Debt consolidation and its cross-country effects: Aggregate and distributional implications

Eleftherios Roumpanis *†
Athens University of Economics and Business

Abstract

This paper builds and solves numerically a New Keynesian DSGE model consisting of two heterogeneous countries participating in a monetary union. We study how public debt consolidation in a country with high debt (like Italy) affects incomes in a country with solid public finances (like Germany); consequently, the emphasis is on the aggregate and distributional implications of debt consolidation, where income heterogeneity of households in both countries, and hence distribution, has to do with the distinction among "capitalists", "private workers" and "public employees". The paper also studies how these implications depend on the specific fiscal policy instrument used for debt consolidation.

*Acknowledgments:I would like to thank my supervisor, Apostolis Philippopoulos, for his support and encouragement during this work

†Corresponding author: Eleftherios Roumpanis, Department of Economics,Athens University of Economics and Business,Athens 10434. e-mail: roubanis@aueb.gr

1 Introduction

The 2008 world crisis has, among other things, brought into the spotlight the need for debt consolidation in several eurozone periphery countries. [1] For reasons related to sustainability and loss of confidence, these countries have been forced to take restrictive fiscal policy measures which have further dampened demand in the short term and hurt especially relatively poor social classes. As a result of lasting recession and its consequent social impact experienced by these countries,[2] the debate about fiscal consolidation has been intensified even more. On the other hand, fiscal policy in eurozone center countries, like Germany, has been neutral. [3] Nevertheless, the recession in the crisis countries has also affected the German economy, which is another reminder of the importance of spillovers in an integrated area like the euro area.

In this paper, we study how public debt consolidation in a country with high debt and sovereign premia and in a country with solid public finances (which can go for mild consolidation) affects each other's incomes. Specifically, we study how public debt consolidation in a country like Italy and in a country like Germany affects each other's level and distribution of incomes as well as how these cross-border effects depend on the fiscal policy mix chosen in each country on how to bring public debt down and, once debt has been reduced, take advantage of the fiscal space created. [4] Hence, the emphasis is on the aggregate and distributional implications of debt consolidation in national levels, where income heterogeneity of households in both countries, and hence distribution, has to do with the distinction among "capitalists", "private workers" and "public employees".

In light of the above, this paper provides a quantitative study of the aggregate and distributional implications of debt consolidation in a New Keynesian DSGE model consisting of two heterogeneous countries forming a currency union. An international asset market allows private agents[5] across countries to borrow from, or lend to, each other and the same market

[1] See e.g. the EEAG Report on the European Economy (2013) by CESifo and EMU-Public Finances (2015) by the European Commission.

[2] This is in particularly true in Cyprus, Greece, Italy, Portugal and Spain.

[3] See e.g. EMU-Public Finances (2015) by the European Commission.

[4] Italy's (public and foreign) debt position, although sizeable in absolute terms, is not one of the worst in the euroarea. Greece, Portugal, Spain, Ireland and Cyprus, are in a worse position (see e.g. the EEAG Report on the European Economy, 2012, by CESifo). However, since these countries have received financial aid from the EC-ECB-IMF, we prefer to use Italy as our euroarea periphery country.

[5] Actually, only capitalists can participate in asset markets.

allows national governments to sell their bonds to foreign private agents. To study distributional implications of debt consolidation on incomes, we obviously need a model with heterogeneous households. There are many types of income heterogeneity in the literature. Here, we focus on the distinctions among ”capitalists”, ”private workers” and ”public employees”. Capitalists are defined to be those households who hold assets, own the private firms and get labor income for their managerial services. Private workers and public employees are defined to be those households that are employed in private and public sector respectively and have labor income only. The labor of employees, together with goods purchased from the private sector, are used by state-owned firm as inputs in the production of public goods and services. On the private production side, firms enjoy some monopoly power and face Rotemberg-type nominal price rigidities, while their productivity is enhanced as the government invest in infrastructure. Regarding macroeconomic policy, being in a monetary union, there is a single monetary policy. On the other hand, the two countries are free to follow independent or national fiscal policies. Introducing a relatively rich public sector in two countries, national fiscal authorities can choose among a number of government spending categories (public consumption, public investment and public wage bill) and taxes (on consumption, capital and labor). Then, following a rule-like approach to policy, we assume that fiscal policy is conducted via simple and implementable feedback policy rules. In particular, we assume that every category of public spending and the tax rates on consumption, capital and labor are all allowed to respond to the inherited public debt-to-GDP ratio as deviation from a policy target.

We solve the above model numerically employing commonly used parameter values and fiscal policy data from Germany (called the home country) and Italy (called the foreign country). As we shall see, the steady state solution of this model can mimic relatively well the key features of the two countries over the euro years and, in particular, the current account deficits in Italy financed by current account surpluses in Germany over 2001-2011 (this is believed to be one of the most important macroeconomic imbalances in Europe today). It is useful to stress that this is achieved by simply allowing for differences in fiscal policy and the degree of patience; the latter means that Italians have been less patient than Germans during the euro period.[6] In turn, we use this solution as a point of departure to study the dynamic evolution of endogenous variables in response to policy reforms, focusing on debt consolidation not only in the high-debt country, namely,

[6] See also Economides et al. (2016) for further details.

Italy, but also in the country with solid public finances, namely, Germany (mild consolidation).

2 A two-country model of a monetary union

This section sets up a New Keynesian DSGE model consisting of two heterogeneous countries populated by heterogeneous households that each economy have a relatively rich public sector and together form a monetary union. We start with an informal description of the model.[7]

2.1 Informal description of the model and discussion of key assumptions

Two countries form a closed system in a New Keynesian setup. [8] In a regime of a currency union, there is a single monetary authority or central bank. In each country, there are heterogeneous households, private and state-owned firms and a national fiscal authority or government.

There are three types of households in each country, called capitalists, private workers and public employees. Capitalists own the private firms, hold capital, money, internationally traded assets, domestic government bonds and also receive labor income for their managerial services. Both private workers and public employees hold money and receive labor income for their labor services. Hence, although all types of households receive labor income, only capitalists can save in the form of physical capital, domestic government bonds and internationally traded assets.

On the production side, in each country there are a state-owned firm and a number of private firms. The state-owned firm uses public employees' labor with goods purchased from private sector in order to produce total public goods and services. Private firms combine capitalists' and private workers' labor with private and public physical capital (public infrastructure) for the production of private goods. Each private firm produces a differentiated tradable private good and, consequently, acts monopolistically facing Rotemberg-type nominal rigidities. Nominal rigidities give a real role to monetary and exchange rate policy, at least in the transition path.

[7]The model is similar to that in Economides et al (2016). However, here we study debt consolidation within a currency union, while that paper compares a currency union to flexible exchange rates and a fiscal union.

[8]The model is a variation of a New Keynesian currency union model. See Okano (2014) for a review of the related literature dating back to Galí and Monacelli (2005, 2008).

In a monetary union, we assume a single monetary policy, but independent national fiscal policies. Both monetary and fiscal policy are conducted by simple implementable state-contingent policy rules. Regarding monetary policy, monetary authority follows a Taylor-type rule for the nominal interest rate. Regarding fiscal policy, in each country there is a relatively rich menu of fiscal policy instruments that, following a rule-like approach, are allowed to respond to the inherited public debt-to-GDP ratio as a deviation from a target value. Introducing a relatively rich public sector in two countries, national fiscal authorities can choose among a number of government spending categories (public consumption, public investment and public wage bill) and taxes (on consumption, capital and labor). Consequently, we assume that every category of public spending and taxes are all allowed to respond to the inherited public debt-to-GDP ratio as deviation from a policy target.

The market for internationally traded assets allows private agents (capitalists) across countries to borrow from, or lend to, each other and it also allows national governments to sell their bonds to foreign private agents(capitalists). [9] In other words, the government in each country can sell its bonds to domestic and foreign capitalists, where the latter, namely, government's borrowing from abroad, takes place via the international asset market. We assume that all international borrowing/lending takes place through a financial intermediary or bank. This financial intermediation requires a transaction, or monitoring, cost proportional to the amount of the nation's debt.[10] This cost creates, in turn, a wedge between the borrowing and the lending interest rate. As a result, when capitalists participate in the international asset market, those in the debtor country face a higher interest rate than their counterparts in the creditor country.[11] To the extent that the bank makes a profit, this profit is rebated lump-sum to capitalists in the creditor country.

As is well-known, systematic borrowing and lending cannot occur in an

[9] See also Forni et al. (2010), Cogan et al. (2013), Erceg and Lindé (2013) and many others.

[10] Instead of using the device of a financial intermediary or bank, we could just assume transaction costs incurred upon borrowers (see e.g. Forni et al. (2010), Cogan et al. (2013), Erceg and Lindé (2013) and many others who assume a transcation cost when agents trade in the international asset market). We prefer however the bank device because we find it to be more intuitive (see also e.g. Curdia and Woodford (2009, 2010) and Benigno et al (2014) although in a closed economy).

[11] That is, here, differences in interest rates across countries are produced by transaction or monitoring costs incurred by the bank. As is known such differences can be produced in various ways including the probability of sovereign default (see Subsection 2.7 below for details).

homogeneous world. Some type of heterogeneity is needed between domestic and foreign agents. A popular way of producing borrowers and lenders has been to assume that agents across countries differ in their patience to consume or, equivalently, in their discount factors; specifically, the discount factor of lenders is higher than that of borrowers or, equivalently, borrowers are more impatient than lenders.[12] It is also well-known that such differences in discount factors need to be combined with an imperfection in the capital market in order to get a well-defined solution;[13] in our model, the capital market imperfection is the transaction, or monitoring, cost of the loan, as said above. Therefore, in our model, the international transaction cost ensures, not only stationarity of foreign asset positions as is typically the case in the literature (see e.g. Schmitt-Grohe and Uribe (2003)), but it also allows for a well-defined solution with different discount factors across different countries.

The solution of the above described model will imply that one country is a net lender and the other is a net borrower in the international asset market and that interest rates are higher in the net debtor country. Given the current account data over the euro years, we will think of the lender country as Germany and the debtor country as Italy. In this case, in equilibrium, the relatively impatient Italians will finance their current account deficits by borrowing funds from the patient Germans who run current account surpluses. This scenario, as said above, is as in the euro period data. It is also consistent with the literature on the interpretation of current accounts in the sense that systematic low saving rates and current account deficits are believed to reflect relatively low patience.[14]

The number of each type of households and their percentages in the population as well as the number of private firms are as follows. The home economy is composed of N^k identical capitalists indexed by $k = 1, 2, ...N^k$, of N^w identical private workers indexed by $w = 1, 2, ..., N^w$, of N^b identical public employees or bureaucrats indexed by $b = 1, 2, ..., N^b$, of N^k private

[12]See also e.g. Benigno et al. (2014). Kiyotaki and Moore (1997) also use a general equilibrium model with two types of agents, creditors and borrowers, who discount the future differently. Note that we could further enrich our model so as the discount factors are formed endogenously; see e.g. Becker and Mulligan (1997) and Doepke and Zilibotti (2008) for an endogenous formation of discount factors depending on income, education, effort, religion, etc. See also e.g. Schmitt-Grohe and Uribe (2003) and Choi et al. (2008) for calibrated models where the discount factor depends on consumption changes.

[13]See also e.g. Benigno et al. (2014). Similarly, in Doepke and Zilibotti (2008), some financial market imperfections are necessary for getting differences in patience across different social classes.

[14]See e.g. Choi et al. (2008).

firms indexed by $h = 1, 2, ..., N^k$. Assuming that each capitalist owns one private firm the total number of capitalists equals that of private firms. Similarly, in the foreign economy, where there are N^{k^*} identical capitalists indexed by $k^* = 1, 2, ...N^{k^*}$, of N^{w^*} identical private workers indexed by $w^* = 1, 2, ..., N^{w^*}$, of N^{b^*} identical public employees indexed by $b^* = 1, 2, ..., N^{b^*}$ of N^{k^*} private firms indexed by $f = 1, 2, ..., N^{k^*}$. For simplicity, we assume that population in both countries,N and N^*, is constant over time and of equal size, $N = N^*$. Furthermore, we assume that the number of capitalists, private workers and public employees in the population are also equal across countries and constant over time, that is $N^k = N^{k^*}$, $N^w = N^{w^*}$ and $N^b = N^{b^*}$, ruling out occupational choice and mobility across groups. Finally, the share of capitalists, private workers and public employees in the population are defined as $v^k \equiv \frac{N^k}{N}$, $v^w \equiv \frac{N^w}{N}$ and $v^b \equiv \frac{N^b}{N}$ respectively.

Below, we present the domestic country. The foreign country will be symmetric except explicitly said. A star will denote the counterpart of a variable or a parameter in the foreign country.

2.2 Households as capitalists

This subsection presents the problem of domestic capitalists, $k = 1, 2, ..., N^k$.

2.2.1 Consumption bundles and expenditures of capitalists

The quantity of each variety h produced at home by domestic private firm h and consumed by each domestic capitalist k is denoted as $c_t^{k,H}(h)$. Using a Dixit-Stiglitz aggregator, the composite of domestic private goods consumed by each k, $c_t^{k,H}$, consists of h varieties and is given by:[15]

$$c_t^{k,H} = \left[\sum_{h=1}^{N^k} \left(\frac{1}{N^k} \right)^{\frac{1}{\phi}} [c_t^{k,H}(h)]^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}} \tag{1}$$

where $\phi > 0$ is the elasticity of substitution across private goods produced in the domestic country.

Similarly, the quantity of each imported variety f produced abroad by foreign private firm f and consumed by each domestic capitalist k is denoted as $c_t^{k,F}(f)$. Using a Dixit-Stiglitz aggregator, the composite of imported

[15] As in e.g. Blanchard and Giavazzi (2003), here we work with summations rather than with integrals.

private goods consumed by each k, $c_t^{k,F}$, consists of f varieties and is given by:

$$c_t^{k,F} = \left[\sum_{f=1}^{N^{k^*}} \left(\frac{1}{N^{k^*}} \right)^{\frac{1}{\phi}} [c_t^{k,F}(f)]^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}} \tag{2}$$

In turn, having defined $c_t^{k,H}$ and $c_t^{k,F}$, k' consumption bundle, c_t^k, is defined as:

$$c_t^k = \frac{\left(c_t^{k,H}\right)^v \left(c_t^{k,F}\right)^{1-v}}{v^v(1-v)^{1-v}} \tag{3}$$

where v is the degree of preference for domestic private goods (if $v > 1/2$, there is a home bias).

Each domestic capitalist k's total consumption expenditure is:

$$p_t c_t^k = p_t^H c_t^{k,H} + p_t^F c_t^{k,F} \tag{4}$$

where p_t is the consumer price index (CPI), p_t^H is the price index of home private tradables, and p_t^F is the price index of foreign private tradables (expressed in domestic currency).

Each domestic capitalist k' total expenditure on home goods and foreign goods are:

$$p_t^H c_t^{k,H} = \sum_{h=1}^{N^k} p_t^H(h) c_t^{k,H}(h) \tag{5}$$

$$p_t^F c_t^{k,F} = \sum_{f=1}^{N^{k^*}} p_t^F(f) c_t^{k,F}(f) \tag{6}$$

2.2.2 Prices and terms of trade

We assume that the law of one price holds meaning that each tradable private good sells at the same price at home and abroad. Thus, $p_t^F(f) = S_t p_t^{H^*}(f)$, where S_t is the nominal exchange rate (where an increase in S_t implies a depreciation) and $p_t^{H^*}(f)$ is the price of variety f produced abroad denominated in foreign currency. Note that the terms of trade are defined as $\frac{p_t^F}{p_t^H}(= \frac{S_t p_t^{H^*}}{p_t^H})$, while the real exchange rate is defined as $\frac{S_t p_t^{H^*}}{p_t}$. In a currency union model, we will exogenously set $S_t \equiv 1$ at all t.

2.2.3 Capitalists' optimization problem

Each capitalist k acts competitively to maximize expected discounted lifetime utility:

$$E_o \sum_{t=0}^{\infty} \beta^t U\left(c_t^k, n_t^k, m_t^k, y_t^g\right) \tag{7}$$

where c_t^k is k's consumption bundle at t as defined above, n_t^k is k's hours of work at t, m_t^k is k's end-of-period real money balances, y_t^g is per capita public goods and services at t, E_o is the rational expectations operator conditional on the current period information set and $0 < \beta < 1$ is the time preference rate.

In our numerical solutions, we use a utility function of the form (see also Gali 2008):

$$U\left(c_t^k, n_t^k, m_t^k, y_t^g\right) = \left[\frac{(c_t^k)^{1-\sigma}}{1-\sigma} - x_n \frac{(n_t^k)^{1+\eta}}{1+\eta} + x_m \frac{(m_t^k)^{1-\mu}}{1-\mu} + x_g \frac{(y_t^g)^{1-\zeta}}{1-\zeta}\right] \tag{8}$$

where x_n, x_m, x_g, σ, η, μ, ζ are standard preference parameters.

The budget constraint of each k (written in real terms) is:

$$\begin{aligned}(1+\tau_t^c)c_t^k + \frac{p_t^H}{p_t}x_t^k + \frac{S_t p_t^*}{p_t}f_t^k + b_t^k + m_t^k =& (1-\tau_t^k)\left[r_t^k \frac{p_t^H}{p_t}k_{t-1}^k + \widetilde{\omega_t}^k\right] + (1-\tau_t^n)w_t^k n_t^k + \\ &+ Q_{t-1}\frac{S_t p_t^*}{p_t}\frac{p_{t-1}^*}{p_t^*}f_{t-1}^k + R_{t-1}\frac{p_{t-1}}{p_t}b_{t-1}^k + \\ &+ \frac{p_{t-1}}{p_t}m_{t-1}^k - \tau_t^{l,k} + \widetilde{\pi}_t^k\end{aligned} \tag{9}$$

where x_t^k is k's real investment at t, f_t^k is the real value of k's end-of-period internationally traded assets at t denominated in foreign currency (if negative, it denotes foreign private debt), b_t^k is k's end-of-period real domestic government bonds at t, r_{t-1}^k is gross real return to inherited capital between $t-1$ and t, k_t^k is k's end-of-period private physical capital, $\widetilde{\omega_t}^k$ is k's real dividends paid by domestic private firms at t, w_t^k is capitalists' real wage rate at t, Q_{t-1} is the gross nominal return to international assets between t-1 and t, $R_{t-1} \geq 1$ is gross nominal return to domestic government bonds between $t-1$ and t, $\tau_t^{l,k}$ is real lump-sum taxes/transfers to each k from the government at t, $\widetilde{\pi}_t^k$ is profits distributed in a lump-sum fashion to each k by the financial intermediary (see below), $0 \leq \tau_t^c \leq 1$ is tax rate on consumption at t, $0 \le \tau_t^k \le 1$ is tax rate on capital income at t, $0 \le \tau_t^n \le 1$ is tax rate on labor income at t. Small letters of quantities denote real variables per capitalist e.g. $f_t^k \equiv \frac{F_t^k}{p_t^*}$, $b_t^k \equiv \frac{B_t^k}{p_t}$, $\widetilde{\omega}_t^k \equiv \frac{\widetilde{\Omega}_t^k}{p_t}$, $\widetilde{\pi}_t^k \equiv \frac{\widetilde{\Pi}_t^k}{p_t}$, small letters of prices denote real variables e.g. $w_t^k \equiv \frac{W_t^k}{p_t}$, capital letters of quantities denote nominal values per capitalist and capital letters of prices denote nominal values. Also, letters with star as superscripts denote the counterpart of a variable in the rest-of-the world, e.g. p_t^* stands for the consumer price index (CPI) abroad.

The motion of private physical capital for each k is:

$$k_t^k = (1-\delta)k_{t-1}^k + x_t^k - \frac{\xi}{2}\left(\frac{k_t^k}{k_{t-1}^k} - 1\right)^2 k_{t-1}^k \tag{10}$$

where $0 < \delta < 1$ is the depreciation rate of capital and $\xi \ge 0$ is a parameter capturing adjustment costs related to physical capital.

Therefore,each capitalist k chooses $\{c_t^k, x_t^k, n_t^k, m_t^k, b_t^k, f_t^k, k_t^k\}_{t=0}^{\infty}$ to maximize Eqs (7) and (8) subject to Eqs. (9) and (10), by taking as given prices $\{r_t^k, w_t^k, Q_t, R_t, p_t, p_t^H p_t^*\}_{t=0}^{\infty}$, dividends $\{\widetilde{\omega}_t^k\}_{t=0}^{\infty}$, profits $\{\widetilde{\pi}_t^k\}_{t=0}^{\infty}$, policy variables $\{S_t, \tau_t^c, \tau_t^n, \tau_t^k, \tau_t^{l,k}\}_{t=0}^{\infty}$, and initial conditions, $\{m_{-1}^k, b_{-1}^k, k_{-1}^k, f_{-1}^k\}$.

The first order conditions include the constraints Eqs. (9),(10), and:

$$\frac{(c_t^k)^{-\sigma}}{(1+\tau_t^c)}\frac{p_t^H}{p_t}\left[1 + \xi\left(\frac{k_t^k}{k_{t-1}^k} - 1\right)\right] = \beta\frac{(c_{t+1}^k)^{-\sigma}}{(1+\tau_{t+1}^c)}\frac{p_{t+1}^H}{p_{t+1}} \times \tag{11}$$
$$\times\left[(1-\delta) + (1-\tau_{t+1}^k)r_{t+1}^k - \frac{\xi}{2}\left(\frac{k_{t+1}^k}{k_t^k} - 1\right)^2 + \xi\left(\frac{k_{t+1}^k}{k_t^k} - 1\right)\frac{k_{t+1}^k}{k_t^k}\right]$$

$$\frac{(c_t^k)^{-\sigma}}{(1+\tau_t^c)}S_t\frac{p_t^*}{p_t} = \beta\frac{(c_{t+1}^k)^{-\sigma}}{(1+\tau_{t+1}^c)}Q_t S_{t+1}\frac{p_{t+1}^*}{p_{t+1}}\frac{p_t^*}{p_{t+1}^*} \tag{12}$$

$$\frac{(c_t^k)^{-\sigma}}{(1+\tau_t^c)} = \beta R_t\frac{p_t}{p_{t+1}}\frac{(c_{t+1}^k)^{-\sigma}}{(1+\tau_{t+1}^c)} \tag{13}$$

$$x_n(n_t^k)^{\eta} = (c_t^k)^{-\sigma}\frac{(1-\tau_t^n)}{(1+\tau_t^c)}w_t^k \tag{14}$$

$$x_m(m_t^k)^{-\mu} = \frac{(c_t^k)^{-\sigma}}{(1+\tau_t^c)} - \beta\frac{(c_{t+1}^k)^{-\sigma}}{(1+\tau_{t+1}^c)}\frac{p_t}{p_{t+1}} \tag{15}$$

Eqs. (11),(12) and (13) are respectively the Euler equations for capital, internationally traded assets and domestic government bonds, Eq. (14) is the optimality condition for work hours and Eq. (15) is the optimality condition for money balances.

Next, each capitalist k chooses $\{c_t^{k,H}, c_t^{k,F}\}$ to minimize its total consumption expenditure, Eq. (4), subject to its consumption bundle, Eq. (3), by taking as given prices, $\{p_t^H, p_t^F\}$, and consumption bundle,$\{c_t^k\}$.

The first order conditions include the consumption bundle of k,Eq. (3), and:

$$\frac{c_t^{k,H}}{c_t^{k,F}} = \frac{v}{1-v}\frac{p_t^F}{p_t^H} \tag{16}$$

which is the optimality condition for sharing the total consumption between domestic and imported private products.

Eq.(3), Eq.(4) and Eq.(16) imply the following relation for consumer price index(CPI):

$$p_t = (p_t^H)^v (p_t^F)^{1-v} \tag{17}$$

Finally, each capitalist k chooses $\{c_t^{k,H}(h), c_t^{k,F}(f)\}$ to minimize the sum of its consumption expenditure on private home goods and private foreign goods, sum of Eqs. (5) and (6), subject to composite of domestic and foreign goods consisting of varieties, Eqs. (1) and (2), by taking as given prices, $\{p_t^H(h), p_t^F(f)\}$, and consumption bundles,$\{c_t^{k,H}\}$ and $\{c_t^{k,F}\}$.

The first order conditions include Eqs. (1), (2) and:

$$c_t^{k,H}(h) = \frac{c_t^{k,H}}{N^k}\left(\frac{p_t^H}{p_t^H(h)}\right)^{\phi} \tag{18}$$

$$c_t^{k,F}(f) = \frac{c_t^{k,F}}{N^{k^*}}\left(\frac{p_t^F}{p_t^F(f)}\right)^{\phi} \tag{19}$$

Plugging Eqs. (18) and (19) into Eqs. (1) and (2) respectively, we get the following relations for price indexes:

$$p_t^H = \left\{\sum_{h=1}^{N^k} \frac{1}{N^k}[p_t^H(h)]^{1-\phi}\right\}^{\frac{1}{1-\phi}} \tag{20}$$

$$p_t^F = \left\{\sum_{f=1}^{N^{k^*}} \frac{1}{N^{k^*}}[p_t^F(f)]^{1-\phi}\right\}^{\frac{1}{1-\phi}} \tag{21}$$

2.3 Households as private workers

This subsection presents the problem of domestic private workers, w=1,2,...,N^w.

2.3.1 Consumption bundles and expenditures of private workers

The quantity of each variety h produced at home by domestic private firm h and consumed by each domestic private worker w is denoted as $c_t^{w,H}(h)$. Using a Dixit-Stiglitz aggregator, the composite of domestic private goods consumed by each w, $c_t^{w,H}$, consists of h varieties and is given by:

$$c_t^{w,H} = \left[\sum_{h=1}^{N^k} \left(\frac{1}{N^k} \right)^{\frac{1}{\phi}} [c_t^{w,H}(h)]^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}} \tag{22}$$

Similarly, the quantity of each imported variety f produced abroad by foreign private firm f and consumed by each domestic private worker w is denoted as $c_t^{w,F}(f)$. Using a Dixit-Stiglitz aggregator, the composite of imported private goods consumed by each w, $c_t^{w,F}$, consists of f varieties and is given by:

$$c_t^{w,F} = \left[\sum_{f=1}^{N^{k^*}} \left(\frac{1}{N^{k^*}} \right)^{\frac{1}{\phi}} [c_t^{w,F}(f)]^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}} \tag{23}$$

In turn, having defined $c_t^{w,H}$ and $c_t^{w,F}$, w' consumption bundle, c_t^w, is defined as:

$$c_t^w = \frac{\left(c_t^{w,H}\right)^v \left(c_t^{w,F}\right)^{1-v}}{v^v(1-v)^{1-v}} \tag{24}$$

Each domestic private worker w's total consumption expenditure is:

$$p_t c_t^w = p_t^H c_t^{w,H} + p_t^F c_t^{w,F} \tag{25}$$

Each domestic private worker w' total expenditure on home goods and foreign goods are respectively:

$$p_t^H c_t^{w,H} = \sum_{h=1}^{N^k} p_t^H(h) c_t^{w,H}(h) \tag{26}$$

$$p_t^F c_t^{w,F} = \sum_{f=1}^{N^{k^*}} p_t^F(f) c_t^{w,F}(f) \tag{27}$$

2.3.2 Private workers' optimization problem

Each private worker w has the same expected lifetime utility and instantaneous utility function as each capitalist k, that are given by (7) and (8) respectively, where now the index is w. Each w acts competitively to maximize expected discounted lifetime utility taking prices and policy as given.

The budget constraint of each w (written in real terms) is:

$$(1+\tau_t^c)c_t^w + m_t^w = (1-\tau_t^n)w_t^w n_t^w + \frac{p_{t-1}}{p_t}m_{t-1}^w - \tau_t^{l,w} \tag{28}$$

where also small letters of prices denote real variables per private worker, e.g. $w_t^w \equiv \frac{W_t^w}{p_t}$. Here c_t^w is w's consumption bundle at t as defined above in Subsection 2.3.1, m_t^w is w's end-of-period real money balances, n_t^w is w's hours of work at t, w_t^w is private workers' real wage rate at t and $\tau_t^{l,w}$ is real lump-sum taxes/transfers to each w from the government at t.

Therefore, each private worker chooses $\{c_t^w, n_t^w, m_t^w\}_{t=0}^{\infty}$ to maximize Eqs. (7) and (8) for w, subject to Eq. (28) , by taking as given prices $\{w_t^w, p_t\}_{t=0}^{\infty}$, policy variables $\{\tau_t^c, \tau_t^n, \tau_t^{l,w}\}_{t=0}^{\infty}$, and initial condition, $\{m_{-1}^w\}$.

The first order conditions include the budget constraint above,Eq. (28), and:

$$\frac{(c_t^w)^{-\sigma}}{x_n(n_t^w)^{\eta}} = \frac{1+\tau_t^c}{(1-\tau_t^n)w_t^w} \tag{29}$$

$$\frac{(c_t^w)^{-\sigma}}{1+\tau_t^c} = \beta\frac{p_t}{p_{t+1}}\left[\frac{(c_{t+1}^w)^{-\sigma}}{1+\tau_{t+1}^c}\right] + x_m(m_t^w)^{-\mu} \tag{30}$$

Eq.(29) is the optimality condition for work hours and Eq.(30) is the optimality condition for real money balances.

Next, each private worker w chooses $\{c_t^{w,H}, c_t^{w,F}\}$ to minimize its total consumption expenditure, Eq. (25), subject to its consumption bundle, Eq. (24), by taking as given prices, $\{p_t^H, p_t^F\}$, and consumption bundle,$\{c_t^w\}$.

The first order conditions include the consumption bundle of w,Eq. (24), and:

$$\frac{c_t^{w,H}}{c_t^{w,F}} = \frac{v}{1-v}\frac{p_t^F}{p_t^H} \tag{31}$$

which is the optimality condition for sharing the total consumption between domestic and imported private products.

Eq.(24), Eq.(25) and Eq. (31) imply the following relation for consumer price index(CPI):

$$p_t = (p_t^H)^v (p_t^F)^{1-v} \tag{32}$$

Finally, each private worker w chooses $\{c_t^{w,H}(h), c_t^{w,F}(f)\}$ to minimize the sum of its consumption expenditure on home goods and foreign goods, sum of Eqs. (26) and (27), subject to composite of domestic and foreign goods consisting of varieties, Eqs. (22) and (23), by taking as given prices, $\{p_t^H(h), p_t^F(f)\}$, and consumption bundles,$\{c_t^{w,H}\}$ and $\{c_t^{w,F}\}$.

The first order conditions include Eqs. (22), (23) and:

$$c_t^{w,H}(h) = \frac{c_t^{w,H}}{N^k}\left(\frac{p_t^H}{p_t^H(h)}\right)^{\phi} \tag{33}$$

$$c_t^{w,F}(f) = \frac{c_t^{w,F}}{N^{k^*}}\left(\frac{p_t^F}{p_t^F(f)}\right)^{\phi} \tag{34}$$

Plugging Eqs. (33) and (34) into Eqs. (22) and (23) respectively, we get the following relations for price indexes:

$$p_t^H = \left\{\sum_{h=1}^{N^k} \frac{1}{N^k}[p_t^H(h)]^{1-\phi}\right\}^{\frac{1}{1-\phi}} \tag{35}$$

$$p_t^F = \left\{\sum_{f=1}^{N^{k^*}} \frac{1}{N^{k^*}}[p_t^F(f)]^{1-\phi}\right\}^{\frac{1}{1-\phi}} \tag{36}$$

2.4 Households as public employees

This subsection presents the problem of domestic public employees, b=1,2,...,N^b.

2.4.1 Consumption bundles and expenditures of public employees

The quantity of each variety h produced at home by domestic private firm h and consumed by each domestic public employee b is denoted as $c_t^{b,H}(h)$. Using a Dixit-Stiglitz aggregator, the composite of domestic private goods consumed by each b, $c_t^{b,H}$, consists of h varieties and is given by:

$$c_t^{b,H} = \left[\sum_{h=1}^{N^k}\left(\frac{1}{N^k}\right)^{\frac{1}{\phi}}[c_t^{b,H}(h)]^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} \tag{37}$$

Similarly, the quantity of each imported variety f produced abroad by foreign private firm f and consumed by each domestic public employee

b is denoted as $c_t^{b,F}(f)$. Using a Dixit-Stiglitz aggregator, the composite of imported private goods consumed by each b, $c_t^{b,F}$, consists of f varieties and is given by:

$$c_t^{b,F} = \left[\sum_{f=1}^{N^{k^*}} \left(\frac{1}{N^{k^*}} \right)^{\frac{1}{\phi}} [c_t^{b,F}(f)]^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}} \tag{38}$$

In turn, having defined $c_t^{b,H}$ and $c_t^{b,F}$, b' consumption bundle, c_t^b, is defined as:

$$c_t^b = \frac{\left(c_t^{b,H}\right)^v \left(c_t^{b,F}\right)^{1-v}}{v^v(1-v)^{1-v}} \tag{39}$$

Each domestic public employee b's total consumption expenditure is:

$$p_t c_t^b = p_t^H c_t^{b,H} + p_t^F c_t^{b,F} \tag{40}$$

Each domestic public employee b' total expenditure on home goods and foreign goods are respectively:

$$p_t^H c_t^{b,H} = \sum_{h=1}^{N^k} p_t^H(h) c_t^{b,H}(h) \tag{41}$$

$$p_t^F c_t^{b,F} = \sum_{f=1}^{N^{k^*}} p_t^F(f) c_t^{b,F}(f) \tag{42}$$

2.4.2 Public employee' optimization problem

Each public employee b has the same expected lifetime utility and instantaneous utility function as each capitalist k, that are given by (7) and (8) respectively, where now the index is b. Each b acts competitively to maximize expected discounted lifetime utility taking prices and policy as given.

The budget constraint of each b (written in real terms) is:

$$(1+\tau_t^c)c_t^b + m_t^b = (1-\tau_t^n) w_t^b n_t^b + \frac{p_{t-1}}{p_t} m_{t-1}^b - \tau_t^{l,b} \tag{43}$$

where also small letters of prices denote real variables, e.g. $w_t^b \equiv \frac{W_t^b}{p_t}$. Here c_t^b is b's consumption bundle at t as defined above in Subsection 2.4.1, m_t^b is b's end-of-period real money balances, n_t^b is b's hours of work at t, w_t^b is public employees' real wage rate at t and $\tau_t^{l,b}$ is real lump-sum taxes/transfers to each b from the government at t.

Assuming that the government exogeneously determines the number of public employees, N^b, their labor, $n_t^b \equiv 1$, and their real wage, w_t^b, then the total public wage bill in real terms divided by the number of capitalists, defined as $\overline{g}_t^w$, equals $\frac{v^b}{v^k} w_t^b n_t^b$. Hence, we can rewrite the budget constraint of b as follows:

$$(1+\tau_t^c)c_t^b + m_t^b = (1-\tau_t^n)\frac{v^k}{v^b}\overline{g}_t^w + \frac{p_{t-1}}{p_t} m_{t-1}^b - \tau_t^{l,b} \tag{44}$$

Therefore, each public employee chooses $\{c_t^b, m_t^b\}_{t=0}^{\infty}$ to maximize Eqs. (7) and (8) for b, subject to Eq. (44), by taking as given prices $\{p_t\}_{t=0}^{\infty}$, policy variables $\{\tau_t^c, \tau_t^n, \tau_t^{l,b}, \overline{g}_t^w\}_{t=0}^{\infty}$, and initial condition, $\{m_{-1}^b\}$.

The first order conditions include the budget constraint above,Eq. (44), and:

$$\frac{(c_t^b)^{-\sigma}}{1+\tau_t^c} = \beta \frac{p_t}{p_{t+1}} \left[\frac{(c_{t+1}^b)^{-\sigma}}{1+\tau_{t+1}^c} \right] + x_m (m_t^b)^{-\mu} \tag{45}$$

Eq.(45) is the optimality condition for real money balances.

Next, each public employee b chooses $\{c_t^{b,H}, c_t^{b,F}\}$ to minimize its total consumption expenditure, Eq. (40), subject to its consumption bundle, Eq. (39), by taking as given prices, $\{p_t^H, p_t^F\}$, and consumption bundle,$\{c_t^b\}$.

The first order conditions include the consumption bundle of b,Eq. (39), and:

$$\frac{c_t^{b,H}}{c_t^{b,F}} = \frac{v}{1-v} \frac{p_t^F}{p_t^H} \tag{46}$$

which is the optimality condition for sharing the total consumption between domestic and imported private products.

Eq.(39), Eq.(40) and Eq. (46) imply the following relation for consumer price index(CPI):

$$p_t = (p_t^H)^v (p_t^F)^{1-v} \tag{47}$$

Finally, each public employee b chooses $\{c_t^{b,H}(h), c_t^{b,F}(f)\}$ to minimize the sum of its consumption expenditure on home goods and foreign goods, sum of Eqs. (41) and (42), subject to composite of domestic and foreign goods consisting of varieties, Eqs. (37) and (38), by taking as given prices, $\{p_t^H(h), p_t^F(f)\}$, and consumption bundles,$\{c_t^{b,H}\}$ and $\{c_t^{b,F}\}$.

The first order conditions include Eqs. (37), (38) and:

$$c_t^{b,H}(h) = \frac{c_t^{b,H}}{N^k}\left(\frac{p_t^H}{p_t^H(h)}\right)^{\phi} \tag{48}$$

$$c_t^{b,F}(f) = \frac{c_t^{b,F}}{N^{k^*}}\left(\frac{p_t^F}{p_t^F(f)}\right)^{\phi} \tag{49}$$

Plugging Eqs. (48) and (49) into Eqs. (37) and (38) respectively, we get the following relations for price indexes:

$$p_t^H = \left\{\sum_{h=1}^{N^k} \frac{1}{N^k}[p_t^H(h)]^{1-\phi}\right\}^{\frac{1}{1-\phi}} \tag{50}$$

$$p_t^F = \left\{\sum_{f=1}^{N^{k^*}} \frac{1}{N^{k^*}}[p_t^F(f)]^{1-\phi}\right\}^{\frac{1}{1-\phi}} \tag{51}$$

2.5 Private firms

This subsection presents the problem of private firms in the domestic economy. There are N^k domestic private firms indexed by $h = 1, 2, ..., N^k$. Each private firm h produces a differentiated tradable good of variety h under monopolistic competition and Rotemberg-type nominal fixities(see Leeper et al, 2013).

2.5.1 Demand for private firms' product

Each domestic private firm h faces demand for its product, $y_t^{H,d}(h)$. The latter comes from domestic households' private consumption and investment, $C_t^H(h)$ and $X_t(h)$, where $C_t^H(h) \equiv \sum_{k=1}^{N^k} c_t^{k,H}(h) + \sum_{w=1}^{N^w} c_t^{w,H}(h) + \sum_{b=1}^{N^b} c_t^{b,H}(h)$ and $X_t(h) \equiv \sum_{k=1}^{N^k} x_t^k(h)$, from the use of private goods the domestic state-owned enterprise as inputs in its production function, denoted as $G_t^c(h)$, from the government's investment,$G_t^i(h)$, from the financial intermediary which is located in the domestic country, denoted as $\Upsilon_t(h)$,[16] and from foreign households' consumption of the domestic good, $C_t^{F^*}(h)$, where $C_t^{F^*}(h) \equiv \sum_{k^*=1}^{N^{k^*}} c_t^{k,F^*}(h) + \sum_{w^*=1}^{N^{w^*}} c_t^{w,F^*}(h) + \sum_{b^*=1}^{N^{b^*}} c_t^{b,F^*}(h)$. Thus, aggregate demand for each good h is:

$$y_t^{H,d}(h) = \left[C_t^H(h) + X_t(h) + G_t^c(h) + G_t^i(h) + \Upsilon_t(h) + C_t^{F^*}(h)\right] \tag{52}$$

Aggregate demand for each good h is associated with production of domestic private firm h according to the following relation:

$$y_t^{H,d}(h) = y_t^H(h) \times \left\{1 - \frac{\phi^p}{2}\left[\frac{p_t^H(f)}{p_{t-1}^H(f)\pi^H} - 1\right]^2 \frac{Y_t^H}{N^k y_t^H(h)}\right\} \tag{53}$$

where $y_t^H(h)$ stands for the production of domestic private firm h, Y_t^H stands for the total domestic private output, π^H stands for the steady state value of the gross domestic goods inflation rate and $\phi^p \geq 0$ is a parameter which determines the degree of nominal price rigidity. The term in the brackets captures the Rotemberg-type pricing cost (See the next Subsection) and reflects the discrepancy between production and demand as one expected in a Rotemberg-type fashion.

Since we have:

$$c_t^{k,H}(h) = \frac{c_t^{k,H}}{N^k}\left(\frac{p_t^H}{p_t^H(h)}\right)^{\phi} \tag{54}$$

$$c_t^{w,H}(h) = \frac{c_t^{w,H}}{N^k}\left(\frac{p_t^H}{p_t^H(h)}\right)^{\phi} \tag{55}$$

$$c_t^{b,H}(h) = \frac{c_t^{b,H}}{N^k}\left(\frac{p_t^H}{p_t^H(h)}\right)^{\phi} \tag{56}$$

$$x_t^k(h) = \frac{x_t^k}{N^k}\left(\frac{p_t^H}{p_t^H(h)}\right)^{\phi} \tag{57}$$

$$G_t^c(h) = \frac{G_t^c}{N^k}\left(\frac{p_t^H}{p_t^H(h)}\right)^{\phi} \tag{58}$$

[16] See also Curdia and Woodford (2009) for a similar modelling of resources consumed by banks; the latter are modelled below. That is, the model requires the bank to use real resources in the period in which the loan is originated.

$$G_t^i(h) = \frac{G_t^i}{N^k}\left(\frac{p_t^H}{p_t^H(h)}\right)^{\phi} \tag{59}$$

$$\Upsilon_t(h) = \frac{\Upsilon_t}{N^k}\left(\frac{p_t^H}{p_t^H(h)}\right)^{\phi} \tag{60}$$

$$c_t^{k,F^*}(h) = \frac{c_t^{k,F^*}}{N^{k^*}}\left(\frac{p_t^{F^*}}{p_t^{F^*}(h)}\right)^{\phi} \tag{61}$$

$$c_t^{w,F^*}(h) = \frac{c_t^{w,F^*}}{N^{k^*}}\left(\frac{p_t^{F^*}}{p_t^{F^*}(h)}\right)^{\phi} \tag{62}$$

$$c_t^{b,F^*}(h) = \frac{c_t^{b,F^*}}{N^{k^*}}\left(\frac{p_t^{F^*}}{p_t^{F^*}(h)}\right)^{\phi} \tag{63}$$

we can rewrite the relation (52) as:

$$y_t^{H,d}(h) = \frac{1}{N^k}\left[C_t^H + X_t + G_t^c + G_t^i + \Upsilon_t + C_t^{F^*}\right] \times \left(\frac{p_t^H}{p_t^H(h)}\right)^{\phi} \tag{64}$$

where $C_t^H \equiv \sum\limits_{k=1}^{N^k} c_t^{k,H} + \sum\limits_{w=1}^{N^w} c_t^{w,H} + \sum\limits_{b=1}^{N^b} c_t^{b,H}$ is total consumption of private home goods, $X_t \equiv \sum\limits_{k=1}^{N^k} x_t^k$ is total private investment, G_t^c denotes total goods and services of private sector that are used by the state-owned enterprise for the production of total public goods and services, G_t^i denotes public infrastructure investment, Υ_t denotes total resources consumed by the financial intermediary and $C_t^{F^*} \equiv \sum\limits_{k^*=1}^{N^{k^*}} c_t^{k,F^*} + \sum\limits_{w^*=1}^{N^{w^*}} c_t^{w,F^*} + \sum\limits_{b^*=1}^{N^{b^*}} c_t^{b,F^*}$ is total consumption of private home goods by households in the foreign country (i.e. domestic country's exports). Also notice that the law of one price implies that in Eqs. (61) and (62):

$$\frac{p_t^{F^*}}{p_t^{F^*}(h)} = \frac{\frac{p_t^H}{S_t}}{\frac{p_t^H(h)}{S_t}} = \frac{p_t^H}{p_t^H(h)} \tag{65}$$

Since aggregate demand,$Y_t^{H,d}$, is:

$$Y_t^{H,d} = \left[C_t^H + X_t + G_t^c + G_t^i + \Upsilon_t + C_t^{F^*}\right] \tag{66}$$

then aggregate demand for each private good h is rewritten as:

$$y_t^{H,d}(h) = \frac{Y_t^{H,d}}{N^k}\left(\frac{p_t^H}{p_t^H(h)}\right)^{\phi} \tag{67}$$

Another equivalent expression of demand for each good h in terms of private production follows:

$$y_t^H(h) = \frac{Y_t^H}{N^k}\left(\frac{p_t^H}{p_t^H(h)}\right)^{\phi} \tag{68}$$

where

$$Y_t^{H,d} \equiv Y_t^H \times \left\{1 - \frac{\phi^p}{2}\left[\frac{p_t^H(f)}{p_{t-1}^H(f)\pi^H} - 1\right]^2 \frac{Y_t^H}{N^k y_t^H(h)}\right\}$$

and with Y_t^H to denote the aggregate domestic private production.

Notice that in the private firms' optimization problem below we should use Eq.(67) as an expression for demand of each good h. However, it is more convenient to work with Eq.(68), to represent demand for each good h, instead of Eq.(67)

2.5.2 Private firms' optimization problem

Nominal profits of private firm h are defined as:

$$\widetilde{\Omega}_t(h) = p_t^H(h)y_t^H(h) - p_t^H r_t^k k_{t-1}(h) - W_t^w n_t^w(h) - W_t^k n_t^k(h) - \frac{\phi^p}{2}\left(\frac{p_t^H(h)}{p_{t-1}^H(h)\pi^H} - 1\right)^2 \frac{p_t^H Y_t^H}{N^k} \tag{69}$$

where $k_{t-1}(h)$ denotes the capital input chosen by private firm h, $n_t^w(h)$ denotes private workers' labor input chosen by private firm h and $n_t^h(h)$ denotes the capitalists' labor input chosen by private firm h. The quadratic cost that the private firm h faces once it changes the price of its product is proportional to the aggregate domestic private output divided by the number of private firms. [17]

[17] This specification of Rotemberg-type cost is similar to that of Leeper et al,2013. Here, working with summations instead of integrals, we should have a pricing cost which is proportional to the aggregate domestic private output divided by the number of private firms. With this modification, we can derive the same NK Philips curve as Leeper at al,2013.

All private firms use the same technology represented by the production function(see also Hornstein et al,2005, and Baxter and King,1993):

$$y_t^H(h) = A_t \left\{ [k_{t-1}(h)]^{\alpha} \left[\{n_t^k(h)\}^{\theta} \{n_t^w(h)^{1-\theta}\} \right]^{1-\alpha} \right\} \left(k_{t-1}^g \right)^{\theta_k} \tag{70}$$

where A_t is an exogenous TFP, $0 < \alpha < 1$ is a technology parameter, k_{t-1}^g denotes the stock of public infrastructure divided by the number of capitalists which is common for all private firms, $0 < \theta_k < 1$ is the output elasticity of public infrastructure for private firm h and $0 < \theta < 1$ labor efficiency parameter of capitalist. We assume a positive θ_k , which implies that the production function has increasing returns with respect to public infrastructure, as in Baxter and King (1993), Basu and Kollmann (2013), Bom and Ligthart (2014),Leduc and Wilson (2013) and Iwata (2013). Notice that we keep CRS over private inputs.

Profit maximization by private firm h is also subject to the demand for its product, Eq.(67)as derived above. But as we have aforementioned, instead of using Eq.(67), we can equivalently use Eq.(68).

Each private firm h chooses its price, $p_t^H(h)$, and its inputs, $k_t^k(h)$, $n_t^k(h)$, $n_t^w(h)$, to maximize the sum of discounted expected real dividends, $\max E_o \sum_{t=0}^{\infty} \Xi_{0,0+t} \frac{\widetilde{\Omega_t}(h)}{p_t}$, subject to Eq. (68) and its production function, Eq. (70). The objective in real terms is given by:

$$\max E_o \sum_{t=0}^{\infty} \Xi_{0,0+t} \left[\frac{p_t^H(h)}{p_t} y_t^H(h) - \frac{p_t^H(h)}{p_t} r_t^k k_{t-1}(h) - w_t^w n_t^w(h) - w_t^k n_t^k(h) - \frac{\phi^p}{2} \left(\frac{p_t^H(h)}{p_{t-1}^H(h)\pi^H} - 1 \right)^2 \frac{p_t^H Y_t^H}{p_t N^k} \right] \tag{71}$$

where $\Xi_{0,0+t}$ is a stochastic discount factor taken as given by the private firm h. This is defined as $\Xi_{0,0+t} = \prod_{i=0}^{t-1} \left\{ \frac{1}{R_i} \right\} = \beta^t \prod_{i=0}^{t-1} \left[\left(\frac{p_i}{p_{i+1}} \right) \left(\frac{1+\tau_i^c}{1+\tau_{i+1}^c} \right) \left(\frac{c_{i+1}^k}{c_i^k} \right)^{-\sigma} \right]$ and arises from Euler for bonds.

2.5.3 Private firms' optimality conditions

Following the related literature, instead of solving the above problem, we follow a two step procedure. We first solve a cost minimization problem, where each private firm h minimizes its cost by choosing factor inputs given technology and prices. The solution will give a minimum nominal cost function, which is a function of factor prices and output produced by the private firm. In turn, given this cost function, each private firm h solves a maximization problem by choosing its price.

Cost minimization problem: In the first stage, we solve a static cost minimization problem, where each h minimizes its cost by choosing its production factor inputs, $k_t^k(h)$,$n_t^k(h)$,$n_t^w(h)$, subject to its production function, Eq.(70), given technology and prices. The cost function is defined in real terms as follows:

$$\min \Psi(y_t^H(h)) = \left[\frac{p_t^H(h)}{p_t} r_t^k k_{t-1}(h) + w_t^w n_t^w(h) + w_t^k n_t^k(h)\right] \tag{72}$$

The solution to the cost minimization problem gives the input demand functions:

$$\frac{p_t^H(h)}{p_t} r_t^k k_{t-1}(h) = mc_t \alpha y_t^H(h) \tag{73}$$

$$w_t^k n_t^k(h) = mc_t \theta(1-\alpha) y_t^H(h) \tag{74}$$

$$w_t^w n_t^w(h) = mc_t (1-\theta)(1-\alpha) y_t^H(h) \tag{75}$$

where $mc_t \equiv \Psi'(y_t^H(h))$ since, by definition, the real marginal cost is the derivative of the associated minimum nominal cost function,$\Psi(y_t^H(h))$, with respect to $y_t^H(h)$.

Summing up the three above equations it arises the following relation for the associated minimum real cost function of h:

$$\Psi(y_t^H(h)) = mc_t y_t^H(h) \tag{76}$$

Where the real marginal cost, mc_t, it can be shown that equals:

$$mc_t = \frac{1}{A_t (k_{t-1}^g)^{\theta_k}} \left[\frac{p_t^H(h)}{p_t} \frac{r_t^k}{\alpha}\right]^{\alpha} \left[\left\{\frac{w_t^k}{\theta(1-\alpha)}\right\}^{\theta} \times \left\{\frac{w_t^w}{(1-\theta)(1-\alpha)}\right\}^{1-\theta}\right]^{1-\alpha} \tag{77}$$

implying that mc_t is common for all private firms since it only depends on prices, parameters and technology which are common for all private firms.

Profit maximization: The solution to the cost minimization problem will give a minimum nominal cost function, which is a function of prices and output produced by the private firm. In turn, given this cost function, each h solves a dynamic maximization problem by choosing its price. Specifically, in the second stage, h chooses its price,$p_t^H(h)$,to maximize the lifetime expected discounted real profits:

$$\max E_o \sum_{t=0}^{\infty} \Xi_{0,0+t} \left[\frac{p_t^H(h)}{p_t} y_t^H(h) - \Psi(y_t^H(h)) - \frac{\phi^p}{2} \left(\frac{p_t^H(h)}{p_{t-1}^H(h)\pi^H} - 1 \right)^2 \frac{p_t^H Y_t^H}{p_t N^k} \right] \tag{78}$$

The above profit maximization is subject to the Eq. (68) which is equivalent to the demand equation that the monopolistically competitive private firm h faces, Eq.(67).

The first order condition gives:

$$(1-\phi)\frac{p_t^H(h)}{p_t} y_t^H(h) + \phi mc_t y_t^H(h) - \phi^p \left[\frac{p_t^H(h)}{p_{t-1}^H(h)\pi^H} - 1 \right] \frac{p_t^H}{p_t} \frac{Y_t^H p_t^H(h)}{N^k p_{t-1}^H(h)\pi^H} =$$
$$\beta \phi^p \left[\left(\frac{p_t}{p_{t+1}} \right) \left(\frac{1+\tau_t^c}{1+\tau_{t+1}^c} \right) \left(\frac{c_{t+1}^k}{c_t^k} \right)^{-\sigma} \right] \left[1 - \frac{p_{t+1}^H(h)}{p_t^H(h)\pi^H} \right] \frac{p_{t+1}^H(h)}{p_t^H(h)\pi^H} \frac{p_{t+1}^H}{p_{t+1}} \frac{Y_{t+1}^H}{N^k} \tag{79}$$

Thus, the behavior of h is summarized by Eqs. (73),(74),(75) and (79).

All private firms solve the identical problem and they will set the same price,$p_t^H(h)$, which implies that $p_t^H(h) = p_t^H$.

2.6 Public sector

We now present the public sector. we start with the government budget constraint and then specify the production function of public goods and services.

2.6.1 Government budget constraint

In the domestic country, the period budget constraint of the ”consolidated” public sector expressed in real terms (per capitalist, per private worker and per public employee) is(See Appendix for details):

$$Q_{t-1}\frac{S_t p_t^*}{p_t}\frac{p_{t-1}^*}{p_t^*}f_{t-1}^g + R_{t-1}\frac{p_{t-1}}{p_t}b_{t-1} + \frac{p_t^H}{p_t}\overline{g}_t^c + \frac{p_t^H}{p_t}\overline{g}_t^i + \overline{g}_t^w + \frac{p_{t-1}}{p_t}m_{t-1} = \tag{80}$$

$$= m_t + \tau_t^c\left[\frac{p_t^H}{p_t}\left(c_t^{k,H} + \frac{v^w}{v^k}c_t^{w,H} + \frac{v^b}{v^k}c_t^{b,H}\right) + \frac{p_t^F}{p_t}\left(c_t^{k,F} + \frac{v^w}{v^k}c_t^{w,F} + \frac{v^b}{v^k}c_t^{b,F}\right)\right] +$$

$$+\tau_t^k\left[r_t^k\frac{p_t^H}{p_t}k_{t-1}^k + \widetilde{\omega_t}^k\right] + \tau_t^n\left[w_t^k n_t^k + \frac{v^w}{v^k}w_t^w n_t^w + \overline{g}_t^w\right] +$$

$$+\left[\tau_t^{l,k} + \frac{v^w}{v^k}\tau_t^{l,w} + \frac{v^b}{v^k}\tau_t^{l,w}\right] + b_t + S_t\frac{p_t^*}{p_t}f_t^g$$

where f_t^g is the end-of-period domestic real public debt held by foreign capitalists and expressed in foreign prices,[18] b_t is the end-of-period domestic real public debt held by domestic capitalists, $\frac{p_t^H}{p_t}\overline{g}_t^c \equiv \frac{p_t^H}{p_t}\frac{G_t^c}{N^k}$ is total public spending on goods and services purchased from the private sector in real terms divided by the number of capitalists, $\frac{p_t^H}{p_t}\overline{g}_t^i \equiv \frac{p_t^H}{p_t}\frac{G_t^i}{N^k}$ is total public investment in infrastructure in real terms divided by the number of capitalists and m_t is the end-of-period stock of real money balances divided by the number of capitalists. All other variables have been defined above.

Here, we model infrastructure as a stock variable assuming that it accumulates like physical private capital (see also Fischer and Turnovsky (1998)). Hence, the stock of public infrastructure divided by the number of capitalists,k_t^g, evolves according to:

$$\overline{g}_t^i = k_t^g - (1-\delta^g)k_{t-1}^g + \frac{\xi^g}{2}\left(\frac{k_t^g}{k_{t-1}^g} - 1\right)^2 k_{t-1}^g \tag{81}$$

where $0 \le \delta^g \le 1$ is the depreciation rate of public infrastructure and $\xi^g \ge 0$ is a parameter capturing adjustment costs related to public infrastructure stock.

Equivalently, if we define total nominal public debt in the domestic country as $D_t \equiv B_t + S_t F_t^g$, so that in real and per capitalist terms $d_t \equiv b_t + \frac{S_t p_t^*}{p_t}f_t^g$, we have $b_t \equiv \lambda_t d_t$ and $\frac{S_t p_t^*}{p_t}f_t^g \equiv (1-\lambda_t)d_t$,where $0 \le \lambda_t \le 1$ denotes the fraction of domestic public debt held by domestic private agents (domestic capitalists) and $0 \le 1-\lambda_t \le 1$ is the fraction of domestic public debt held

[18]That is, since the returns to bonds held by domestic capitalists and to the same bonds held by foreign capitalists can differ, our modelling implies that the government bond market can be segmented.

by foreign private agents(foreign capitalists). Then, the above government budget constraint is rewritten as:

$$Q_{t-1}\frac{S_t p_t^*}{p_t}\frac{p_{t-1}^*}{p_t^*}\frac{p_{t-1}}{S_{t-1}p_{t-1}^*}(1-\lambda_{t-1})d_{t-1}+R_{t-1}\frac{p_{t-1}}{p_t}\lambda_{t-1}d_{t-1}+\frac{p_t^H}{p_t}\overline{g}_t^c+\frac{p_t^H}{p_t}\overline{g}_t^i+\overline{g}_t^w+\frac{p_{t-1}}{p_t}m_{t-1}= \tag{82}$$

$$=m_t+\tau_t^c\left[\frac{p_t^H}{p_t}\left(c_t^{k,H}+\frac{v^w}{v^k}c_t^{w,H}+\frac{v^b}{v^k}c_t^{b,H}\right)+\frac{p_t^F}{p_t}\left(c_t^{k,F}+\frac{v^w}{v^k}c_t^{w,F}+\frac{v^b}{v^k}c_t^{b,F}\right)\right]+\tau_t^k\left[r_t^k\frac{p_t^H}{p_t}k_{t-1}^k+\widetilde{\omega_t}^k\right]+$$
$$+\tau_t^n\left[w_t^k n_t^k+\frac{v^w}{v^k}w_t^w n_t^w+\overline{g}_t^w\right]+\left[\tau_t^{l,k}+\frac{v^w}{v^k}\tau_t^{l,w}+\frac{v^b}{v^k}\tau_t^{l,b}\right]+d_t$$

where $d_t \equiv \frac{D_t}{p_t N^k}$.

Therefore, as in e.g. Alesina et al. (2002), we include the four main types of government spending (purchases of goods and services from the private sector, public investment in infrastructure, public wages, and transfers to individuals). We also include the three main types of taxes (taxes on consumption, capital income and labor).

In each period, one of ($\tau_t^c, \tau_t^k, \tau_t^n, \overline{g}_t^c, \overline{g}_t^i, \overline{g}_t^w, \tau_t^{l,k}, \tau_t^{l,w}, \tau_t^{l,b}, \lambda_t, d_t, v_t^b$)[19] need to adjust to satisfy the government budget constraint in the domestic country. We assume, except otherwise said, that this role is played by the end-of period total public debt, d_t. [20]

Similarly, in the foreign country, the period budget constraint of the "consolidated" public sector expressed in real terms (per capitalist, per

[19]Actually, in this model the percantage of public employees in the domestic country remains constant over time, for this reason the time subscript can be eliminated.

[20]That is, we treat the share of public debt held by foreign private agents, $(0 \leq 1-\lambda_t \leq 1)$, as an exogenous variable. In our model, there is a single international asset subject to a single transaction cost. Thus, since we do not allow for separate international asset markets (one for private and one for public), we need an extra assumption to get a solution and this is provided by treating λ_t as an exogenous variable in each country (it will be set as in the data average). Alternatively, we could assume that private agents (capitalists) in each country can separately invest in foreign private assets and foreign government bonds (rather than in a single international asset). But, as is known, this modelling would lead to a non-well specified system (a kind of portfolio indeterminacy), except if one is willing to assume different transaction costs in different asset markets. In the latter case, portfolio shares could be determined but their solution would depend on the parameterization of the associated transaction cost function. This would not be different from treating λ_t exogenously in the first place.

private worker and per public employee) is(See Appendix for details):

$$Q^*_{t-1}\frac{p_t}{S_t p^*_t}\frac{p_{t-1}}{p_t}f^{g*}_{t-1}+R^*_{t-1}\frac{p^*_{t-1}}{p^*_t}b^*_{t-1}+\frac{p^{H*}_t}{p^*_t}\overline{g}^{c*}_t+\frac{p^{H*}_t}{p^*_t}\overline{g}^{i*}_t+\overline{g}^{w*}_t+\frac{p^*_{t-1}}{p^*_t}m^*_{t-1}= \tag{83}$$

$$=m^*_t+\tau^{c*}_t\left[\frac{p^{H*}_t}{p^*_t}\left(c^{k,H*}_t+\frac{v^{w*}}{v^{k*}}c^{w,H*}_t+\frac{v^{b*}}{v^{k*}}c^{b,H*}_t\right)+\frac{p^{F*}_t}{p^*_t}\left(c^{k,F*}_t+\frac{v^{w*}}{v^{k*}}c^{w,F*}_t+\frac{v^{b*}}{v^{k*}}c^{b,F*}_t\right)\right]+$$
$$+\tau^{k*}_t\left[r^{k*}_t\frac{p^{H*}_t}{p^*_t}k^{k*}_{t-1}+\widetilde{\omega}^{k*}_t\right]+\tau^{n*}_t\left[w^{k*}_t n^{k*}_t+\frac{v^{w*}}{v^{k*}}w^{w*}_t n^{w*}_t+\overline{g}^{w*}_t\right]+$$
$$+\left[\tau^{l,k*}_t+\frac{v^{w*}}{v^{k*}}\tau^{l,w*}_t+\frac{v^{b*}}{v^{k*}}\tau^{l,b*}_t\right]+b^*_t+\frac{p_t}{S_t p^*_t}f^{g*}_t$$

where, as we have aforementioned, a star denotes the counterpart of a variable or a parameter in the foreign country.

Let denote D^*_t to be the total foreign public debt in foreign currency. This can be held by foreign private agents(capitalists), $B^*_t=\lambda^*_t D^*_t$, and by domestic private agents(capitalists), $\frac{F^{g*}_t}{S_t}=(1-\lambda^*_t)D^*_t$. Then, we have:

$$Q^*_{t-1}\frac{p_t}{S_t p^*_t}\frac{p_{t-1}}{p_t}\frac{S_{t-1}p^*_{t-1}}{p_{t-1}}(1-\lambda^*_{t-1})d^*_{t-1}+R^*_{t-1}\frac{p^*_{t-1}}{p^*_t}\lambda^*_{t-1}d^*_{t-1}+\frac{p^{H*}_t}{p^*_t}\overline{g}^{c*}_t+\frac{p^{H*}_t}{p^*_t}\overline{g}^{i*}_t+\overline{g}^{w*}_t+\frac{p^*_{t-1}}{p^*_t}m^*_{t-1}= \tag{84}$$

$$=m^*_t+\tau^{c*}_t\left[\frac{p^{H*}_t}{p^*_t}\left(c^{k,H*}_t+\frac{v^{w*}}{v^{k*}}c^{w,H*}_t+\frac{v^{b*}}{v^{k*}}c^{b,H*}_t\right)+\frac{p^{F*}_t}{p^*_t}\left(c^{k,F*}_t+\frac{v^{w*}}{v^{k*}}c^{w,F*}_t+\frac{v^{b*}}{v^{k*}}c^{b,F*}_t\right)\right]+$$
$$+\tau^{k*}_t\left[r^{k*}_t\frac{p^{H*}_t}{p^*_t}k^{k*}_{t-1}+\widetilde{\omega}^{k*}_t\right]+\tau^{n*}_t\left[w^{k*}_t n^{k*}_t+\frac{v^{w*}}{v^{k*}}w^{w*}_t n^{w*}_t\overline{g}^{w*}_t\right]+$$
$$+\left[\tau^{l,k*}_t+\frac{v^{w*}}{v^{k*}}\tau^{l,w*}_t+\frac{v^{b*}}{v^{k*}}\tau^{l,b*}_t\right]+d^*_t$$

Similarly, in each period, one of ($\tau^{c*}_t,\tau^{k*}_t,\tau^{n*}_t,\overline{g}^{c*}_t,\overline{g}^{i*}_t,\overline{g}^{w*}_t,\tau^{l,k*}_t,\tau^{l,w*}_t,\tau^{l,b*}_t,\lambda^*_t,d^*_t,v^{b*}_t$)[21] need to adjust to satisfy the government budget constraint in the foreign country. We also assume, except otherwise said, that this role is played by the end-of period total public debt, d^*_t.

[21] Actually, in this model the percantage of public employees in the foreign country remains constant over time, for this reason the time subscript can be eliminated.

2.6.2 State-owned enterprise

Following most of the related literature,[22] we assume that total public goods and services, Y_t^g, are produced using goods and services purchased from the private sector, G_t^c , and total public employment, L_t^g. In particular, following e.g. Linnemann (2009) and Economides et al. (2013, 2014), we use a Cobb-Douglas production function of the form:

$$Y_t^g = A\left(G_t^c\right)^{\theta_g}\left(L_t^g\right)^{1-\theta_g} \tag{85}$$

where $0 \leq \theta_g \leq 1$ is a technology parameter. Notice that both private and public good production face the same TFP; this is because we do not want our results to be driven by exogenous factors. The total cost of public production, $G_t^c + w_t^b L_t^g$, is financed by the government through taxes and bonds (see the government budget constraint (80) above).

2.7 World financial intermediary

We use a simple and popular model of financial frictions (see e.g. Uribe and Yue (2006), Curdia and Woodford (2009 and 2010) and Benigno et al. (2014)). International borrowing, or lending, takes place through a financial intermediary or bank. This intermediary is located in the home country. It plays a traditional role only, collecting deposits from lenders and lending the funds to borrowers.

In particular, the bank raises funds from domestic private agents (capitalists), $(f_t^k - f_t^g)$, at the rate Q_t and lends to foreign agents, $(f_t^{g*} - f_t^{k*})$, at the rate Q_t^*. [23] In addition, the bank faces operational costs, which are increasing and convex in the volume of the loan, $(f_t^{g*} - f_t^{k*})$). The profit of the bank is revenue minus cost where revenue is net of transaction costs. Thus, the profit written in real and per capitalist terms in the domestic country is given by (details are in Appendix):

[22] See Economides et al. (2014) for details and a review of the literature on the production function of public goods.

[23] Here f_t^k is each domestic capitalist's foreign assets denominated in foreign currency, and f_t^g is real and per capitalist public foreign debt (i.e. public debt held by foreign capitalists) in the domestic country; similarly in the foreign country. Then, if it so happens that $(f_t^k - f_t^g)$ is positive, it denotes net foreign assets in the home country and if it so happens that $(f_t^{g*} - f_t^{k*})$ is positive, it denotes net foreign liabilities in the foreign country. In equilibrium, $(f_t^{g*} - f_t^{k*}) + \frac{S_t p_t^*}{p_t}(f_t^g - f_t^k) = 0$. Appendix provides details.

$$\widetilde{\pi}_t \equiv Q^*_{t-1}\left[\frac{p_{t-1}}{p_t}(f^{g*}_{t-1}-f^{k*}_{t-1})-\frac{\psi}{2}\frac{p^H_{t-1}}{p^H_t}\frac{p^H_t}{p_t}(f^{g*}_{t-1}-f^{k*}_{t-1})^2\right]-Q_{t-1}S_t\frac{p^*_t}{p_t}\frac{p^*_{t-1}}{p^*_t}\left(f^k_{t-1}-f^g_{t-1}\right) \tag{86}$$

where $\frac{\psi}{2}\left(f^{g*}_{t-1}-f^{k*}_{t-1}\right)^2$ is a real and per capitalist cost function and $\psi \geq 0$ is a parameter (see Subsection 3.1 below for its value). The first term in the brackets on the RHS is the bank's return on the loan net of transaction costs, while the last term is payments to the savers. [24]

At each t, the bank chooses the volume of its loan taking Q_t and Q^*_t as given. The optimality condition is (details are in Appendix):

$$Q^*_{t-1} = \frac{Q_{t-1}\frac{S_t}{S_{t-1}}}{1-\psi\frac{p^H_{t-1}}{p_{t-1}}(f^{g*}_{t-1}-f^{k*}_{t-1})} \tag{87}$$

where, in a currency union,$S_t \equiv 1$; thus, $Q^*_t > Q_t$ which means that borrowers pay a sovereign premium.

It needs to be said that the implied property in equation (87) - namely, that the interest rate, at which the country borrows from the rest of the world, is increasing in the nation's total foreign debt - is supported by a number of empirical studies (see e.g. EMU-Public Finances (2012) by the European Commission). It should also be said that a similar type of endogeneity of the country premium can be produced by several other models, including models of default risk.[25]

2.8 Decentralized Equilibrium (given Policy)

We now combine all the above to solve for a symmetric Decentralized Equilibrium (DE) for any feasible policy. The DE is defined to be a sequence of allocations, prices and policies such that: (i) every type of household maximize utility; (ii) every private firm maximize profit; (iii) the state-owned

[24] As in e.g. Curdia and Woodford (2009 and 2010), any resources consumed by the bank for the monitoring of its financial operations will be part of the aggregate demand for the Dixit-Stiglitz composite good (details are in Appendix).

[25] Default risk reflects the fear of repudiation of debt obligations but also the fear of new wealth taxes with retroactive effect on debt repayments (see Alesina et al. (1992) for an early study). As Corsetti et al. (2013) point out, there are two approaches to sovereign default. The first approach models it as a strategic choice of the government (see e.g. Eaton and Gersovitz (1981), Arellano (2008) and many others). The second approach assumes that default occurs when debt exceeds an endogenous fiscal limit (see e.g. Bi (2012) and many others).

enterprise produces the public goods and services; (iv) the world financial intermediary maximizes profit; (v) all constraints, including the government budget constraint and the balance of payments, are satisfied; and (vi) all markets clear, including the international asset market.

To proceed with the solution, we need to define the policy regime. Regarding the conduct of monetary policy, we assume that a Taylor-type rule is used for determining nominal interest rate, R_t, in a regime similar to that of eurozone (See below for details). Regarding fiscal policy, we assume that, in the transition, except otherwise said , $\tau_t^c, \tau_t^k, \tau_t^n, \tau_t^{l,k}, \tau_t^{l,w}, \tau_t^{l,b}, \bar{g}_t^c, \bar{g}_t^i, \bar{g}_t^w, f_t^g$ and v_t^b, are set exogenously, while the end-of-period total public debt,b_t^k, follows residually from the government budget constraint. The same holds for fiscal policy instruments of the foreign country.

Appendix H.2 presents the dynamic DE system. It consists of 63 equations in 63 variables $[c_t^k, c_t^{k,H}, c_t^{k,F}, c_t^w, c_t^{w,H}, c_t^{w,F}, c_t^b, c_t^{b,H}, c_t^{b,F}, n_t^k, w_t^k, n_t^w, w_t^w, m_t^k, m_t^w, m_t^b, r_t^k, k_t^k, k_t^g, x_t^k, b_t^k, f_t^k, Q_t, y_t^g, y_t^H, mc_t, \widetilde{\omega}_t^k, \upsilon_t, \widetilde{\pi}_t^k, p_t, p_t^H, p_t^F, c_t^{k*}, c_t^{k,H*}, c_t^{k,F*}, c_t^{w*}, c_t^{w,H*}, c_t^{w,F*}, c_t^{b*}, c_t^{b,H*}, c_t^{b,F*}, n_t^{k*}, w_t^{k*}, n_t^{w*}, w_t^{w*}, m_t^{k*}, m_t^{w*}, m_t^{b*}, r_t^{k*}, k_t^{k*}, k_t^g, x_t^{k*}, b_t^{k*}, R_t^*, f_t^{k*}, Q_t^*, y_t^{g*}, y_t^{H*}, mc_t^*, \widetilde{\omega}_t^{k*}, p_t^*, p_t^{H*}, p_t^{F*}]_{t=0}^{\infty}$. This is given the independently set monetary and fiscal policy instruments,$[R_t, \tau_t^c, \tau_t^k, \tau_t^n, \tau_t^{l,k}, \tau_t^{l,w}, \tau_t^{l,b}, \bar{g}_t^c, \bar{g}_t^i, \bar{g}_t^w, f_t^g, v_t^b, \tau_t^{c*}, \tau_t^{k*}, \tau_t^{n*}, \tau_t^{l,k*}, \tau_t^{l,w*}, \tau_t^{l,b*}, \bar{g}_t^{c*}, \bar{g}_t^{i*}, \bar{g}_t^{w*}, f_t^{g*}, v_t^{b*}]_{t=0}^{\infty}$, exogenous variables $[S_t, A_t, A_t^*]_{t=0}^{\infty}$, and initial conditions for the state variables,$[k_{-1}^k, k_{-1}^g, f_{-1}^k, b_{-1}^k, R_{-1}, m_{-1}^k, m_{-1}^w, m_{-1}^b, k_{-1}^{k*}, k_{-1}^{g*}, f_{-1}^{k*}, b_{-1}^{k*}, R_{-1}^*, m_{-1}^{k*}, m_{-1}^{w*}, m_{-1}^{b*}]$.

2.9 Monetary and fiscal policy

We now specify monetary and fiscal policy rules.

2.9.1 Single monetary policy rule in a monetary union

If we had flexible exchange rates, the exchange rate would be an endogenous variable and the two countries' nominal interest rates, R_t and R_t^*, could be free to be set independently by the national monetary authorities, say, to follow national Taylor-type rules. Here, by contrast, to mimic the eurozone regime, we assume that only one of the nominal interest rates, say R_t, can follow a Taylor-type rule, while R_t^* is an endogenous variable replacing the exchange rate which becomes an exogenous policy variable (this modelling, where the union's central bank uses one of national governments' interest rates as its policy instrument, is similar to that in e.g. Galí and Monacelli

(2008) and Benigno and Benigno (2008)).[26]

In particular, we assume a single monetary feedback policy rule of the form:

$$log\left(\frac{R_t}{R}\right) = \phi_\pi\left[\tilde{\eta} log\left(\frac{\pi_t}{\pi}\right) + (1-\tilde{\eta}) log\left(\frac{\pi_t^*}{\pi^*}\right)\right] + \qquad (88)$$
$$+\phi_y\left[\tilde{\eta} log\left(\frac{y_t^H}{y^H}\right) + (1-\tilde{\eta}) log\left(\frac{y_t^{H*}}{y^{H*}}\right)\right]$$

where π_t and π_t^* are inflation rate of CPI in domestic and foreign country respectively, which are defined $\pi_t \equiv \frac{p_t}{p_{t-1}}$ and $\pi_t \equiv \frac{p_t^*}{p_{t-1}^*}$, $\phi_\pi \geq 0$ and $\phi_y \geq 0$ are respectively feedback monetary policy coefficients on price inflation and the output gap, $0 \leq \tilde{\eta} \leq 1$ is the political weight given to the domestic country relative to the foreign country (see subsection 3.1 below for the value of this parameter), variables without time subscripts denote policy targets (in the case of monetary policy, the policy targets are simply the steady state values of the corresponding variables) and,finally, a star denotes the counterpart of a variable in the foreign country.

2.9.2 National fiscal policy rules

Countries can follow independent fiscal policies. As in the case of monetary policy above, we focus on simple feedback rules meaning that national fiscal authorities react to a small number of easily observable macroeconomic indicators. In particular, in each country, we allow all the main spending-tax policy instruments, namely, the ratio of real government spending on private goods and services to real GDP, defined as s_t^g , the ratio of real government spending on investment to real GDP, defined as s_t^i , the ratio of real public wage bill to real GDP, defined as s_t^w , the ratio of real total government transfers to real GDP, denoted as s_t^l, and the tax rates on consumption, capital income and labor income, τ_t^c, τ_t^k and τ_t^n, to react to the public debt-to-GDP ratio as deviation from a target value according to simple linear rules:[27]

$$s_t^g = s^g - \gamma_l^g \left(l_{t-1} - l\right) \qquad (89)$$

[26]For various ways of modelling monetary policy in a monetary union, see e.g. Dellas and Tavlas (2005) and Collard and Dellas (2006).

[27]For similar rules, see e.g Schmitt-Grohé and Uribe (2007) and Cantore et al. (2015). See also EMU-Public Finances (2011) by the European Commission for similar fiscal reaction functions used in practice.

$$s_t^i = s^i - \gamma_l^i (l_{t-1} - l) \tag{90}$$

$$s_t^w = s^w - \gamma_l^w (l_{t-1} - l) \tag{91}$$

$$s_t^l = s^l + \gamma_l^l (l_{t-1} - l) \tag{92}$$

$$\tau_t^c = \tau^c + \gamma_l^c (l_{t-1} - l) \tag{93}$$

$$\tau_t^k = \tau^k + \gamma_l^k (l_{t-1} - l) \tag{94}$$

$$\tau_t^n = \tau^n + \gamma_l^n (l_{t-1} - l) \tag{95}$$

where l_{t-1} is the beginning-of-period government liabilities as share of GDP (defined below), γ_l^q for $q \equiv$ (g,i,w,l,c,k,n), are respectively feedback fiscal policy coefficients on public liabilities gap, and variables without time subscripts denote policy targets. Notice that, in the above rules, a policy target value (like $s^g, s^i, s^w, s^l, \tau^c, \tau^k, \tau^n, l, y^H$) will be the steady state value of the corresponding variable. This value will depend on whether we are in the status quo economy, or in a reformed economy. For example, as further discussed in Section 4 below, the debt policy target, l, can be either the average public debt-to-GDP ratio in the data (this will be the benchmark case without reforms where fiscal policy adjusts so as to keep the public debt ratio at its average value) or it can be set to a value less than in the data (this will be the case of debt consolidation where fiscal policy systematically brings public debt down over time). It should be recalled that a negative value of s_t^l denotes transfers, so a positive γ_l^l means that transfers fall when public liabilities rise above their target.

From the government budget constraint, public liabilities at the end of period t expressed in real and per capita terms are (see Appendix for details):

$$l_{t-1} \equiv \frac{R_{t-1}\lambda_{t-1}D_{t-1} + Q_{t-1}\frac{S_t}{S_{t-1}}(1-\lambda_{t-1})D_{t-1}}{p_{t-1}^H y_{t-1}^H} \tag{96}$$

Fiscal policy in the foreign country is modelled similarly.

2.10 Exogenous variables

In this subsection, we define the exogenous variables. Regarding the exogenously set policy instruments, we set the nominal exchange rate S_t at 1 (under fixed exchange rates) and the fraction of domestic public debt in total public debt held by domestic capitalists, λ_t, are set at their data averages values at all t (see Subsection .. below). As we present in detail in Appendix H.3, instead of working with nominal exchange rate, we can work with gross rate of exchange rate depreciation, defined as $\epsilon_t \equiv \frac{S_t}{S_{t-1}}$, and assume that its value is equal to 1 at all t.

Finally, TFP,A_t, remains constant over time and equal to 1.

Again, exogenous variables of foreign country are determined similarly.

2.11 Final Equilibrium system

We now combine all the above to solve for a symmetric Decentralized Equilibrium (DE) for any feasible policy. The DE is defined to be a sequence of allocations, prices and policies such that: (i) every type of household maximize utility; (ii) every private firm maximize profit; (iii) the state-owned enterprise produces the public goods and services; (iv) the world financial intermediary maximizes profit; (v) all constraints, including the government budget constraint and the balance of payments, are satisfied; and (vi) all markets clear, including the international asset market; (vii) policy instruments are set by feedback rules.

This equilibrium system is presented in detail in Appendix H. It consists of 79 equations in 79 variables, $[c_t^k, c_t^{k,H}, c_t^{k,F}, c_t^w, c_t^{w,H}, c_t^{w,F}, c_t^b, c_t^{b,H}, c_t^{b,F}, n_t^k, w_t^k, n_t^w, w_t^w, m_t^k, m_t^w, m_t^b, r_t^k, k_t^k, k_t^g, x_t^k, f_t^k, Q_t, y_t^H, y_t^g, mc_t, \widetilde{\omega}_t^k, \upsilon_t, \widetilde{\pi}_t^k, \pi_t, \pi_t^H, TT_t, d_t, R_t, s_t^g, s_t^i, s_t^w, s_t^l, \tau_t^c, \tau_t^k, \tau_t^n, l_t, c_t^{k*}, c_t^{k,H*}, c_t^{k,F*}, c_t^{w*}, c_t^{w,H*}, c_t^{w,F*}, c_t^{b*}, c_t^{b,H*}, c_t^{b,F*}, n_t^{k*}, w_t^{k*}, n_t^{w*}, w_t^{w*}, m_t^{k*}, m_t^{w*}, m_t^{b*}, r_t^{k*}, k_t^{k*}, k_t^{g*}, x_t^{k*}, R_t^*, f_t^{k*}, Q_t^*, y_t^{H*}, y_t^{H*}, mc_t^*, \widetilde{\omega}_t^{k*}, \pi_t^*, \pi_t^{H*} d_t^*]$. This is for given the exogenous variables, $[\epsilon_t, \lambda_t, \lambda_t^*, A_t, A_t^*, v_t^b, v_t^{b*}]$, as defined in Subsection 2.10, the values of the feedback (monetary and fiscal) policy coefficients in the policy rules as defined in Subsection 2.9 and initial conditions for the state variables.

2.12 Plan of the rest of the paper

Our main goal in this paper is to evaluate the implications of various hypothetical debt consolidation policies. We will therefore work as follows. First, using commonly employed parameter values and fiscal data from Germany and Italy, we will numerically solve the above model. This is in the next section. In turn, to the extent that the steady state solution of this model is empirically relevant (meaning that it can mimic the data averages over the euro area period of study), we will use this regime - defined as the status quo - as a point of departure in order to evaluate the implications of various debt consolidation policies. A description of policy experiments and a discussion of the solution methodology are in Section 4, while numerical solutions are in Section 5.

3 Data, parameteres and solution of the status quo model

This section solves numerically the above model by using annual data from Germany and Italy over the period 2001-2011. We start in 2001 because this year marked the introduction of the euro and we stop at 2011 because the year 2012 marked the beginning of fiscal consolidation efforts in Italy (see e.g. EMU-Public Finances (2015) by the European Commission).

3.1 Parameter values and fiscal policy variables

The baseline parameter values and the data averages of fiscal policy variables, used in the numerical solution of the above model, are listed in Tables 1 and 2 respectively. The time unit is meant to be a year. The two countries can differ only in their discount factors (see β and β^* in Table 1), fiscal policy variables (see the fiscal policy instruments in Table 2) and some parameters related to public sector (see $v^b, v^{b*}, \theta_g, \theta_g^*$ in Table 1). In all other respects, the two countries are assumed to be symmetric. Interestingly, as said above, these two differences will prove to be enough to give a steady state solution close to the data averages during 2001-2011.

Regarding parameter values, the model's key parameters are the discount factors in the two countries, β and β^*, and the cost coefficient driving the wedge between the borrowing and the lending interest rate, ψ. The values of these parameters are calibrated to match the real interest rates and the net foreign asset position of the two countries in the time period under consideration. In particular, the values of β and β^* follow from the Euler equations in the two countries which, at the steady state, are reduced to:

$$\beta Q/\pi = 1 \tag{97}$$

$$\beta^* Q^*/\pi^* = 1 \qquad (98)$$

where Q/π and Q^*/π^* are the real interest rates in the two countries.[28] Since $Q/\pi < Q^*/\pi^*$ in the data over the period under consideration, it follows $\beta = 0.9833 > \beta^* = 0.9780$. That is, the Germans are more patient than the Italians.

In turn, the optimality condition of the bank, Eq. (87), written at the steady state, is (as said, $S \equiv 1$ in a currency union):

$$Q^* = \frac{Q}{1 - \psi \frac{p^H}{p}(f^{g*} - f^{k*})} \qquad (99)$$

from which the value of the parameter is calibrated.

Some parameter values related to public sector change across countries. In particular, the percentage of public employees in Germany and Italy are set at values 0.16 and 0.2 respectively, which are close to data. Furthermore, in Germany the share of private goods in public production, θ_g, is defined as $\frac{s^g}{s^g + s^w}$ and takes the value 0.3, while in Italy this parameter, θ_g^*, is defined in a similar way, $\frac{s^{g*}}{s^{g*} + s^{w*}}$, and takes the value 0.35.

All other parameter values, as listed in Table 1, are the same across countries and are set at values commonly used in related studies. Let us briefly discuss some of them. We start by setting the value of the political weight, $\tilde{\eta}$, at the "neutral" value of 0.5. In addition, θ_k and θ_k^*, which stand for the output elasticity of public infrastructure are both set at 0.05, as in Baxter and King, AER,1993. Leeper et al., JME, 2010, report that there is a lack of consensus on the productivity of public capital in the literature and, consequently, in their experiments they assigned to this parameter the value 0.05 which also used in Baxter and King (1993). The parameters θ and θ^*, standing for capitalists' labor efficiency parameters in each country, are calibrated so that we obtain a reasonable value for the ratio of capitalist's wage to private worker's wage, $\frac{w^k}{w^w}$ and $\frac{w^{k*}}{w^{w*}}$, which, in our model, equals 1.7 in Germany and 1.6 in Italy. We set Rotemberg's price adjustments cost parameters ,ϕ^p and ϕ^{p*}, at 1.56 which correspond to a probability approximately 20 per cent a firm not to be able to reset its price each year in a Calvo pricing model (see Keen and Wang (2007)).

Regarding fiscal policy variables in two countries as defined in Subsection 2.9.2 above, the steady state government spending-to-GDP ratios (i.e. public

[28] Here, $\pi_t \equiv \frac{p_t}{p_{t-1}}$ and $\pi_t^* \equiv \frac{p_t^*}{p_{t-1}^*}$ (see Appendix for detailed definitions of all variables).

consumption, public spending and public wage bill as share of GDP) and tax rates are set to their average values in the data in Germany and Italy over 2001-2011 (see Table 2). In particular, as a measure of s^g and s^{g*}, which serve as arguments in households' utility function and hence are typically thought of as public consumption, we use data on total government consumption spending on goods and services. Furthermore, we use data on public investment as a measure of s^i and s^{i*} and on public wage bill as a measure of s^w and s^{w*}. Measure of s^l and s^{l*}, that captures transfer payments as share of GDP, follows residually from each country's government budget constraint. It is also worth to mention that the lump-sum taxes/transfers are distributed to each class of household in each country according to their shares in the population. As tax rates, τ^c,τ^{c*},τ^k,τ^{k*},τ^n and τ^{n*} we use the associated effective tax rates (or what Eurostat calls implicit tax rates).

Regarding policy instruments along the transition, they react to deviations of macroeconomic indicators from their steady state values. [29] As for fiscal (tax-spending) policy instruments along the transition, they can respond to the inherited public debt as a deviation from a steady state value, where this reaction is quantified by the feedback policy coefficients in the policy rules (89)-(95). In our experiments we use only one fiscal instrument at a time in each country to respond to debt imbalances by setting the associated feedback policy coefficient on debt gap at 0.1 [30] (e.g. $\gamma_l^g = 0.1$ and $\gamma_l^{g*} = 0.1$), while we switch off the feedback policy coefficient on debt gap of other fiscal policy instruments. In (almost) all cases studied, other things equal, the above fiscal policy can guarantee a unique transition path. As for monetary policy, we assume a Taylor-type rule for the nominal interest rate of the monetary union that aggressively react on each country's inflation meaning that the associated feedback policy coefficients (ϕ_π and ϕ_π^*) are set both at 1.5, while the feedback monetary policy coefficients on output gap are set at 0.5. As we discuss below, aggressive reaction of nominal interest rate on each country's inflation can guarantee determinacy.

[29] Since policy instruments react to deviations of macroeconomic indicators from their steady state values, feedback policy coefficients do not play any role in steady state solutions. Also, recall that "money is neutral" in the long run, so that the monetary policy regime also do not matter to the real economy at the steady state.

[30] These values are close to those found by optimized policy rules in related studies (see e.g. Schmitt-Grohé and Uribe (2007) and Philippopoulos et al. (2014)). They are also consistent with calibrated or estimated values by previous research(see e.g. Leeper et al.(2009), Forni et al.(2010), Coenen et al. (2013), Cogan et al.(2013),Erceg and Linde(2013)).

Table 1: Parameter values

Parameter	Home	Foreign	Description
v^k,v^{k*}	0.20	0.20	share of capitalists in population
v^w,v^{w*}	0.64	0.60	share of private workers in population
v^b,v^{b*}	0.16	0.20	share of public employees in population
α,α^*	0.3	0.3	share of private physical capital in production
θ_k,θ_k^*	0.05	0.05	output elasticity of public infrastructure
θ,θ^*	0.26	0.26	labor efficiency parameter of capitalist
θ_g,θ_g^*	0.30	0.35	share of private goods in public production
β,β^*	0.9833	0.9780	time discount factor
v,v*	0.5	0.5	home goods bias in consumption
μ,μ^*	3.42	3.42	money demand elasticity in utility
δ,δ^*	0.1	0.1	private capital depreciation rate
δ^g,δ^{g*}	0.1	0.1	public capital depreciation rate
ϕ^p,ϕ^{p*}	1.56	1.56	Rotemberg's price adjustments cost parameter
ϕ,ϕ^*	6	6	price elasticity of demand
η,η^*	1	1	inverse of Frisch labor supply elasticity
σ,σ^*	1	1	inverse of elasticity of substitution in consumption
ζ,ζ^*	1	1	elasticity of public consumption in utility
$\tilde{\eta}$	0.5	0.5	political weight in union-wide policies
ψ	0.072	-	cost parameter in international borrowing
χ_m,χ_m^*	0.001	0.001	preference parameter related to real money balances
χ_n,χ_n^*	5	5	preference parameter related to work effort
χ_g,χ_g^*	0.1	0.1	preference parameter related to public spending
ξ,ξ^*	0.01	0.01	adjustment cost parameter of private physical capital
ξ^g,ξ^{g*}	0.01	0.01	adjustment cost parameter of public physical capital
A,A*	1	1	TFP level

Table 2: Fiscal policy variables (2001-2011 data averages)

Variable	Home	Foreign	Description
τ^c, τ^{c*}	0.1934	0.1756	consumption tax rate
τ^k, τ^{k*}	0.2041	0.3118	capital income tax rate
τ^n, τ^{n*}	0.3833	0.421	labor income tax rate
s^g, s^{g*}	0.185	0.193	government consumption spending as share of GDP
s^i, s^{i*}	0.021	0.029	government investment spending as share of GDP
s^w, s^{w*}	0.078	0.104	public wage bill as share of GDP
λ, λ^*	0.52	0.61	share of public debt held by domestic agents

Note: The data source is Eurostat.

3.2 Steady state solution in the status quo model

The equilibrium system was defined in Subsection 2.9 and the associated steady state follows simply if we assume that variables do not change over time (details are in Appendix). Table 3 presents the steady state solution when parameters and policy instruments are set at the values in Tables 1 and 2. It is worth pointing out that, since policy instruments react to deviations of macroeconomic indicators from their steady state values, feedback policy coefficients do not play any role in steady state solutions. In this steady state solution, the debt-to-GDP ratio and fiscal policy instruments in both countries are set as data averages over 2001-2011 and lump-sum transfer payment as share of GDP comprise the residually determined public financing variable in each country. Table 3 also presents some key ratios in the German and Italian data and, as can be seen, the respective ratios implied by the steady state solution are close to their values in the data. This steady state solution will serve as a point of departure to study various policy experiments. That is, in what follows, we will depart from this solution to study the implications of various policy experiments.

We report (and this is confirmed below) that an exogenous reduction in public debt implies a lower sovereign premium and this leads to higher output in both countries; this can provide a first justification for our fiscal consolidation experiments.

3.3 Transition dynamics and determinacy

It is well recognized that the interaction between fiscal and monetary policy, and, in particular, the magnitude of the associated feedback policy coef-

Table 3: "Status quo" steady state solution

Variables	Description	Home	Data	Foreign	Data
y^H, y^{H*}	output	0.8952		0.7863	
c^k, c^{k*}	consumption of capitalist	0.2142		0.2064	
c^w, c^{w*}	consumption of private worker	0.0824		0.0843	
c^b, c^{b*}	consumption of public employee	0.0660		0.0680	
n^k, n^{k*}	labor of capitalist	0.2492		0.2451	
n^w, n^{w*}	labor of worker	0.3788		0.3736	
k^k, k^{k*}	physical capital	1.5226		1.1045	
w^k, w^{k*}	real wage rate of capitalist	0.5163		0.5136	
w^w, w^{w*}	real wage rate of worker	0.3020		0.3196	
r^k, r^{k*}	real return to capital	0.1470		0.1780	
$Q^* - Q$	interest rate premium			0.0055	0.0055
$\frac{c^k+\frac{v^w}{v^k}c^w+\frac{v^b}{v^k}c^b}{y^H TT^{v-1}}, \frac{c^{k*}+\frac{v^{w*}}{v^{k*}}c^{w*}+\frac{v^{b*}}{v^{k*}}c^{b*}}{y^{H*} TT^{1-v^*}}$	total consumption as share of GDP	0.6257		0.6351	
$\frac{k}{y^H}, \frac{k^*}{y^{H*}}$	physical capital as share of GDP	1.7009		1.4045	
$\frac{d}{y^H TT^{v-1}}, \frac{d^*}{y^{H*} TT^{1-v^*}}$	total public debt as share of GDP	0.6861	0.6861	1.08	1.08
$\frac{\frac{(1-\lambda)d}{TT^{v-1}} - f^k TT^{v^*}}{y_t^H}, \frac{\frac{(1-\lambda^*)d^*}{TT^{1-v^*}} - f^{k*} TT^{-v}}{y_t^{H*}}$	total foreign debt as share of GDP	-0.0930		0.0950	
y^k, y^{k*}	income of capitalist	0.3584		0.3230	
y^w, y^{w*}	income of private worker	0.0824		0.0843	
y^b, y^{b*}	income of public employee	0.0660		0.0680	

Note: Parameters and policy variables as in Tables 1 and 2 ficients in the policy rules, are crucial to determinacy (see e.g. Leith and Wren-Lewis, 2008). This is also the case in our paper. In particular, when we assume that fiscal policy instruments remain constant at their data average values in Table 2 without any reaction to public debt and there is no interest rate policy reaction to inflation, the model, when approximated around the status quo steady state solution, exhibits dynamic instability meaning that there is no convergence to the steady state solution reported above. In other words, policy can guarantee a unique transition path, only when at least one fiscal policy instrument in each country ($s_t^g, \tau_t^c, \tau_t^k, \tau_t^n$ and $s_t^{g*}, \tau_t^{c*}, \tau_t^{k*}, \tau_t^{n*}$) reacts to public liabilities. The magnitude of these reactions lies within a range of critical minimum and maximum non-zero values. These critical values differ across different fiscal policy instruments. And all this holds when monetary policy satisfies the so-called Taylor principle, meaning that the single nominal interest rate reacts aggressively to inflation. In sum, determinacy requires stabilizing fiscal reaction to inherited public debt and monetary reaction to inflation; this holds in all cases studied below.

4 Description of policy experiments and solution strategy

This section defines in detail our policy experiments and explains the solution strategy. Numerical results will be presented in Section 5. In our main thought experiment, we will depart from the status quo steady state solution (in other words, the initial values of the predetermined variables will be those found by the steady state solution in Table 3) and compute the equilibrium transition path as we travel towards a new reformed steady state (policy reforms are defined in Subsection below). Transition dynamics from the status quo steady state to a new steady state will be driven by debt consolidation policies in the high-debt country. Along this transition, regarding public debt consolidation, we experiment with one fiscal policy instrument at a time, which means that we allow only one national fiscal policy instrument to react to its policy target and switch off all other instruments. The feedback monetary and fiscal policy coefficients of the instrument(s) used along the transition path -by the single monetary authority as well as by the one national fiscal authority - are chosen ad hoc. Regarding transition results, we will compute first-order approximate solutions, around the associated steady state.

4.1 National fiscal policies and reforms in the main policy experiment

In our main thought experiment, the domestic country (defined to be Germany) with solid public finances is assumed not to take any fiscal consolidation measure meaning that in this country, we depart from, follow and end up at, the same tax-spending position, which is as in the average data in Germany (however, as explained below, the new steady state solution will differ from the status quo solution because of fiscal consolidation in the foreign country). Specifically, fiscal policy in the domestic country is defined as follows: (a) All exogenously set tax-spending national policy instruments remain at the same value (data average value) in both the status quo steady state and the new steady state. (b) Along the transition to the new steady state, all tax-spending national policy instruments are not allowed to react to deviations from policy targets, but they remain unchanged in their steady state values. (c) All the time, namely, both during the transition and in the steady state, the public debt serves as the residually determined public financing instrument closing the within-period government budget constraint.

In our main thought experiment, the role of fiscal policy in the foreign country (defined to be Italy) is to improve resource allocation by bringing down its public debt-to-GDP ratio over time. This is typically called "debt consolidation" in the related literature (see Wren-Lewis (2010)). Specifically, in our main thought experiment, fiscal policy in the foreign country (i.e. Italy) is defined as follows: (a) In the new reformed steady state, the country's output share of public debt is exogenously set at the target value of 90% (recall that it was around 110% of GDP in the status quo steady state solution in Subsection 3.2). (b) In this new reformed steady state, since the country's public debt has been reduced and thus fiscal space has been created relative to the status quo, fiscal spending can be increased and/or tax rates can be cut, depending on which fiscal policy instrument is assumed to follow residually to close the government budget constraint. This is known as the long-term fiscal gain from debt consolidation. Here, we will report aggregate results only for the case in which the fiscal space created by debt reduction is used to reduce the capital tax rate, and distributional results for the case in which the fiscal space created by debt reduction is used to reduce the consumption tax rate; the former has been found to be the most efficient way of making use of the fiscal space created and is consistent with the Chamley-Judd well known normative result that the limiting capital tax rate should be zero. To put it differently, our solutions confirm, as in most of the literature, that the impact of debt consolidation depends on expectations about how the fiscal space will be used in the future and it is expectations of a cut in the capital tax rate that appear to have a long-lasting beneficial effect on investment and output. (c) Along the transition to the new reformed steady state, the national tax-spending policy instruments are allowed to react to deviations from policy targets in an ad hoc way. Given that the new debt policy target is set at a value lower than in the status quo (i.e. we depart from 110% but the policy target in Italy's feedback fiscal policy rules is 90%), this requires lower public spending, and/or higher tax rates, during the early phase of the transition period. This is known as the short-term fiscal pain of debt consolidation.

This intertemporal tradeoff, between short-term fiscal pain and medium-term fiscal gain, also implies that the implications of consolidation depend heavily on the public financing policy instruments used, namely, which policy instrument adjusts endogenously to accommodate the exogenous change in fiscal policy (see also e.g. Leeper et al., 2009, and Davig and Leeper, 2011). Specifically, these implications depend both on which policy instrument bears the cost of adjustment in the early period of adjustment and on which policy instrument is anticipated to reap the benefit, once consolidation has been achieved. In the policy experiments we consider below, we experiment with fiscal policy mixes, which means that the fiscal authority of the high-debt country (Italy) can use various instruments in the transition and in the steady state.

4.2 Solution methodology and comparison

4.2.1 Reference regime

To evaluate the implications of debt consolidation as defined above, we need to compare them to a reference regime. Here, we find it natural to use as a reference regime the case without debt consolidation for both countries other things equal(status quo steady state).

4.2.2 Methodology

As we have mentioned above, we experiment with fiscal policy mixes, which means that fiscal authority in the high debt country is allowed to use different instruments in the transition and in the steady state. Having chosen the fiscal policy instrument used in the new steady state, fiscal authority choose which fiscal policy instrument will be used in the transition and,thus, its feedback policy coefficient on national public debt will take the ad hoc value 0.1 (see equations 89-95 above for Italy). The corresponding coefficients of all other fiscal policy instruments are switched off.

In particular, we work as follows. We first solve and compare the status quo steady state solution to the steady state of reformed solutions for every case of adjusting fiscal policy instrument. We will then study the transitional results. The latter means that we log-linearize the model around the new steady-state solution of each reformed economy and then check its saddle-path stability when we use as initial conditions for the state variables their values in the status quo steady state.

In all cases, we will study both aggregate and distributional implications. Regarding aggregate outcomes, we look, for instance, at per capita output. Regarding distribution, we compute separately the income of the representative capitalist vis-à-vis that of the representative worker. The above values are then compared to their respective values had we remained in the status quo permanently.

5 Main results

As expected, had tax-spending policy in both countries remained unchanged as in the data averages over 2001-2011, the model would be dynamically unstable. In other words, some type of fiscal reaction (spending cuts and/or tax rises) to public debt imbalances was necessary for restoring dynamic stability. [Actually, in both countries at least one instrument should react to public debt imbalances].

In what follows below, we assume that both the foreign economy(Italy) and the domestic one (Germany) take consolidation measures.

5.1 Aggregate results(efficiency)

5.1.1 Steady state results

We start with comparison of each new reformed steady state solution with that of the SQ steady state. Recall that in the SQ steady state, fiscal policy instruments were set as in the data and public debt followed residually, while in a new reformed steady state public debt has been cut at 90% in Italy and at 60% in Germany so that one of the fiscal policy instruments follows residually meaning that in each country one of the s^g, s^i, s^w is allowed to rise or one of the τ^k,τ^n,τ^c is allowed to fall. Tables 4 and 5 report the aggregate implications of all possible public financing cases in a new reformed steady state for Germany and Italy respectively. That is, we investigate how the aggregate implications of debt consolidation in steady state for both countries depend on the public financing policy instrument used for exploiting the fiscal space created by debt consolidation in each one of them.

Table 4 reports values of GDP in the domestic country in each new reformed steady state. Specifically, the entries of this table represent values of GDP in the domestic country depending on which fiscal policy instrument (columns) is used by the domestic country in the new steady state. Similarly, Table 5 reports values of GDP in the foreign country in each new reformed steady state. Specifically, the entries of this table represent values of GDP in the foreign country depending on which fiscal policy instrument (columns) is used by the foreign country in the new steady state.

The conclusions that can be made by the Tables 4 and 5 are reported below:

1) Debt consolidation in Germany (country with solid public finance) is productive in the new reformed steady state under all cases studied. In terms of efficiency, the best way of using the fiscal space created by debt consolidation is to rise the public investment spending. This policy is also pareto efficient in the new reformed steady state relative to SQ steady state (see Table 6).

2) Debt consolidation in Italy (high-debt country) is productive in the new reformed steady state under all cases studied. In terms of efficiency, the best way of using the fiscal space created by debt consolidation is to rise public investment spending. This policy is also pareto efficient in the new reformed steady state relative to SQ steady state (see Table 6).

5.1.2 Transition results

From now on, to study the aggregate implications in the transition, we focus only on the case where the fiscal space created by debt consolidation in the high-debt country (Italy) and the country with solid public finance (Germany) is used to rise their public investment spending.

Tables 8 and 9 in this Subsection report present values of GDP in the domestic and the foreign country respectively T periods after debt consolidation starts taking place in both countries. In Tables 8 and 9, the entries of these tables represent the present value of GDP for a specific T (columns) in the domestic and foreign country respectively depending on which fiscal policy instrument (rows) is used for debt reduction in each one of them in

Table 4: Total output (GDP) in steady state(SS).

Residual instrument in the domestic economy	τ^k	τ^n	τ^c	s^g	s^i	s^w
GDP in the domestic economy	0.8972	0.8965	0.8958	0.8958	0.8997	0.8958

Steady state value of the total output in the status quo (SQ) for domestic economy is 0.8952.
Note: The above values are barely affected by other country's choice of fiscal policy instrument in steady state.

Table 5: Total output (GDP) in steady state(SS).

Residual instrument in the foreign economy	τ^{k*}	τ^{n*}	τ^{c*}	s^{g*}	s^{i*}	s^{w*}
GDP in the foreign economy	0.7919	0.7896	0.7879	0.7879	0.7946	0.7879

Steady state value of the total output in the status quo (SQ) for foreign economy is 0.7863.
Note: The above values are barely affected by other country's choice of fiscal policy instrument in steady state.

Table 6: Incomes in steady state (SS) when the fiscal space created by debt consolidation is used by the domestic and foreign economy to rise their public investment spending (s^i and s^{i*}) .

	Domestic economy	Foreign economy
Capitalist's income	0.3595(0.3584)	0.3232 (0.3230)
Private worker's income	0.0829 (0.0824)	0.0851 (0.0843)
Public employee's income	0.0664 (0.0660)	0.0686 (0.0680)

Note: In brackets they are reported the associated SQ steady state values.

Table 7: Incomes in steady state (SS) when the fiscal space created by debt consolidation is used by the domestic and foreign economy to reduce their labor tax rate (τ^n and τ^{n*}).

	Domestic economy	Foreign economy
Capitalist's income	0.3586 (0.3584)	0.3216 (0.3230)
Private worker's income	0.0829 (0.0824)	0.0851 (0.0843)
Public employee's income	0.0664 (0.0660)	0.0686 (0.0680)

Note: In brackets they are reported the associated SQ steady state values.

the transition. (As said above, we focus on the case where the fiscal space created by debt consolidation in Italy and Germany is used to rise their public investment spending.)

Table 8 implies that, in terms of efficiency, the best policy mix in Germany would be to use public consumption spending cuts (s^g) to bring public debt down during the early period of fiscal pain and, once debt has been reduced, to use the fiscal space created by debt consolidation to rise public investment spending (s^i). And all this holds given Italy uses the fiscal space created by debt consolidation to rise public investment spending (s^{i*}) and, during the early period of fiscal pain, to use public consumption spending cuts (s^{g*}) to bring public debt down.

Table 9 implies that, in terms of efficiency, the best policy mix in Italy would be to use public consumption spending cuts (s^{g*}) to bring public debt down during the early period of fiscal pain and, once debt has been reduced, to use the fiscal space created by debt consolidation to rise public investment spending (s^{i*}). And all this holds given Germany uses the fiscal space created by debt consolidation to increase public investment spending (s^i) and, during the early period of fiscal pain, to use public consumption spending cuts (s^g) to bring public debt down.

Yet, the above policy mixes, when followed by both countries simultaneously, are productive (see Tables 8 and 9) for both countries and pareto efficient only for Germany along the transition to the new reformed steady state (see Tables 10 and 11). All this is relative to the status quo.

5.2 Distributional implications (equity)

In this subsection, we study the distributional implications of debt consolidation as measured by the net income of private worker and public employee relative to that of capitalist. [31] And all this vis-à-vis the status quo.

In steady state one way to avoid a rise in inequality in both countries is to use the fiscal space created by debt consolidation to reduce their tax rate

[31] In the domestic country the net income of capitalist is defined as $y_t^k \equiv -\tau_t^c c_t^k + (1-\tau_t^k)\left[r_t^k TT_t^{v-1} k_{t-1}^k + \widetilde{\omega_t}^k\right] + (1-\tau_t^n) w_t^k n_t^k + (Q_{t-1}-1) TT_t^{v+v^*-1} \frac{1}{\pi_t^*} f_{t-1}^k + (R_{t-1}-1)\frac{1}{\pi_t}\lambda_{t-1} d_{t-1} - v^k s_t^l y_t^H TT_t^{v-1} + \widetilde{\pi}_t^k$, private worker is defined as $y_t^w \equiv -\tau_t^c c_t^w + (1-\tau_t^n) w_t^w n_t^w - v^k s_t^l y_t^H TT_t^{v-1}$ and public employee is defined as $y_t^b \equiv -\tau_t^c c_t^b + (1-\tau_t^n)\frac{v^k}{v^b} s_t^w TT_t^{v-1} y_t^H - v^k s_t^l y_t^H TT_t^{v-1}$. In the foreign country the net income of capitalist is defined as $y_t^{k*} \equiv -\tau_t^{c*} c_t^{k*} + (1-\tau_t^{k*})\left[r_t^{k*} TT_t^{1-v^*} k_{t-1}^{k*} + \widetilde{\omega_t}^{k*}\right] + (1-\tau_t^{n*}) w_t^{k*} n_t^{k*} + (Q_{t-1}^*-1) TT_t^{1-v-v^*} \frac{1}{\pi_t} f_{t-1}^{k*} + (R_{t-1}^*-1)\frac{1}{\pi_t^*}\lambda_{t-1}^* d_{t-1}^* - v^{k*} s_t^{l*} y_t^{H*} TT_t^{1-v^*}$, private worker is defined as $y_t^{w*} \equiv -\tau_t^{c*} c_t^{w*} + (1-\tau_t^{n*}) w_t^{w*} n_t^{w*} - v^{k*} s_t^{l*} y_t^{H*} TT_t^{1-v^*}$ and public employee is defined as $y_t^{b*} \equiv -\tau_t^{c*} c_t^{b*} + (1-\tau_t^{n*})\frac{v^{k*}}{v^{b*}} s_t^{w*} TT_t^{1-v^*} y_t^{H*} - v^{k*} s_t^{l*} y_t^{H*} TT_t^{1-v^*}$.

Table 8: Present value of total output (**GDP**) in the domestic economy over different T periods, Y_T, after debt consolidation starts taking place, **when this economy uses the fiscal space created by debt consolidation to rise the public investment spending ($\mathbf{s}^i$)**.

Adj. Instr.	Y_5	Y_{10}	Y_{20}	Y_{40}	Y_{60}	Y_{80}	Y_∞
τ^k	4.3343	8.3271	15.3682	26.3528	34.2040	39.8120	51.9288
τ^n	4.3258	8.3236	15.3695	26.3595	34.2124	39.8208	51.9376
τ^c	4.3428	8.3408	15.3897	26.3832	34.2365	39.8448	51.9616
s^g	4.3463	8.3475	15.3990	26.3953	34.2496	39.8581	51.9750
s^i	4.3190	8.2948	15.3245	26.3156	34.1710	39.7798	51.8967
s^w	4.3413	8.3393	15.3884	26.3819	34.2353	39.8436	51.9604
SQ	4.3288	8.3081	15.3285	26.2736	34.0889	39.6692	51.7251

Note: Every row represents a different case depending on which fiscal policy instrument is used by the domestic country to bring debt down during the early period of fiscal pain. At the same time, by assumption, the foreign country uses the fiscal space created by debt consolidation to rise the public investment spending and, during the early period of fiscal pain, uses public consumption cuts to bring public debt down.

Table 9: Present value of total output (**GDP**) in the foreign economy over different T periods,Y_T^*, after debt consolidation starts taking place, **when this economy uses the fiscal space created by debt consolidation to rise its public investment spending ($\mathbf{s}^{i*}$)**.

Adj. Instr.	Y_5^*	Y_{10}^*	Y_{20}^*	Y_{40}^*	Y_{60}^*	Y_{80}^*	Y_∞^*
τ^{k*}	3.7532	7.1267	12.8621	21.1528	26.4768	29.8909	35.5527
τ^{n*}	3.7390	7.1218	12.8734	21.1803	26.5084	29.9231	35.5849
τ^{c*}	3.7733	7.1647	12.9216	21.2306	26.5584	29.9730	35.6348
s^{g*}	3.7785	7.1763	12.9418	21.2566	26.5847	29.9991	35.6608
s^{i*}	3.7460	7.1072	12.8334	21.1422	26.4730	29.8876	35.5493
s^{w*}	3.7705	7.1598	12.9153	21.2235	26.5515	29.9662	35.6280
SQ	3.7625	7.1289	12.8360	21.0623	26.3344	29.7132	35.3158

Note: Every row represents a different case depending on which fiscal policy instrument is used by the foreign country to bring debt down during the early period of fiscal pain. At the same time, by assumption, the domestic country uses the fiscal space created by debt consolidation to reduce the public investment spending and, during the early period of fiscal pain, uses capital taxes to bring public debt down.

Table 10: Present values of household *j'* income in the domestic economy over various T periods after debt consolidation starts taking place, Y_T^j, **when this economy uses the fiscal space created by debt consolidation to rise the public investment spending (s^i) and, during the transition, bring debt down through public consumption spending cuts (s^g).**

	Y_5^j	Y_{10}^j	Y_{20}^j	Y_{40}^j	Y_{60}^j	Y_{80}^j	Y_∞^j
PVs of domestic capitalist k's income ($j = k$)	1.7410 (1.7331)	3.3368 (3.3263)	6.1535 (6.1371)	10.5470 (10.5192)	13.6853 (13.6482)	15.9264 (15.8824)	20.768 (20.709)
PVs of domestic private worker w's income ($j = w$)	0.4020 (0.3985)	0.7715 (0.7649)	1.4216 (1.4112)	2.4350 (2.4188)	3.1588 (3.1383)	3.6756 (3.6521)	4.792 (4.762)
PVs of domestic public employee b's income ($j = b$)	0.3220 (0.3193)	0.6180 (0.6128)	1.1389 (1.1305)	1.9507 (1.9378)	2.5306 (2.5142)	2.9446 (2.9258)	3.839 (3.815)

Note: (i) By assumption, the foreign country uses the fiscal space created by debt consolidation to rise the public investment spending and, during the early period of fiscal pain, uses public consumption cuts to bring public debt down. (ii) Results without debt consolidation in parentheses.

Table 11: Present values of household *j'* income in the foreign economy over various T periods after debt consolidation starts taking place, Y_T^{j*}, **when this economy uses the fiscal space created by debt consolidation to rise the public investment spending (s^{i*}) and, during the transition, bring debt down through public consumption spending cuts(s^{g*}).**

	Y_5^{j*}	Y_{10}^{j*}	Y_{20}^{j*}	Y_{40}^{j*}	Y_{60}^{j*}	Y_{80}^{j*}	Y_∞^{j*}
PVs of foreign capitalist k's income ($j = k$)	1.5373 (1.5453)	2.9123 (2.9279)	5.2519 (5.2719)	8.6313 (8.6505)	10.7979 (10.8158)	12.1865 (12.2035)	14.489 (14.505)
PVs of foreign private worker w's income ($j = w$)	0.4037 (0.4032)	0.7669 (0.7640)	1.3835 (1.3755)	2.2735 (2.2571)	2.8439 (2.8221)	3.2095 (3.1841)	3.816 (3.785)
PVs of foreign public employee b's income ($j = b$)	0.3255 (0.3252)	0.6184 (0.6161)	1.1157 (1.1094)	1.8335 (1.8203)	2.2936 (2.2760)	2.5884 (2.5680)	3.077 (3.052)

Note: (i) By assumption, the domestic country uses the fiscal space created by debt consolidation to rise the public investment spending and, during the early period of fiscal pain, uses public consumption cuts to bring public debt down. (ii) Results without debt consolidation in parentheses.

on labor (see Table 12). The above policy is pareto efficient only in domestic country though (see Table 7).

Given that both countries use their fiscal space created by debt consolidation to finance a reduction in labor tax rate, one way to avoid a rise in inequality in the transition in both countries is to use capital taxes to bring public debt down.

Conclusively, searching for policy mixes so that we can avoid a rise in inequality in both countries we make the following conclusion: One way to avoid a rise in inequality in both countries is to use the fiscal space created by debt consolidation to reduce the tax rate on labor, and during the early period of fiscal pain to use capital taxes to bring public debt down.

6 Robustness

In process

7 Concluding remarks and possible extensions

In process

Appendix A Households as capitalists

This appendix provides details and the solution of domestic capitalist's problem. Each domestic capitalist k solves an inter-temporal problem, in which he acts competitively to maximize expected discounted lifetime utility, and an intra-temporal one, in which he minimizes the consumption expenditures.

A.1 Capitalist's problem

Inter-temporal problem: Each domestic capitalist $k = 1, 2, ..., N^k$ acts competitively to maximize expected discounted lifetime utility taking prices and policy as given. Each k maximizes expected discounted lifetime utility:

$$E_o \sum_{t=0}^{\infty} \beta^t U\left(c_t^k, n_t^k, m_t^k, y_t^g\right) \tag{100}$$

where c_t^k is k's consumption bundle at t as defined below in the intra-temporal problem,Eq. (107), n_t^k is k's hours of work at t, m_t^k is k's end-of-period real money balances, y_t^g is per capita public goods and services at t, E_o is the rational expectations operator conditional on the current period information set and $0 < \beta < 1$ is the time preference rate.

In our numerical solutions, we use a utility function of the form (see also Gali 2008):

$$U\left(c_t^k, n_t^k, m_t^k, y_t^g\right) = \left[\frac{(c_t^k)^{1-\sigma}}{1-\sigma} - x_n \frac{(n_t^k)^{1+\eta}}{1+\eta} + x_m \frac{(m_t^k)^{1-\mu}}{1-\mu} + x_g \frac{(y_t^g)^{1-\zeta}}{1-\zeta}\right] \tag{101}$$

where x_n, x_m, x_g, σ, η, μ, ζ are standard preference parameters.

The budget constraint of each k (written in real terms) is:

$$(1+\tau_t^c)c_t^k + \frac{p_t^H}{p_t}x_t^k + \frac{S_t p_t^*}{p_t}f_t^k + b_t^k + m_t^k = (1-\tau_t^k)\left[r_t^k \frac{p_t^H}{p_t}k_{t-1}^k + \widetilde{\omega_t}^k\right] + (1-\tau_t^n)w_t^k n_t^k + \tag{102}$$

$$+ Q_{t-1}\frac{S_t p_t^*}{p_t}\frac{p_{t-1}^*}{p_t^*}f_{t-1}^k + R_{t-1}\frac{p_{t-1}}{p_t}b_{t-1}^k +$$

$$+ \frac{p_{t-1}}{p_t}m_{t-1}^k - \tau_t^{l,k} + \widetilde{\pi_t}^k$$

where x_t^k is k's real investment at t, f_t^k is the real value of k's end-of-period internationally traded assets at t denominated in foreign currency (if negative, it denotes foreign private debt), b_t^k is k's end-of-period real domestic government bonds at t, r_{t-1}^k is gross real return to inherited capital between $t-1$ and t, k_t^k is k's end-of-period private physical capital, $\widetilde{\omega_t}^k$ is k's real dividends paid by domestic private firms at t, w_t^k is capitalists' real wage rate at t, Q_{t-1} is the gross nominal return to international assets between t-1 and t, $R_{t-1} \geq 1$ is gross nominal return to domestic government bonds between $t-1$ and t, $\tau_t^{l,k}$ is real lump-sum taxes/transfers to each k from the government at t, $\widetilde{\pi_t}^k$ is profits distributed in a lump-sum fashion to each k by the financial intermediary (see below), $0 \leq \tau_t^c \leq 1$ is tax rate on consumption at t, $0 \leq \tau_t^k \leq 1$ is tax rate on capital income at t, $0 \leq \tau_t^n \leq 1$ is tax rate on labor income at t, p_t is the consumer price index (CPI), p_t^H is the price index of home tradables. Small letters of quantities denote real variables per capitalist e.g. $f_t^k \equiv \frac{F_t^k}{p_t^*}$, $b_t^k \equiv \frac{B_t^k}{p_t}$, $\widetilde{\omega_t}^k \equiv \frac{\widetilde{\Omega_t}^k}{p_t}$, $\widetilde{\pi_t}^k \equiv \frac{\widetilde{\Pi_t}^k}{p_t}$, small letters of prices denote real variables e.g. $w_t^k \equiv \frac{W_t^k}{p_t}$, capital letters of quantities denote nominal values per capitalist and capital letters of prices denote nominal values. Also, letters with star as superscripts denote the counterpart of a variable in the rest-of-the world, e.g. p_t^* stands for the consumer price index (CPI) abroad.

The motion of private physical capital for each k is:

$$k_t^k = (1-\delta)k_{t-1}^k + x_t^k - \frac{\xi}{2}\left(\frac{k_t^k}{k_{t-1}^k} - 1\right)^2 k_{t-1}^k \tag{103}$$

where $0 < \delta < 1$ is the depreciation rate of capital and $\xi \geq 0$ is a parameter capturing adjustment costs related to physical capital.

Therefore, in the inter-temporal problem, each domestic capitalist k chooses $\{c_t^k, x_t^k, n_t^k, m_t^k, b_t^k, f_t^k, k_t^k\}_{t=0}^{\infty}$ to maximize Eqs (100) and (101) subject to Eqs. (102) and (103), by taking as given prices $\{r_t^k, w_t^k, Q_t, R_t, p_t, p_t^H p_t^*\}_{t=0}^{\infty}$, dividends $\{\widetilde{\omega_t}^k\}_{t=0}^{\infty}$, profits $\{\widetilde{\pi_t}^k\}_{t=0}^{\infty}$, policy variables $\{S_t, \tau_t^c, \tau_t^n, \tau_t^k, \tau_t^{l,k}\}_{t=0}^{\infty}$, and initial conditions, $\{m_{-1}^k, b_{-1}^k, k_{-1}^k, f_{-1}^k\}$.

Intra-temporal problem: Each k minimizes the following total consumption expenditure:

$$p_t c_t^k = p_t^H c_t^{k,H} + p_t^F c_t^{k,F} \tag{104}$$

where p_t^F is the price index of foreign tradables (expressed in domestic currency).

Each domestic capitalist k' total consumption expenditure is split into total expenditure on private home goods and private foreign goods as follows:

$$p_t^H c_t^{k,H} = \sum_{h=1}^{N^k} p_t^H(h) c_t^{k,H}(h) \tag{105}$$

$$p_t^F c_t^{k,F} = \sum_{f=1}^{N^{k^*}} p_t^F(f) c_t^{k,F}(f) \tag{106}$$

The consumption bundle of k is defined as:

$$c_t^k = \frac{\left(c_t^{k,H}\right)^v \left(c_t^{k,F}\right)^{1-v}}{v^v (1-v)^{1-v}} \tag{107}$$

where $c_t^{k,H}$ is the composite domestic private good produced in the domestic country and consumed by domestic capitalist k, $c_t^{k,F}$ is the composite imported private good produced abroad and consumed by domestic capitalist k and v is the degree of preference for domestic private goods (if $v > 1/2$, there is a home bias).

The quantity of each variety h produced at home by domestic private firm h and consumed by each domestic capitalist k is denoted as $c_t^{k,H}(h)$. Using a Dixit-Stiglitz aggregator, the composite of domestic private goods consumed by each k, $c_t^{k,H}$, consists of h varieties and is given by:

$$c_t^{k,H} = \left[\sum_{h=1}^{N^k} \left(\frac{1}{N^k} \right)^{\frac{1}{\phi}} [c_t^{k,H}(h)]^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}} \tag{108}$$

where $\phi > 0$ is the elasticity of substitution across private goods produced in the domestic country.

Similarly, the quantity of each imported variety f produced abroad by foreign private firm f and consumed by each domestic capitalist k is denoted as $c_t^{k,F}(f)$. Using a Dixit-Stiglitz aggregator, the composite of imported private goods consumed by each k, $c_t^{k,F}$, consists of f varieties and is given by:

$$c_t^{k,F} = \left[\sum_{f=1}^{N^{k^*}} \left(\frac{1}{N^{k^*}} \right)^{\frac{1}{\phi}} [c_t^{k,F}(f)]^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}} \tag{109}$$

Therefore, in the intra-temporal problem, each domestic capitalist k chooses $\{c_t^{k,H}, c_t^{k,F}\}$ to minimize Eq. (104) subject to Eq. (107), by taking as given prices, $\{p_t^H, p_t^F\}$, and consumption bundle,$\{c_t^k\}$. Next, each capitalist k chooses $\{c_t^{k,H}(h), c_t^{k,F}(f)\}$ to minimize the sum of its consumption expenditure on private home goods and private foreign goods, sum of Eqs. (105) and (106), subject to composite of private domestic and private foreign goods consisting of varieties, Eqs. (108) and (109), by taking as given prices, $\{p_t^H(h), p_t^F(f)\}$, and consumption bundles,$\{c_t^{k,H}\}$ and $\{c_t^{k,F}\}$.

A.2 Capitalist k's optimality conditions

Each k acts competitively taking prices and policy as given.

Inter-temporal problem: The first order conditions include the constraints, Eqs. (102) and (103), and:

$$\frac{(c_t^k)^{-\sigma}}{(1+\tau_t^c)}\frac{p_t^H}{p_t}\left[1+\xi\left(\frac{k_t^k}{k_{t-1}^k}-1\right)\right]=\beta\frac{(c_{t+1}^k)^{-\sigma}}{(1+\tau_{t+1}^c)}\frac{p_{t+1}^H}{p_{t+1}}\times \tag{110}$$

$$\times\left[(1-\delta)+(1-\tau_{t+1}^k)r_{t+1}^k-\frac{\xi}{2}\left(\frac{k_{t+1}^k}{k_t^k}-1\right)^2+\xi\left(\frac{k_{t+1}^k}{k_t^k}-1\right)\frac{k_{t+1}^k}{k_t^k}\right]$$

$$\frac{(c_t^k)^{-\sigma}}{(1+\tau_t^c)}S_t\frac{p_t^*}{p_t}=\beta\frac{(c_{t+1}^k)^{-\sigma}}{(1+\tau_{t+1}^c)}Q_tS_{t+1}\frac{p_{t+1}^*}{p_{t+1}}\frac{p_t^*}{p_{t+1}^*} \tag{111}$$

$$\frac{(c_t^k)^{-\sigma}}{(1+\tau_t^c)}=\beta R_t\frac{p_t}{p_{t+1}}\frac{(c_{t+1}^k)^{-\sigma}}{(1+\tau_{t+1}^c)} \tag{112}$$

$$x_n(n_t^k)^{\eta}=(c_t^k)^{-\sigma}\frac{(1-\tau_t^n)}{(1+\tau_t^c)}w_t^k \tag{113}$$

$$x_m(m_t^k)^{-\mu}=\frac{(c_t^k)^{-\sigma}}{(1+\tau_t^c)}-\beta\frac{(c_{t+1}^k)^{-\sigma}}{(1+\tau_{t+1}^c)}\frac{p_t}{p_{t+1}} \tag{114}$$

Eqs. (110),(111) and (112) are respectively the Euler equations for capital, internationally traded assets and domestic government bonds and Eq.(113) is the optimality condition for work hours and Eq.(114) is the optimality condition for real money balances.

Intra-temporal problem: The first order conditions include the consumption bundle of k,Eq. (107), and:

$$\frac{c_t^{k,H}}{c_t^{k,F}} = \frac{v}{1-v}\frac{p_t^F}{p_t^H} \tag{115}$$

$$c_t^{k,H}(h) = \frac{c_t^{k,H}}{N^k}\left(\frac{p_t^H}{p_t^H(h)}\right)^{\phi} \tag{116}$$

$$c_t^{k,F}(f) = \frac{c_t^{k,F}}{N^{k^*}}\left(\frac{p_t^F}{p_t^F(f)}\right)^{\phi} \tag{117}$$

Eq. (115) is the optimality condition for sharing the total consumption between domestic and imported private products, Eqs. (116) and (117) are demand equations of domestic capitalist for varieties produced at home and abroad respectively.

Plugging Eqs. (116) and (117) into Eqs. (108) and (109) respectively, we get the following relations for price indexes:

$$p_t^H = \left\{\sum_{h=1}^{N^k}\frac{1}{N^k}[p_t^H(h)]^{1-\phi}\right\}^{\frac{1}{1-\phi}} \tag{118}$$

$$p_t^F = \left\{\sum_{f=1}^{N^{k^*}}\frac{1}{N^{k^*}}[p_t^F(f)]^{1-\phi}\right\}^{\frac{1}{1-\phi}} \tag{119}$$

Yet, Eqs. (104), (107) and (115) imply the following relation for consumer price index(CPI):

$$p_t = (p_t^H)^v(p_t^F)^{1-v} \tag{120}$$

Appendix B Households as private workers

This appendix provides details and the solution of domestic private worker's problem. Each domestic private worker w solves an inter-temporal problem, in which he acts competitively to maximize expected discounted lifetime utility, and an intra-temporal one, in which he minimizes the consumption expenditures.

B.1 Private worker's problem

Inter-temporal problem: Each domestic private worker $w = 1, 2, ..., N^w$ has the same expected lifetime utility and instantaneous utility function as each domestic capitalist k, that are given by (100) and (101) respectively, where now the index is w. Each w acts competitively to maximize expected discounted lifetime utility taking prices and policy as given.

The budget constraint of each w is in real terms:

$$(1+\tau_t^c)c_t^w + m_t^w = (1-\tau_t^n)w_t^w n_t^w + \frac{p_{t-1}}{p_t}m_{t-1}^w - \tau_t^{l,w} \tag{121}$$

where also small letters of prices denote real variables, e.g. $w_t^w \equiv \frac{W_t^w}{p_t}$, and small letters of quantities denote real variables per private worker, e.g. $\tau_t^{l,w} \equiv \frac{T_t^{l,w}}{N^w p_t}$. Here c_t^w is w's consumption bundle at t as defined below in the intra-temporal problem, $Eq.$(125), m_t^w is w's end-of-period real money balances, n_t^w is w's hours of work at t, w_t^w is private workers' real wage rate at t and $\tau_t^{l,w}$ is real lump-sum taxes/transfers to each w from the government at t.

Therefore, in the inter-temporal problem, each domestic private worker w chooses $\{c_t^w, n_t^w, m_t^w\}_{t=0}^{\infty}$ to maximize Eqs. (100) and (101) for w, subject to Eq. (121), by taking as given prices $\{w_t^w, p_t\}_{t=0}^{\infty}$, policy variables $\{\tau_t^c, \tau_t^n, \tau_t^{l,w}\}_{t=0}^{\infty}$, and initial conditions, $\{m_{-1}^w\}$.

Intra-temporal problem: Each w minimizes the following total consumption expenditure:

$$p_t c_t^w = p_t^H c_t^{w,H} + p_t^F c_t^{w,F} \tag{122}$$

Each domestic private worker w' total consumption expenditure is split into total expenditure on private home goods and private foreign goods as follows:

$$p_t^H c_t^{w,H} = \sum_{h=1}^{N^k} p_t^H(h) c_t^{w,H}(h) \tag{123}$$

$$p_t^F c_t^{w,F} = \sum_{f=1}^{N^{k^*}} p_t^F(f) c_t^{w,F}(f) \tag{124}$$

The consumption bundle of w is defined as:

$$c_t^w = \frac{\left(c_t^{w,H}\right)^v \left(c_t^{w,F}\right)^{1-v}}{v^v(1-v)^{1-v}} \tag{125}$$

where $c_t^{w,H}$ is the composite domestic private good produced in the domestic country and consumed by domestic private worker w and $c_t^{w,F}$ is the composite imported private good produced abroad and consumed by domestic private worker w.

The quantity of each variety h produced at home by domestic private firm h and consumed by each domestic private worker w is denoted as $c_t^{w,H}(h)$. Using a Dixit-Stiglitz aggregator, the composite of domestic private goods consumed by each w, $c_t^{w,H}$, consists of h varieties and is given by:

$$c_t^{w,H} = \left[\sum_{h=1}^{N^k} \left(\frac{1}{N^k} \right)^{\frac{1}{\phi}} [c_t^{w,H}(h)]^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}} \tag{126}$$

Similarly, the quantity of each imported variety f produced abroad by foreign private firm f and consumed by each domestic private worker w is denoted as $c_t^{w,F}(f)$. Using a Dixit-Stiglitz aggregator, the composite of imported private goods consumed by each w, $c_t^{w,F}$, consists of f varieties and is given by:

$$c_t^{w,F} = \left[\sum_{f=1}^{N^{k*}} \left(\frac{1}{N^{k*}} \right)^{\frac{1}{\phi}} [c_t^{w,F}(f)]^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}} \tag{127}$$

Therefore, in the intra-temporal problem, each private worker w chooses $\{c_t^{w,H}, c_t^{w,F}\}$ to minimize Eq. (122) subject to Eq. (125), by taking as given prices, $\{p_t^H, p_t^F\}$, and consumption bundle,$\{c_t^w\}$. Next, each private worker w chooses $\{c_t^{w,H}(h), c_t^{w,F}(f)\}$ to minimize the sum of its consumption expenditure on private home goods and private foreign goods, sum of Eqs. (123) and (124), subject to composite of domestic and foreign private goods consisting of varieties, Eqs. (126) and (127), by taking as given prices, $\{p_t^H(h), p_t^F(f)\}$, and consumption bundles,$\{c_t^{w,H}\}$ and $\{c_t^{w,F}\}$.

B.2 Private worker w's optimality conditions

Each w acts competitively taking as given prices and policy.

Inter-temporal problem: The first order conditions include the budget constraint, Eq. (121), and:

$$\frac{(c_t^w)^{-\sigma}}{x_n (n_t^w)^\eta} = \frac{1 + \tau_t^c}{(1 - \tau_t^n) w_t^w} \tag{128}$$

$$\frac{(c_t^w)^{-\sigma}}{1+\tau_t^c} = \beta \frac{p_t}{p_{t+1}} \left[\frac{(c_{t+1}^w)^{-\sigma}}{1+\tau_{t+1}^c} \right] + x_m (m_t^w)^{-\mu} \tag{129}$$

Intra-temporal problem: The first order conditions include the consumption bundle of w,Eq. (125), and:

$$\frac{c_t^{w,H}}{c_t^{w,F}} = \frac{v}{1-v} \frac{p_t^F}{p_t^H} \tag{130}$$

$$c_t^{w,H}(h) = \frac{c_t^{w,H}}{N^k} \left(\frac{p_t^H}{p_t^H(h)} \right)^{\phi} \tag{131}$$

$$c_t^{w,F}(f) = \frac{c_t^{w,F}}{N^{k^*}} \left(\frac{p_t^F}{p_t^F(f)} \right)^{\phi} \tag{132}$$

Eq. (130) is the optimality condition for sharing the total consumption between domestic and imported private products, Eqs. (131) and (132) are demand equations of domestic worker for varieties of private goods produced at home and abroad respectively.

Plugging Eqs. (131) and (132) into Eqs. (126) and (127) respectively, we get the following relations for price indexes:

$$p_t^H = \left\{ \sum_{h=1}^{N^k} \frac{1}{N^k} [p_t^H(h)]^{1-\phi} \right\}^{\frac{1}{1-\phi}} \tag{133}$$

$$p_t^F = \left\{ \sum_{f=1}^{N^{k^*}} \frac{1}{N^{k^*}} [p_t^F(f)]^{1-\phi} \right\}^{\frac{1}{1-\phi}} \tag{134}$$

Yet, Eqs. (122), (125) and (130) imply the following relation for consumer price index(CPI):

$$p_t = (p_t^H)^v (p_t^F)^{1-v} \tag{135}$$

Appendix C Households as public employees

This appendix provides details and the solution of domestic public employee's problem. Each domestic public employee b solves an inter-temporal problem, in which he acts competitively to maximize expected discounted lifetime utility, and an intra-temporal one, in which he minimizes the consumption expenditures.

C.1 Public employee's problem

Inter-temporal problem: Each domestic public employee $b = 1, 2, ..., N^b$ has the same expected lifetime utility and instantaneous utility function as each domestic capitalist k, that are given by (100) and (101) respectively, where now the index is b. Each b acts competitively to maximize expected discounted lifetime utility taking prices and policy as given.

The budget constraint of each b is in real terms:

$$(1+\tau_t^c)c_t^b + m_t^b = (1-\tau_t^n)w_t^b n_t^b + \frac{p_{t-1}}{p_t}m_{t-1}^b - \tau_t^{l,b} \tag{136}$$

where also small letters of prices denote real variables, e.g. $w_t^b \equiv \frac{W_t^b}{p_t}$, and small letters of quantities denote real variables per public employee, e.g. $\tau_t^{l,b} \equiv \frac{T_t^{l,b}}{N^b p_t}$. Here c_t^b is b's consumption bundle at t as defined below in the intra-temporal problem, $Eq.$(141), m_t^b is b's end-of-period real money balances, n_t^b is b's hours of work at t, w_t^b is public employees' real wage rate at t and $\tau_t^{l,b}$ is real lump-sum taxes/transfers to each b from the government at t.

Assuming that the government exogeneously determines the number of public employees, N^b, their labor, $n_t^b \equiv 1$, and their real wage, w_t^b, then the total public wage bill in real terms divided by the number of capitalists, defined as $\overline{g}_t^w$, equals $\frac{v^b}{v^k}w_t^b n_t^b$. Hence, we can rewrite the budget constraint of b as follows:

$$(1+\tau_t^c)c_t^b + m_t^b = (1-\tau_t^n)\frac{v^k}{v^b}\overline{g}_t^w + \frac{p_{t-1}}{p_t}m_{t-1}^b - \tau_t^{l,b} \tag{137}$$

Therefore, in the inter-temporal problem, each domestic public employee b chooses $\{c_t^b, m_t^b\}_{t=0}^{\infty}$ to maximize Eqs. (100) and (101) for b, subject to Eq. (137), by taking as given prices $\{w_t^b, p_t\}_{t=0}^{\infty}$, policy variables $\{\tau_t^c, \tau_t^n, \tau_t^{l,b}\}_{t=0}^{\infty}$, and initial conditions, $\{m_{-1}^b\}$.

Intra-temporal problem: Each b minimizes the following total consumption expenditure:

$$p_t c_t^b = p_t^H c_t^{b,H} + p_t^F c_t^{b,F} \tag{138}$$

Each domestic public employee b' total consumption expenditure is split into total expenditure on private home goods and private foreign goods as follows:

$$p_t^H c_t^{b,H} = \sum_{h=1}^{N^k} p_t^H(h) c_t^{b,H}(h) \tag{139}$$

$$p_t^F c_t^{b,F} = \sum_{f=1}^{N^{k^*}} p_t^F(f) c_t^{b,F}(f) \tag{140}$$

The consumption bundle of b is defined as:

$$c_t^b = \frac{\left(c_t^{b,H}\right)^v \left(c_t^{b,F}\right)^{1-v}}{v^v(1-v)^{1-v}} \tag{141}$$

where $c_t^{b,H}$ is the composite domestic private good produced in the domestic country and consumed by domestic public employee b and $c_t^{b,F}$ is the composite imported private good produced abroad and consumed by domestic public employee b.

The quantity of each variety h produced at home by domestic private firm h and consumed by each domestic public employee b is denoted as $c_t^{b,H}(h)$. Using a Dixit-Stiglitz aggregator, the composite of domestic private goods consumed by each b, $c_t^{b,H}$, consists of h varieties and is given by:

$$c_t^{b,H} = \left[\sum_{h=1}^{N^k} \left(\frac{1}{N^k}\right)^{\frac{1}{\phi}} [c_t^{b,H}(h)]^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} \tag{142}$$

Similarly, the quantity of each imported variety f produced abroad by foreign private firm f and consumed by each domestic public employee b is denoted as $c_t^{b,F}(f)$. Using a Dixit-Stiglitz aggregator, the composite of imported private goods consumed by each b, $c_t^{b,F}$, consists of f varieties and is given by:

$$c_t^{b,F} = \left[\sum_{f=1}^{N^{k^*}} \left(\frac{1}{N^{k^*}}\right)^{\frac{1}{\phi}} [c_t^{b,F}(f)]^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} \tag{143}$$

Therefore, in the intra-temporal problem, each public employee b chooses $\{c_t^{b,H}, c_t^{b,F}\}$ to minimize Eq. (138) subject to Eq. (141), by taking as given prices, $\{p_t^H, p_t^F\}$, and consumption bundle,$\{c_t^b\}$. Next, each public employee b chooses $\{c_t^{b,H}(h), c_t^{b,F}(f)\}$ to minimize the sum of its consumption expenditure on private home goods and private foreign goods, sum of Eqs. (139) and (140), subject to composite of domestic and foreign private goods consisting of varieties, Eqs. (142) and (143), by taking as given prices, $\{p_t^H(h), p_t^F(f)\}$, and consumption bundles,$\{c_t^{b,H}\}$ and $\{c_t^{b,F}\}$.

C.2 Public employee b's optimality conditions

Each b acts competitively taking as given prices and policy.

Inter-temporal problem: The first order conditions include the budget constraint, Eq. (137), and:

$$\frac{(c_t^b)^{-\sigma}}{1+\tau_t^c} = \beta \frac{p_t}{p_{t+1}} \left[\frac{(c_{t+1}^b)^{-\sigma}}{1+\tau_{t+1}^c} \right] + x_m (m_t^b)^{-\mu} \tag{144}$$

Intra-temporal problem: The first order conditions include the consumption bundle of b,Eq. (141), and:

$$\frac{c_t^{b,H}}{c_t^{b,F}} = \frac{v}{1-v} \frac{p_t^F}{p_t^H} \tag{145}$$

$$c_t^{b,H}(h) = \frac{c_t^{b,H}}{N^k} \left(\frac{p_t^H}{p_t^H(h)} \right)^{\phi} \tag{146}$$

$$c_t^{b,F}(f) = \frac{c_t^{b,F}}{N^{k^*}} \left(\frac{p_t^F}{p_t^F(f)} \right)^{\phi} \tag{147}$$

Eq. (145) is the optimality condition for sharing the total consumption between domestic and imported private products, Eqs. (146) and (147) are demand equations of public employee for varieties of private goods produced at home and abroad respectively.

Plugging Eqs. (146) and (147) into Eqs. (142) and (143) respectively, we get the following relations for price indexes:

$$p_t^H = \left\{ \sum_{h=1}^{N^k} \frac{1}{N^k} [p_t^H(h)]^{1-\phi} \right\}^{\frac{1}{1-\phi}} \tag{148}$$

$$p_t^F = \left\{ \sum_{f=1}^{N^{k^*}} \frac{1}{N^{k^*}} [p_t^F(f)]^{1-\phi} \right\}^{\frac{1}{1-\phi}} \tag{149}$$

Yet, Eqs. (138), (141) and (145) imply the following relation for consumer price index(CPI):

$$p_t = (p_t^H)^v (p_t^F)^{1-v} \tag{150}$$

Appendix D Private firms

This appendix provides details and the solution of the private firm's problem in the domestic country. There are $h = 1, 2, ..., N^k$ domestic private firms. Each firm h produces a differentiated good of variety h under monopolistic competition and Rotemberg-type nominal fixities(see Leeper et al, 2013).

D.1 Demand for the private firm's product

Each domestic private firm h faces demand for its product, $y_t^{H,d}(h)$. The latter comes from domestic households' private consumption and investment, $C_t^H(h)$ and $X_t(h)$, where $C_t^H(h) \equiv \sum_{k=1}^{N^k} c_t^{k,H}(h) + \sum_{w=1}^{N^w} c_t^{w,H}(h) + \sum_{b=1}^{N^b} c_t^{b,H}(h)$ and $X_t(h) \equiv \sum_{k=1}^{N^k} x_t^k(h)$, from the use of private goods by the domestic state-owned enterprise as inputs in its production function, denoted as $G_t^c(h)$, from the government's investment,$G_t^i(h)$, from the financial intermediary which is located in the domestic country, denoted as $\Upsilon_t(h)$,[32] and from foreign households' consumption of the domestic good, $C_t^{F^*}(h)$, where $C_t^{F^*}(h) \equiv \sum_{k^*=1}^{N^{k^*}} c_t^{k,F^*}(h) + \sum_{w^*=1}^{N^{w^*}} c_t^{w,F^*}(h) + \sum_{b^*=1}^{N^{b^*}} c_t^{b,F^*}(h)$. Thus, aggregate demand for each good h is:

$$y_t^{H,d}(h) = \left[C_t^H(h) + X_t(h) + G_t^c(h) + G_t^i(h) + \Upsilon_t(h) + C_t^{F^*}(h)\right] \tag{151}$$

Aggregate demand for each good h is associated with production of domestic private firm h according to the following relation:

$$y_t^{H,d}(h) = y_t^H(h) \times \left\{1 - \frac{\phi^p}{2}\left[\frac{p_t^H(f)}{p_{t-1}^H(f)\pi^H} - 1\right]^2 \frac{Y_t^H}{N^k y_t^H(h)}\right\} \tag{152}$$

where $y_t^H(h)$ stands for the production of domestic private firm h, Y_t^H stands for the total domestic private output, π^H stands for the steady state value of the gross domestic goods inflation rate and $\phi^p \geq 0$ is a parameter which determines the degree of nominal price rigidity. The term in the brackets captures the Rotemberg-type pricing cost (See the next Subsection) and

[32] See also Curdia and Woodford (2009) for a similar modelling of resources consumed by banks; the latter are modelled below. That is, the model requires the bank to use real resources in the period in which the loan is originated.

reflects the discrepancy between production and demand as one expected in a Rotemberg-type fashion.

Since we have:

$$c_t^{k,H}(h) = \frac{c_t^{k,H}}{N^k}\left(\frac{p_t^H}{p_t^H(h)}\right)^{\phi} \tag{153}$$

$$c_t^{w,H}(h) = \frac{c_t^{w,H}}{N^k}\left(\frac{p_t^H}{p_t^H(h)}\right)^{\phi} \tag{154}$$

$$c_t^{b,H}(h) = \frac{c_t^{b,H}}{N^k}\left(\frac{p_t^H}{p_t^H(h)}\right)^{\phi} \tag{155}$$

$$x_t^{k}(h) = \frac{x_t^{k}}{N^k}\left(\frac{p_t^H}{p_t^H(h)}\right)^{\phi} \tag{156}$$

$$G_t^{c}(h) = \frac{G_t^{c}}{N^k}\left(\frac{p_t^H}{p_t^H(h)}\right)^{\phi} \tag{157}$$

$$G_t^{i}(h) = \frac{G_t^{i}}{N^k}\left(\frac{p_t^H}{p_t^H(h)}\right)^{\phi} \tag{158}$$

$$\Upsilon_t(h) = \frac{\Upsilon_t}{N^k}\left(\frac{p_t^H}{p_t^H(h)}\right)^{\phi} \tag{159}$$

$$c_t^{k,F^*}(h) = \frac{c_t^{k,F^*}}{N^{k^*}}\left(\frac{p_t^{F^*}}{p_t^{F^*}(h)}\right)^{\phi} \tag{160}$$

$$c_t^{w,F^*}(h) = \frac{c_t^{w,F^*}}{N^{k^*}}\left(\frac{p_t^{F^*}}{p_t^{F^*}(h)}\right)^{\phi} \tag{161}$$

$$c_t^{b,F^*}(h) = \frac{c_t^{b,F^*}}{N^{k^*}}\left(\frac{p_t^{F^*}}{p_t^{F^*}(h)}\right)^{\phi} \tag{162}$$

we can rewrite the relation (151) as:

$$y_t^{H,d}(h) = \frac{1}{N^k}\left[C_t^H + X_t + G_t^c + G_t^i + \Upsilon_t + C_t^{F^*}\right] \times \left(\frac{p_t^H}{p_t^H(h)}\right)^{\phi} \tag{163}$$

where $C_t^H \equiv \sum_{k=1}^{N^k} c_t^{k,H} + \sum_{w=1}^{N^w} c_t^{w,H} + \sum_{b=1}^{N^b} c_t^{b,H}$ is total consumption of private home goods, $X_t \equiv \sum_{k=1}^{N^k} x_t^k$ is total private investment, G_t^c denotes total goods and services of private sector that are used by the state-owned enterprise for the production of total public goods and services, G_t^i denotes public infrastructure investment, Υ_t denotes total resources consumed by the financial intermediary and $C_t^{F^*} \equiv \sum_{k^*=1}^{N^{k^*}} c_t^{k,F^*} + \sum_{w^*=1}^{N^{w^*}} c_t^{w,F^*} + \sum_{b^*=1}^{N^{b^*}} c_t^{b,F^*}$ is total consumption of private home goods by households in the foreign country (i.e. domestic country's exports). Also notice that the law of one price implies that in Eqs. (160),(161) and (162):

$$\frac{p_t^{F^*}}{p_t^{F^*}(h)} = \frac{\frac{p_t^H}{S_t}}{\frac{p_t^H(h)}{S_t}} = \frac{p_t^H}{p_t^H(h)} \tag{164}$$

Since aggregate demand, $Y_t^{H,d}$, is:

$$Y_t^{H,d} = \left[C_t^H + X_t + G_t^c + G_t^i + \Upsilon_t + C_t^{F^*}\right] \tag{165}$$

then aggregate demand for each good h is rewritten as:

$$y_t^{H,d}(h) = \frac{Y_t^{H,d}}{N^k}\left(\frac{p_t^H}{p_t^H(h)}\right)^{\phi} \tag{166}$$

Another equivalent expression of demand for each good h in terms of private production follows:

$$y_t^H(h) = \frac{Y_t^H}{N^k}\left(\frac{p_t^H}{p_t^H(h)}\right)^{\phi} \tag{167}$$

where

$$Y_t^{H,d} \equiv Y_t^H \times \left\{1 - \frac{\phi^p}{2}\left[\frac{p_t^H(f)}{p_{t-1}^H(f)\pi^H} - 1\right]^2 \frac{Y_t^H}{N^k y_t^H(h)}\right\}$$

and with Y_t^H to denote the aggregate domestic private production.

Notice that in the private firms' optimization problem below we should use Eq.(166) as an expression for demand of each good h. However, it is more convenient to work with Eq.(167), to represent demand for each good h, instead of Eq.(166)

D.1.1 Private firms' optimization problem

Nominal profits of private firm h are defined as:

$$\widetilde{\Omega_t}(h) = p_t^H(h)y_t^H(h) - p_t^H r_t^k k_{t-1}(h) - W_t^w n_t^w(h) - W_t^k n_t^k(h) - \frac{\phi^p}{2}\left(\frac{p_t^H(h)}{p_{t-1}^H(h)\pi^H} - 1\right)^2 \frac{p_t^H Y_t^H}{N^k} \tag{168}$$

where $k_{t-1}(h)$ denotes the capital input chosen by private firm h, $n_t^w(h)$ denotes private workers' labor input chosen by private firm h and $n_t^h(h)$ denotes the capitalists' labor input chosen by private firm h. The quadratic cost that the private firm h faces once it changes the price of its product is proportional to the aggregate domestic private output divided by the number of private firms. [33]

All private firms use the same technology represented by the production function(see also Hornstein et al,2005, and Baxter and King,1993):

$$y_t^H(h) = A_t\left\{[k_{t-1}(h)]^\alpha \left[\{n_t^k(h)\}^\theta \{n_t^w(h)^{1-\theta}\}\right]^{1-\alpha}\right\}\left(k_{t-1}^g\right)^{\theta_k} \tag{169}$$

where A_t is an exogenous TFP, $0 < \alpha < 1$ is a technology parameter, k_{t-1}^g denotes the stock of public infrastructure divided by the number of capitalists which is common for all private firms, $0 < \theta_k < 1$ is the output elasticity of public infrastructure for private firm h and $0 < \theta < 1$ labor efficiency parameter of capitalist. We assume a positive θ_k , which implies that the production function has increasing returns with respect to public infrastructure, as in Baxter and King (1993), Basu and Kollmann (2013), Bom and Ligthart (2014),Leduc and Wilson (2013) and Iwata (2013). Notice that we keep CRS over private inputs.

Profit maximization by private firm h is also subject to the demand for its product, Eq.(166)as derived above. But as we have aforementioned, instead of using Eq.(166), we can equivalently use Eq.(167).

Each private firm h chooses its price, $p_t^H(h)$, and its inputs, $k_t^k(h)$, $n_t^k(h)$, $n_t^w(h)$, to maximize the sum of discounted expected real dividends, $\max E_o \sum_{t=0}^{\infty} \Xi_{0,0+t} \frac{\widetilde{\Omega_t}(h)}{p_t}$, subject to Eq. (167) and its production function, Eq. (169). The objective in

[33]This specification of Rotemberg-type cost is similar to that of Leeper et al,2013. Here, working with summations instead of integrals, we should have a pricing cost which is proportional to the aggregate domestic private output divided by the number of private firms. With this modification, we can derive the same NK Philips curve as Leeper at al,2013.

real terms is given by:

$$\max E_o \sum_{t=0}^{\infty} \Xi_{0,0+t} \left[\frac{p_t^H(h)}{p_t} y_t^H(h) - \frac{p_t^H(h)}{p_t} r_t^k k_{t-1}(h) - w_t^w n_t^w(h) - w_t^k n_t^k(h) - \frac{\phi^p}{2} \left(\frac{p_t^H(h)}{p_{t-1}^H(h)\pi^H} - 1 \right)^2 \frac{p_t^H Y_t^H}{p_t N^k} \right] \tag{170}$$

where $\Xi_{0,0+t}$ is a stochastic discount factor taken as given by the private firm h. This is defined as $\Xi_{0,0+t} = \prod_{i=0}^{t-1} \left\{ \frac{1}{R_i} \right\} = \beta^t \prod_{i=0}^{t-1} \left[\left(\frac{p_i}{p_{i+1}} \right) \left(\frac{1+\tau_i^c}{1+\tau_{i+1}^c} \right) \left(\frac{c_{i+1}^k}{c_i^k} \right)^{-\sigma} \right]$ and arises from Euler for bonds.

D.1.2 Private firms' optimality conditions

Following the related literature, instead of solving the above problem, we follow a two step procedure. We first solve a cost minimization problem, where each private firm h minimizes its cost by choosing factor inputs given technology and prices. The solution will give a minimum nominal cost function, which is a function of factor prices and output produced by the private firm. In turn, given this cost function, each private firm h solves a maximization problem by choosing its price.

Cost minimization problem: In the first stage, we solve a static cost minimization problem, where each h minimizes its cost by choosing its production factor inputs, $k_t^k(h)$,$n_t^k(h)$,$n_t^w(h)$, subject to its production function, Eq.(169), given technology and prices. The cost function is defined in real terms as follows:

$$\min \Psi(y_t^H(h)) = \left[\frac{p_t^H(h)}{p_t} r_t^k k_{t-1}(h) + w_t^w n_t^w(h) + w_t^k n_t^k(h) \right] \tag{171}$$

The solution to the cost minimization problem gives the input demand functions:

$$\frac{p_t^H(h)}{p_t} r_t^k k_{t-1}(h) = mc_t \alpha y_t^H(h) \tag{172}$$

$$w_t^k n_t^k(h) = mc_t \theta (1-\alpha) y_t^H(h) \tag{173}$$

$$w_t^w n_t^w(h) = mc_t (1-\theta)(1-\alpha) y_t^H(h) \tag{174}$$

where $mc_t \equiv \Psi'(y_t^H(h))$, since by definition the real marginal cost is the derivative of the associated minimum nominal cost function,$\Psi(y_t^H(h))$, with respect to $y_t^H(h)$.

Summing up the three above equations it arises the following relation for the associated minimum real cost function of h:

$$\Psi(y_t^H(h)) = mc_t y_t^H(h) \tag{175}$$

Where the real marginal cost, mc_t, it can be shown that equals:

$$mc_t = \frac{1}{A_t (k_{t-1}^g)^{\theta_k}} \left[\frac{p_t^H(h)}{p_t} \frac{r_t^k}{\alpha} \right]^{\alpha} \left[\left\{ \frac{w_t^k}{\theta(1-\alpha)} \right\}^{\theta} \times \left\{ \frac{w_t^w}{(1-\theta)(1-\alpha)} \right\}^{1-\theta} \right]^{1-\alpha} \tag{176}$$

implying that mc_t is common for all private firms since it only depends on prices, parameters and technology which are common for all private firms.

Profit maximization: The solution to the cost minimization problem will give a minimum nominal cost function, which is a function of prices and output produced by the private firm. In turn, given this cost function, each h solves a dynamic maximization problem by choosing its price. Specifically, in the second stage, h chooses its price,$p_t^H(h)$,to maximize the lifetime expected discounted real profits:

$$\max E_o \sum_{t=0}^{\infty} \Xi_{0,0+t} \left[\frac{p_t^H(h)}{p_t} y_t^H(h) - \Psi(y_t^H(h)) - \frac{\phi^p}{2} \left(\frac{p_t^H(h)}{p_{t-1}^H(h)\pi^H} - 1 \right)^2 \frac{p_t^H Y_t^H}{p_t N^k} \right] \tag{177}$$

The above profit maximization is subject to the Eq. (167) which is equivalent to the demand equation that the monopolistically competitive private firm h faces, Eq.(166).

The first order condition gives:

$$(1-\phi)\frac{p_t^H(h)}{p_t} y_t^H(h) + \phi mc_t y_t^H(h) - \phi^p \left[\frac{p_t^H(h)}{p_{t-1}^H(h)\pi^H} - 1 \right] \frac{p_t^H}{p_t} \frac{Y_t^H p_t^H(h)}{N^k p_{t-1}^H(h)\pi^H} = \beta\phi^p \left[\left(\frac{p_t}{p_{t+1}} \right) \left(\frac{1+\tau_t^c}{1+\tau_{t+1}^c} \right) \left(\frac{c_{t+1}^k}{c_t^k} \right)^{-\sigma} \right] \left[1 - \frac{p_{t+1}^H(h)}{p_t^H(h)\pi^H} \right] \frac{p_{t+1}^H(h)}{p_t^H(h)\pi^H} \frac{p_{t+1}^H}{p_{t+1}} \frac{Y_{t+1}^H}{N^k} \tag{178}$$

Thus, the behavior of h is summarized by Eqs. (172),(173),(174) and (178).

All private firms solve the identical problem and they will set the same price,$p_t^H(h)$, which implies that $p_t^H(h) = p_t^H$.

Appendix E Government budget constraint

This Appendix presents the government budget constraint in some detail. We start by presenting the domestic government's budget constraint in nominal and aggregate terms:

$$Q_{t-1}S_tF^g_{t-1} + R_{t-1}B_{t-1} + p^H_tG^c_t + p^H_tG^i_t + p_tG^w_t + M_{t-1} = \tag{179}$$

$$= M_t + \tau^c_t\left[p^H_tC^H_t + p^F_tC^F_t\right] + \tau^k_t\left[r^k_tp^H_tK^k_{t-1} + p_t\widetilde{\Omega}_t\right] +$$

$$+ \tau^n_t\left[W^k_t\widetilde{N_t}^k + W^w_t\widetilde{N_t}^w + +p_tG^w_t\right] + \tag{180}$$

$$+ \left[p_tT^{l,k}_t + p_tT^{l,w}_t + p_tT^{l,b}_t\right] + B_t + S_tF^g_t$$

where F^g_t is the end-of-period nominal public debt held by foreign agents and expressed in foreign currency, B_t is the end-of-period nominal public debt held by domestic agents, G^w_t is the total public wage bill in real terms, M_t is the end-of-period stock of nominal money balances, $C^F_t \equiv \sum\limits_{k=1}^{N^k} c^{k,F}_t + \sum\limits_{w=1}^{N^w} c^{w,F}_t + \sum\limits_{b=1}^{N^b} c^{b,F}_t$, $K^k_{t-1} \equiv \sum\limits_{k=1}^{N^k} k^k_{t-1}$, $\widetilde{\Omega}_t \equiv \sum\limits_{k=1}^{N^k} \omega^k_t$, $\widetilde{N_t}^k \equiv \sum\limits_{k=1}^{N^k} n^k_t$, $\widetilde{N_t}^w \equiv \sum\limits_{w=1}^{N^w} n^w_t$, $T^{l,k}_t \equiv \sum\limits_{k=1}^{N^k} \tau^{l,k}_t$, $T^{l,w}_t \equiv \sum\limits_{w=1}^{N^w} \tau^{l,w}_t$, $T^{l,b}_t \equiv \sum\limits_{b=1}^{N^b} \tau^{l,b}_t$. (the rest of the variables have been defined above).

Then, dividing by the current CPI, p_t, and the constant number of capitalists, N^k, we get the government budget constraint in real terms (per capitalist, per private worker and per public employee):

$$Q_{t-1}\frac{S_tp^*_t}{p_t}\frac{p^*_{t-1}}{p^*_t}f^g_{t-1} + R_{t-1}\frac{p_{t-1}}{p_t}b_{t-1} + \frac{p^H_t}{p_t}\overline{g}^c_t + \frac{p^H_t}{p_t}\overline{g}^i_t + \overline{g}^w_t + \frac{p_{t-1}}{p_t}m_{t-1} = \tag{181}$$

$$= m_t + \tau^c_t\left[\frac{p^H_t}{p_t}\left(c^{k,H}_t + \frac{v^w}{v^k}c^{w,H}_t + \frac{v^b}{v^k}c^{b,H}_t\right) + \frac{p^F_t}{p_t}\left(c^{k,F}_t + \frac{v^w}{v^k}c^{w,F}_t + \frac{v^b}{v^k}c^{b,F}_t\right)\right] + \tau^k_t\left[r^k_t\frac{p^H_t}{p_t}k^k_{t-1} + \widetilde{\omega_t}^k\right] +$$

$$+\tau^n_t\left[w^k_tn^k_t + \frac{v^w}{v^k}w^w_tn^w_t + \overline{g}^w_t\right] + \left[\tau^{l,k}_t + \frac{v^w}{v^k}\tau^{l,w}_t + \frac{v^b}{v^k}\tau^{l,b}_t\right] + b_t + S_t\frac{p^*_t}{p_t}f^g_t$$

where small letters denote real, per capitalist, per private worker and per public employee quantities, namely, $f^g_t \equiv \frac{F^g_t}{p^*_tN^k}$, $b_t \equiv \frac{B_t}{p_tN^k}$, $m_t \equiv \frac{M_t}{p_tN^k}$, $\overline{g}^c_t \equiv \frac{G^c_t}{N^k}$, $\overline{g}^i_t \equiv \frac{G^i_t}{N^k}$, $\overline{g}^w_t \equiv \frac{G^w_t}{N^k}$.

Here, we model infrastructure as a stock variable assuming that it accumulates like physical private capital (see also Fischer and Turnovsky (1998)). Hence, the stock of public infrastructure divided by the number of capitalists,k_t^g, evolves according to:

$$\overline{g}_t^i = k_t^g - (1-\delta^g)k_{t-1}^g + \frac{\xi^g}{2}\left(\frac{k_t^g}{k_{t-1}^g} - 1\right)^2 k_{t-1}^g \tag{182}$$

where $0 \leq \delta^g \leq 1$ is the depreciation rate of public infrastructure and $\xi^g \geq 0$ is a parameter capturing adjustment costs related to public infrastructure stock.

For convenience, let $D_t \equiv B_t + S_t F_t^g$ denote the total nominal public debt issued by the domestic government. This debt can be held by domestic private agents(capitalists), $\lambda_t D_t$, and by foreign private agents(capitalists), $(1-\lambda_t)D_t$, where $0 \leq \lambda_t \leq 1$.[34] Then, the above government budget constraint is rewritten as:

$$Q_{t-1}\frac{S_t p_t^*}{p_t}\frac{p_{t-1}^*}{p_t^*}\frac{p_{t-1}}{S_{t-1}p_{t-1}^*}(1-\lambda_{t-1})d_{t-1} + R_{t-1}\frac{p_{t-1}}{p_t}\lambda_{t-1}d_{t-1} + \frac{p_t^H}{p_t}\overline{g}_t^c + \frac{p_t^H}{p_t}\overline{g}_t^i + \overline{g}_t^w + \frac{p_{t-1}}{p_t}m_{t-1} = \tag{183}$$

$$= m_t + \tau_t^c\left[\frac{p_t^H}{p_t}\left(c_t^{k,H} + \frac{v^w}{v^k}c_t^{w,H} + \frac{v^b}{v^k}c_t^{b,H}\right) + \frac{p_t^F}{p_t}\left(c_t^{k,F} + \frac{v^w}{v^k}c_t^{w,F} + \frac{v^b}{v^k}c_t^{b,F}\right)\right] + \tau_t^k\left[r_t^k\frac{p_t^H}{p_t}k_{t-1}^k + \widetilde{\omega}_t^k\right] +$$

$$+\tau_t^n\left[w_t^k n_t^k + \frac{v^w}{v^k}w_t^w n_t^w + \overline{g}_t^w\right] + \left[\tau_t^{l,k} + \frac{v^w}{v^k}\tau_t^{l,w} + \frac{v^b}{v^k}\tau_t^{l,b}\right] + d_t$$

where $d_t \equiv \frac{D_t}{p_t N^k}$.

Therefore, as in e.g. Alesina et al. (2002), we include the four main types of government spending (purchases of goods and services from the private sector, public investment in infrastructure, public wages, and transfers to individuals). We also include the three main types of taxes (taxes on consumption, capital income and labor).

In each period, one of ($\tau_t^c, \tau_t^k, \tau_t^n, \overline{g}_t^c, \overline{g}_t^i, \overline{g}_t^w, \tau_t^{l,k}, \tau_t^{l,w}, \tau_t^{l,b}, \lambda_t, v_t^b, d_t, v_t^b$) need to adjust to satisfy the government budget constraint.

[34] Public debt differs from foreign debt. The end-of-period total public debt, written in nominal terms, is $D_t = B_t + S_t F_t^g$, where $B_t = \lambda_t D_t$ is domestic government bonds held by domestic capitalists and $S_t F_t^g = (1-\lambda_t)D_t$ denotes domestic government bonds held by foreign investors. On the other hand, the country's end-of-period net foreign debt, written in nominal terms, is $S_t(F_t^g - F_t^k) = (1-\lambda_t)D_t - S_t F_t^k$, where F_t^k is foreign assets held by domestic capitalists (if negative, it denotes liabilities).

Similarly, the government budget constraint in real terms (per capitalist, per private worker and per public employee) in the foreign country is:

$$Q^*_{t-1}\frac{p_t}{S_t p^*_t}\frac{p_{t-1}}{p_t}f^{g*}_{t-1} + R^*_{t-1}\frac{p^*_{t-1}}{p^*_t}b^*_{t-1} + \frac{p^{H*}_t}{p^*_t}\overline{g}^{c*}_t + \frac{p^{H*}_t}{p^*_t}\overline{g}^{i*}_t + \overline{g}^{w*}_t + \frac{p^*_{t-1}}{p^*_t}m^*_{t-1} = \tag{184}$$

$$= m^*_t + \tau^{c*}_t\left[\frac{p^{H*}_t}{p^*_t}\left(c^{k,H*}_t + \frac{v^{w*}}{v^{k*}}c^{w,H*}_t + \frac{v^{b*}}{v^{k*}}c^{b,H*}_t\right) + \frac{p^{F*}_t}{p^*_t}\left(c^{k,F*}_t + \frac{v^{w*}}{v^{k*}}c^{w,F*}_t + \frac{v^{b*}}{v^{k*}}c^{b,F*}_t\right)\right] +$$
$$+\tau^{k*}_t\left[r^{k*}_t\frac{p^{H*}_t}{p^*_t}k^{k*}_{t-1} + \widetilde{\omega}^{k*}_t\right] + \tau^{n*}_t\left[w^{k*}_t n^{k*}_t + \frac{v^{w*}}{v^{k*}}w^{w*}_t n^{w*}_t + \overline{g}^{w*}_t\right] +$$
$$+\left[\tau^{l,k*}_t + \frac{v^{w*}}{v^{k*}}\tau^{l,w*}_t + \frac{v^{b*}}{v^{k*}}\tau^{l,b*}_t\right] + b^*_t + \frac{p_t}{p^*_t S_t}f^{g*}_t$$

The stock of public infrastructure divided by the number of capitalists in the foreign country, k^{g*}_t, evolves according to:

$$\overline{g}^{i*}_t = k^{g*}_t - (1-\delta^{g*})k^{g*}_{t-1} + \frac{\xi^{g*}}{2}\left(\frac{k^{g*}_t}{k^{g*}_{t-1}} - 1\right)^2 k^{g*}_{t-1} \tag{185}$$

Let denote D^*_t to be the total foreign public debt in foreign currency. This can be held by foreign private agents(capitalists), $B^*_t = \lambda^*_t D^*_t$, and by domestic private agents(capitalists), $\frac{F^{g*}_t}{S_t} = (1-\lambda^*_t)D^*_t$. Then, we have:

$$Q^*_{t-1}\frac{p_t}{S_t p^*_t}\frac{p_{t-1}}{p_t}\frac{S_{t-1}p^*_{t-1}}{p_{t-1}}(1-\lambda^*_{t-1})d^*_{t-1} + R^*_{t-1}\frac{p^*_{t-1}}{p^*_t}\lambda^*_{t-1}d^*_{t-1} + \frac{p^{H*}_t}{p^*_t}\overline{g}^{c*}_t + \frac{p^{H*}_t}{p^*_t}\overline{g}^{i*}_t + \overline{g}^{w*}_t + \frac{p^*_{t-1}}{p^*_t}m^*_{t-1} = \tag{186}$$

$$= m^*_t + \tau^{c*}_t\left[\frac{p^{H*}_t}{p^*_t}\left(c^{k,H*}_t + \frac{v^{w*}}{v^{k*}}c^{w,H*}_t + \frac{v^{b*}}{v^{k*}}c^{b,H*}_t\right) + \frac{p^{F*}_t}{p^*_t}\left(c^{k,F*}_t + \frac{v^{w*}}{v^{k*}}c^{w,F*}_t + \frac{v^{b*}}{v^{k*}}c^{b,F*}_t\right)\right] +$$
$$+\tau^{k*}_t\left[r^{k*}_t\frac{p^{H*}_t}{p^*_t}k^{k*}_{t-1} + \widetilde{\omega}^{k*}_t\right] + \tau^{n*}_t\left[w^{k*}_t n^{k*}_t + \frac{v^{w*}}{v^{k*}}w^{w*}_t n^{w*}_t + \overline{g}^{w*}_t\right] +$$
$$+\left[\tau^{l,k*}_t + \frac{v^{w*}}{v^{k*}}\tau^{l,w*}_t + \frac{v^{b*}}{v^{k*}}\tau^{l,b*}_t\right] + d^*_t$$

In each period, one of ($\tau^{c*}_t, \tau^{k*}_t, \tau^{n*}_t, \overline{g}^{c*}_t, \overline{g}^{i*}_t, \overline{g}^{w*}_t, \tau^{l,k*}_t, \tau^{l,w*}_t, \tau^{l,b*}_t, \lambda^*_t, v^{b*}_t, d^*_t, v^{b*}_t$) need to adjust to satisfy the government budget constraint.

Appendix F State-owned enterprise

Following most of the related literature,[35] we assume that total public goods and services, Y_t^g, are produced using goods and services purchased from the private sector, G_t^c , and total public employment, L_t^g. In particular, following e.g. Linnemann (2009) and Economides et al. (2013, 2014), we use a Cobb-Douglas production function of the form:

$$Y_t^g = A\left(G_t^c\right)^{\theta_g}\left(L_t^g\right)^{1-\theta_g} \tag{187}$$

where $0 \le \theta_g \le 1$ is a technology parameter. Notice that both private and public good production face the same TFP; this is because we do not want our results to be driven by exogenous factors. The total cost of public production, $G_t^c + w_t^b L_t^g$, is financed by the government through taxes and bonds (see the government budget constraint (183) above).

Appendix G Financial intermediary

The profit of the international financial intermediary from loans between $t-1$ and t is distributed at time t. In nominal and aggregate terms, this profit is defined as:[36]

$$Q_{t-1}^*\left[(F_{t-1}^{g*} - F_{t-1}^{k*}) - \frac{\psi}{2}N^{k*}p_{t-1}^H(f_{t-1}^{g*} - f_{t-1}^{k*})^2\right] - Q_{t-1}S_t\left(F_{t-1}^k - F_{t-1}^g\right) \tag{188}$$

where $F_t^k \equiv \sum\limits_{k=1}^{N^k} p_t f_t^k$ are aggregate nominal international assets held by private agents (capitalists) in the domestic and foreign country respectively, $F_t^{k*} \equiv \sum\limits_{k*=1}^{N^{k*}} p_t^* f_t^{k*}$ are aggregate nominal foreign public debt in the domestic

[35] See Economides et al. (2014) for details and a review of the literature on the production function of public goods. We report that our results are robust to adding public capital, whose changes are financed by public investment spending, as a third factor into the production function of public goods. Again, see Economides et al. (2014) for the role of public capital in public good production functions.

[36] Thus, at the beginning of period t, agents carry over assets and liabilities from period $t-1$. Borrowers honor their preexisting obligations to lenders. In particular, in the international capital market, where transactions take place via the bank, the bank receives interest income from borrowers and pays off the lenders. The latter is the interest payments that the bank promised at $t-1$ to pay at t. The bank also pays the monitoring cost associated with these transactions.

and foreign country respectively, and $\frac{\psi}{2}(f^{g*}_{t-1} - f^{k*}_{t-1})^2$ is a per capitalist real cost function. Where f^{g*}_{t-1} and f^{k*}_{t-1} are respectively foreign public debt and foreign private assets respectively in per capitalist real terms and ψ is a cost parameter. That is, at any t, $f^{g}_{t} \equiv \frac{F^g_t}{p^*_t N^k}$ and analogously for f^{g*}_t. Then, if $\left(F^k_{t-1} - F^g_{t-1}\right)$ is positive (respectively negative), it denotes the net asset (respectively liability) position of the domestic country in the world financial market, and similarly for $\left(F^{k*}_{t-1} - F^{g*}_{t-1}\right)$ in the foreign country. Notice that the real resources used by the bank are assumed to be consumed at the same time the interest payments/income are repaid/received, namely at time t, rather then when the loan contract was originated, namely at time $t-1$.

Then, dividing by the current CPI, p_t, and the constant population size of capitalists, N^k, the real and per capitalist profit, defined as $\widetilde{\pi}_t$, is:

$$\widetilde{\pi}_t \equiv Q^*_{t-1}\left[\frac{p_{t-1}}{p_t}(f^{g*}_{t-1} - f^{k*}_{t-1}) - \frac{\psi}{2}\frac{p^H_{t-1}}{p^H_t}\frac{p^H_t}{p_t}(f^{g*}_{t-1} - f^{k*}_{t-1})^2\right] - Q_{t-1}S_t\frac{p^*_t}{p_t}\frac{p^*_{t-1}}{p^*_t}\left(f^k_{t-1} - f^g_{t-1}\right) \tag{189}$$

Recall that we have assumed that $N^k = N^{k*}$ in Subsection 2.1 in the main text.

Since, in equilibrium, international borrowing equals international lending at each t, namely, $F^{g*}_t - F^{k*}_t = S_t(F^k_t - F^g_t)$ in nominal and aggregate terms, or $f^{g*}_t - f^{k*}_t = S_t\frac{p^*_t}{p_t}(f^k_t - f^g_t)$ in real and per capitalist terms, so that $f^{g*}_{t-1} - f^{k*}_{t-1} = S_{t-1}\frac{p^*_{t-1}}{p_{t-1}}(f^k_{t-1} - f^g_{t-1})$, this is rewritten as:

$$\widetilde{\pi}_t = Q^*_{t-1}\left[\frac{p_{t-1}}{p_t}(f^{g*}_{t-1} - f^{k*}_{t-1}) - \frac{\psi}{2}\frac{p^H_{t-1}}{p^H_t}\frac{p^H_t}{p_t}(f^{g*}_{t-1} - f^{k*}_{t-1})^2\right] - Q_{t-1}\frac{S_t}{S_{t-1}}\frac{p_{t-1}}{p_t}\left(f^{g*}_{t-1} - f^{k*}_{t-1}\right) \tag{190}$$

If the volume of the loan, $(f^{g*}_{t-1} - f^{k*}_{t-1})$, is chosen optimally by the financial intermediary, the first-order condition is:

$$Q^*_{t-1} = \frac{Q_{t-1}\frac{S_t}{S_{t-1}}}{1 - \psi\frac{p^H_{t-1}}{p_{t-1}}(f^{g*}_{t-1} - f^{k*}_{t-1})} \tag{191}$$

In what follows, we define $Q^*_{t-1}\frac{\psi}{2}\frac{p^H_{t-1}}{p^H_t}(f^{g*}_{t-1} - f^{k*}_{t-1})^2 \equiv \upsilon_t$, where recall from above the aggregate demand $Y^{H,d}_t = \left[C^H_t + X_t + G^c_t + G^i_t + \Upsilon_t + C^{F^*}_t\right]$ in real total terms or, equivalently,

$y_t^{H,d} = \Big[\Big(c_t^{k,H} + \frac{v^w}{v^k} c_t^{w,H} + \frac{v^b}{v^k} c_t^{b,H}\Big) + x_t^k + \overline{g}_t^c + \overline{g}_t^i + v_t + \Big(c_t^{k,F*} + \frac{v^{w*}}{v^{k*}} c_t^{w,F*} + \frac{v^{b*}}{v^{k*}} c_t^{b,F*}\Big)\Big]$ in per capitalist, per private worker and per public employee terms. Recall that $S_t = 1$ in a currency union regime and the assumptions $N^k = N^{k*}$ and $N^w = N^{w*}$ we have made in Subsection 2.1 in the main text.

Appendix H Equilibrium in the status quo economy

This Appendix presents in some detail the status quo equilibrium system, given feedback policy coefficients. We will work in steps.

H.1 Market clearing conditions and the balance of payments

In the domestic economy, the market-clearing conditions in the capital market, the labor markets, the money market, the domestic government bond market and the domestic dividend market are respectively (and similarly in the foreign country):

$$\sum_{k=1}^{N^k} k_{t-1}^k = \sum_{h=1}^{N^k} k_{t-1}(h)$$

$$\sum_{k=1}^{N^k} n_t^k = \sum_{h=1}^{N^k} n_t^k(h)$$

$$\sum_{w=1}^{N^w} n_t^w = \sum_{h=1}^{N^k} n_t^w(h)$$

$$\sum_{b=1}^{N^b} n_t^b = L_t^g \equiv 1$$

$$\sum_{k=1}^{N^k} m_t^k + \sum_{w=1}^{N^w} m_t^w + \sum_{b=1}^{N^b} m_t^b = \sum_{k=1}^{N^k} m_t$$

$$\sum_{k=1}^{N^k} b_t^k = \sum_{k=1}^{N^k} b_t$$

$$\sum_{k=1}^{N^k} \omega_t^k = \sum_{h=1}^{N^k} \frac{\widetilde{\Omega}_t(h)}{p_t}$$

The market-clearing condition for the profits made by the international financial intermediary (these profits are distributed to households in the domestic economy only who also bear the associated costs) is:

$$\sum_{k=1}^{N^k} \widetilde{\pi}_t^k = \sum_{k=1}^{N^k} \widetilde{\pi}_t$$

Regarding the balance of payments in each country, this is obtained by adding the constraints of households, firms and the government in the country. Then, the balance of payments in the domestic country (written in real terms per capitalist, per private worker and per public employee) is:

$$\frac{p_t^H}{p_t}\left\{\left[c_t^{k,H} + \frac{v^w}{v^k} c_t^{w,H} + \frac{v^b}{v^k} c_t^{b,H}\right] + x_t^k + \overline{g}_t^c + \overline{g}_t^i\right\} + \frac{p_t^F}{p_t}\left[c_t^{k,F} + \frac{v^w}{v^k} c_t^{w,F} + \frac{v^b}{v^k} c_t^{b,F}\right] = \tag{192}$$

$$S_t \frac{p_t^*}{p_t}(f_t^g - f_t^k) - Q_{t-1} \frac{S_t p_t^*}{p_t} \frac{p_{t-1}^*}{p_t^*}(f_{t-1}^g - f_{t-1}^k) + \widetilde{\pi}_t^k + \frac{1}{N^k} \sum_{h=1}^{N^k} \frac{p_t^H(h) y_t^H(h)}{p_t} -$$

$$-\frac{1}{N^k} \sum_{h=1}^{N^k} \left[\frac{\phi^p}{2}\left(\frac{p_t^H(h)}{p_{t-1}^H(h)\pi^H} - 1\right)^2 \frac{p_t^H}{p_t} \frac{Y_t^H}{N^k}\right]$$

where are variables have been defined above.

It can be shown, by solving the Eq. (167), $y_t^H(h) = \frac{Y_t^H}{N^k}\left(\frac{p_t^H}{p_t^H(h)}\right)^{\phi}$, with respect to $p_t^H(h)^{-\phi}$, and plugging this term into the domestic price index, $p_t^H = \left\{\sum_{h=1}^{N^k} \frac{1}{N^k}[p_t^H(h)]^{1-\phi}\right\}^{\frac{1}{1-\phi}}$, that $p_t^H Y_t^H = \sum_{h=1}^{N^k} p_t^H(h) y_t^H(h)$. Furthermore, due to symmetry in private firms' problem, price of every private firm h, $p_t^H(h)$, will be common for all firms and this, given the relation for domestic price index above, implies $p_t^H(h) = p_t^H$. Hence, the term of the third line and the last term of the second line on the RHS of the balance of payments above are $\frac{\phi^p}{2}\left(\frac{p_t^H}{p_{t-1}^H \pi^H} - 1\right)^2 \frac{p_t^H}{p_t} y_t^H$ and $\frac{p_t^H}{p_t} y_t^H$ respectively, where $y_t^H \equiv \frac{Y_t^H}{N^k}$ is per capitalist domestic absorption. Therefore, the balance of payments in the domestic economy is:

$$\frac{p_t^H}{p_t}\left\{\left[c_t^{k,H}+\frac{v^w}{v^k}c_t^{w,H}+\frac{v^b}{v^k}c_t^{b,H}\right]+x_t^k+\overline{g}_t^c+\overline{g}_t^i-y_t^H\left[1-\frac{\phi^p}{2}\left(\frac{p_t^H}{p_{t-1}^H\pi^H}-1\right)^2\right]\right\}+ \tag{193}$$

$$+\frac{p_t^F}{p_t}\left[c_t^{k,F}+\frac{v^w}{v^k}c_t^{w,F}+\frac{v^b}{v^k}c_t^{b,F}\right]=S_t\frac{p_t^*}{p_t}(f_t^g-f_t^k)-Q_{t-1}\frac{S_tp_t^*}{p_t}\frac{p_{t-1}^*}{p_t^*}(f_{t-1}^g-f_{t-1}^k)+\widetilde{\pi}_t^k$$

where recall that the resources used by the financial intermediary, $\upsilon_t\equiv Q_{t-1}^*\frac{\psi}{2}\frac{p_{t-1}^H}{p_t^H}(f_{t-1}^{g*}-f_{t-1}^{k*})^2$, are paid by the domestic country, so that

$$y_t^H\left[1-\frac{\phi^p}{2}\left(\frac{p_t^H}{p_{t-1}^H\pi^H}-1\right)^2\right]=\left[\left(c_t^{k,H}+\frac{v^w}{v^k}c_t^{w,H}+\frac{v^b}{v^k}c_t^{b,H}\right)+x_t^k+\overline{g}_t^c+\overline{g}_t^i+\upsilon_t+\left(c_t^{k,F*}+\frac{v^{w*}}{v^{k*}}c_t^{w,F*}+\frac{v^{b*}}{v^{k*}}c_t^{b,F*}\right)\right]$$

and where $\widetilde{\pi}_t\equiv Q_{t-1}^*\frac{p_{t-1}}{p_t}(f_{t-1}^{g*}-f_{t-1}^{k*})-\frac{p_t^H}{p_t}\upsilon_t-Q_{t-1}S_t\frac{p_t^*}{p_t}\frac{p_{t-1}^*}{p_t^*}\left(f_{t-1}^k-f_{t-1}^g\right)$. If, in turn, we add $\frac{p_t^H}{p_t}\upsilon_t$ on both sides of the balance of payments above, we have $\frac{p_t^H}{p_t}\left\{\left[c_t^{k,H}+\frac{v^w}{v^k}c_t^{w,H}+\frac{v^b}{v^k}c_t^{b,H}\right]+x_t^k+\overline{g}_t^c+\overline{g}_t^i+\upsilon_t-y_t^H\left[1-\frac{\phi^p}{2}\left(\frac{p_t^H}{p_{t-1}^H\pi^H}-1\right)^2\right]\right\}=-\frac{p_t^H}{p_t}[c_t^{k,F*}+\frac{v^{w*}}{v^{k*}}c_t^{w,F*}+\frac{v^{b*}}{v^{k*}}c_t^{b,F*}]$ so that the terms $\frac{p_t^F}{p_t}\left[c_t^{k,F}+\frac{v^w}{v^k}c_t^{w,F}+\frac{v^b}{v^k}c_t^{b,F}\right]-\frac{p_t^H}{p_t}\left[c_t^{k,F*}+\frac{v^{w*}}{v^{k*}}c_t^{w,F*}+\frac{v^{b*}}{v^{k*}}c_t^{b,F*}\right]$ on the LHS is the trade balance.

Working similarly, we get the balance of payments in the foreign country:

$$\frac{p_t^{H*}}{p_t^*}\left\{\left[c_t^{k,H*}+\frac{v^{w*}}{v^{k*}}c_t^{w,H*}+\frac{v^{b*}}{v^{k*}}c_t^{b,H*}\right]+x_t^{k*}+\overline{g}_t^{c*}+\overline{g}_t^{i*}-y_t^{H*}\left[1-\frac{\phi^{p*}}{2}\left(\frac{p_t^{H*}}{p_{t-1}^{H*}\pi^{H*}}-1\right)^2\right]\right\}+ \tag{194}$$

$$+\frac{p_t^{F*}}{p_t^*}\left[c_t^{k,F*}+\frac{v^{w*}}{v^{k*}}c_t^{w,F*}+\frac{v^{b*}}{v^{k*}}c_t^{b,F*}\right]=\frac{1}{S_t}\frac{p_t}{p_t^*}(f_t^{g*}-f_t^{k*})-Q_{t-1}^*\frac{p_t}{S_tp_t^*}\frac{p_{t-1}}{p_t}(f_{t-1}^{g*}-f_{t-1}^{k*})$$

where now

$$y_t^{H*}\left[1-\frac{\phi^{p*}}{2}\left(\frac{p_t^{H*}}{p_{t-1}^{H*}\pi^{H*}}-1\right)^2\right]=\left[\left(c_t^{k,H*}+\frac{v^{w*}}{v^{k*}}c_t^{w,H*}+\frac{v^{b*}}{v^{k*}}c_t^{b,H*}\right)+x_t^{k*}+\overline{g}_t^{c*}+\overline{g}_t^{i*}+\left(c_t^{k,F}+\frac{v^w}{v^k}c_t^{w,F}+\frac{v^b}{v^k}c_t^{b,F}\right)\right].$$

Finally, as said above, the market clearing condition in the market of internationally traded assets is (written in real and per capitalist terms):

$$f_t^{g*}-f_t^{k*}=S_t\frac{p_t^*}{p_t}(f_t^k-f_t^g)$$

which means that net foreign liabilities in the foreign country (the LHS) are equal to net foreign assets in the domestic country (the RHS).

H.2 Decentralized equilibrium (given policy)

We now combine all the above to solve for a symmetric Decentralized Equilibrium (DE) for any feasible policy. The DE is defined to be a sequence of allocations, prices and policies such that: (i) every type of household maximize utility; (ii) every private firm maximize profit; (iii) the state-owned enterprise produces the public goods and services; (iv) the world financial intermediary maximizes profit; (v) all constraints, including the government budget constraint and the balance of payments, are satisfied; and (vi) all markets clear, including the international asset market.

The DE is summarized by the following conditions (quantities are in real terms per capitalist, per private worker and per public employee):

$$x_n(n_t^k)^\eta (c_t^k)^\sigma = \frac{(1-\tau_t^n)}{(1+\tau_t^c)} w_t^k \tag{D1}$$

$$\frac{(c_t^k)^{-\sigma}}{(1+\tau_t^c)} S_t \frac{p_t^*}{p_t} = \beta \frac{(c_{t+1}^k)^{-\sigma}}{(1+\tau_{t+1}^c)} Q_t S_{t+1} \frac{p_{t+1}^*}{p_{t+1}} \frac{p_t^*}{p_{t+1}^*} \tag{D2}$$

$$\frac{(c_t^k)^{-\sigma}}{(1+\tau_t^c)} \frac{p_t^H}{p_t}\left[1+\xi\left(\frac{k_t^k}{k_{t-1}^k}-1\right)\right] = \beta \frac{(c_{t+1}^k)^{-\sigma}}{(1+\tau_{t+1}^c)} \frac{p_{t+1}^H}{p_{t+1}} \times$$
$$\times\left[(1-\delta)+(1-\tau_{t+1}^k)r_{t+1}^k - \frac{\xi}{2}\left(\frac{k_{t+1}^k}{k_t^k}-1\right)^2 + \xi\left(\frac{k_{t+1}^k}{k_t^k}-1\right)\frac{k_{t+1}^k}{k_t^k}\right] \tag{D3}$$

$$x_m(m_t^k)^{-\mu} = \frac{(c_t^k)^{-\sigma}}{(1+\tau_t^c)} - \beta \frac{p_t}{p_{t+1}} \frac{(c_{t+1}^k)^{-\sigma}}{(1+\tau_{t+1}^c)} \tag{D4}$$

$$\frac{(c_t^k)^{-\sigma}}{(1+\tau_t^c)} = \beta R_t \frac{p_t}{p_{t+1}} \frac{(c_{t+1}^k)^{-\sigma}}{(1+\tau_{t+1}^c)} \tag{D5}$$

$$k_t^k = (1-\delta)k_{t-1}^k + x_t^k - \frac{\xi}{2}\left(\frac{k_t^k}{k_{t-1}^k}-1\right)^2 k_{t-1}^k \tag{D6}$$

$$\frac{c_t^{k,H}}{c_t^{k,F}} = \frac{v}{1-v}\frac{p_t^F}{p_t^H} \tag{D7}$$

$$c_t^k = \frac{\left(c_t^{k,H}\right)^v \left(c_t^{k,F}\right)^{1-v}}{v^v(1-v)^{1-v}} \tag{D8}$$

$$\left[\left(c_t^{k,H} + \frac{v^w}{v^k}c_t^{w,H} + \frac{v^b}{v^k}c_t^{b,H}\right) + x_t^k + \overline{g}_t^c + \overline{g}_t^i + \upsilon_t + \left(c_t^{k,F*} + \frac{v^{w*}}{v^{k*}}c_t^{w,F*} + \frac{v^{b*}}{v^{k*}}c_t^{b,F*}\right)\right] = \tag{195}$$

$$= y_t^H\left[1 - \frac{\phi^p}{2}\left(\frac{p_t^H}{p_{t-1}^H\pi^H} - 1\right)^2\right] \tag{D9}$$

$$x_n(n_t^w)^\eta(c_t^w)^\sigma = \frac{(1-\tau_t^n)}{(1+\tau_t^c)}w_t^w \tag{D10}$$

$$x_m(m_t^w)^{-\mu} = \frac{(c_t^w)^{-\sigma}}{1+\tau_t^c} - \beta\frac{p_t}{p_{t+1}}\left[\frac{(c_{t+1}^w)^{-\sigma}}{1+\tau_{t+1}^c}\right] \tag{D11}$$

$$\frac{c_t^{w,H}}{c_t^{w,F}} = \frac{v}{1-v}\frac{p_t^F}{p_t^H} \tag{D12}$$

$$c_t^w = \frac{\left(c_t^{w,H}\right)^v \left(c_t^{w,F}\right)^{1-v}}{v^v(1-v)^{1-v}} \tag{D13}$$

$$(1+\tau_t^c)c_t^w + m_t^w = \frac{p_{t-1}}{p_t}m_{t-1}^w + (1-\tau_t^n)w_t^w n_t^w - \tau_t^{l,w} \tag{D14}$$

$$x_m(m_t^b)^{-\mu} = \frac{(c_t^b)^{-\sigma}}{1+\tau_t^c} - \beta\frac{p_t}{p_{t+1}}\left[\frac{(c_{t+1}^b)^{-\sigma}}{1+\tau_{t+1}^c}\right] \tag{D15}$$

$$\frac{c_t^{b,H}}{c_t^{b,F}} = \frac{v}{1-v}\frac{p_t^F}{p_t^H} \tag{D16}$$

$$c_t^b = \frac{\left(c_t^{b,H}\right)^v \left(c_t^{b,F}\right)^{1-v}}{v^v(1-v)^{1-v}} \tag{D17}$$

$$(1+\tau_t^c)c_t^b + m_t^b = \frac{p_{t-1}}{p_t}m_{t-1}^b + (1-\tau_t^n)\frac{v^k}{v^b}\overline{g}_t^w - \tau_t^{l,b} \tag{D18}$$

$$\frac{p_t^H}{p_t}r_t^k k_{t-1}^k = mc_t\alpha y_t^H \tag{D19}$$

$$w_t^k n_t^k = mc_t \theta (1-\alpha) y_t^H \tag{D20}$$

$$\frac{v^w}{v^k} w_t^w n_t^w = mc_t (1-\theta)(1-\alpha) y_t^H \tag{D21}$$

$$y_t^H = A_t \left\{ [k_{t-1}^k]^\alpha \left[\{n_t^k\}^\theta \times \left\{ \frac{v^w}{v^k} n_t^w \right\}^{1-\theta} \right]^{1-\alpha} \right\} \left(k_{t-1}^g \right)^{\theta_k} \tag{D22}$$

$$\widetilde{\omega_t}^k = \frac{p_t^H}{p_t} y_t^H - mc_t y_t^H - \frac{\phi^p}{2} \left(\frac{p_t^H}{p_{t-1}^H \pi^H} - 1 \right)^2 \frac{p_t^H}{p_t} y_t^H \tag{D23}$$

$$Q_{t-1} \frac{S_t p_t^*}{p_t} \frac{p_{t-1}^*}{p_t^*} f_{t-1}^g + R_{t-1} \frac{p_{t-1}}{p_t} b_{t-1}^k + \frac{p_t^H}{p_t} \overline{g}_t^c + \frac{p_t^H}{p_t} \overline{g}_t^i + \overline{g}_t^w + \frac{p_{t-1}}{p_t} \left[m_{t-1}^k + \frac{v^w}{v^k} m_{t-1}^w + \frac{v^b}{v^k} m_{t-1}^b \right] =$$
$$= \left[m_t^k + \frac{v^w}{v^k} m_t^w + \frac{v^b}{v^k} m_t^b \right] + \tau_t^c \left[\frac{p_t^H}{p_t} \left(c_t^{k,H} + \frac{v^w}{v^k} c_t^{w,H} + \frac{v^b}{v^k} c_t^{b,H} \right) + \frac{p_t^F}{p_t} \left(c_t^{k,F} + \frac{v^w}{v^k} c_t^{w,F} + \frac{v^b}{v^k} c_t^{b,F} \right) \right] +$$
$$+ \tau_t^k \left[r_t^k \frac{p_t^H}{p_t} k_{t-1}^k + \widetilde{\omega_t}^k \right] + \tau_t^n \left[w_t^k n_t^k + \frac{v^w}{v^k} w_t^w n_t^w + \overline{g}_t^w \right] + \left[\tau_t^{l,k} + \frac{v^w}{v^k} \tau_t^{l,w} + \frac{v^b}{v^k} \tau_t^{l,b} \right] + b_t^k + S_t \frac{p_t^*}{p_t} f_t^g \tag{D24}$$

$$\frac{p_t^H}{p_t} \left\{ \left[c_t^{k,H} + \frac{v^w}{v^k} c_t^{w,H} + \frac{v^b}{v^k} c_t^{b,H} \right] + x_t^k + \overline{g}_t^c + \overline{g}_t^i - y_t^H \left[1 - \frac{\phi^p}{2} \left(\frac{p_t^H}{p_{t-1}^H \pi^H} - 1 \right)^2 \right] \right\} +$$
$$+ \frac{p_t^F}{p_t} \left[c_t^{k,F} + \frac{v^w}{v^k} c_t^{w,F} + \frac{v^b}{v^k} c_t^{b,F} \right] = S_t \frac{p_t^*}{p_t} (f_t^g - f_t^k) - Q_{t-1} \frac{S_t p_t^*}{p_t} \frac{p_{t-1}^*}{p_t^*} (f_{t-1}^g - f_{t-1}^k) + \widetilde{\pi}_t^k \tag{D25}$$

$$p_t = (p_t^H)^v (p_t^F)^{1-v} \tag{D26}$$

$$p_t^F = S_t p_t^{H*} \tag{D27}$$

$$(1-\phi)\frac{p_t^H}{p_t}y_t^H + \phi mc_t y_t^H - \phi^p\left[\frac{p_t^H}{p_{t-1}^H\pi^H}-1\right]\frac{p_t^H}{p_t}\frac{y_t^H p_t^H}{p_{t-1}^H\pi^H} =$$

$$\beta\phi^p\left[\left(\frac{p_t}{p_{t+1}}\right)\left(\frac{1+\tau_t^c}{1+\tau_{t+1}^c}\right)\left(\frac{c_{t+1}^k}{c_t^k}\right)^{-\sigma}\right]\left[1-\frac{p_{t+1}^H}{p_t^H\pi^H}\right]\frac{p_{t+1}^H}{p_t^H\pi^H}\frac{p_{t+1}^H}{p_{t+1}}y_{t+1}^H \tag{D28}$$

$$\widetilde{\pi}_t^k \equiv Q_{t-1}^*\left[\frac{p_{t-1}}{p_t}(f_{t-1}^{g*}-f_{t-1}^{k*}) - \frac{\psi}{2}\frac{p_{t-1}^H}{p_t^H}\frac{p_t^H}{p_t}(f_{t-1}^{g*}-f_{t-1}^{k*})^2\right] - Q_{t-1}S_t\frac{p_t^*}{p_t}\frac{p_{t-1}^*}{p_t^*}\left(f_{t-1}^k - f_{t-1}^g\right) \tag{D29}$$

$$Q_{t-1}^* = \frac{Q_{t-1}\frac{S_t}{S_{t-1}}}{1-\psi\frac{p_{t-1}^H}{p_{t-1}}(f_{t-1}^{g*}-f_{t-1}^{k*})} \tag{D30}$$

$$v_t \equiv Q_{t-1}^*\frac{\psi}{2}\frac{p_{t-1}^H}{p_t^H}(f_{t-1}^{g*}-f_{t-1}^{k*})^2 \tag{D31}$$

$$y_t^g = A\left(v^k\overline{g}_t^c\right)^{\theta_g}\left(v^b\right)^{1-\theta_g} \tag{D32}$$

$$\overline{g}_t^i = k_t^g - (1-\delta^g)k_{t-1}^g + \frac{\xi^g}{2}\left(\frac{k_t^g}{k_{t-1}^g}-1\right)^2 k_{t-1}^g \tag{D33}$$

$$x_n^*(n_t^{k*})^{\eta*}(c_t^{k*})^{\sigma*} = \frac{(1-\tau_t^{n*})}{(1+\tau_t^{c*})}w_t^{k*} \tag{D34}$$

$$\frac{(c_t^{k*})^{-\sigma*}}{(1+\tau_t^{c*})}\frac{1}{S_t}\frac{p_t}{p_t^*} = \beta^*\frac{(c_{t+1}^{k*})^{-\sigma*}}{(1+\tau_{t+1}^{c*})}Q_t^*\frac{1}{S_{t+1}}\frac{p_{t+1}}{p_{t+1}^*}\frac{p_t}{p_{t+1}} \tag{D35}$$

$$\frac{(c_t^{k*})^{-\sigma*}}{(1+\tau_t^{c*})}\frac{p_t^{H*}}{p_t^*}\left[1+\xi^*\left(\frac{k_t^{k*}}{k_{t-1}^{k*}}-1\right)\right] = \beta^*\frac{(c_{t+1}^{k*})^{-\sigma*}}{(1+\tau_{t+1}^{c*})}\frac{p_{t+1}^{H*}}{p_{t+1}^*}\times$$

$$\times\left[(1-\delta^*)+(1-\tau_{t+1}^{k*})r_{t+1}^{k*} - \frac{\xi^*}{2}\left(\frac{k_{t+1}^{k*}}{k_t^{k*}}-1\right)^2 + \xi^*\left(\frac{k_{t+1}^{k*}}{k_t^{k*}}-1\right)\frac{k_{t+1}^{k*}}{k_t^{k*}}\right] \tag{D36}$$

$$x_m^*(m_t^{k*})^{-\mu*} = \frac{(c_t^{k*})^{-\sigma*}}{(1+\tau_t^{c*})} - \beta^*\frac{p_t^*}{p_{t+1}^*}\frac{(c_{t+1}^{k*})^{-\sigma*}}{(1+\tau_{t+1}^{c*})} \tag{D37}$$

$$\frac{(c_t^{k*})^{-\sigma*}}{(1+\tau_t^{c*})} = \beta^* R_t^* \frac{p_t^*}{p_{t+1}^*} \frac{(c_{t+1}^{k*})^{-\sigma*}}{(1+\tau_{t+1}^{c*})} \tag{D38}$$

$$k_t^{k*} = (1-\delta^*)k_{t-1}^{k*} + x_t^{k*} - \frac{\xi^*}{2}\left(\frac{k_t^{k*}}{k_{t-1}^{k*}} - 1\right)^2 k_{t-1}^{k*} \tag{D39}$$

$$\frac{c_t^{k,H*}}{c_t^{k,F*}} = \frac{v^*}{1-v^*}\frac{p_t^{F*}}{p_t^{H*}} \tag{D40}$$

$$c_t^{k*} = \frac{\left(c_t^{k,H*}\right)^{v*}\left(c_t^{k,F*}\right)^{1-v*}}{v^{*v^*}(1-v^*)^{1-v^*}} \tag{D41}$$

$$\left[\left(c_t^{k,H*} + \frac{v^{w*}}{v^{k*}}c_t^{w,H*} + \frac{v^{b*}}{v^{k*}}c_t^{b,H*}\right) + x_t^{k*} + \overline{g}_t^{c*} + \overline{g}_t^{i*} + \left(c_t^{k,F} + \frac{v^w}{v^k}c_t^{w,F} + \frac{v^b}{v^k}c_t^{b,F}\right)\right] = \\ = y_t^{H*}\left[1 - \frac{\phi^{p*}}{2}\left(\frac{p_t^{H*}}{p_{t-1}^{H*}\pi^{H*}} - 1\right)^2\right] \tag{D42}$$

$$x_n^*(n_t^{w*})^{\eta*}(c_t^{w*})^{\sigma*} = \frac{(1-\tau_t^{n*})}{(1+\tau_t^{c*})}w_t^{w*} \tag{D43}$$

$$x_m^*(m_t^{w*})^{-\mu*} = \frac{(c_t^{w*})^{-\sigma*}}{1+\tau_t^{c*}} - \beta^*\frac{p_t^*}{p_{t+1}^*}\left[\frac{(c_{t+1}^{w*})^{-\sigma*}}{1+\tau_{t+1}^{c*}}\right] \tag{D44}$$

$$\frac{c_t^{w,H*}}{c_t^{w,F*}} = \frac{v^*}{1-v^*}\frac{p_t^{F*}}{p_t^{H*}} \tag{D45}$$

$$c_t^{w*} = \frac{\left(c_t^{w,H*}\right)^{v*}\left(c_t^{w,F*}\right)^{1-v*}}{v^{*v^*}(1-v^*)^{1-v^*}} \tag{D46}$$

$$(1+\tau_t^{c*})c_t^{w*} + m_t^{w*} = \frac{p_{t-1}^*}{p_t^*}m_{t-1}^{w*} + (1-\tau_t^{n*})w_t^{w*}n_t^{w*} - \tau_t^{l,w*} \tag{D47}$$

$$x_m^*(m_t^{b*})^{-\mu*} = \frac{(c_t^{b*})^{-\sigma*}}{1+\tau_t^{c*}} - \beta^*\frac{p_t^*}{p_{t+1}^*}\left[\frac{(c_{t+1}^{b*})^{-\sigma*}}{1+\tau_{t+1}^{c*}}\right] \tag{D48}$$

$$\frac{c_t^{b,H*}}{c_t^{b,F*}} = \frac{v^*}{1-v^*}\frac{p_t^{F*}}{p_t^{H*}} \tag{D49}$$

$$c_t^{b*} = \frac{\left(c_t^{b,H*}\right)^{v*}\left(c_t^{b,F*}\right)^{1-v*}}{v^{*v^*}(1-v^*)^{1-v^*}} \tag{D50}$$

$$(1+\tau_t^{c*})c_t^{b*} + m_t^{b*} = \frac{p_{t-1}^*}{p_t^*}m_{t-1}^{b*} + (1-\tau_t^{n*})\frac{v^{k*}}{v^{b*}}\overline{g}_t^{w*} - \tau_t^{l,b*} \tag{D51}$$

$$\frac{p_t^{H*}}{p_t^*}r_t^{k*}k_{t-1}^{k*} = mc_t^*\alpha^* y_t^{H*} \tag{D52}$$

$$w_t^{k*}n_t^{k*} = mc_t^*\theta^*(1-\alpha^*)y_t^{H*} \tag{D53}$$

$$\frac{v^{w*}}{v^{k*}}w_t^{w*}n_t^{w*} = mc_t^*(1-\theta^*)(1-\alpha^*)y_t^{H*} \tag{D54}$$

$$y_t^{H*} = A_t^*\left\{[k_{t-1}^{k*}]^{\alpha^*}\left[\{n_t^{k*}\}^{\theta^*} \times \left\{\frac{v^{w*}}{v^{k*}}n_t^{w*}\right\}^{1-\theta^*}\right]^{1-\alpha^*}\right\}\left(k_{t-1}^{g*}\right)^{\theta_k^*} \tag{D55}$$

$$\widetilde{\omega_t}^{k*} = \frac{p_t^{H*}}{p_t^*}y_t^{H*} - mc_t^* y_t^{H*} - \frac{\phi^{p*}}{2}\left(\frac{p_t^{H*}}{p_{t-1}^{H*}\pi^{H*}} - 1\right)^2 \frac{p_t^{H*}}{p_t^*}y_t^{H*} \tag{D56}$$

$$Q_{t-1}^*\frac{p_t}{S_t p_t^*}\frac{p_{t-1}}{p_t}f_{t-1}^{g*} + R_{t-1}^*\frac{p_{t-1}^*}{p_t^*}b_{t-1}^{k*} + \frac{p_t^{H*}}{p_t^*}\overline{g}_t^{c*} + \frac{p_t^{H*}}{p_t^*}\overline{g}_t^{i*} + \overline{g}_t^{w*} + \frac{p_{t-1}^*}{p_t^*}\left[m_{t-1}^{k*} + \frac{v^{w*}}{v^{k*}}m_{t-1}^{w*} + \frac{v^{b*}}{v^{k*}}m_{t-1}^{b*}\right] = \tag{D57}$$

$$= \left[m_t^{k*} + \frac{v^{w*}}{v^{k*}}m_t^{w*} + \frac{v^{b*}}{v^{k*}}m_t^{b*}\right] + \tau_t^{n*}\left[w_t^{k*}n_t^{k*} + \frac{v^{w*}}{v^{k*}}w_t^{w*}n_t^{w*} + \frac{v^{b*}}{v^{k*}}w_t^{b*}n_t^{b*}\right] +$$

$$+\tau_t^{c*}\left[\frac{p_t^{H*}}{p_t^*}\left(c_t^{k,H*} + \frac{v^{w*}}{v^{k*}}c_t^{w,H*} + \frac{v^{b*}}{v^{k*}}c_t^{b,H*}\right) + \frac{p_t^{F*}}{p_t^*}\left(c_t^{k,F*} + \frac{v^{w*}}{v^{k*}}c_t^{w,F*} + \overline{g}_t^{w*}\right)\right] +$$

$$+\tau_t^{k*}\left[r_t^{k*}\frac{p_t^{H*}}{p_t^*}k_{t-1}^{k*} + \widetilde{\omega_t}^{k*}\right] + \left[\tau_t^{l,k*} + \frac{v^{w*}}{v^{k*}}\tau_t^{l,w*} + \frac{v^{b*}}{v^{k*}}\tau_t^{l,b*}\right] + b_t^{k*} + \frac{1}{S_t}\frac{p_t}{p_t^*}f_t^{g*}$$

$$\frac{p_t^{H*}}{p_t^*}\left\{\left[c_t^{k,H*}+\frac{v^{w*}}{v^{k*}}c_t^{w,H*}+\frac{v^{b*}}{v^{k*}}c_t^{b,H*}\right]+x_t^{k*}+\overline{g}_t^{c*}+\overline{g}_t^{i*}-y_t^{H*}\left[1-\frac{\phi^{p*}}{2}\left(\frac{p_t^{H*}}{p_{t-1}^{H*}\pi^{H*}}-1\right)^2\right]\right\}+$$

$$+\frac{p_t^{F*}}{p_t^*}\left[c_t^{k,F*}+\frac{v^{w*}}{v^{k*}}c_t^{w,F*}+\frac{v^{b*}}{v^{k*}}c_t^{b,F*}\right]=\frac{1}{S_t}\frac{p_t}{p_t^*}(f_t^{g*}-f_t^{k*})-Q_{t-1}^*\frac{p_t}{S_t p_t^*}\frac{p_{t-1}}{p_t}(f_{t-1}^{g*}-f_{t-1}^{k*}) \tag{D58}$$

$$p_t^*=(p_t^{H*})^{v*}(p_t^{F*})^{1-v*} \tag{D59}$$

$$p_t^{F*}=\frac{p_t^H}{S_t} \tag{D60}$$

$$(1-\phi^*)\frac{p_t^{H*}}{p_t^*}y_t^{H*}+\phi^* mc_t^* y_t^{H*}-\phi^{p*}\left[\frac{p_t^{H*}}{p_{t-1}^{H*}\pi^{H*}}-1\right]\frac{p_t^{H*}}{p_t^*}\frac{y_t^{H*}p_t^{H*}}{p_{t-1}^{H*}\pi^{H*}}=$$

$$\beta^*\phi^{p*}\left[\left(\frac{p_t^*}{p_{t+1}^*}\right)\left(\frac{1+\tau_t^{c*}}{1+\tau_{t+1}^{c*}}\right)\left(\frac{c_{t+1}^{k*}}{c_t^{k*}}\right)^{-\sigma*}\right]\left[1-\frac{p_{t+1}^{H*}}{p_t^{H*}\pi^{H*}}\right]\frac{p_{t+1}^{H*}}{p_t^{H*}\pi^{H*}}\frac{p_{t+1}^{H*}}{p_{t+1}^*}y_{t+1}^{H*} \tag{D61}$$

$$y_t^{g*}=A^*\left(v^{k*}\overline{g}_t^{c*}\right)^{\theta_g^*}\left(v^{b*}\right)^{1-\theta_g^*} \tag{D62}$$

$$\overline{g}_t^{i*}=k_t^{g*}-(1-\delta^{g*})k_{t-1}^{g*}+\frac{\xi^{g*}}{2}\left(\frac{k_t^{g*}}{k_{t-1}^{g*}}-1\right)^2 k_{t-1}^{g*} \tag{D63}$$

Thus, we have a system of 63 equations [(D1)-(D63)] in the 63 following endogeneous variables

$$[c_t^k, c_t^{k,H}, c_t^{k,F}, c_t^w, c_t^{w,H}, c_t^{w,F}, c_t^b, c_t^{b,H}, c_t^{b,F}, n_t^k, w_t^k, n_t^w, w_t^w, m_t^k, m_t^w, m_t^b, r_t^k, k_t^k, k_t^g, x_t^k, b_t^k,$$

$$f_t^k, Q_t, y_t^g, y_t^H, mc_t, \widetilde{\omega}_t^k, v_t, \widetilde{\pi}_t^k, p_t, p_t^H, p_t^F, c_t^{k*}, c_t^{k,H*}, c_t^{k,F*}, c_t^{w*}, c_t^{w,H*}, c_t^{w,F*}, c_t^{b*}, c_t^{b,H*},$$

$$c_t^{b,F*}, n_t^{k*}, w_t^{k*}, n_t^{w*}, w_t^{w*}, m_t^{k*}, m_t^{w*}, m_t^{b*}, r_t^{k*}, k_t^{k*}, k_t^{g*}, x_t^{k*}, b_t^{k*}, R_t^*, f_t^{k*}, Q_t^*, y_t^{g*}, y_t^{H*},$$

$$mc_t^*, \widetilde{\omega}_t^{k*}, p_t^*, p_t^{H*}, p_t^{F*}]_{t=0}^{\infty}$$

Conclusively, the Decentralized Equilibrium is a sequence of

$$[c_t^k, c_t^{k,H}, c_t^{k,F}, c_t^w, c_t^{w,H}, c_t^{w,F}, c_t^b, c_t^{b,H}, c_t^{b,F}, n_t^k, w_t^k, n_t^w, w_t^w, m_t^k, m_t^w, m_t^b, r_t^k, k_t^k, k_t^g, x_t^k, b_t^k,$$

$$f_t^k, Q_t, y_t^g, y_t^H, mc_t, \widetilde{\omega_t}^k, v_t, \widetilde{\pi_t}^k, p_t, p_t^H, p_t^F, c_t^{k*}, c_t^{k,H*}, c_t^{k,F*}, c_t^{w*}, c_t^{w,H*}, c_t^{w,F*}, c_t^{b*}, c_t^{b,H*},$$

$$c_t^{b,F*}, n_t^{k*}, w_t^{k*}, n_t^{w*}, w_t^{w*}, m_t^{k*}, m_t^{w*}, m_t^{b*}, r_t^{k*}, k_t^{k*}, k_t^{g*}, x_t^{k*}, b_t^{k*}, R_t^*, f_t^{k*}, Q_t^*, y_t^{g*}, y_t^{H*},$$

$$mc_t^*, \widetilde{\omega_t}^{k*}, p_t^*, p_t^{H*}, p_t^{F*}]_{t=0}^{\infty}$$

satisfying the equations [(D1)-(D63)], given:

a) exogenous variables $[S_t, A_t, A_t^*,]_{t=0}^{\infty}$,

b) initial conditions for state variables $[k_{-1}^k, k_{-1}^g, f_{-1}^k, b_{-1}, R_{-1}, m_{-1}^k, m_{-1}^w, m_{-1}^b, k_{-1}^{k*}, k_{-1}^{g*}, f_{-1}^{k*}, b_{-1}^*, R_{-1}^*, m_{-1}^{k*}, m_{-1}^{w*}, m_{-1}^{b*}]$,

c) policy.

H.3 Transformed variables

We first express prices in rate form. We define 6 new variables, which are the gross domestic CPI inflation rate $\pi_t \equiv \frac{p_t}{p_{t-1}}$, the gross foreign CPI inflation rate,$\pi_t^* \equiv \frac{p_t^*}{p_{t-1}^*}$,the gross domestic goods inflation rate for the domestic economy $\pi_t^H \equiv \frac{p_t^H}{p_{t-1}^H}$, the gross domestic goods inflation rate for the foreign economy $\pi_t^{H*} \equiv \frac{p_t^{H*}}{p_{t-1}^{H*}}$, the gross rate of exchange rate depreciation,$\epsilon_t \equiv \frac{S_t}{S_{t-1}}$ and the terms of trade $TT_t \equiv \frac{p_t^F}{p_t^H} = \frac{S_t p_t^{H*}}{p_t^H}$.[37] The equation ($D60$) implies that $\frac{p_t^{F*}}{p_t^{H*}} = \frac{1}{TT_t}$. Plugging p_t^{F*} of the latter equation into other equations, this unknown eliminates and the system of equations is reduced by one equation. Yet, as we have shown in Appendix E for the domestic economy, we can replace f_t^g and b_t^k with $\frac{p_t}{S_t p_t^*}(1-\lambda_t)d_t$ and $\lambda_t d_t$ respectively. Similarly, we have shown in the same Appendix for the foreign economy that we can replace f_t^{g*}

[37]Thus, $\frac{TT_t}{TT_{t-1}} = \frac{\frac{S_t}{S_{t-1}} \frac{p_t^{H*}}{p_{t-1}^{H*}}}{\frac{p_t^H}{p_{t-1}^H}} = \frac{\epsilon_t \pi_t^{H*}}{\pi_t^H}$

and b_t^{k*} with $\frac{S_t p_t^*}{p_t}(1-\lambda_t^*)d_t^*$ and $\lambda_t^* d_t^*$ respectively. Hence, in what follows, we use the 10 new variables π_t,π_t^*,π_t^H, π_t^{H*}, ϵ_t,TT_t, λ_t, d_t,λ_t^*, d_t^*, instead of the 11 variables p_t,p_t^*,p_t^H,p_t^{H*}, S_t,p_t^F,p_t^{F*}, f_t^g, b_t^k,f_t^{g*}, b_t^{k*}, meaning that the variables are reduced by one, as it happens with the number of equations.

Also, for convenience and comparison with the data, we express fiscal and public finance variables as shares of real GDP,$\frac{p_t^H}{p_t}y_t^H$. In particular, using the definitions above, the total public spending on goods and services purchased from the private sector in real terms divided by the number of capitalists, $\frac{p_t^H}{p_t}\overline{g}_t^c$, can be written as ratio of real GDP, as $\frac{p_t^H}{p_t}\overline{g}_t^c = s_t^g \frac{p_t^H}{p_t} y_t^H$, where s_t^g denotes the output share of government spending on private goods and services. The total public investment in infrastructure in real terms divided by the number of capitalists,$\frac{p_t^H}{p_t}\overline{g}_t^i$, can be written as ratio of real GDP, as $\frac{p_t^H}{p_t}\overline{g}_t^i = s_t^i \frac{p_t^H}{p_t} y_t^H$, where s_t^i denotes the output share of public investment. The total public wage bill in real terms divided by the number of capitalists, $\overline{g}_t^w$, can be written as ratio of real GDP, as $\overline{g}_t^w = s_t^w \frac{p_t^H}{p_t} y_t^H$, where s_t^w denotes the output share of total public wage bill. The total lump-sum taxes/transfers in real terms,$\left[\tau_t^{l,k} + \frac{v^w}{v^k}\tau_t^{l,w} + \frac{v^b}{v^k}\tau_t^{l,b}\right]$, can be written as ratio of real GDP, as $\left[\tau_t^{l,k} + \frac{v^w}{v^k}\tau_t^{l,w} + \frac{v^b}{v^k}\tau_t^{l,b}\right] = s_t^l \frac{p_t^H}{p_t} y_t^H$, where as s_t^l are defined the lump-sum taxes/-transfers as share of output. Assuming that the government imposes/gives to the class of capitalists, private workers and public employees a percentage of the total lump-sum taxes/transfers according to their shares in the population. From the above, it arises that $\tau_t^{l,k} = \tau_t^{l,w} = \tau_t^{l,b} = v^k s_t^l \frac{p_t^H}{p_t} y_t^H$. We work similarly for the foreign country.

Finally, given the above, notice that we make use of the following equations:

$$TT_t = \frac{p_t^F}{p_t^H} = S_t \frac{p_t^{H*}}{p_t^H} = \frac{p_t^{H*}}{p_t^{F*}}$$

$$\frac{p_t^H}{p_t} = TT_t^{v-1}$$

$$\frac{p_t^{H*}}{p_t^*} = TT_t^{1-v^*}$$

$$\frac{p_t^F}{p_t} = TT_t^v$$

$$\frac{p_t^{F*}}{p_t^*} = TT_t^{-v^*}$$

$$S_t \frac{p_t^*}{p_t} = TT_t^{v+v^*-1}$$

$$\overline{g}_t^c = s_t^g y_t^H$$

$$\overline{g}_t^i = s_t^i y_t^H$$

$$\overline{g}_t^w = s_t^w TT_t^{v-1} y_t^H$$

$$\tau_t^{l,k} = \tau_t^{l,w} = \tau_t^{l,b} = v^k s_t^l TT_t^{v-1} y_t^H$$

H.4 Final equations

Using the above, we now present the final non-linear stochastic system (given feedback policy coefficients).

The domestic country is summarized by the following equations:

$$x_n (n_t^k)^\eta (c_t^k)^\sigma = \frac{(1-\tau_t^n)}{(1+\tau_t^c)} w_t^k \tag{D1'}$$

$$\frac{(c_t^k)^{-\sigma}}{(1+\tau_t^c)} TT_t^{v+v^*-1} = \beta \frac{(c_{t+1}^k)^{-\sigma}}{(1+\tau_{t+1}^c)} Q_t TT_{t+1}^{v+v^*-1} \frac{1}{\pi_{t+1}^*} \tag{D2'}$$

$$\frac{(c_t^k)^{-\sigma}}{(1+\tau_t^c)} TT_t^{v-1} \left[1+\xi\left(\frac{k_t^k}{k_{t-1}^k}-1\right)\right] = \beta \frac{(c_{t+1}^k)^{-\sigma}}{(1+\tau_{t+1}^c)} TT_{t+1}^{v-1} \times$$
$$\times \left[(1-\delta)+(1-\tau_{t+1}^k) r_{t+1}^k - \frac{\xi}{2}\left(\frac{k_{t+1}^k}{k_t^k}-1\right)^2 + \xi\left(\frac{k_{t+1}^k}{k_t^k}-1\right)\frac{k_{t+1}^k}{k_t^k}\right] \tag{D3'}$$

$$x_m (m_t^k)^{-\mu} = \frac{(c_t^k)^{-\sigma}}{(1+\tau_t^c)} - \beta \frac{1}{\pi_{t+1}} \frac{(c_{t+1}^k)^{-\sigma}}{(1+\tau_{t+1}^c)} \tag{D4'}$$

$$\frac{(c_t^k)^{-\sigma}}{(1+\tau_t^c)} = \beta R_t \frac{1}{\pi_{t+1}} \frac{(c_{t+1}^k)^{-\sigma}}{(1+\tau_{t+1}^c)} \tag{D5'}$$

$$k_t^k = (1-\delta)k_{t-1}^k + x_t^k - \frac{\xi}{2}\left(\frac{k_t^k}{k_{t-1}^k} - 1\right)^2 k_{t-1}^k \tag{D6'}$$

$$\frac{c_t^{k,H}}{c_t^{k,F}} = \frac{v}{1-v} TT_t \tag{D7'}$$

$$c_t^k = \frac{\left(c_t^{k,H}\right)^v \left(c_t^{k,F}\right)^{1-v}}{v^v(1-v)^{1-v}} \tag{D8'}$$

$$\left[\left(c_t^{k,H} + \frac{v^w}{v^k}c_t^{w,H} + \frac{v^b}{v^k}c_t^{b,H}\right) + x_t^k + s_t^g y_t^H + s_t^i y_t^H + \upsilon_t + \left(c_t^{k,F*} + \frac{v^{w*}}{v^{k*}}c_t^{w,F*} + \frac{v^{b*}}{v^{k*}}c_t^{b,F*}\right)\right] = \tag{D9'}$$

$$= y_t^H\left[1 - \frac{\phi^p}{2}\left(\frac{\pi_t^H}{\pi^H} - 1\right)^2\right]$$

$$x_n(n_t^w)^\eta(c_t^w)^\sigma = \frac{(1-\tau_t^n)}{(1+\tau_t^c)} w_t^w \tag{D10'}$$

$$x_m(m_t^w)^{-\mu} = \frac{(c_t^w)^{-\sigma}}{1+\tau_t^c} - \beta\frac{1}{\pi_{t+1}}\left[\frac{(c_{t+1}^w)^{-\sigma}}{1+\tau_{t+1}^c}\right] \tag{D11'}$$

$$\frac{c_t^{w,H}}{c_t^{w,F}} = \frac{v}{1-v} TT_t \tag{D12'}$$

$$c_t^w = \frac{\left(c_t^{w,H}\right)^v \left(c_t^{w,F}\right)^{1-v}}{v^v(1-v)^{1-v}} \tag{D13'}$$

$$(1+\tau_t^c)c_t^w + m_t^w = \frac{1}{\pi_t}m_{t-1}^w + (1-\tau_t^n)w_t^w n_t^w - v^k s_t^l y_t^H TT_t^{v-1} \tag{D14'}$$

$$x_m(m_t^b)^{-\mu} = \frac{(c_t^b)^{-\sigma}}{1+\tau_t^c} - \beta\frac{1}{\pi_{t+1}}\left[\frac{(c_{t+1}^b)^{-\sigma}}{1+\tau_{t+1}^c}\right] \tag{D15'}$$

$$\frac{c_t^{b,H}}{c_t^{b,F}} = \frac{v}{1-v} TT_t \tag{D16'}$$

$$c_t^b = \frac{\left(c_t^{b,H}\right)^v \left(c_t^{b,F}\right)^{1-v}}{v^v (1-v)^{1-v}} \tag{D17'}$$

$$(1+\tau_t^c)c_t^b + m_t^b = \frac{1}{\pi_t} m_{t-1}^b + (1-\tau_t^n)\frac{v^k}{v^b} s_t^w TT_t^{v-1} y_t^H - v^k s_t^l y_t^H TT_t^{v-1} \tag{D18'}$$

$$TT_t^{v-1} r_t^k k_{t-1}^k = mc_t \alpha y_t^H \tag{D19'}$$

$$w_t^k n_t^k = mc_t \theta (1-\alpha) y_t^H \tag{D20'}$$

$$\frac{v^w}{v^k} w_t^w n_t^w = mc_t (1-\theta)(1-\alpha) y_t^H \tag{D21'}$$

$$y_t^H = A_t \left\{ [k_{t-1}^k]^\alpha \left[\{n_t^k\}^\theta \times \left\{ \frac{v^w}{v^k} n_t^w \right\}^{1-\theta} \right]^{1-\alpha} \right\} \left(k_{t-1}^g\right)^{\theta_k} \tag{D22'}$$

$$\widetilde{\omega_t}^k = TT_t^{v-1} y_t^H - mc_t y_t^H - \frac{\phi^p}{2} \left(\frac{\pi_t^H}{\pi^H} - 1 \right)^2 TT_t^{v-1} y_t^H \tag{D23'}$$

$$\begin{aligned} & Q_{t-1} TT_t^{v+v^*-1} \frac{1}{\pi_t^*} TT_{t-1}^{1-v-v^*} [1-\lambda_{t-1}] d_{t-1} + R_{t-1} \frac{1}{\pi_t} \lambda_{t-1} d_{t-1} + s_t^g y_t^H TT_t^{v-1} + \\ & + s_t^i y_t^H TT_t^{v-1} + s_t^w y_t^H TT_t^{v-1} + \frac{1}{\pi_t} \left(m_{t-1}^k + \frac{v^w}{v^k} m_{t-1}^w + \frac{v^b}{v^k} m_{t-1}^b \right) = \\ & = \left(m_t^k + \frac{v^w}{v^k} m_t^w + \frac{v^b}{v^k} m_t^b \right) + \tau_t^c \left[TT_t^{v-1} \left(c_t^{k,H} + \frac{v^w}{v^k} c_t^{w,H} + \frac{v^b}{v^k} c_t^{b,H} \right) + \right. \\ & \left. + TT_t^v \left(c_t^{k,F} + \frac{v^w}{v^k} c_t^{w,F} + \frac{v^b}{v^k} c_t^{b,F} \right) \right] + \tau_t^k \left[r_t^k TT_t^{v-1} k_{t-1}^k + \widetilde{\omega_t}^k \right] + \\ & + \tau_t^n \left[w_t^k n_t^k + \frac{v^w}{v^k} w_t^w n_t^w + s_t^w y_t^H TT_t^{v-1} \right] + s_t^l y_t^H TT_t^{v-1} + d_t \end{aligned} \tag{D24'}$$

$$TT_t^{v-1}\left\{\left[c_t^{k,H}+\frac{v^w}{v^k}c_t^{w,H}+\frac{v^b}{v^k}c_t^{b,H}\right]+x_t^k+s_t^g y_t^H+s_t^i y_t^H-y_t^H\left[1-\frac{\phi^p}{2}\left(\frac{\pi_t^H}{\pi^H}-1\right)^2\right]\right\}+ \tag{D25'}$$

$$+TT_t^v\left[c_t^{k,F}+\frac{v^w}{v^k}c_t^{w,F}+\frac{v^b}{v^k}c_t^{b,F}\right]=TT_t^{v+v^*-1}(TT_t^{1-v-v^*}[1-\lambda_t]d_t-f_t^k)-$$

$$-Q_{t-1}TT_t^{v+v^*-1}\frac{1}{\pi_t^*}(TT_{t-1}^{1-v-v^*}[1-\lambda_{t-1}]d_{t-1}-f_{t-1}^k)+\widetilde{\pi}_t^k$$

$$\frac{\pi_t}{\pi_t^H}=\left(\frac{TT_t}{TT_{t-1}}\right)^{1-v} \tag{D26'}$$

$$\frac{TT_t}{TT_{t-1}}=\epsilon_t\frac{\pi_t^{H*}}{\pi_t^H} \tag{D27'}$$

$$(1-\phi)TT_t^{v-1}y_t^H+\phi mc_t y_t^H-\phi^p\left[\frac{\pi_t^H}{\pi^H}-1\right]TT_t^{v-1}\frac{y_t^H\pi_t^H}{\pi^H}=$$

$$\beta\phi^p\left[\left(\frac{1}{\pi_{t+1}}\right)\left(\frac{1+\tau_t^c}{1+\tau_{t+1}^c}\right)\left(\frac{c_{t+1}^k}{c_t^k}\right)^{-\sigma}\right]\left[1-\frac{\pi_{t+1}^H}{\pi^H}\right]\frac{\pi_{t+1}^H}{\pi^H}TT_{t+1}^{v-1}y_{t+1}^H \tag{D28'}$$

$$\widetilde{\pi}_t^k\equiv Q_{t-1}^*\left[\frac{1}{\pi_t}([1-\lambda_{t-1}^*]d_{t-1}^*TT_{t-1}^{v+v^*-1}-f_{t-1}^{k*})-\frac{\psi}{2}\frac{1}{\pi_t^H}TT_t^{v-1}([1-\lambda_{t-1}^*]d_{t-1}^*TT_{t-1}^{v+v^*-1}-f_{t-1}^{k*})^2\right]$$

$$-Q_{t-1}TT_t^{v+v^*-1}\frac{1}{\pi_t^*}\left(f_{t-1}^k-TT_{t-1}^{1-v-v^*}[1-\lambda_{t-1}]d_{t-1}\right) \tag{D29'}$$

$$Q_{t-1}^*=\frac{Q_{t-1}\epsilon_t}{1-\psi TT_{t-1}^{v-1}([1-\lambda_{t-1}^*]d_{t-1}^*TT_{t-1}^{v+v^*-1}-f_{t-1}^{k*})} \tag{D30'}$$

$$\upsilon_t\equiv Q_{t-1}^*\frac{\psi}{2}\frac{1}{\pi_t^H}([1-\lambda_{t-1}^*]d_{t-1}^*TT_{t-1}^{v+v^*-1}-f_{t-1}^{k*})^2 \tag{D31'}$$

$$y_t^g=A\left(v^k s_t^g y_t^H\right)^{\theta_g}\left(v^b\right)^{1-\theta_g} \tag{D32'}$$

$$s_t^i y_t^H=k_t^g-(1-\delta^g)k_{t-1}^g+\frac{\xi^g}{2}\left(\frac{k_t^g}{k_{t-1}^g}-1\right)^2 k_{t-1}^g \tag{D33'}$$

Next, the foreign country is summarized by the following equations:

$$x_n^*(n_t^{k*})^{\eta*}(c_t^{k*})^{\sigma*} = \frac{(1-\tau_t^{n*})}{(1+\tau_t^{c*})}w_t^{k*} \tag{D34'}$$

$$\frac{(c_t^{k*})^{-\sigma*}}{(1+\tau_t^{c*})}TT_t^{1-v-v^*} = \beta^*\frac{(c_{t+1}^{k*})^{-\sigma*}}{(1+\tau_{t+1}^{c*})}Q_t^*TT_{t+1}^{1-v-v^*}\frac{1}{\pi_{t+1}} \tag{D35'}$$

$$\frac{(c_t^{k*})^{-\sigma*}}{(1+\tau_t^{c*})}TT_t^{1-v^*}\left[1+\xi^*\left(\frac{k_t^{k*}}{k_{t-1}^{k*}}-1\right)\right] = \beta^*\frac{(c_{t+1}^{k*})^{-\sigma*}}{(1+\tau_{t+1}^{c*})}TT_{t+1}^{1-v^*}\times$$

$$\times\left[(1-\delta^*)+(1-\tau_{t+1}^{k*})r_{t+1}^{k*}-\frac{\xi^*}{2}\left(\frac{k_{t+1}^{k*}}{k_t^{k*}}-1\right)^2+\xi^*\left(\frac{k_{t+1}^{k*}}{k_t^{k*}}-1\right)\frac{k_{t+1}^{k*}}{k_t^{k*}}\right] \tag{D36'}$$

$$x_m^*(m_t^{k*})^{-\mu*} = \frac{(c_t^{k*})^{-\sigma*}}{(1+\tau_t^{c*})}-\beta^*\frac{1}{\pi_{t+1}^*}\frac{(c_{t+1}^{k*})^{-\sigma*}}{(1+\tau_{t+1}^{c*})} \tag{D37'}$$

$$\frac{(c_t^{k*})^{-\sigma*}}{(1+\tau_t^{c*})} = \beta^*R_t^*\frac{1}{\pi_{t+1}^*}\frac{(c_{t+1}^{k*})^{-\sigma*}}{(1+\tau_{t+1}^{c*})} \tag{D38'}$$

$$k_t^{k*} = (1-\delta^*)k_{t-1}^{k*}+x_t^{k*}-\frac{\xi^*}{2}\left(\frac{k_t^{k*}}{k_{t-1}^{k*}}-1\right)^2k_{t-1}^{k*} \tag{D39'}$$

$$\frac{c_t^{k,H*}}{c_t^{k,F*}} = \frac{v^*}{1-v^*}\frac{1}{TT_t} \tag{D40'}$$

$$c_t^{k*} = \frac{\left(c_t^{k,H*}\right)^{v*}\left(c_t^{k,F*}\right)^{1-v*}}{v^{*v^*}(1-v^*)^{1-v^*}} \tag{D41'}$$

$$\left[\left(c_t^{k,H*}+\frac{v^{w*}}{v^{k*}}c_t^{w,H*}+\frac{v^{b*}}{v^{k*}}c_t^{b,H*}\right)+x_t^{k*}+s_t^{g*}y_t^{H*}+s_t^{i*}y_t^{H*}+\left(c_t^{k,F}+\frac{v^w}{v^k}c_t^{w,F}+\frac{v^b}{v^k}c_t^{b,F}\right)\right]= \tag{196}$$

$$=y_t^{H*}\left[1-\frac{\phi^{p*}}{2}\left(\frac{\pi_t^{H*}}{\pi^{H*}}-1\right)^2\right] \tag{D42'}$$

$$x_n^*(n_t^{w*})^{\eta*}(c_t^{w*})^{\sigma*} = \frac{(1-\tau_t^{n*})}{(1+\tau_t^{c*})}w_t^{w*} \tag{D43'}$$

$$x_m^*(m_t^{w*})^{-\mu*} = \frac{(c_t^{w*})^{-\sigma*}}{1+\tau_t^{c*}} - \beta^* \frac{1}{\pi_{t+1}^*}\left[\frac{(c_{t+1}^{w*})^{-\sigma*}}{1+\tau_{t+1}^{c*}}\right] \tag{D44'}$$

$$\frac{c_t^{w,H*}}{c_t^{w,F*}} = \frac{v^*}{1-v^*}\frac{1}{TT_t} \tag{D45'}$$

$$c_t^{w*} = \frac{\left(c_t^{w,H*}\right)^{v*}\left(c_t^{w,F*}\right)^{1-v*}}{v^{*v^*}(1-v^*)^{1-v^*}} \tag{D46'}$$

$$(1+\tau_t^{c*})c_t^{w*} + m_t^{w*} = \frac{1}{\pi_t^*}m_{t-1}^{w*} + (1-\tau_t^{n*})w_t^{w*}n_t^{w*} - v^{k*}s_t^{l*}y_t^{H*}TT_t^{1-v^*} \tag{D47'}$$

$$x_m^*(m_t^{b*})^{-\mu*} = \frac{(c_t^{b*})^{-\sigma*}}{1+\tau_t^{c*}} - \beta^* \frac{1}{\pi_{t+1}^*}\left[\frac{(c_{t+1}^{b*})^{-\sigma*}}{1+\tau_{t+1}^{c*}}\right] \tag{D48'}$$

$$\frac{c_t^{b,H*}}{c_t^{b,F*}} = \frac{v^*}{1-v^*}\frac{1}{TT_t} \tag{D49'}$$

$$c_t^{b*} = \frac{\left(c_t^{b,H*}\right)^{v*}\left(c_t^{b,F*}\right)^{1-v*}}{v^{*v^*}(1-v^*)^{1-v^*}} \tag{D50'}$$

$$(1+\tau_t^{c*})c_t^{b*}+m_t^{b*} = \frac{1}{\pi_t^*}m_{t-1}^{b*}+(1-\tau_t^{n*})\frac{v^{k*}}{v^{b*}}s_t^{w*}TT_t^{1-v^*}y_t^{H*}-v^{k*}s_t^{l*}y_t^{H*}TT_t^{1-v^*} \tag{D51'}$$

$$TT_t^{1-v^*}r_t^{k*}k_{t-1}^{k*} = mc_t^*\alpha^*y_t^{H*} \tag{D52'}$$

$$w_t^{k*}n_t^{k*} = mc_t^*\theta^*(1-\alpha^*)y_t^{H*} \tag{D53'}$$

$$\frac{v^{w*}}{v^{k*}}w_t^{w*}n_t^{w*} = mc_t^*(1-\theta^*)(1-\alpha^*)y_t^{H*} \tag{D54'}$$

$$y_t^{H*} = A_t^*\left\{[k_{t-1}^{k*}]^{\alpha^*}\left[\{n_t^{k*}\}^{\theta^*}\times\left\{\frac{v^{w*}}{v^{k*}}n_t^{w*}\right\}^{1-\theta^*}\right]^{1-\alpha^*}\right\}\left(k_{t-1}^{g*}\right)^{\theta_k^*} \tag{D55'}$$

$$\widetilde{\omega_t}^{k*} = TT_t^{1-v^*} y_t^{H*} - mc_t^* y_t^{H*} - \frac{\phi^{p*}}{2}\left(\frac{\pi_t^{H*}}{\pi^{H*}} - 1\right)^2 TT_t^{1-v^*} y_t^{H*} \quad \text{(D56')}$$

$$\begin{aligned}
&Q_{t-1}^* TT_t^{1-v-v^*} \frac{1}{\pi_t}[1-\lambda_{t-1}^*] d_{t-1}^* TT_{t-1}^{v+v^*-1} + R_{t-1}^* \frac{1}{\pi_t^*} \lambda_{t-1}^* d_{t-1}^* + s_t^{g*} y_t^{H*} TT_t^{1-v^*} + \\
&s_t^{i*} y_t^{H*} TT_t^{1-v^*} + s_t^{w*} y_t^{H*} TT_t^{1-v^*} + \frac{1}{\pi_t^*}\left(m_{t-1}^{k*} + \frac{v^{w*}}{v^{k*}} m_{t-1}^{w*} + \frac{v^{b*}}{v^{k*}} m_{t-1}^{b*}\right) = \quad \text{(D57')} \\
&= \tau_t^{c*}\left[TT_t^{1-v^*}\left(c_t^{k,H*} + \frac{v^{w*}}{v^{k*}} c_t^{w,H*} + \frac{v^{b*}}{v^{k*}} c_t^{b,H*}\right) + TT_t^{-v^*}\left(c_t^{k,F*} + \frac{v^{w*}}{v^{k*}} c_t^{w,F*} + \frac{v^{b*}}{v^{k*}} c_t^{b,F*}\right)\right] + \\
&+ \left(m_t^{k*} + \frac{v^{w*}}{v^{k*}} m_t^{w*} + \frac{v^{b*}}{v^{k*}} m_t^{b*}\right) + \tau_t^{k*}\left[r_t^{k*} TT_t^{1-v^*} k_{t-1}^{k*} + \widetilde{\omega_t}^{k*}\right] + \\
&+ \tau_t^{n*}\left[w_t^{k*} n_t^{k*} + \frac{v^{w*}}{v^{k*}} w_t^{w*} n_t^{w*} + s_t^{w*} y_t^{H*} TT_t^{1-v^*}\right] + s_t^{l*} y_t^{H*} TT_t^{1-v^*} + d_t^*
\end{aligned}$$

$$\begin{aligned}
&TT_t^{1-v^*}\left\{\left[c_t^{k,H*} + \frac{v^{w*}}{v^{k*}} c_t^{w,H*} + \frac{v^{b*}}{v^{k*}} c_t^{b,H*}\right] + x_t^{k*} + s_t^{g*} y_t^{H*} + s_t^{i*} y_t^{H*} - y_t^{H*}\left[1 - \frac{\phi^{p*}}{2}\left(\frac{\pi_t^{H*}}{\pi^{H*}} - 1\right)^2\right]\right\} + \\
&\quad \text{(D58')} \\
&+ TT_t^{-v^*}\left[c_t^{k,F*} + \frac{v^{w*}}{v^{k*}} c_t^{w,F*} + \frac{v^{b*}}{v^{k*}} c_t^{b,F*}\right] = \\
&= TT_t^{1-v-v^*}([1-\lambda_t^*] d_t^* TT_t^{v+v^*-1} - f_t^{k*}) - Q_{t-1}^* TT_t^{1-v-v^*} \frac{1}{\pi_t}([1-\lambda_{t-1}^*] d_{t-1}^* TT_{t-1}^{v+v^*-1} - f_{t-1}^{k*})
\end{aligned}$$

$$\frac{\pi_t^*}{\pi_t^{H*}} = \left(\frac{TT_t}{TT_{t-1}}\right)^{v^*-1} \quad \text{(D59')}$$

$$\begin{aligned}
&(1-\phi^*) TT_t^{1-v^*} y_t^{H*} + \phi^* mc_t^* y_t^{H*} - \phi^{p*}\left[\frac{\pi_t^{H*}}{\pi^{H*}} - 1\right] TT_t^{1-v^*} \frac{y_t^{H*} \pi_t^{H*}}{\pi^{H*}} = \\
&\beta^* \phi^{p*}\left[\left(\frac{1}{\pi_{t+1}^*}\right)\left(\frac{1+\tau_t^{c*}}{1+\tau_{t+1}^{c*}}\right)\left(\frac{c_{t+1}^{k*}}{c_t^{k*}}\right)^{-\sigma*}\right]\left[1 - \frac{\pi_{t+1}^{H*}}{\pi^{H*}}\right] \frac{\pi_{t+1}^{H*}}{\pi^{H*}} TT_{t+1}^{1-v^*} y_{t+1}^{H*} \quad \text{(D60')}
\end{aligned}$$

$$y_t^{g*} = A^*\left(v^{k*} s_t^{g*} y_t^{H*}\right)^{\theta_g^*}\left(v^{b*}\right)^{1-\theta_g^*} \quad \text{(D61')}$$

$$s_t^{i*} y_t^{H*} = k_t^{g*} - (1-\delta^{g*}) k_{t-1}^{g*} + \frac{\xi^{g*}}{2}\left(\frac{k_t^{g*}}{k_{t-1}^{g*}} - 1\right)^2 k_{t-1}^{g*} \tag{D62'}$$

We finally have the feedback monetary and fiscal policy rules:

$$\begin{aligned} log\left(\frac{R_t}{R}\right) = \phi_\pi \left[\tilde{\eta} log\left(\frac{\pi_t}{\pi}\right) + (1-\tilde{\eta}) log\left(\frac{\pi_t^*}{\pi^*}\right)\right] + \\ + \phi_y \left[\tilde{\eta} log\left(\frac{y_t^H}{y^H}\right) + (1-\tilde{\eta}) log\left(\frac{y_t^{H*}}{y^{H*}}\right)\right] \end{aligned} \tag{D63'}$$

$$l_{t-1} \equiv \frac{R_{t-1}\lambda_{t-1} d_{t-1} TT_{t-1}^{1-v} + Q_{t-1}\epsilon_t (1-\lambda_{t-1}) d_{t-1} TT_{t-1}^{1-v}}{y_{t-1}^H} \tag{D64'}$$

$$s_t^g = s^g - \gamma_l^g (l_{t-1} - l) \tag{D65'}$$

$$s_t^i = s^i - \gamma_l^i (l_{t-1} - l) \tag{D66'}$$

$$s_t^w = s^w - \gamma_l^w (l_{t-1} - l) \tag{D67'}$$

$$s_t^l = s^l + \gamma_l^l (l_{t-1} - l) \tag{D68'}$$

$$\tau_t^c = \tau^c + \gamma_l^c (l_{t-1} - l) \tag{D69'}$$

$$\tau_t^k = \tau^k + \gamma_l^k (l_{t-1} - l) \tag{D70'}$$

$$\tau_t^n = \tau^n + \gamma_l^n (l_{t-1} - l) \tag{D71'}$$

$$l_{t-1}^* \equiv \frac{R_{t-1}^*\lambda_{t-1}^* d_{t-1}^* TT_{t-1}^{v^*-1} + Q_{t-1}^*\frac{1}{\epsilon_t}(1-\lambda_{t-1}^*) d_{t-1}^* TT_{t-1}^{v^*-1}}{y_{t-1}^{H*}} \tag{D72'}$$

$$s_t^{g*} = s^{g*} - \gamma_l^{g*} (l_{t-1}^* - l^*) \tag{D73'}$$

$$s_t^{i*} = s^{i*} - \gamma_l^{i*} (l_{t-1}^* - l^*) \tag{D74'}$$

$$s_t^{w*} = s^{w*} - \gamma_l^{w*}(l_{t-1}^* - l^*) \quad \text{(D75')}$$

$$s_t^{l*} = s^{l*} + \gamma_l^{l*}(l_{t-1}^* - l^*) \quad \text{(D76')}$$

$$\tau_t^{c*} = \tau^{c*} + \gamma_l^{c*}(l_{t-1}^* - l^*) \quad \text{(D77')}$$

$$\tau_t^{k*} = \tau^{k*} + \gamma_l^{k*}(l_{t-1}^* - l^*) \quad \text{(D78')}$$

$$\tau_t^{n*} = \tau^{n*} + \gamma_l^{n*}(l_{t-1}^* - l^*) \quad \text{(D79')}$$

Therefore, we have 79 equations in total. We also have 79 endogenous variables, which are $[c_t^k, c_t^{k,H}, c_t^{k,F}, c_t^w, c_t^{w,H}, c_t^{w,F}, c_t^b, c_t^{b,H}, c_t^{b,F}, n_t^k, w_t^k, n_t^w, w_t^w, m_t^k, m_t^w, m_t^b, r_t^k, k_t^k, k_t^g, x_t^k, f_t^k, Q_t, y_t^H, y_t^g, mc_t, \widetilde{\omega_t}^k, v_t, \widetilde{\pi_t}^k, \pi_t, \pi_t^H, TT_t, d_t]$ and $[R_t, s_t^g, s_t^i, s_t^w, s_t^l, \tau_t^c, \tau_t^k, \tau_t^n, l_t]$ for the home country, and $[c_t^{k*}, c_t^{k,H*}, c_t^{k,F*}, c_t^{w*}, c_t^{w,H*}, c_t^{w,F*}, c_t^{b*}, c_t^{b,H*}, c_t^{b,F*}, n_t^{k*}, w_t^{k*}, n_t^{w*}, w_t^{w*}, m_t^{k*}, m_t^{w*}, m_t^{b*}, r_t^{k*}, k_t^{k*}, k_t^{g*}, x_t^{k*}, R_t^*, f_t^{k*}, Q_t^*, y_t^{H*}, y_t^{H*}, mc_t^*, \widetilde{\omega_t}^{k*}, \pi_t^*, \pi_t^{H*} d_t^*]$ and $[s_t^{g*}, s_t^{i*}, s_t^{w*}, s_t^{l*}, \tau_t^{c*}, \tau_t^{k*}, \tau_t^{n*}, l_t^*]$ for the foreign country. This is given the exogenous variables, $[\epsilon_t, \lambda_t, \lambda_t^*, A_t, A_t^*, v^b, v^{b*}]$, initial conditions for the state variables and the values of the feedback (monetary and fiscal) policy coefficients in the policy rules.

Conclusively, we have a system of 79 equations [(D1')-(D79')] in the 79 following endogenous variables

$$[c_t^k, c_t^{k,H}, c_t^{k,F}, c_t^w, c_t^{w,H}, c_t^{w,F}, c_t^b, c_t^{b,H}, c_t^{b,F}, n_t^k, w_t^k, n_t^w, w_t^w, m_t^k, m_t^w, m_t^b, r_t^k, k_t^k, k_t^g,$$

$$x_t^k, f_t^k, Q_t, y_t^H, y_t^g, mc_t, \widetilde{\omega_t}^k, v_t, \widetilde{\pi_t}^k, \pi_t, \pi_t^H, TT_t, d_t, R_t, s_t^g, s_t^i, s_t^w, s_t^l, \tau_t^c, \tau_t^k, \tau_t^n, l_t,$$

$$c_t^{k*}, c_t^{k,H*}, c_t^{k,F*}, c_t^{w*}, c_t^{w,H*}, c_t^{w,F*}, c_t^{b*}, c_t^{b,H*}, c_t^{b,F*}, n_t^{k*}, w_t^{k*}, n_t^{w*}, w_t^{w*}, m_t^{k*}, m_t^{w*}, m_t^{b*},$$

$$r_t^{k*}, k_t^{k*}, k_t^{g*}, x_t^{k*}, R_t^*, f_t^{k*}, Q_t^*, y_t^{H*}, y_t^{H*}, mc_t^*, \widetilde{\omega_t}^{k*}, \pi_t^*, \pi_t^{H*} d_t^*]$$

Table 12: Income of private worker and public employee relative to that of capitalist in each country in steady state (SS) when the fiscal space created by debt consolidation is used by the domestic and foreign economy to reduce their labor tax rate (τ^n and τ^{n*}).

	Domestic economy	Foreign economy
Private worker's net income relative to capitalist's net income $\left(\frac{y^w}{y^k} or \frac{y^{w*}}{y^{k*}}\right)$	0.2311 (0.2299)	0.2646 (0.2609)
Public employee's net income relative to capitalist's net income $\left(\frac{y^b}{y^k} or \frac{y^{b*}}{y^{k*}}\right)$	0.1851 (0.1842)	0.2132 (0.2104)

Note: In brackets they are reported the associated SQ steady state values.