Random Encounters and Information Diffusion about Markets

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Abstract

International openness enhances social interactions between citizens of different countries or regions, and *vice versa*. Social exchanges in turn increase the trade flows between countries. Accordingly, these meetings influence markets and prices. We analyze the interaction between the increased mobility following openness between two different countries and the corresponding effects on market outcomes. The main result of our analysis consists in showing that market prices tend at the limit to align with the duopoly solution. Nonetheless, this convergence can take two different paths depending on the size asymmetry between the countries.

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1 Introduction

Many Italians nowadays agree that a Belgian beer as *Stella Artois* is of a higher quality than an Italian beer as *Peroni*, and many Belgians prefer *San Pellegrino* water to *Spa Reine.* Before the European Common Market existed, fewer consumers in Italy knew about Belgian Brewers' beers, whereas fewer consumers in Belgium knew about *San Pellegrino.* But, after the increased mobility of European citizens following the opening of the markets, citizens located in Belgium and in Italy *met much more frequently* and shared their personal consumption experiences.[1]

This example reveals that a vehicle through which information about the existence of goods, their prices and their quality, comes from social meetings among agents. When these agents belong to different countries or regions, openness increases the frequency of such meetings. Interpersonal exchanges in turn increase trade flows between countries. Accordingly, these interactions influence national markets and prices.

In this paper, we analyze how these *non-market* interactions, intensified by the opening of trade between two different countries, affect market quantities and prices in both countries. More precisely, we consider initially two countries with different population densities. Countries have each a national market on which a domestic firm sells a domestic good. These two goods are further assumed to be vertically differentiated commodities. As long as these markets remain strictly national, consumers in each country remain ignorant about the existence of the substitute commodity produced in the other country. There is no mutual influence between their respective markets: each national firm is a monopolist on its national market. Now imagine that a common market is decided, starting to link these two goods, and entailing thereby the birth and the outgrowth of mutual interactions among the agents in the two asymmetric sized countries. Assuming a random encounter model as in Lazear (1995), progressively, and according to the number and frequency of these interactions, mutual experiences of consumption habits are exchanged. While at the start several consumers in each country are still remaining ignorant about the existence of the foreign product, the exchange of consumption experiences among their citizens reinforces the process of competition between the national and the foreign substitute. This magnifies competition between these products, transforming slowly but surely two national monopolies into a single duopoly market in vertically differentiated products.

How these monopoly markets at the start metamorphose into a single common duopoly market is precisely the object of this paper. In particular, we are interested in exploring how prices change along the sequence of equilibria generated by these dynamics of interactions: does the common market drive the initial monopoly prices to the duopoly market solution with vertically

[1] At least this was experienced by the coauthors of this paper !

differentiated products? This question can be precisely formulated and answered in the model proposed thereafter.

The above exercise aims at intertwining the literature about markets with the growing trend of research about the role played by social interactions. Here we assume that the existence and the exchange possibilities of national products is progressively revealed to citizens of different countries by the individual relations developing among them. These relations are enhanced through time due to the existence of a common market allowing consumers of different countries to share their consumption experiences. This view is typically developed in the field of behavioral microeconomics (see for instance, Lazear, 1999, or Bowles, 2004). This paper brings together theoretical tools inspired from industrial organisation (Gabszewicz and Thisse, 1979, Gabszewicz et al.. 1981), behavioral microeconomics (Camerer, 2003), and international trade (Mercenier and Schmitt, 1995; Boccard and Wauthy, 2006; Chatterjee and Raychaudhuri, 2004). However, the problem of information about the existence of goods and their markets has been mostly neglected in the literature in which information deals mainly with the price and the quality of the goods. By contrast, our paper is mainly concerned with the existence of goods since we assume that knowledge about the good implies knowledge about its price and quality.

The main outcome of our analysis consists in showing that the market price tends, at the limit, to align with the duopoly solution. Surprisingly, this convergence can take two different paths. When countries sizes are relatively similar, interpersonal meetings between consumers of each country are relatively frequent so that a significant size of the population in both becomes aware of the other country's good. Then, competition between national goods becomes vivid very quickly in the common market. It follows from this that the market evolution from monopoly to duopoly occurs already in the first period, although prices take some time to adjust to their full information level. On the contrary, when countries differ significantly in size, meetings of consumers are rare and thus the information diffusion about the existence of the foreign commodity is much slower. As a consequence, in spite of the opening of the two monopoly markets, competition does not succeed to drive the monopoly price to the duopoly price before a significant period of time: this determines a situation of "*nearly-monopoly*". However, we show that there exists a time period at which the informed consumers are sufficiently numerous to turn the nearly monopoly into a duopoly. Finally, we analyse how profitable openness is for firms and consumers. We identify a range of parameters in which openness is beneficial for the high quality firms whereas it is detrimental for the low quality firm.

Our results obtain under the crucial, but classical, assumption that all markets in each period are covered. This helps to keep the knowledge transmission process among consumers tractable and to model the dynamic price selection by firms as a Markovian strategy profile.

The paper is organized as follows. The next section presents the model in autarky and under full information hypothesis. It also explains the information diffusion mechanism. Section 3 provides the market solution for period one and Section 4 develops the multi-period market solution. Section 5 analyses openness profitability and section 6 concludes.

2 The model

Consider a two-country-two-good model, where country $i = 1$ produces good 1 and country $i = 2$ good 2. Heterogeneous consumers in each country are indexed by θ and uniformly distributed over the interval $[a, b]$, with $a > 0$ and $b < \infty$. The parameter θ captures the consumers' heterogeneous willingness to pay for the good: the higher θ, the higher the utility obtained when consuming the product. Each consumer can either buy one unit of a given product or not buy at all. Formally, consumer's utility is given by

$$U(\theta) = \begin{cases} \theta u_i - p_i \text{ if he buys variant } i \\ 0 \text{ if he refrains from buying.} \end{cases} \tag{1}$$

where u_i denotes the variant of each good $i = 1, 2$ and p_i its market price. Let s denote the total mass of consumers living in country *one* and $(1 - s)$ that of country *two*.[2] Let denote C_1 the population of country *one* and C_2 the population of country *two*.

Initially the consumers of each country are served by a monopolistic firm (or by a national industry acting as a monopolist) selling its good at a price that *covers* the entire domestic market.[3] Therefore, at period $t = 0$, the two countries are autarchies producing a good, consumed only in their own country. For simplicity, in what follows we assume zero costs of production.[4]

At period $t = 1$, the governments of the two countries decide to sign an agreement that opens the two countries to unrestricted trade and people's circulation. The reasons why this agreement is signed are assumed exogenous to the model.[5] Starting from period 1, consumers have the chance to meet consumers of the foreign country and share their consumption experience. We assume that these social interactions may arise for any reason we may think of (work, friendship, schooling,

[2] Note here that the total population of the two countries is normalized to 1 so that s and $(1 - s)$ simply express the fraction of people living in country one and two, respectively. While consumers' population of the two countries is assumed different (except when $s = 0.5$), their preferences distribution (degree of heterogeneity), expressed by the support of consumers' willingness to pay for goods, is assumed equal in both countries. Assuming different supports in the two countries would not alter the qualitative results of the model.

[3] Admittedly, the assumption of covered market is made for simplicity. Uncovered markets make the analysis of price dynamics intractable.

[4] Introducing a production cost depending on product quality would make the analysis more cumbersome without improving the model intuitions.

[5] Nonetheless in Section 4 we analyse when the agreement would be desirable for the two economies and thus endogenously selected even by firms and consumers.

romantic or simply vacations). Whatever the reason, when two people meet, we assume that they exchange information about the goods they consume. Only then, some consumers become acquainted to both goods and they acknowledge them as vertically differentiated in accordance to (1). Without loss of generality, we assume that good 1 produced by country *one* is of lower quality than good 2 produced by country *two*, namely $u_2 > u_1$. Openness improves personal meetings and implies trade exchanges. Since the aim of this paper is to draw attention on the information process occurring through personal meetings, we neglect the information diffusion through prices and, consequently, we neglect the role of trade costs normalizing them to zero.[6]

2.1 Market solution in autarky

Before the markets open, the two countries are assumed to live in a regime of autarky. Under the assumption of fully covered market, the consumer located at a, i.e., $\theta = a$, will consume the good 1 only if

$$au_1 - p_1 \geq 0$$

from where we get the monopoly price at time $t = 0$, p_1^M (compatible with a fully covered market) as

$$\left(p_1^M\right)^* = au_1. \tag{2}$$

Similarly, for country two we obtain that

$$\left(p_2^M\right)^* = au_2. \tag{3}$$

Before openness, populations do not mix and, hence, they only purchase the domestic good paying the monopoly price. The corresponding demand functions $\{D_1\}_{t=0}$ and $\{D_2\}_{t=0}$ for each firm are respectively:

$$\{D_1\}_{t=0} = s(b-a) \text{ and } \{D_2\}_{t=0} = (1-s)(b-a),$$

with profits

$$\{\Pi_1\}_{t=0} = au_1 s(b-a) \text{ and } \{\Pi_2\}_{t=0} = au_2(1-s)(b-a).$$

[6] Introducing the trade cost in the model would slow down the process of transformation of the two monopolies in duopolies but convergence will eventually take place.

2.2 Market solution under full information

If after the trade opening everyone living in country i *would meet instantaneously everyone* of country j, then all consumers would immediately become fully informed about the existence of the two goods. Consequently, the two markets would be segmented among the consumers of good one and good two where the marginal consumer (indifferent whether to buy good one or two) in each country would be:

$$\theta(p_1, p_2) = \frac{p_2 - p_1}{u_2 - u_1}.$$

Then, with perfectly informed consumers, the demand for good one, $D_1(p_1, p_2)$, and two, $D_2(p_1, p_2)$, would be, respectively:

$$D_1(p_1, p_2) = \frac{p_2 - p_1}{u_2 - u_1} - a \quad \text{and} \quad D_2(p_1, p_2) = b - \frac{p_2 - p_1}{u_2 - u_1},$$

yielding, the following equilibrium prices at every period $t = 1, 2, ...$

$$p_1^* = \frac{1}{3}(u_2 - u_1)(b - 2a) \tag{4}$$

$$p_2^* = \frac{1}{3}(u_2 - u_1)(2b - a). \tag{5}$$

The positivity of full information prices (4)-(5) is implied by the condition $b > 2a$ which, from here on, we assume to hold. Then, the marginal consumer, will locate at

$$\theta^* = \frac{1}{3}(a + b).$$

Moreover, for the markets to be fully covered at the duopoly prices, we need to ensure that *all* consumers receive positive utility from purchasing both goods at the duopoly prices under full information. This occurs when the following condition holds:

$$\frac{u_1}{u_2} > \frac{b - 2a}{b + a}. \tag{6}$$

The corresponding profits for firms 1 and 2 at the duopoly solution obtain, respectively as:

$$\begin{aligned}
\Pi_1^* &= \frac{1}{9}(u_2 - u_1)(2a - b)^2, \\
\Pi_2^* &= \frac{1}{9}(u_2 - u_1)(a - 2b)^2.
\end{aligned}$$

However, since not *all consumers* meet at once, when the two countries open their markets, at every period there may exist consumers with different levels of knowledge about the two goods. To see this in more detail, in the next section we analyse how the demands in each country evolve over time, after the two countries open to trade.

2.3 Information diffusion under social interaction

We now describe the evolution of the demand functions of each good when the two countries open to meetings among agents living in different countries and to trade. In particular, at each period there exist three types of consumers in each country:

Definition 1 ***Uninformed,*** *the consumers who are only aware of the domestic good but ignore the existence of the foreign good.*

Definition 2 ***Informed,*** *the consumers who, having met a consumer of the other country, have become informed about the existence of both goods.*

Finally, using a definition borrowed from evolutionary game theory, we can define

Definition 3 ***Mutants,*** *the consumers who after becoming informed find convenient to update their consumption choice, switching from one good to another.*

Notice that the trade and social interaction openness among the consumers of the two countries brings two important consequences: *(1)* the two goods become available in both markets at negligible trade costs; *(2) mutants* originating from social interactions modify the demand function faced by each firm.

We assume the following process of knowledge transmission among consumers. In each period, every consumer randomly meets one consumer, who can, therefore, either be a foreign consumer or a consumer of her own country, as in Lazear (1995, 1999). Given the mass of consumers s and $(1-s)$ in the two countries, the probability that one consumer of country *one* meets one consumer of her/his own country at period 1, remaining *uninformed* of the existence of the other good, is simply given by

$$\Pr\left\{(i \in C_1) \cap (j \in C_1)\right\}_{t=1} = s^2.$$

Similarly, the probability that an individual of country 2 meets someone of her/his own country, thus remaining *uninformed* of country *one* good, is given by

$$\Pr\left\{(i \in C_2) \cap (j \in C_2)\right\}_{t=1} = (1-s)^2.$$

Thus, it follows that the probability that the consumers of the two countries become *informed,* in period 1, is given by

$$\Pr\left\{(i \in C_1) \cap (j \in C_2)\right\}_{t=1} = 1 - s^2 - (1-s)^2 = 2s\,(1-s)\,.$$

A similar knowledge transmission process occurs in all subsequent periods t. In what follows, we analyse how the set of informed and uninformed consumers in each country evolve over time, after the two countries open to trade. The population dynamics of these two subsets defines the demand for each good. For ease of exposition, we start with the analysis of the first period and then present the extension to any number of period t.

3 Market solution in period one

At period $t = 1$, agents of different countries have the possibility to meet and, hence, mutants may appear within each consumer population. We start by building the demand function faced by the firm producing good 1. All consumers of country one, whose willingness to pay lays in the interval $(a, \theta(p_1, p_2))$, whether uninformed or informed, given their low willingness to pay, will continue to buy good one in period one. Their mass is given by $s[\theta(p_1, p_2) - a]$, as in Fig. 1. In this figure, the vertical axis indicates the mass of population corresponding to each country, while the horizontal axis is the support of the populations' willingness to pay in country one and two.

The uninformed consumers in country one who could potentially consume good two but who do not meet anyone from country two (occurring with probability $s \cdot s$) are of mass $s^2[b - \theta(p_1, p_2)]$. These agents will, therefore, continue to consume good 1 at period one.

Finally, the consumers of country two with a willingness to pay laying in interval $(a, \theta(p_1, p_2))$ and becoming informed of good 1, with probability $s(1 - s)$, will become mutants switching from good 2 to good 1. Their mass is given by $s(1 - s)(\theta(p_1, p_2) - a)$. We represent these population movements in Figure 1 below:

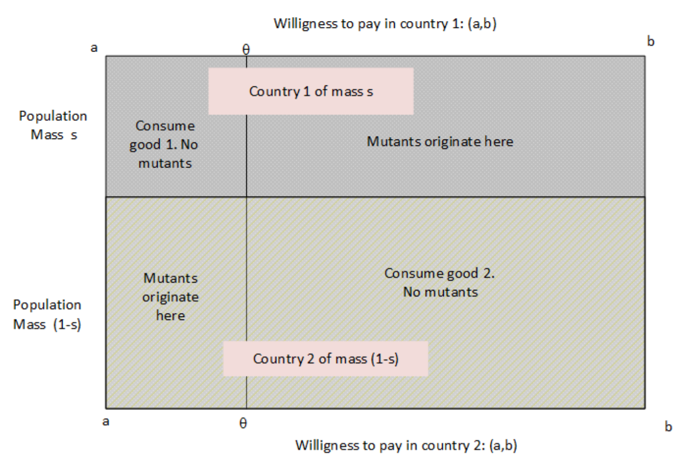


Figure 1: Willingness to pay and Mutants in period 1.

In particular, notice that the *mutants* can arise only among individuals having a level of willingness to pay laying in the interval $(\theta - a)$ in country two (and $(b - \theta)$ in country one).

It follows that the demand functions for good 1 in period 1 is

$$\{D_1(t)\}_{t=1} = s(\theta(p_1, p_2) - a) + s^2(b - \theta(p_1, p_2)) + s(1 - s)(\theta(p_1, p_2) - a), \tag{7}$$

We turn now to the demand function faced by the firm producing good 2. Similarly to the above, all consumers whose willingness to pay lays in the interval $[b - \theta(p_1, p_2)]$ in country *two*, whether uninformed or informed, will continue to demand good *two*. Their mass is $(1 - s)[b - \theta(p_1, p_2)]$. The mass of consumers in country *two* who would consume good 1 but are uninformed is $(1 - s)^2[b - \theta(p_1, p_2)]$. Finally, the informed consumers of country 1 with willingness to pay in the interval $b - \theta(p_1, p_2)$, i. e. the *mutants* of country *one*, are of mass $s(1 - s)[b - \theta(p_1, p_2)]$.

Therefore the demand function for good 2 in period 1 is:

$$\{D_2(p_1(t), p_2(t), t)\}_{t=1} = (1 - s)(b - \theta(p_1, p_2)) + (1 - s)^2(\theta(p_1, p_2) - a) + s(1 - s)(b - \theta(p_1, p_2)).$$

Thus, if firms decide their price at every period, they set prices to maximize profits:

$$\begin{aligned} \{\Pi_1(t)\}_{t=1} &= \{D_1(p_1(t), p_2(t), t) \cdot p_1(t)\}_{t=1}, \\ \{\Pi_2(t)\}_{t=1} &= \{D_2(p_1(t), p_2(t), t) \cdot p_2(t)\}_{t=1}. \end{aligned}$$

Concavity of firms' profits functions with respect to their own prices, allows us to solve the system of FOCs for an interior maximum and to obtain the profit maximizing prices $\{p_1^*(t)\}_{t=1}$ and $\{p_2^*(t)\}_{t=1}$ as:

$$\begin{aligned} \{p_1^*(t)\}_{t=1} &= \frac{1}{6}\frac{(u_2 - u_1)[(1 + s^2)b - (1 - s^2 + 2s)a]}{s(1 - s)}, \\ \{p_2^*(t)\}_{t=1} &= \frac{1}{6}\frac{(u_2 - u_1)[(2 - s^2)b - (2 - 2s + s^2)a]}{s(1 - s)}. \end{aligned} \tag{8}$$

It is easy to check that the positivity of the profit maximizing prices simply follows from condition $b > 2a$ assumed above for prices positivity in a duopoly with perfectly informed consumers.[7]

At period 1 the corresponding marginal consumer is, therefore, given by

$$\{\theta^*(t)\}_{t=1} = \frac{1}{6}\frac{b - a(1 - 4s) - 2s^2(a + b)}{s(1 - s)}, \tag{9}$$

[7] Indeed, $\{p^t\}_{t=1}$ is positive iff $b > a(1 - s^2 + 2s)/(1 + s^2)$. The coefficient $(1 - s^2 + 2s)/(1 + s^2)$ is strictly smaller than 2 for any $s \in (0, 1)$ and, therefore, $b > 2a$ is a more binding condition.

and the corresponding equilibrium profits are:

$$\{\Pi_1^*(t)\}_{t=1} = \frac{(u_2 - u_1)\left(b - a + as^2 + bs^2 - 2as\right)^2}{18s\,(1-s)}$$
$$\{\Pi_2^*(t)\}_{t=1} = \frac{(u_2 - u_1)\left(2a - 2b + as^2 + bs^2 - 2as\right)^2}{18s\,(1-s)}$$

Let us define below two threshold values for the parameter expressing the fraction of country's one population,

$$s' = \frac{1}{2}\frac{3b - 2a - \sqrt{5b^2 - 6ab + 2a^2}}{2b - a} \text{ and } s'' = \frac{\sqrt{2b^2 - 6ab + 5a^2} - a}{2b - 4a}.$$

Then, the expression for the marginal consumer $\{\theta^*(t)\}_{t=1}$ takes a value at the interior of interval $[a, b]$ iff

$$s' < s < s''. \tag{10}$$

In addition, the above interval of values of s guarantees that $\{p_2^*(t)\}_{t=1}$ exceeds $\{p_1^*(t)\}_{t=1}$, as expected in a model of vertical differentiation.

When the value of s is within the interval (10), we can use Fig. 2 to plot period 1 profit maximizing prices for firm 1 and 2 as a function of s

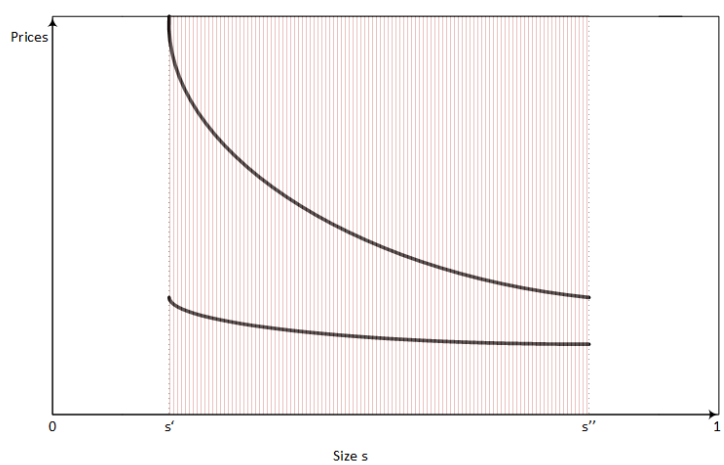


Figure 2: Optimal prices in period 1 as a function of s.

Two properties of these period 1 profit-maximizing prices are worth noticing. These properties hold in every period and shape the dynamics of the market solutions. The first property concerns the nonmonotonic and convex behavior of both prices with respect to the level of countries' size asymmetry. Indeed, when the two countries are exactly symmetric, namely for $s = 0.5$, the marginal consumer at period $t = 1$ exactly coincides with the one at the full information scenario. However, even so, the prices of the two goods are nevertheless higher than under full information, due to the existing informational frictions. The convexity of prices is caused by the frequency of meetings implied by the countries' size asymmetry. For high country size asymmetry, namely for very small or very large s, prices are high. This occurs because a high population asymmetry implies small chances of meeting other-country consumers and ultimately small chances for *mutants* to appear. Thus, for high size asymmetry, firms do not incur into dramatic reduction in the corresponding demand functions as effect of trade openness. By contrast, when the population asymmetry between the two countries is low (which occurs for intermediate values of s) chances for the consumers of the two countries to meet is high and, hence, also the chances for *mutants* to appear. It follows that the market power of the two firms get quickly reduced and their equilibrium prices become substantially smaller since already in the first period of trade openness and social interaction.

The second property of prices concerns the relationship between price gap and level of s. It can be noticed that when s is very small the difference between prices is much larger than when s is big. When the mass of population living in the country producing the low quality good is large (s large), the price competition coming from the social interactions has a strong and immediate negative impact on prices. In particular, due to the large size of the low quality market, the price gap is squeezed down. By contrast, when s is very small, implying a very large high quality market, the price gap is big. This is not all. The price of the high quality good is very high due to the high relative size of its market. In turn, this enlarges the price gap.

It remains to clarify the type of market solution for values of s that do not belong to the interval (10). Two scenarios may occur: *(i)* $\{\theta^*(t)\}_{t=1} = b$ in correspondence of $s < s''$ and *(ii)* $\{\theta^*(t)\}_{t=1} = a$ in correspondence of $s > s'$. More specifically, in *(i)*, the country producing the low quality good is very small and the price gap associated to the duopoly solution, as we saw it in figure 2, is very high. This drives the position of the marginal consumer outside the interval $[a, b]$ exceeding the threshold b. This implies that no informed consumers will buy good 2 and rather prefer to buy good 1, whereas only *uninformed* consumers of country 2 will continue to buy good 2. Consequently, this coincides with a corner solution in which $\{\theta^*(t)\}_{t=1} = b$. In *(ii)* instead, the country producing the low quality good is very large and the price gap associated to the duopoly solution turns out to be small. Again, this drives the position of the marginal consumer outside the interval $[a, b]$ being smaller than the threshold a. Now, this implies that no informed consumer will buy good 1 and rather prefer to buy good 2, whereas only *uninformed* consumers of country 1 will continue to buy good 1. Similarly to case *(i)*, this corresponds to a corner solution with $\{\theta^*(t)\}_{t=1} = a$.

In the following sections, we will denote these two cases as *nearly-monopolies* of *type-one* and *type two.*

4 Multi-period market solution

4.1 Knowledge Transmission over Time

To characterize the levels of prices set by the firms and the associated quantities sold over time, we need to specify the process of knowledge transmission. The evolution of the mass of informed, uninformed and mutants in every period and every country allows to analyze the market equilibrium prices at each period t and their convergence, if any, for $t \to \infty$.

As we will clarify in more detail below, in our setting the process of knowledge transmission is somehow independent on prices set by the firms and this allows to treat the firms strategies as *Markovian*, since the firms' payoffs depend in any period t only on the current history and not on all previous histories. In particular, if we denote the mass of *uninformed* consumers on good i at time t as U_i^t for $i = 1, 2$, and similarly, that of consumers becoming *informed* at time t, I_i^t. As explained above, the total mass of informed consumers about good 1 at period 1 is given by

$$I_1^0 + I_1^1 = s + (1-s)s = s\,(2-s)\,.$$

By the same reasoning, the mass of *newly* informed consumers on good 1 at $t = 2$, denoted I_1^2 is,

$$I_1^2 = \Pr\left(\left\{i \in \left(I_1^0 \cup I_1^1\right)\right\} \cap \left\{j \in U_1^1\right\}\right) = s\,(2-s)\,(1-s)^2\,.$$

The cumulative mass of consumers who are informed on good 1 at period 2 is, therefore, given by

$$\begin{aligned}\mathcal{I}_1^2 &= I_1^1 + I_1^2 = s\,(2-s) + s\,(2-s)\,(1-s)^2 \\ &= s\,(2-s)\,(1+(1-s)^2),\end{aligned}$$

and the proportion of uninformed consumers of country *two* (who can potentially demand good 1) are:

$$\mathcal{U}_1^2 = 1 - \mathcal{I}_1^2 = (1-s)^4\,.$$

In *Appendix* A, we develop the general expression for $\mathcal{I}_1^t$ and $\mathcal{U}_1^t$ at any period t, which can be shown to be, respectively:

$$\mathcal{I}_1^t = 1 - (1-s)^{2^t} \text{ and } \mathcal{U}_1^t = (1-s)^{2^t}\,.$$

In an analogous way, the set of informed and uninformed agents about good 2 at time t is equal to:

$$\mathcal{I}_2^t = 1 - s^{2^t} \text{ and } \mathcal{U}_2^t = s^{2^t}\,.$$

As a result, in any period t, the demand functions D_1 and D_2 for the two goods are simply:

$$D_1(p_1(t), p_2(t)) = \left(1 - (1-s)^{2^t}\right)(\theta(p_1(t), p_2(t)) - a) + s^{2^t}(b - \theta(p_1(t), p_2(t))), \tag{11}$$

$$D_2(p_1(t), p_2(t)) = \left(1 - s^{2^t}\right)(b - \theta(p_1(t), p_2(t))) + (1-s)^{2^t}(\theta(p_1(t), p_2(t)) - a). \tag{12}$$

4.2 Equilibrium Concept

As in a standard multi-period setting the two firms can be assumed to maximize their intertemporal profits as

$$\max_{p_1(1), p_1(2).,...} \sum_{t=1}^{\infty} \delta^{t-1} \Pi_1 \left(\mathbf{p}_1(t), \mathbf{p}_2(t) \left| \mathcal{I}_1^t \right.\right), \tag{13}$$
$$\max_{p_2(1), p_2(2).,...} \sum_{t=1}^{\infty} \delta^{t-1} \Pi_2 \left(\mathbf{p}_1(t), \mathbf{p}_2(t) \left| \mathcal{I}_2^t \right.\right),$$

where $\mathbf{p}_1(t) = (p_1(1), p_1(2)., ...)$ and $\mathbf{p}_2(t) = (p_2(1), p_2(2)., ...)$ denote the profile of optimal per-period prices set by the firms. These are, in turn, conditional on the level of information $\mathcal{I}_i^t$ about good 1 or 2 existing at every period t and with δ being a common discount factor that, for simplicity, we set equal to 1. Since the process of knowledge transmission does not directly depend on $(\mathbf{p}_1(t), \mathbf{p}_2(t))$, it follows that the informational state $\mathcal{I}_i^t$ at every period t is independent of $\mathbf{p}_i(t)$ for every $i = 1, 2$, and the price profile of every firm is *Markovian*, only depending on the current state variables. The appropriate equilibrium concept is, therefore, a *Markov perfect equilibrium* (Maskin and Tirole, 2001), as defined below.

Definition 4 *In our setting he profile $(\mathbf{p}_1^*(t), \mathbf{p}_2^*(t))$ is a Markov perfect equilibrium if, in every period t,*

$$\Pi_i^* \left(\{p_i^*(t)\}_t, \{p_j^*(t)\}_t\right) \geq \Pi_i \left(\{p_i'(t)\}_t, \{p_j^*(t)\}_t\right),$$

for every $\{p_i'(t)\}_t \neq \{p_i^*(t)\}_t \in [0, \infty)$ and $i = 1, 2$, $j \neq i$.

The above equilibrium implies that at every period a firm maximizes its current profit, given the only payoff relevant state variable of that period, namely the fraction of consumers ready to buy its good.[8]

We will see below how, in our setting, only two types of Markov perfect equilibria (MPE) are possible, one interior, which we denote *duopoly equilibrium*, whose prices corresponds to duopoly

[8] The Markovian nature of firms' price strategies descends here from the fact that the only feature of the game history which matters for the firms at every period t is the mass of informed and uninformed consumers who are willing to patronize their product in that period. For a more detailed analysis of Markovian games see Maskin and Tirole (2001), p. 192.

prices with information frictions, and *nearly-monopoly* prices, where only residual uninformed consumers continue to patronize one firm, and where, therefore, firms are implicitly playing the role of monopolists for uninformed consumers of their own country. As explained below, the existence of one or the other equilibrium crucially depends on countries' population asymmetries.

4.3 Duopoly equilibrium

Let us now proceed with the characterization of an interior MPE of the model, namely the duopoly equilibrium with information frictions.

Given the demands (11)-(12) of the two firms in every period, the *duopoly equilibrium* is simply given by the price profile (whenever interior) which maximizes the firms' profit functions at every period t:

$$\begin{aligned} \Pi_1\left(p_1\left(t\right), p_2(t)\right) &= \left[\left(1-\left(1-s\right)^{2^t}\right)\left(\theta(p_1(t),p_2(t))-a\right)+s^{2^t}\left(b-\theta(p_1(t),p_2(t))\right)\right]p_1, \\ \Pi_2\left(p_1\left(t\right), p_2(t)\right) &= \left[\left(1-s^{2^t}\right)\left(b-\theta(p_1(t),p_2(t))\right)+\left(1-s\right)^{2^t}\left(\theta(p_1(t),p_2(t))-a\right)\right]p_2. \end{aligned}$$

By the concavity of $\Pi_i(t)$ in $p_i(t)$ for $i=1,2$, the solution of the system of first order conditions yields the following unique equilibrium prices at every period t,

$$p_1^*\left(t\right) = \frac{1}{3}\left(u_2-u_1\right)\frac{b-2a+a\left(1-s\right)^{2^t}+bs^{2^t}}{1-\left(1-s\right)^{2^t}-s^{2^t}}, \tag{14}$$

$$p_2^*\left(t\right) = \frac{1}{3}\left(u_2-u_1\right)\frac{2b-a-a\left(1-s\right)^{2^t}-bs^{2^t}}{1-\left(1-s\right)^{2^t}-s^{2^t}}. \tag{15}$$

Notice that, since expressions (14)-(15) are monotonically decreasing in t for any s, both the positivity of equilibrium prices and the fully covered market assumption are guaranteed at any period by the condition on the support of consumers' preferences: $b>2a$.

Moreover, it is easy to see that the above equilibrium prices converge over time to their counterparts in a duopoly with vertically differentiated goods and fully informed agents, that we can write as

$$\lim_{t\rightarrow\infty}\left\{p_1^*(t)\right\}_t = p_1^* \text{ and } \lim_{t\rightarrow\infty}\left\{p_2^*(t)\right\}_t = p_2^*.$$

As additional findings, we can check whether the relationship existing between prices and countries' populations observed for period 1 extend, in general, to the multi-period setting. In particular, we highlighted how the relationship between prices and countries' sizes is not so straightforward: prices are nonmonotone, they are convex in s, and the price gap is large for s very small and small for s very large. Non-monotonicity and convexity are both due to the frequency of social interaction in every period; the behavior of price gap depends on which country, (either the biggest or the smallest) is producing which quality. This defines the intensity of competition.

By expressions (14)-(15) obtained in the duopoly equilibrium, we can state the following proposition.

Proposition 1 *Duopoly equilibrium prices at every period t are nonmonotone and convex withe respect to s before converging to the full information case. The price of good 1 (good 2) reaches its maximum value when the size of the country producing the low quality good is very large (very small).*

Proof. Before converging to the full information case, it is easy to see that in both expressions (14)-(15) the denominator is concave in s reaching a maximum for $s = 0.5$. In addition, the numerator of $p_1^*(t)$ can be either convex or concave in s depending on t, while the numerator of $p_2^*(t)$ is convex in s for any t. Both these facts directly imply that prices are nonmonotone in s for any t.

From the property that the product of two convex functions is a convex function and that the ratio of 1 over a concave function is itself a convex function directly proves that $p_2^*(t)$ is convex in s. Convexity of p_1^t is guarantied by the condition $b > 2a$. Indeed, the sign of second derivative of $p_1^*(t)$ is given by the sign of the expression $-(1-s)^{2^t}\left(s + s^{2^t}\right) + 2(1-s)s^{2^t}$ which is negative for $s' < s < s''$ given $b > 2a$. Finally, since the numerator of $p_1^*(t)$ (respectively $p_2^*(t)$) assumes highest (lowest) values in the neighborhood of $s = 1$ ($s = 0$) coupled with the symmetry of the denominator around $s = 0.5$, leads to the result that the price of good 1 (good 2) reaches its maximum value when the size of the country producing the low quality good is very large (very small), for any given $t = 1, 2, ..., T$. ■

From the above proposition it follows that:

Corollary 1 *Every period price gap $d(t) = (p_2^*(t) - p_1^*(t))$ as a function of s reaches its highest value when the size of the country producing the low quality good is relatively small and decreases monotonically with it.*

Proof. By using expressions (14)-(15), it is straightforward to see that the magnitude of the price gap in every period depends on $d(t) = a + b - 2a(1-s)^{2^t} - 2bs^{2^t}$, which is monotonically decreasing in s, for any $t > 0$. ■

Thus, all properties of the duopoly equilibrium prices observed for the first period, hold true for any period t. The forces that explains such properties are again: the impact of the population size asymmetry on the frequency of social interactions which determines prices convexity. The more symmetric countries' populations, the more frequent meetings, the more intense competition and thus the lower equilibrium prices. By contrast, the higher countries' size asymmetry, the smaller chances for mutants to appear, and therefore, the softer price competition leading to higher equilibrium prices. As an illustration, we represent in Fig 3, optimal prices corresponding to period 1 and 2. The curves in bold represent optimal prices in period 1. It is worth noticing not only how prices change as s increases, but also how both prices decrease as times go by. Last, Fig 3 highlights how the set of admissible values $s \in [s', s'']$ for which a *duopoly equilibrium* takes place, expands over time.

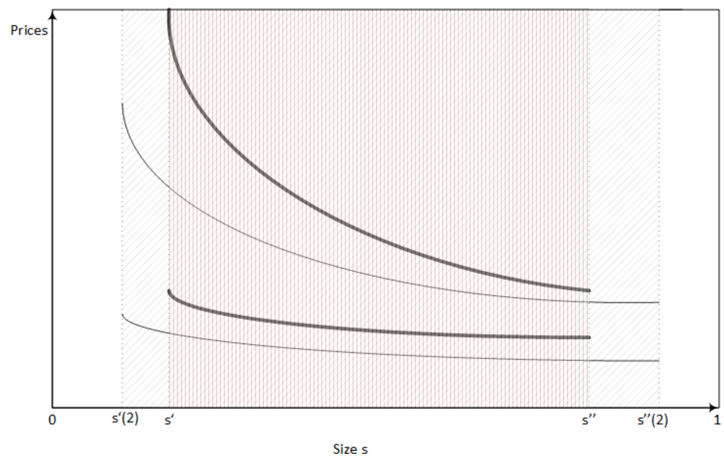


Figure 3: Optimal prices in period 1 and period 2 as a function of s.

Finally, the price gap dynamics is determined by which country (either the biggest or the smallest) is producing which quality. On the one side, if the low quality is produced in the large country, the information about the existence of a cheap good will spread fast enlarging the market for the low quality good. At the same time, the information about the existence of a higher quality good will spread, but at a lower pace. On the other side, if the low quality good is produced in the smaller country, the information about the existence of a higher quality good will spread faster, increasing the number of mutants in the smaller country that switch to the higher quality good consumption. Such dynamics make the equilibrium prices relatively low in the former scenario, and high in the latest scenario. In Fig 4, we depict the price gap as a function of s. As claimed in Corollary 1, the price gap decreases monotonically with s, but interestingly, for s exceeding 0.5, the price gap in period 2 may exceed the price gap in period 1. Depending on how large is s, the price gap in period 3 may be still larger than price gap in period 2 and so on, till to a certain finite period t, in which the price gap converges to the price gap of the full information duopoly, namely $1/3(u_2 - u_1)(a + b)$. This property shows the complex interplay between the size of the country producing the low quality and the frequency of social interactions that determines the speed of knowledge transmission. More specifically,

Corollary 2 *For $s < 0.5$, the price gap monotonically decreases in t and converges to the price gap of full information duopoly for $t \to \infty$, whereas, for $s > 0.5$, it monotonically increases in t and converges to the price gap of full information duopoly for $t \to \infty$.*

Proof. The difference $\Delta = d^t(s) - d^*(s)$ between the price gap in period t and the price gap under full information can be expressed as $\Delta = \frac{1}{3}(u_2 - u_1)(b-a)\frac{(1-s)^{2^t} - s^{2^t}}{1-(1-s)^{2^t} - s^{2^t}}$, whose sign is positive (negative) for any t whether s is lower (higher) than 0.5. Furthermore, $\lim_{t\to\infty} \Delta = 0$. ■

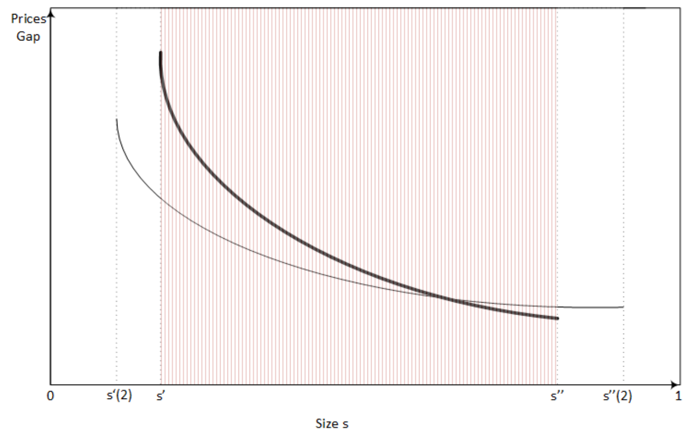


Figure 4: Price gap in period 1 and period 2 as a function of s.

Notice that if the size of the two countries is symmetric ($s = 0.5$), thus the price gap between firms is equal to the price gap of the duopoly under full information immediately after opening trade at $t = 1$ and remains at this level thereafter, even though optimal prices require some periods to converge to the duopoly prices under full information.

Given every period duopoly prices (14)-(15), the equilibrium marginal consumer in any period t is simply obtained as:

$$\theta^*(t) = \frac{1}{3}\frac{a+b-2\left(a(1-s)^{2^t} + bs^{2^t}\right)}{1-(1-s)^{2^t} - s^{2^t}}. \tag{16}$$

It is important now to recall that the above equilibrium prices are *duopoly* equilibrium prices if and only if they induce a marginal consumer whose location lies at the interior of the support of consumers' willingness to pay $[a, b]$. For period $t = 1$, we have defined two thresholds $s'$and s'' of country sizes that guarantee that the location of the marginal consumer at $t = 1$ lies inside $[a, b]$. Similarly, for any period t, we can implicitly define two intertemporal thresholds $s'(t)$ and $s''(t)$ by solving $\theta^*(t) - a = 0$ and $b - \theta^*(t) = 0$ in s, correspondingly, for any t. Given the fundamentals of the model, these thresholds are key to determine the type of equilibria arising from the model: *duopoly vs. nearly-monopoly.*

Before describing in details the nearly monopoly equilibrium, we highlight the dynamics of the location of the marginal consumer over time.

The next proposition describes how the country size asymmetry interplays with the effect of time to determine the location of the marginal consumer.

Lemma 1 *The marginal consumer $\theta^*(t)$ dynamics is a function of the countries' size distribution. In particular, there exist three cases: (i) for $s = 0.5$, $\theta^*(t)$ converges to the full information duopoly θ^* already at $t = 1$; (ii) for $s < 0.5$, $\theta^*(t)$ is monotonically decreasing in t and converges to $\theta^*(t)$ for $t \to \infty$; and finally, (iii) for $s > 0.5$, $\theta^*(t)$ is monotonically increasing in t and converges to θ^* for $t \to \infty$.*

Proof. Evaluating the expression (16) at $t = 1$ and $s = 0.5$, we obtain θ^*, and it remains constant at θ^* for any t. Note also that the sign of the partial derivative of $\theta^*(t)$ with respect to t is the sign of expression $\ln(1-s)(1-s)^{2^t}\left[1-2s^{2^t}\right] - s^{2^t}(\ln s)\left[1-2(1-s)^{2^t}\right]$, which has positive sign for $s > 0.5$ and negative for $s < 0.5$. Finally, $\lim_{t\to\infty}\theta^*(t) = \theta^*$. ■

Furthermore, the above expression for $\theta^*(t)$ lies inside the interval $[a, b]$ for $s \in (s'(t), s''(t))$. Clearly, the fact that $\theta^*(t)$ lies in $[a, b]$ embodies the condition that $p_2^*(t) > p_1^*(t)$, because otherwise, if $p_2^*(t)$ would be smaller than $p_1^*(t)$, no consumer would buy good 2 and $\theta(t)$, in turn, would not lie within the interval $[a, b]$.

Finally, we can summarize the conditions defining the interior duopoly equilibrium:

Proposition 2 *At any given t, if $s \in (s'(t), s''(t))$, namely for a not too high country size asymmetry, the MPE prices of the multi-period market correspond to the duopoly prices given by (14)-(15).*

To conclude this section, the profit function of the firms at any period t are:

$$\begin{aligned}
\Pi_1^*(t) &= \frac{1}{9}(u_2 - u_1)\frac{\left(b - 2a + a(1-s)^{2^t} + bs^{2^t}\right)^2}{1-(1-s)^{2^t} - s^{2^t}}, \\
\Pi_2^*(t) &= \frac{1}{9}(u_2 - u_1)\frac{\left(2b - a - a(1-s)^{2^t} - bs^{2^t}\right)^2}{1-(1-s)^{2^t} - s^{2^t}}.
\end{aligned}$$

These expression will be useful to analyse the profitability of trade openness examined in Section 5.

4.4 Nearly-monopoly equilibrium

A careful analysis of the MPE prices reveals that in our intertemporal setting, two substantially different equilibria may arise, one interior, that we have labelled *duopoly equilibrium*, and another one, which we have defined *nearly-monopoly* equilibrium. Let us discuss here some of the features of nearly-monopoly equilibria.

The analysis of period one equilibrium prices has revealed that a duopoly equilibrium can arise only when $s \in (s'(t), s''(t))$, namely when the two countries are not "too asymmetric" in terms of their populations. Alternatively, the nearly monopoly equilibrium can occur when $s \notin (s'(t), s''(t))$, namely when the two countries are extremely asymmetric in size. Two different types of nearly-monopoly equilibria may actually arise. The first occurs for $s < s'(t)$, namely when the low quality good is produced in one country that, compared to the other, is very small. Then, using expression (16), the marginal consumer location falls at its lowest bound a just because, by construction, cannot be smaller than a. This implies that all consumers of country 1, whether informed, prefer to buy good 2. Accordingly, the firms' demand functions in period t write as

$$
\begin{aligned}
D_1(t) &= \mathcal{U}_2^t (b-a) = s^{2^t} (b-a), \\
D_2(t) &= \mathcal{I}_2^t (b-a) = \left(1 - s^{2^t}\right) (b-a).
\end{aligned}
$$

Only the uninformed consumers of country 1 keep buying good 1, whereas all informed consumers prefer to buy good 2. The corresponding prices are given by monopoly prices (2) and (3) which are the highest fully covered market prices.

The second type of nearly-monopoly arises when $s > s'(t)$, namely, when the low quality good is produced in a very large country (compared to the country producing the high quality good). Then, using expression (16), the marginal consumer locates at the upper bound b because by construction cannot be higher than b. This implies that all consumers of country 2, when informed, will buy good 1 and the demand functions for each firm at period t write as

$$
\begin{aligned}
D_1(t) &= \mathcal{I}_1^t (b-a) = \left(1 - (1-s)^{2^t}\right) (b-a), \\
D_2(t) &= \mathcal{U}_1^t (b-a) = \left(1 - s^{2^t}\right) (b-a).
\end{aligned}
$$

Given the above expressions, the nearly monopoly prices are simply given by the monopoly prices (2) and (3). We summarize below these results.

Proposition 3 *At a given period t, for $s \notin (s'(t), s''(t))$, namely if the two countries size asymmetry is sufficiently high, the fully covered equilibrium prices of the multi-period market coincides with the nearly-monopoly prices (2) and (3).*

The one described in Proposition 3 is a situation in which either the size of country one or country two are proportionally too big compared to that of the other country. In turn, this makes the price gap between the two goods either too big or two small for one of the goods to remain attractive for consumers. Only uninformed consumers continue to patronize the good produced by the larger country, whose mass progressively shrinks over time. Thus, the model reveals that the knowledge transmission among the two countries ultimately plays a balancing role in the market. When the time goes by and the mass of informed consumers increases, the excessive number of people living in the bigger country and only purchasing the domestic good progressively shrinks, thus driving the price gap once again within a reasonable range, and allowing a duopoly market equilibrium to arise. This property of the nearly monopoly equilibrium to be a transitory phenomenon is expressed in the next proposition.

Proposition 4 *Nearly monopoly equilibria may only last for a finite number of periods after which the market transforms into a duopoly.*

Proof. The switch from the equilibrium that we denote *nearly-monopoly* to the duopoly equilibrium occurs by definition when the value of the marginal consumer $\theta^*(t)$ returns inside the interval $[a, b]$, meaning that, once again, both firms face the competition of the firm located in the other country. We proved above that $\lim_{t\to\infty} \{\theta^*(t)\}_t = \frac{1}{3}(a+b)$, meaning that over time, when the information on the existence of the two goods is fully unveiled to the consumers living in the two countries, the market ends up behaving as a full information duopoly, with the marginal consumer $\theta(t)$ laying at the interior of the interval of consumers' willingness to pay $[a, b]$. In addition, the sequence $\{\theta^*(t)\}_t$ is either monotonically decreasing (increasing) in t for $s > 0.5$ ($s < 0.5$) or constant, for $s = 0.5$. Thus, there does necessarily exist a finite period $t = T$ for which $\theta^*(t)$ reaches the interior of the interval $[a, b]$, either from the right or from the left (respectively for $s < s'$ and $s > s''$). This return inside interval $[a, b]$ brings to an end the phenomenon of nearly monopoly. ■

A monopoly equilibrium arises when the size s is quite far from $s = 0.5$, meaning that the asymmetry in the two countries' populations is quite high. A very small size s causes two forces into place, which in turn determine that $p_2^*(t)$ is too high compared to $p_1^*(t)$. First, because of the very high number of consumers who know only the high quality good, the high quality firm is not under much competitive pressure from the bottom quality one and, thus, does not lose market share even when setting prices which are relatively high. Second, the small size s determines a low chances of knowledge transmission, keeping high the market power of the high quality firm. This second force involves size asymmetry and works similarly to the monopoly equilibrium arising when s is very large. In this case, the low quality good is only purchased by ignorant consumers living in country 1. Proposition 4 show that, as time goes by, the nearly-monopoly equilibrium can never persist. This is because the increasing number of mutants switching from one market to the other over time, decreases the demand share of nearly-monopolists given by $(1-s)^{2^t}$ or s^{2^t}.

Trade openness, therefore, always breaks national monopolies, either in a quick or in a slow pace, which ultimately depends on the fraction of people living in the two countries.

5 Is Openness Profitable ?

In this section, we analyse how openness affects consumers and firms as compared to the autarky outcomes. Are there winners and losers? Are firms and the consumers better off as the two countries open to trade and exchanges? In our setup, the answer is certainly positive for consumers. They have the chance to choose among vertically differentiated goods and both prices decrease as a consequence of competition.

By contrast, openness has an ambiguous overall effect on firms profits because exchanges determine two contrasting effects on profits. On the one hand, openness enlarges markets served for each firm. Some consumers of country one will meet consumers of country two and will become mutants consuming good 2 and thus enlarging the market share of good 2, as shown in Fig 1. Yet, some consumers of country two will also meet consumers of country one and some may become mutants consuming good 1, and thus increasing the demand for good 1. Clearly, how much each firm see its demand expanding depends on the how many social interactions take place and how different are the level of willingness to pay u_1 and u_2.

On the other hand, openness leads to a more intense competition as consumers learn about the two goods. Again, the intensity of competition depends on the intensity of interactions that is ultimately determined by countries' size asymmetry, as well as, the level of u_1 and u_2.

To identify the sign of the overall effect for every period t, we build an analysis consisting on three steps. (i) First, for each firm, we compare the monopoly profits in period $t = 0$ with the profits in period $t = 1$ under a duopoly market. (ii) Second, for each firm, we compare profits in period $t = 0$ with profits in period T, period in which the market structure has converged at the duopoly market with full information. (iii) Then, in the third step, using the properties of monotonicity of profit function with respect to time, we analyse the convergence, if any, of the profit function of each firm at T. We will limit the analysis in this section to the case where openness leads to a duopoly equilibrium in $t = 1$, neglecting the nearly-monopoly solution. Analysing the duopoly scenario is sufficient to identify the salient features of openness on firms' profit.

Starting with the first step analysis, let us define

$$\breve{u} \equiv \frac{1}{18}\frac{\left(b-a+as^2+bs^2-2as\right)^2}{s^2\left(1-s\right)\left(b-a\right)a} \text{ and } \hat{u} \equiv \frac{1}{18}\frac{\left(2a-2b+as^2+bs^2-2as\right)^2}{s\left(1-s\right)^2\left(b-a\right)a}$$

where s satisfies the condition $s' < s < s''$. Then, a direct comparison of profits for the two firms in period 1 and period 0, gives:

$$\{\Pi_1(t)\}_{t=1} - \{\Pi_1(t)\}_{t=0} \quad > \quad 0 \Leftrightarrow u_1 < \frac{\check{u}}{(1+\check{u})} u_2$$
$$\{\Pi_2(t)\}_{t=1} - \{\Pi_2(t)\}_{t=0} \quad > \quad 0 \Leftrightarrow u_1 < \frac{\hat{u}-1}{\hat{u}} u_2.$$

Then, openness to trade and exchanges is profitable, in $t = 1$, to both firms iff

$$u_1 < \min\left\{\frac{\check{u}}{(1+\check{u})} u_2; \frac{\hat{u}-1}{\hat{u}} u_2\right\}. \tag{17}$$

By contrast, none of the firms has advantages from openness and would rather prefer closed economies in $t = 1$, iff

$$u_1 > \max\left\{\frac{\check{u}}{(1+\check{u})} u_2; \frac{\hat{u}-1}{\hat{u}} u_2\right\}. \tag{18}$$

Finally, the position of the firms towards free exchanges is asymmetric with one being favorable and the other being against in $t = 1$ if

$$\min\left\{\frac{\check{u}}{(1+\check{u})} u_2; \frac{\hat{u}-1}{\hat{u}} u_2\right\} < u_1 < \max\left\{\frac{\check{u}}{(1+\check{u})} u_2; \frac{\hat{u}-1}{\hat{u}} u_2\right\}.$$

In absence of strong size asymmetry between countries (i.e. $s' < s < s''$), openness is profitable, in the short run, to both firms only when the utility from consuming the low quality good is quite smaller (i.e. (17) holds) than the quality of consuming the high quality good because openness enlarges the market share of both firms. By contrast, if the utility from consuming the low quality good is not very different from the utility of consuming the high quality good (i.e. (18) holds), then openness to trade leads to a more intense competition that is detrimental for both firms. And, when the utility from consuming good 1 has intermediate values, then, one of the two firms disfavors openness to exchanges. More specifically, exchanges are detrimental for the profits of the low quality good if $\frac{\check{u}}{(1+\check{u})} < \frac{\hat{u}-1}{\hat{u}}$; whereas, openness is detrimental for the profits of the high quality good if $\frac{\check{u}}{(1+\check{u})} > \frac{\hat{u}-1}{\hat{u}}$. It is easily verifiable that both inequalities are possible depending on the value of s.

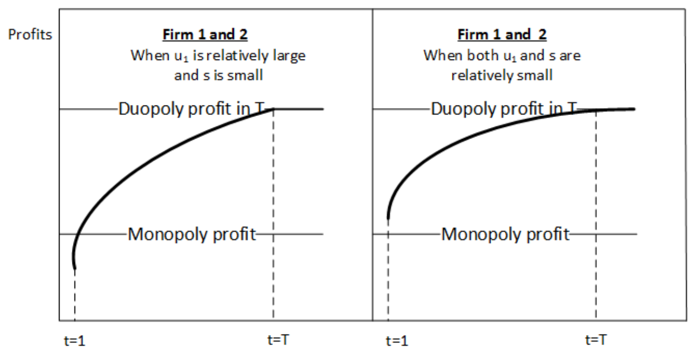
We turn now to the second step analysis, where we compare profits under monopoly in period $t = 0$ with the profits of duopoly under full information, finding that for firm 2 duopoly profits are always higher than the monopoly profit, under the assumption of fully covered market (see appendix B). For firm 1, the same hold as long as size asymmetry satisfies the condition $s' < s < \bar{s}$[9]. By contrast, monopoly profits are higher than duopoly profits if $\bar{s} < s < s''$.

[9]The threshold $\bar{s} = \frac{1}{9}\frac{(2a-b)^2}{a(b-a)}\frac{(u_2-u_1)}{u_1}$ is defined in Appendix B. It can be shown that $\bar{s}$ can lay between s' and s''.

Finally, in the last step, as shown in appendix B, we verify that the per-period profit $\{\Pi_i^*(t)\}_t$ of each firm $i = 1, 2$, converges to the full information duopoly:

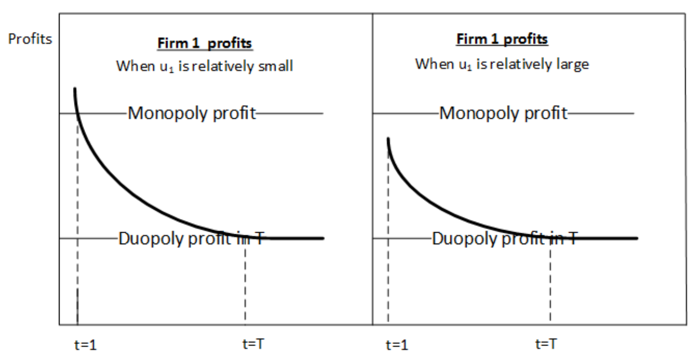
$$\lim_{t \to \infty} \{\Pi_i^*(t)\}_t = \Pi_i^*.$$

Using the results of this three-step analysis, we can now built the dynamics of firms' profits. Graphically, we obtain the following possibilities represented in Figure 5 and 6. When the size asymmetry belongs to the range: $s' < s < \bar{s}$, then the profits' dynamics of both firms are represented in Figure 5.



Profit dynamics for both firms for relatively small s

Last, when size asymmetry lies in the interval $\bar{s} < s < s''$, and only for firm 1, entry can be detrimental as shown in Figure 6. In this range of size asymmetry, firm 1 certainly looses from entry in the long run while its profits may increase due to openness in the short run.



Profit dynamics for firm 1 for relatively large s.For firm 2, profits dynamics is as in Fig 5.

Hence, we can summarize our results in the following proposition:

Proposition 5 *Assume a duopoly equilibrium occurs in period $t = 1$. (i) When $s' < s < \bar{s}$, openness is profitable in every t, for both firms, if u_1 is sufficiently large (17 holds). Openness can be detrimental for few periods then returning profitable for both firms if u_1 is sufficiently small*

(18 holds). (ii) Instead, when $\bar{s} < s < s''$ and u_1 is sufficiently small (18 holds), then, openness is detrimental in every period for the firm producing the low quality good. Openness can be profitable for some periods then returning detrimental for the low quality firm. For the high quality firm, the same profit dynamics occurs as described in (i).

Proof. In Appendix B. ■

Recall that the opening of trade has two effects on the firms' profits. On the one side, there is a competition effect, since firms face a new foreign competitor. On the other side, there is a market expansion effect, since with trade liberalization firms can sell in an additional market. In general, the high quality firm, which is furthermore located in the large country, tends to gain more from free trade. However, low quality firms located in the small country might still benefit from trade, because they can sell in a larger market than the domestic one. This happens when the asymmetry in size between the countries is very pronounced. The opposite might happen to the low quality good firm operating in a country with relatively high population size ($\bar{s} < s < s''$). Competition effect is larger than the market enlarging effect leading to detrimental effects on firms 1' profits. For the high quality firm, openness remains profitable also in the latest scenario because information diffusion enlarges its market size offsetting the competition effect.

6 Concluding Remarks

This paper introduces a multi-period setting to explore the effect of interpersonal social interaction among individuals of different countries, the trade flows occurring among these countries and the prices of exchanged goods, assumed here vertically differentiated. In particular, we explore how prices change along the sequence of equilibria generated by the dynamics of individual interaction and we wonder whether the common market drives the initial monopoly prices to the duopoly market solution. Our analysis shows that at the limit the market prices tends to align with the standard duopoly solution. However, the convergence can take two different paths: if countries sizes are relatively similar, interpersonal meetings between consumers of each country are relatively frequent and the market evolution from monopoly to duopoly occurs already at the first period. On the contrary, if the countries differ significantly in size, meetings of consumers are rare and, thus, competition does not succeed to drive the monopoly price to the duopoly price before a significant period of time (denoted *nearly-monopoly*). However, there always exists a time period at which the informed consumers are sufficiently numerous to drive the market to the standard duopoly configuration.

It is worth noticing that it is possible to extend our analysis to other market contexts than vertical differentiation markets. For instance, we can use our approach for analysing the opening of initially two closed markets for a homogeneous good. This analysis has been performed by the authors in Gabszewicz et al. 2017.

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Appendix A: Population dynamics

A.1.The mass of informed and uninformed agents on good 1 over time

Let denote I_1^t the *new* proportion of *agent* who have become *informed* on good 1 at time t and U_1^t the proportion of those still remaining uninformed. Let us also denote $\mathcal{I}_1^1$ (resp. $\mathcal{U}_1^t$) the total fraction of all agents in the two country informed (resp. uninformed) on good 1 existence at time t.

At $t = 0$ only a proportion s of agents knows about good 1 (monopoly): $\mathcal{I}_1^0 = s$, $\mathcal{U}_1^0 = 1 - s$;

At $t = 1$, an additional proportion of agents become informed on good 1, and, therefore:

$$\mathcal{I}_1^1 = s + (1-s)s = s\,(2-s)\,,$$

and

$$\mathcal{U}_1^1 = 1 - (s + (1-s)s) = (1-s)^2\,.$$

At $t = 2$, the newly informed agents on good 1 are:

$$I_1^2 = \Pr\left\{\left(\mathcal{I}_1^1 \cap \mathcal{U}_1^1\right)\right\} = s\,(2-s)\,(1-s)^2\,,$$

and, hence

$$\mathcal{I}_1^2 = \mathcal{I}_1^1 + I_1^2 = s\,(2-s) + s\,(2-s)\,(1-s)^2 = s\,(2-s)\,(1+(1-s)^2),$$

$$\mathcal{U}_1^2 = 1 - s\,(2-s)\,(1+(1-s)^2) = (1-s)^4\,.$$

At $t = 3$, the agents becoming informed on good 1 are given by:

$$\begin{aligned} I_1^3 &= \Pr\left\{\left(\mathcal{I}_1^2 \cap \mathcal{U}_1^2\right)\right\} = \left(s\,(2-s)\,(1+(1-s)^2)\right)(1-s)^4 = \\ &= \left(s^2-2s+2\right)(2-s)\,(s-1)^4\,s. \end{aligned}$$

and,

$$\begin{aligned} \mathcal{I}_1^3 &= \mathcal{I}_1^2 + I_1^3 = s\,(2-s)\,(1+(1-s)^2) + \left(s^2-2s+2\right)(2-s)\,(s-1)^4\,s = \\ &= s\,(2-s)\left(s^2-2s+2\right)\left(6s^2-4s-4s^3+s^4+2\right). \end{aligned}$$

Since

$$\mathcal{U}_1^3 = 1 - \left(s\,(2-s)\left(s^2-2s+2\right)\left(6s^2-4s-4s^3+s^4+2\right)\right) = (1-s)^8\,,$$

the total proportion of informed agents at $t = 3$ on good 1 is obtained as

$$\mathcal{I}_1^3 = 1 - \mathcal{U}_1^3 = 1 - (1-s)^8 .$$

At $t = 4$, the proportion of newly informed consumers on good 1 is given by:

$$\begin{aligned} I_1^4 &= \Pr\left\{\left(\mathcal{I}_1^3 \cap \mathcal{U}_1^3\right)\right\} = (1-s)^8 \left(1-(1-s)^8\right) = \\ &= \left(6s^2 - 4s - 4s^3 + s^4 + 2\right)\left(s^2 - 2s + 2\right)(2-s)\, s\,(s-1) . \end{aligned}$$

which once again shows that

$$\begin{aligned} \mathcal{I}_1^4 &= \mathcal{I}_1^3 + I_1^4 = 1 - (1-s)^8 + \left(6s^2 - 4s - 4s^3 + s^4 + 2\right) \\ \left(s^2 - 2s + 2\right)(2-s)\, s\,(s-1)^8 &= s\,(2-s)\left(s^2 - 2s + 2\right) \times \\ &\quad \times \left(28s^2 - 8s - 56s^3 + 70s^4 - 56s^5 + 28s^6 - 8s^7 + s^8 + 2\right) \\ &\quad \times (6s^2 - 4s - 4s^3 + s^4 + 2), \end{aligned}$$

and

$$\begin{aligned} \mathcal{U}_1^4 &= 1 - s\,(2-s)\left(s^2 - 2s + 2\right) \times \\ &\quad \times \left(28s^2 - 8s - 56s^3 + 70s^4 - 56s^5 + 28s^6 - 8s^7 + s^8 + 2\right) \\ \times \left(6s^2 - 4s - 4s^3 + s^4 + 2\right) &= (1-s)^{16} . \end{aligned}$$

This implies that

$$\begin{aligned} \mathcal{U}_1^4 &= (1-s)^{16} , \\ \mathcal{I}_1^4 &= 1 - (1-s)^{16} . \end{aligned}$$

The above expressions show that the same information path occurs at every period and, in general, for good 1 the total fraction of informed and uninformed agents at any arbitrary period t is simply given by:

$$\mathcal{I}_1^t = 1 - (1-s)^{2^t} \text{ and } \mathcal{U}_1^t = (1-s)^{2^t} .$$

Morever, notice that, consistently, for any $s \in (0,1)$,

$$\begin{aligned} \mathcal{I}_1^\infty &= \lim_{t\to\infty} \left\{1 - (1-s)^{2^t}\right\} = 1 \text{ and} \\ \mathcal{U}_1^\infty &= \lim_{t\to\infty} \left\{(1-s)^{2^t}\right\} = 0. \end{aligned}$$

A.2. The mass of informed and uninformed agents on good 2 over time

As above, let us denote $\mathcal{I}_2^t$ (resp $\mathcal{U}_2^0$) the total proportion of informed agents on good 2 at time t.

At $t = 0$ only a fraction $(1 - s)$ of agents is aware of good 2 (monopoly) and, therefore, $\mathcal{I}_2^0 = 1 - s$. $\mathcal{U}_2^0 = s$.

At $t = 1$: $\mathcal{I}_2^1 = \mathcal{I}_2^0 + I_2^1 = (1 - s) + (1 - s)s = 1 - s^2$, $\mathcal{U}_2^1 = s^2$;

At $t = 2$, the proportion of newly informed consumers is given by:

$$I_2^1 = \Pr\left\{\left(\mathcal{I}_2^1 \cap \mathcal{U}_2^1\right)\right\} = s^2\left(1 - s^2\right),$$

and

$$\begin{aligned} \mathcal{I}_2^2 &= \left(1 - s^2\right) + s^2\left(1 - s^2\right) = 1 - s^4 \\ \mathcal{U}_2^2 &= 1 - 1 - s^4 = s^4. \end{aligned}$$

At $t = 3$, the set of newly informed is given by:

$$I_2^3 = \Pr\left\{\left(\mathcal{I}_2^2 \cap \mathcal{U}_2^2\right)\right\} = \left(1 - s^4\right)s^4,$$

and

$$\begin{aligned} \mathcal{I}_2^3 &= 1 - s^4 + \left(1 - s^4\right)s^4 = 1 - s^8; \\ \mathcal{U}_2^3 &= 1 - 1 - s^8 = s^8. \end{aligned}$$

At $t = 4$, the newly informed consumers are given by:

$$I_2^4 = \Pr\left\{\left(\mathcal{I}_2^3 \cap \mathcal{U}_2^3\right)\right\} = \left(1 - s^8\right)s^8,$$

so that

$$\mathcal{I}_1^4 = \mathcal{I}_2^3 + I_2^4 = 1 - s^8 + \left(1 - s^8\right)s^8 = 1 - s^{16}$$

and

$$\mathcal{U}_2^4 = s^{16}.$$

Thus, in general, for good 2, the total fraction of informed and uninformed agents show a recurrent path and, in any period t is just equal to:

$$\mathcal{I}_2^t = 1 - s^{2^t} \text{ and } \mathcal{U}_2^t = s^{2^t}.$$

Finally, notice that, again, consistently, for any $s \in (0, 1)$:

$$\begin{aligned} \mathcal{I}_2^\infty &= \lim_{t\to\infty}\left\{1 - s^{2^t}\right\} = 1 \\ \mathcal{U}_1^\infty &= \lim_{t\to\infty}\left\{s^{2^t}\right\} = 0. \end{aligned}$$

7 Appendix B: Analysis of openness profitability

The analysis of step 1 is presented in Section (5).

Step 2: we sign the differences $\{\Pi_i\}_{t=0} - \{\Pi_i\}_{t=T}$, $i = 1, 2$.

For firm 1, directly comparing the profits $\{\Pi_1\}_{t=0}$ and $\{\Pi_1\}_{t=T}$, we have that the monopoly profit in $t = 0$ is higher than full information duopoly profit at $t = T$ iff

$$s > \bar{s} \equiv \frac{1}{9} \frac{(2a-b)^2}{a(b-a)} \frac{(u_2 - u_1)}{u_1} \tag{19}$$

This threshold remains within the interval $s' < s < s''$.

For firm 2, the difference $\{\Pi_2\}_{t=0} - \{\Pi_2\}_{t=T}$ is positive if

$$s < s'' \equiv \frac{1}{9} \frac{(4b-5a)(b-2a)u_2 - (a-2b)^2 u_1}{au_2(b-a)} \tag{20}$$

We prove that this conditon is not compatible with $s' < s < s''$, because $s'' < s'$. We take the difference $s' - s''$ writing $b = ak$ with $k > 2$, obtaining:

$$\frac{(9-9k)\sqrt{2-6k+5k^2} - 111k + 87k^2 - 16k^3 + 38}{36k^2 - 54k + 18} + \frac{1}{9} u_1 \frac{(2k-1)^2}{u_2(k-1)}.$$

If this is positive at the smallest value of u_1, namely $u_1 = \frac{k-2}{k+1} u_2$, then, it will always be positive. At the smallest value for u_1 the difference is

$$\frac{1}{6} \frac{(3-3k^2)\sqrt{2-6k+5k^2} - 33k + 12k^2 + 5k^3 + 14}{(k+1)(2k-1)(k-1)},$$

which is always positive for $k > 2$.

Step 3: Finally, we check how the duopoly profit behaves as a function of t. More precisely, it can be checked easily that

$$\begin{aligned}
\lim_{t\to\infty} \{\Pi_1\}_t &= \lim_{t\to\infty} \left(\frac{1}{9}(u_2 - u_1) \frac{\left(b - 2a + a(1-s)^{2^t} + bs^{2^t}\right)^2}{1 - (1-s)^{2^t} - s^{2^t}} \right) = \frac{1}{9}(u_2 - u_1)(2a-b)^2, \\
\lim_{t\to\infty} \{\Pi_2\}_t &= \lim_{t\to\infty} \left(\frac{1}{9}(u_2 - u_1) \frac{\left(2b - a - a(1-s)^{2^t} - bs^{2^t}\right)^2}{1 - (1-s)^{2^t} - s^{2^t}} \right) = \frac{1}{9}(u_2 - u_1)(2b-a)^2.
\end{aligned}$$