Profit Sharing and firms' strategies

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May 2015

Abstract

Profit sharing, with one form or the other, has been in wide use all over the world. Literature lacks a holistic approach to profit sharing, and has not made the obvious connection between price (of labor) discrimination, vertical (union-firm) relations and profit sharing. We show that profit sharing schemes are mathematically similar with two part tariffs. We set up a four stage game, in order to endogenize both the decision of the union to bargain central coordinated or decentralized, and the decision of the firm to introduce a profit sharing scheme or not. We find that it is always optimal for unions to bargain centrally, and that firms choosing their remuneration scheme, will fall into a prisoner's dilemma.
JEL Classification: **J20, J30, J40, L10, M50**
Keywords: union oligopoly; two-part tariffs; bargaining; profit sharing;

1 Introduction

Profit sharing schemes are a type of remuneration schemes, in which part of a worker's salary is based on the profits made by the firm. Profit sharing schemes can be paid in cash, stocks or other forms, and can be paid annually or be kept by the firm, and be given to workers at the form of a pension. In practise, profit sharing is quite complex, and contains a set of different elements (OECD 1995). Nevertheless, profit sharing, with one form or the other, has been in wide use all over the world (Dhillon and Petrakis 2001). A survey of the largest 1250 global corporations (Weeden et al. 1998) found that 33% of them, offered some short of profit sharing schemes to all employees, and 11% had plans to put in place a broad-based profit sharing scheme. Profit sharing is particularly pronounced in the high-technology sector.

A quick question emerges; why should a firm introduce a profit sharing scheme? An answer could be a paper of Sesil, Kroumova, Blasi et al. (2002), who studied 229 United States major "New Technology" Firms (i.e. pharmaceuticals, semiconductors, software, telecommunications & high-technology manufacturing), all offering broad-based profit sharing plans. Using multivariate analysis with panel data found that, in contrast with their non-profit sharing counterparts, profit sharing firms' productivity increased 4%, total shareholder returns increase by 2%, and profit levels jump by about 14%. These gains are after dilution effect is taken into account.

Arriving at similar results, Kruse (1992), uses data from almost 3000 United States firms (separated only in manufacturing firms and non-manufacturing firms), from 1971 to 1985, and

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shows that the introduction of a profit sharing scheme is (statistically significant) associated with a productivity increase of 2.8% to 3.5% for manufacturing firms, and 2.5% to 4.2% for non-manufacturing firms. In a different country, Germany, Kraft and Ugarkovic (2005), using data from more than 2000 German firms from 1998 to 2002, show that the introduction of a profit sharing system improves profitability.

Then, a new question arises; if this thing is so good, then why not all firms have not already introduce a profit sharing scheme? Literature on the motive of introduction of profit sharing (OECD 1995) suggests that several groups of variables such as e.g. firm size, organizational structure, industrial relations, labor and legal institutions, and the external environment, affects the introduction of profit sharing. Furthermore, Petrakis and Vlassis (2004) have proven that (under the assumptions of their model) technological asymmetries among same sector firms may be one driving force behind the variation in remuneration bargain. This plethora of factors makes the introduction of a profit sharing scheme a difficult matter to assess and account, and thus to introduce. Some researchers (like Kruse 1992) suggest that only the most profitable and most productive firms introduce profit sharing schemes, in order to align firm and workers interests, and through this alignment to reach new, higher levels of profitability and market share.

Weitzman (1984), a pioneer of profit sharing schemes, states that profit sharing makes the cost of labor completely flexible and gives firms the incentive to hire as many workers as are willing to take jobs. According to him, a firm that pays workers a fixed wage plus a share of profits, views workers as comparable to salespeople paid commissions. Since employing more salespeople, should increase total sales, firms that apply profit sharing should want to hire as many salespeople as will accept jobs. Sales and profits will rise even as the increased number of salespeople drives down sales per employee and the earnings of workers. Firms will also have the incentive to hang onto workers if the demand for the firm's output goes down, leading to Weitzman's (1984) prediction that an economy of profit sharing firms will have lower levels of unemployment and greater macroeconomic stability.

Away from all the pros stated above, there has been some common sense based cons, such as: a)a lack of an effective monitoring mechanism can create a free rider problem, in which some employees carry the burden of hard work and drive profits into higher levels, while other shirk, but enjoy the same profit share, b)profit sharing focuses solely into the goal of profitability, superseding other important measures of a firm, c)for smaller companies, or for companies with volatile profits, this can lead workers to a worse off position, comparing to a fixed base wage.

The exact type of the remuneration contract between unions and firms is of great importance not only to labor economics, but also to strategic management. The goal of using the proper remuneration contract in order to align the interests of both workers and the firm, is clearly a managerial decision. Real life remuneration contracts can come to many different forms. While in some industries (e.g. heavy industries workers, or workers in retail industry) the common remuneration scheme is a fixed base wage, in some other industries (e.g. High Tech / Silicon Valley firms) remuneration schemes are more complex, and often include profit sharing, stock sharing, and other kind of bonuses, which are pretty difficult to track and account.

The main objective of this paper is to endogenize two thinks: first, the decision of the unions to bargain individually (decentralized version) or to form a grand coalition of workers (central coordinated version), and second the decision of the firm to give or not to give a profit sharing scheme to its workers. Furthermore, we show that profit sharing schemes are mathematically similar to two part tariffs, i.e. wage discrimination works in the same way as price discrimination. In two part tariff instruments, the two parties, as a first step, via bargaining, use a variable fee

to maximize each others' piece of the pie, and as a second step use a fixed fee to maximize the pie as a whole.

Under the same reasoning, in profit sharing the two parties (employees and employers, or if you prefer firm and union) bargain using the profit share as an instrument of maximizing each others piece of the profits, and later they bargain over the fixed wages in order to maximize the profits as a whole (the right fixed base wage leads to the right cost of production which leads to the right quantity sold). This mathematical similarity creates bridges between the two research fields, which could yield interesting results.

Our model resembles the model of Sorensen (1992). In his paper, he analyzes when it is optimal for firms to introduce a profit sharing scheme. He sets a three stage game, in which two firms producing a single type homogeneous product, and two unions, one for each firm, as a single labor supplier for that firm. In stage one firms decide the remuneration system (fixed base wage versus profit share), in stage two there is a determination of wages and profit share via a Nash bargaining product style negotiation among firms and unions, and in stage three there is a Cournot style competition among firms, which determines output, prices, and employment levels. Sorensen (1992) checks only one case, the decentralized version, in contrast with our paper, in which we check two versions (decentralized and central coordinated versions).

We have added a first ("pre-") stage, in which unions separately but simultaneously decide whether to bargain individually or form coalition. The idea is to explore whether coalitions favor profit sharing or not. We assume that the decision of forming or not alliances is a decision that changes more difficult in time than the decision of changing remuneration schemes, because the decision of remuneration scheme depends on each firm's management, and can be change easily, but the decision of forming a grand coalition is something that needs greater acceptance and delegation, and probably legal support, and as such is something more difficult to undo.

Looking further in literature, we have to mention the research of Takami and Nakamura (2012), who tried to endogenize the order of moves. In their paper, they use a classical duopoly model, with firms compete in quantities, and the monopoly union assumption, i.e. unions determine wages while firms determine output and employment. Between the two firms, one firm applies profit sharing scheme, while the other does not apply profit sharing scheme, and its remuneration scheme consists of a fixed base wage. Their model consists of three stages; in stage 1 the two unions of the firms decide whether to move simultaneously (to determine wages) or in a Stackelberg way (leader-follower). In stage 2, the two unions have seen the outcome of stage 1, and they maximize their remuneration scheme. In stage 3 the two firms compete Cournot-style over quantities, determining output (and employment $q_i = L_i$). They end up with the proposition that the only Stackelberg equilibrium that emerges in stage 1 (and affects the other two stages) is one union to move first and the other second. It is not an equilibrium to move simultaneously, and it is beneficial for the union that receives more than 75% of firm's profits as a profit share (something quite unusual!), while for profit shares less than 75% it is Stackelberg equilibrium to move first the union with the fixed base wage.

Goeddeke (2010) analyzes the emergence of profit sharing schemes, when wages are negotiated in a (fully) centralized or decentralized way. She is based upon Sorensen (1992) model, but it extends it by using n-opoly structure (in stead of duopoly), and (fully) centralized bargains. She founds that in n-opoly, under decentralized bargains, firms have incentives to replace fixed base wage schemes with profit sharing schemes. But when the majority of firms (depends on magnitude of n in n-opoly) adopts profit sharing schemes, it is in the collective interest of both firms and unions to move back to fixed base wages. Also, she shows that the existence of profit

sharing scheme creates incentives for firms to bargain independently, and not centrally.

Negotiations between firm and union over the profit share are not common in all industrial countries. Poole (1988), surveys British firms, and shows that the decision about the level (percentage) of profit shares made by the managers in 97.7% of the cases studied, while only 0.7% was made under bargain. This assumption is behind the research made by Pemberton (1991); he assumes that the very existence and extent of profit sharing cannot be a process of bargain between firm and union, but in real world is part of firm's profit maximization strategy. He proves that in the absence of product subsidies, profit sharing schemes will not be profitable for the firm, but if firms' facing heterogenous demand, and have different technologies, there are incentives for firms to apply a profit share. However, in France, since 1967, there is a legal obligation for firms employing more than 50 workers, to apply some short of profit share, calculated on a basis of predetermined profit sharing formula. In 2011 this legal obligation was reinforces and became wider, requiring firms to also pay a "social" dividend, if their net profits were higher than last year.

Modeling the union-firm relation as a bilateral monopoly, in which union is the upstream supplier of labor (as a production input) to a downstream firm is not a new one. We can trace it even before Horn and Wolinsky (1988), who stated:

> Perhaps the most familiar example is that of a firm with a unionized labor force. In this case, although both the firm and the labor union may have some substitution possibilities, considerations such as investments in specific human capital ,hiring costs, and legal barriers may lock them into bilateral monopoly relations.

The rest of the paper is as follows: in the second section, we describe the stages of the game, in the extensive third section we solve the game for all of its stages using backwards induction, and we close this research with concluding results and references.

2 The Game

2.1 Game outline

Before we begin with the presentation of the game, we must state that game timing reflects the idea that long run decisions, such as the forming of grand coalitions of workers or the forming of grand confederation of employers, may have considerable strategic effects in short run decisions, such as the employment decision made by a firm. This timing captures the idea that some variables (such as employment) are easier to change and are greatly affected by other variables, which may be much more difficult to alter.

This timing is standard in literature, and allows us to capture the contract forms' commitment value (see e.g. Milliou and Petrakis 2007). We are viewing the relationship between a firm and its union, as a relationship between a downstream buyer of labor, and an upstream supplier of labor.

2.1.1 Stage 1

The merger stage. Using terminology from vertical related markets literature, it is the stage where upstream "labor unions" decide whether to merge horizontally or not. We use these two different types of bargain (centralized and decentralized), because we encounter them both in real businesses. In fact, labor market institutions show high variability as of the level of wage negotiations. In United States, United Kingdom, Australia, Canada, and Japan, wages are usually bargained on a decentralized firm level between unions and firms. In contrast, in Germany, Austria, Belgium, Greece and Scandinavian countries, bargain take place either on an sector-wide level or even at a national level (Petrakis and Vlassis 2004, and Goeddeke 2010).

In a macroeconomic view, Jackman, Pissarides and Savouri (1990) shows that the degree of wage bargaining centralization clearly affects long-run unemployment and inflation rates. So, it is not only microeconomic reasons to impose a research over the endogenous emergence of centralized or decentralized bargains, but also macroeconomic reasons. There are two possible outcomes; a)both unions bargain in an "one-to-one" decentralized form with their respective firm (decentralized version), and b)unions form a central (sector-wide) coalition of workers, and bargain with both firms (central coordinated version).

2.1.2 Stage 2

The contract type stage. The downstream "firm" decides whether to offer a fixed base wage as a "wholesale" price of labor, or a two part tariff scheme made from a fixed base wage plus a profit share. This is a take it or leave it offer form the firm to the union. In case of deciding not to implement a profit sharing scheme, they bargain (in stage 3) with their union over a fixed base wage w_i. In case of deciding to implement a profit sharing scheme, they bargain (in stage 3) with their union over both a fixed base wage w_i and a profit sharing scheme a_i, as a percentage over firm's i net profits. There are four possible outcomes:

a)both firms implement a profit sharing scheme,

b)both firms implement fixed base wage schemes, and

c) & d)a double symmetrical outcome (due to the assumptions of our model) were one firm applies a profit sharing scheme, while the other applies a fixed base wage scheme.

The big question for each firm's management is how to best react by applying (or not) a profit sharing scheme, under different actions taken by its direct rival firm. The matter of a firm providing a single take it or leave it offer to the union, or a menu of offers is not fully explored. As Peters (2003) proves:

> "Complex negotiations can result in interesting new behavior in common agency when there are many principals [...] With symmetric information, pure strategy equilibria in take it or leave it contracts are robust against the possibility that principals might offer far more complicated communication schemes. The role of communication is then to support new kinds of equilibrium outcomes."

We can apply his work in our case as follows; under symmetric information during negotiations between the firm and the union, the existence of a single offer or a menu of offers does not move

or alters all equilibria; a single take it or leave it offer creates some equilibria, while a menu of choices just adds some more equilibria, always under a pure strategy assumption (all workers have the same remuneration scheme).

2.1.3 Stage 3

The bargaining stage. In stage 3 unions (or grand union) and firms bargain over the remuneration scheme. The main goal of solving this stage is to find, under all possible remuneration schemes and under all possible forms of centralized/decentralized bargain, Nash equilibria values of wages w_i^* and of profit sharing (if applicable) a_i^* for $i = 1, 2$, as functions of exogenous variables only. Exogenous variables, in our model are non labor cost c, inverse demand function parameters α, β, unemployment benefit w_0 and bargain powers b, d. Furthermore, profits π_i^* and quantities sold q_i^* as has been found in previous stage, are now "double" Nash equilibria values π_i^{**}, q_i^{**} because we have substitute labor cost w_i with the wage w_i^* as it came up from Nash bargain product maximization.

2.1.4 Stage 4

The competition stage.

In the spirit of Singh and Vives (1984) and Sorensen (1992), we solve a duopoly, with two firms (having one union each) competing on quantities (Cournot-style). We assume a two-factor Leontief technology, were firms produce under constant returns to scale, and the amount of capital K_i is fixed in the short run, and large enough not to induce zero marginal product of labor L_i.

$$q_i = \min\{K_i, L_i\} \Rightarrow q_i = L_i \ , \quad i = 1, 2 \tag{1}$$

Each firm produces a single type of a homogeneous good. The inverse demand function has the form:

$$P = \alpha - \beta * (q_i + q_j) = \alpha - \beta * Q, \quad \text{where:} Q = q_i + q_j \tag{2}$$

Q is the total market quantity sold by firms, and q_i, q_j are the quantities sold by each firm respectively. Because of the assumption $q_i = L_i$, , Q symbolizes total employment$L$. We assume a constant marginal cost per product, being the sum of a (non-labor) cost c per quantity q_i produced, and a labor cost (wage rate) w_i per unit of labor L_i. Firms assumed to have no other costs or income, so they have a (*gross* [1]) profit function equal to:

$$\pi_i = P * q_i - w_i * L_i - c * q_i = (P - w_i - c) * q_i \ , \quad i = 1, 2 \ \& \ i \neq j \tag{3}$$

[1] To avoid any misinterpretation, please note that as gross profits we mark profits before profit share (if applicable), and as net profits we mark profits after profits share (if applicable). In case of using a fixed base wage as remuneration scheme, then gross and net profits are the same. This assumption holds for the rest of the paper.

Firm's decision makers are assumed to be risk-neutral, thus their (*net*) profits, which is their utility function, can be modelled by the following:

with profit share a_i[2] (notice the superscript P)

$$V_i^P = (1 - a_i) * \pi_i, \quad i = 1, 2 \tag{4}$$

without profit share (notice the superscript W)

$$V_i^W = \pi_i, \quad i = 1, 2 \tag{5}$$

Union is also assumed to be risk-neutral, thus their utility function can be modeled by the following:

$$\begin{aligned} &\text{with profit share } a_i \\ U_i^P =&((w_i - w_0) + \frac{a_i * \pi_i}{L_i}) * L_i = (w_i - w_0) * q_i + a_i * \pi_i \ , \qquad i = 1, 2 \\ &\text{without profit share} \\ U_i^W =&(w_i - w_0) * L_i = (w_i - w_0) * q_i \ , \qquad i = 1, 2 \end{aligned} \tag{6}$$

where w_i is the fixed base wage, as it endogenously emerges from union-firm bargains (see stage 3), and $w_0 > 0$ is the (exogenous, and assumed constant) unemployment benefit. We use the specific utility function based on Pemberton (1988), who shows that a rent-maximising union is equivalent to a 'managerial union' with union leaders who are interested in employment, and union members (modeled by the median worker) who are interested in excess wages. This stage includes the so called manager's right to manage. The set up of our game is based on the assumption that unions bargain with firms over wages, but the level of employment is a privilege of firm's management.

2.2 Solving the Game

2.2.1 Stage 4

We will solve the game using backwards induction. This stage of the game is common to all versions of bargain (centralized or decentralized). It shows the Cournot-style competition in product market between the two firms, despite the case of forming (or not forming) a grand confederation of employers in order to bargain centrally with their unions. In this stage, wages are treated as exogenous constants, because they are brought here (as Nash equilibria variables) from previous stages of the game, in which they are treated as endogenous variables.

The reasoning behind this stage is that in Cournot-style competition, firm i maximizes its profit function, with respect to product quantity $q_i = L_i \ , \ i = 1, 2$. From the first order condition we get:

[2]Note that profit share a_i is a percentage, from 0% to 100%, not to be confused with the Greek letter α used in inverse demand function, which is a positive constant (>0).

$$
\begin{aligned}
q_i^* = & L_i^* = \frac{1}{3*\beta} * (\alpha - c + w_j - 2 * w_i), \qquad j \neq i = 1,2 \\
\pi_i^* = & \frac{1}{9*\beta} * (\alpha - c + w_j - 2 * w_i)^2 = \beta * (q_i^*)^2, \quad j \neq i = 1,2
\end{aligned} \tag{7}
$$

In centralized (coordinated) version of the game, in stage 3, there is an outside option for the grand coalition of workers to block the access to labor force to one firm, making the other firm a monopolist in stage 4. Under this possibility, it is helpful to mention quantities and profits under this monopoly option:

$$
\begin{aligned}
q_{iMONOPOLIST}^* = & L_{iMONOPOLIST}^* = \frac{1}{2\beta} * (\alpha - c - w_i) \ , \qquad i = 1,2 \\
\pi_{iMONOPOLIST}^* = & \frac{1}{4*\beta} * (\alpha - c - w_i)^2 = \beta * (q_{iMONOPOLIST}^*)^2 \ , \qquad i = 1,2
\end{aligned} \tag{8}
$$

Mention that the other firm has exit the sector, having: $q_j = L_j = \pi_j = 0, \ \ j \neq i$.

2.2.2 Stage 3

Decentralized Version *Two individual firms versus two individual unions.*

In this version, unions have decided to bargain in a decentralized fashion (stage 1), i.e. individually and independently one from the other. So, we are facing two couples of firm and its respective union, bargaining simultaneously for their own remuneration scheme.

► **Both firms use fixed base wages.** We suppose that both firms (in stage 2) decide to offer a "take it or leave it" offer of only a fixed base wage to their unions. In this case, Nash bargain products are:

$$
\begin{aligned}
NBP_1^{DW} = & (V_1^W)^b * (U_1^W)^{1-b} = (\pi_1)^b * [(w_1 - w_0) * q_1]^{1-b} \\
NBP_2^{DW} = & (V_2^W)^d * (U_2^W)^{1-d} = (\pi_2)^d * [(w_2 - w_0) * q_2]^{1-d}
\end{aligned} \tag{9}
$$

where DW symbolizes **D**ecentralized bargain for the union (decided at stage 1), and fixed base **W**age schemes for both firms applied (decided at stage 2).

The power $b \in (0,1)$ used as a superscript, is the bargaining power of the firm, thus $1-b$ is the bargaining power of the union. We use a different power $d \in (0,1)$ for the second firm, so in general $b \neq d$, but we ease this assumption later.

Maximizing Nash bargain products over fixed base wages w_i, and solving the system of equations, we get:

$$
\begin{aligned}
w_1^* = & \frac{(c-\alpha) * (b-1) * (d-5) + 2 * w_0 * (-5 - 3 * b + d * (b-1))}{-15 + b * (d-1) - d} \\
w_2^* = & \frac{(c-\alpha) * (d-1) * (b-5) + 2 * w_0 * (-5 - 3 * d + b * (d-1))}{-15 + b * (d-1) - d}
\end{aligned} \tag{10}
$$

which can be easily seen that are symmetrical (not equal) as of bargaining powers $b, d \in (0,1)$ [3].

Price, Total quantity sold (which is equal to total employment), Nash quantities per firm (which are equal to employment per firm), (net) profits per firm, total consumer surplus and unions' welfare are:

$$\begin{aligned}
q_1^{**} =& L_1^* = \frac{2*(1+b)*(d-5)*(c+w_0-\alpha)}{3*\beta*(15+b+d-b*d)} \\
q_2^{**} =& L_2^* = \frac{2*(1+d)*(b-5)*(c+w_0-\alpha)}{3*\beta*(15+b+d-b*d)} \\
\pi_1^{**} =& \frac{4*(d-5)^2*(b+1)^2*(c+w_0-\alpha)^2}{9*\beta*(15+b+d-b*d)^2} \\
\pi_2^{**} =& \frac{4*(b-5)^2*(d+1)^2*(c+w_0-\alpha)^2}{9*\beta*(15+b+d-b*d)^2} \\
Price:\ P^* =& \frac{4*(-5+b*(d-2)-2*d)*(c+w_0)-(b-5)*(d-5)*\alpha}{3*b*(d-1)-3(15+d)} \\
Quantity:\ Q^* =& L^* = \frac{4*(-5+b*(d-2)-2*d)*(c+w_0-\alpha)}{3*\beta*(15+b+d-b*d)} \\
Consumer\ Surplus:\ CS =& \frac{8*(-5+b*(d-2)-2*d)^2*(c+w_0-\alpha)^2}{9*\beta^2*(15+b+d-b*d)^2} \\
Union\ 1\ Welfare:\ UW_1 =& \frac{2*(1-b)*(1+b)*(d-5)^2*(c+w_0-\alpha)^2}{3*\beta*(15+b+d-b*d)^2} \\
Union\ 2\ Welfare:\ UW_2 =& \frac{2*(1-d)*(1+d)*(b-5)^2*(c+w_0-\alpha)^2}{3*\beta*(15+b+d-b*d)^2}
\end{aligned} \tag{11}$$

► **Both firms use profit sharing schemes.** In stage 1 firms and unions have decided to bargain separately and independently (decentralized version). In stage 2, both firms have decided to introduce profit sharing schemes. Both negotiations are simultaneous and observable. In this case, in stage 3, Nash bargain products will have the form:

$$\begin{aligned}
NBP_1^{DP} =& (V_1^P)^b * (U_1^P)^{1-b} = ((1-a_1)*\pi_1^*)^b * [(w_1-w_0)*q_1^* + a_1*\pi_1^*]^{1-b} \\
NBP_2^{DP} =& (V_2^P)^d * (U_2^P)^{1-d} = ((1-a_2)*\pi_2^*)^d * [(w_2-w_0)*q_2^* + a_2*\pi_2^*]^{1-d}
\end{aligned} \tag{12}$$

where DP symbolizes **D**ecentralized bargain for the union (decided at stage 1), and **P**rofit sharing schemes for both firms applied (decided at stage 2).

Maximizing Nash bargain product NBP_i^{DP} with respect to a_i, we get:

$$\begin{cases} \frac{\partial NBP_1^{DP}}{\partial a_1} = 0 \\ \frac{\partial NBP_2^{DP}}{\partial a_2} = 0 \end{cases} \Rightarrow \begin{cases} a_1^* = 1-b+\frac{3*b*(w_1-w_0)}{(c-\alpha+2*w_1-w_2)} \\ a_2^* = 1-d+\frac{3*d*(w_2-w_0)}{(c-\alpha+2*w_2-w_1)} \end{cases} \tag{13}$$

[3] In general: $c+w_0 < \alpha \Rightarrow c+w_0-\alpha < 0$

Writing down Excess Joint Profits as the sum of profits made by the firm and the union we get:

$$\begin{aligned} ExcessJointProfits_1 &= V_1^P + U_1^P = (w_1 - w_0) * q_1^* + \pi_1^* \\ ExcessJointProfits_2 &= V_2^P + U_2^P = (w_2 - w_0) * q_2^* + \pi_2^* \end{aligned} \tag{14}$$

and replacing a_i^* at NBP_i^{DP} we get:

$$\begin{aligned} NBP_1^{DP}|_{a_1=a_1^*} =& NBP_1^{DP*} = \\ =& \frac{(\frac{1}{b}-1)^{-b} * (1-b)}{9 * \beta} * (c + 3 * w_0 - w_1 - w_2 - \alpha) * (c + 2 * w_1 - w_2 - \alpha) \\ NBP_2^{DP}|_{a_2=a_2^*} =& NBP_2^{DP*} = \\ =& \frac{(\frac{1}{d}-1)^{-d} * (1-d)}{9 * \beta} * (c + 3 * w_0 - w_1 - w_2 - \alpha) * (c + 2 * w_2 - w_1 - \alpha) \end{aligned} \tag{15}$$

if we divide Excess Joint Profits with maximized Nash bargain product with respect to profit share we find that they are analogous:

$$\frac{ExcessJointProfits_1}{NBP_1^{DP*}} = \frac{\left(\frac{1}{b}-1\right)^b}{1-b} \quad , \quad \frac{ExcessJointProfits_2}{NBP_2^{DP*}} = \frac{\left(\frac{1}{d}-1\right)^d}{1-d} \tag{16}$$

so, profit share works (mathematically) just like a two-part tariff contract. It is well known and proven (O'Brien and Shaffer 1992) that in Nash bargaining games with transfer payments, the Nash bargaining solution can be found in two steps; First, the two bargain sides choose marginal transfer prices (in our case profit share a_i) in order to maximize the surplus to be divided, and second they negotiate over fixed fees (in our case fixed base wages w_i) in order to optimally transfer this surplus among the two bargain sides.

Proposition 2.1. *The functional form of production function, profits and demand does not alter the mathematical equivalence of two part tariffs and profit sharing.*

Proof. Let us assume that quantity sold equals labor $q = q(L)$ and that the profits of the firm during stage 4 are equal to π. We do not assume any particular competition style (like Cournot or Bertrand). Then, the utility function of the union is: $U = (w - w_0) * L + a * \pi$ and the utility function of the firm is: $V = (1 - a) * \pi$. Nash bargain product will be:

$$NBP = V^b * U^{1-b} = [(1-a) * \pi]^b * [(w - w_0) * L + a * \pi]^{1-b} \ , \ b \in (0, 1)$$

equals bargain power. Maximizing Nash bargain product over profit sharing will get:

$$\frac{\partial NBP}{\partial a} = 0 \Rightarrow a^* = (1-b) + b * \frac{L}{\pi} * (w - w_0).$$

Substituting this Nash equilibrium profit share a^* back to Nash bargain product we have:

$$NBP|_{a=a^*} = NBP^* = (-1+b)^{1-b} * b^b * (\pi + (w - w_0) * L).$$

Excess Joint Profits of the firm are equal to the sum of utilities:

$$ExcessJointProfits = U + V = (w - w_0) * L + a * \pi + (1 - a) * \pi = \pi + (w - w_0) * L,$$

so: $$\frac{ExcessJointProfits}{NBP^*} = \frac{(\frac{1}{b} - 1)^b}{1 - b}$$ □

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Using the methodology described above, we will get the Nash equilibrium fixed base wage w_i^* directly from NBP_i^{DP*} using first order conditions:

$$w_1^* = \frac{1}{5} * (c + 6 * w_0 - \alpha) \quad , \quad w_2^* = \frac{1}{5} * (c + 6 * w_0 - \alpha) \tag{17}$$

so, by substituting w_i^* back to profit shares:

$$a_1^* = 1 - \frac{b}{2} \quad , \quad a_2^* = 1 - \frac{d}{2} \tag{18}$$

.

In this case, Nash quantities (which are equal to labor units), Nash profits, Nash price, Nash total quantity (which is equal to total employment), union welfare, and Nash Consumer Surplus are:

$$\begin{aligned}
q_1^{**} = L_1^* = q_2^{**} = L_2^* &= \frac{2}{5 * \beta} * (\alpha - c - w_0) \\
Gross\ Profits: \ \pi_1^{**} = \pi_2^{**} &= \frac{4}{25 * \beta} * (\alpha - c - w_0)^2 \\
Net\ Profits_1: \ (1 - a_1^*) * \pi_1^{**} &= \frac{2 * b}{25 * \beta} * (\alpha - c - w_0)^2 \\
Net\ Profits_2: \ (1 - a_2^*) * \pi_2^{**} &= \frac{2 * d}{25 * \beta} * (\alpha - c - w_0)^2 \\
Price: \ P^* &= \frac{1}{5} * (4 * c + 4 * w_0 + \alpha) \\
Quantity: \ Q^* = L^* &= \frac{4}{5 * \beta} * (\alpha - c - w_0) \\
Consumer\ Surplus: \ CS &= \frac{8}{25 * \beta^2} * (\alpha - c - w_0)^2 \\
Union\ 1\ Welfare: \ UW_1 &= \frac{(2 - b) * b - 2}{25 * \beta} * (c + w_0 - \alpha)^2 \\
Union\ 2\ Welfare: \ UW_2 &= \frac{(2 - d) * d - 2}{25 * \beta} * (c + w_0 - \alpha)^2
\end{aligned} \tag{19}$$

▶ **One firm uses profit sharing scheme, and the other firm uses only fixed base wage.** Without any loss of generality, we can assume that it is firm 1 that uses profit sharing scheme, and it is firm 2 that uses fixed base wage. Nash bargain products will be:

$$\begin{aligned}
NBP_1^{DM} &= (V_1^P)^b * (U_1^P)^{1-b} = ((1 - a_1) * \pi_1)^b * [(w_1 - w_0) * q_1 + a_1 * \pi_1]^{1-b} \\
NBP_2^{DM} &= (V_2^W)^d * (U_2^W)^{1-d} = (\pi_2)^d * (w_2 - w_0) * q_2]^{1-d}
\end{aligned} \tag{20}$$

,

where DM symbolizes **D**ecentralized bargain for the union (decided at stage 1), and **M**ixed remuneration schemes for both firms applied (decided at stage 2), i.e. one firm applies profit shares while the other firm applies fixed wage. Maximizing Nash bargain product NBP_i^{DM} with respect to a_i, we get:

$$\begin{aligned} &\frac{\partial NBP_1^{DM}}{\partial a_1} = 0 \Rightarrow \ a_1^* = 1 - b + \frac{3 * b * (w_1 - w_0)}{(c - \alpha + 2 * w_1 - w_2)} \\ &\frac{\partial NBP_2^{DM}}{\partial w_2} = 0 \Rightarrow \ w_2^* = \frac{1}{4} * ((1 - d) * (w_1^* - c + \alpha) + 2 * w_0 * (d + 1)) \end{aligned} \tag{21}$$

Just for firm 1 (which applies profit share), we write down Excess Joint Profits as the sum of profits made by the firm and the union we get:

$$ExcessJointProfits_1 = V_1^P + U_1^P = (w_1 - w_0) * q_1^* + \pi_1^* \tag{22}$$

and replacing a_1^* at NBP_1^{DM}:

$$\begin{aligned} NBP_1^{DM}|_{a_1} = a_1^* = NBP_1^{DM*} = \\ = \frac{(\frac{1}{b} - 1)^{-b} * (1 - b)}{9 * \beta} * (c + 3 * w_0 - w_1 - w_2 - \alpha) * (c + 2 * w_1 - w_2 - \alpha) \end{aligned} \tag{23}$$

if we divide Excess Joint Profits with maximized Nash bargain product with respect to profit share we find that they are analogous:

$$\frac{ExcessJointProfits_1}{NBP_1^{DM*}} = \frac{(\frac{1}{b} - 1)^b}{1 - b} \tag{24}$$

.

Using the same methodology as on previous case:

$$\frac{\partial NBP_1^{DM*}}{\partial w_1} = 0 \Rightarrow w_1^* = \frac{1}{4} * (c + 6 * w_0 - w_2^* - \alpha) \tag{25}$$

.

Using w_2^*, we solve the system, and also we replace Nash wages to profit share a_1^*:

$$\begin{aligned} a_1^* =& 1 - \frac{b}{2} \\ w_1^* =& \frac{1}{d - 17} * ((c - \alpha) * (d - 5) + 2 * w_0 * (d - 11)) \\ w_2^* =& \frac{1}{d - 17} * (3 * (\alpha - c) * (d - 1) - 2 * w_0 * (d + 7)) \end{aligned} \tag{26}$$

.

Notice that only profit share a_1^* is a function of b bargain power of firm 1 (which applies profit sharing scheme), while both wages w_1^*, w_2^* are independent of b and they both depend only on d which is the bargain power of the firm 2 (does not apply profit sharing scheme; it pays with fixed base wage).

In this case, Nash quantities (which are equal to labor units), Nash profits, Nash price, Nash total quantity (which is equal to total employment), union welfare, and Nash Consumer Surplus are:

$$\begin{aligned}
q_1^{**} = L_1^* &= \frac{2*(5-d)}{(d-17)*\beta} * (c - \alpha + w_0) \\
q_2^{**} = L_2^* &= \frac{2*(1+d)}{(d-17)*\beta} * (c - \alpha + w_0) \\
Gross\ Profits_1:\ \pi_1^{**} &= \frac{4*(d-5)^2}{(d-17)^2*\beta} * (c - \alpha + w_0)^2 \\
Net\ Profits_1:\ (1-a_1^*)*\pi_1^{**} &= \frac{2*b*(d-5)^2}{(d-17)^2*\beta} * (c - \alpha + w_0)^2 \\
Gross(=Net)\ Profits_2:\ \pi_2^* &= \frac{4*(d+1)^2}{(d-17)^2*\beta} * (c - \alpha + w_0)^2 \\
Price:\ P^* &= \frac{12*(c+w_0) + (5-d)*\alpha}{17-d} \\
Quantity:\ Q^* = L^* &= \frac{12}{(d-17)*\beta} * (c - \alpha + w_0) \\
Consumer\ Surplus:\ CS &= \frac{72}{(d-17)^2*\beta^2} * (c - \alpha + w_0)^2 \\
Union\ 1\ Welfare:\ UW_1 &= \frac{((2-b)*b-2)*(d-5)^2}{(d-17)^2*\beta} * (c + w_0 - \alpha)^2 \\
Union\ 2\ Welfare:\ UW_2 &= \frac{6*(1-d^2)}{(d-17)^2*\beta} * (c + w_0 - \alpha)^2
\end{aligned} \tag{27}$$

Central (coordinated) version *Two individual firms versus a grand coalition of workers.*

In this version, unions join their forces to form a grand coalition of worker's unions. Firms still negotiate independently one from the other. This gives to the grand union a very powerful weapon; the outside option. As an outside option, in general terms, we mention a payoff a player of a game reserves if bargaining fails.

In our case, if negotiations between the grand coalition of workers and one firm fails, then the grand coalition of workers deny the supply of labor force in this particular firm and give all of their labor force to the other firm.

As in the, previous mentioned, decentralized version of the game, also in this central (coordinated) version, firms have the upper hand to apply the remuneration scheme, in a take-it-or-leave-it style offer. Nevertheless, in all cases, the grand coalition of workers holds tight his outside option, i.e. not to give labor force to a firm, making the rival firm a monopolist[4].

[4]The solution of a grand union not giving labor force to neither firms, gives a grand union welfare function of zero, which can be improved by just giving a single labor unit to just one firm, so it is neither a realistic assumption nor a stable solution.

The grand coalition of workers, as an upstream monopolist of labor, bargains with the two downstream firms simultaneously and separately (independently). This assumtion is standard in similar multi contract situations (see e.g. Hart and Tirole 1990), but there is always the possibility of sequential bargain, which we will not be discussed in this paper. Again, based on the latter reference, the opportunistic behavior between grand coalition and one firm, creates incentives to be their bargain kept secret from the other firm. We obtain a unique equilibrium by imposing pairwise proofness on the equilibrium contracts, which is close to the passive beliefs assumptions, mentioned in Hart and Tirole (1990).

▶ **Both firms use fixed base wage** Each single firm bargains independently and simultaneously with the (sector-wide) grand union of workers. In stage 2 both firms have decided to use as a remuneration scheme a fixed base wage, so in stage 3 the only bargain subject is the amount of this fixed base wage. In the grand union utility function we count its outside option, as mentioned above.

The Nash bargain product will be:

$$\begin{aligned} NBP_1^{CW} &= (\pi_1)^b * [Total\ Union\ Gains - Outside\ Option_1]^{1-b} \\ Outside\ Option_1 &= (w_2 - w_0) * q_{2MONOPOLIST} \\ q_{2MONOPOLIST} &= \frac{1}{2\beta} * (\alpha - c - w_2) \\ \pi_{2MONOPOLIST} &= \frac{1}{4\beta} * (\alpha - c - w_2)^2 \end{aligned}$$

$$\begin{aligned} NBP_2^{CW} &= (\pi_2)^d * [Total\ Union\ Gains - OutsideOption_2]^{1-d} \\ Outside\ Option_2 &= (w_1 - w_0) * q_{1MONOPOLIST} \\ q_{1MONOPOLIST} &= \frac{1}{2\beta} * (\alpha - c - w_1) \\ \pi_{1MONOPOLIST} &= \frac{1}{4\beta} * (\alpha - c - w_1)^2 \end{aligned} \tag{28}$$

while:

$$Total\ Union\ Gains:\ UW_G^W = (w_1^* - w_0) * q_1^{**} + (w_2^* - w_0) * q_2^{**}$$

Maximizing Nash bargain products over fixed base wages w_1 and w_2, and solving the system of equations, we get:

$$\begin{aligned} w_1^* &= \frac{1}{6} * (3 * (-c + w_0 - \alpha) + (2 * b + d) * (c + w_0 - \alpha)) \\ w_2^* &= \frac{1}{6} * (3 * (-c + w_0 - \alpha) + (b + 2 * d) * (c + w_0 - \alpha)) \end{aligned} \tag{29}$$

Price, Total quantity sold (which is equal to total employment), Nash quantities per firm (which are equal to employment per firm), (net) profits per firm, total consumer surplus and unions' welfare are[5]:

[5] $3 - b^2 - b * d - d^2 > 0,\ \forall\ b, d \in (0, 1)$

$$
\begin{aligned}
q_1^{**} = L_1^* &= \frac{-(1+b)}{6*\beta} * (c + w_0 - \alpha) \\
q_2^{**} = L_2^* &= \frac{-(1+d)}{6*\beta} * (c + w_0 - \alpha) \\
(Net =) Gross\ Profits:\ \pi_1^* &= \frac{(1+b)^2}{36*\beta} * (c + w_0 - \alpha)^2 \\
(Net =) Gross\ Profits_2:\ \pi_2^* &= \frac{(1+d)^2}{36*\beta} * (c + w_0 - \alpha)^2 \\
Price:\ P^* &= \frac{1}{6} * ((2+b+d) * (c + w_0 - \alpha) + 6 * \alpha) \\
Quantity:\ Q^* = L^* &= \frac{-(2+b+d)}{6*\beta} * (c + w_0 - \alpha) \\
Consumer\ Surplus:\ CS &= \frac{(2+b+d)^2}{72*\beta^2} * (c + w_0 - \alpha)^2 \\
Total\ Union\ Gains:\ UW_G^W &= \frac{(3 - b^2 - b*d - d^2)}{18*\beta} * (c + w_0 - \alpha)^2
\end{aligned}
\tag{30}
$$

► **Both firms use profit sharing schemes.** In case of both firms apply profit sharing schemes as their remuneration schemes, Nash bargain products will have the form:

$$
\begin{aligned}
NBP_1^{CP} &= ((1 - a_1) * \pi_1)^b * [Total\ Union\ Gains - Outside\ Option_1]^{1-b} \\
Outside\ Option_1 &= (w_2 - w_0) * q_{2MONOPOLIST} + a_2 * \pi_{2MONOPOLIST} \\
q_{2MONOPOLIST} &= \frac{1}{2\beta} * (\alpha - c - w_2) \\
\pi_{2MONOPOLIST} &= \frac{1}{4\beta} * (\alpha - c - w_2)^2
\end{aligned}
$$

$$
\begin{aligned}
NBP_2^{CP} &= ((1 - a_2) * \pi_2)^d * [Total\ Union\ Gains - Outside\ Option_2]^{1-d} \\
Outside\ Option_2 &= (w_1 - w_0) * q_{1MONOPOLIST} + a_1 * \pi_{1MONOPOLIST} \\
q_{1MONOPOLIST} &= \frac{1}{2\beta} * (\alpha - c - w_1) \\
\pi_{1MONOPOLIST} &= \frac{1}{4\beta} * (\alpha - c - w_1)^2 \\
&\text{while:} \\
Total\ Union\ Gains:\ UW_G^{CP} &= (w_1 - w_0) * q_1 + (w_2 - w_0) * q_2 + a_1 * \pi_1 + a_2 * \pi_2
\end{aligned}
\tag{31}
$$

Maximizing Nash bargain product NBP_i^{CP} with respect to a_i:

$$
\left\{ \begin{array}{l} \frac{\partial NBP_1^{CP}}{\partial a_1} = 0 \\ \frac{\partial NBP_2^{CP}}{\partial a_2} = 0 \end{array} \right. \Rightarrow \left\{ \begin{array}{l} a_1^* = \frac{2*(8 - b*(-2 + 10*d - \frac{3*d*(w_0 + w_1 - 2*w_2)}{-c + w_1 - 2*w_2 + \alpha} - \frac{12*(w_0 + d*w_0 - d*w_1 - w_2)}{-c - 2*w_1 + w_2 + \alpha}))}{16 + b*d*(-25 + \frac{36*(w_1 - w_2)^2}{(c + 2*w_1 - w_2 - \alpha)*(-c + w_1 - 2*w_2 + \alpha)})} \\ a_2^* = \frac{2*(8 - d*(-2 + 10*b - \frac{3*b*(w_0 + w_2 - 2*w_1)}{-c + w_2 - 2*w_1 + \alpha} - \frac{12*(w_0 + d*w_0 - d*w_2 - w_1)}{-c - 2*w_2 + w_1 + \alpha}))}{16 + b*d*(-25 + \frac{36*(w_1 - w_2)^2}{(c + 2*w_2 - w_1 - \alpha)*(-c + w_2 - 2*w_1 + \alpha)})} \end{array} \right.
\tag{32}
$$

Writing down Excess Joint Profits as the sum of profits made by both the firm and the grand union we get:

$$ExcessJointProfits_i = V_i^P + U_i^P = (w_i - w_0) * q_i^* + \pi_i^* \quad , i = 1, 2 \tag{33}$$

and replacing a_i^* at NBP_i^{CP}:

$$\begin{aligned} NBP_1^{CP}|_{a_1=a_1^*} &= NBP_1^{CP*} = \\ &= \frac{(\frac{1}{b}-1)^{-b} * (-1+b)}{36 * \beta} * (c + 2 * w_1 - w_2 - \alpha) * \\ &\quad * [(-4 + 5 * a_2^*) * c - 6 * w_0 + 4 * (\alpha + w_1) - 2 * (a_2^* * w_1 - w_2) + \\ &\quad + a_2^* * (7 * w_2 - 5 * \alpha)] \end{aligned} \tag{34}$$

$$\begin{aligned} NBP_2^{CP}|_{a_2=a_2^*} &= NBP_2^{CP*} = \\ &= \frac{(\frac{1}{d}-1)^{-d} * (-1+d)}{36 * \beta} * (c + 2 * w_2 - w_1 - \alpha) * \\ &\quad * [(-4 + 5 * a_1^*) * c - 6 * w_0 + 4 * (\alpha + w_2) - 2 * (a_1^* * w_2 - w_1) + \\ &\quad + a_1^* * (7 * w_1 - 5 * \alpha)] \end{aligned}$$

if we divide Excess Joint Profits with maximized Nash bargain product with respect to profit share we find that they are analogous:

$$\frac{ExcessJointProfits_1}{NBP_1^{CP*}} = \frac{(\frac{1}{b}-1)^b}{1-b} \quad , \quad \frac{ExcessJointProfits_2}{NBP_2^{CP*}} = \frac{(\frac{1}{d}-1)^d}{1-d} \tag{35}$$

We have proven here, that also under the centralized (coordinated) case, profit share works (mathematically) just like a two-part tariff contract. Using the same methodology as in previous cases, we will get the Nash equilibrium fixed base wage w_i^* directly from NBP_i^{CP*} using first order conditions:

$$\begin{cases} \frac{\partial NBP_1^{CP*}}{\partial w_1} = 0 \\ \frac{\partial NBP_2^{CP*}}{\partial w_2} = 0 \end{cases} \Rightarrow \begin{cases} w_1^* = \frac{(c-\alpha)*(1-2*a_2^*)+3*w_0+2*w_2^*-4*w_2^**a_2^*}{4-2*a_2^*} \\ w_2^* = \frac{(c-\alpha)*(1-2*a_1^*)+3*w_0+2*w_1^*-4*w_1^**a_1^*}{4-2*a_1^*} \end{cases} \tag{36}$$

and substituting a_1^*, a_2^* from equations above, and solving for w_1^*, w_2^*, and by substituting w_1^* and w_2^* back to profit shares:

$$a_1^* = 2 + \frac{4 * (4+b)}{-16 + b * d} \quad , \quad a_2^* = 2 + \frac{4 * (4+d)}{-16 + b * d} \tag{37}$$

In this case, Nash quantities (which are equal to labor units), Nash profits, Nash price, Nash total quantity (which is equal to total employment), Consumer Surplus, and Total Union Gains are:

$$q_1^{**} = L_1^* = \frac{(c + w_0 - \alpha)}{2 * \beta} \frac{(4 + b) * d * (16 - b * d)}{-64 * d + 8 * b * (d - 4) * (2 + d) + b^2 * d * (8 + 3 * d)}$$

$$q_2^{**} = L_2^* = \frac{(c + w_0 - \alpha)}{2 * \beta} \frac{(4 + d) * b * (16 - b * d)}{-64 * d + 8 * b * (d - 4) * (2 + d) + b^2 * d * (8 + 3 * d)}$$

$$Gross\ Profits_1 :\ \pi_1^{**} = \frac{(c + w_0 - \alpha)^2}{4 * \beta} * \frac{(4 + b)^2 * d^2 * (16 - b * d)^2}{(-64 * d + 8 * b * (d - 4) * (2 + d) + b^2 * d * (8 + 3 * d))^2}$$

$$Gross\ Profits_2 :\ \pi_2^{**} = \frac{(c + w_0 - \alpha)^2}{4 * \beta} * \frac{(4 + d)^2 * b^2 * (16 - b * d)^2}{(-64 * d + 8 * b * (d - 4) * (2 + d) + b^2 * d * (8 + 3 * d))^2}$$

$$Net\ Profits_1 :\ (1 - a_1^*) * \pi_1^{**} = \frac{(c + w_0 - \alpha)^2}{4 * \beta} *$$

$$\frac{b * (4 + b)^2 * d^2 * (4 + d) * (16 - b * d)}{(-64 * d + 8 * b * (d - 4) * (2 + d) + b^2 * d * (8 + 3 * d))^2}$$

$$Net\ Profits_2 :\ (1 - a_2^*) * \pi_2^{**} = \frac{(c + w_0 - \alpha)^2}{4 * \beta} *$$

$$\frac{b^2 * (4 + b) * d * (4 + d)^2 * (16 - b * d)}{(-64 * d + 8 * b * (d - 4) * (2 + d) + b^2 * d * (8 + 3 * d))^2}$$

$$P^* = \frac{(-16 + b * d) * (2 * d + b * (2 + d)) * (c + w_0) + 2 * \alpha * (-16 * d + b * (-16 + 3 * d^2 + b * d * (3 + d))}{-64 * d + 8 * b * (d - 4) * (2 + d) + b^2 * d * (8 + 3 * d)}$$

$$Q^* = L^* = \frac{(c + w_0 - \alpha) * (16 - b * d) * (2 * d + b * (2 + d))}{\beta * (-64 * d + 8 * b * (d - 4) * (2 + d) + b^2 * d * (8 + 3 * d))}$$

$$CS = \frac{(c + w_0 - \alpha)^2 * (16 - b * d)^2 * (2 * d + b * (2 + d))^2}{2 * \beta^2 * (-64 * d + 8 * b * (d - 4) * (2 + d) + b^2 * d * (8 + 3 * d))^2}$$

$$UW_G^{CP} = \frac{(c + w_0 - \alpha)^2}{2 * \beta} *$$

$$\frac{(-16 + b * d) * (2 * d + b * (2 + d)) * (-64 * d + b^2 * d * (16 + 5 * d) + 16 * b * (-4 + d + d^2))}{(-64 * d + 8 * b * (d - 4) * (2 + d) + b^2 * d * (8 + 3 * d))^2} \tag{38}$$

► **One firm uses profit sharing scheme, and the other firm uses only fixed base wage.** In this double symmetric case, one firm (let firm 1 be that firm) decides (at stage 2) to offer a profit sharing scheme, while the other firm (say firm 2) decides (at stage 2) to offer a fixed base wage. At stage 1, firms have decided to bargain individually and independently one from the other, while their unions have decided to bargain centrally, forming a grand coalition of workers' unions (central coordinated case).

Following similar reasoning as in decentralized version of the game we will prove that Excess Joint Profits for firm 1 (which gives profit shares) are analogous to Nash Bargain Product, and we will state wages and profits for these two firms.

Nash bargain products will be:

$$
\begin{aligned}
NBP_1^{CM} &= ((1-a_1)*\pi_1)^b * [Total\ Union\ Gains - Outside\ Option_1]^{1-b} \\
Outside\ Option_1 &= (w_2 - w_0) * q_{2MONOPOLIST} \\
q_{2MONOPOLIST} &= \frac{1}{2\beta} * (\alpha - c - w_2)\ ,\ \pi_{2MONOPOLIST} = \frac{1}{4\beta} * (\alpha - c - w_2)^2 \\
NBP_2^{CM} &= (\pi_2)^d * [Tota\ lUnion\ Gains - Outside\ Option_2]^{1-d} \\
Outside\ Option_2 &= (w_1 - w_0) * q_{1MONOPOLIST} + a_1 * \pi_{1MONOPOLIST} \\
q_{1MONOPOLIST} &= \frac{1}{2\beta} * (\alpha - c - w_1)\ ,\ \pi_{1MONOPOLIST} = \frac{1}{4\beta} * (\alpha - c - w_1)^2 \\
Total\ Union\ Gains:\ UW_G^{CM} &= (w_1 - w_0) * q_1 + (w_2 - w_0) * q_2 + a_1 * \pi_1
\end{aligned}
\tag{39}
$$

Remember that CM stands for **C**entral coordinated union bargain, and **M**ixed remuneration schemes firms. These symbols summarize stages 1 and 2. Note the outside option which is present in (sector-wide grand) union's utility function, and is a characteristic of the central (coordinated) bargain.

Maximizing Nash bargain product NBP_i^{CM} with respect to a_i:

$$
\begin{aligned}
&\frac{\partial NBP_1^{CM}}{\partial a_1} = 0 \Rightarrow a_1^* = 1 - b + \frac{3 * b * (w_0 - 2 * w_1 + w_2)}{2 * (-c + \alpha - 2 * w_1 + w_2)} \\
&\frac{\partial NBP_2^{CM}}{\partial w_2} = 0 \Rightarrow \\
&w_2^* = \frac{(c-\alpha) * (3 + 2 * a_1^* + 3 * d * (a_1^* - 1)) - 3 * w_0 * (1+d) + w_1 * (-6 + a_1^* * (4 + 3 * d))}{2 * (a_1^* - 6)}
\end{aligned}
\tag{40}
$$

Just for firm 1 (which applies profit share), we write down Excess Joint Profits as the sum of profits made by the firm and the union we get:

$$
ExcessJointProfits_1 = V_1^P + U_1^P = (w_1 - w_0) * q_1^* + \pi_1^* \tag{41}
$$

and replacing a_1^* at NBP_1^{CM}:

$$
\begin{aligned}
NBP_1^{CM}|_{a_1 = a_1^*} &= NBP_1^{CM*} = \\
&= \frac{(\frac{1}{b} - 1)^{-b} * (1-b)}{18 * \beta} * (2 * c + 3 * w_0 - 2 * w_1 + w_2 - 2 * \alpha) * (c + 2 * w_1 - w_2 - \alpha)
\end{aligned}
\tag{42}
$$

if we divide Excess Joint Profits with maximized Nash bargain product with respect to profit share we find that they are analogous:

$$
\frac{ExcessJointProfits_1}{NBP_1^{CM*}} = \frac{(\frac{1}{b} - 1)^b}{1-b} \tag{43}
$$

.

Using the same methodology as on previous case:

$$\frac{\partial NBP_1^{CM*}}{\partial w_1} = 0 \Rightarrow w_1^* = \frac{1}{4} * (c + 3 * w_0 + 2 * w_2^* - \alpha) \tag{44}$$

and using w_2^*, we solve the system of three equations with three unknown variables, i.e.:

$$\begin{aligned} a_1^* &= 1 - \frac{b}{2} \\ w_1^* &= \frac{2 * w_0 * (6 + d) + b * ((c - \alpha) * (1 + d) + *w_0)}{12 - d * (b - 2)} \\ w_2^* &= \frac{w_0 * (12 + 4 * b + d * (2 + 3 * b)) - (c - \alpha) * (2 * (6 + d) - b * (4 + 5 * d))}{24 - 2 * d * (b - 2)} \end{aligned} \tag{45}$$

In this case, Nash quantities (which are equal to labor units), Nash profits, Nash price, Nash total quantity (which is equal to total employment), union welfare, Nash Consumer Surplus, and grand union's welfare are:

$$\begin{aligned} q_1^{**} = L_1^* =& \frac{-(c - \alpha + w_0)}{2 * \beta} \\ q_2^{**} = L_2^* =& \frac{-(c - \alpha + w_0)}{2 * \beta} * \frac{2 * b * (1 + d)}{d * (2 - b) + 12} \\ Gross\ Profits_1:\ \pi_1^{**} =& \frac{(c - \alpha + w_0)^2}{4 * \beta} \\ Net\ Profits_1:\ (1 - a_1^*) * \pi_1^{**} =& \frac{b * (c - \alpha + w_0)^2}{8 * \beta} \\ Net = Gross\ Profits_2:\ \pi_2^* =& \frac{b * (c - \alpha + w_0)^2}{8 * \beta} * \frac{8 * b * (d + 1)^2}{(d * (b - 2) - 12)^2} \\ Price:\ P^* =& \alpha + \frac{(c + w_0 - \alpha) * (b * (2 + d) + 2 * (6 + d))}{24 - 2 * d * (b - 2)} \\ Quantity:\ Q^* = L^* =& \frac{(c - \alpha + w_0)}{2 * \beta} * \frac{b * (2 + d) + 2 * (6 + d)}{d * (b - 2) - 12} \\ Consumer\ Surplus:\ CS =& \frac{(c - \alpha + w_0)^2}{8 * \beta^2} * \frac{[b * (2 + d) + 2 * (6 + d)]^2}{(d * (b - 2) - 12)^2} \\ Grand\ Union\ Welfare:\ UW_G^{CM} =& \frac{(c + w_0 - \alpha)^2}{8 * \beta} * \frac{16 * b * (9 + b) + 8 * b * d * (12 + b)}{(d * (b - 2) - 12)^2} + \\ &+ \frac{d^2 * (2 + b) * (-4 + b * (8 + b)) - 96 * (3 + d)}{(d * (b - 2) - 12)^2} \end{aligned} \tag{46}$$

2.2.3 Stage 2

In our analysis, all four members of the bargaining game (two firms and two unions) can observe other members' equilibrium pricing. So, under a "take it or leave it" offer from firm i to union i, we can employ subgame perfect equilibrium in the next stage 3. This is because, for a member of the bargaining game to choose profit share and fixed base wage optimally, each firm must have certain beliefs about his rivals' non-linear contracts (two part tariffs) and pricing.

As stated in (O'Brien and Shaffer 1992), when a member of a Nash bargaining game can observe his rivals' non-linear contracts, the joint profit maximizing outcome can be supported in a subgame perfect equilibrium. It is stated that two part tariffs suffice, thus we don't involve more complex non-linear forms of contracts. Fixed fees (in our case fixed base wage w_i) are enough to transfer the surplus. This result relies on contract observation, whereby each member of the bargaining game (in our case the two firms and the two unions) can observe and can calculate his own and his rivals' equilibrium pricing decision under two part tariffs. Once observability is dropped, a member of the bargaining game must form beliefs about his rivals' contracts, so subgame perfection cannot be employed.

In this stage of the game, firms (or better, the management of these firms) decide whether to implement a profit sharing scheme or not. This has to do with the profitability offered by one remuneration scheme or the other. In this stage, we take for granted that firms and unions at the previous stage 1, have decided to form grand coalitions and/or grand confederations or not. So, in our choice here, we have to check all different versions of centralized or decentralized bargain.

Solving stage 2 in Decentralized version. We start from a case of neither firm has profit sharing scheme, i.e. both give fixed base wages. Firm 1 wishes to implement a profit sharing scheme. It has to have either absolute bargain power over its corresponding union, or it has to rely partly on bargain power of firm 2. As the bargain power of firm 2 rises, firm 1 needs less and less bargain power over its corresponding union to move from fixed base wage to profit sharing scheme. Until now, firm 2 uses fixed base wage. Shaded area represents the following inequality:

$$\text{Net Profits}^{W}_{firm1} < \text{Net Profits}^{M}_{firm1} \Leftrightarrow 48*\sqrt{2b}*(1+b)+9*b^2*(d-1)+d-17-2*b*(55+d) > 0 \tag{47}$$

Proposition 2.2. *When both firms introduce profit sharing schemes, firm 1 has lower profits compare to the case of only firm 1 has introduce profit sharing scheme, and firm 2 pays fixed base wage, i.e.:* $NetProfits^{P}_{1} < NetProfits^{M}_{1}$

Proof. Net Profits$^{P}_{firm1}$ <Net Profits$^{M}_{firm1} \Leftrightarrow \frac{2*b}{25*\beta} * (\alpha - c - w_0)^2 < \frac{2*b*(d-5)^2}{(d-17)^2*\beta} * (c - \alpha + w_0)^2 \Leftrightarrow \frac{1}{25} < \frac{(d-5)^2}{(d-17)^2}$ True for $d \in (0, 1)$. □

Proposition 2.3. *When firm 1 has introduce profit sharing scheme, and firm 2 still uses fixed base wage, firm 2 has always lower profits, i.e.:* $NetProfits^{M}_{2} < NetProfits^{M}_{1}$

Proof. Firm 1 has introduce profit sharing scheme, so inequality $48 * \sqrt{2b} * (1 + b) + 9 * b^2 * (d - 1) + d - 17 - 2 * b * (55 + d) > 0$ holds. Then: Net Profits$^{M}_{firm2}$ <Net Profits$^{M}_{firm1} \Leftrightarrow$

$\frac{4*(d+1)^2}{(d-17)^2*\beta} * (c-\alpha+w_0)^2 < \frac{2*b*(d-5)^2}{(d-17)^2*\beta} * (c-\alpha+w_0)^2 \Leftrightarrow 2*(d+1)^2 < b*(d-5)^2$ True for $b, d \in (0,1)$ and under the assumption of the first inequality. □

Proposition 2.4. *It is always optimal for both firms not to introduce profit sharing schemes, rater than both introduce profit sharing schemes, i.e.:* $NetProfits_1^P < NetProfits_1^W$

Proof. Net Profits$^P_{firm1}$ <Net Profits$^W_{firm1} \Leftrightarrow \frac{2*b}{25*\beta} * (\alpha - c - w_0)^2 < \frac{4*(d-5)^2*(b+1)^2*(c+w_0-\alpha)^2}{9*\beta*(15+b+d-b*d)^2} \Leftrightarrow$ $\frac{9}{50} < \frac{(d-5)^2*(b+1)^2}{b*(15+b+d-b*d)^2}$ True for $b, d \in (0,1)$. □

The scenario seems to be as follows: when both firms use fixed base wage as their remuneration scheme, it is always optimal (=more profits) for firm 1 to introduce profit sharing scheme. To do so, firm 1 needs great bargain power (which lowers as the bargain power of firm 2 grows), because this will mean that workers will receive lower fixed base wage This lowers the profits of firm 2 (which has not introduce, at the time, a profit sharing scheme), so it creates incentives (=profit maximization) for firm 2 to introduce profit sharing. At the end, when both firm use profit sharing, they both enjoy lower profits compare to the case of both using fixed wages. Let us examine this scenario more closely.

Having answer the question of when a firm can apply a profit sharing scheme, clearly a profit driven decision, we are going to answer the following questions, which sum up to the phrase "What happens to market structure, and to the competition among firms when one firm introduces a profit sharing scheme?". Even if this question is hard to answer, mostly because the assumptions of our model enforce a certain structure related functional form of the answer, we try to shed some light.

Proposition 2.5. *When only one firm (say firm 1) introduces profit sharing scheme, and the other (say firm 2) pays fixed base wage, compare to the case of both paying in fixed base wage, the following hold:*

a)Price of the homogeneous good rise,

b)Total Output which equals Total Employment rises,

c)Consumer Surplus is rising,

d)Both Union welfare are lesser than the previous state, but the welfare of the union of the firm that still pays fixed base wage (firm 2) is bigger than the welfare of the union of the firm that introduced profit sharing scheme (firm 1),

e)Profits for the firm that introduces profit sharing (say firm 1) rises,

f)Wage for the firm that introduces profit sharing (firm 1) is falling,

g)Profits for the firm that does not introduce profit sharing (say firm 2) are falling, while

h)Wages for the firm that does not introduce profit sharing (firm 2) are rising[6]*, and*

i)Wage of the firm that use fixed base wage (firm 2) is bigger than the wage of the firm that use profit sharing (firm 1),

[6]Similar positive spillover effect in wages have been found by Sorensen (1992) and Oswald (1979).

j)Selling quantities (and employment) for the firm that does not introduce profit sharing (firm 2) are falling.

Proofs are quite easy, the reader has only to solve a simple inequality. It is obvious that the firm that does not introduce profit sharing suffers from less sales, less profits, and bigger wages. This economic weapon of strategic introduction of profit sharing creates an economic suffocation to the firm that does not introduce a profit sharing. Even though firm 2 pays more fixed base wage, because of the lesser selling quantities, is forced to employ less workers than firm 1. With fixed base wages, unions enjoy more welfare than the case of profit sharing introduction.

Proposition 2.6. *When both firm introduce profit sharing schemes, compare to the case of only one firm applies profit sharing and the other applies fixed base wage, the following hold:*

a)Prices are falling,

b)Total Output which equals Total Employment rises,

c)Consumer Surplus is rising,

d)Welfare of the union of the firm that had profit sharing in the previous state (firm 1) is rising, but the welfare of the union of the firm that just now introduced profit sharing (firm 2) is falling, compare to the previous state. The comparison between the two union welfare is now based only to the comparison between the two bargain powers b, d*,*

e)Profits for the firm that had profit sharing in the previous state (firm 1) are falling,

f) Wages for the firm that had profit sharing in the previous state (firm 1) are rising,

g)Profits for the firm that did not had profit sharing in the previous state and just now introduced it (firm 2) are bigger now if and only if $d > 41 - 6 * \sqrt{46} \approx 0.3060$*,*

h)Wages for the firm that did not had profit sharing in the previous state and just now introduced it (firm 2) are falling,

i)Wages between two firms can be compared now based to the comparison between the two bargain powers b, d*,*

j)Selling quantities (and employment) for the firm that had profit share (firm 1) are falling, while for the firm that did not had profit share but just now introduced it (firm 2) are rising.

Trying to simplify the model, in order to acquire more handy results, we set the two bargain powers to be equal, i.e. $b = d$. This assumption has a rational base; the is no clue why the two bargain powers should be different. Firms sell homogeneous products in the same market, have same costs and profit functions, unions have the same objectives and so on. There is no reason the two bargain powers to be quite different, but there are many reasons they should be quite close, so an equality assumption will bring insignificant changes in a simple model.

Proposition 2.7. *Setting the two bargain powers equal* $b = d$*, the following three Nash equilibria hold:*

a)Both firms have incentives (bigger profits) to remain in fixed base wages as remuneration schemes, if their bargain power is lower than $b < 0.6265$*,*

b)Both firms have incentives (bigger profits) to introduce and maintain profit sharing schemes, if their bargain power is bigger than $b > 0.6265$.

As a Proof, we will examine the incentives of a firm to deviate from a profit share or fixed wage decision. Setting $b = d$ we will end up in a single numeric value of b in which deviation is not possible. All these numeric values will construct the line of the proposition above. Algebraic manipulations used are trivial and easy to reproduce. Let us first restate profit functions in case of equal bargain powers $b = d$:

$$\begin{aligned} NetProfits_1^W = NetProfits_2^W =& \frac{4 * (1+b)^2 * (c - \alpha + w_0)^2}{9 * (3+b)^2 * \beta} \\ NetProfits_1^M =& \frac{2 * b * (5-b)^2 * (c - \alpha + w_0)^2}{(17-b)^2 * \beta} \\ NetProfits_2^M =& \frac{4 * (1+b)^2 * (c - \alpha + w_0)^2}{(17-b)^2 * \beta} \\ NetProfits_1^P = NetProfits_2^P =& \frac{2 * b * (c - \alpha + w_0)^2}{25 * \beta} \end{aligned} \tag{48}$$

All possible deviations that could ruin Nash Equilibria are the following:

1)Given that firm 2 has introduced a profit sharing scheme, while firm 1 has already a profit sharing scheme, does firm 1 has incentives (i.e. bigger profits) to move back and switch to fixed base wage?

The answer is "yes" but for $b < 0.3060$.

2)Given that firm 1 and firm 2 give both fixed base wages, does firm 1 has incentives (i.e. bigger profits) to switch to profit sharing scheme?

The answer is "yes" but for $b > 0.6265$.

3)Given that firm 2 has not introduce profit sharing (gives fixed base wage), does firm 1 has incentives to switch from profit sharing to fixed base wage?

The answer is "yes" but for $b < 0.6265$.

4)Given that firm 1 has introduced a profit sharing scheme, does firm 2 has incentives to move from fixed base wage to profit sharing scheme?

The answer is "yes" but for $b > 0.3060$.

The following diagram will help us visualize the deviations stated above. We state deviations as D1, D2, D3 and D4.

Note that for small bargain power $b < 0.3060$ a firm that has introduced a profit share (in a previous moment, in which had greater bargain power), now has incentives (bigger profits) to stop using profit share, and fall back to a fixed base wage. Also, if a firm use fixed base wage as remuneration scheme, but faces a rival firm that uses profit share, it is optimal to jump into a profit sharing scheme if and only if it has bargain power greater than $b > 0.3060$.

As it is vividly illustrated in the diagram above, profits are different, under different circumstances. We have set a constant value for variables like α, β, c, w_o and we have set $b = d$. Under

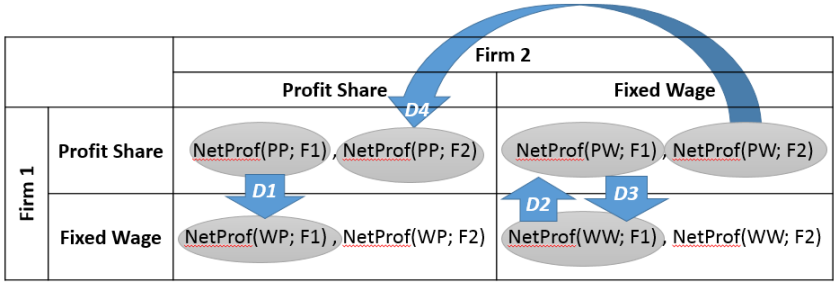

Figure 1

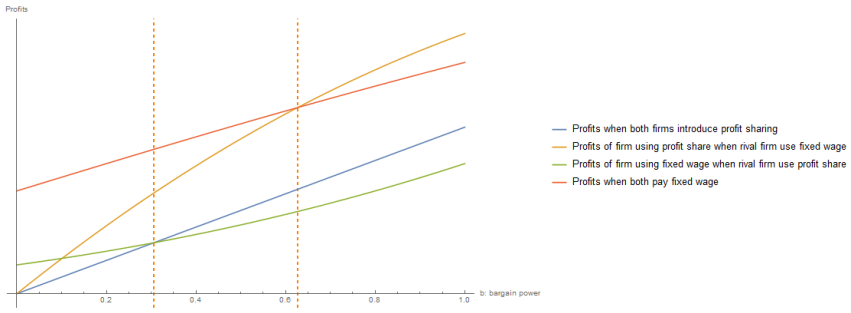

Figure 2

these assumptions, we have plotted profits as a function of bargain power b only. The vertical dashed lines are set at $b = 0.3060$ and $b = 0.6265$, the two thresholds of bargain power. Once again, it is easy to see that for $b < 0.6265$ profits for both firms when they use fixed base wage are bigger than profits under other scenarios. Furthermore, for $b < 0.3060$ profits for both firms when they both use profit share are the least among other scenarios.

The story that the diagram above is saying is more or less as follows; we are in decentralized version of bargain between firms and unions. Both firms start from the point where they use fixed base wage as their remuneration scheme. Someone dropped the idea about a different style of remuneration scheme, called profit sharing, and firm's management is willing to give it a try. Both firms have no incentives (i.e. more profits) to introduce a profit sharing scheme, up to the point where they have bargain power more than 0.6265. Consider it not as an absolute threshold, but more as a level of bargain power.

Then one firm, let us say firm 1, realizes that has more profit if it introduce a profit sharing scheme, so it jumps to the upper line of profits (upper versus lower as we see them beyond the second dashed vertical line); suddenly the other firm, say firm 2, suffers lesser profits, as it falls to the last profit line; This negative externality from firm 1, affects firm 2, and managers of the last firm, count and measure that if they also introduce a profit sharing scheme, both firm 1 and firm 2 will meet at the third line of profits, and so they introduce it. To do so, they do not need so much bargain power over their union as firm 1 when first introduced a profit share (b=0.6265), but now they need less than half of it (b=0.3060), a much more easier goal to achieve.

Now this part is very crucial. Firms can not deviate unilaterally, because they will end up will less profits. They have to cooperate and both deviate to fixed wage, unless something changes in bargain powers and unions become stronger. But in the special case where the two firms can negotiate and form some sort of cartel, it is their best interest to move back to fixed base wage, because both of them will enjoy more profits.

Solving stage 2 in central (coordinated) version. In central (coordinated) version, as it can be seen from equations above, things became much more complicated. This makes our analysis more difficult; it is not easy to extract results. In this case, more now than in the decentralized version, the equality of bargain powers $b = d$ makes more sense. We have a grand coalition of workers, a strong sector wide union, facing two firms. Grand union has outside options, and it is practically the monopolist of labor in the sector. This makes difficult for one firm to have more bargain power than the other, probably both firms have the same (weak) bargain power.

Grand union, as a monopolist of labor, has the right to deny access to labor force for one firm or the other, and still has positive welfare. But the firm that had been denied the labor force, is forced to move out and exit the sector. If this sector wide, strong, grand union can not face these two firms with the same way (i.e. with the same bargain power), having the upper hand in the bargain, then who is it? We will try to extract as general propositions as possible, but nevertheless, the assumption $b = d$ as we argued is not far from reality.

Proposition 2.8. *Start from the scenario of both firms (say firm 1 and firm 2) apply fixed base wage as their remuneration scheme. When one firm introduce profit sharing scheme (say firm 1), while the other remains in fixed wage (say firm 2), and under the assumptions of the model, the following hold:*

1)Wages for the firm that introduced profit sharing scheme (say firm 1) are lower than the case of both firms giving fixed wage,

2)Profits for firm 1 (applying profit share) are bigger than the case of both firms apply fixed wage, if an only if $b > \frac{1}{2}$,

3)Prices are lower in this case, than the case of both firms giving fixed wage, if and only if $\sqrt{49 + b * (10 + b)} > 5 + b + 2 * d$,

4)Total quantity sold (=total employment) is bigger now, than the previous case of both firms giving fixed wage, if and only if $\sqrt{49 + b * (10 + b)} > 5 + b + 2 * d$,

5)Consumer Surplus is bigger now, than the previous case of both firms giving fixed wage, if and only if $\sqrt{49 + b * (10 + b)} > 5 + b + 2 * d$,

6)Profits of firm 2 (fixed wage) are less than the previous case (both firms give fixed wage),

7)Wage of firm 2 (fixed wage) are less than the previous case (both firms give fixed wage),

8)Wage of firm 2 (fixed wage) is higher than the wage of firm 1 (introduced a profit sharing scheme),

9)Quantities sold (=firm's employment $q_i = L_i$*) by firm 2 (fixed wage) are less than the previous case (both firms give fixed wage), even though firm 2 gives more fixed base wage.*

In the shaded area above, we find pairs of bargain powers (b,d) in which moving one firm from fixed wage to profit share, while the other still gives fixed wage, creates more employment, more quantity sold, more consumer surplus, and cheaper prices. In other words, the shaded area above is the inequality $\sqrt{49 + b * (10 + b)} > 5 + b + 2 * d$.

This clearly shows that in order to have all these positive externalities from the introduction of a profit sharing scheme, bargain power of at least one firm must be absolute.

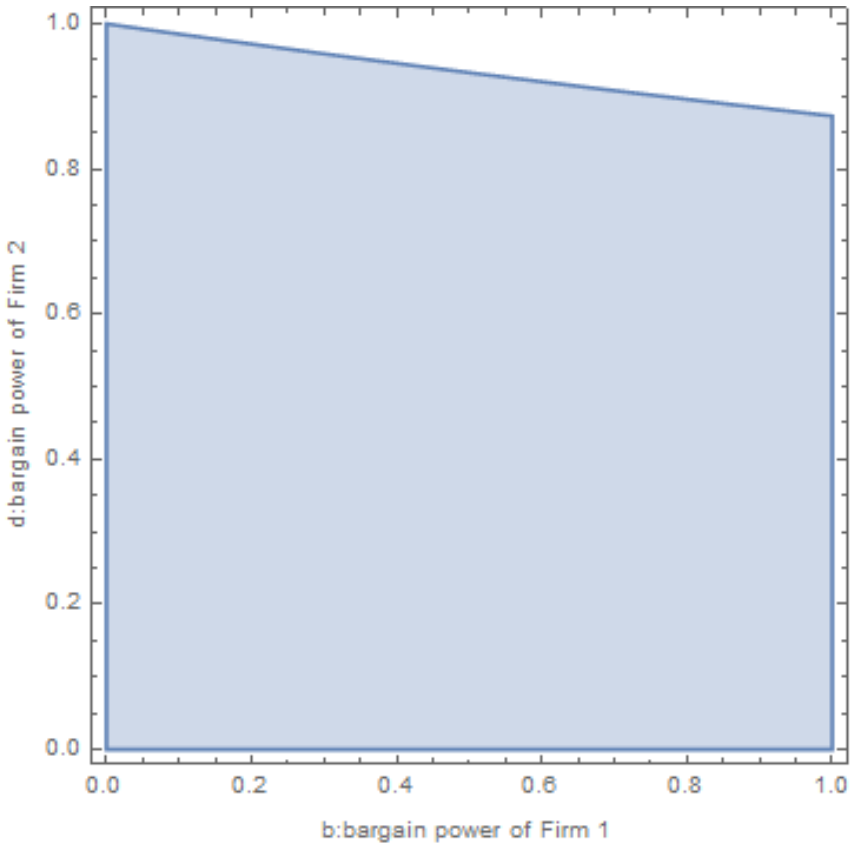

Figure 3

The strange result is that in the central (coordinated) case, it is easiest for firms to introduce a profit sharing scheme, but all the positive externalities are much more difficult to pass to consumers. Namely, in the decentralized case, firms needed bargain power >0.6265 in order to introduce a profit sharing scheme that could create more profits for them; now this threshold falls to bargain power >0.5, something like 20% lower bargain power. So, when a firm is facing a grand, sector wide, coalition of workers, needs about 20% less bargain power to find profitable to introduce profit share.

Having in mind that it is proven above that profit sharing is mathematically equivalent to two part tariffs, it seems that a decentralized union is more reluctant to accept a profit sharing scheme than a sector-wide grand union. More discussion over the subject, and more comparisons between versions, will be made later in this paper.

Proposition 2.9. *Start from the scenario of one firm give profit sharing scheme (say firm 1), while the other firm give fixed base wage (say firm 2), as their remuneration scheme. When both firms apply profit sharing schemes, and under the assumptions of the model, the following hold:*

1) Wages for firm 1 are bigger now than the case of only firm 1 applied profit sharing scheme, while firm 2 applied fixed wage,

2) Wages for firm 2 are now (when both firms apply profit share) less than the previous case, when one firm applied profit share (firm 1) and the other applied fixed wage (firm 2),

3) Quantity sold by firm 1 (and thus firm 1 employment) is less now, when both firms apply profit share, than the previous case,

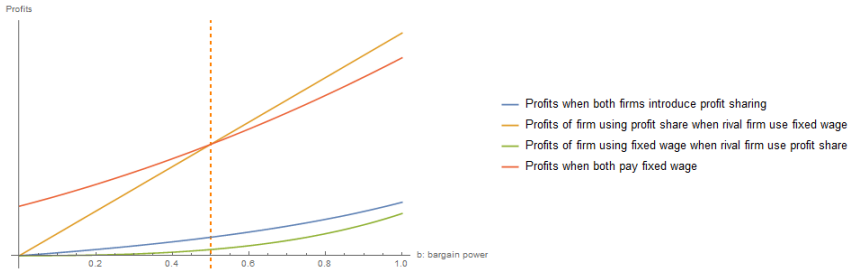

Figure 4

4)Quantity sold by firm 2(and thus firm 2 employment) is more now than the previous case,

5)For equal bargain powers $b = d$, Grand Union Welfare in case of both firms introducing a profit sharing scheme is higher than the case of both firms using fixed base wage.

Having in mind that $b = d$, we draw four profit functions, one for each case in this version. The diagram above shows these four different profit functions; notice the dashed vertical line which is set to 0.50. For computational reasons, and without any loss of generality, we set constant values to the other exogenous variables such as c, w_0, α, β. As it can be seen, these variables have nothing to do with the points of intersection between profit functions.

This diagram tells us a different story from the diagram in the previous decentralized version. Both firms negotiate (in a non-cooperative manner) with a single, sector wide, grand union of workers. Both firms initiate using a fixed base wage scheme as their remuneration scheme. As long as no firm has bargain power more than 0.50, they both are better off by paying fixed wage.

When one firm, say firm 1, reaches a level of bargain power that is equal to or more than 0.50, has more profits if unilaterally switch to profit sharing, so it moves to the uppest profit line (as we count them beyong the vertical dashed line set to bargain power 0.50). Suddenly, the second firm, say firm 2, that remained in using fixed wage as their remuneration scheme, feels all the negative externality of the action taken by firm 1. It suffers less profits, and has less quantity sold (so less employment) even if it pays its workers more money as a fixed wage.

It takes immediate action, and the management of firm 2 decides to introduce also a profit sharing scheme. This is possible for any level of bargain power from firm 2 (easy to check for any $b \in (0, 1)$), something quite different from the previous decentralized case. When this happens, both firms apply profit sharing schemes so both move to the third profit line in the diagram. This is a huge step down for firm 1, and a small step up for firm 2. No firm can deviate solely; it will fall to the bottom line among the four profit lines, giving a free ride up to the top for the other firm.

In this case, also, it is obvious to discuss a situation where both firms cooperate and negotiate over the case of returning back to the fixed wage remuneration scheme. If there is a proper communication between the two firms, lack of assymetry information (which could turn deterministic decisions to bayesian), and a suitable legal framework, then there is a good change for both firms to increase their profitability by simultaneously move back to fixed wage. This case is to be studied side by side with the analogous discussion made in decentralized version of the game, propably in another paper.

2.2.4 Stage 1

In this stage we are concerned about the formation or not of grand coalition of unions. Unions will have incentives to bargain centrally if and only if they can achieve higher levels of welfare for their members, i.e. bigger wages.

Proposition 2.10. *Under the assumptions of the model, the following hold:*

1) When both firms give fixed base wage, wages are higher in Central Coordinated version than in Decentralized version,

2) When both firms give profit sharing schemes plus a fixed base wage, wages are higher in Central Coordinated version than in Decentralized version,

3) When both firms give profit sharing schemes plus a fixed base wage, profit shares are higher in Central Coordinated version than in Decentralized version,

4) When both firm give profit sharing schemes plus a fixed base wage, in Central Coordinated version wages are higher than unemployment benefit w_0, but in Decentralized version wages are lower than unemployment benefit w_0.,

5) When both firm give profit sharing schemes plus a fixed base wage, Quantity total, which equals total employment, is higher in Decentralized version than in Central Coordinated version,

6) When both firms give fixed base wage, Consumer Surplus is higher in Decentralized Version, than in Central Coordinated version,

7) When both firm give profit sharing schemes plus a fixed base wage, Consumer Surplus is higher in Decentralized Version, than in Central Coordinated version,

8) Setting the two bargain powers equal, $b = d$, and under a fixed base wage remuneration scheme, the Grand Union Welfare of Central Coordinated version is bigger than the sum of Welfares of the two separate Unions in Decentralized version,

9) When both firms give profit sharing schemes plus a fixed base wage, product price is higher in Central Coordinated Version than in Decentralized,

10) When both firms give fixed base wage, product price is higher in Central Coordinated Version than in Decentralized,

11) When both firms give fixed base wage, Profits (of either firm) are higher in Decentralized version than in Centralized version.

We see clearly that in any version of remuneration scheme, central coordinated version of bargain gives union members a bigger welfare. When only one grand coalition of workers bargain with firms over wages and possibly over profit shares, it achieves higher levels of welfare than the decentralized case. So, in this stage, without any second thoughts, unions will negotiate centrally.

3 Conclusions

We have seen that profit sharing schemes do exist in business world, with one form or the other. We have seen certain micro and macro researches which indicate that profit sharing schemes can help individual firms achieve higher levels of productivity, and an economy to reduce unemployment.

We have endogenize the firm's decision to give or not to give profit shares, and we have also endogenize the union's decision to bargain individually or to form grand coalition.

We have proven that it is always optimal for unions to unite their power, and form a grand union, and thus bargain centrally with their firms. This creates a supeadditivity in their welfare; the sum of the two union's welfare under decentralized version is less than the welfare of the grand union in central coordinated version. In any case, the grand union gets more wages and more profit shares for their members than the single union.

We have proven that under the default state of both firms giving fixed base wage, it is a prisoner's dilemma for one firm to give profit share. At the same time, the other firm will suffers less profits without making any move, so it will decide to alter it's remuneration scheme to profit share also. This will create a situation were both firms will have less profits than in the default state. Also, we have seen that under central coordinated version, it is up to 20% more easy to implement a profit share than under decentralized version.

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