finance may be one of the key determinants of

[1]Schumpeter (1942) has pointed out that firms that operate in highly concentrated markets are more able to innovate but this conjecture has never been formally analyzed.

innovation success. Hence, the quality of the financial system is likely to be important for securing outside.

Innovation projects are by definition risky, complex, involve intangible assets and their outcome is realized in the long-run. As competition increases, firms' profits decrease, so firms are less able to both self finance their innovation projects and have access to the necessary external funds due to their inability to convince the lenders that they are credible borrowers. This element of credibility is introduced through the (nonverifiable) effort that firms put on their innovation projects. Following the main framework of Holmstrom and Tirole (1997), the ability of firms to have access to external financing in order to invest in a project is affected by moral hazard concerns on the side of investors. In order to convince lenders to provide the necessary funds for their innovation projects, firms should signal their credibility by pledging in their projects sufficient amount of their own assets (collateral). Since, product market competition adversely affects the available pledgeable assets held by firms, it introduces some limits in their ability to be credible borrowers and therefore to finance their investments. These limits can become the main driving force for $R\&D$ activities if firms are financial constrained. Therefore, in financial constrained firms, the "ability-effect" dominates the "incentive-effect" and an increase in product market competition decreases firm level $R\&D$ investments.

I introduce the "ability-effect" and the above moral hazard model in the context of the step by step innovation of Aghion, Bloom, Blundel, Griffith and Howitt (2005) and I identify the important impact of financial constraints on the relationship between competition and innovation. In their seminal contribution, Aghion, Bloom, Blundel, Griffith and Howitt develop an endogenous growth model where they consider a population of industries each of which consists of two firms. Industries can either be in the neck-and-neck state where there is no technological gap between the two firms of the industry or in the unleveled state where there is a technological gap as the one firm, the leader is one step ahead its competitor, the follower. Their paper focuses on the "incentive-effect" and predicts an inverted-U relationship between competition and innovation.

By introducing the "ability-effect" into this framework we observe a shift of the peak of the inverted-U curve to lower levels of competition. This finding suggests that the "ability-effect" limits the "escape-competition" effect and that market power provides a stable platform that facilitates risky $R\&D$ investments (in line to what Schumpeter has proposed but never formalized in a model framework). If we only consider financial constrained firms, then the relationship between competition and innovation may be completely driven by the "ability-effect" suggesting a negative relationship between competition and innovation.

This study suggests that policy reforms which have as a main goal to increase product market competition (such as liberalization, the creation and development of the EU Single Market, privatization, ect...) may have negative impact on investments in $R\&D$ at periods of crises. At least, if they are not accompanied by the necessary credit reforms that correct potential market failures (e.g. credit rationing) generated by asymmetric information and adverse economic market conditions.

In conclusion, this paper attempts to provide a link between real economy and financial sector in order to develop an insight on how firms' market performance and investments in

$R\&D$ are affected by product market competition. The potential contribution of this study is to introduce an alternative way through which product market competition affects investments in $R\&D$. Understanding how competition affects the ability of firms to innovate given their financial constraints may lead to important implications for policies that can be implemented to reinforce innovation and economic growth, especially during periods of economic turmoil when financial constraints are more prevalent. The main policy message of this work is that policies that are designed to increase product market competition should be linked to credit reforms such that investments of firms in $R\&D$ will not decrease.

2 The Model

2.1 Framework of the Model

Consider an economy with a continuum of industries indexed by $j \in [0, 1]$. In each industry there are two firms that compete in the product market and produce an aggregate output Q_{jt} at each period t. Each firm produces using labor as the only input according to a constant-returns production function and takes the wage rate as given. Hence, the unit costs of production of the two firms are independent from the quantities produced.

Moreover, there is a unit mass of identical infinitely-lived consumers with a logarithmic instantaneous utility function:

$$u(y_t) = \ln y_t = \int_0^1 lnQ_{jt}dj \tag{1}$$

where y_t is the consumption good produced at each date t. This good is produced using as an input the aggregate output Q_{jt}.

These logarithmic preferences imply that in equilibrium consumers spend the same proportion of their income on the aggregate output of each industry j at all periods t. If we normalize this amount to unity, the demand functions facing the two firms in industry j and period t, q_{Ajt} and q_{Bjt} are given by the following maximization problem of the representative household:

$$\max_{q_{Ajt}, q_{Bjt}} \{Q_{jt} = q_{Ajt} + q_{Bjt}\}$$
$$s.t. p_{Ajt}q_{Ajt} + p_{Bjt}q_{Bjt} = 1$$

where p_{Ajt} and p_{Bjt} are the prices set by the two firms. I assume that the demand functions that are derived from this maximization problem are downward slopping.

Let $k_i \in K$ be a non-negative integer parameter that characterizes the technology level of each firm i in each industry j, where $i = \{A, B\}$. K is the set of all available technologies which is assumed to be ordered in terms of the efficiency of the technology, $K = (k^1, k^2, ...)$, with $k^x < k^{x+1}$ for every positive integer x. Firms can invest in $R\&D$ having as an objective to acquire the next available technology level in K (step-by-step innovation). The innovator has a comparative advantage as it becomes more efficient with the adoption of the higher technology with respect to its competitor. Specifically, as k_i increases, firm i can produce more output using the same amount of input (same number of employees)[2]. So, firms have incentives to engage in $R\&D$ in order to improve their production process and reduce their relevant costs. To be more formal for the analysis that follows, we consider that one unit of labor currently employed by firm i generates an output flow equal to γ^{k_i}, where $\gamma > 0$ is a parameter that measures the size of a leading-edge innovation[3].

It immediately follows that the state of each industry at any period t is fully characterized by the integers l and m, where l is the technology level of the most efficient firm (in terms of production) and m is the technology gap between the two firms in an industry j. Then, the equilibrium profit of each firm depends only on the technological gap m but not on absolute levels of technology[4]. We denote by π_m (π_{-m}) the profit of a firm that is m steps ahead (behind) its rival. We restrict our attention to the case where the technology gap between the duopolists in each industry is either zero or one ($m = \{0, 1\}$). This is because the consideration of higher technology gaps does not allow us to derive closed form solutions. Furthermore, as Aghion et. al. (2001) illustrates, the results remain qualitatively the same when higher technology gaps are included in the analysis. This restriction about the technology gap m is introduced in the model by assuming that if the leader innovates, the follower can automatically learn and copy the leader's previous technology (at no cost) and thereby remains only one step behind. Since, the technology gap determines the profitability of the duopolists, in the unleveled state, the leader does not have any incentive to innovate. In contrast, the follower has incentives to innovate in order to get the same technology as the leader and improve its position in the product market. In this case, the industry moves from the unleveled to the neck-and-neck state.

Hence, we have two types of industries, the ones that are in the neck-and-neck state with no technology gap (m=0) between the duopolists and the others in the unleveled state with technology gap 1 (m=1). In the unleveled state, the firm in the technological frontier is the leader and the other firm the follower. If firm A in a neck-and-neck industry with technology $k_A = k^x$ innovates then it adopts the next technology in K, $k_A = k^{x+1}$ and becomes the technological leader while the other firm is now behind the technological frontier (with technology $k_B = k^x$) and is therefore the follower. In other words, as a firm in the neck-and-neck state innovates, its industry moves to the unleveled state where the innovator is the leader and the other firm is the follower. The reverse also holds. In the unleveled industry, if firm A is the leader with technology $k_A = k^{x+1}$ and firm B the follower with technology $k_B = k^x$, then, the latter by innovating can adopt the same technology as the leader ($k_B = k^{x+1}$) and the industry moves to the neck-and-neck state[5].

[2]It is totally equivalent to consider that the adoption of the next technology k_i reduces the unit production cost of the innovator relative to its competitor, or that it leads to an improvement of the quality of innovator's product(s).

[3]As we will see below, the size of innovation is important for the relationship between competition and innovation when we include in the analysis the firms' financial constraints.

[4]This property facilitates the comparative analysis between industries below.

[5]We do not consider the possibility of leapfrogging here. The industries move step by step from the neck-and-neck state to the unleveled and vice versa.

2.2 Product Market Competition

As measure of competition we consider the degree of collusion between the two firms in the neck-and-neck state[6]. Let $\Delta \in [\frac{1}{2}, 1]$ be the parameter that indicates the level of competition in the industry. Then, $(1-\Delta)$ is the respective degree of collusion. Let π_1, π_{-1} the equilibrium profits of the leader and the follower in the unleveled state, respectively and $\pi_0 = \pi_{-1} + (1-\Delta)(\pi_1 - \pi_{-1})$ be the equilibrium profit of a firm in the neck-and-neck state such that $\pi_1 > \pi_{-1} \geq 0$. Note that $\frac{\pi_1 - \pi_0}{\pi_0 - \pi_{-1}}$ is increasing in Δ for every $\Delta \in [\frac{1}{2}, 1)$. In words, an increase in competition benefits the firms in the technological frontiers in comparison to less efficient firms. So, the reallocation effect à la Boone (2008) that can be viewed as a condition for a good and robust measure of competition, from a theoretical point of view, is satisfied by our measure Δ. If competition is intense, $\Delta = 1$, each firm earns profit flow π_{-1} equal to the laggards profit flow in the unleveled state. In the opposite case, if competition is relaxed, $\Delta = \frac{1}{2}$, each firm earns profit flow $\pi_{-1} + \frac{1}{2}(\pi_1 - \pi_{-1})$ as the two firms share, in the same proportion, a collusion rent derived from technology gap $\pi_1 - \pi_{-1}$. See Appendix A for an alternative specification of product market competition.

The profitability of firms at different states can be justified as follows. If firms compete in prices then in the neck-and-neck state where we have symmetry firms set the same equilibrium price and get the same profit. Under perfect competition ($\Delta = 1$), if the products produced by the two firms are homogeneous then $\pi_{-1} = 0$ and firms earn zero profit (Bertrand outcome with homogeneous goods). If the two products are not perfect substitutes, perfect competition leads to profit $\pi_{-1} > 0$ for both firms. In the unleveled state, if goods are homogeneous, the leader, in equilibrium set a price slightly below the marginal cost of the follower, and earns profit $\pi_1 > 0$ while the follower earns zero profit due to its production efficiency disadvantage. Note that π_1 only depends on the technology gap of the two firms. If products are not homogeneous then the difference in production efficiency leads to some positive rents for the follower, while the leader is still the most profitable firm in the industry. The degree of collusion in the neck-and-neck state refers to the intensity of the market conduct of the two firms taking the degree of substitutability between the two firms as given. Since the product market production decisions are repeated in every period, folk theorem applies and prices higher to the perfect price competition case can be sustained (tacit collusion).

2.3 *R&D* Investment

Firms need to invest in *R&D* in order to innovate. But, investment is risky and can be either successful or not. Successful innovation corresponds to the case in which the innovator moves one step ahead by adopting the next technology in K. If investment fails, the firm remains with the same technology as in the pre-innovation stage. The amount that each firm invests in *R&D* has an impact on the probability that the innovation project is successful. A leader (a neck-and-neck firm) can move one technological step ahead by

[6]Results remain qualitatively the same if we consider as a measure of competition another index that refers to the product market conduct of the two firms.

spending amount $\psi(n_1) = \frac{n_1^2}{2}$ ($\psi(n_0) = \frac{n_0^2}{2}$) in $R\&D$ and exerting effort e on the $R\&D$ project. In this case, its innovation rate (or $R\&D$ intensity) will be given by the hazard rate $n_1 e$ ($n_0 e$). The follower can move one step ahead with hazard rate $h \geq 0$ even if it spends nothing in $R\&D$ and exerts no effort. Hence, parameter h can be interpreted as the rate at which the follower can imitate the leader or as the leader's innovation spillover effect[7]. So, if the follower spends $\frac{n_{-1}^2}{2}$ and exerts effort e, it innovates with hazard rate $n_{-1}e + h$. Since the leader does not have any incentive to innovate in equilibrium, $n_1 = 0$. Let $e_m \in \{e_L, e_H\}$, where $m = -1, 0$ and $1 \geq e_H > e_L \geq 0$. Therefore, each of the firms that invests in $R\&D$ can exert either high (e_H) or low (e_L) effort on the innovation project[8]. By exerting low effort, firms enjoy private benefit $B > 0$ which is common knowledge but the probability of success of the innovation project is lower, since $e_L < e_H$. So, firms have to decide how much they will invest and how much effort they will exert on the project. We assume that when firms invest using only internal funds to cover their $R\&D$ cost, they always behave in equilibrium[9].

While in the unleveled state, in each period, there can only be one investment in $R\&D$ by the follower, in the neck-and-neck state both firms invest in $R\&D$ as they have incentives to become the leader of the industry. We assume that investment in $R\&D$ can be successful only for the one of the two rivals[10].

In each period there is only one good $R\&D$ project that leads to successful innovation. This project belongs to the portfolio of projects to be investigated by only one of the two firms. Firms are unable to discriminate among the quality of the available projects before they invest on them. By increasing $\frac{n_0^2}{2}$, each neck-and-neck firm accommodates more projects under consideration that are within its portfolio. So, the probability to consider the good project (given that it is in its portfolio) increases. Since the good project can be in each portfolio with probability 1/2 (symmetry), the $R\&D$ intensity $n_0 e_m \in [0, \frac{1}{2}]$. So, $n_0 e_0 = \frac{1}{2}$ corresponds to the case that the neck-and-neck firm investigates all the projects its in portfolio. The same applies for the unleveled state, so that $n_{-1}e_{-1} \in [0, \frac{1}{2}]$. In the case that the good project belongs to the portfolio of the leader, then since the leader does not have any incentive to innovate, the probability that he will investigate the new project is zero.

[7]From policy point of view, this parameter h depends on the design of the property rights system implemented in the economy. It could be interpreted as the efficacy of patent protection.

[8]The analysis below can easily be extended to the case that effort e is a continuous variable on $[0, 1]$ with similar qualitative and quantitative results.

[9]Investment using only internal funds and shirking is clearly a dominated strategy by the innovating firm.

[10]While neck-and-neck firms can simultaneously invest, they cannot simultaneously move one step ahead by adopting the next available technology in K. Therefore, successful innovation always improves the market position of the innovator for at least one period.

2.4 The Financial Contract

Firms may be unable to cover their *R&D* investment expenses by internal funds only. They may have to raise external funds in order to invest. If this is the case, a financial contract between the firm (neck-and-neck firms or follower in the unleveled state) and investors needs to be signed. For simplicity we focus on the bilateral contract between an entrepreneurial firm and an investor but the analysis can easily be extended in the case of multiple investors[11]. The supply of funds is assumed to be infinite in order to focus on the moral hazard concerns on the firm's side. Both parties are risk neutral and protected by limited liability.

Consider a neck-and-neck firm that needs to raise funds equivalent to $\frac{n_0^2}{2}$ in order to move one technological step ahead with innovation rate $n_0 e$. The investor designs a contract that determines the terms of borrowing to the firm taking into account the required assets for its investment on the new technology of cost $\frac{n_0^2}{2}$. In particular, the contract specifies how much each side should invest and how much it should be paid as a function of the project outcome. We consider two possible outcomes of this investment that are perfectly observed by both parties. If innovation is successful the firm moves to the technological frontier while if it is not it remains with the same technology it had before their investment took place. Since the contract is signed only when the neck-and-neck firm does not have internal funds to finance the project by itself[12], it should be $\frac{n_0^2}{2} \geq \pi_0$. Let the effort e that the entrepreneur exerts on the project be nonverifiable. The contract specifies:

- The amount A_0 should be invested by the entrepreneur. Therefore, the investor should plug in the project amount $\frac{n_0^2}{2} - A_0$
- The repayment P_0 of the investor if innovation is successful

There is no repayment for the investor if the project fails (due to limited liability). Hence, the amount A can be perceived as the collateral that the firm puts on the project.

This contract introduces moral hazard considerations through the nonverifiable effort e of the innovating firm. High effort by the entrepreneur leads to high *R&D* intensity $n_0 e_H$ with which the next technological level is reached. The cost of innovation becomes particularly relevant when the firm borrows from the investors because it has increased incentives to shirk. Shirking becomes attractive especially in the case that A_0 is small, so the entrepreneur has to rely heavily on external financing. Since, in this case, the entrepreneur does not have a large stake of the project it finds attractive to exert low effort as the potential private benefit from shirking may exceed its share of the project's expected return when it behaves[13].

[11]Competition among multiple investors in supplying the necessary funds for investment in *R&D* does not change the main results of this study about the effects of product market competition on innovation.

[12]External finance is more expensive than internal finance due to the interest rate.

[13]This is similar to the Holmstrom and Tirole (1997) main finding that poorly capitalized firms do not have access to financial markets. If firms have to rely extensively on external funds, they may prefer to shirk and enjoy a private benefit since, in the case they behave they have to give the larger part of the investment return to their investors in case that the project is successful.

The investor seeks to design the contract in a way that maximizes its expected payoff and at the same time satisfies the incentive compatibility (IC) for entrepreneur. This implies that it has to adjust the amount $\frac{n_0^2}{2} - A_0$ it is going to plug in the project and its repayment P in an appropriate way.

Obviously, an optimal contact is to ask the entrepreneur to invest all its current profit in the $R\&D$ project[14]: $A_0 = \pi_0$[15].

The same applies to followers in the unleveled state when they do not have sufficient funds to finance by themselves their $R\&D$ investments. If A_{-1} is the amount a laggard firm invests on the project, then an optimal contract will be $A_{-1} = \pi_{-1}$.

2.5 Timing of the Model

The timing of the model is as follows:

- During period t, each firm with a technology gap m with its rival, at an industry whose level of competition is Δ makes its product market production decisions and earns profitability π_m.
- Each firm with profitability π_m decides how much will invest on $R\&D$, $\frac{n_m^2}{2}$. It may rely on external financing if it cannot cover all its cost by itself and is able to secure external funds (if there exists an IC and IR contract). If this is the case, the contract is signed and investment is made.
- Each firm privately decides how much effort will put on the innovation project.
- At the beginning of period $t+1$, the return of the investment is realized. In case of successful innovation, the firm moves one technological level ahead and improves its market profitability. If a contract had been signed, repayment to the lender takes place.

2.6 Financial Constraints and Product Market Competition

Consider again the case of a neck-and-neck firm. Without loss of generality we can restrict our attention to the case that $e_H = 1$ and $e_L = 0$. In the case the financial contract is signed, it should be that the investor earns non-negative payoff:

$$\delta n_0 e P - \left[\frac{n_0^2}{2} - \pi_0\right] \geq 0 \tag{2}$$

[14] Alternatively, we could consider that A_0 takes a more general form of a function of profit. Since, it is intuitive that an increase in Δ reduces the profit, A_0 should be decreasing in Δ. It is this key effect of competition that is important and not the exact expression of A_0.

[15] In the analysis that follows, we focus on this simple optimal contract because it curries all the necessary aspects of the borrowing conditions of firms and illustrates how these conditions are affected by product market competition without introducing unnecessary complications.

where δ is the common discount factor. The incentive compatibility (IC) constraint is:

$$n_0(V_1 - P) + \overline{n}_0(V_{-1}) + (1 - n_0 - \overline{n}_0)V_0 \geq n_0 B + \overline{n}_0(V_{-1}) + (1 - \overline{n}_0)V_0$$

or

$$V_1 - V_0 - B \geq P \tag{3}$$

where $\overline{n}_0$ is the innovation intensity of the competitor, V_m denotes the firm's steady state value of being currently in the state with technology gap m with its competitor and B is the private benefit of the firm from shirking. In other words, B is the measure of credibility of the borrower or an indicator of the intensity of the financial constraints are. The lower the B, the easier will be for the firm to have access to external financing. While the investor should ask sufficient high P such that its participation constraint (2) is satisfied, P should not be so high that violates the IC constraint (3) and induces the firm to shirk[16]. The part of investment financed by firm, π_0 (collateral) indicates the firm's interest for the success of the investment. When the collateral is high, the entrepreneur has more incentives to behave. Hence, the lender will choose the highest possible P for which the contract is incentive compatible. The constraint (3), in equilibrium, will be binding. Solving it for P and substitute it to (2) we conclude to:

$$\frac{n_0^2}{2} - n_0\delta\left(V_1 - V_0 - B\right) - \pi_0 \leq 0 \tag{4}$$

This weak inequality introduces the financial constraints into neck-and-neck firm's innovation maximization problem. The firm chooses how much it will spend in $R\&D$ having as an upper limit the $R\&D$ expenditure that satisfies (4) with equality. Since, firm's current profit is decreasing in Δ, an increase in competition decreases the firm's ability to have access to the necessary funds for investment in $R\&D$. In particular, its internal funds go down as the firm has less profit and therefore, it is more likely that the entrepreneur has to rely on external financing in order to innovate. In this case, moral hazard concerns introduce an issue of credibility for the borrower. Higher competition reduces the range of P values that the investor should be promised through a contract that simultaneously satisfy participation (for investor) and IC constraints. If the firms choose to invest a lot without having the necessary internal funds, they will not be able to attract external funds as they will not appear in the eyes of investors as credible borrowers. Hence, an increase in competition reduces the credibility of the entrepreneur and consequently, its ability to borrow. In other words, it limits the ability of the firm to invest in $R\&D$.

This "ability effect" is an additional way in which product market competition can affect innovation. Its interaction with the impact of product market competition on the incentives of firms to innovate ("incentive effect") completes the picture of the relationship between competition and innovation.

[16]Clearly, shirking leaves the investor with negative payoff.

Following a completely analogous reasoning for the follower in the unleveled state, the financial constraint becomes:

$$\frac{n_{-1}^2}{2} - (n_{-1} + h)\delta\,(V_0 - V_{-1} - B) - \pi_{-1} \leq 0 \tag{5}$$

Note that an increase in h relaxes the constraint (5) of the follower. As imitation becomes easier and the intellectual property is not well protected, the probability of innovation by the follower increases without any additional cost reducing the moral hazard concerns of investor.

3 Individual Innovation Intensities

In order to investigate the impact of the financial constraints on the relationship between competition and innovation, we firstly consider the case where financial constraints are absent, as a benchmark case. In this way, we capture the "incentive effect" of product market competition on innovation ignoring its potential "ability effect". Incentives for innovation can be defined as the difference between post-innovation profit and profit in the case no investment in $R\&D$ is made.

3.1 Unconstrained Markov-Stationary Equilibrium

Following Aghion et. al. (2005), we assume that the equilibrium innovation rates n_0 and n_{-1} are determined by the necessary conditions for a symmetric Markov-stationary equilibrium in which each firm chooses its $R\&D$ intensity seeking to maximize expected discounted profits. The following proposition applies:

Proposition 1. *In the unconstrained Markov stationary equilibrium, the research intensity by each neck-and-neck firm is:*

$$n_0^* = -\left(\frac{1-\delta}{\delta} + h\right) + \sqrt{\left(\frac{1-\delta}{\delta} + h\right)^2 + 2\Delta(\pi_1 - \pi_{-1})} \tag{6}$$

and is decreasing in product market competition, while, the research intensity by a follower is:

$$n_{-1}^* = -\left(\frac{1-\delta}{\delta} + h + n_0^*\right) + \sqrt{\left(\frac{1-\delta}{\delta} + h\right)^2 + n_0^{*2} + 2(\pi_1 - \pi_{-1})} \tag{7}$$

and is decreasing in product market competition.

Note that n_0^* and n_{-1}^* are decreasing in h. Intuitively, as the rate of imitation increases, laggard firms invest less in $R\&D$ as they have more chances to catch up the firms in the technological frontier at not cost. At the same time, since, the value of being the leader is reduced, so neck-and-neck firms have reduced incentives to innovate and become the leaders.

Moreover, as competition increases, the benefit from innovation for a neck-and-neck firm increases as the firm wants to escape from competition and its resulting low profit ("escape-competition effect"). In contrast, an increase in competition reduces the post-innovation profit π_0^C of a successful innovation for the follower, so, its incentives to innovate decrease ("Schumpeterian effect"). So, the "incentive effect" can be either positive or negative.

3.2 Constrained Markov-Stationary Equilibrium

The financial constraints (4) and (5) define an upper threshold for the $R\&D$ intensities of the neck-and-neck firm and the follower, respectively, when they have to rely on external financing in order to innovate. The first step is to identify when these constraints become relevant.

If firms invest on $R\&D$ using only internal funds then in equilibrium they choose (6) and (7). When firms cannot reach these equilibrium intensities by themselves and they have to rely on external financing, moral hazard concerns become relevant. On the one side, firms prefer to rely extensively on external financing and invest a lot on $R\&D$ in order to increase the probability of successful innovation[17]. On the other side, the lender is not willing to borrow a large amount due to its fear that the borrower may shirk. In equilibrium, constraints (4) and (5) are binding and determine the equilibrium $R\&D$ intensities when entrepreneurs rely on external financing. Let $\hat{\Delta}$ be the level of competition above which neck-and-neck firms have to rely on external financing in order to invest in $R\&D$ and $\overline{\Delta}$ the level of competition below which the laggard firms need to borrow in order to invest.

It can be proven that $\overline{\Delta} > \hat{\Delta}$. So, the constrained Markov equilibrium can be separated in three regions of Δ:

- Low competition region, $\Delta \in [\frac{1}{2}, \hat{\Delta}]$: Followers in the unleveled state are financially constrained
- Intermediate competition region $\Delta \in [\hat{\Delta}, \overline{\Delta}]$: Both followers and neck-and-neck firms are financially constrained
- Intense competition region, $\Delta \in [\overline{\Delta}, 1]$: Neck-and-neck firms are financially constrained.

Note that the optimal $R\&D$ intensities of the unconstrained problem (hereafter, first best) are increasing in the profitability gap between the pre-innovation state and the post-innovation state (when $R\&D$ investment is successful). When competition is low, in terms of profitability, neck-and-neck firms are relatively close to the firm in the technological frontier in the unleveled state. Therefore, in equilibrium, the first best cost of innovation is not so high and they can cover it by internal funds. On the other hand, the laggard firm

[17]If they can invest with the money of someone else and since they are protected by limited liability, they will be happy to do it.

is relatively far from the neck-and-neck profit level. In equilibrium, it has to invest a lot in *R&D* but it is unable to cover the cost using only internal funds. It has to borrow from the lender and it is financially constrained.

For intermediate level of competition, the profit distance between the neck-and-neck firms and the leader has increased, so neck-and-neck firms have to undertake more costly investments, which cannot finance only with internal funds. They have to rely on external financing and therefore they are financially constrained. The laggard firm undertakes a less costly investment since their profit gap with the neck-and-neck state has been reduced, but this profit gap is still relatively high and financial constraints are still relevant.

For high levels of competition, neck-and-neck firms have moved far way from the profit of the leader. In addition, due to the increased product market competition, they earn limited amount of profit. They are unable to cover the innovation cost by themselves and they are financially constrained. In contrast, the laggard firm is now in close profit distance with the neck-and-neck state. Therefore, its equilibrium first best cost of innovation is relatively small and despite the high level of competition, it can cover it by itself.

In other words, competition primary affects the profitability of the firms (direct effect) and through it the equilibrium costs of innovation (indirect effect). The interaction of these two effects lead to the separation of the equilibrium space to the three regions mentioned above.

By determining the equilibrium *R&D* intensities for each of these three regions we conclude to the following proposition:

Proposition 2. *In the constrained Markov stationary equilibrium, the R&D intensities are:*

- *For $\Delta \in [\frac{1}{2}, \hat{\Delta}]$, the equilibrium R&D intensity for neck-and-neck firms is $n_0^{un} = n_0^*$ which is in increasing Δ while, the R&D intensity of laggard firms n_{-1}^c is decreasing in Δ.*
- *For $\Delta \in [\hat{\Delta}, \overline{\Delta}]$, the equilibrium R&D intensity of neck-and-neck firms n_0^c is decreasing in Δ. The equilibrium R&D intensity of laggards firms is still n_{-1}^{cc} (and it is decreasing in Δ).*
- *For $\Delta \in [\overline{\Delta}, 1]$, the equilibrium R&D intensity of neck-and-neck firms is n_0^{cc} while the equilibrium R&D intensity of followers is n_{-1}^{un}. They are both decreasing in Δ.*

This proposition leads us to the following predictions:

Corollary 3. *In the constrained Markov stationary equilibrium, the R&D intensity of each neck-and-neck firm has an inverted-U shaped relationship with product market competition:*

- *For $\Delta \in [\frac{1}{2}, \hat{\Delta}]$, the equilibrium R&D intensity is increasing in Δ*
- *For $\Delta \in [\hat{\Delta}, 1]$, the equilibrium R&D intensity is decreasing in Δ*

Corollary 4. *In the constrained Markov stationary equilibrium, the research intensity of each laggard firm in the unleveled state is decreasing in product market competition.*

So, the inclusion of financial constraints change the way that product market competition affect the $R\&D$ intensities by limiting the impact of "escape-competition" effect in the neck-and-neck state. Product market competition has a direct impact reducing the ability of firms to concentrate the necessary funds in order to innovate. This "ability" effect is present in both industry states and moves in the same direction with the Schumpeterian effect. But, it opposes "escape competition effect" that we meet in neck-and-neck states. When product market crosses threshold $\hat{\Delta}$, financial constraints become binding for neck-and-neck firms and therefore, the "ability" effect dominates over the "escape-competition" effect.

When the level of competition is below $\overline{\Delta}$, laggard firms are financially constrained and the "ability effect" dominates. When Δ exceeds this threshold, $R\&D$ intensity are determined by the "Schumpeterian" effect. Since, these effects are both negative, the $R\&D$ intensity is decreasing in product market competition for the whole range of values of Δ.

4 Average Innovation Intensities

Having determined the effects of competition to individual $R\&D$ intensities, we proceed by deriving the relationship between competition and innovation in the economy. Changes in product market competition have an impact on the (steady-state) fraction of industries that are in the neck-and-neck or the unleveled state and therefore to the effect ("escape-competition", Schumpeterian and "ability") that in equilibrium determines this relationship.

Let μ_1 (μ_0) be the steady state probability of being in the unleveled (neck-and-neck) state. Obviously, it should be $\mu_0 + \mu_1 = 0$. If n_0 (n_{-1}) is the equilibrium $R\&D$ intensity in the neck-and-neck (unleveled) state, then, in each period t, the steady-state probability that a sector moves from being unleveled to leveled is $\mu_1(n_{-1}^* + h)$, while, the probability that it moves in the opposite direction is $\mu_0 n_0^*$. In steady state, these two probabilities must be equal: $\mu_1(n_{-1} + h) = 2\mu_0 n_0$. Hence, the aggregate flow of innovations will be:

$$I = 2\mu_0 n_0 + \mu_1(n_{-1} + h) = \frac{4n_0(n_{-1} + h)}{2n_0 + n_{-1} + h} \tag{8}$$

For favor of comparison, as a starting point we assume that financial constraints are absent and firms are free to choose the unconstrained Markov equilibrium quantities (6) and (7). In this case, the following proposition holds:

Proposition 5. *The average innovation intensity follows an inverted-U pattern for* $\Delta_{peak} \in [\frac{1}{2}, 1]$, *where* Δ_{peak} *is defined by the relationship*

$$n_0(\Delta = \Delta_{peak}) = \frac{\sqrt{\left(\frac{1-\delta}{\delta} + h\right)^2 + 2(\pi_1 - \pi_{-1})}}{3}$$

It increases with competition for $\Delta \in [\frac{1}{2}, \Delta_{peak}]$ and it decreases with competition for $\Delta \in (\Delta_{peak}, 1]$. if $\Delta_{peak} < \frac{1}{2}$, then the average intensity I is monotonously decreasing in Δ, while when $\Delta_{peak} > 1$, it is monotonously increasing in Δ.

The inverted-U pattern can be explained as follows: When competition is low, firms in the neck-and-neck state do not have incentives to innovate, as their profit level is relatively close to the profit level of the leader in the unleveled state. In contrast, followers in the unleveled state have high incentives to innovate and move to the neck-and-neck state due to the high profitability. Therefore, the industry will be quick to leave the unleveled state and slow to leave the neck-and-neck state. It will spend the most of its time in the neck-and-neck state where the "escape-competition" effect dominates resulting an increasing average intensity I in competition. In contrast, for high levels of product market competition, followers have little incentives to innovate, while neck-and-neck rivals have increased incentives to innovate in order to escape from the high level of competition. The industry moves quickly from the neck-and-neck state to the unleveled one and slowly to the opposite direction. So, the industry is for more time in the unleveled state where the Schumpeterian effect dominates and I decreases with product market competition.

The consideration of financial constraints has an impact on the shape of the inverted-U curve. Note that since $\frac{1}{2} < \hat{\Delta} < 1$, it cannot be the case that I is increasing in Δ for high values of product market competition. In this region (intermediate and high competition region) the ability effect in the neck-and-neck state dominates the "escape-competition" effect and therefore, I is decreasing in competition. The intensity of financial constraints (which can be measured by the private benefit B) determines the shape of the curve. As B increases, the threshold $\hat{\Delta}$ moves to lower values of product market competition. We conclude again to an inverted-U pattern with a peak value at Δ^c_{peak} which is decreasing in B. For sufficiently high B (above a particular threshold $\hat{B}$), it is $\Delta^c_{peak} < \Delta_{peak}$, so financial constraints shift the peak of the curve to lower levels of competition.

Proposition 6. *In the constrained Markov equilibrium, the average R&D intensity I follows an inverted-U pattern with peak Δ^c_{peak} which is decreasing in B. If $B > \hat{B}$, then $\Delta^c_{peak} < \Delta_{peak}$. Otherwise, $\Delta^c_{peak} = \Delta_{peak}$.*

The shift of the peak can be easily illustrated using the constraint (4). In the first best, as competition increases the left hand side increases. How fast it increases depends on δB. The higher the intensity of financial constraints B is, the faster the constraint will become binding (so it will become binding at lower values of Δ) and the negative "ability-effect" will prevail. So, high B corresponds to the case that the "ability-effect" dominates earlier and therefore the peak shifts to lower values of Δ.

5 Venture Capital Financing-Summary (Under Construction)

While the financial contract as defined above refers to debt financing the study can be extended to capture the option of venture capital financing. In this case, the lender becomes active in the management of the firm and is in charge in selecting its level of $R\&D$ expenditure n_m. The proceeds from the investment are appropriated by the venture capitalist who leave a payment to the entrepreneur so that he behaves diligently by exerting high effort in the project. Results (need to be typed) show that venture capital financing reduces the negative impact ("ability-effect") of competition on innovation as credit constraints are not so important anymore. The impact of competition again depends on the moral hazard concerns which now affect the payment by the venture capitalist to the entrepreneur when the investment is successful. The intensity of moral hazard concerns determines to what extent the peak of the competition-innovation curve shifts to the left.

6 Empirical Part (Under Construction)

Parallel to the completion of the theoretical model, I am currently working on testing empirically its predictions. In the following lines I provide some information on the data set I am using for testing my theoretical predictions. The derivation of results is in progress and not included in this version of the paper.

6.1 Data

The data comes from Fiben, a large French firm-level panel database constructed by the Bank of France. Fiben is based on fiscal documents, including balance sheet and $P\&L$ statement, and thus contains detailed information on both, flow and stock accounting variables. A subsample of Fiben, called Centrale des Bilans, is available for a lower number of firms and includes additional information directly collected by the Bank of France. The time unit in this database is the year.

The FiBen database includes all French firms which sales at least equal to 75,000 euros or with credit outstanding of at least 38,000 euros; annual accounting data is then available for about 200,000 firms. In 2004, Fiben covered 80% of the firms with 20 to 500 employees, and 98% of those employing more than 500 employees[18].

We then restrict our sample by looking only at firms that have at least one year a positive $R\&D$ investment; our sample is unbalanced and includes about 16,699 firms over the period 1994-2004. A same firm appears in our database during a seven year period on average.

[18] More than 50% of the firms in Fiben have less than 20 employees. However, these firms are under-represented in Fiben since their sales rarely exceed the required amount.

The main $R\&D$ variable used here is the $R\&D$ investment. $R\&D$ investment is defined as the fraction of $R\&D$ expenditures that the firms are allowed to capitalize.

The main competition measure is the Lerner index. A Lerner index for each firm can be built using these data. We only observe sectoral price provided by the INSEE, but we have detailed information on costs. The Lerner index is supposed to measure the market power of the firm by the difference between price and marginal costs (which equals the negative inverse of demand elasticity). Since neither price nor marginal costs are available at the firm level, for each firm i and year t, we compute the index based of the following formula[19]:

$$Lerner_{i,t} = \frac{Operating \quad profits_{i,t} - Cost \quad of \quad capital_{i,t}}{Sales_{i,t}}$$

(see Aghion et al., 2005). The Fiben database contains very detailed balance sheet information that enables to compute these Lerner indicators. The Lerner index by definition takes values between 0 and 1. Our competition measure is one minus the average of this across each firm i within the sector s for each time t:

$$comp_{s,t} = 1 - \frac{1}{N_{s,t}} \sum_{i \in s} Lerner_{i,t} \tag{9}$$

A value of 1 indicates perfect competition (price equals marginal cost) while values below 1 indicate some degree of market power.

As a starting measure of financial constraints I consider firms' annual payment incidents, namely the cases that firms do not manage to repay their financial obligations. Payment incidents collected by Banque de France and are communicated among all the financial intermediaries in the French market. Firms that have payment incidents at a particular year do not have access to external financing for the two subsequent years.

Empirical investigation of the theoretical predictions required the estimation of the impact of the interaction between product market competition and financial constraints on the $R\&D$ investments. If first and second order interactions terms have a clear impact on $R\&D$ investments, then, we can establish an additional channel through which product market competition affects innovation.

7 Discussion

Financial constraints can have a real impact on the relationship between competition and innovation. In particular depending on their intensity they can shift the peak of the relationship to lower levels of competition which means that the aggregate innovation intensity is decreasing for wider range of levels of competition. Some policy suggestions can be drawn from this result. First of all, product market reforms that result an increase in competition should be accompanied by credit reforms when competition distorts $R\&D$ investments taken by firms.

[19]The cost of capital is assumed to be 0.085 for all firms and time periods and the capital stock is measured using the "Mairesse" method.

Second, at times that financial constraints are expected to be more prevalent (e.g. economic downturns), the negative "ability effect" of competition is more likely to dominates. This suggests that priority for the policy institutions should be to make sure that firms can secure financing before they decise to increase market competition.

Different forms of financing can have a different impact on the inverted-U relationship, depending on the agency costs that are linked to. Venture capital financing is likely to reduce the private benefit of shirking as the venture capitalist has an active role in the management of the firm. Therefore, in the language of this model, firms that are financed in this way are expected to have low B and therefore the shift in the curve to the right should be so large.

Industrial policy in terms of tax incentives and tax cuts for firms that invest in $R\&D$ can reduce the ability constraint. In the context of innovation industrial policy should be perceived as complement of competition policy so that the financial constraints of the innovative firms may be relaxed.

Throughout this paper I assumed that the measure of competition is exogenous in order to establish a clear casual relationship between product market competition and financial constraints. This can be implicitly satisfied if firms collude tacitly and not explicit. Explicit collusion may be a choice of firms so that they can escape from their financial constraints and enjoy high profitability which will allow them to invest in $R\&D$ and improve their market position.

The results can be sustained even if we have more than one investors present in the model. Since the financial contract has very general figures of the financial market, the results carry on even if we have many investors in a competitive framework.

The shift in the peak can be observed even in the case of homogeneous products ($\pi_{-1} = 0$). In this case, the follower have to rely on external financing for all possible levels of competition Δ. Therefore, in the constrained equilibrium we have to discriminate only between two regions, the low competition region in which (4) is not binding and the high competition region in which (4) becomes binding the the negative "ability-effect" dominates.

Because I wanted to focus on a stationary equilibrium, I imposed the assumption that firms can only pledge in their innovation project their current profit flow. This is clearly a simplification. Firms in a more realistic framework can save and aggregate their tangible assets as time passes. Intuitively, an increase in product market competition has a negative impact in the ability of firms to innovate. There is not any reason this effect not to apply in the savings of firms. Therefore, if we replace the profit flow of firms with any function of profitability we expect that the impact of product market competition will be the similar.

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A The Measure of Product Market Competition

At first sight it may sound strange that competition affects only the neck-and-neck state but not the unleveled one. This simplification is innocent without any effect to our results below. To see this, consider that competition (degree of collusion) affects the profit in both types of industries (neck-and-neck and unleveled). Specifically, adopt the alternative specification with π_0^C to be the equilibrium profit of a firm in the neck-and-neck state and π_1^C, π_{-1}^C the equilibrium profits of the leader and the follower in the unleveled state, respectively. Then, the following definitions apply:

$$\begin{aligned} \pi_0^C &= (1-\Delta)\pi_1 \\ \pi_1^C &= \pi_1 + (1-\Delta)\pi_{-1} \\ \pi_{-1}^C &= (1-\Delta)\pi_{-1} \end{aligned} \tag{10}$$

where, $\pi_1 > \pi_{-1} > 0$. These profit flows depend on the technology gap m and on the level of competition within each industry. In the absence of collusion ($\Delta = 1$), firms compete à la Bertrand. In the neck-and-neck state, firms earn zero profit since they have the same marginal cost of production, while in the unleveled state, the follower makes zero profit and the leader earns π_1 by setting its price equal to the marginal cost of the follower. Collusion allows the firms to increase their equilibrium profits above their Bertrand levels. In the case of maximal collusion ($\Delta = \frac{1}{2}$) each neck-and-neck firm earns $\frac{1}{2}\pi_1$ while the total industry profit is π_1. For the unleveled state, maximal collusion allows the follower to have positive profit $\frac{\pi_{-1}}{2}$ while the leader's profit increases to $\pi_1 + \frac{\pi_{-1}}{2}$ and the total industry profit becomes $\pi_1 + \pi_{-1}$. In both types of industries we assume that the extra profit flows generated by collusion are equally shared between firms. For the unleveled state, firms collude under side payments. The leader carries out all the production and sells at price that is higher than the marginal cost of production of the follower. As a return, follower gets a share over the extra generated profits from collusion. Namely, firms bargain how to share the extra profit flow with equal bargaining power and considering as a threat point the Bertrand competition outcome. The consideration of other collusive technologies such as market sharing or market division while affects the equilibrium profit flows in the unleveled state, does not change the way in which competition affects profit flows and therefore does not change the predictions and implications of the model. Our Nash bargaining problem can be defined as:

$$\max_{\pi_1^C, \pi_{-1}^C} \{(\pi_1^C - \pi_1)\pi_{-1}^C\}$$

$$s.t. \pi_1^C \geq \pi_1$$

$$\pi_{-1}^C \geq 0$$

$$\pi_1^C + \pi_{-1}^C = \pi_1 + \pi_{-1}$$

where π_{-1} is the total industry extra benefit from maximal collusion. This alternative specification of competition satisfies the Boone (2008) criterion for a good measure of competition. What it matters are not the exact profit flows themselves, but the relative effect of Δ on neck-and-neck state profit with respect to its effect on the unleveled state profit. Neck-and-neck state profit moves further from the leader's profit and closer to the follower's profit as Δ increases. Since, this effect by our simple product market competition framework above, there is no need to introduce competition in the unleveled state.

B Proof of Proposition 1

To solve for the equilibrium $R\&D$ intensities we use Bellman equations. Let V_m denote the firm's steady state value of being currently in the state with technology gap m with its competitor. If δ is the discount factor, then the Bellman equations can be written as following:

$$\begin{aligned} V_0 &= \max_{n_0}\left\{\pi_0 - \frac{n_0^2}{2} + \delta[n_0 V_1 + \overline{n}_0 V_{-1} + (1 - n_0 - \overline{n}_0)V_0]\right\} \\ V_{-1} &= \max_{n_{-1}}\left\{\pi_{-1} - \frac{n_{-1}^2}{2} + \delta[(n_{-1}+h)V_0 + (1 - n_{-1} - h)V_{-1}]\right\} \\ V_1 &= \pi_1 + \delta[(n_{-1}+h)V_0 + (1 - n_{-1} - h)V_1] \end{aligned}$$

or

$$\begin{aligned} (1-\delta)V_0 &= \max_{n_0}\left\{\pi_0 - \frac{n_0^2}{2} + \delta[n_0(V_1 - V_0) - \overline{n}_0(V_0 - V_{-1})]\right\} \\ (1-\delta)V_{-1} &= \max_{n_{-1}}\left\{\pi_{-1} - \frac{n_{-1}^2}{2} + \delta[(n_{-1}+h)(V_0 - V_{-1})]\right\} \\ (1-\delta)V_1 &= \pi_1 - \delta[(n_{-1}+h)(V_1 - V_0)] \end{aligned}$$

Solving the first two maximization problems we derive:

$$\begin{aligned} n_0^* &= \delta(V_1 - V_0) \\ n_{-1}^* &= \delta(V_0 - V_{-1}) \end{aligned}$$

where, $\overline{n}_0$ is the $R\&D$ intensity of the competitior. Using the fact that in equilibrium and due to symmetry $n_0 = \overline{n}_0$, we can derive the reduced form equations:

$$\begin{aligned} \frac{(n_0^*)^2}{2} + n_0^*\left(\frac{1-\delta}{\delta} + h\right) - (\pi_1 - \pi_0) &= 0 \\ \frac{(n_{-1}^*)^2}{2} - \frac{(n_0^*)^2}{2} + n_{-1}^*\left(\frac{1-\delta}{\delta} + h + n_0^*\right) - (\pi_0 - \pi_{-1}) &= 0 \end{aligned}$$

Solving this system of equations we conclude to (6) and (7).

C Proof of Proposition 2

For low level of competition, the profitability of the neck-and-neck state is closer to the leader's profit flow and far away from the follower's one. Therefore, n_{-1}^* is relatively high and the follower needs to rely on external financing since, $\frac{(n_{-1}^*)^2}{2} > \pi_{-1}$. Let this be true for $\Delta \in [\frac{1}{2}, \overline{\Delta})$, where $\overline{\Delta}$ is the level of competition for which $\frac{(n_{-1}^*)^2}{2} = \pi_{-1}$. For $\Delta \in (\overline{\Delta}, 1]$

firm is able to finance its $R\&D$ investment with its internal funds and borrowing does not take place. For $\Delta \in [\frac{1}{2}, \overline{\Delta})$ the maximization problem for the follower is

$$\begin{aligned}(1-\delta)V_{-1} &= \max_{n_{-1}} \{\delta(n_{-1}+h)B\} \\ s.t. & \quad (5)\end{aligned}$$

Clearly, in equilibrium (5) is binding.

For high level of competition, the profit flow in the neck-and-neck state is far away from the leader's profitability. Therefore, neck-and-neck firms need to invest a lot to reach the first best level of $R\&D$ investment. So, they need to rely on external financing since their profitability is low. There is a threshold level of competition $\hat{\Delta}$ (which is given by solving $\frac{(n_0^*)^2}{2} = \pi_0$) above which neck-and-neck firms need to borrow in order to innovate. For $\Delta \in (\hat{\Delta}, 1]$ neck-and-neck firms need to borrow and their maximization problem is:

$$\begin{aligned}(1-\delta)V_0 &= \max_{n_0} \{\delta n_0 B - \overline{n}_0(V_0 - V_{-1})\} \\ s.t. & \quad (4)\end{aligned}$$

Again, in equilibrium, (4) will be binding.

After some algebra, it can be proven that for large innovation steps similar to the ones considered by Aghion et. al. (2005), we have $\hat{\Delta} < \overline{\Delta}$. Hence, there is an intermediate region such that firms in both states are financially constrained, $\Delta \in (\hat{\Delta}, \overline{\Delta})$.

So, for different levels of competition we can distinquish three different regions:

- Low competition region, $\Delta \in [\frac{1}{2}, \hat{\Delta}]$: The neck-and-neck firms can self finance their $R\&D$ investments but the followers need to borrow. In this case we need to solve the system of equations:

$$\begin{aligned}(1-\delta)V_{-1} &= \max_{n_{-1}} \{\delta(n_{-1}+h)B\} \\ s.t. & \quad (5) \\ (1-\delta)V_0 &= \max_{n_0} \left\{\pi_0 - \frac{n_0^2}{2} + \delta[n_0(V_1 - V_0) - \overline{n}_0(V_0 - V_{-1})]\right\} \\ (1-\delta)V_1 &= \pi_1 - \delta[(n_{-1}+h)(V_1 - V_0)]\end{aligned}$$

  from where we conclude to the first best $R\&D$ intensity for the neck-and-neck sector, n_0^*, and to the constrained (second best) n_{-1}^c which is decreasing in Δ.

- Intermediate competition region, $\Delta \in (\hat{\Delta}, \overline{\Delta})$: In this case the system of equations to be solved is

$$\begin{aligned}(1-\delta)V_{-1} &= \max_{n_{-1}} \{\delta(n_{-1}+h)B\} \\ s.t. & \quad (5) \\ (1-\delta)V_0 &= \max_{n_0} \{\delta n_0 B - \overline{n}_0(V_0 - V_{-1})\} \\ s.t. & \quad (4) \\ (1-\delta)V_1 &= \pi_1 - \delta[(n_{-1}+h)(V_1 - V_0)]\end{aligned}$$

from which we conclude to constrained $R\&D$ intensities n_0^c and n_{-1}^{cc} that are both decreasing in Δ.

- Intense competition region, $\Delta \in [\overline{\Delta}, 1]$: The system to be solved is

$$
\begin{aligned}
(1-\delta)V_0 &= \max_{n_0} \{\delta n_0 B - \overline{n}_0(V_0 - V_{-1})\} \\
s.t. &\quad (4) \\
(1-\delta)V_{-1} &= \max_{n_{-1}} \left\{\pi_{-1} - \frac{n_{-1}^2}{2} + \delta[(n_{-1}+h)(V_0 - V_{-1})]\right\} \\
(1-\delta)V_1 &= \pi_1 - \delta[(n_{-1}+h)(V_1 - V_0)]
\end{aligned}
$$

from which we conclude to n_0^{cc} and n_{-1}^{un} that are both decreasing in Δ