

# Truthful Equilibria in Generalized Common Agency Models

Ilias Boultzis\*

July 3, 2019

**Keywords:** common agency, truthful equilibria, externalities, efficiency  
JEL classification: C72, D44, D43, D72

## Abstract

In this paper I discuss truthful equilibria in common agency models. Specifically, I provide general conditions under which truthful equilibria are plausible, easy to calculate and efficient. These conditions generalize similar results in the literature and allow the use of truthful equilibria in novel economic applications. Moreover, I provide two such applications. The first application is a market game in which multiple sellers sell a uniform good to a single buyer. The second application is a lobbying model in which there are externalities in contributions between lobbies. This last example indicates that externalities between principals do not necessarily prevent efficient equilibria. In this regard, this paper provides a set of conditions, under which, truthful equilibria in common agency models with externalities are efficient.

## 1 Introduction

The common agency model is a game in which many principals share a common agent. In this model, the principals submit bids to the agent, so as to influence his actions. In order to achieve their goal, the principals

---

\*Department of Economics, Athens University of Economics and Business, e-mail: [boultzis@gmail.com](mailto:boultzis@gmail.com). I would like to thank Apostolis Philippopoulos for our discussions, in an early stage of this paper and Katerina Katsandredaki for her assistance in the editing of this paper. The usual disclaimer applies.

condition their bids on the agent's actions. Economists apply this model in many areas of economic research. One of these areas is lobbying. Specifically, in lobbying models, the bids are contributions that lobbies (principals) offer to a government official (agent) in return for favourable policy decisions. Other fields of application, include industrial organization, public economics e.t.c.

Common agency models often have many equilibria. However, truthful equilibria are probably the most popular among them. In these equilibria, the principals use truthful strategies. These strategies exactly reflect the principals' true preferences over the agent's possible actions. Following Bernheim and Whinston (1986), Dixit et al. (1997) argued that truthful equilibria are focal because they have three important properties: they are easy to calculate, Pareto efficient and plausible. The last property means that the best response set of the principals always contains a truthful strategy.

However, despite their merits, truthful equilibria can not be used in all common agency models. This is so, because they are motivated by lobbying games. Following this motivation, truthful equilibria are used in applications that belong in the general setting associated with lobbying models. This setting consists of three key assumptions. First, the utility of the principals is decreasing in their bids. Second, the utility of the principals depends only on own bids. Third, the utility of the agent is increasing in all bids offered by the principals. These three assumptions define the standard common agency model discussed by Bernheim and Whinston (1986) and Dixit et al. (1997). Furthermore, under these assumptions, Dixit et al. (1997) prove that truthful equilibria are easy to calculate efficient and plausible.

Nevertheless, many applications of common agency do not fit this restricted framework. Thus, a question arises. Is it possible to find a broader set of conditions under which truthful equilibria are valid? In this paper, I attempt to answer this question. Specifically, I provide general conditions under which truthful equilibria can be used. These conditions generalize the results of Dixit et al. (1997) and allow the use of truthful equilibria in novel economic applications.

In this respect, I find that conflict of interests between principals and agent is crucial. For example, truthful strategies in lobbying imply that lobbies contribute more in exchange for the policies they prefer. However, this course of action does not make any sense if the government official dislikes the contributions he receives. Thus, truthful equilibria work, only if the principals and the agent have conflicting interests over bids.

I formulate this idea in the following manner. Consider any feasible allocation of principals' bids and agent's actions. Then, there is conflict of interests, if for a given agent's action, there is no reallocation of bids that

makes both principals and agent better off. This property always holds in the standard model, in which the utility of the agent is increasing in bids, while the utility of the principals is decreasing in the same variable. However, I identify two more cases that satisfy conflict of interests. For both these cases, I prove that truthful equilibria are efficient, easy to calculate and plausible.

The first case is the reverse of the standard model. As an example consider a group of sellers (principals), selling a uniform good to a single buyer (agent). First, each seller presents the buyer with a price menu. The prices in this menu depend on the amount of the good that the buyer wishes to buy. Then, the buyer decides on the quantity he buys from each seller. Typically, the utility of the sellers increases, while the utility of the buyers decreases following a rise in prices. Thus, there is conflict of interests over prices, which are the bids in this example.

The second case discusses externalities in bids, among principals. These externalities, occur naturally when principals are interrelated in other ways besides sharing a common agent. For example, think of a model in which two duopolists buy an input from a monopolist. The duopolists, announce the quantities that they are willing to buy at each price and the monopolist chooses the selling price. In this situation the duopolists are the principals and the monopolist is the common agent. However, the profits of each duopolist also depend on the output of the other. In turn, each output depends on the corresponding input. Thus, externalities in bids, in this case inputs, emerge.

In general, externalities in bids can destroy conflict of interests. For example, consider a lobbying game with positive externalities ( greater contributions by principal  $j$ , mean more utility for principal  $i$  ). In this case, an increase in all contributions might not only increase the utility of the government official, but also the utility of the lobbies. Specifically, following an increase in contributions, the lobbies face two opposing effects on their utility. First, a direct negative effect, which is due to the increase in own contributions. Second, an indirect positive effect because of the increase in other contributions. If the indirect effect dominates the direct effect, an increase in all contributions increases the utility of both principals and agent. This situation violates conflict of interests.

However, it turns out that under certain conditions this problem does not occur. In particular, if the externalities are relatively small, the influence of own bids on the utility of the principals dominates the cross effect of the other bids. As a result, the conflict of interests between principals and agent survives, despite externalities. The same is true for negative externalities, regardless of their size, if the utility functions of the principals are symmetric and quasi-concave with respect to bids. Under these conditions, I show that

truthful equilibria have all three properties identified by Dixit et al. (1997), including efficiency.

In this regard, my model belongs in the strand of literature that explores the robustness of truthful equilibria to various extensions of the standard model. Dixit et al. (1997) pioneered this literature by extending the model of Bernheim and Whinston (1986) to general utility functions. Other important examples of this literature include Bergemann and Välimäki (2003) who consider dynamic common agency, Prat and Rustichini (2003) who discuss multiple agents and Martimort and Stole (2009b) who consider asymmetric information. Furthermore, my paper relates to the literature discussing common agency with externalities in bids. Examples of this literature are Peters (2001), Martimort and Stole (2002), Peters and Szentes (2012), Szentes (2014) and Galperti (2015). The papers in this strand of literature discuss models with asymmetric information. In this context, they allow bids to depend on the bidding strategies of the other principals. They use this assumption to derive powerful insights on the nature of equilibria in such games. These papers relate to my work to the extent that their results also apply to symmetric information models. In this respect, my contribution is that truthful equilibria can be efficient, even if bids do not depend on other bidding strategies.

The rest of the paper is organised as follows. Section 2 describes the model, section 3 presents the key results and section 4 considers economic applications. Section 5 discusses the general issue of efficiency of equilibria and the relationship between my paper and the existing literature on externalities. Section 6 concludes. Finally, appendix A contains the most important proofs and calculations, while appendix B, which is not intended for publication, contains the rest of the proofs and some examples.

## 2 Model

### 2.1 Setting

Before I continue with the details of the model, let me introduce some notation that I use in the rest of the paper. The index  $i$  runs from 1 to  $n$ . Furthermore, except when otherwise stated, I use Latin letters in the following manner. Consider for example the lower-case Latin letter “ $x$ ”. Then,  $x_i$  is a real number,  $x$  is the vector of all  $x_i$ ,  $x_{-i}$  is the vector containing all members of  $x$  except  $x_i$ ,  $\tilde{x} = \sum_i x_i$  and  $\tilde{x}_{-j} = \sum_{i \neq j} x_i$ . Moreover, I use the symbol  $x_i(\cdot)$  to describe a function  $x_i : Z \rightarrow R$ , such that  $x_i = x_i(z)$ , when using such a set and such a function, is appropriate. The symbols  $x(\cdot)$  and

$x_{-i}(\cdot)$  describe the respective vectors of functions. Finally, if  $x, y$  are two vectors, then  $x \geq y$  means that  $x_i \geq y_i$  for all  $i$ , while  $x > y$  means that  $x_i \geq y_i$  for all  $i$  and there exists at least one  $i$ , such that  $x_i > y_i$ .

I turn now to the model. Consider a common agency model with one agent and  $n$  principals. Following Dixit et al. (1997) I discuss here what is known as a public, delegated common agency model<sup>1</sup>.

**Agent.** The agent chooses an element  $a$  from the set  $A$ . The set  $A$  reflects budget, institutional or other constraints that depend on specific applications. For example, if the agent is a government choosing a tax rate, the set  $A$  is the interval  $[0, 1]$ . Likewise, if the agent is a buyer, buying a quantity of a good from each of the principals, the set  $A$  is  $R_+^n$ . Henceforth, I refer to  $a$  as the agent's action.

The utility function of the agent is a function:

$$u_0 : A \times R^n \rightarrow R \quad \text{such that} \quad u_o = u_0(a, b).$$

The vector  $b \in R^n$ , is the vector of bids that the principals submit to the agent in order to influence the choice of  $a$ . This utility function is strictly monotonous with respect to all bids and continuous with respect to all its elements.

**Principals.** On the other hand, each principal chooses a bidding function:

$$b_i : A \rightarrow R \quad \text{such that} \quad b_i = b_i(a),$$

in order to influence the agent. These bidding functions meet appropriate restrictions. Specifically, there exist two functions  $\bar{b}_i : A \rightarrow R$  and  $\underline{b}_i : A \rightarrow R$ , which are uniformly bounded above and below by  $b_{max}, b_{min} \in R$  respectively and satisfy the inequality  $\bar{b}_i(a) \geq \underline{b}_i(a)$  for all  $a \in A$ . These function define feasible bids:

**Definition 1. Feasibility:** A bid  $b_i \in R$  is feasible relative to  $a \in A$ , if  $b_i \in [\underline{b}_i(a), \bar{b}_i(a)]$ . Moreover, a bidding function  $b_i(\cdot)$  is feasible, if  $b_i(a)$  is feasible relative to  $a$ , for all  $a \in A$ .

Additionally, the vector  $b \in R^n$  is feasible relative to  $a \in A$  if all  $b_i$  are feasible relative to  $a$ . In this case I say that the pair  $(a, b)$  is feasible. A feasible pair  $(a, b)$  is **symmetric** if  $b_i = b_j$  for all  $i, j$ . Similarly, the vector of bidding functions  $b(\cdot)$  is feasible, if all its elements are feasible. Moreover, if  $b(\cdot)$  is feasible and  $a \in A$ , I say that  $(a, b(\cdot))$  and  $(a, b_i(\cdot))$  are feasible. A feasible pair  $(a, b(\cdot))$  is **symmetric** if  $b_i(\cdot) = b_j(\cdot)$  for all  $i, j$ .

---

<sup>1</sup>For the meaning of this terms see Martimort and Stole (2009a).

Feasibility restrictions in bids reflect application specific constraints. For example, if the principals are lobbies offering campaign contributions, the bids must be positive and not exceed the budget constraint of the lobby. Likewise, if the principals are sellers and the bids are selling prices, the bids should be greater than the cost of acquisition and smaller than the buyer's reservation price.

Now I turn to the utility functions of the principals. Specifically, the utility function of principal  $i$  is a function:

$$u_i : A \times R^n \rightarrow R \quad \text{such that} \quad u_i = u_i(a, b).$$

This utility function is strictly monotonous with respect to own bids  $b_i$  and continuous with respect to both  $a$  and  $b$ .

**Model features.** Finally, the timing of the model is standard. There are two stages. In stage one, the principals submit simultaneously their bidding functions. In stage two, the agent chooses his action and the bids are realised.

Following the analysis above, my model is fully described by the number of principals  $n$ , the  $n + 1$  utility functions, the set  $A$  and the restrictions in bids  $(\underline{b}_i(\cdot), \bar{b}_i(\cdot), b_{max}, b_{min})$ . Henceforth, I use the term game, whenever I refer to a common agency model defined in such a way.

This game extends the model by Dixit et al. (1997) in two ways. First, the utility of each principal also depends on the bids of all other principals. Thus, my model allows for externalities among principals. Second, I make no prior assumptions, regarding the effect of bids on utility, other than monotonicity. These extensions broaden the range of applications to which truthful equilibria apply. I discuss these issues again in 2.4 which provides further specification of the model and in section 4 which discusses economic applications.

Let me now turn to the equilibrium of the game.

## 2.2 Equilibrium

Since the game above has two stages, I concentrate on subgame perfect Nash equilibria. Definitions 2 and 3 bellow apply the familiar notions of best response and subgame perfect Nash equilibrium to the present setting <sup>2</sup>.

**Definition 2. Best response:** A feasible bidding function  $b_i(\cdot)$ , belongs in the best response set of principal  $i$ , to the feasible bidding functions  $b_{-i}(\cdot)$  of the other principals, if:

There exists an  $a' \in \arg \max_{a \in A} u_0(a, b(a))$ , such that there does not exist a

---

<sup>2</sup>On the definitions 2 and 3 see Ko (2011)

feasible pair  $(a^*, b_i^*(\cdot))$ , such that  $u_i(a^*, b_i^*(a^*), b_{-i}(a^*)) > u_i(a', b(a'))$  and  $a^* \in \arg \max_{a \in A} u_0(a, b_i^*(a), b_{-i}(a))$ .

**Definition 3. *Equilibrium*:** A feasible pair  $(a^o, b^o(\cdot))$  is an equilibrium if:

- a)  $a^o \in \arg \max_{a \in A} u_0(a, b^o(a))$  and
- b) for all  $i$ , there does not exist a feasible pair  $(a', b_i(\cdot))$ , such that  $a' \in \arg \max_{a \in A} u_0(a, b_i(a), b_{-i}^o(a))$  and  $u_i(a', b_i(a'), b_{-i}^o(a')) > u_i(a^o, b(a^o))$ .

Let me now turn to the notion of truthful equilibrium.

## 2.3 Truthful equilibrium

Truthful equilibria refine the equilibria described in definition 3. They were introduced by Bernheim and Whinston (1986) for quasi linear utility functions and were generalized to all utility functions by Dixit et al. (1997). In truthful equilibria the principals submit truthful bidding functions. These functions exactly reflect changes in the utility of the principals that follow from changes in the actions of the agent. Thus, truthful bidding functions reveal the true preferences of the principals. The two definitions that follow adapt these ideas to my setting.

Let  $(a, b(\cdot))$  be a feasible pair and  $u_i^* \in R$ . Consider the equation  $u_i^* = u_i(a, \phi_i, b_{-i}(a))$ , with respect to  $\phi_i$ . Since  $b(\cdot)$  is a vector of feasible bidding functions,  $b_{-i}(a)$  exists for all  $a \in A$ . Furthermore, because the utility of the principals is monotonous in own bids, this equation always has a unique solution. This solution defines a function  $\phi_i : A \rightarrow R$  such that  $\phi_i = \phi_i(a; u_i^*, b_{-i}(\cdot))$ . Then, I define truthful responses as follows:

**Definition 4. *Truthful response*:** A bidding function  $b_i^T : A \rightarrow R$ , is a truthful response of principal  $i$  to the feasible bidding functions  $b_{-i}(\cdot)$  of the other principals, relative to the constant  $u_i^*$ , if :

a)

$$b_i^T = \begin{cases} \underline{b}_i(a) & \text{if } u_i(a, \underline{b}_i(a), b_{-i}(a)) < u_i^* \\ \phi_i(a; u_i^*, b_{-i}(\cdot)) & \text{if } u_i(a, \bar{b}_i(a), b_{-i}(a)) \leq u_i^* \leq u_i(a, \underline{b}_i(a), b_{-i}(a)) \\ \bar{b}_i(a) & \text{if } u_i^* < u_i(a, \bar{b}_i(a), b_{-i}(a)) \end{cases}$$

and  $u_i(\cdot)$  is strictly decreasing in own bids, or

b)

$$b_i^T = \begin{cases} \underline{b}_i(a) & \text{if } u_i(a, \underline{b}_i(a), b_{-i}(a)) > u_i^* \\ \phi_i(a; u_i^*, b_{-i}(\cdot)) & \text{if } u_i(a, \bar{b}_i(a), b_{-i}(a)) \geq u_i^* \geq u_i(a, \underline{b}_i(a), b_{-i}(a)) \\ \bar{b}_i(a) & \text{if } u_i^* > u_i(a, \bar{b}_i(a), b_{-i}(a)) \end{cases}$$

and  $u_i(\cdot)$  is strictly increasing in own bids.

Definition 4 states that truthful responses are equal to the expression  $\phi_i(a; u_i^*, b_{-i}(\cdot))$  except when this expression violates lower or upper feasibility bounds. In such cases the truthful responses are equal to these bounds. Therefore, truthful responses are by construction feasible bidding functions<sup>3</sup>.

Now I can turn to the definition of truthful equilibrium.

**Definition 5. Truthful equilibrium:** Let  $(a^o, b^o(\cdot))$  be an equilibrium of the game and  $u^o = u(a^o, b^o(a))$ . This equilibrium is truthful, if for all  $i$ , the equilibrium bidding function  $b_i^o(\cdot)$ , is a truthful response of principal  $i$  to the equilibrium bidding functions  $b_{-i}^o(\cdot)$  of the other principals, relative to his equilibrium utility level  $u_i^o$ .

Let me clarify the notion of truthful equilibrium. Consider any feasible pair  $(a, b(\cdot))$  and let  $u = u(a, b(a))$ , be the vector of corresponding utility levels. Then, definition 4 determines the truthful response, for each principal, to the bidding functions of the other principals, relative to  $u_i$ . This operation defines a mapping from the  $n$ -dimensional space of feasible bidding functions to itself. Now consider an equilibrium pair  $(a^o, b^o(\cdot))$  and let  $u^o = u(a^o, b^o(a))$ . If  $b^o(\cdot)$  is a fixed point in the mapping above, then it is a truthful equilibrium<sup>4</sup>.

Next, I discuss the structure of the game.

---

<sup>3</sup>Ko (2011) explains the advantages of definition 4 when compared to  $b_i^T = \min\{\bar{b}_i(a), \max\{\underline{b}_i(a), \phi_i(a; \bar{u}_i, b_{-i}(\cdot))\}\}$  which was used by Dixit et al. (1997).

<sup>4</sup>The existence of such a fixed point deals with the issue of infinite regress that can appear in common agency with externalities. For the problem of infinite regress in common agency see Peters (2001), Martimort and Stole (2002) and more recently Szentes (2014) and Galperti (2015). In these papers the bidding functions explicitly depend on other bidding functions. Therefore, infinite regress can appear because the bidding function of principal  $i$  depends on the bidding function of principal  $j$ , which in turn depends on the bidding function of principal  $i$  and so on. My setting does not allow for explicit dependence on other bidding functions. However, the principals form guesses about the other bidding functions which also depend on the guesses that the other principals form and so on.

## 2.4 Additional assumptions

Here, I provide some additional assumptions that often characterize applications.

### **Assumption A. *Opposing monotonicity***

**A1. Lobbying.** The utility of all principals is strictly decreasing in own bids and the utility of the agent is strictly increasing in all bids. Moreover,  $\underline{b}_i(a) = b_{min}$ , for all  $i$  and  $a \in A$ .

**A2. Market.** The utility of all principals is strictly increasing in own bids and the utility of the agent is strictly decreasing in all bids. Moreover,  $\bar{b}_i(a) = b_{max}$ , for all  $i$  and  $a \in A$ .

Assumption A1 is often satisfied in lobbying models. In these models the principals are lobbies and their bids are usually campaign contributions to politicians. In such a case, the politicians like receiving contributions while the lobbies dislike paying them. Moreover, contributions must be non-negative and therefore, the common lower bound  $b_{min}$  is zero. Assumption A2 is more relevant in market games, in which the principals are sellers of a homogeneous good and their bids are selling prices. In this case, the sellers like high prices while the buyer has the opposite feelings. Furthermore, the upper bound of the bids is the buyer's reservation price.

Henceforth, when I refer to assumptions A1 and A2, I use the terms lobbying and market monotonicity.

### **Assumption B. *No externalities***

The utility of the principals takes the form  $u_i : A \times R \rightarrow R$  such that  $u_i = u_i(a, b_i)$ .

Assumption B describes a special case without externalities in bids. The combination of lobbying monotonicity and no externalities defines the standard common agency game discussed by Dixit et al. (1997).

### **Assumption C. *Conflict of interests at (a, b)***

Let  $(a, b)$  be a feasible pair.

If there exists a feasible pair  $(a, b')$  such that  $u_0(a, b') > u_0(a, b)$  then there exists an  $i$  such that  $u_i(a, b') < u_i(a, b)$  and

if there exists a feasible pair  $(a, b')$  such that  $u(a, b') > u(a, b)$  then  $u_0(a, b') < u_0(a, b)$ .

Assumption C states that for a given agent's action, it is impossible to change bids, in a way that makes both the principals and the agent better off. Therefore, this assumption introduces conflict of interests between principals and agent, over bids. However, assumption C is redundant in games without externalities. This is so, because in such games, opposing monotonicity suffices to achieve the aforementioned conflict.

In order to explain the role of externalities in this issue, let me consider a game with lobbying monotonicity. First, assume that there are no externalities. Also, fix the agent's action at a certain level and consider an increase in all bids. Then, the utility of the agent increases. Moreover, since  $u_i = u_i(a, b_i)$ , the utility of all principals decreases. Thus, there exists conflict of interests. Alternatively, consider a similar game with externalities. In this case, the utility of principal  $i$  is  $u_i = u_i(a, b_i, b_{-i})$ . Furthermore, assume that the utility of all principals is increasing in all the elements of  $b_{-i}$ . Then, two conflicting effects emerge. On the one hand, the increase in own bids has a negative effect on the utility of the principals. On the other hand, the increase in the other bids has a positive effect for all principals. If the positive cross effect dominates the negative own effect, for all principals, then conflict of interests is violated. In such a case, an increase in all bids increases the utility of all principal and the agent. Assumption C appropriately restricts externalities to disallow such situations. This assumption along with assumption D that follows guarantee the efficiency of truthful equilibria.

**Assumption D. *Deep pockets***

**D 1. *Weak deep pockets at  $(a^o, b^o(\cdot))$ .***

Let  $(a^o, b^o(\cdot))$  be a truthful equilibrium and  $(a^*, b^*)$  be a feasible pair such that  $u_0(a^*, b^*) \geq u_0(a^o, b^o(a^o))$  and  $u(a^*, b^*) \geq u(a^o, b^o(a^o))$  with at least one strict inequality, then

$$u(a^o, b^o(a^o)) \geq u(a^*, b^o(a^*)).$$

**D 2. *Strong deep pockets***

There exists  $\underline{u}_i \in R$  such that:

**D 2.1.** If the utility of all principals is strictly decreasing in own bids then  $u_i(a, \bar{b}_i(a), b_{-i}(a)) = \underline{u}_i \leq u_i(a, b(a))$  for all  $i, a \in A$  and feasible  $b_{-i}(\cdot)$ .

**D 2.2.** If the utility of all principals is strictly increasing in own bids then  $u_i(a, \underline{b}_i(a), b_{-i}(a)) = \underline{u}_i \leq u_i(a, b(a))$  for all  $i, a \in A$  and feasible  $b_{-i}(\cdot)$ .

The term “deep pockets” is due to Ko (2011) who uses a similar assumption. Furthermore, Dixit et al. (1997) also employ a version of strong deep pockets. Specifically, Dixit et al. (1997) assume that there is a subsistence utility level  $\underline{u}_i$  and define  $\bar{b}_i(\cdot)$  implicitly through  $u_i(a, \bar{b}_i(a)) = \underline{u}_i$ . If a game satisfies strong deep pockets then it also satisfies weak deep pockets.

However, as I show later on, many applications satisfy weak deep pockets directly.

Definition 6 introduces some terms that I use in assumptions D and E below.

**Definition 6. Game structure:**

**a)** A game is **differentiable** if all utility functions are differentiable with respect to all bids.

**b)** A game is **cumulative** if the utility functions of the agent and the principals are as follows:

$$u_0 : A \times R \rightarrow R \text{ such that } u_0 = u_0(a, \tilde{b}) \text{ and}$$

$$u_i : A \times R^2 \rightarrow R \text{ such that } u_i = u_i(a, b_i, \tilde{b}_{-i}) \text{ for all } i.$$

**c)** A game exhibits **negative externalities** if it is differentiable, cumulative and the utility of all principals is strictly decreasing in the total bids of the other principals.

**d)** A game exhibits **positive externalities** if it is differentiable, cumulative and the utility of all principals is strictly increasing in the total bids of the other principals.

**e)** A game is **symmetric** if  $u_i(\cdot) = u_j(\cdot)$ ,  $\underline{b}_i(\cdot) = \underline{b}_j(\cdot)$  and  $\bar{b}_i(\cdot) = \bar{b}_j(\cdot)$  for all  $i, j$ .

**f)** A game is **quasi-concave** if it is cumulative and the utility functions of all principals are quasi-concave with respect to own and other bids.

**Assumption E. Small externalities**

Either the game exhibits negative externalities and  $\left| \frac{\partial u_i}{\partial b_i} \right| > \left| \frac{\partial u_i}{\partial \tilde{b}_{-i}} \right|$  for all  $i$  and all feasible  $(a, b)$  or

the game exhibits positive externalities and  $\left| \frac{\partial u_i}{\partial b_i} \right| > (n - 1) \left| \frac{\partial u_i}{\partial \tilde{b}_{-i}} \right|$  for all  $i$  and all feasible  $(a, b)$ .

Assumption E is a special case of conflict of interests. Specifically, small externalities describe a situation in which the effect of own bids appropriately dominates the respective cross effects. In this respect, think of  $(n - 1) \left| \frac{\partial u_i}{\partial \tilde{b}_{-i}} \right|$

as the total cross effect and of  $\left| \frac{\partial u_i}{\partial \tilde{b}_{-i}} \right|$  as the average cross effect. These two effects coincide when  $n = 2$ . As it turns out, this restriction in externalities achieves the conflict between agent and principals which is necessary to sat-

isfy assumption c.

**Assumption F. *Symmetric externalities***

The game is symmetric, quasi concave and exhibits negative externalities.

Proposition 1 that follows explains the relationship between assumptions A-F.

**Proposition 1.**

(i) Strong deep pockets imply weak deep pockets at all feasible pairs  $(a, b)$ .

(ii) The combination of opposing monotonicity and no externalities, implies conflict of interests at all feasible pairs  $(a, b)$  and weak deep pockets at all truthful equilibria.

(iii) The combination of small externalities and lobbying monotonicity implies conflict of interests at all feasible pairs  $(a, b)$  and weak deep pockets at all truthful equilibria.

(iv) The combination of symmetric externalities and lobbying monotonicity implies conflict of interests at all symmetric pairs  $(a, b)$  and weak deep pockets at all symmetric truthful equilibria.

Proof of (ii): Dixit et al. (1997) prove that lobbying monotonicity and no externalities imply conflict of interests and deep pockets during their proof of the efficiency of truthful equilibria<sup>5</sup>. In appendix B.1, I provide a similar proof using market instead of lobbying monotonicity.

Proofs of (i), (iii) and (iv): see appendix A.1.

Conflict of interests and deep pockets are the backbone of the efficiency result that I derive in the next section.

### 3 Results

Dixit et al. (1997), like Bernheim and Whinston (1986) before them, argue that truthful equilibria are focal, because they share three key properties. Namely, truthful equilibria are plausible, easy to calculate and efficient.

Dixit et al. (1997) arrive at this result under the assumptions of lobbying monotonicity and no externalities. Here I generalize their argument by providing a broader set of conditions under which it is valid. The four propositions that follow achieve this task.

---

<sup>5</sup>For the proof see Dixit et al. (1996)

**Proposition 2. *Plausibility***

Consider a game that exhibits opposing monotonicity. Then, the best response set of principal  $i$  to the bidding functions of the other principals  $b_{-i}(\cdot)$  always contains a truthful response.

Proof: see appendix B.2.

**Proposition 3. *Calculation***

Consider a game that exhibits opposing monotonicity. Then, the feasible pair  $(a^o, b^o(\cdot))$  is an equilibrium if and only if (i) and (ii) below are true:

(i)  $a^o \in \arg \max_{a \in A} u_0(a, b(a))$

(ii) lobbying monotonicity implies that  $u_0(a^o, b^o(a^o)) = \max_{a \in A} u_0(a, b_{min}, b_{-i}(a))$

and

market monotonicity implies that  $u_0(a^o, b^o(a^o)) = \max_{a \in A} u_0(a, b_{max}, b_{-i}(a))$

Proof: See appendix B.2.

According to propositions 2 and 3 opposing monotonicity implies plausibility for truthful equilibria and easy calculation for all equilibria. These results are true regardless of other characteristics of the game i.e. the existence of externalities. In more detail, proposition 2 states that the principals stand to lose nothing from responding truthfully to any bidding function chosen by the other principals. Also, the intuition behind proposition 3 is standard. In particular, principal  $i$  submits a bid that matches the agent's outside option. Principal  $i$  has no motive to improve his bid any further. Moreover, although proposition 3 applies to all equilibria, it is particularly helpful in the calculation of truthful equilibria. In this respect, I provide examples in section 4 and appendices B.3 and B.4.

Now I turn to efficiency.

**Proposition 4. *Efficiency***

Consider a game which exhibits conflict of interests and weak deep pockets at  $(a^o, b^o(\cdot))$  which is a truthful equilibrium of this game. Then, there does not exist a feasible pair  $(a^*, b^*)$ , such that  $u(a^*, b^*) \geq u(a^o, b^o(a^o))$  and  $u_0(a^*, b^*) \geq u_0(a^o, b^o(a^o))$ , with at least one strict inequality.

Proof: See appendix A.2

Proposition 4 states that truthful equilibria implement an allocation which is Pareto efficient for all participants of the game (principals and agent). Definition 7 and proposition 5 below summarize the results so far.

**Definition 7. *Validity***

A truthful equilibrium of a game is valid, if it satisfies propositions 3 and 4 and the game satisfies proposition 2.

**Proposition 5. *Results***

a) All truthful equilibria are valid in all games that satisfy one of the following:

(i) lobbying monotonicity and no externalities

(ii) market monotonicity and no externalities<sup>6</sup>

(iii) lobbying monotonicity and small externalities<sup>7</sup>

b) Symmetric truthful equilibria are valid in games that satisfy symmetric externalities and lobbying monotonicity.

Proof: Follows directly from propositions 1-4.

Part (a-i) of proposition 5 restates the argument by Dixit et al. (1997) in favour of truthful equilibria in the standard model, while parts (a-ii), (a-iii) and (b) generalize this argument in different settings. The intuition behind proposition 5 is straightforward for the case of market monotonicity and no externalities. This case is the reverse of the standard model by Dixit et al. (1997) and thus its motivation is similar. In the case of small externalities the intuition is also simple. Specifically, if the externalities are small they have no effect. I discuss this issue further in section 5. For the case of symmetric externalities I provide examples that highlight the role of both symmetry and quasi-concavity in achieving conflict of interests, in appendix B.6.

Proposition 5 describes 4 general settings in which truthful equilibria are relevant. However, other such settings might also exist. In this respect, interested researchers can directly check whether a specific application satisfies any or all of propositions 2-4. The proof of proposition 1 can act as a guide in this endeavour.

In the next section I discuss economic applications of proposition 5.

---

<sup>6</sup>Chiesa and Denicolò (2009) also suggest that truthful equilibria can be used in such games. However, they do not give a formal proof.

<sup>7</sup>Dasgupta and Maskin (2000) also find that small externalities lead to efficient equilibria in their study of efficient equilibria in Vickrey auctions. I am indebted to professor de Frutos for pointing that to me.

## 4 Applications

### 4.1 Lobbying

This is a case of lobbying monotonicity and no externalities. I provide this example in order to show that the standard model is consistent with my setting.

The principals are lobbies lobbying the government (agent) for a lump sum tax. For simplicity I assume that each of the  $n$  lobbies has one member and that all individuals in the economy form a lobby. The utility function of lobby  $i$  is:

$$u_i = u_i(c_i, g).$$

The variables  $c_i$  and  $g$  stand for private and public good consumption respectively. The function  $u_i(\cdot)$  is strictly increasing in both goods. The budget constraint of lobby  $i$  is:

$$e_i = c_i + t + b_i.$$

In this constraint  $e_i$  is the endowment,  $t$  is the lump-sum tax and  $b_i$  is the contribution that aims to influence the decision of the government on  $t$ . The government uses the tax to finance the public good. Thus,

$$g = nt.$$

Under this assumptions the utility of lobby  $i$  becomes:

$$u_i = u_i(e_i - t - b_i, nt).$$

Thus, the optimal tax for each lobby depends on its endowment  $e_i$ . This asymmetry rationalizes lobbying.

The utility function of the government is

$$u_0 = u_0((t - t^*)^2, b).$$

The variable  $t^*$  represents a tax target for the government. The utility of the government is strictly decreasing in the first argument and strictly increasing in all elements of the second.

This model is consistent with my setting. Specifically, the bounds that define feasible bids are:

$$\underline{b}_i(t) = 0 = b_{min} \quad \text{and} \quad \bar{b}_i(t) = e_i - t \leq e_i = b_{max}.$$

Furthermore, the utility of the principals decreases, while the utility of the agents increases with contributions. Therefore, this model satisfies lobbying monotonicity and since it lacks externalities, it also satisfies proposition 5.

This type of models is very popular in the literature and has been studied extensively<sup>8</sup>. I refer to it here for the sake of completeness. I turn now to an application of market monotonicity.

## 4.2 Market application

This is a case of market monotonicity and no externalities.

Assume  $n = 2$ . The two principals are sellers who sell a homogeneous good. The agent is a buyer who has decided to buy  $\bar{q}$  units of the good. For example, think of a government which is in the market for a fixed number of warships. First, the two sellers submit an offer for a unit price that depends on the quantity that the buyer buys from each seller. Then, the buyer decides how much to buy from each seller. This model is known in the literature as split-award procurement<sup>9</sup>.

The profit function of seller  $i$  is:

$$\Pi_i = p_i q_i - c q_i^2.$$

Here,  $p_i$  is the unit price,  $q_i$  stands for quantity and  $c > 0$  is a cost parameter.

The buyer wants to allocate  $\bar{q}$  between the two sellers in a way that minimizes his expenditure. Thus, the utility of the buyer is:

$$u_0 = -p_1 q_1 - p_2 (\bar{q} - q_1).$$

The reservation price for the buyer is  $\bar{p}$ . You can think of the reservation price as the cost of not buying a unit or as the unit price offered by an outside source.

The bounds that define feasibility for prices are :

$$\underline{p}_i(q_i) = c q_i \geq 0 = p_{min} \quad \text{and} \quad \bar{p}_i(q_i) = \bar{p}.$$

These conditions reflect the fact that prices must be positive, must not be less than the average cost and must not exceed the reservation price. Furthermore, assume  $\bar{p} > 3c\bar{q}$ .

This model satisfies market monotonicity and lacks externalities. Therefore it satisfies proposition 5. As I show in appendix B.3, solving for a truthful equilibrium yields the following symmetric outcome:

$$q_i = \frac{\bar{q}}{2}, \quad p_i = \frac{3c\bar{q}}{2}, \quad \Pi_i = \frac{c\bar{q}^2}{2}$$

---

<sup>8</sup>Early examples of this literature are Grossman and Helpman (1994), Dixit (1996), Persson (1998), Mitra (1999) e.t.c. More recent examples are Campante and Ferreira (2007), Aidt and Hwang (2008) and Esteller-Moré et al. (2012) among many others.

<sup>9</sup>See Anton and Yao (1989) and Chiesa and Denicolò (2009).

while the price functions are

$$p_i(q_i) = \begin{cases} \frac{c\bar{q}^2}{2q_i} + cq_i & \text{if } q_i \geq \frac{\bar{p} - \sqrt{\bar{p}^2 - 2c^2\bar{q}^2}}{2c} \\ \bar{p} & \text{if } q_i < \frac{\bar{p} - \sqrt{\bar{p}^2 - 2c^2\bar{q}^2}}{2c} \end{cases}$$

In this model, total cost decreases with the number of producers, because of the increasing marginal cost. Specifically, the total cost of producing quantity  $\bar{q}$  of the good is  $c\bar{q}^2$  if there is only one producer, while it is cut in half, if there are two producers. Consequently, a familiar result emerges.

The profits of each principal equal the decrease in the total cost due to his participation in production. This result is standard in common agency games with quasi-linear utility functions. In this respect, Bergemann and Välimäki (2003) show that in such cases, each principal receives his contribution to the social surplus.

I turn now to an application with externalities.

### 4.3 Lobbying with production externalities

This is a case of lobbying monotonicity and small externalities. In this application I consider a simple variation of the model by Alesina and Rodrik (1994). Specifically, I introduce lobbying to a two period AK model.

#### 4.3.1 Setting

There are three types of agents in the economy: individuals, firms and the government.

Individuals form lobbies that try to influence government policy in favour of their members. In order to achieve this goal, lobbies offer contributions to the government. Lobbies condition these contributions on the choice of policy.

In terms of the common agency game, the lobbies are the principals, the government is the agent, while contributions paid by the lobbies are the principals' bids. Firms do not participate in lobbying.

Let me now describe the economy starting with the firms.

Following Romer (1986), I consider a large number of identical firms with production function

$$y_h = k_h^a k_{av}^{1-a} L_h^{1-a}.$$

In this function,  $k_h$  and  $L_h$  are the capital and labour used by firm  $h$ , while  $k_{av}$  is the average capital across firms. Production takes place only in the second period.

All individuals participate in lobbies. The individuals form two lobbies ( $i = 1, 2$ ) on the basis of their initial endowment. Specifically, each member of lobby  $i$  has an initial capital endowment equal to  $k_i^o$ . Without loss of generality I assume that  $k_1^o < k_2^o$ . Furthermore, lobby  $i$  has  $m_i$  members which is a large number, while total population equals  $m$  ( $m = m_1 + m_2$ ).

Each individual lives for two periods. In the first period he pays a proportional tax on his capital endowment. He uses the after tax endowment to consume, to lobby the government and to invest in capital<sup>10</sup>. The firms use this capital in production during the second period. In the second period, the individuals work and thus earn a wage along with the return of their capital investment. Moreover, capital depreciates fully after the second period.

In this respect the representative member of lobby  $i$  faces the following budget constraints :

$$c_i^1 = (1 - t_i)k_i^o - b_i - k_i^1 \quad \text{and} \quad c_i^2 = rk_i^1 + wL_i.$$

In these constraints  $C_i^j$  is the consumption in period  $j$ ,  $b_i$  is the contribution to the government,  $k_i^1$  is the investment in capital,  $L_i$  is the individual labour supply,  $r$  is the return to capital and  $w$  is the wage.

Finally, the intertemporal utility function of an individual in lobby  $i$  is

$$u_i = (c_i^1)(c_i^2)^\delta,$$

in which  $\delta \in (0, 1)$  is a parameter.

The government taxes capital endowments in order to collect a total amount of taxes equal to  $\tilde{T}$ . In order to achieve this goal it chooses a tax rate  $t_i$  for each group. Thus, the tax rates satisfy the following constraint:

$$m_1 t_1 k_1^o + m_2 t_2 k_2^o = \tilde{T}.$$

Furthermore, I assume that each group can pay the whole amount of tax itself. Therefore  $\tilde{T} \leq m_1 k_1^o$ . I define  $\bar{t}_i = \frac{\tilde{T}}{m_i k_i^o}$ , which is the maximum tax rate for the members of lobby  $i$ .

Other than collecting taxes, the government's objective is the maximization of contributions from lobbies. Therefore, the utility function of the government is:

$$u_0 = m_1 b_1 + m_2 b_2.$$

Next, I consider the equilibrium of the model.

---

<sup>10</sup>Campante and Ferreira (2007) justify the idea that lobbies contribute in the first period.

### 4.3.2 Equilibrium

The sequence of events in the model is as follows: First, the lobbying game determines contributions and tax rates (political equilibrium). Then, the market determines the prices and the allocation of resources (private sector equilibrium). Following this timing, I start with private sector equilibrium.

For simplicity, I assume that each individual supplies inelastically one unit of labour and that the number of firms equals the number of individuals. Thus,  $L_i = L_h = 1$ . Then, standard profit maximization by the firms yields:

$$r = aA \quad \text{and} \quad w = (1 - a)Ak_{av}.$$

I turn now to the individuals. The individuals know the size of taxes and contributions when they make their investment decision, since the lobbying game is resolved first. Furthermore, individual investment is relatively small, thus individuals ignore its effect on average capital. Then, the problem of the individuals is to choose an investment level  $k_i^1$  that maximizes their utility, subject to their budget constraints and the given level of taxes, contributions and average capital. Solving this problem yields

$$c_i^2 = aA\delta c_i^1 \tag{1}$$

$$c_i^1 = [(1 - t_i)k_i^o - b_i] \frac{1}{1 + \delta} + \frac{\delta}{1 + \delta} \frac{1 - a}{1 + a\delta} (k_{av}^o - b_{av} - \tilde{T}_{av}). \tag{2}$$

In the conditions above,  $k_{av}^o$  and  $b_{av}$  are the average endowments and contributions across the economy and  $\tilde{T}_{av} = \frac{\tilde{T}}{m}$ . Average contributions appear here because they affect the wage through average capital.

Now I turn to the lobbying game. The objective of each lobby is to maximize the utility of its members. Substituting equation 1 in the individual utility function yields:

$$u_i = (aA\delta)^\delta (c_i^1)^{(1+\delta)}.$$

In this expression,  $c_i^1$  is given by equation 2. Unlike their members though lobbies are big players. Thus, they recognize that average contributions are jointly determined by the contributions of both lobbies. This observation introduces externalities.

This problem fits my general setting. First, the contributions are appropriately bounded. In particular, they must satisfy:

$$0 \leq b_i(t_i) \leq (1 - t_i)k_i^o \leq k_i^o.$$

Second, the derivatives of interest are:

$$\frac{\partial u_0}{\partial b_i} = m_i > 0 \quad (3)$$

$$\frac{\partial u_i}{\partial b_i} = \frac{\partial u_i}{\partial c_i^1} \frac{\partial c_i^1}{\partial b_i} = \frac{\partial u_i}{\partial c_i^1} \left( -\frac{1}{(1+\delta)} - \frac{\delta}{(1+\delta)} \frac{1-a}{1+a\delta} s_i \right) < 0 \quad (4)$$

$$\frac{\partial u_i}{\partial b_j} = \frac{\partial u_i}{\partial c_i^1} \frac{\partial c_i^1}{\partial b_j} = \frac{\partial u_i}{\partial c_i^1} \left( -\frac{\delta}{(1+\delta)} \frac{1-a}{1+a\delta} s_j \right) < 0. \quad (5)$$

In the equations above,  $s_i$  is the share of the members of lobby  $i$  in the population. Furthermore, the combination of equations 3 and 4 implies lobbying monotonicity. Finally, the combination of equations 4 and 5 implies that  $|\frac{\partial u_i}{\partial b_i}| > |\frac{\partial u_i}{\partial b_j}|$ , which is the condition for small externalities. Therefore, this model satisfies proposition 5.

In appendix B.4, I show that the following constitute a truthful equilibrium of this application:

$$b_i(t_i) = \frac{\tilde{T}}{m_i} - t_i k_i^o, \quad \text{and} \quad t_i \in [0, \bar{t}_i] \quad \text{s.t.} \quad m_1 t_1 k_1^o + m_2 t_2 k_2^o = \tilde{T}.$$

This equilibrium has two familiar properties. First, the government receives, in the form of contributions, all the benefit that it generates through favourable policy for a principal. Second, the distribution of the burden between taxes and contributions for each principal is not determined. These two properties often characterize the truthful equilibria of models without externalities and quasilinear utilities. They are a direct result of the efficiency of truthful equilibria and the structure of this particular application, which assigns all the “bargaining” power to the agent. Since small externalities do not disrupt these characteristics, these properties still hold here<sup>11</sup>.

#### 4.4 Lobbying and public goods

I turn now to a case of lobbying monotonicity and symmetric externalities. In particular I introduce lobbying to a local public goods model inspired by Persson and Tabellini (1994).

Consider a federal country with  $n$  identical states, each populated by one individual. In this country, there is a federal government with the sole

<sup>11</sup>On this issue see Dixit et al. (1997). This application is quasilinear up to a monotonic transformation. Specifically, the utility of the lobbies along with equation 2 imply that  $u_i^* = [(1-t_i)k_i^o - b_i] \frac{1}{1+\delta} + \frac{\delta}{1+\delta} \frac{1-a}{1+a\delta} (k_{av}^o - b_{av} - \tilde{T}_{av})$ , in which  $u_i^* = (\frac{u_i}{(aA\delta)^\delta})^{\frac{1}{1+\delta}}$ .

purpose to redistribute income across states. This government imposes a tax or subsidy  $t_i$  to each of the states. These taxes satisfy:

$$\sum_i t_i = 0.$$

Each state offers to the federal government a bribe  $b_i$  in order to affect the choice of  $t_i$ . In this respect the utility of the federal government is given by:

$$u_0 = \sum_i b_i.$$

Moreover, there are two goods in the economy. These goods are the private good  $C$  and the public good  $G$ . Each state finances the public good by offering a voluntary contribution  $g_i$ . Thus the total amount of public good equals:

$$G = \sum_i g_i.$$

An example of such a public good is public safety. Each state decides independently how much to spend on the security of its airport. However, since terrorists who arrive in one state can move freely to all states, this spending affects safety in the whole country.

Total spending in each state is financed by an endowment  $e$ . This endowment plus or minus the federal transfer  $t_i$  is used for private good consumption, contribution to the public good and bribes. Therefore, the budget constraint for each state is:

$$e + t_i = C_i + b_i + g_i,$$

where  $t_i \in [-e, e]$ . Finally the utility in state  $i$  is :

$$u_i = C_i G.$$

The timing in this model is as follows: First, a common agency game determines bribes and transfers. Then, the two states decide on their public good contributions simultaneously.

Following this timing, I start by determining the equilibrium public good contributions. Define,  $e_i = e + t_i - b_i$ . Then, the unique Nash equilibrium yields  $g_i = e_i - \frac{\sum_i e_i}{n+1}$  and  $G = \frac{\sum_i e_i}{n+1}$ , which implies that:

$$u_i = \left( \frac{\sum_i e_i}{n+1} \right)^2.$$

Now I can turn to the common agency game. In this respect I notice that  $b_i$  is appropriately bounded since:

$$0 \leq b_i \leq e + t_i \leq 2e.$$

Moreover, the derivatives of interest are as follows:

$$\frac{\partial u_i}{\partial b_i} = -2G < 0 \tag{6}$$

$$\frac{\partial u_i}{\partial \tilde{b}_{-i}} = -2G < 0 \quad \text{negative externality} \tag{7}$$

$$\tag{8}$$

This game exhibits symmetric externalities and lobbying monotonicity. Therefore truthful equilibria are valid. Solving for a truthful equilibrium yields:

$$b_1 = b_2 = 0 \quad \text{and} \quad t_i \in [-e, e] \quad \text{s.t.} \quad \sum_i t_i = 0.$$

In this model the agent has no “bargaining power”. Thus, as in similar models without externalities he ends with nothing. Specifically, since the utility of each state depends on total income, the government can not affect it, by redistributive transfers. Thus, there is no room for bribes. The intuition here is exactly the opposite to the previous application.

## 5 Discussion

In this section I discuss games that do not satisfy propositions 2-4. Specifically, I investigate the existence of efficient equilibria in such games. As it turns out, efficient equilibria might or might not exist depending on the specifics of the model. I discuss this issue with the help of two examples. These examples also clarify the relationship between the paper in hand and the existing literature on common agency with externalities. I start with an example in which there are no efficient equilibria.

### Example 1

Assume there are two principals ( $i = 1, 2$ ) with utility functions  $u_i = a - b_i + \gamma b_j$ , in which  $a \in [0, 1]$  is the agent’s action,  $b_i \in [0, a]$  is the bid of principal  $i$  and  $\gamma$  is a positive parameter. The utility of the agent is  $u_0 = b_1 + b_2$ .

First, I assume  $\gamma = 2$ . In this case, the example satisfies opposing monotonicity, but violates small externalities. Moreover, as I show in appendix B.5 there exists a unique symmetric and efficient allocation:  $a = 1$ ,  $b_1 = b_2 = 1$  which yields  $u_0 = u_1 = u_2 = 2$ . Henceforth, I call this allocation, allocation

A. However, allocation A is not an equilibrium. To see this, consider any bidding functions such that  $b_i(1) = 1$  for both  $i$  and think of the following deviation by principal 1:  $b_1 = 0,5$  if  $a = 1$  and  $b_1 = 0$  for all  $a \neq 1$ . Then, no matter what is the bidding function of principal 2, the agent chooses  $a = 1$  and total bids decrease. Moreover, as I also discuss in appendix B.5, there are no efficient equilibria in this example. On the contrary, there is a unique symmetric inefficient equilibrium which is intuitive<sup>12</sup>. Specifically, consider the following allocation, which I name allocation B:  $a = 1$ ,  $b_1 = b_2 = 0$ . This allocation yields  $u_0 = 0$ ,  $u_1 = u_2 = 1$  and can be supported as an equilibrium, by the constant bidding functions  $b_i(a) = 0$  for all  $a \in [0, 1]$ .

Allocations A and B illustrate the cause of efficiency failure in common agency with externalities. This cause is a prisoners' dilemma. Specifically, the principals can benefit from committing to high bids, as in allocation A. However, this commitment is not viable. This is so, because each principal has the motive to unilaterally deviate from any such "agreement" and offer smaller bids. This deviation generates a race to the bottom which leads to the inefficient equilibrium B.

This prisoners' dilemma also characterizes the discrete examples provided by Peters (2001) and Martimort and Stole (2002). These authors, observe that externalities in common agency models might lead to inefficient equilibria. Following this observation, they suggest an extension of the common agency model. This extension, allows bids to depend on the bidding functions of the other principal and restores efficiency. This idea in terms of example 1 is as follows: Allocation A is not an equilibrium because principal 1 deviates. However, if the bids of one principal depend on the bidding function of the other, then principal 2 can use his bidding function to punish the deviating behaviour and support the equilibrium.

In contrast to this approach, I bypass the solution of the efficiency issue altogether. Instead, I notice that in certain cases a prisoners' dilemma does not appear. Indeed, if  $\gamma = 0.5$ , example 1 satisfies proposition 5. Thus, if a truthful equilibrium exists it is efficient. Specifically, allocation B is such an equilibrium. This result, follows from the fact that if  $\gamma = 0.5$ , the principals can not benefit from high bids. In general, small externalities do not allow for prisoner dilemmas and therefore, lead to efficient equilibria. A similar argument holds for symmetric externalities.

Now I turn to an example which violates proposition 5, but nevertheless has an efficient equilibrium.

---

<sup>12</sup>Nevertheless, there exist many asymmetric inefficient equilibria that are supported by implausible off equilibrium strategies. I provide an example in appendix B.5.

**Example 2**

Consider a variation of example 1 in which the utility of the agent is  $u_0 = -(b_1 + b_2)$  and  $\gamma = 2$ . This example violates both opposing monotonicity and small externalities. Yet, allocation B can still be supported as an efficient equilibrium, by the constant bidding functions  $b_i(a) = 0$  for all  $a$ .

In this case, allocation B is efficient because the agent dislikes bids. In any other allocation with  $a = 1$  and positive bids, the principals might be better off but the position of the agent deteriorates. This is in contrast to example 1, in which both principals and agent get worse off as a result of a decrease in bids. Thus, in example 2, the prisoners' dilemma between principals remains, however it does not hinder efficiency, due to the characteristics of the agent's utility function.

Yet, although allocation B is an efficient equilibrium it is not also a truthful one. This is so, because the structure of truthful bidding functions renders them meaningless without opposing monotonicity. Indeed, in such cases truthful bidding functions fail, since they imply that the principals offer to the agent something he dislikes. In Boultzis (2015) I discuss an example of this situation.

## 6 Conclusions

In this paper I provide a set of conditions under which truthful equilibria are valid in common agency models. These conditions generalize the work of Dixit et al. (1997) and Bernheim and Whinston (1986) on this issue. Furthermore, I identify two new families of economic applications to which these conditions apply. In this regard, this paper shows that the scope of truthful equilibria is broader than believed so far.

In terms of future research my results can be useful in two ways. First, the conditions listed in proposition 5 apply in a wide variety of economic models. The validity of truthful equilibria in these models provides for a simple and intuitive way of solving an otherwise difficult problem. Second, proposition 2-4 can help identify additional settings in which truthful equilibria are valid in the future. Thus, the scope of truthful equilibria might be extended even further.

# A Appendix

## A.1 Proof of proposition 1

### Proof of (i)

I show that the first version of strong deep pockets (D.2.1) implies weak deep pockets (D.1). Let  $(a^o, b^o(\cdot))$  be a feasible pair, in which  $b^o(\cdot)$  is a set of truthful bidding functions relative to the utility level  $u^o$ . Also, let  $(a^*, b^*)$  be a feasible pair. Because of strong deep pockets,  $u_i(a^*, b^o(a^*)) \geq u_i(a^*, \bar{b}_i(a^*), b_{-i}^o(a^*))$  for all  $a \in A$ . Then, the definition of truthful bidding functions implies either a)  $b_i^o(a^*) = \phi(a^*; u^o; b_{-i}^o(\cdot))$  or b)  $b_i^o(a^*) = \bar{b}_i(a^*)$ . If a holds, then  $u_i(a^*, b^o(a^*)) = u^o$ , while if b holds, then  $u_i(a^*, b^o(a^*)) < u^o$ . Taking a and b together yields  $u^o = u_i(a^o, b^o(a^o)) \geq u_i(a^*, b^o(a^*))$ , for all  $i$ . Since this inequality holds for all truthful bidding functions and feasible pairs, it also holds for the equilibrium bidding functions and the feasible pair in the definition of strong deep pockets. The proof that the second version of weak deep pockets implies strong deep pockets is essentially the same. Q.E.D.

### Proof of (iii)

#### Part 1.

First, I show that lobbying monotonicity and small externalities imply conflict of interests.

The total differential of the utility of the principals is  $du_i = \frac{\partial u_i}{\partial b_i} db_i + \frac{\partial u_i}{\partial \tilde{b}_{-i}} d\tilde{b}_{-i}$ . Rearranging terms yields:  $du_i = (\frac{\partial u_i}{\partial b_i} - \frac{\partial u_i}{\partial \tilde{b}_{-i}}) db_i + \frac{\partial u_i}{\partial \tilde{b}_{-i}} d\tilde{b}$ .

In order to proceed I need to show two things. First, that when the agent's utility increases ( $d\tilde{b} > 0$ ), there is at least one principal who becomes worse off ( $du_i < 0$  for at least one  $i$ ).

I start with negative externalities ( $\frac{\partial u_i}{\partial \tilde{b}_{-i}} < 0$ ). In this case small externalities imply that  $\frac{\partial u_i}{\partial b_i} - \frac{\partial u_i}{\partial \tilde{b}_{-i}} < 0$ . Then, since  $d\tilde{b} > 0$  there is at least an  $i$  such that  $db_i > 0$ . As a result,  $du_i < 0$  for at least one principal.

Now, I turn to positive externalities ( $\frac{\partial u_i}{\partial \tilde{b}_{-i}} > 0$ ). Consider the principal with the largest increase in bids. Then, for this principal  $n db_i \geq d\tilde{b}$ . Substituting this expression in the expression for  $du_i$  yields  $du_i \leq (\frac{\partial u_i}{\partial b_i} + (n - 1) \frac{\partial u_i}{\partial \tilde{b}_{-i}}) db_i < 0$ . The last inequality follows from small externalities, since  $\frac{\partial u_i}{\partial b_i} + (n - 1) \frac{\partial u_i}{\partial \tilde{b}_{-i}} < 0$ .

The second thing I need to show is that if all principals are weakly better off and at least one is strictly better off ( $du > 0$ ), then the utility of the agent

decreases ( $d\tilde{b} < 0$ ). Assume the contrary,  $d\tilde{b} \geq 0$ . Then, if  $db_i = 0$  for all  $i$  it must be that  $du_i = 0$  for all  $i$ , which is a contradiction. Thus, for at least an  $i$ , it must be that  $db_i > 0$ . Then, for this principal the analysis above implies that  $du_i < 0$ , which is a contradiction.

**Part 2.**

In this part, I show that small externalities along with opposing monotonicity imply weak deep pockets. Again I consider only lobbying monotonicity, since the proof for the case of market monotonicity is essentially the same. I start with a proof of a lemma that I will use later on in the proof.

**Lemma 1**

Consider a game that exhibits small negative externalities and lobbying monotonicity. Then, the following is true:

If  $b, b'$  are feasible for some  $a \in A$  and there exists an  $i$ , such that  $b'_i > b_i$  and  $u_i(a, b'_i, \tilde{b}_{-i}) > u_i(a, b_i, \tilde{b}_{-i})$  then,  $\tilde{b}' < \tilde{b}$ .

Proof of lemma 1:

Assume the contrary:  $\tilde{b}' \geq \tilde{b} \Rightarrow b'_i - b_i \geq \tilde{b}_{-i} - \tilde{b}'_{-i} > 0$ . The last inequality follows from  $b'_i > b_i$ ,  $u_i(a, b'_i, \tilde{b}'_{-i}) > u_i(a, b_i, \tilde{b}_{-i})$  and negative externalities. Define  $\kappa = b'_i - b_i > 0$ . Then, there exists  $\lambda \in (0, 1]$  such that  $\tilde{b}_{-i} - \tilde{b}'_{-i} = \lambda\kappa$ . Substituting  $\kappa$  and  $\lambda$  in the utility function of principal  $i$ , yields:  $u_i(a, b'_i, \tilde{b}'_{-i}) = u_i(a, b_i + \kappa, \tilde{b}_{-i} - \lambda\kappa)$ . Then,  $\frac{\partial u_i}{\partial \kappa} = \frac{\partial u_i}{\partial b_i} - \lambda \frac{\partial u_i}{\partial b_{-i}} < 0$ . The last inequality is due to small negative externalities. Viewing  $u_i(\cdot)$  as a function of  $\kappa$  and using  $\frac{\partial u_i}{\partial \kappa} < 0$  yields  $u_i(0) > u_i(b'_i - b_i)$  or  $u_i(a, b_i, \tilde{b}_{-i}) > u_i(a, b'_i, \tilde{b}'_{-i})$  which is a contradiction. This concludes the proof of the lemma.

I proceed now with the rest of the proof of part 2.

The properties of  $(a^*, b^*)$  and  $(a^o, b^o(a^o))$  imply that  $u_0(a^*, \tilde{b}^*) \geq u_0(a^o, \tilde{b}^o(a^o)) \geq u_0(a^*, \tilde{b}^o(a^*))$ . In turn, this chain of inequalities implies that  $\tilde{b}^* \geq \tilde{b}^o(a^*)$ . Assume there is an  $i$ , such that  $u_i(a^*, b^o(a^*)) > u_i(a^o, b^o(a^o))$ . Then, if  $\tilde{b}^* = \tilde{b}^o(a^*)$  it follows that  $u_0(a^*, \tilde{b}^*) = u_0(a^o, \tilde{b}^o(a^o)) = u_0(a^*, \tilde{b}^o(a^*))$ . In this case there is a contradiction since the pair  $(a^o, b^o(\cdot))$  violates trivially the definition of equilibrium. Specifically, it follows from  $u_0(a^*, \tilde{b}^o(a^*)) = u_0(a^o, \tilde{b}^o(a^o))$  that  $a^* \in \arg \max_{a \in A} u_0(a, \tilde{b}^o(a))$ . This last observation, along with  $u_i(a^*, b^o(a^*)) > u_i(a^o, b^o(a^o))$  imply that  $a^o$  can not be part of an equilibrium. Therefore,  $\tilde{b}^* > \tilde{b}^o(a^*)$ .

Since the utility of the agent is strictly increasing in total bids,  $u_0(a^*, \tilde{b}^*) > u_0(a^*, \tilde{b}^o(a^*))$ . Then, following the first part of this proof, there exists an  $i$  such that  $u_i(a^*, b^o(a^*)) > u_i(a^*, b^*) \geq u_i(a^o, b^o(a^o))$ . For this  $i$ , the definition of truthful responses implies that  $b_i^o(a^*) = \bar{b}_i(a^*) \geq b_i^*$ .

I consider two cases. First, I consider positive externalities. Since  $u_i(a^*, b^o(a^*)) > u_i(a^*, b^*)$  and  $b_i^o(a^*) \geq b_i^*$ , it must also be that  $\tilde{b}_{-i}^o(a^*) \geq \tilde{b}_{-i}^*$  with at least one

strict inequality. However, in this case  $\tilde{b}^o(a^*) > \tilde{b}^*$ , which is a contradiction.

Second, I turn to negative externalities. In this case, I start by observing that  $b_i^o(a^*) \geq b_i^*$  does not hold for all  $i$ , because if it does, it follows that  $\tilde{b}^o(a^*) \geq \tilde{b}^*$ , which is a contradiction. Thus, there is at least an  $i$ , such that  $b_i^o(a^*) < b_i^* \leq \bar{b}_i(a^*)$ . Then, the definition of truthful responses implies that  $u_i(a^*, b^o(a^*)) < u_i(a^o, b^o(a^o)) \leq u_i(a^*, b^*)$ . Finally, using lemma 1 yields  $\tilde{b}^o(a^*) > \tilde{b}^*$ , which is a contradiction. Thus  $u(a^o, b^o(a^o)) \geq u(a^*, b^o(a^*))$ . Q.E.D.

## Proof of (iv)

### Conflict of interests.

I prove first that lobbying monotonicity and symmetric externalities imply conflict of interests at symmetric allocations.

Consider a game that exhibits lobbying monotonicity and symmetric externalities and let  $(a, b^*)$ ,  $(a, b)$  be two feasible pairs and also let  $(a, b)$  be symmetric. Then I will prove that, (i) if  $\tilde{b}^* > \tilde{b}$  there is an  $i$  such that  $u_i(a, b^*) < u_i(a, b)$  and (ii) if  $u(a, b^*) > u(a, b)$  then  $\tilde{b}^* < \tilde{b}$ .

I start with the proof of (i). I assume the contrary,  $u_i(a, b^*) \geq u_i(a, b)$ . I notice that it can not be that  $b_i^* > b_i$  for all  $i$ , because in this case  $\tilde{b}_{-i}^* > \tilde{b}_{-i}$  and therefore negative externalities along with lobbying monotonicity imply that  $u_i(a, b^*) < u_i(a, b)$ , for all  $i$ . Furthermore, it can not be that  $b_i^* \leq b_i$  for all  $i$ , since in this case  $\tilde{b}^* \leq \tilde{b}$ . Thus, for all principals either  $b_i^* = b_i + \xi_i$  or  $b_i^* = b_i - \mu_i$  for some  $\xi_i \geq 0$  and  $\mu_i > 0$ . Moreover,  $\sum_i (\xi_i - \mu_i) > 0$  because  $\tilde{b}^* < \tilde{b}$ .

Because of symmetry,  $u_i(a, b^*) \geq u_i(a, b)$  for all  $i$ , implies  $u_k(a, b_i^*, \tilde{b}_{-i}^*) \geq u_k(a, b_k, \tilde{b}_{-k})$  for all  $i$ . Then, the quasi-concavity of  $u_k(\cdot)$  yields  $u_k(a, \frac{1}{n} \sum_i b_i^*, \frac{1}{n} \sum_i \tilde{b}_{-i}^*) \geq u_k(a, b_k, \tilde{b}_{-k})$ . However,  $\frac{1}{n} \sum_i b_i^* = \frac{1}{n} \sum_i b_i + \frac{1}{n} \sum_i (\xi_i - \mu_i) = b_k + \frac{1}{n} \sum_i (\xi_i - \mu_i) > b_k$ . Also,  $\frac{1}{n} \sum_i \tilde{b}_{-i}^* = \frac{n-1}{n} \sum_i b_i + \frac{n-1}{n} \sum_i (\xi_i - \mu_i) = \tilde{b}_{-k} + \frac{n-1}{n} \sum_i (\xi_i - \mu_i) > \tilde{b}_{-k}$ . Then, because of negative externalities  $u_k(a, \frac{1}{n} \sum_i b_i^*, \frac{1}{n} \sum_i \tilde{b}_{-i}^*) < u_k(a, b_k, \tilde{b}_{-k})$ , which is a contradiction.

I turn now to (ii). Specifically, I must show that if  $u(a, b^*) > u(a, b)$  then  $\tilde{b}^* < \tilde{b}$ . Assume the contrary,  $\tilde{b}^* \geq \tilde{b}$ . If  $\tilde{b}^* > \tilde{b}$  then the analysis above yields a contradiction. Therefore,  $\tilde{b}^* = \tilde{b}$ . Using symmetry,  $u(a, b^*) > u(a, b)$  implies that  $u_k(a, b_i^*, \tilde{b}_{-i}^*) \geq u_k(a, b_k, \tilde{b}_{-k})$  for all  $i$ , with at least one strict inequality. Then, the quasi-concavity of  $u_k(\cdot)$  yields  $u_k(a, \frac{1}{n} \sum_i b_i^*, \frac{1}{n} \sum_i \tilde{b}_{-i}^*) > u_k(a, b_k, \tilde{b}_{-k})$ . However,  $\frac{1}{n} \sum_i b_i^* = \frac{\tilde{b}^*}{n} = \frac{\tilde{b}}{n} = b_k$ . Also,  $\frac{1}{n} \sum_i \tilde{b}_{-i}^* = \frac{(n-1)\tilde{b}^*}{n} = \frac{(n-1)\tilde{b}}{n} = (n-1)b_k = \tilde{b}_{-k}$ . These equalities yield  $u_k(a, \frac{1}{n} \sum_i b_i^*, \frac{1}{n} \sum_i \tilde{b}_{-i}^*) = u_k(a, b_k, \tilde{b}_{-k})$ , which is a contradiction.

### Weak deep pockets

I continue to prove that symmetric externalities and opposing monotonicity imply weak deep pockets. In this regard consider such a game and let  $(a^o, b^o(\cdot))$  be a symmetric equilibrium and  $(a^*, b^*)$  be a feasible pair such that  $u_0(a^*, b^*) \geq u_0(a^o, b^o(a^o))$  and  $u(a^*, b^*) \geq u(a^o, b^o(a^o))$  with at least one strict inequality. Then,  $u(a^o, b^o(a^o)) \geq u(a^*, b^o(a^*))$ .

I start by assuming the opposite. That is, there exists an  $i$  for which  $u_i(a^*, b^o(a^*)) > u_i(a^o, b^o(a^o))$ . Consider first the case  $u_0(a^*, b^*) > u_0(a^o, b^o(a^o))$ . Then,  $u_0(a^*, b^*) > u_0(a^o, b^o(a^o)) \geq u_0(a^*, b^o(a^*))$  implies that  $u_0(a^*, b^*) > u_0(a^*, b^o(a^*))$ . Therefore, because of conflict of interests there exists an  $i$  such that  $u_i(a^*, b^o(a^*)) > u_i(a^*, b^*)$ . Following the definition of truthful responses, for this  $i$ , it follows that  $b_i^o(a^*) = \bar{b}_i(a^*)$ . Then, symmetry implies that  $b_i^o(a^*) = \bar{b}_i(a^*)$  for all  $i$ . Therefore,  $\tilde{b}^o(a^*) \geq \tilde{b}^*$  and  $u_0(a^*, b^o(a^*)) \geq u_0(a^*, b^*)$  which is a contradiction. The last inequality follows from the fact that the game is cumulative.

Now I turn to the case  $u_0(a^*, b^*) = u_0(a^o, b^o(a^o))$ . In this case, if  $u_0(a^o, b^o(a^o)) > u_0(a^*, b^o(a^*))$  the analysis above can be repeated. Thus,  $u_0(a^*, b^*) = u_0(a^o, b^o(a^o)) = u_0(a^*, b^o(a^*))$ . This equality implies that  $a^*$  also maximizes the utility of the agent when the principals submit  $b^o(\cdot)$ . Moreover, because of assuming the contrary in the beginning of the proof, there exists an  $i$  such that  $u_i(a^*, b^o(a^*)) > u_i(a^o, b^o(a^o))$ . The existence of this principal leads to a contradiction, since  $a^o$  violates the definition of equilibrium. Q.E.D.

## A.2 Proof of proposition 4

Assume the contrary is true. Then, there exists a feasible pair  $(a^*, b^*)$  and an equilibrium of the game  $(a^o, b^o(\cdot))$ , such that  $u_0(a^*, b^*) \geq u_0(a^o, b^o(a^o))$  and  $u(a^*, b^*) \geq u(a^o, b^o(a^o))$ , with at least one strict inequality. These inequalities yield:

$$u_0(a^*, b^*) \geq u_0(a^o, b^o(a^o)) \geq u_0(a^*, b^o(a^*))$$

and

$$u(a^*, b^*) \geq u(a^o, b^o(a^o)) \geq u(a^*, b^o(a^*)).$$

The last inequality in the first expression above holds because  $(a^o, b^o(\cdot))$  is an equilibrium. Also, the last inequality in the second expression is due to deep pockets.

The two chains of inequalities above imply that  $u_0(a^*, b^*) \geq u_0(a^*, b^o(a^*))$  and  $u(a^*, b^*) \geq u(a^*, b^o(a^*))$ , with at least one strict inequality. Then, there is a contradiction. Indeed, if  $u_0(a^*, b^*) > u_0(a^*, b^o(a^*))$  it follows from conflict of interests that there is an  $i$ , such that  $u_i(a^*, b^*) \geq u_i(a^*, b^o(a^*))$ , which

contradicts the second chain of inequalities. Alternatively, if  $u(a^*, b^*) > u(a^*, b^o(a^*))$  conflict of interests implies that  $u_0(a^*, b^*) > u_0(a^*, b^o(a^*))$ , which contradicts the first chain of inequalities. Q.E.D.

## References

- Aidt, Toke S, and Uk Hwang. 2008. “On the internalization of cross-national externalities through political markets: the case of labour standards”. *Journal of Institutional and Theoretical Economics JITE* 164 (3): 509–533.
- Alesina, Alberto, and Dani Rodrik. 1994. “Distributive politics and economic growth”. *The quarterly journal of economics* 109 (2): 465–490.
- Anton, James J, and Dennis A Yao. 1989. “Split awards, procurement, and innovation”. *The RAND Journal of Economics*: 538–552.
- Bergemann, Dirk, and Juuso Välimäki. 2003. “Dynamic common agency”. *Journal of Economic Theory* 111 (1): 23–48.
- Bernheim, B Douglas, and Michael D Whinston. 1986. “Menu auctions, resource allocation, and economic influence”. *The quarterly journal of economics* 101 (1): 1–31.
- Boultzis, Ilias. 2015. “Common agency with caring agents”. *Economics Letters* 126:71–74.
- Campante, Filipe R, and Francisco HG Ferreira. 2007. “Inefficient lobbying, populism and oligarchy”. *Journal of Public Economics* 91 (5-6): 993–1021.
- Chiesa, Gabriella, and Vincenzo Denicolò. 2009. “Trading with a common agent under complete information: A characterization of Nash equilibria”. *Journal of Economic Theory* 144 (1): 296–311.
- Dasgupta, Partha, and Eric Maskin. 2000. “Efficient auctions”. *The Quarterly Journal of Economics* 115 (2): 341–388.
- Dixit, Avinash. 1996. “Special-interest lobbying and endogenous commodity taxation”. *Eastern Economic Journal* 22 (4): 375–388.
- Dixit, Avinash, Gene M Grossman, and Elhanan Helpman. 1997. “Common agency and coordination: General theory and application to government policy making”. *Journal of political economy* 105 (4): 752–769.
- Dixit, Avinash, Gene Grossman, and Elhanan Helpman. 1996. *Common Agency and Coordination: General Theory and Application to Tax Policy*. Tech. rep. CEPR Discussion Papers.

- Esteller-Moré, Alejandro, Umberto Galmarini, and Leonzio Rizzo. 2012. “Vertical tax competition and consumption externalities in a federation with lobbying”. *Journal of Public Economics* 96 (3-4): 295–305.
- Galperti, Simone. 2015. “Common agency with informed principals: Menus and signals”. *Journal of Economic Theory* 157:648–667.
- Grossman, Gene M, and Elhanan Helpman. 1994. “Protection for sale”. *The American Economic Review* 84 (4): 833.
- Ko, Chiu Yu. 2011. “Menu auctions with non transferable utilities and budget constraints”.
- Martimort, David, and Lars Stole. 2009a. “Market participation in delegated and intrinsic common-agency games”. *The RAND journal of economics* 40 (1): 78–102.
- . 2009b. “Selecting equilibria in common agency games”. *Journal of Economic Theory* 144 (2): 604–634.
- . 2002. “The revelation and delegation principles in common agency games”. *Econometrica* 70 (4): 1659–1673.
- Mitra, Devashish. 1999. “Endogenous lobby formation and endogenous protection: a long-run model of trade policy determination”. *American Economic Review* 89 (5): 1116–1134.
- Persson, Torsten. 1998. “Economic policy and special interest politics”. *The Economic Journal* 108 (447): 310–327.
- Persson, Torsten, and Guido Tabellini. 1994. “Does centralization increase the size of government?” *European Economic Review* 38 (3-4): 765–773.
- Peters, Michael. 2001. “Common agency and the revelation principle”. *Econometrica* 69 (5): 1349–1372.
- Peters, Michael, and Balázs Szentes. 2012. “Definable and contractible contracts”. *Econometrica* 80 (1): 363–411.
- Prat, Andrea, and Aldo Rustichini. 2003. “Games played through agents”. *Econometrica* 71 (4): 989–1026.
- Romer, Paul M. 1986. “Increasing returns and long-run growth”. *Journal of political economy* 94 (5): 1002–1037.
- Szentes, Balázs. 2014. “Contractible contracts in common agency problems”. *The Review of Economic Studies* 82 (1): 391–422.

## B Appendix

### B.1 Proof of proposition 1

#### Proof of (ii)

I consider market monotonicity. The fact that market monotonicity implies conflict of interests in models without externalities is obvious. In what follows I prove that it also implies weak deep pockets.

Define  $u^o = u(a^o, b^o(a^o))$ . Then,  $u(a^*, b^*) \geq u(a^o, b^o(a^o)) = u(a^*, \phi(a^*; u^o))$ . Because  $u(\cdot)$  is increasing in own bids it must be that  $b_i^* \geq \phi_i(a^*; u^o)$  for all  $i$ . Furthermore, since  $b_i^*$  is feasible it follows that  $b_{max} \geq b_i^* \geq \underline{b}_i(a^*)$ . Therefore,  $b_{max} \geq \phi_i(a^*; u^o)$ . This last inequality and the definition of truthful responses imply that either  $b_i^o(a^*) = \phi_i(a^*; u^o)$  or  $b_i^o(a^*) = \underline{b}_i(a^*)$ . In turn, these equalities along with the feasibility of  $b_i^*$  imply that  $b_i^* \geq b_i^o(a^*)$  for all  $i$ . Then, the fact that  $u_0(\cdot)$  is decreasing in all bids yields:  $u_0(a^*, b^o(a^*)) \geq u_0(a^*, b^*) \geq u_0(a^o, b^o(a^o))$ . If any of these two inequalities is strict then we have a contradiction since  $(a^o, b^o(\cdot))$  is an equilibrium. Thus,  $u_0(a^*, b^o(a^*)) = u_0(a^*, b^*) = u_0(a^o, b^o(a^o))$ . Now, if there exists an  $i$  such that  $u_i(a^*, b_i^o(a^*)) > u_i^o$  we arrive at a contradiction since the pair  $(a^o, b^o(\cdot))$  violates trivially the definition of equilibrium. Indeed, in such a case it follows from  $u_0(a^*, b^o(a^*)) = u_0(a^o, b^o(a^o))$  that  $a^* \in \arg \max_{a \in A} u_0(a, b_i^o(a), b_{-i}^o(a))$ . Thus,  $a^o$  can not be part of an equilibrium. Therefore,  $u(a^o, b^o(a^o)) \geq u(a^*, b^o(a^*))$ . Q.E.D.

### B.2 Proof of propositions 2 and 3

#### Proof of proposition 2

I prove proposition 2 under the assumption of market monotonicity. The proof for lobbying monotonicity is very similar.

Let  $b_i^o(\cdot)$  be a best response of principal  $i$  to the bidding functions  $b_{-i}^o(\cdot)$  of the other principals. Then, there exists  $a^o \in \arg \max_{a \in A} u_0(a, b^o(a))$ , such that there does not exist a feasible pair  $(a^*, b_i^*(\cdot))$ , such that  $u_i(a^*, b_i^*(a^*), b_{-i}^o(a^*)) > u_i(a^o, b^o(a^o))$  and  $a^* \in \arg \max_{a \in A} u_0(a, b_i^*(a), b_{-i}^o(a))$ .

Define,  $u_i^o = u_i(a^o, b^o(a^o))$  and the truthful response of principal  $i$  to the bidding functions of the other principals relative to  $u_i^o$  as  $b_i^T(a; b_{-i}(\cdot); u_i^o)$ . For the shake of simplicity in the remaining of the proof I suppress the other arguments and write  $b_i^T(a)$ . Finally, I define the set  $A' = \arg \max_{a \in A} u_0(a, b_i^T(a), b_{-i}^o(a))$ .

If  $a^o \in A'$  then  $b_i^T(a)$  is trivially a best response since  $b_i^T(a^o) = b_i^o(a^o)$ .

I turn now to the case in which  $a^o \notin A'$ . Assume that  $a' \in A'$ . Then, I claim that  $u_0(a, b_i^T(a'), b_{-i}^o(a')) > u_0(a', b^o(a'))$ . If not, then  $u_0(a', b^o(a')) \geq$

$u_0(a, b_i^T(a'), b_{-i}^o(a')) > u_0(a, b_i^T(a^o), b_{-i}^o(a^o)) = u_0(a^o, b^o(a^o))$ . The inequality follows from the fact that  $a^o \notin A$  while  $a' \in A$  and the equality from the definition of  $b_i^T(\cdot)$ . Thus,  $u_0(a', b^o(a')) > u_0(a^o, b^o(a^o))$  which is a contradiction because  $a^o \in \arg \max_{a \in A} u_0(a, b^o(a))$ .

Furthermore, because of market monotonicity the utility of the agent is decreasing in all bids. Therefore,  $u_0(a, b_i^T(a'), b_{-i}^o(a')) > u_0(a', b^o(a'))$  implies that  $b_i^T(a') < b_i^o(a') \leq \bar{b}_i(a')$ . Then, following the definition of truthful responses either  $b_i^T(a') = \phi_i(a'; u_i^o; b_{-i}(\cdot))$  or  $b_i^T(a') = \underline{b}_i(a')$ . The last result combined with the increasing utility of the principals in own bids yields  $u_i(a, b_i^T(a'), b_{-i}^o(a')) \geq u_i^o$  which proves that  $b_i^T(\cdot)$  is a best response. Q.E.D.

### Proof of proposition 3

I prove proposition 3 under the assumption of market monotonicity. The proof for lobbying monotonicity is very similar. I start with the following lemma.

#### Lemma

A feasible pair  $(a^o, b^o(\cdot))$  is an equilibrium if and only if:

(i)  $a^o \in \arg \max_{a \in A} u_0(a, b^o(a))$

(ii) For all  $i$ ,  $(a^o, b_i^o(a^o)) \in \arg \max_{(a, b_i)} u_i(a, b_i, b_{-i}^o(a))$  subject to  $a \in A$ ,  $b_i = b_i(a)$

for some feasible bidding function  $b_i(\cdot)$  and  $u_0(a, b_i, b_{-i}^o(a)) \geq \max_{a' \in A} u_0(a', b_{max}, b_{-i}^o(a'))$ .

Proof of the Lemma:

Necessity:

Assume that  $(a^o, b^o(\cdot))$  is an equilibrium but it does not solve the maximization problem in condition (ii) of the lemma. Then, there exists an  $i$  and a feasible pair  $(a^*, b_i^*)$  which satisfies the constraints in condition (ii) and yields  $u_i(a^*, b_i^*, b_{-i}^o(a^*)) > u_i(a^o, b^o(a^o))$ .

In this case though, can show that  $(a^o, b^o(\cdot))$  is not an equilibrium which is a contradiction. In order to show this contradiction I need to prove that there exists a feasible bidding function  $b_i^*(\cdot)$  such that  $b_i^*(a^*) = b_i^*$  and  $a^* \in \arg \max_{a \in A} u_0(a, b_i^*(a), b_{-i}^o(a))$ .

Since  $(a^*, b_i^*)$  satisfies the constraints in condition (ii), there exists a feasible bidding function  $\hat{b}_i^*(\cdot)$  such that  $\hat{b}_i^*(a^*) = b_i^*$ . Define the function  $\varphi_i : A \rightarrow R$  implicitly, through  $u_0(a, \varphi_i, b_{-i}^o(a)) = u_0(a^*, b_i^*, b_{-i}^o(a^*))$ . The function  $\varphi_i(\cdot)$  always exists because the utility of the agent is strictly decreasing in all bids. Furthermore,  $\varphi_i(a^*) = b_i^*$ .

Moreover, because of condition (ii),  $u_0(a^*, b_i^*, b_{-i}^o(a^*)) \geq \max_{a' \in A} u_0(a', b_{max}, b_{-i}^o(a'))$  and therefore  $u_0(a, \varphi_i(a), b_{-i}^o(a)) = u_0(a^*, b_i^*, b_{-i}^o(a^*)) \geq \max_{a' \in A} u_0(a', b_{max}, b_{-i}^o(a')) \geq$

$u_0(a, b_{max}, b_{-i}^o(a))$  for all  $a \in A$ .

Since the agent's utility is strictly decreasing in all bids, this chain of inequalities implies that  $\varphi_i(a) \leq b_{max}$ . Define  $b_i^*(a) = \max[\varphi_i(a), \hat{b}_i(a)]$ . Due to  $\varphi_i(a) \leq b_{max}$  and the fact that  $\hat{b}_i(\cdot)$  is feasible,  $b_i^*(a) \in [\underline{b}_i(a), b_{max}]$  for all  $a \in A$  and therefore  $b_i^*(\cdot)$  is also feasible. Furthermore,  $\hat{b}_i(a^*) = \varphi_i(a^*) = b_i^*$  and therefore  $b_i^*(a^*) = b_i^*$ .

Finally, I observe that

$$u_0(a^*, b_i^*(a^*), b_{-i}^o(a^*)) = u_0(a^*, b_i^*, b_{-i}^o(a^*)) = u_0(a, \varphi_i(a), b_{-i}^o(a)) \geq u_0(a, b_i^*(a), b_{-i}^o(a))$$

for all  $a \in A$ . The last inequality follows from the definition of  $b_i^*(\cdot)$  which implies that  $b_i^*(a) \geq \varphi_i(a)$ . This chain confirms that  $a^* \in \arg \max_{a \in A} u_0(a, b_i^*(a), b_{-i}^o(a))$

which concludes the proof for necessity.

Sufficiency:

Assume that  $(a^o, b_i^o(a^o))$  is a feasible pair that solves the maximization problem in condition (ii) of the lemma but is not part of an equilibrium and that  $b_i^o(\cdot)$  is a feasible bidding function. Then, there exists a feasible pair  $(a^*, b_i^*(\cdot))$  such that

$$\text{a) } a^* \in \arg \max_{a \in A} u_0(a, b_i^*(a), b_{-i}^o(a)) \text{ and}$$

$$\text{b) } u_i(a^*, b_i^*(a^*), b_{-i}^o(a^*)) > u_i(a^o, b_i^o(a^o))$$

$$\text{However, in this case } u_0(a^*, b_i^*(a^*), b_{-i}^o(a^*)) \geq u_0(a, b_i^*(a), b_{-i}^o(a)) \geq u_0(a, b_{max}, b_{-i}^o(a)).$$

The first inequality is due to (a) above, while the second inequality is because  $b_i^*(\cdot)$  is feasible and therefore  $b_i^*(a) \leq b_{max}$  for all  $a \in A$ . As a result,  $(a^*, b_i^*(a^*))$  satisfies the constraints in condition (ii) of the lemma, which implies that  $(a^o, b_i^o(a^o))$  does not solve the maximization problem. This contradiction concludes the proof for necessity and the lemma.

Now I proceed with the rest of the proof.

Let  $(a^o, b^o(\cdot))$  be an equilibrium of the game. Consider principal  $i$ . I will show that the inequality in condition (ii) of the lemma, holds as an equality. Assume the contrary. Then, if the equilibrium bid equals the maximum bid or  $b_i^o(a^o) = b_{max}$ , the contradiction is obvious. If on the other hand  $b_i^o(a^o) < b_{max}$ , the pair  $(a^o, b_i^o(a^o))$  does not solve the maximization problem in the lemma. Indeed, in this case there exists a feasible pair  $(a^o, b_i^*)$  such that  $b_i^* > b_i^o(a^o)$ , which satisfies the constraints in condition (ii) of the lemma and yields  $u_i(a^o, b_i^*, b_{-i}^o(a^o)) > u_i(a^o, b_i^o(a^o))$ . In order to conclude the proof, I need to show that there exists a feasible bidding function  $b_i^*(\cdot)$  such that  $b_i^*(a^o) = b_i^*$ .

In this respect consider the function  $b_i^* = \begin{cases} b_i^o(a) & \text{if } a \neq a^o \\ b_i^* & \text{if } a = a^o \end{cases}$ . Q.E.D.

### B.3 Market application

Solving the profit function with respect to  $p_i$  yields:  $p_i = \frac{\Pi + cq_i^2}{q_i}$ . When both sellers offer this price function the agent chooses the quantity that minimizes his spending. The problem he solves is the following:

$$\max_{q_1 \in [0, \bar{q}]} -p_1 q_1 - p_2 (\bar{q} - q_1)$$

The solution to this problem yields:  $q_i = \frac{\bar{q}}{2}$

Then, I use proposition 3 to calculate the equilibrium  $\Pi_i$ . I guess and verify that when seller  $j$  submits the reservation price the buyer buys all the quantity from principal  $i$ . Under this guess, proposition 3 yields  $-2\Pi_i - c\frac{\bar{q}^2}{2} = -\Pi - c\bar{q}^2$ , which in turn yields  $\Pi_i = c\frac{\bar{q}^2}{2}$ . I substitute  $\Pi_i$  in the expression for  $p_i$  and get  $\frac{c\bar{q}^2}{2q_i} + cq_i$ . Then, I solve the equation  $\bar{p} = \frac{c\frac{\bar{q}^2}{2} + c(q_i^*)^2}{q_i^*}$  with respect to  $q_i^*$  in order to calculate the quantity  $q_i^*$  at which the price hits the upper bound. This exercise yields:  $q_i^* = \frac{\bar{p} - \sqrt{\bar{p}^2 - 2c^2\bar{q}^2}}{2c}$ . In this way I obtain the price function that I provide in the main text. Moreover, the assumption  $\bar{p} > 3c\bar{q}$  implies that  $\frac{\bar{q}}{2} > q_i^*$ . and  $p_i(\bar{q}) < \bar{p}$ , which verifies the guess above. Finally,  $p_i(q_i) \geq cq_i$  for all  $q_i \in [0, \bar{q}]$ , which confirms the proposed equilibrium price function.

### B.4 Lobbying with externalities

I will guess and verify that  $b_i = \frac{\tilde{T}}{m_i} - t_i k_i^o$  is part of a truthful equilibrium. Under the above guess  $b = \frac{1}{m}(m_1 b_1 + m_2 b_2) = \frac{\tilde{T}}{m}$ . Now I ask two questions. First, does a constant utility level  $u_i$  exists, such that  $b_i = \frac{\tilde{T}}{m_i} - t_i k_i^o$  is a truthful response of principal  $i$  to the average contribution  $\frac{\tilde{T}}{m}$  relative to the constant  $u_i$ ?

Since the utility of the members of lobby  $i$  is  $u_i = (aA\delta)^\delta (c_i^1)^{(1+\delta)}$  I can concentrate on finding a constant level of consumption  $c_i^1$  that answers the question above. I guess that this consumption level is  $c_i^1 = \frac{1}{(1+\delta)} [k_i^o - \frac{\tilde{T}}{m} + \frac{1-a}{1+a\delta} \delta (k_i^o - \frac{2\tilde{T}}{m})]$ .

I proceed by substituting this guess in the LHS of equation 2. Then I solve for  $b_i$ . This operation yields  $b_i = \frac{\tilde{T}}{m_i} - t_i k_i^o$ . Thus, the utility level that answers the first question is  $u_i = (aA\delta)^\delta ((\frac{1}{1+\delta})(k_i^o - \frac{\tilde{T}}{m} + \frac{1-a}{1+a\delta} \delta (k_i^o - \frac{2\tilde{T}}{m})))^{(1+\delta)}$ . Furthermore, the assumptions  $\bar{t}_i = \frac{\tilde{T}}{m_i k_i^o}$  and  $\tilde{T} \leq m_1 k_1^o$  imply that  $0 \leq \frac{\tilde{T}}{m_i} - t_i k_i^o \leq (1 - t_i) k_i^o$  for all  $t_i \in [0, \bar{t}_i]$ .

I turn now to the second question. Is the utility level above an equilibrium utility level?

If this is the case then the corresponding contribution functions must satisfy proposition 3 (calculation). Indeed, if lobby  $i$  is the only one offering contributions then the government sets  $t_i = 0$ . In this case total contributions received by the government are  $m_i b_i = \tilde{T}$ . On the other hand if both lobbies contribute the government receives  $mb = \tilde{T}$ . Since these two quantities are the same proposition 3 is satisfied and therefore the suggested allocation is a truthful equilibrium.

## B.5 Example 1

First I show that the unique symmetric efficient allocation is  $a = 1, b_1 = b_2 = 1$ . Obviously in any efficient allocation  $a = 1$ . Assume there exists an efficient allocation such that  $b_i < 1$  for both  $i$ . Then, define  $\mu = 1 - \max\{b_1, b_2\} > 0$ . Consequently, if both the principals increase their bids at  $a = 1$  by  $\mu$  the utility of both the principals and the agent increases. Thus, in all efficient allocations  $a = 1$  and at least for one principal  $b_i = 1$ . In turn, this point implies that there is only one symmetric efficient allocation in which  $a = 1$  and for both principals  $b_i = 1$ .

Now I show that there are no efficient equilibria. Consider an efficient allocation. In any such allocation  $a = 1$  and at least for one principal  $b_i = 1$ . Without loss of generality assume that  $b_1 = 1$ . Then there are two cases:  $b_2(1) > 0$  and  $b_2(1) = 0$ . If  $b_2 = \lambda > 0$  then principal 1 can deviate to  $b_1 = \begin{cases} 1 - \frac{\lambda}{2} & \text{if } a = 1 \\ 0 & \text{if } a \neq 1 \end{cases}$ . In this case the agent still chooses  $a = 1$  and the utility of principal 1 increases regardless of his initial off equilibrium strategy.

If instead  $b_2(1) = 0$  principal 1 earns  $u_1 = 0$ . Then, principal 1 can deviate to  $b_1 = 0$  for all  $a \in [0, 1]$ . In this case, if there exists an  $a \in [0, 1]$  such that  $b_2(a) > 0$  the utility of principal 1 is  $u_1 = a + 2b_2 > 0$ . If on the other hand  $b_2 = 0$  for all  $a \in [0, 1]$  then the agent chooses  $a = 1$  (since he is indifferent among all  $a$ ) and principal 1 earns utility  $u_1 = 1 > 0$ .

Finally, as an example of an asymmetric inefficient equilibrium consider the following:  $b_1 = \begin{cases} 0.2 & \text{if } a = 1 \\ 0 & \text{if } a \neq 1 \end{cases}$ ,  $b_2 = \begin{cases} 0.2 & \text{if } a = 0.2 \\ 0 & \text{if } a \neq 0.2 \end{cases}$ . These bidding functions yield  $a = 1, b_1 = 0.2, b_2 = 0, u_0 = 0.2, u_1 = 0.8, u_2 = 1.4$ .

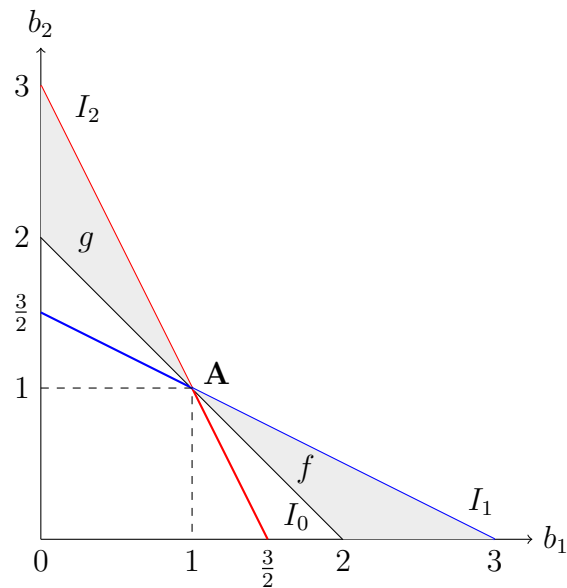
## B.6 Symmetric externalities

Examples 1 and 2 highlight the role of symmetry.

### Example 1

Consider a game with two principals in which  $u_0 = b_1 + b_2$ ,  $u_i = a - b_i - 2b_j$  for both  $i$ ,  $a \in [0, 1]$  and  $b_i(0) \in [0, 5]$  for both  $i$ . This game is symmetric. Define allocation  $A$  as  $a = 0$  and  $b_1 = b_2 = 1$ . For this allocation  $u_0 = 2$  and  $u_1 = u_2 = -3$ .

Graph 1 describes this situation. In this graph  $I_0, I_1$  and  $I_2$  are the indifference curves for the agent and the two principals that go through point  $A$ . The utility of the principals increases at allocations that lie to the south west of point  $A$ , while the utility of the agent at allocations that lie to the north east of the same point. In this regard the shaded area  $f$  depicts the allocations that make both principal 1 and the agent better off than in  $A$ . In a similar manner, the shaded area  $g$  depicts the same allocations for the agent and principal 2. The fact that the areas  $f$  and  $g$  intersect only at point  $A$  implies conflict of interests.



**Graph 1**

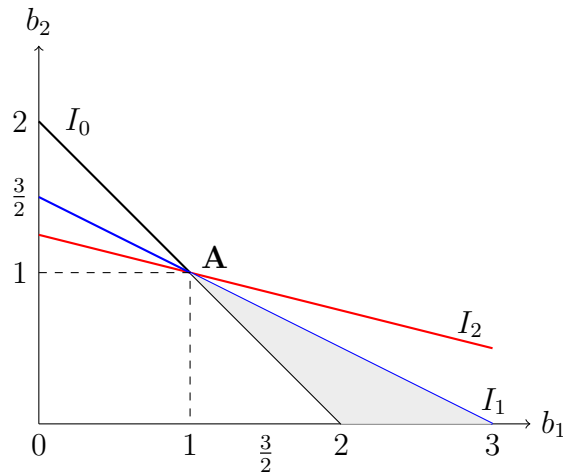
Symmetry and quasi-concavity.

### Example 2

Let me now consider an asymmetric example in which there is no conflict of interests. Consider a variation of the previous game in which  $u_0 = b_1 + b_2$ ,

$u_1 = a - b_1 - 4b_2$  and  $u_2 = a - 2b_2 - b_1$ . Define allocation  $A$  as  $a = 0$ ,  $b_1 = 1$  and  $b_2 = 1$ . This allocation yields  $u_0 = 2$ ,  $u_1 = -5$  and  $u_2 = -3$ .

Graph 2 reproduces graph 1 for this example. However, in this case, because of asymmetry, there are allocations that make both principals and the agent better off than in  $A$ . The shaded area in graph 2 depicts these allocations. The existence of such allocations violates conflict of interests.



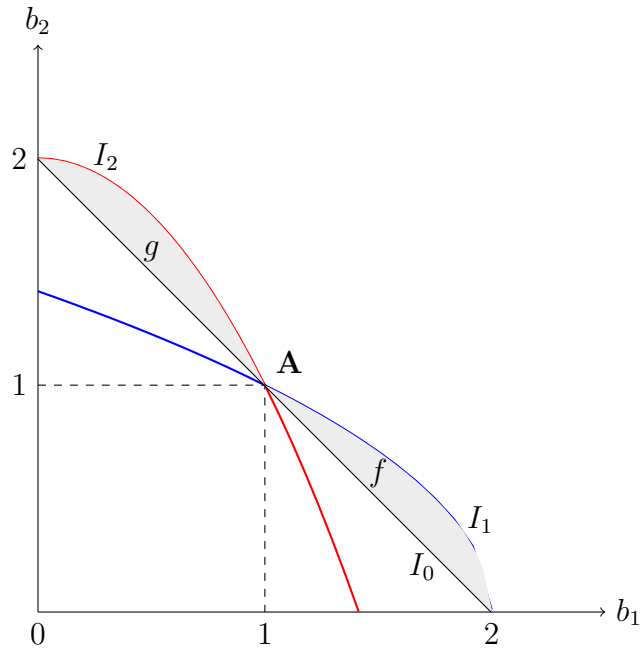
**Graph 2**

Quasi-concavity without symmetry.

Now I turn to quasi-concavity. Examples 3 and 4 that follow highlight its role.

**Example 3**

Consider a game in which  $u_0 = b_1 + b_2$ ,  $u_1 = a - b_1 - b_2^2$ ,  $u_2 = a - b_2 - b_1^2$ ,  $a \in [0, 1]$  and  $b_i(0) \in [0, 3]$  for both  $i$ . This game is both symmetric and quasi-concave. Define allocation  $A$  as  $a = 0$  and  $b_1 = b_2 = 1$ , which yields  $u_0 = 2$ ,  $u_1 = -2$  and  $u_2 = -2$ . Then, graph 3 depicts the respective indifference curves. As in graph 1 the shaded areas  $f$  and  $g$  depict the allocations that make the agent and one of the principals better off than in  $A$ . Again, the fact that  $f$  and  $g$  intersect only in  $A$ , implies conflict of interests.



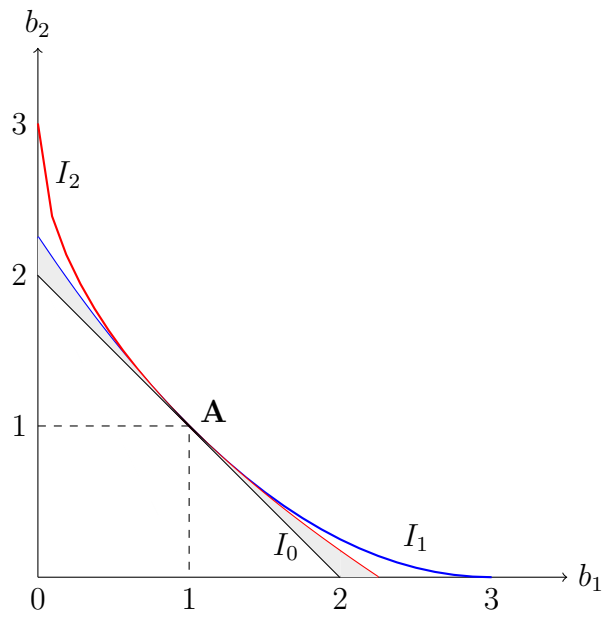
**Graph 3**

Symmetry and quasi-concavity (2).

I turn now to example 4.

**Example 4**

Consider a variation of example 3 in which  $u_0 = b_1 + b_2$ ,  $u_1 = a - b_1 - 2\sqrt{b_2}$  and  $u_2 = a - b_2 - 2\sqrt{b_1}$ . This game is symmetric but not quasi concave. Graph 4 reproduces graph 3 for this example. The shaded area depicts the set of allocations that make both the principals and the agent better off than in  $A$ . The fact that this set is not empty violates conflict of interests.



**Graph 4**  
Symmetry without quasi-concavity.