# Natural Resource Management: A Network Perspective

Efthymia Kyriakopoulou* Anastasios Xepapadeas†

July 10, 2018

**Abstract**

This paper studies the role of networks in the management of natural resources by assuming a finite number of agents who exploit a specific natural resource. Harvesting is subject to three external effects, namely resource stock externalities, crowding externalities, and collaboration spillovers. We show that the structure of the network determines both the equilibrium and the optimal harvesting amount and the corresponding value of the network. The centrality of each player – determined by the presence of active collaboration links with other agents – is crucial when determining the market and the optimal harvest. We then allow individual agents to make decisions about creating or eliminating cooperation links, which endogenizes the structure of the network and is proved to affect total harvesting and aggregate welfare. Conservation plans are shown to change the regulator's objective and increase even further the gap between the decentralized and the optimal outcomes. We show that the optimal policy depends explicitly on the structure of the network and the 'centrality' of the associated agents. Finally, introducing heterogeneity is proved to affect both the value of the network and the incentives to create cooperation links.

**JEL classification:** D85, H23, Q30.

**Keywords:** Environmental externalities, networks, natural resource management, optimal network structure.

*Department of Economics, Swedish University of Agricultural Sciences, Sweden. Efthymia.kyriakopoulou@slu.se

†Department of International and European Economic Studies, Athens University of Economics and Business, Greece and Department of Economics, University of Bologna, Italy. xepapad@aueb.gr

# 1 Introduction

Resource appropriators are usually not capable of designing rules that will secure the sustainable use of the resource over time. However, they are all jointly affected by everything each one of them is doing. Resource use is so interconnected that all agents should take into account the choices of the rest of the agents when taking individual decisions. This dependence of individual outcomes on group behavior is known as "peer effects" in the literature of social networks. In the context of natural resources, this intragroup externality may imply both positive and negative effects. For example, if a fisherman occupies a good fishing site, the second fisherman arriving at the same location will have to find another site. If, however, the two fishermen coordinate and agree in advance on who will occupy this site and who will move to another site, neither of them will have to spend time and energy on finding another location or fighting for the first one. Along the same lines, when a fisherman allocates time and material to improving some fishing area, those sites will be used by other fishermen who will also enjoy the effort and money spent by the first user. In this paper, we use a resource model that takes into account these intragroup externalities and analyze the resulting dependence of individual outcomes on group behavior.

Social network models are used to analyze non-market interactions. Thus, environmental and resource management issues – where externalities play a central role – can be clearly considered in the context of a social network. The nodes of the network could be polluting firms, or firms adopting clean technologies, agents harvesting an exhaustible or a renewable resource, countries emitting greenhouse gases and adopting mitigation or adaptation policies, or consumers engaging in polluting or pollution-reducing activities.

A particular characteristic of environmental networks is the fact that these networks can be characterized by strategic heterogeneity. That is, the network includes both strategic complementarities, emerging when the marginal payoff of an agent is increasing in the actions of her neighbors, and strategic substitutabilities, when the marginal payoff of an agent is decreasing in the actions of her neighbors. The first case may emerge in links indicating cost-reducing technology agreements and cooperation among agents. Examples of strategic substitutability could be the case of congestion effects, or increasing search or extraction costs when the stock of a resource exploited by the network is depleted. The study of this potential heterogeneity in a network context is shown to provide new insights in terms of market outcomes and policies to attain the social optimum.

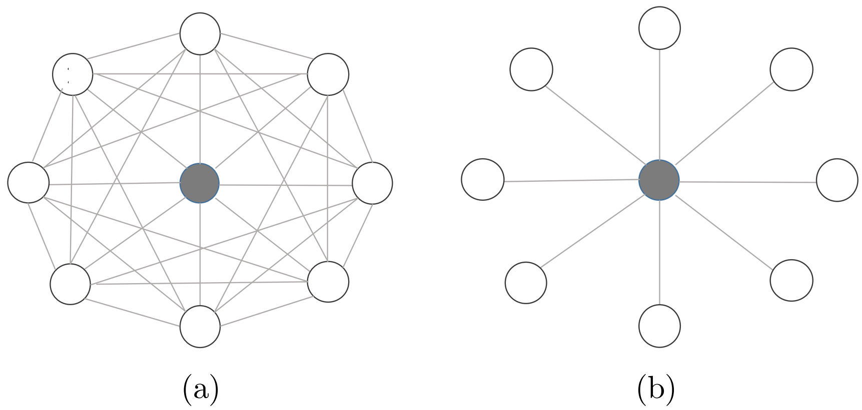



Figure 1: White nodes: resource appropriators. Gray node: resource. (a) Collective action, (b) Independent action

In this paper, we ask how the network structure, i.e., the links between the resource appropriators, affects the exploitation of the resource. The key fact here is that resource users are tied together as long as they continue to share the same resource. As pointed out by Ostrom (1990) in her seminal analysis about collective action in common pool resources (CPR), the physical interdepence does not disappear when the government or management regulators enforce effective institutional rules. The only thing that changes is the result that the appropriators obtain. Ostrom (1990) shows that when appropriators exploit a CPR independently (Figure 1(b)), the total benefit obtained is lower compared to what they would achieve if they had decided to coordinate their strategies (Figure 1(a)). This can lead either to lower returns or, in the worst case, in the destruction of the CPR. Thus, collective action is important in achieving the highest possible joint return.[1]

The way social networks affect human behavior in a natural resource framework is analyzed in a recent PNAS paper (Barnes et al., 2016). The authors show how homophily and the formation of social ties with homogeneous agents can directly impact marine ecosystems and impede the diffusion of sustainable behaviors. More precisely, they use data on fishers' information-sharing networks and observe fishing behaviors to show that social networks directly affect sustainable environmental outcomes. Network analysis can thus shed more light on the interdependence of resource appropriators by

[1]Collective action does not need to be associated with formal organizations. As pointed out in Ostrom (2004), collective action can be informal, where local networks or local goups organize and coordinate local action in order to achieve specific purposes.

analyzing the incentives of individual users to connect with the other users or not.

In this paper, the different results under collective, subgroup or independent action are analyzed and policies that take into account the specific network structure are designed so as to promote the formation of the optimal network structure. This is particularly important in a CPR problem since the problem that CPR appropriators face is how to organize. In other words, how to change the situation where appropriators act independently to a new one, where they adopt a more collective action approach and enjoy higher joint benefits while at the same time reducing their joint harm. Formally, in this paper we study stable network structures in equilibrium, as well as at the optimum, and determine policies that can close the gap between the optimal and the market network structures.

More specifically, in this paper we consider a network consisting of a finite number of agents located at distinct spatial locations who exploit a depletable resource which is located at a site different than the locations of the agents. The exploited quantity of the resource is subject to three external effects: (i) *Resource stock externalities*, which occur when total private cost increases when resource stock decreases, (ii) *Crowding externalities*, which imply that cost is increasing in the harvesting of co-appropriators, and (iii) *collaboration or knowledge spillovers*, which occur when agents are involved in some kind of collaboration activities by coordinating with the rest of the agents and thereby decreasing the negative effect of crowding externalities.[2] The existence of externalities implies that the market outcome differs from the optimal one. More precisely, in equilibrium the involved agents maximize their private profits. On the contrary, at the optimum, the social planner maximizes the private value of the network, which is the sum of the agents' payoffs, but at the same time she also takes into account the scarce nature of the resource by explicitly considering its conservation value. This could facilitate the discussions regarding conservation versus development, which is based on the fact that natural resources should not be treated in the same way as producible goods. Therefore one of the contributions of this paper is to show how the network structure and the associated payoffs are affected by this special nature of natural resources.

The creation of collaboration links between the appropriators is represented by a network: a link between two appropriators indicates that those two agents collaborate and agree on specific strategies that have to be followed when exploiting the resource.

[2]For a discussion on external effects characterizing natural resource exploitation see Smith (1968, 1969).

Congestion externalities are assumed to affect all users, regardless of whether they collaborate or not, and increase with the number of appropriators. Collaboration links could thus be interpreted as efforts to reduce the magnitude of the congestion externality. Taking turns to exploit the resource, for example, requires some minimum collaboration among the agents, but at the same time, it reduces congestion. The two opposing effects are illustrated in Figure 1. In Figure 1(b), we can see the effect of congestion externalities: the more users exploit the resource, the higher the implied congestion. In Figure 1 (a), we can observe the case where all appropriators collaborate with each other. So apart from the fact that they all exploit the same resource (as in Figure 1(b)), they also collaborate with each other, which results in a different network structure.

Following Ballester et al. (2006), we decompose the different effects into three components. The first one is the idiosyncratic component which captures the concavity of the payoff function into own effort (the private cost of exploiting the resource). The second one is the global interaction component, which captures the congestion force in our model, or in more general terms the strategic substitutability in efforts across all players. The last component includes the local interaction network, which reflects the existence of potential collaboration links between two nodes and, in general terms, it shows the strategic complementarities in efforts that differ across pairs of players. These local complementaries are captured by the network structure.

Our network game has an interior Nash equilibrium that is proportional to the Bonacich network centrality. Using this Bonacich measure allows us to express the relation between the equilibrium strategic behavior and the network topology. More precisely, we show that the individual agent's equilibrium outcome is related to the player's position in the local interactions network. We also show that the aggregate equilibrium effort increases with the density and with the number of local interactions.

More generally, the purpose of this paper is to make a step toward the study of environmental issues in a network context, by studying a network associated with the exploitation of a depletable resource which is characterized by strategic heterogeneity. Our purpose is to study the market outcomes associated with specific network structures and eventually characterize the most efficient market structure. We also characterize the socially optimal network structure and examine policies under which market outcomes will reproduce the social optimal structure.

Our results have important policy implications. The main advantage and contribution of the network approach is that by identifying the efficient structure, desirable or non-desirable links among the agents can be determined and policies can therefore be designed not just to control the level of externalities but also to control for the desired link structure among agents under strategic complementarities and substitutabilities. Disregarding the structure of the network – when this structure determines its value – is shown to result in inefficient policies.

The rest of the paper is organized as follows. The next section presents the related literature. Section 3 presents the model and solves for the market outcome. Section 4 solves for the optimum, while in section 5, we describe the dynamics of network formation. In section 6 we apply the theory in a small numbers network. Section 7 extends the model by introducing heterogeneity and the final section concludes the paper.

## 2 Related Literature

In order to study the dependence of individual outcomes on group behavior, we use the tools and the theory of social networks. The main areas where social networks have been studied up to now include financial networks, labour markets, development economics, exchange theory, bargaining, and trade.[3] However, despite the critical importance of social networks on environmental issues, network theory has been neglected in environmental and resource economics literature. There are, though, some exceptions of recent theoretical contributions that include more explicit network structures in their analysis. In this context, Conley and Udry (2001) study the role of social networks on the adoption of innovative, environmentally-friendly technologies by firms and show that high network density leads to the development of new technology and the diffusion of more sustainable management practices. In a more recent paper, Günther and Hellman (2017) study the stability of International Environmental Agreements (IEA) when pollution has both a global and a local effect, where local pollution spillovers are represented by a network structure. Their main finding is that for stable IEAs to exist, the network structure needs to be balanced.

Closer to the present study is İlkiliç (2011), who uses a network model of common property resources with multiple sources and users to study how the exploitation of each source by a different number of users, as well as the connection of each user with one or more sources, leads to different levels of extraction by users and outflows

[3]For overviews, see Ioannides (2012), Jackson (2014), Jackson and Zenou (2015), and Jackson et al. (2017).

from sources. Contrary to our paper where the network represents the collaboration links between the different appropriators, in İlkiliç (2011) the links connect users to a different number of sources. In other words, İlkiliç (2011) studies the right of different users to exploit a different number of sources, while here, we explore the incentives of appropriators to collaborate with the rest of the users and show how denser networks affect aggregate harvesting and aggregate profits.

Barnes et al. (2016) study how the formation of social networks affects fishers' behavior and actions and show that fishers form subgroups with people of the same ethnicity, which is strongly correlated with shark bycatch. In particular, their analysis suggests that enhanced communication across segregated groups could have prevented the incidental catch of over 46,000 sharks between 2008 and 2012 in a single commercial fishery. Their results suggest that having a better understanding of social interactions across resource users groups could potentially lead to more sustainable environmental outcomes. For a broad discussion on how networks can be used in order to facilitate the study of environmental issues see Currarini et al. (2016). The authors explain that the application of network economics to environmental and resource issues is still in its infancy and there are a lot of issues that need to be explored by using this new approach.

# 3 Cake-Eating and Competitive Exploitation in a Network

## 3.1 Nash Equilibrium and Bonacich Centrality

We consider a depletable resource of fixed stock $S$ that is exploited by $n$ agents. Exploitation lasts one period and harvest depends on the amount of effort, $E$, that is made during harvesting and on the size of the resource stock, $S$. Then, the harvest function is given by:

$$H(E,S) = qES \tag{1}$$

where $q$ is the "catchability-coefficient," which can translate one unit of effort into one unit of harvest. Solving (1) with respect to $E$ will give us:

$$E_i = \frac{H_i}{qS}$$

When the resource is more scarce, the agents need to put more effort into harvesting it, which increases the private cost of harvesting. Thus, the pay-off of agent $i$ will be:

$$u_i = \underbrace{pH_i - \frac{\gamma}{2}H_i^2 - \frac{1}{2}\frac{\beta}{(qS)^2}H_i^2}_{\text{"own" net profit}} \underbrace{-\gamma\sum_{j\neq i}^{n}H_iH_j}_{\text{global interaction effect}} + \underbrace{\delta\sum_{j=1}^{n}g_{ij}H_iH_j}_{\text{local interaction effect}} , \tag{2}$$

where $p$ is the price of the resource which is taken as exogenous. Along with the "own" net benefits of harvest, appropriators have to consider two more effects: the global interaction effect and the local interaction effect. The first one is interpreted as a global substitutability effect, which in our case creates some type of congestion. The more the rest of the agents exploit the resource, the lower my benefit is. The second effect reflects the local complementarity component. That is, collaboration links between the agents decrease the magnitude of the congestion externality, since agents, for example, could coordinate and agree to take turns when exploiting the resource.[4] In a resource context, it is more probable to have $\gamma > \delta$, meaning that collaboration between appropriators will reduce the magnitude of the congestion externality. However, it is possible to have (and interesting to explore) the case where $\delta > \gamma$, which implies that collaboration outweighs congestion.

We set $g_{ii} = 0$ and $0 \leq g_{ij} \leq 1$, for $i \neq j$. For simplicity, $g_{ij}$ will be either 0 when there is no link between nodes $i$ and $j$, or 1 when agents $i$ and $j$ decide to collaborate. Rearranging (2), the maximization problem can be written as:

$$\max_{H_i} \; pH_i - \frac{1}{2}\left(\frac{\beta}{(qS)^2} - \gamma\right)H_i^2 - \gamma\sum_{j=1}^{n}H_iH_j + \delta\sum_{j=1}^{n}g_{ij}H_iH_j$$

$$\text{subject to} \sum_{j=1}^{n}H_j \leq S \;,\; H_j \geq 0.$$

[4]Here, we assume that $\delta$ captures the collaboration benefits, net of collaboration costs. Alternatively, we could explicitly assume that the creation of collaboration links is costly and the cost to create or keep a collaboration link is equal to $c$. In this case, net collaboration benefits would be given by: $\delta - c$.

The Kuhn-Tucker conditions imply the following first-order necessary conditions:

$$p-\frac{\beta}{q^{2}S^{2}}H_{i}-\gamma\sum_{j=1}^{n}H_{j}+\delta\sum_{j=1}^{n}g_{ij}H_{j}\leq 0\ ,\ \ H_{j}\geq 0$$

$$\mu_{j}\left(S-\sum_{j=1}^{n}H_{j}\right)=0\ ,\ \ \mu_{j}\geq 0. \tag{3}$$

To simplify the exposition, we assume for the rest of the paper that the resource is not exhausted in equilibrium, so that the corresponding Lagrangian multipliers $\mu_j, j = 1, ...n$ are zero. For interior solutions equilibrium harvesting solves

$$H_{i}^{*}=\frac{p-\gamma\sum_{j=1}^{n}H_{j}^{*}+\delta\sum_{j=1}^{n}g_{ij}H_{j}^{*}}{\frac{\beta}{q^{2}S^{2}}+\gamma}. \tag{4}$$

We will now define the Bonacich network centrality measure that will be used in the equilibrium analysis. For this purpose, we will use the n-square adjacency matrix $\mathbf{G}$ that keeps track of all the direct connections in the network $\mathbf{g}$. Thus, $g_{ij}$ will show if agents $i$ and $j$ are *directly* connected $(g_{ij} = 1)$ or not $(g_{ii} = 0)$. In order to take into account the *indirect* connection, we need to define the matrix $\mathbf{G}^k$, which is the $k$th power of $\mathbf{G}$, with coefficient $g_{ij}^{[k]}$, where $k$ is some integer. More specifically, $g_{ij}^{[k]} \geqslant 0$ measures the number of paths of length $k \geq 1$ in $\mathbf{g}$ from $i$ to $j$. We can then define the matrix

$$\mathbf{M}(\mathbf{g},a)=[\mathbf{I}-a\mathbf{G}]^{-1}=\sum_{k=0}^{+\infty}a^{k}\mathbf{G}^{k}$$

where $a \geq 0$ is a scalar that takes low enough values. This parameter gives relatively lower weight to longer paths. The coefficients $m_{ij}(g, a) = \sum_{k=0}^{+\infty} a^k g_{ij}^{[k]}$ of the $\mathbf{M}(\mathbf{g}, a)$ matrix count the number of paths that start from $i$ and end in $j$, while the paths of length $k$ are weighted by $a^k$.

We can now define the Bonacich centrality of network $\mathbf{g}$ which is $\mathbf{b}(\mathbf{g},a) = [\mathbf{I}-a\mathbf{G}]^{-1}\cdot \mathbf{1}$. The Bonacich centrality of node $i$ is then $b_i(\mathbf{g}, a) = \sum_{j=1}^{n} m_{ij}(\mathbf{g}, a)$, which is the sum of all self-loops and all the other loops, i.e., $b_i(\mathbf{g}, a) = m_{ii}(\mathbf{g}, a) + \sum_{\substack{j=1\\ j\neq i}}^{n} m_{ij}(\mathbf{g}, a)$.

Let us assume that $\lambda_1(\mathbf{G})$ is the largest eigenvalue of $\mathbf{G}$, with $\lambda_1(\mathbf{G}) > 0$. Then if $\frac{\beta}{q^2S^2} > \delta\lambda_1(\mathbf{G})$, we can show that there is a unique Nash equilibrium.[5] Let us

[5]The condition $\frac{\beta}{q^2S^2} > \delta\lambda_1(\mathbf{G})$ requires that local complementarities are small enough compared to own concavity, that prevents multiple equilibria and also guarantees that $\left(\frac{\beta}{q^2S^2}\mathbf{I} - \delta\mathbf{G}\right)$ is invertible (see Ballester et al., 2006).

first define the square matrix of aggregate cross-effects: $\mathbf{\Sigma} = [\sigma_{ij}]$, as follows: $\mathbf{\Sigma} = -\frac{\beta}{q^2S^2}\mathbf{I} - \gamma\mathbf{U} + \delta\mathbf{G}$, where $\mathbf{U}$ denotes the n-square matrix of ones. An interior Nash equilibrium in pure strategies $\mathbf{H}^* \in \mathbb{R}^n_+$ is such that $\frac{\partial u_i}{\partial H_i(\mathbf{H}^*)} = 0$ and $H_i^* > 0$ for all $i = 1, ..., n$. If such an equilibrium exists, then it solves:

$$-\mathbf{\Sigma} \cdot \mathbf{H} = \left[\frac{\beta}{q^2S^2}\mathbf{I} + \gamma\mathbf{U} - \delta\mathbf{G}\right] \cdot \mathbf{H} = p \cdot \mathbf{1}. \tag{5}$$

The matrix $\frac{\beta}{q^2S^2}\mathbf{I} + \gamma\mathbf{U} - \delta\mathbf{G}$ is generically nonsingular and the above equation has a unique generic solution in $\mathbb{R}^n_+$, denoted by $\mathbf{H}^*$. Since $\mathbf{U}$ is an n-square matrix of ones, $\mathbf{U} \cdot \mathbf{H}^* = H^*\mathbf{1}$. Then,

$$\begin{aligned}\left[\frac{\beta}{q^2S^2}\mathbf{I} + \gamma\mathbf{U} - \delta\mathbf{G}\right] \cdot \mathbf{H}^* &= p \cdot \mathbf{1} \\ \frac{\beta}{q^2S^2}\left[\mathbf{I} - \frac{\delta q^2S^2}{\beta}\mathbf{G}\right] \cdot \mathbf{H}^* &= (p - \gamma H^*) \cdot \mathbf{1}\end{aligned}$$

By inverting the matrix, we get:

$$\begin{aligned}\frac{\beta}{q^2S^2}\mathbf{H}^* &= (p - \gamma H^*)\left[\mathbf{I} - \frac{\delta q^2S^2}{\beta}\mathbf{G}\right]^{-1} \cdot \mathbf{1} \\ \frac{\beta}{q^2S^2}\mathbf{H}^* &= (p - \gamma H^*)\mathbf{b}\left(\mathbf{g}, \frac{\delta q^2S^2}{\beta}\right) \\ \mathbf{H}^*(\mathbf{\Sigma}) &= \frac{p}{\frac{\beta}{q^2S^2} + \gamma b\left(\mathbf{g}, \frac{\delta q^2S^2}{\beta}\right)}\mathbf{b}\left(\mathbf{g}, \frac{\delta q^2S^2}{\beta}\right)\end{aligned} \tag{6}$$

where $b\left(\mathbf{g}, \frac{\delta q^2S^2}{\beta}\right)$ is the sum of the unweighted Bonacich centralities of all players:

$$b\left(\mathbf{g}, \frac{\delta q^2S^2}{\beta}\right) = b_1\left(\mathbf{g}, \frac{\delta q^2S^2}{\beta}\right) + b_2\left(\mathbf{g}, \frac{\delta q^2S^2}{\beta}\right) + .... + b_n\left(\mathbf{g}, \frac{\delta q^2S^2}{\beta}\right)$$

Expression (6) shows that more connected individuals extract a higher quantity of the resource, compared to less connected agents.

**Proposition 1** *Let* $\mathbf{g}$ *and* $\mathbf{g}'$ *be symmetric such that* $\mathbf{g}' \geq \mathbf{g}$*. Then,* $\sum_i H_i^*(\mathbf{g}') > \sum_i H_i^*(\mathbf{g})$*, if* $\delta\lambda_1(\mathbf{g}) < \frac{\beta}{q^2S^2}$ *and* $\delta\lambda_1(\mathbf{g}') < \frac{\beta}{q^2S^2}$.

**Proof.** Proposition 1 shows that a more dense network leads to higher aggregate resource use. To prove this, let us assume that we have a partly connected network, $\mathbf{g}$,

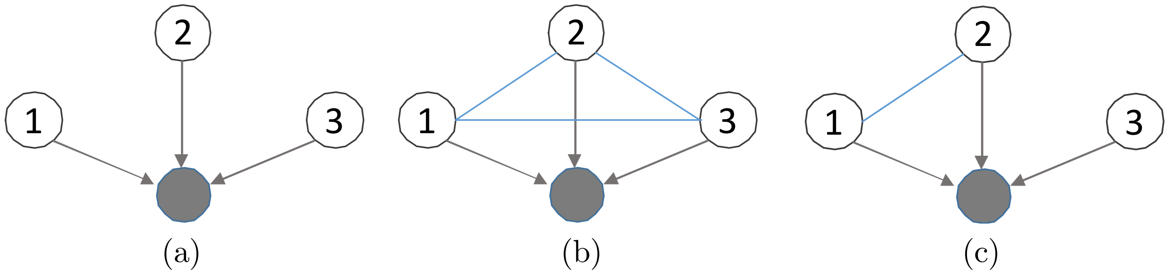



Figure 2: A 3-agent resource network

where agents $i = 1, ...., m$ are connected and $j = m+1, ...., n$ are unconnected. Then, from (6), we can calculate individual harvesting for two representative agents of the two groups, say agent 1 and $m+1$. In particular, $H_1^*(\mathbf{g}) = \frac{p}{\frac{\beta}{q^2S^2}+\gamma b\left(\mathbf{g},\frac{\delta q^2 S^2}{\beta}\right)} b_1\left(\mathbf{g}, \frac{\delta q^2 S^2}{\beta}\right)$ and $H_{m+1}^*(\mathbf{g}) = \frac{p}{\frac{\beta}{q^2S^2}+\gamma b\left(\mathbf{g},\frac{\delta q^2 S^2}{\beta}\right)}$, since the Bonacich centrality of the unconnected agent is equal to 1, while $b_1\left(\mathbf{g}, \frac{\delta q^2 S^2}{\beta}\right) > 1$. If agent $m+1$ now connects to agents $i = 1, ...., m$, the two groups of connected and unconnected agents will be $i = 1, ...., m+1$ and $j = m+2, ...., n$ respectively. This is a denser network, say $\mathbf{g}'$, since agent $m+1$ is now connected to all agents $i = 1, ...., m$. The new harvesting amounts of the two agents are now: $H_1^*(\mathbf{g}') = \frac{p}{\frac{\beta}{q^2S^2}+\gamma b\left(\mathbf{g}',\frac{\delta q^2 S^2}{\beta}\right)} b_1\left(\mathbf{g}', \frac{\delta q^2 S^2}{\beta}\right)$ and $H_{m+1}^*(\mathbf{g}') = \frac{p}{\frac{\beta}{q^2S^2}+\gamma b\left(\mathbf{g}',\frac{\delta q^2 S^2}{\beta}\right)} b_{m+1}\left(\mathbf{g}', \frac{\delta q^2 S^2}{\beta}\right)$, which are clearly higher than before. The same is true for the sum of the associated harvesting amounts of the two networks: $\sum_i H_i^*(\mathbf{g}') > \sum_i H_i^*(\mathbf{g})$. This is because in a denser network, appropriators form more collaboration links with their competitors, which allows them to reduce the magnitude of the congestion effect. Thus, market efficiency leads to higher aggregate harvesting in $\mathbf{g}'$ than in $\mathbf{g}$. ■

## 3.2 Examples

### 3.2.1 A 3-agent star network

To illustrate our results, we continue with a simple example of three unconnected agents who exploit the resource, as illustrated in Figure 2(a). Here, we assume that there is no local interaction effect, meaning that resource appropriators act independently. The arrows in Figure 2 show that all three agents are exploiting the same resource, creating some congestion externality.

Notice that since the agents are not connected, the adjacency matrix $\mathbf{G}$ is a $(3 \times 3)$

zero matrix, so $\mathbf{M} = \mathbf{M}^{-1} = \mathbf{I}$ and the Bonacich centrality vector is the vector of ones $\mathbf{b} = (1, 1, 1)$. Using (6), we can now compute the agents' harvesting amounts (with $b = 3$) :

$$H_i^* = \frac{pq^2 S^2}{\beta + 3\gamma q^2 S^2} \tag{7}$$

In the case where there is no congestion, or in the case of a single user, price equals private marginal cost, $p = \beta \left( \frac{H_i}{q^2 S^2} \right)$, which leads to $H_i^{NC} = \frac{pq^2 S^2}{\beta}$. It is easy to see that congestion makes harvesting more costly, which leads to a reduction in the equilibrium amount of harvesting:

$$\frac{dH_i^*}{d\gamma} = -\frac{3pq^4 S^4}{(\beta + 3\gamma q^2 S^2)^2} < 0$$

Thus, $H_i^* < H_i^{NC}$, for any $\gamma > 0$.

Individual profits under the presence of this negative externality are given by:

$$u_i^* = \frac{p^2 q^2 S^2 (\beta + \gamma q^2 S^2)}{2(\beta + 3\gamma q^2 S^2)^2} \tag{8}$$

while aggregate profits are given by:

$$u_T^* = \frac{3p^2 q^2 S^2 (\beta + \gamma q^2 S^2)}{2(\beta + 3\gamma q^2 S^2)^2} \tag{9}$$

#### 3.2.2 A 3-agent full network

Let us now analyze the case illustrated in Figure 2(b) above, where all agents are linked to each other, creating collaboration links. The adjacency matrix of this network is given by:

$$\mathbf{G} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

and the corresponding Bonacich centralities are given by the vector:

$$\mathbf{b} = \left( \frac{\beta}{\beta - 2\delta q^2 S^2}, \frac{\beta}{\beta - 2\delta q^2 S^2}, \frac{\beta}{\beta - 2\delta q^2 S^2} \right)$$

Harvesting for each agent is symmetric and given by:

$$\widehat{H}_i = \frac{pq^2 S^2}{\beta + (3\gamma - 2\delta) q^2 S^2} \tag{10}$$

It is easy to see that $\widehat{H}_i > H_i^*$, meaning that the fully connected network leads to higher use of the resource than the unconnected network. Individual equilibrium payoffs are given by:

$$\hat{u}_i = \frac{p^2 q^2 S^2 (\beta + \gamma q^2 S^2)}{2(\beta + (3\gamma - 2\delta) q^2 S^2)^2} \tag{11}$$

while aggregate profits are given by:

$$\hat{u}_T = \frac{3p^2 q^2 S^2 (\beta + \gamma q^2 S^2)}{2(\beta + (3\gamma - 2\delta) q^2 S^2)^2} \tag{12}$$

We can now illustrate Proposition 1 by considering the aggregate harvesting, $H_T^* = \sum_{i=1}^{3} H_i^*$, and profits of the star network (9) and the corresponding ones of the full network, $\widehat{H}_T = \sum_{i=1}^{3} \widehat{H}_i$, and (12). Not surprisingly, $\widehat{H}_T > H_T^*$ and $\hat{u}_T > u_T^*$, which confirms the fact that denser networks generate more aggregate profits that correspond to a higher harvesting amount than less dense networks.

#### 3.2.3 Partly connected network: Two types of agents

Lastly, we study the case illustrated in Figure 2(c) where two of the agents (say 1 and 2) create a collaboration link while the third agent acts independently. The adjacency matrix $\mathbf{G}$ in this case is:

$$\mathbf{G} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

and the vector of Bonacich centralities of the three players is:

$$\mathbf{b} = \left( \frac{\beta}{\beta - \delta q^2 S^2}, \frac{\beta}{\beta - \delta q^2 S^2}, 1 \right)$$

Harvesting for symmetric agents 1 and 2 is:

$$\tilde{H}_{1,2} = \frac{\beta p q^2 S^2}{\beta^2 + (3\gamma - \delta)\beta q^2 S^2 - \gamma \delta q^4 S^4}$$

while for agent 3:

$$\tilde{H}_3 = \frac{p q^2 S^2 (\beta - \delta q^2 S^2)}{\beta^2 + (3\gamma - \delta)\beta q^2 S^2 - \gamma \delta q^4 S^4}$$

Individual payoffs are given by:

$$\tilde{u}_{1,2} = \frac{\beta^2 p^2 q^2 S^2 (\beta + \gamma q^2 S^2)}{2(\beta^2 + (3\gamma - \delta)\beta q^2 S^2 - \gamma\delta q^4 S^4)^2}$$

$$\tilde{u}_3 = \frac{p^2 q^2 S^2 (\beta + \gamma q^2 S^2)(\beta - \delta q^2 S^2)^2}{2(\beta^2 + (3\gamma - \delta)\beta q^2 S^2 - \gamma\delta q^4 S^4)^2}$$

Notice that:

$$\tilde{u}_3 = \tilde{u}_{1,2} \frac{(\beta - \delta q^2 S^2)^2}{\beta^2} < \tilde{u}_{1,2}$$

which shows that, as expected, the unconnected agent enjoys lower profits in equilibrium compared to the connected agents, while aggregate profits are:

$$\tilde{u}_T = \frac{p^2 q^2 S^2 (\beta + \gamma q^2 S^2)(3\beta^2 - 2\beta\delta q^2 S^2 + \delta^2 q^4 S^4)}{2(\beta^2 + (3\gamma - \delta)\beta q^2 S^2 - \gamma\delta q^4 S^4)^2}$$

For appropriators 1 and 2, individual profits in the fully connected network (b) are higher than those in the partly connected network (c), which in turn, are higher than the ones associated with the unconnected network (a), i.e., $\hat{u}_{1,2} > \tilde{u}_{1,2} > u^*_{1,2}$. What is interesting here is the case of the third appropriator. While the full connected network structure gives her the highest profits, ($\hat{u}_3 > \tilde{u}_3$ and $\hat{u}_3 > u^*_3$), the comparison between profits in the partly connected network and profits in the unconnected network leads to a more surprising result. More precisely, for the "independent" appropriator, profits in the unconnected network are unambiguously higher than profits in the partly connected network (i.e., $u^*_3 > \tilde{u}_3$). This is explained by the fact that the first two agents enjoy higher profits resulting from higher harvesting when they connect, which puts the last agent in a disadvantageous position. This leads to the following result.

**Proposition 2** *Assume a partly connected network, where $g_{ij} = 1$, for some pairs of agents and $g_{ij} = 0$ for some other pairs. Then, stronger local interactions (i.e., higher $\delta$ value), will increase the individual profits of connected agents (say $i$), $\partial\tilde{u}_i/\partial\delta > 0$, while the opposite is true for the individual profits of unconnected agents (say $j$), $\partial\tilde{u}_j/\partial\delta < 0$. Aggregate profits always increase with stronger local interactions ($\partial\tilde{u}_T/\partial\delta > 0$).*

***Proof.*** *The proof is available in Appendix A.1 for a 3-agent network and is numerically shown to hold for an 8-agent network in section 7 (Table 1).* ■

# 4 Optimal Cake-Eating in the Network

## 4.1 Conservation and Optimal Use of the Resource

When the global interaction effect is larger than the local interaction effect, the negative congestion externality dominates and there is too much extraction at the Nash equilibrium compared to the social optimum outcome, since individuals ignore the negative externality that their effort has on others. Also, the appropriators do not consider the scarce nature of the resource which leads to even further increases in the gap between the market and the optimal harvest. Then, there is room for government intervention which can take the form of a Pigouvian tax.

Let us assume that the regulator's objective is to maximize the sum of individual payoffs and at the same time consider the welfare loss that is associated with the reduction in the stock of the resource. If $\kappa > 0$ denotes the (per unit) value of the unharvested resource then the social planner chooses $H_1, H_{2,}...., H_n$ to maximize total welfare:

$$\max_{\mathbf{H}^o} W^o(\mathbf{H}^o, \mathbf{g}) = \max_{H_i} \sum_{i=1}^{n} u_i(H_i^o, g) + \kappa \left( S - \sum_{i=1}^{n} H_i^o \right)$$

The first-order necessary conditions for an interior solution are:

$$p - \kappa - \frac{\beta}{q^2 S^2} H_i - \gamma H_i - 2\gamma \sum_{\substack{j=1 \\ j \neq i}}^{n} H_j + 2\delta \sum_{j=1}^{n} g_{ij} H_j = 0$$

$$H_i^o = \frac{p - \kappa - 2\gamma \sum_{j=1}^{n} H_j + 2\delta \sum_{j=1}^{n} g_{ij} H_j}{\frac{\beta}{q^2 S^2} + \gamma} \tag{13}$$

It is clear that for $\gamma > \delta$, $H_i^o < H_i^*$, i.e., equilibrium individual resource use is higher than the optimal value.

In matrix form,

$$\mathbf{H}^o(\mathbf{\Sigma}) = \frac{p - \kappa}{\frac{\beta}{q^2 S^2} + 2\gamma b \left( \mathbf{g}, \frac{2\delta q^2 S^2}{\beta} \right)} \mathbf{b} \left( \mathbf{g}, \frac{2\delta q^2 S^2}{\beta} \right) \tag{14}$$

The higher the interest of the regulator with respect to the conservation of the resource ($\kappa > 0$), the lower the optimal individual (and aggregate) harvesting amount, since $\frac{\partial H_i^o}{\partial \kappa} = -\frac{q^2 S^2}{\beta + \gamma q^2 S^2} < 0$. This captures the fact that society has a benefit from preserving part of the natural resource for future use, which implies lower harvesting at the optimum. Another interesting observation concerns the "stock effect": $\frac{\partial H_i^D}{\partial S} > \frac{\partial H_i^o}{\partial S} > 0$, where $H_i^D$ and $H_i^o$ are the optimal harvesting amounts when $\kappa = 0$ and $\kappa > 0$, respectively. This shows that increases in the stock of the resource will increase $H_i^D$ to a larger extent compared to the "optimal harvesting under preservation", $H_i^o$. In the second case, the trade-off between the benefit of harvesting a higher amount of the resource versus the benefit of preserving part of it leads to a lower resource exploitation.

Note that the solution of the optimal cake-eating (14) is conditional on the existing network structure. In other words, in the maximization problem above the regulator does not choose the density of the network since the network structure is taken as given. Thus, the question remains: is there any socially optimal network structure? To answer this question, the social planner should determine, in addition to the optimal harvesting, an optimal link configuration, such as:

$$\{g_{ij}^*\}, \ i, j = 1, ...., n, \ g_{ij} = 0.$$

Thus, the full social optimum, including the optimal network structure, will be determined by the solution to the problem:

$$\begin{aligned} &\max_{\mathbf{H},\mathbf{g}} W^o(\mathbf{H}, \mathbf{g}) = \max_{H_i, g_{ij}} \sum_{i=1}^{n} u_i(H_i, g) + \kappa \left( S - \sum_{i=1}^{n} H_i \right) \\ &\text{subject to} \qquad g_{ij} = 0, \quad g_{ij} \in \{0, 1\}, \quad i, j = 1, ..., n, \quad i \neq j. \end{aligned} \tag{15}$$

This is a concave mixed-integer, nonlinear optimization problem (MI-NLOP) and its solutions can be obtained using numerical optimization methods. In section 7 below, we solve this problem numerically and derive both the optimal harvesting amount and the optimal network structure.

## 4.2 Optimal Policy

As discussed above, the optimal policy, in the case where the net congestion externality is positive, will take the form of a Pigouvian tax. This tax will depend explicitly on the network centrality of the agent and will internalize the net congestion externality that an agent imposes on the rest of the agents when harvesting a unit of the resource. In other words, an agent who is much more engaged in coordination/collaboration efforts trying to reduce the negative congestion effect will have to pay a lower amount in the form of taxation than an agent who acts more "independently".

If $\tau_i^o$ denotes the optimal tax per unit of the harvest, it is clear from the comparison between the equilibrium and the optimal FOC that the optimal tax is given by:

$$\tau_i^o = \gamma \sum_{j=1}^{n} H_j^o - \delta \sum_{j=1}^{n} g_{ij} H_j^o + \kappa \tag{16}$$

for $\kappa \geq 0$.

The optimal policy instrument that can be used to reproduce the optimal outcome as an equilibrium solution is an agent-specific tax. The tax will depend explicitly on the network centrality of the agent and will internalize the net congestion externality that an agent imposes on the rest of the agents when harvesting a unit of the resource. In terms of equation (16), more central agents are connected to a higher number of appropriators, which increases the second part of the RHS of equation (16). This implies a lower agent-specific tax as compared to the tax imposed on a "less-connected" agent.

The timing will be as follows: first, the government will announce a tax per unit of resource extracted that will be equal to $\tau_i^o(\mathbf{H}) \geq \mathbf{0}$, for each individual $i = 1, ..., n$. Then $i$ will choose the amount of the resource use that will maximize her payoff:

$$\begin{aligned} u_i &= (p - \tau_i^o) H_i - \frac{1}{2}\left(\frac{\beta}{q^2 S^2} - \gamma\right) H_i^2 - \sum_{j=1}^{n} \gamma H_i H_j + \delta \sum_{j=1}^{n} g_{ij} H_i H_j \\ &= pH_i - \frac{1}{2}\left(\frac{\beta}{q^2 S^2} - \gamma\right) H_i^2 - 2\sum_{j=1}^{n} \gamma H_i H_j + 2\delta \sum_{j=1}^{n} g_{ij} H_i H_j \end{aligned} \tag{17}$$

This allows the social planner to restore the optimum as an equilibrium outcome and to "punish" collaborative agents relatively less. The optimal policy is stricter in case the regulator follows a conservation plan that takes into account the value of the unharvested resource ($\kappa > 0$).

To determine the tax and express it in terms of Bonacich centrality, we follow the process below. When the regulator imposes a tax $\tau_i$ per unit $H_i$, the agents take the tax rate as given and maximize their profits. Then, the Nash equilibrium ($\mathbf{H}^*$) under a non-uniform tax should solve:

$$-\mathbf{\Sigma} \cdot \mathbf{H}^* = \left[\frac{\beta}{q^2 S^2}\mathbf{I} + \gamma \mathbf{U} - \delta \mathbf{G}\right] \cdot \mathbf{H}^* = p - \tau$$

where $\tau$ is the vector of agent-specific taxes, $\tau = (\tau_1, ...\tau_n)$ .

Then, in Appendix A.2 we show that the harvesting amount is given by:

$$\mathbf{H}^*(\mathbf{\Sigma}) = \frac{1}{\frac{\beta}{q^2S^2} + \gamma b\left(\mathbf{g}, \frac{\delta q^2 S^2}{\beta}\right)}\left[p\mathbf{b}\left(\mathbf{g}, \frac{\delta q^2 S^2}{\beta}\right) - \mathbf{b}_\tau\left(\mathbf{g}, \frac{\delta q^2 S^2}{\beta}\right)\right]$$

where $\mathbf{b}_\tau\left(\mathbf{g}, \frac{\delta q^2 S^2}{\beta}\right) = \left[\mathbf{I} - \frac{2\delta q^2 S^2}{\beta}\mathbf{G}\right]^{-1} \cdot \tau = \mathbf{M}(\mathbf{g}, \frac{\delta q^2 S^2}{\beta}) \cdot \tau$ is the weighted Bonacich centrality.[6]

Then the optimal tax should solve:

$$\begin{aligned}
&\mathbf{H}^*(\tau) = \mathbf{H}^o, or \\
&\frac{1}{\frac{\beta}{q^2S^2} + \gamma b\left(\mathbf{g}, \frac{\delta q^2 S^2}{\beta}\right)}\left[p\mathbf{b}\left(\mathbf{g}, \frac{\delta q^2 S^2}{\beta}\right) - \mathbf{M}(\mathbf{g}, \frac{\delta q^2 S^2}{\beta}) \cdot \tau\right] \\
&= \frac{p}{\frac{\beta}{q^2S^2} + 2\gamma b\left(\mathbf{g}, \frac{2\delta q^2 S^2}{\beta}\right)}\mathbf{b}^0\left(\mathbf{g}, \frac{2\delta q^2 S^2}{\beta}\right).
\end{aligned}$$

Using the optimal values for $\mathbf{H}^*, \mathbf{H}^o$, we obtain:

$$\mathbf{H}^* - \mathbf{H}^o = \frac{\mathbf{M}(\mathbf{g}, \frac{\delta q^2 S^2}{\beta}) \cdot \tau}{\frac{\beta}{q^2S^2} + \gamma b\left(\mathbf{g}, \frac{\delta q^2 S^2}{\beta}\right)}$$

and finally the vector of the optimal tax is:

$$\tau = \left[\frac{\beta}{q^2S^2} + \gamma b\left(\mathbf{g}, \frac{\delta q^2 S^2}{\beta}\right)\right]\left[\mathbf{M}(\mathbf{g}, \frac{\delta q^2 S^2}{\beta})\right]^{-1}(\mathbf{H}^* - \mathbf{H}^o) \tag{18}$$

It is clear from both (16)and (18) that the optimal policy is an agent-specific tax, which depends on the centrality of each agent. This leads to the following proposition:

**Proposition 3** *Assuming that $\delta\lambda_1(\mathbf{g}) < \frac{\beta}{q^2S^2}$, then a tax of the form $\tau_i^o = \gamma\sum_{j=1}^n H_j^o - \delta\sum_{j=1}^n g_{ij}H_j^o$ will implement the optimal allocation as an equilibrium outcome.*

**Proof.** When the regulator announces a tax per unit of resource extracted which is equal to $\tau_i^o(\mathbf{g})$, then agents maximize their payoffs given by (17). It is easy to see that the first-order condition derived by (17) is the same as the optimal FOC given by (13).

[6] This is in line with Remark 1 of Ballester et al. (2006) which explains the case of heterogeneous agents.

That is, the equilibrium harvesting amount when an agent-specific tax is enforced, $\tau_i^o(\mathbf{g})$, is equal to the optimal harvesting amount. ■

**Proposition 4** *The optimal harvesting as an equilibrium outcome can only be achieved by using the agent-specific taxation described in Proposition 3. Uniform taxes that produce the same revenues do not lead to the optimal amount of harvesting.*

**Proof.** Agents can be heterogeneous with respect to the number of links they form with their co-appropriators, which implies that the individual net congestion externality is different for different agents. However, the optimal taxation should fully internalize this externality, which means that different levels of the associated external effect lead to different tax levels. Also, the derived equilibrium level of resource exploitation is unique, and any other level of taxation will not satisfy the zero profit condition for the same amount of the resource harvested, and will not constitute an equilibrium outcome. ■

Note that the optimal policy, as described above, will restore the first best in a given network structure. However, it will not induce the optimal network structure. If the existing stable market structure for the network is different from the optimal structure then the regulator will have to implement the optimal structure with other instruments (e.g., command and control).

An alternative policy instrument that is proposed in similar contexts is the use of permits. In terms of our model, if permits are to be used then the number of permits should restrict the use of the resource and the optimal number of permits should be equal to $\mathbf{H}^o$. In other words, the number of permits will be equal to the total amount of harvesting at the optimum. However, permits trading (if allowed) would result in an equilibrium price level which would be the same for all the agents. On the contrary, in the case of taxation, appropriators pay an agent-specific per-unit tax, which, as has been extensively explained above, differs with respect to their centrality in the network. Thus, it is clear that tradable permits can only be used as a second-best instrument.

## 4.3 Example

### 4.3.1 A 3-agent star network

The difference between the market and the optimal outcomes can be illustrated in the small numbers network. Let us use the 3-agent example that was presented above (section 3.2) and derive the optimum in the presence of the global interaction effect. In this case, the regulator will choose $H_1, H_2, H_3$, to maximize total welfare. Then, the optimal use of the resource is given by:

$$H_i^S = \frac{pq^2S^2}{\beta + 5\gamma q^2 S^2}, \tag{19}$$

and the optimal level of aggregate profits:

$$W(H_i^S) = \frac{3p^2q^2S^2}{2(\beta + 5\gamma q^2 S^2)^2} \tag{20}$$

Comparing the market (7) and the optimal harvesting (19) in a network where there is only one negative externality, namely congestion, it is easy to see that harvesting is lower at the optimum. Agents should reduce the amount of the resource they are exploiting in order to reduce the negative effects that are imposed on the rest of the agents. A tax per unit of output that will fully internalize the damage caused by congestion will increase the cost of harvesting and will reduce the use of the resource to the optimal level.

#### 4.3.2 A 3-agent full network

Let us now turn to the case where all externalities are present and agents have created collaboration links between each other. The regulator takes into account these positive local interactions and the optimal use of the resource is given by:

$$\hat{H}_i^S = \frac{pq^2S^2}{\beta + (5\gamma - 4\delta) q^2 S^2} \tag{21}$$

while the optimal value of the network is given by:

$$W(\hat{H}_i^S) = \frac{3pq^2S^2(\beta(4\delta + p) + (5\gamma p - 8\delta p + 4\delta(5\gamma - 4\delta))q^2S^2)}{2(\beta + (5\gamma - 4\delta)q^2S^2)^2} \tag{22}$$

It is interesting to again compare the optimal use of the resource, with the decentralized use in the corresponding case, $\widehat{H}_i = \frac{pq^2S^2}{\beta + (3\gamma - 2\delta)q^2S^2}$. If the global interaction effect, $\gamma$, is higher than the local interaction effect, $\delta$, the use of the resource will be smaller at the optimum than in the decentralized case, $\hat{H}_i^S < \hat{H}_i$. Internalizing the net congestion effect leads to lower use of the resource at the optimum. Notice that the optimal value of the network (or else aggregate welfare) under the presence of two externalities (given by 22) is higher than the optimal value of the network under congestion externality (described by 20) which clearly shows that the regulator has an incentive to enforce policies that will promote the collaboration between the agents.

#### 4.3.3 Optimal Taxation

We can now calculate the optimal taxation for all the network structures presented in Figure 2. This tax will restore the optimum for a given network structure. In the case of symmetric agents, in a star network (Figure 2(a)), the optimal tax is the same for all individuals and is equal to:

$$t_S = \frac{2\gamma p q^2 S^2}{\beta + 5\gamma q^2 S^2}$$

while in the full network (Figure 2(b)), the optimal tax is clearly lower than the one imposed on the appropriators of the star network, or:

$$t_F = \frac{2(\gamma - \delta) p q^2 S^2}{\beta + (5\gamma - 4\delta) q^2 S^2} < t_S$$

In the partly connected network (Figure 2(c)), the tax imposed on the connected agents (1 and 2) is different than the one imposed on the unconnected agent. In particular, the connected agents have to pay a per unit of harvest tax, equal to:

$$t^{1,2} = \frac{2pq^2 S^2 \gamma(\beta - q^2 S^2 \gamma) - pq^2 S^2 \delta(\beta - q^2 S^2 \gamma)}{\Omega}$$

where $\Omega = \beta^2 + 2\beta q^2 S^2 (2\gamma - \delta) - q^4 S^4 \gamma(5\gamma + 2\delta)$, while the unconnected agent will pay a higher tax given by:

$$t^3 = \frac{2pq^2 S^2 \gamma(\beta - q^2 S^2 \gamma)}{\Omega} > t^{1,2}$$

The higher tax punishes agent 3 in the partly connected network for the externality she imposes on the rest of the agents, without making any effort to reduce the magnitude of the externality (since $g_{i3} = 0,$ for $i = 1, 2$). In other words, agent 3 now has a stronger incentive to create a link with the other two appropriators since non-collaboration implies a higher cost. The comparison of agent-specific taxes in the three network structures gives: $t_S > t^{1,2} > t_F$ for agents 1 and 2, and $t^3 > t_S > t_F$ for agent 3. It is clear that it is more costly for agent 3 to stay unconnected in the partly connected network as compared to the star network.

# 5 Dynamics of Network Formation

In this section we examine whether the appropriators in our resource network will decide to create or sever links. We develop a simplified framework where at the beginning of the period the agents, given full information about congestion costs and cooperation spillovers, are going to decide to create new links or sever existing cooperation links. Their decisions will determine the structure of the network at the end of the period.[7]

The same approach can be applied to determine the socially efficient network structure. The regulator decides at the beginning of the period to retain or sever cooperation links using as an objective the maximization of aggregate payoffs plus the conservation value of the resource. The structure of the network at the end of the period will characterize the socially efficient network structure. If this structure is different from the efficient "market structure" the regulator has an incentive to intervene in order to provide incentive schemes which will attain the socially efficient market structure.

## 5.1 Market Network Structure

Assume that at the beginning of the period the network is star-shaped, that is, no cooperation links exist. For simplicity, we will use the 3-agent example presented above, but our results can be generalized in an $n-$agent network. Let $u_i^* \left(\mathbf{H}^S \mid -ij\right)$, $i, j = 1, 2, 3$ , $i \neq j$, denote the maximized payoff of each agent, given that no links exist, with $\mathbf{H}^S$ the vector of profit-maximizing harvesting when the network is star-shaped. For cooperation to emerge it should be profitable for both agents. Then, the following results can be stated, where $\mathbf{H}$ denotes the vector of profit-maximizing harvesting at each network structure, $+ij$ means that the link $ij$ is created and $-ij$ means that the link is not created, for $i, j = 1, 2, 3$ , $i \neq j$.

[7] It should be noted that this not a fully dynamic set-up. A fully dynamic set-up would consider a multi-period problem and examine the convergence to a stable network structure. Such a problem has been studied by Watts (2001) and Jackson and Watts (2002). In these papers, the payoff functions were, however, relatively simple and allowed analytic results. We regard our approach to the problem as a first-order approach with the development of a fully dynamic model being the next stage of our research.

a If

$$u_1(\mathbf{H} \mid +12, -13) > u_1^*\left(\mathbf{H}^S \mid -ij\right) \text{ and } u_1(\mathbf{H} \mid +12, -13) > u_1(\mathbf{H} \mid +ij),$$
$$u_2(\mathbf{H} \mid +21, -23) > u_2^*\left(\mathbf{H}^S \mid -ij\right) \text{ and } u_2(\mathbf{H} \mid +21, -23) > u_2(\mathbf{H} \mid +ij),$$

then, there will be a link between 1 and 2 but not with 3.

Note that it is possible to have:

$$\begin{aligned} & u_3(\mathbf{H} \mid +ij) > u_3^*\left(\mathbf{H}^S \mid -ij\right) \\ \text{or,} \quad & u_3(\mathbf{H} \mid +13, -23) > u_3^*\left(\mathbf{H}^S \mid -ij\right) \\ \text{or,} \quad & u_3(\mathbf{H} \mid +23, -13) > u_3^*\left(\mathbf{H}^S \mid -ij\right) \end{aligned}$$

but there will be no cooperation with 3 since this is not profitable for the other agents. Any equilibrium should satisfy the *pairwise stability* condition, which accounts for the mutual approval of both agents. Similar inequalities hold in the case where any two agents find it profitable to connect without including the third one.

b If

$$u_i(\mathbf{H} \mid +ij) > u_i^*\left(\mathbf{H}^S \mid -ij\right), i, j = 1, 2, 3\ , i \neq j,$$

then, all agents create cooperation links.

c If

$$u_i(\mathbf{H} \mid +ij) < u_i^*\left(\mathbf{H}^S \mid -ij\right), i, j = 1, 2, 3\ , i \neq j,$$

no cooperation links are created and the star-shaped network will remain until the end of the period.

The networks created in cases (a)-(c) are pairwise stable in the sense of Jackson and Wolinsky (1996). The efficiency of the network depends on the maximization of the individual payoffs:

$$u_h^M = \sum_{i=1}^{3} u_{ih}\ , h = a, b, c$$

The strongly efficient network is the one that implies the maximum aggregate payoffs, or:

$$u_h^M = \arg\max_h \sum_{i=1}^{3} u_{ih}\ , h = a, b, c$$

In our resource problem, as has been shown above, denser networks always increase aggregate harvesting and payoffs (see Proposition 1), since they decrease the magnitude of the congestion externality. Thus, when agents are homogeneous, the strongly efficient network is always the fully connected one. Introducing heterogeneity (see sections 6 and 7) can lead to different outcomes.

In case (a), if

$$\left[u_3\left(\mathbf{H} \mid +13, -23\right) - u_3^*\left(\mathbf{H}^S \mid -ij\right)\right] > \left[u_1\left(\mathbf{H} \mid +12, -13\right) - u_1\left(\mathbf{H} \mid +ij\right)\right]$$
$$\left[u_3\left(\mathbf{H} \mid +23, -13\right) - u_3^*\left(\mathbf{H}^S \mid -ij\right)\right] > \left[u_2\left(\mathbf{H} \mid +21, -23\right) - u_2\left(\mathbf{H} \mid +ij\right)\right]$$

agent 3 could bribe agents 1 and 2 in order to convince them to cooperate with her.

# 6 Extensions: Introducing Heterogeneity

## 6.1 Costly Transportation

Let us assume now that the transportation of the resource is costly and the cost depends on the geographical distance between the agent and the resource. Until now, we had assumed that the agents involved in the exploitation of the resource were homogeneous. Costly transportation introduces heterogeneity in the model, since agents who locate further away from the resource have to pay a higher transportation cost, which increases the total cost of harvesting, affects their harvesting decisions and decreases their payoffs.

Let $l_i \in (0, 1]$ represent the location of the agent $i$, defined as her distance from the resource, and $\xi$ represents the marginal transportation cost.

Then, using the notation $\hat{p}_i = p - \xi l_i$, equation (5) becomes:

$$-\mathbf{\Sigma} \cdot \mathbf{H} = \left[\frac{\beta}{q^2 S^2}\mathbf{I} + \gamma \mathbf{U} - \delta \mathbf{G}\right] \cdot \mathbf{H} = \hat{\mathbf{p}}$$

The matrix $\frac{\beta}{q^2 S^2}\mathbf{I} + \gamma \mathbf{U} - \delta \mathbf{G}$ is generically nonsingular and the above equation has a unique generic solution in $\mathbb{R}^n_+$, denoted by $\mathbf{H}^*$. Since $\mathbf{U}$ is an n-square matrix of ones,

$\mathbf{U} \cdot \mathbf{H}^* = H^* \mathbf{1}$. Then,

$$\left[\frac{\beta}{q^2 S^2} \mathbf{I} + \gamma \mathbf{U} - \delta \mathbf{G}\right] \cdot \mathbf{H}^* = \hat{\mathbf{p}}$$
$$\frac{\beta}{q^2 S^2} \left[\mathbf{I} - \frac{\delta q^2 S^2}{\beta} \mathbf{G}\right] \cdot \mathbf{H}^* = \hat{\mathbf{p}} - \gamma H^* \cdot \mathbf{1} \tag{23}$$

By inverting the matrix (23) and doing some calculations that are described in Appendix A.3, we get:

$$\mathbf{H}^*(\boldsymbol{\Sigma}) = \frac{1}{\frac{\beta}{q^2 S^2} + \gamma b\left(\mathbf{g}, \frac{\delta q^2 S^2}{\beta}\right)} \mathbf{b}_{\hat{\mathbf{p}}}\left(\mathbf{g}, \frac{\delta q^2 S^2}{\beta}\right) \tag{24}$$

where $b\left(\mathbf{g}, \frac{\delta q^2 S^2}{\beta}\right)$ is the sum of the unweighted Bonacich centralities of all players and $\mathbf{b}_{\hat{\mathbf{p}}}\left(\mathbf{g}, \frac{\delta q^2 S^2}{\beta}\right)$ is the weighted Bonacich centrality:

$$b_{\hat{p}_i}\left(\mathbf{g}, \frac{\delta q^2 S^2}{\beta}\right) = \left[\mathbf{I} - \frac{\delta q^2 S^2}{\beta} \mathbf{G}\right]^{-1} (p - \xi l_i) =$$
$$p b_i\left(\mathbf{g}, \frac{\delta q^2 S^2}{\beta}\right) + \xi \mathbf{M}(\mathbf{g}, \frac{\delta q^2 S^2}{\beta}) \cdot \mathbf{l}$$

The equilibrium condition (24) shows that higher transportation cost increases the cost of harvesting and leads to lower use of the resource. This implies that agents who are located further away find it profitable to harvest a smaller amount of the resource compared to those that are located closer to the resource.

It is interesting to explore whether transportation costs and heterogeneity will make the formation of links easier, meaning that the agents will have a higher incentive to collaborate with each other. In order to do so we need to compare individual payoffs in different cases. If $\bar{u}_i$, $u_i^{**}$ denotes the payoff of agent $i$ in a fully connected and unconnected network, respectively, with heterogeneity then transportation costs make the formation of links easier iff:

$$\bar{u}_i(\mathbf{H} \mid +ij) - u_i^{**}\left(\mathbf{H}^S \mid -ij\right) > \hat{u}_i(\mathbf{H} \mid +ij) - u_i^{*}\left(\mathbf{H}^S \mid -ij\right)$$

That is, heterogeneity creates a larger difference between the payoffs of the fully connected network and the corresponding ones of the star network ($u_i^{**}$). In terms of the inequality above, the LHS shows the difference between the payoffs of the connected and the unconnected network in case of heterogeneity, while the RHS shows the same difference in case all agents locate at an equal distance from the resource (or more generally, when transportation is not costly or the per unit cost is not very high). Numerical simulations will give a clearer idea of whether heterogeneity could facilitate the creation of links between the agents.

Using the three agent example analyzed above, we can compare the case of costly transportation to one that involves no cost. In particular, the comparison between the payoffs of the connected network in both of these cases leads to the following condition:

$$\begin{gathered}\bar{u}_i\left(\mathbf{H} \mid +ij\right) > \hat{u}_i\left(\mathbf{H} \mid +ij\right) \Rightarrow \\ \frac{l_i}{\sum_{j\neq i} l_j} < \frac{\gamma-\delta}{\left(\frac{\beta}{q^2 S^2}+2\gamma-\delta\right)} \\ l_i < \lambda \sum\nolimits_{j\neq i} l_j \end{gathered} \tag{25}$$

where $\lambda = (\gamma-\delta)/\left(\frac{\beta}{q^2S^2}+2\gamma-\delta\right)$ and $0<\lambda<1$. Inequality (25) shows that costly transportation can, surprisingly, be more profitable for agent $i$ than non-costly transportation, iff the distance between agent $i$ and the resource is smaller than the sum of the distance between the rest of the agents and the resource, multiplied by a positive number $\lambda$. It is interesting to notice that $\lambda$ is increasing in $\gamma$ and decreasing in $\delta$. That is, it is easier for agent $i$ to obtain higher profits under costly transportation when the negative congestion effect is stronger or when she enjoys lower benefits from collaborating with the rest of the agents. In general, costly transportation can be shown to be beneficial for the agent who is located closer to the resource. Note that the per unit transportation cost, $\xi$, does not affect this result at all. The only thing that matters is how much "*closer*" to the resource agent $i$ is located compared to the co-appropriators.

## 6.2 The size of firms

A second interesting case that introduces heterogeneity in the model is the case where the agents, or the exploiting firms, are of different size. Let us assume here that there is one big firm, the "leader" ($L$) and a number of smaller ones ($F$) of equal size, the "followers." In terms of our modeling, this will lead to different private costs. In particular, the marginal cost of the larger firm will be lower than the corresponding cost of the smaller firms, i.e., $\beta_L < \beta_F$.

If such an equilibrium exists, then it solves:[8]

$$-\mathbf{\Sigma} \cdot \mathbf{H} = \left[ \frac{1}{q^2 S^2} \mathbf{B} + \gamma \mathbf{U} - \delta \mathbf{G} \right] \cdot \mathbf{H} = p \cdot \mathbf{1}.$$

where

$$\mathbf{B} = \begin{pmatrix} \beta_L & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \beta_F \end{pmatrix}$$

In order to solve the problem, we define the matrix $\mathbf{X}$ such that $\mathbf{B} = \mathbf{X} + \mathbf{I}$. Then,

$$-\mathbf{\Sigma} \cdot \mathbf{H} = \left[ \frac{1}{q^2 S^2} (\mathbf{X} + \mathbf{I}) + \gamma \mathbf{U} - \delta \mathbf{G} \right] \cdot \mathbf{H} = p \cdot \mathbf{1}.$$

$$\left[ \frac{1}{q^2 S^2} \mathbf{X} + \frac{1}{q^2 S^2} \left[ \mathbf{I} - q^2 S^2 \delta \mathbf{G} \right] \right] \cdot \mathbf{H} = (p - \gamma H^*) \cdot \mathbf{1} \qquad (26)$$

By using simple algebra, we end up with the equilibrium condition:[9]

$$\mathbf{H}^* = p \Phi^{-1} \mathbf{b} \left( \mathbf{g}, \delta q^2 S^2 \right) \qquad (27)$$

where $\Phi$ denotes the matrix: $\Phi = \left[ \frac{1}{q^2 S^2} \mathbf{M}^{-1} \mathbf{X} + \left( \frac{1}{q^2 S^2} + \gamma H^* b \right) \mathbf{I} \right]$

The numerical exercise of section 7 will provide interesting insights into the incentives of the two types of agents to connect or not. More specifically, we will see that if the leader has a large competitive advantage, by being able to produce at a much lower cost (in which case $\beta_L$ is significantly lower than $\beta_F$), and uses advance technology, meaning that smaller firms would benefit more from collaborating with the leader rather than the opposite ($\delta_L < \delta_F$), then there is no incentive for the large firm to collaborate with the competitors. In this case, the leading firm will keep exploiting a large part of the resource without being willing to share any knowledge with the followers.

If the regulator wants to protect the smaller appropriators and works toward the formation of a fully connected network, she will have to impose a stricter regulation to the large firm, in the form of taxation, which will fully internalize the negative congestion effects that she imposes on the rest of the appropriators. This is in line with our policy analysis above.

[8] Here the condition becomes: $\frac{\beta_L}{q^2 S^2} > \delta \lambda_1(\mathbf{G})$.

[9] The steps are available in Appendix A.4

# 7 Numerical Example

## 7.1 Symmetric agents

Let us provide some more insightful examples by using an 8-agent network and solve for the market and the optimal solution numerically. In Figure 3, we consider four different network structures: the star network (a), the case where there is only one collaboration link between two agents (b), the case where one agent is connected to the rest of the agents (c), and the full network (d). Table 1 gives the Bonacich measure, the individual harvesting amount and the corresponding individual profits for the four network structures. It is easy to observe that all the agents enjoy the highest profits in the full network, apart from agent 1 whose profits are higher in network (c). This network, though, is not pairwise stable, since all agents will find it profitable to connect with each other which will lead to network (d), which is the pairwise stable equilibrium of this game.

Table 1

| Agent Type | $b_i$ | $H_i$ | $u_i$ | $b_i$ | $H_i$ | $u_i$ |
|---|---|---|---|---|---|---|
| | | (a) | | | (b) | |
| 1 | 1 | 0.30242 | 0.22041 | 1.0020 | 0.30304 | 0.22132 |
| 2 | 1 | 0.30242 | 0.22041 | 1 | 0.30241 | 0.22040 |
| | | (c) | | | (d) *Equilibrium* | |
| 1 | 1.0146 | 0.30680 | **0.22778** | 1.01148 | 0.30674 | 0.22770 |
| 2 | 1.0021 | 0.30302 | 0.22129 | 1.01148 | 0.30674 | **0.22770** |

By solving the full optimal problem given by (15) numerically, where the planner will determine both the optimal harvesting amount and the optimal link configuration, we find that the optimal network structure is the equilibrium one, or else the full network. This is the case for any value of $\kappa \geq 0$. When the equilibrium network structure coincides with the optimal one, the planner only needs to control for the level of individual harvest by implementing the optimal agent-specific tax. In this numerical example, the optimal, individual harvesting amount is 0.30242 (or 0.28225 in conservation, with $\kappa = 0.1$) $< H_i^{(d)}$.

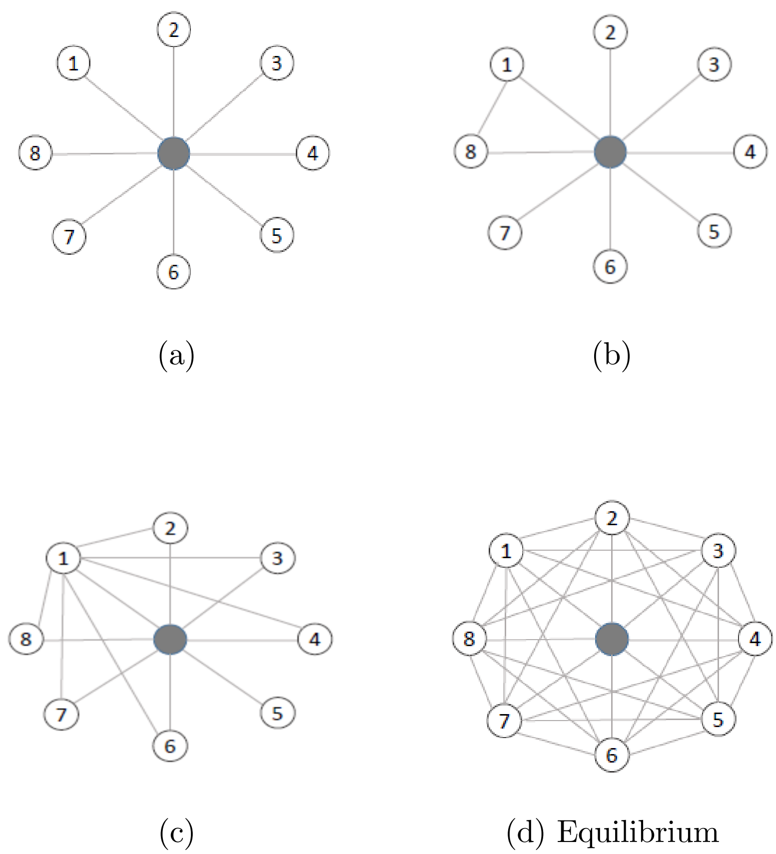



Figure 3: Symmetric Agents: Parameter Values: S=5, q=0.1, betta=1.2, delta=0.01, gamma=0.02, p=1.5

## 7.2 Asymmetric agents

## 7.3 The size of the firms

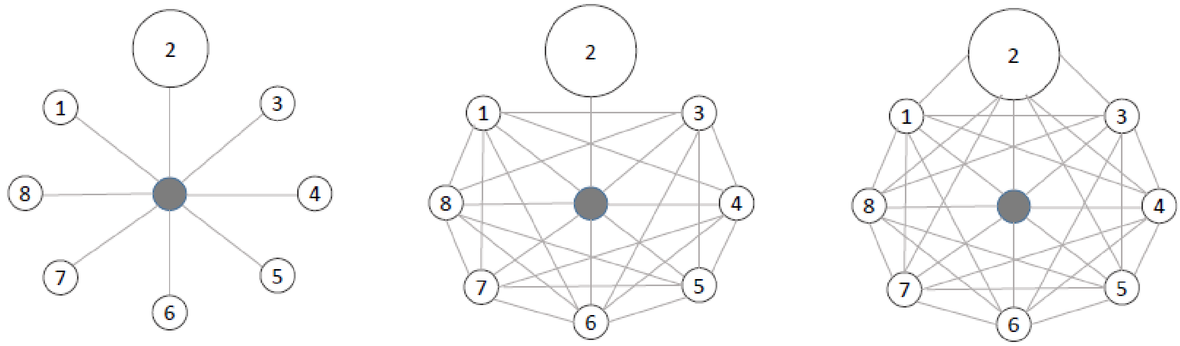
Not surprisingly, in the case of symmetric agents, the full network is shown to be the pairwise stable equilibrium, as well as the optimal network configuration. We will now show that introducing asymmetry in the game could lead to different results, where the market structure is different from the optimal one. In such a case, as explained above, the planner, along with the agent-specific taxation that aims to restrict individual harvesting, should use some other policy instrument to create or sever collaboration links between the agents.

Table 2

| Agent Type | $H_i$ | $u_i$ | $H_i$ | $u_i$ | $H_i$ | $u_i$ |
|---|---|---|---|---|---|---|
| | (a) | | (b) *Equilibrium* | | (c) | |
| 1 | 0.1813 | 0.1318 | 0.1826 | 0.1337 | 0.1841 | **0.1359** |
| 2 | 1.2087 | **0.8911** | 1.2085 | 0.8909 | 1.2084 | 0.8908 |

Figure 4 illustrates the case where there is a big player in the market (agent 2) and seven smaller resource users. The leading firm has a cost advantage, and thus her private cost of harvesting is lower than the corresponding one of the rest of the agents, $\beta_2 < \beta_i$, $i = 1, 3, ...8$. Also, the benefit of the larger player when collaborating with the rest of the agents is lower, indicating the fact that she already uses some more advance technology in the harvesting of the resource and thus the exchange of knowledge benefits more firms of smaller size ($\delta_2 < \delta_i$, $i = 1, 3, ...8$).

The results in this case are interesting. As shown in Table 2, individual profits for smaller users are higher in the case of the full network (c), followed by the corresponding ones in the partly connected network (where small users are connected only to each other without including the big player), while profits are lower in the star network (a). This is not the case, though, for the big firm. In particular, the big player enjoys the highest profits in the star network. It is not profitable for her to connect to any smaller user, while the highest profits for her are associated with the network where there are no collaboration links between any agent, *i.e.*, $u_2^{(a)} > u_2^{(b)} > u_2^{(c)}$. However, the star network violates the pairwise stability condition. This is why the profits of the rest of the agents are higher in the full network, followed by the partly-connected network, and then by the star network, *i.e.*, $u_i^{(a)} < u_i^{(b)} < u_i^{(c)}$. So, although the big player will decide not to connect with the rest of the agents (which impedes the creation of a full




(a) (b) Equilibrium (c)

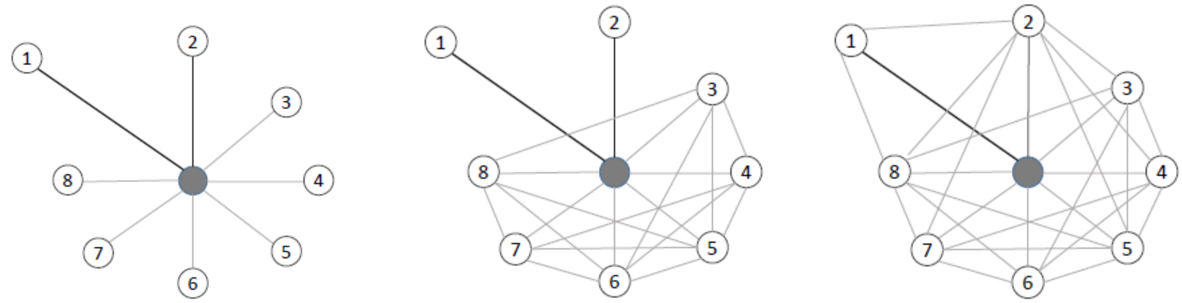
Figure 4: Asymmetric Agents: Firms of Different Size. Parameter Values: S=5, q=0.1, bettai=1.2, betta2=0.3, deltai=0.01, delta2=0.0001, gamma=0.02, p=1.5, i=1,3,...,8

network), the small players will still have an incentive to collaborate with each other. Thus, the pairwise stable equilibrium configuration is the partly connected network (b).

Deriving numerically the optimal network structure, we observe that the full network is the optimal one. In this case (with $\kappa = 0$), the optimal harvesting amount of the small agents is equal to $0.1820 < H_i^{(b)}$ and the corresponding one for the big agent is $1.178 < H_2^{(b)}$. The planner will have to come up with policies that will incentivize the big player to create collaboration links with the smaller ones (such as a subsidy per new link created).

### 7.3.1 The effect of geographical distance

When the transportation of the resource is costly, the distance between the exploiting agent and the resource matters. Here, we explore the case where two of the agents are located further away, while the remaining six agents are located at an equal distance from the resource. More specifically, we assume that the distance between agent 1 and the resource is the longest ($l_1 = 0.5$), followed by agent 2 who is located a bit closer ($l_2 = 0.3$) and then the rest of the agents positioned even closer to the resource ($l_i = 0.1$, $i = 3, ..., 8$).




(a) (b) (c) Equilibrium

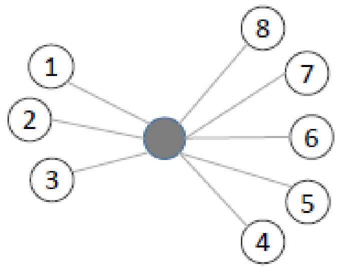
Figure 5: Asymmetric Agents: Costly Transportation. Parameter Values: S=5, q=0.1, betta=1.2, delta=0.01, gamma=0.02, p=1.5, l1=0.5, l2=0.3, l3=0.1

Table 3

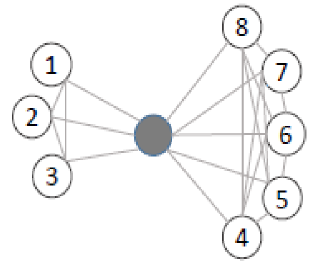
| Agent Type | $H_i$ | $u_i$ | $H_i$ | $u_i$ | $H_i$ | $u_i$ |
|---|---|---|---|---|---|---|
| | (a) | | (b) | | (c) *Equilibrium* | |
| 1 | 0.1994 | 0.0958 | 0.1993 | 0.0957 | 0.2033 | **0.0996** |
| 2 | 0.2411 | 0.1401 | 0.2410 | 0.1400 | 0.2449 | **0.1446** |
| 3 | 0.2827 | 0.1865 | 0.2856 | 0.1966 | 0.2865 | **0.1978** |

The results, which correspond to the network configurations shown in Figure 5, are presented in Table 3. More precisely, when the agents that are located closer to the resource create collaboration links, the profits of the unconnected agents decrease ($u_i^{(b)} < u_i^{(a)}$, for $i = 1, 2$), while the profits of the connected ones increase ($u_i^{(b)} > u_i^{(a)}$, for $i = 3, ..., 8$). In this case, the unconnected agents have an incentive to connect to the rest of the agents and enjoy higher profits, since $u_i^{(c)} > u_i^{(a),(b)}$, i.e., network (c) implies higher profits for all the agents. It is shown that the full connected network is also the optimal network structure.

Note that here we have assumed that any cost that is associated with the creation of collaboration links between the appropriators does not depend on the distance between them. If, however, connecting to a distant appropriator implies higher cost for the ones that are located closer to the resource (or closer to some other group of appropriators), then the incentives to collaborate become weaker. In such a case, we might observe the formation of subgroups, which is the case illustrated in Figure 6. In particular, in the case where the benefits of collaboration are outweighed by the costs of connecting to some more distant appropriators, the agents find it profitable to connect only to nearby




(a)




(b) Equilibrium

Figure 6: Asymmetric Agents: Costly Coordination. Parameter Values: S=5, q=0.1, betta=1.2, betta=1.2, delta(A)=0.01, delta(B)=0.01, gamma=0.02, p=1.5

resource users. Thus, in our specific numerical example, we observe the formation of two subgroups, which include agents that are located in the "neighborhood." The subgroup formation is more profitable for all the agents of the two groups ($u_i^{(b)} > u_i^{(a)}$ in Table 4). In such a case, the planner should subsidize the "costly collaboration" in order to attain a full network of appropriators.

Table 4

| Agent Type | $b_i$ | $H_i$ | $u_i$ | $b_i$ | $H_i$ | $u_i$ |
|---|---|---|---|---|---|---|
| | | (a) | | | (b) *Equilibrium* | |
| 1 | 1 | 0.3024 | 0.220 | 1.0041 | 0.3036 | **0.222** |
| 4 | 1 | 0.3024 | 0.220 | 1.0084 | 0.3048 | **0.224** |

# 8 Conclusion

In this paper, we use network theory to study the market and the optimal outcomes in a natural resource management problem. The externalities characterizing the exploitation of the resource under study, in the form of collaboration links, crowding effects, and resource stock effects, along with the conservation plan of the regulator, open the gap between decentralized and optimal harvesting. Different network structures have been studied and show how harvesting and aggregate welfare are affected. We also analyze the incentives of harvesting agents with respect to the formation or elimination of cooperative links, which determine the equilibrium network configurations. We show that homogeneous agents always have a strong incentive to move from an unconnected to a fully connected network, which is not necessarily the case when agents are heterogeneous.

Heterogeneity with respect to the distance of each agent to the resource, which implies higher or lower transportation costs for the agent who is located further away or closer, respectively, changes the harvesting amounts and the individual and aggregate payoffs. This also leads to different views regarding how profitable or not the cooperation between the harvesting agents is. We also show that the size of the firms matters. Thus, large appropriators, with lower extraction costs and advance technology, have weaker or no incentives to connect to the rest of the appropriators. Thus, heterogeneity leads to different equilibrium network configurations, as compared to the socially optimum ones. Finally, we show that the special nature of natural resources, when taken into account in the planner's problem, opens the gap between equilibrium and optimal harvesting. In particular, the more the regulator focuses on the conservation of the resource, the lower the optimal harvesting amount and the larger the gap between the market and the optimal harvesting.

These approaches help us examine and determine the optimal structure of the resource network. When the optimal network structure has been identified, we will be able to design policies that are more in line with the scarce nature of the resources, prevent the overexploitation and promote the formation or the elimination of collaboration links between the appropriators. We recognize that the resource use is not a static problem. However, this is a first attempt to use the theory of network economics in order to study a natural resource management problem. Resource dynamics can be explicitly introduced in the problem by assuming, for example, that

$$S_{t+1} - S_t = F\left(S_t\right) - \sum_{j=1}^{n} H_j\left(t\right), \quad S_0 \text{ given,}$$

where $F\left(S_t\right)$ is a standard resource-growth function. The network for the entire planning horizon can be regarded in this case as a multilayer network (e.g., Boccaletti et al., 2014). For the multilayer network, each time period $t \geq 0$ can be interpreted as representing a different network layer and each node on a layer can represent an appropriator at a given point in time who maximizes discounted profits. Intralayer connections are links among agents at a given point in time and can thus be interpreted as collaboration links as in the static network. Interlayer connections, on the other hand, follow the path of each harvester for $t \geq 0$. Severance of an interlayer connection for an agent $i$ can be interpreted as having the agent stop harvesting. The dynamic network problem can in principle be analyzed using dynamic optimizations methods. The dynamic framework introduces new interesting issues which relate to the way that intralayer and interlayer links evolve in socially optimal solutions and market equilibrium. This undoubtedly is an interesting area for further research.

# References


[1] Ballester, C., Calvó-Armengol, A., Zenou, Y., 2006. Who's who in Networks. Wanted: The Key Player. *Econometrica*, 74 (5): 1403-1417.

[2] Barnes, M.L., Lynham, J., Kalberg, K., Leung P., 2016. Social Networks and Environmental Outcomes. *Proceedings of the National Academy of Sciences*, 113 (23): 6466-6471.

[3] Boccaletti, S., Bianconi, G., Criado, R., del Genio, C.I., Gómez-Gardeñes, J., Romance, M., Sendiña-Nadal, I., Wang, Z., Zanin, M., 2014. The Structure and Dynamics of Multilayer Networks. *Physics Reports*, 544 (1): 1-122.

[4] Conley T., Udry C., 2001. Social Learning through Networks: The Adoption of New Agricultural Technologies in Ghana. *American Journal of Agricultural Economics*, 83(3): 668-673.

[5] Currarini, S., Marchiori, C. and Tavoni, A., 2016. Network Economics and the Environment: Insights and Perspectives. *Environmental and Resource Economics*, 65 (1), pp. 159-189.

[6] Günther M., Hellmann T., 2017. International Environmental Agreements for Local and Global Pollution. *Journal of Environmental Economics and Management*, 81: 38-58.

[7] İlkiliç, R., 2011. Networks of Common Property Resources. *Economic Theory*, 47: 105-134.

[8] Ioannides, Y.M., 2012. From Neighborhood to Nations: The Economics of social Interactions. Princeton University Press, Princeton.

[9] Jackson, M.O., 2014. Networks in the Understanding of Economic Behaviors. *Journal of Economic Perspectives*, 28, 3-22.

[10] Jackson, M.O., Rogers, B.W., Zenou, Y., 2017. The Economic Consequences of Social Network Structure. *Journal of Economic Literature* 55(1), 49-95.

[11] Jackson, M., Watts, A., 2002. The Evolution of Social and Economic Networks. *Journal of Economic Theory*, 106: 265-295.

[12] Jackson, M., Wolinsky, A., 1996. A Strategic Model of Social and Economic Networks. *Journal of Economic Theory*, 71: 44-74.

[13] Jackson, M., Zenou, Y., 2015. Games on Networks. In: Young, P., Zamir, S. (Eds.), In: Handbook of Game Theory, vol. 4. Elsevier, Amsterdam, pp. 91–157.

[14] Ostrom E., 1990. Governing the Commons: the Evolution of Institutions for Collective Action. Cambridge University Press, New York.

[15] Ostrom E., 2004. Understanding Collective Action. In: Meinzen-Dock R., Gregorio M Di (eds) Collective action and property rights for sustainable development, 2020 vision for food, agriculture and the environment. Focus 11, IFPRI International Food Policy Research Institute, Washington.

[16] Smith, V., L., 1968. Economics of Production from Natural Resources. *The American Economic Review*, 58(3): 409-431.

[17] Smith, V., L., 1969. On Models of Commercial Fishing. *Journal of Political Economy*, 77(2): 181-198.

[18] Watts, A., 2001. A Dynamic Model of Network Formation. *Games and Economic Behavior*, 34: 331-341.

# Apendix A.1

**Local Interactions Effect**

We prove that stronger local interactions increase the profits of connected agents (say $i$) and decrease the profits of unconnected agents (say $j$).

From section 3.2.3, the profits of the agents $i$ and $j$ (in a 3-agent network) are given by:

$$\tilde{u}_i = \frac{\beta^2 p^2 q^2 S^2 (\beta + \gamma q^2 S^2)}{2\Delta^2}$$

$$\tilde{u}_j = \frac{p^2 q^2 S^2 (\beta + \gamma q^2 S^2)(\beta - \delta q^2 S^2)^2}{2\Delta^2}$$

where $\Delta > 0$. Taking the derivative wrt $\delta$ :

$$\frac{\partial \tilde{u}_i}{\partial \delta} = \frac{\beta^2 p^2 q^2 S^2 (\beta + \gamma q^2 S^2)(\beta q^2 S^2 + \gamma q^4 S^4)}{\Delta^3} > 0,$$

and

$$\frac{\partial \tilde{u}_j}{\partial \delta} = -\frac{2\beta\gamma p^2 q^6 S^6 (\beta + \gamma q^2 S^2)(\beta - \delta q^2 S^2)}{\Delta^3} < 0,$$

since $(\beta - \delta q^2 S^2) > 0$.

# Apendix A.2

**Optimal Taxation, Bonacich Centrality**

We show how we derive the equilibrium harvesting amount under taxes in matrix form. The Nash equilibrium under a non-uniform tax solves:

$$-\mathbf{\Sigma} \cdot \mathbf{H}^o = \left[\frac{\beta}{q^2 S^2}\mathbf{I} + \gamma \mathbf{U} - \delta \mathbf{G}\right] \cdot \mathbf{H}^o = p - \tau$$

$$\tau = (\tau_1, ...\tau_n)$$

Then,

$$\left[\frac{\beta}{q^2 S^2}\mathbf{I} + \gamma \mathbf{U} - \delta \mathbf{G}\right] \cdot \mathbf{H}^* = p - \tau$$

$$\frac{\beta}{q^2 S^2}\left[\mathbf{I} - \frac{\delta q^2 S^2}{\beta}\mathbf{G}\right] \cdot \mathbf{H}^* = (p - \gamma H^*) \cdot \mathbf{1} - \tau$$

By inverting the matrix, we get:

$$\begin{aligned}
\frac{\beta}{q^2S^2}\mathbf{H}^* &= (p-\gamma H^*)\left[\mathbf{I}-\frac{\delta q^2S^2}{\beta}\mathbf{G}\right]^{-1}\cdot\mathbf{1}-\left[\mathbf{I}-\frac{\delta q^2S^2}{\beta}\mathbf{G}\right]^{-1}\cdot\tau\\
\frac{\beta}{q^2S^2}\mathbf{H}^* &= (p-\gamma H^*)\mathbf{b}\left(\mathbf{g},\frac{\delta q^2S^2}{\beta}\right)-\mathbf{b}_\tau\left(\mathbf{g},\frac{\delta q^2S^2}{\beta}\right)\\
\left[\frac{\beta}{q^2S^2}+\gamma b\left(\mathbf{g},\frac{\delta q^2S^2}{\beta}\right)\right]\mathbf{H}^* &= p\mathbf{b}\left(\mathbf{g},\frac{\delta q^2S^2}{\beta}\right)-\mathbf{b}_\tau\left(\mathbf{g},\frac{\delta q^2S^2}{\beta}\right)\\
\mathbf{H}^*(\mathbf{\Sigma}) &= \frac{1}{\frac{\beta}{q^2S^2}+\gamma b\left(\mathbf{g},\frac{\delta q^2S^2}{\beta}\right)}\left[p\mathbf{b}\left(\mathbf{g},\frac{\delta q^2S^2}{\beta}\right)-\mathbf{b}_\tau\left(\mathbf{g},\frac{\delta q^2S^2}{\beta}\right)\right]
\end{aligned} \tag{28}$$

where $\mathbf{b}_\tau\left(\mathbf{g},\frac{\delta q^2S^2}{\beta}\right)=\left[\mathbf{I}-\frac{2\delta q^2S^2}{\beta}\mathbf{G}\right]^{-1}\cdot\tau=\mathbf{M}(\mathbf{g},\frac{\delta q^2S^2}{\beta})\cdot\tau$ is the weighted Bonacich centrality (look also at Remark 1 of Ballester et al., 2006).

# Apendix A.3

**Costly Transportation**

We show how we get the equilibrium harvesting amount in matrix form.

By inverting the matrix (23), we get:

$$\begin{aligned}
\frac{\beta}{q^2S^2}\mathbf{H}^* &= \left[\mathbf{I}-\frac{\delta q^2S^2}{\beta}\mathbf{G}\right]^{-1}\hat{\mathbf{p}}-\gamma H^*\left[\mathbf{I}-\frac{\delta q^2S^2}{\beta}\mathbf{G}\right]^{-1}\cdot\mathbf{1}\\
\frac{\beta}{q^2S^2}\mathbf{H}^* &= \mathbf{b}_{\hat{\mathbf{p}}}\left(\mathbf{g},\frac{\delta q^2S^2}{\beta}\right)-\gamma H^* b\left(\mathbf{g},\frac{\delta q^2S^2}{\beta}\right)\\
\left[\frac{\beta}{q^2S^2}+\gamma b\left(\mathbf{g},\frac{\delta q^2S^2}{\beta}\right)\right]\mathbf{H}^* &= \mathbf{b}_{\hat{\mathbf{p}}}\left(\mathbf{g},\frac{\delta q^2S^2}{\beta}\right)\\
\mathbf{H}^*(\mathbf{\Sigma}) &= \frac{1}{\frac{\beta}{q^2S^2}+\gamma b\left(\mathbf{g},\frac{\delta q^2S^2}{\beta}\right)}\mathbf{b}_{\hat{\mathbf{p}}}\left(\mathbf{g},\frac{\delta q^2S^2}{\beta}\right)
\end{aligned} \tag{29}$$

# Apendix A.4

**The size of firms**

In order to solve for the equilibrium harvesting amount in the case where firms are of heterogeneous size, we define $\mathbf{M}(\mathbf{g},a)=[\mathbf{I}-q^2S^2\delta\mathbf{G}]$ and multiply (26) by $\mathbf{M}^{-1}$ to obtain:

$$\left[\frac{1}{q^2S^2}\mathbf{M}^{-1}\mathbf{X} + \frac{1}{q^2S^2}\right] \cdot \mathbf{H} = \ (p - \gamma H^*) \cdot \mathbf{b}\left(\mathbf{g}, \delta q^2 S^2\right)$$

$$\left[\frac{1}{q^2S^2}\mathbf{M}^{-1}\mathbf{X} + \frac{1}{q^2S^2} + \gamma H^* b\left(\mathbf{g}, \delta q^2 S^2\right)\right] \cdot \mathbf{H} = p\mathbf{b}\left(\mathbf{g}, \delta q^2 S^2\right)$$

$$\left[\frac{1}{q^2S^2}\mathbf{M}^{-1}\mathbf{X} + \left(\frac{1}{q^2S^2} + \gamma H^* b\right)\mathbf{I}\right] \cdot \mathbf{H} = p\mathbf{b}\left(\mathbf{g}, \delta q^2 S^2\right)$$

If we define the matrix $\Phi = \left[\frac{1}{q^2S^2}\mathbf{M}^{-1}\mathbf{X} + \left(\frac{1}{q^2S^2} + \gamma H^* b\right)\mathbf{I}\right]$, the equilibrium harvesting becomes:

$$\mathbf{H}^* = p\Phi^{-1}\mathbf{b}\left(\mathbf{g}, \delta q^2 S^2\right).$$