

Endogenous Forward Guidance*

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Abstract

We propose a novel framework where forward guidance (FG) is endogenously determined. Our model assumes that a monetary authority solves an optimal policy problem under commitment at the zero-lower bound. FG derives from two sources: 1. from committing to keep interest rates low at the exit of the liquidity trap to stabilize inflation today (e.g. Eggertsson and Woodford) and 2. from debt sustainability concerns, when the planner takes into account the consolidated budget constraint in optimization. Our model is tractable and admits an analytical solution for interest rates in which 1 and 2 show up as separate arguments that enter additively to the standard Taylor rule.

In the case where optimal policy reflects debt sustainability concerns monetary policy becomes subservient to fiscal policy, and needs to partially give up on the objective of stabilizing inflation and the output gap to satisfy the consolidated budget. We show analytically that this regime is isomorphic to a 'passive monetary /active fiscal policy' regime (e.g. Leeper (1991)). Instead, under no debt concerns, monetary policy is 'active' and fiscal policy ensures debt sustainability.

During liquidity trap (LT) episodes the presence of debt concerns, makes monetary policy ineffective in stabilizing inflation. Committing to keep interest rates low for a long period leads to deflation rather than to an increase inflation rates. Under no debt concerns, however, keeping interest rates low increases inflation and helps the economy escape from the LT.

We embed our theory into a DSGE model and estimate it with US data. Our findings suggest that FG during the Great Recession most likely did not reflect debt sustainability concerns, rather policy reflected a strong commitment to stabilize inflation and the output gap. Our quantitative findings are thus broadly consistent with the view that the evolution of debt aggregates had a rather small impact on monetary policy in the Great Recession.

Keywords: Bayesian estimation, DSGE model, Fiscal policy, Forward Guidance, Inflation, Liquidity trap, Monetary policy.

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1 Introduction

What forces lie behind the Federal Reserve’s policy to keep interest rates close to the zero lower bound (ZLB) for several years following the burst of the Great Recession in 2008-9? A standard view in monetary theory is that a central bank which focuses on stabilizing inflation and the output gap, should commit to keep interest rates at the ZLB for a long period of time, as this can mitigate the negative effects of shocks on output and inflation (Eggertsson and Woodford (2003)). However, the post-2009 developments in monetary policy, most notably the purchases of substantial amounts of long term government bonds by the Federal Reserve, coupled with the mounting debt and deficit levels in the US economy, have also led many economists to think that ‘keeping interest rates low’ was (at least partially) driven by the (implicit) objective to contain debt servicing costs.

Which of these alternative views is more plausible? A large body of recent literature has used medium-scale DSGE models to structurally identify key forces behind the behavior of macroeconomic aggregates in the Great Recession, focusing primarily on the lack of deflation in US data during this period (the so called missing deflation puzzle). In some of these studies (e.g., Del Negro et al. (2015); Kollman et al. (2016); Fratto and Uhlig (2014)) government debt aggregates are of little relevance for monetary policy. Instead, other work (e.g., Bianchi and Melosi (2017)) emphasizes the importance of the interactions between monetary and fiscal policies in explaining the data.

These medium-scale models, however, summarize monetary policy in simple (ad hoc) Taylor rules. Since these rules typically imply interest rates that revert rapidly to mean levels, to match the data, these papers augment the rules with exogenous shocks which make interest rates deviate from the value implied by the Taylor rule. Typically, the shocks are interpreted as a model-consistent form of forward guidance (FG).

As such, the models cannot inform us about whether keeping the interest rate low is a policy response of a central bank which focuses on inflation stabilization, and beyond this, of one, which may have debt sustainability concerns, the latter being motivated by a desire to keep the costs of debt refinancing low. Our contribution in this paper is to provide a framework which can be used to answer this question. We propose a model of endogenous optimal interest rates at the ZLB, which nests both the case where the Fed’s policies reflect debt sustainability concerns and the case where they do not. Our model is tractable, enabling us to derive interest rates in closed form, which in turn allows us to develop several new analytical insights about the efficacy of forward guidance in liquidity trap (LT) episodes. Moreover, the model can be easily embedded in a medium scale DSGE; to demonstrate this we perform a quantitative evaluation of optimal policies in the Great Recession.

Our theoretical framework is developed in Section 2 and assumes that a Ramsey planner under commitment sets allocations to minimize the deviations of inflation, output and interest rates from their respective target levels, subject to the standard set of dynamic equations which define the competitive equilibrium. In the baseline version of the Ramsey policy equilibrium, in which we assume that the Fed’s policies are also driven by debt sustainability concerns, we assume that this set also includes the consolidated budget constraint which determines the value of net debt in the hands of the private sector. Taxes are distortionary and, moreover, tax policy is assumed to be exogenous to the planner’s problem; it follows a simple rule which determines the tax rate as a function of lagged debt, a standard assumption in the DSGE literature (e.g. Leeper (1991)).

Our first analytical result derives the interest rate rule that emerges from optimal policy in this model. We show that the policy function is similar to the interest rate rule assumed in the recent DSGE literature: the short-term rate is expressed as the sum of a Taylor rule component (a function of inflation and output growth) and “forward guidance shocks”. In our model these shocks are endogenous and expressed as the sum of two components: The first component represents commitment to keep interest rates low at the exit from a LT episode (as in e.g. Eggertsson and Woodford (2003, 2006)) and is captured by the lags of the Lagrange multiplier attached to the occasionally binding ZLB constraint. The second component measures the impact of past shocks to the consolidated budget constraint, capturing the planner’s commitment to “twist interest rates” in order to satisfy the intertemporal budget (e.g. Lustig, Sleet and Yeltekin (2008), Faraglia, Marcet, Oikonomou and Scott (2016), henceforth FMOS). Since debt is distortionary, ours is a model of

optimal policy under incomplete markets (as in e.g. Aiyagari et al. (2002), FMOS) and the Lagrange multiplier on the consolidated budget constraint is a state variable. Interest rate twisting is captured by the lags of this multiplier.

As an alternative to this benchmark model we consider a Ramsey policy where the consolidated budget does not enter into the constraint set. In this case, endogenous forward guidance emerges only from the impact of the occasionally-binding ZLB and the model does not feature any interest rate twisting effects from shocks to the intertemporal budget. By switching on and off the consolidated budget from the Ramsey program in this way, we are able to contrast the properties of equilibria where monetary policy has concerns over debt sustainability with equilibria where monetary policy has none of such concerns.

We then focus on the properties of these two models in the neighborhood of the steady state (Section 3). We first study the monetary/fiscal policy interactions aiming to identify what types of fiscal policies 'justify' to consider a monetary policy that exhibits debt sustainability concerns and satisfies the consolidated budget (and what types of policies do not). We show analytically that in the debt concerns model the rational expectations equilibrium is (locally) unique if fiscal policy does not respond strongly to the deviations of government debt from its steady state level and debt becomes an explosive process. In contrast, in the case of 'no debt concerns', where the consolidated budget does not enter in optimization, taxes need to strongly adjust to high debt levels and debt becomes a mean reverting process.

We interpret these findings in light of the analysis of Leeper (1991). As in Leeper (1991) determinacy in our model requires an 'active/passive' or 'passive/active' policy mix for the equilibrium to be unique. Fiscal policy is 'active' when the Fed has 'debt concerns', and 'passive' when it does not.

Leeper's classification of monetary policy into 'active' and 'passive', hinges on the response of interest rates to inflation, and this is not straightforward to map into our baseline model in the presence of interest rate twists. We show, however, using a mixture of analytical results and simulations, that the policy rule under debt concerns is equivalent to a standard 'passive money' rule, exhibiting anemic responses of interest rates to inflation. The no debt concerns policy rule is equivalent to an 'active' monetary policy.

This finding, which shows that Leeper's analysis can be derived in an optimal Ramsey policy framework, is new to the literature, and also it makes easy to explain the models' properties and the responses to shocks that they predict. A standard feature of 'passive money' models is that they magnify the impact of shocks on interest rates output and inflation; the macroeconomic volatility predicted by these models tends to be higher.¹ This is precisely what happens under debt concerns; the monetary authority (partially) gives up on the goal of stabilizing interest rates, output and inflation, to satisfy the intertemporal budget; this leads to higher volatility. In contrast, volatility is much lower under no debt concerns, where monetary policy focuses on stabilizing macroeconomic variables.

In Section 4 we turn to the properties of optimal policies in the LT. Our key finding is that, under debt concerns, FG is ineffective in stabilizing output and inflation, and generally keeping interest rates low (at the ZLB) for a long period, does not lead to positive inflation rates at the onset of the episode, and may even lead to a sharp deflation. In contrast, under no debt concerns FG is effective, and promising lower future rates increases inflation (as in Eggertsson and Woodford (2003)).

What explains the ineffectiveness of FG under debt concerns? It is now well known, that under 'passive' monetary policy an exogenous drop in interest rates (eventually) makes inflation turn negative; this is what Sims (2011) and Cochrane (2018) refer to as the 'stepping on a rake' phenomenon. Our debt concerns model has this property, and as we show, promising to lower the interest rate impinges a drop in the price level.

Wouldn't it then be preferable to promise an increase in the interest rate rather than a drop, in order to avoid deflation? In our optimal policy model, this is the case when in LTs the market value of debt outstanding increases more than the present discounted value of surpluses that compensate for debt. This in turn requires that LTs are accompanied by large fiscal shocks and lead to a sharp

¹See e.g. Bianchi and Melosi (2017) and Bianchi and Ilut (2018).

drop tax revenues or a rise in spending levels. In the simplistic version of the model we consider in section 4 this is not the case; promising to raise interest rates destabilizes the consolidated budget.

In Section 5 we embed our optimal policy framework into a medium-scale DSGE model. Our quantitative model extends the baseline with preferences exhibiting habit formation, shocks to TFP, markup shocks, government transfers, etc. The model has a rich enough structure to match the US data; it is broadly similar to the DSGE models without capital used in Bianchi and Ilut (2017), Bianchi and Melosi (2017) among others. We estimate our quantitative model with standard Bayesian techniques and doing so we follow the trail of the recent DSGE literature which uses observations prior to the Great Recession to estimate the posterior distributions of structural parameters (e.g. Del Negro et al. (2015), De Graeve and Theodoridis (2017), Chen et al (2012) and others).

We then use the model to investigate the implications of optimal monetary policies under 'debt concerns' and 'no debt concerns' in the Great Recession. We consider a simple "forecasting" exercise in which we recover the realized vector of disturbances that drove the US economy to the LT trap in the first quarter of 2009, and simulate the paths of macroeconomic variables for several quarters thereafter. This experiment enables us to evaluate the ability of the models to predict interest rates that remain persistently at the ZLB, through their endogenous propagation mechanisms, and at the same time capture the lack of deflation observed.

We find that under both 'debt concerns' and 'no debt concerns', our framework is able to generate positive inflation rates. In the case of no debt concerns this due to the Fed's policy of keeping interest rates at the ZLB for a long period, which generates higher inflation expectations at the exit from the LT (e.g. Eggertsson and Woodford (2003)). The no debt concerns model therefore matches the data well. In the debt concerns model, however, positive inflation rates are due to the large fiscal shocks and fast increasing debt levels that the US economy experienced in 2008-9. Because inflation and interest rates positively comove under in this model, interest rates escape from the ZLB after one period, the US economy is effectively not in a LT during the Great Recession. Thus the debt concerns model is not able to match jointly the persistence of interest rates and the positive rates of inflation observed.

To test the robustness of these results we repeat our forecasting exercise assuming that monetary policy begins to echo debt concerns after 2009. Our findings remain.

Related literature

This paper is related to several strands of the literature. First, as discussed previously, our analysis is motivated by the wideheld view that monetary policy in the Great Recession begun to take into account the evolution of debt aggregates. This view is analogous to the view that unbacked fiscal deficits during the late 1970s made difficult for monetary policy to disinflate the US economy under the "fiscally led" regime which prevailed at that time.² As in the late 1970s, at the onset of the Great Recession, we have seen a large run-up in US government debt and sizable deficits.³ As a result, concerns over the ability of the fiscal authority to generate sufficient surpluses have resurfaced. When fiscal policy cannot sufficiently adjust taxes to debt, monetary policy must ensure debt solvency.

For many, the unconventional policies of the Great Recession are in fact the likely turning point for the relation between monetary and fiscal policies. This view was succinctly summarized by President Draghi in the Lamfalussy lecture at the National Bank of Belgium in 2018. He said "*The crisis has also prompted questions about the relationship between monetary and fiscal policies. First, it is claimed that, once central banks use unconventional measures that entail buying large volumes of government bonds, they cross the boundary into fiscal policy. In doing so, they exceed their mandates and the scope of their independence.*"

²See for example Bianchi and Ilut (2017) and also Sims's (2011) well known "stepping on a rake" analysis.

³Public debt over GDP basically doubled in the US since 2009, following a series of tax cuts and hikes in spending levels which led to some of the largest deficit levels ever recorded. For example, in 2009 the government's deficit to GDP ratio was the largest since WWII. In 2010 it was the second largest. In 2010 the CBO, in its budget outlook report, projected that the ratio of debt held by the public to GDP would stabilize at around 70 percent (already the highest level since the 1950s). Today public debt is expected to continue to grow unless significant increases in tax rates and decreases in spending are introduced (see the 2018-2028 outlook).

Several papers have explored this. Del Negro and Sims (2015) use a microfounded model to study the issue of the solvency of a central bank when it accumulates long term assets on its balance sheet and characterize the implications for monetary policy. Cochrane (2018), building on Cochrane (2001) and Sims (2011), presents a fiscal theory of the price level microfoundation of quantitative easing and forward guidance. Bhattarai et al. (2015) use an optimal Ramsey policy program to propose a microfoundation of the so called signalling theory of quantitative easing, that buybacks of long-term bonds made the Fed more willing to keep interest rates low during the recession.

Our focus in this paper is not on quantitative easing per se.⁴ However, implicitly we adopt the view that quantitative easing is a likely turning point for monetary policy in the US. In our quantitative experiment in Section 5 we take the 1st quarter of 2009 (which marks the start of QE) to be the date where policy begins to take into account the consolidated budget. Moreover, in our debt concerns model we let the planner choose the quantity of (net) long term debt held by private agents. This can be seen as being broadly consistent with the Fed's buybacks of long bonds from the secondary market. Cochrane (2018) models quantitative easing in a similar manner.

Related to our paper is also the work of Bianchi and Melosi (2017) who provide evidence that policy uncertainty increased during the recession, when the passive monetary /active fiscal regime resurfaced. In their model, LTs lead to a rise in the debt-to-GDP ratio and a rise in inflation, when monetary policy becomes subservient to fiscal policy. The main difference between our paper and theirs is that we consider a model of optimal monetary policy whereas they assume that an exogenous change in the regime (an exogenous shock to the interest rate rule) kept interest rates at the ZLB during the recession. Our model endogenizes interest rates, however, this requires to make assumptions regarding the conduct of monetary policy. For instance, assuming full commitment is probably key to the success of the no debt concerns model in matching the data (see the final subsection of the paper for a discussion). Whether or not relaxing this assumption changes our results significantly, remains to be explored.

Our model also draws heavily from the literature, which studies optimal monetary policy at the ZLB and in a variety of economic environments (e.g. Eggertsson and Woodford (2003, 2006), Jung et al. (2005), Adam and Billi (2006, 2008), Bhattarai et al. (2015), Bouakez et al. (2018)). These models also give rise to endogenous FG and some of them consider the interactions between monetary and fiscal policies as we do. However, the focus of these papers is mainly theoretical and generally disconnected from the DSGE literature. Crucially, because these models are highly non-linear they characterize optimal policies through first order conditions, which are not straightforward to map into simple interest rate rules comparable to the ones assumed in the DSGE literature. As discussed previously, a contribution of our paper is to provide an optimal policy microfoundation for the ad hoc "forward guidance shocks" assumed in recent DSGE models. To accomplish this, we cast our optimal policy problem in the linear quadratic (LQ) framework of Giannoni and Woodford (2003), Benigno and Woodford (2003) and others, this enables to derive microfounded Taylor rules. We augment the LQ framework with the ZLB constraint and with a fiscal block assuming that taxes are distortionary and government debt is of long-term maturity.

Finally, our paper is more broadly related to numerous studies on optimal policies under the Ramsey equilibrium. Schmitt-Grohé and Uribe (2004), Faraglia et al (2013), Lustig et al. (2008), and Siu (2004) consider the case of coordinated monetary and fiscal policies with distortionary taxation and incomplete markets. FMOS extend the optimal fiscal policy model of Aiyagari et al. (2002) to the case of long-term government debt. Unlike in these papers our tax policies are not optimal; we assume that taxes follow an exogenous rule, which relates the level of taxes to the lagged value of debt. This assumption makes our framework fit better the DSGE literature. However, the findings of these papers continue to be relevant here. For example, our microfoundation of FG in

⁴Also there is no explicit modelling of the Fed's balance sheet here, like for example in Del Negro and Sims (2015). We follow the trail of numerous macro papers which use the consolidated budget in models of optimal policy (e.g. Schmitt-Grohé and Uribe (2004), Faraglia et al (2013), Lustig et al (2009) and others).

Bhattarai et al (2015) show that the optimal policies which emerge from the Ramsey program when optimization is subject to the consolidated budget, are similar to policies when optimization is subject to the Fed's balance sheet, when the planner targets the net worth of the central bank.

the debt concerns model draws heavily from the interest rate twisting concept developed in FMOS. In that paper, interest rate twisting emerges when the fiscal authority distorts the tax schedule to influence long bond prices. In our model it emerges because the monetary authority distorts the path of inflation to satisfy the consolidated budget.

2 Theoretical Framework

We begin with a simple setup of the Ramsey policy equilibrium which allows us to illuminate key forces driving interest rates in our model. In the baseline version of the Ramsey policy we let the planner choose interest rates, inflation and output, subject to the consolidated budget constraint (of the Fed and the government). We refer to this model as the 'debt concerns' model. The aim is to capture a scenario under which the planner (Fed) has concerns over the sustainability of debt. We show analytically that in this case the optimal interest rate rule has three components: First, a standard Taylor rule component, which links interest rates to realized inflation, output growth and lagged values of interest rates. Second, a component which represents commitment to keep interest rates low at the exit from a LT episode (as in e.g. Eggertsson and Woodford (2003, 2006)). Third, a component which measures the impact of past promises made by the planner to alter interest rates in the face of shocks to the consolidated intertemporal budget constraint. The third component is an "interest twisting effect" which emerges because debt is distortionary and long-term as in FMOS.

As an alternative to the baseline, we consider the case where the planner has "no debt concerns" and does not account for the consolidated budget constraint in optimization. The optimal policy rule in this case is identical to the baseline in terms of the first two components, but features no interest rate twisting impacts.

Endogenous FG emerges in our model because of these extra components which make interest rates deviate from the Taylor rule. Our definition of FG is consistent with recent papers in the DSGE literature in which interest rate rules are of the following form: ⁵

$$\hat{i}_t^{DSGE} = \max\left\{\mathcal{T}_t^{DSGE} + \sum_{l=0}^M \epsilon_{t-l}^{l,DSGE}, -\hat{i}^*\right\} \quad (1)$$

where $M > 0$. \mathcal{T}_t^{DSGE} represents the Taylor rule component in these papers, a function of inflation, output growth and lags of the interest rate. ϵ_t^0 is a standard monetary policy shock announced in period t and applying to policy in that period, whereas the vector $\left(\epsilon_{t-1}^1, \dots, \epsilon_{t-M}^M\right)$ summarizes shocks which have been revealed in past periods and apply to the policy rule in t . These shocks capture *forward guidance* in monetary policy.

We show below that the optimal policy rules in our model are of the form (1). The shocks $\left(\epsilon_{t-1}^1, \dots, \epsilon_{t-M}^M\right)$ are endogenous and, moreover, they are functions of economic fundamentals and of the structure of the Fed's optimal policy program. In the case of 'no debt concerns' these shocks reflect the Fed's commitment to keep interest rates low after a liquidity trap. In the case of 'debt concerns' they also reflect interest rate twisting in response to shocks to the consolidated budget.

2.1 The baseline model

The building blocks of our model are a standard NK Phillips curve, an IS (Euler) equation which prices a short nominal bond, and the consolidated budget constraint which determines the value of (net) debt held by the private sector. The model is a simplified version of the New-Keynesian (NK) model that we will later employ in estimation and which we formally describe in Section 3. For

⁵See for example Laseen and Swensson (2011), Campbell et al (2012, 2016), Del Negro et al (2015), De Graeve and Theodoridis (2017) and De Graeve, Ilbas and Wouters (2017) and others.

brevity, we summarize here the competitive equilibrium in the linearized version of the model; for detailed derivations we refer the reader to the online appendix.

Let \hat{x} denote the log deviation of variable x from its steady state value, \bar{x} . The competitive equilibrium equations are the following:

$$\hat{\pi}_t = \kappa_1 \hat{Y}_t + \kappa_2 \hat{\tau}_t - \kappa_3 \hat{G}_t + \beta E_t \hat{\pi}_{t+1}, \quad (2)$$

where $\kappa_1 \equiv -\frac{(1+\eta)\bar{Y}}{\theta}(\gamma_h + \sigma\frac{\bar{Y}}{C}) > 0$, $\kappa_2 \equiv -\frac{(1+\eta)\bar{Y}}{\theta}\frac{\bar{\tau}}{(1-\bar{\tau})} > 0$, $\kappa_3 \equiv -\frac{(1+\eta)}{\theta\bar{Y}}\sigma\frac{\bar{G}}{C} > 0$,

$$\hat{i}_t = E_t \left(\hat{\pi}_{t+1} - \hat{\xi}_{t+1} + \hat{\xi}_t - \sigma \left[\frac{\bar{Y}}{C} (\hat{Y}_t - \hat{Y}_{t+1}) - \frac{\bar{G}}{C} (\hat{G}_t - \hat{G}_{t+1}) \right] \right) \quad (3)$$

$$\hat{i}_t \geq -\frac{1}{\beta} + 1 \equiv -i^* \quad (4)$$

$$\begin{aligned} & \frac{\beta\bar{b}_\delta}{1-\beta\delta}\hat{b}_{t,\delta} + \bar{b}_\delta \sum_{j=1}^{\infty} \beta^j \delta^{j-1} \left[E_t \left(-\sigma \left(\frac{\bar{Y}}{C} \hat{Y}_{t+j} - \frac{\bar{G}}{C} \hat{G}_{t+j} \right) - \sum_{l=1}^j \hat{\pi}_{t+l} + \hat{\xi}_{t+j} \right) \right] \\ & + \frac{\bar{\tau}(1+\eta)\bar{Y}}{\eta} \left((\gamma_h + 1)\hat{Y}_t + \hat{\xi}_t + \frac{\hat{\tau}_t}{1-\bar{\tau}} \right) - \bar{G} \left(\hat{G}_t - \sigma \frac{\bar{Y}}{C} \hat{Y}_t + \sigma \frac{\bar{G}}{C} \hat{G}_t + \hat{\xi}_t \right) + \bar{b}_\delta \sigma \left(\frac{\bar{Y}}{C} \hat{Y}_t - \frac{\bar{G}}{C} \hat{G}_t \right) - \bar{b}_\delta \hat{\xi}_t \\ & = \frac{\bar{b}_\delta}{1-\beta\delta} (\hat{b}_{t-1,\delta} - \hat{\pi}_t) + \delta\bar{b}_\delta \sum_{j=1}^{\infty} \beta^j \delta^{j-1} E_t \left(-\sigma \left(\frac{\bar{Y}}{C} \hat{Y}_{t+j} - \frac{\bar{G}}{C} \hat{G}_{t+j} \right) - \sum_{l=1}^j \hat{\pi}_{t+l} + \hat{\xi}_{t+j} \right) \end{aligned} \quad (5)$$

(2) is the Phillips curve at the heart of our model. \hat{Y}_t is the output gap and $\hat{\tau}$ denotes a distortionary tax levied on the labor income of households. \hat{G}_t denotes government spending in t . Parameters η and θ denote the elasticity of substitution across differentiated products and the degree of price stickiness respectively.⁶

(3) is the log-linear Euler equation which prices a short term nominal asset. $\hat{\xi}$ is a standard preference shock which affects the relative valuation of current vs. future utility by the household. A drop in $\hat{\xi}$ makes the household relatively patient, willing to substitute current for future consumption. (4) is the ZLB constraint on the short-term nominal interest rate.

(5) is the consolidated budget constraint. As Bianchi and Melosi (2017) we assume that debt is issued in a perpetuity bond with decaying coupons where δ denotes the decay factor. Short debt is in zero net supply. Finally, parameter σ denotes the inverse of the intertemporal elasticity of substitution and γ_h is the inverse of the Frisch elasticity of labor supply.

Tax policies in our model follow a simple rule of the form:

$$\hat{\tau}_t = \rho_\tau \hat{\tau}_{t-1} + (1 - \rho_\tau) \phi_{\tau,b} \hat{b}_{t-1,\delta} + \epsilon_{\tau,t} \quad (6)$$

where $\epsilon_{\tau,t}$ is a shock to the tax rate and $\phi_{\tau,b}$ measures the feedback effect of debt issued in $t-1$ on the tax rate in t .

2.2 Ramsey Policy

The Ramsey policy chooses inflation, output and interest rate sequences to maximize the following objective

$$-\frac{1}{2} \sum_{t=0}^{\infty} \beta^t E_0 \left\{ \hat{\pi}_t^2 + \lambda_Y \hat{Y}_t^2 + \lambda_i \hat{i}_t^2 \right\} \quad (7)$$

subject to equations (2) to (5) and given (6). λ_Y and λ_i govern the relative weights attached to output gap and interest rate stabilization by the planner. For brevity, we leave it to the online appendix

⁶We assume price adjustment costs as in Rotemberg (1989). θ governs the magnitude of these costs. When θ equals zero, prices are fully flexible.

to derive the Lagrangian function that we solve to find the optimal policies. Letting $\psi_{\pi,t}$ be the multiplier attached to the Phillips curve constraint, $\psi_{i,t}$, $\psi_{ZLB,t}$ and $\psi_{gov,t}$ the analogous multipliers attached to the Euler equation, the ZLB constraint and the consolidated budget respectively, the first order conditions for the optimum are given by:

$$-\hat{\pi}_t + \Delta\psi_{\pi,t} - \frac{\psi_{i,t-1}}{\beta} + \frac{\bar{b}_\delta}{1-\beta\delta} \sum_{l=0}^{\infty} \delta^l \Delta\psi_{gov,t-l} = 0 \quad (8)$$

$$-\lambda_Y \hat{Y}_t - \psi_{\pi,t} \kappa_1 + \sigma \frac{\bar{Y}}{C} (\psi_{i,t} - \frac{\psi_{i,t-1}}{\beta}) + \sigma \frac{\bar{Y}}{C} \bar{b}_\delta \sum_{l=0}^{\infty} \delta^l \Delta\psi_{gov,t-l} + \omega_Y \psi_{gov,t} = 0 \quad (9)$$

$$-\lambda_i \hat{i}_t + \psi_{i,t} + \psi_{ZLB,t} = 0 \quad (10)$$

$$\frac{\beta \bar{b}_\delta}{1-\beta\delta} \left(\psi_{gov,t} - E_t \psi_{gov,t+1} \right) - \mathcal{I}_P \beta \phi_{\tau,b} (1-\rho_\tau) E_t \left(\sum_{j=0}^{\infty} (\rho_\tau \beta)^j (\kappa_2 \psi_{\pi,t+1+j} - \frac{\bar{\tau} \bar{Y} (1+\eta)}{\eta(1-\bar{\tau})} \psi_{gov,t+1+j}) \right) = 0 \quad (11)$$

$$\psi_{ZLB,t} \geq 0 \quad \text{and} \quad (\hat{i}_t + i^*) \psi_{ZLB,t} = 0 \quad (12)$$

where $\omega_Y \equiv \bar{G} \sigma \frac{\bar{Y}}{C} + \frac{\bar{\tau}(1+\eta)}{\eta} \bar{Y} (1+\gamma_h)$. (8) is the FONC with respect to $\hat{\pi}_t$; (9), (10), (11) are first order conditions with respect to \hat{Y}_t , \hat{i}_t and $\hat{b}_{t,\delta}$ respectively. (12) is the complementary slackness condition for the ZLB constraint. Finally, \mathcal{I}_P is an indicator function that takes the value 1 when the planner internalizes the feedback effect of debt on taxes in equation (6) and is zero otherwise. For the remainder of the paper we will set $\mathcal{I}_P = 0$.⁷

2.2.1 Optimal interest rate rules

Combining the first order conditions (8), (9) and (10) and following the argument of Giannoni and Woodford (2001) we can derive analytically the optimal interest rate rule which emerges from the Ramsey program. The following proposition summarizes the result:

Proposition 1. *The optimal interest rate rule which derives from the Ramsey program is:*

$$\hat{i}_t = \max\{\mathcal{T}_t + \mathcal{D}_t + \mathcal{Z}_t, -i^*\} \quad (13)$$

$$\mathcal{T}_t \equiv \phi_\pi \hat{\pi}_t + \phi_Y \Delta \hat{Y}_t + \phi_i \hat{i}_{t-1} + \frac{1}{\beta} \Delta \hat{i}_{t-1}$$

with $\phi_\pi = \frac{\kappa_1 \bar{C}}{\lambda_i \sigma \bar{Y}}$, $\phi_i = (1 + \frac{\kappa_1 \bar{C}}{\beta \sigma \bar{Y}})$ and $\phi_Y = \frac{\lambda_Y \bar{C}}{\sigma \lambda_i \bar{Y}}$,

$$\mathcal{D}_t = -\frac{\bar{C}}{\bar{Y}} \frac{\kappa_1}{\lambda_i \sigma} \frac{\bar{b}_\delta}{1-\beta\delta} \sum_{l=0}^{\infty} \delta^l \Delta\psi_{gov,t-l} - \frac{\bar{b}_\delta}{\lambda_i} \sum_{l=0}^{\infty} \delta^l \left(\Delta\psi_{gov,t-l} - \Delta\psi_{gov,t-l-1} \right) - \frac{\bar{C} \omega_Y}{\bar{Y} \sigma \lambda_i} \Delta\psi_{gov,t} \quad (14)$$

and

$$\mathcal{Z}_t = -\frac{1}{\lambda_i} \left(1 + \frac{1}{\beta} + \frac{\kappa_1 \bar{C}}{\sigma \beta \bar{Y}} \right) \psi_{ZLB,t-1} + \frac{1}{\beta \lambda_i} \psi_{ZLB,t-2}. \quad (15)$$

Proof See appendix.

As equation (13) shows there are three distinct components in the interest rate rule. The first, \mathcal{T}_t , is a standard Taylor rule component which links interest rates to inflation, output growth and lagged values of the interest rate. The impact of these variables on the interest rate policy rule depends on

⁷This assumption is made to keep with the rest of the literature on fiscal/monetary policy interactions which assumes independent policies.

the weights λ_i , λ_Y which capture the output and interest rate stabilization objectives of the planner. It also depends on the structural parameters σ , κ_1 and β .

The second component, \mathcal{Z}_t , is a pull factor which relates interest rates to the “bindness” of the ZLB constraint. If for example $\hat{i}_{t-1} = i^*$ then (from complementary slackness) we have $\psi_{ZLB,t-1} > 0$. In this case \mathcal{Z}_t becomes negative and so the planner keeps the interest rate in t lower than the value implied by the Taylor rule component (and partially reverses the effect in $t + 1$). The impact of this channel on the level of interest rates depends on λ_i , with higher values of λ_i leading to a lower impact, since the planner’s objective to minimize the deviation of the interest rate from its target becomes stronger.

Finally, the term \mathcal{D}_t measures the impact of changes in the value of the multiplier $\psi_{gov,t}$ on the optimal interest rate path. To analyze this term use equation (11) setting $\mathcal{I}_P = 0$. We have

$$\psi_{gov,t+1} = \psi_{gov,t} + \epsilon_{t+1,G} \quad (16)$$

In other words, the multiplier $\psi_{gov,t}$ is a random walk, and $\epsilon_{t+1,G}$ denotes a mean zero, i.i.d shock to the value of the multiplier. We can now write (14) as:

$$\mathcal{D}_t = -\frac{\bar{C}}{\bar{Y}} \frac{\kappa_1}{\lambda_i \sigma} \frac{\bar{b}_\delta}{1 - \beta \delta} \sum_{l=0}^{\infty} \delta^l \epsilon_{t-l,G} - \frac{\bar{b}_\delta}{\lambda_i} \sum_{l=0}^{\infty} \delta^l \left(\epsilon_{t-l,G} - \epsilon_{t-l-1,G} \right) - \frac{\bar{C} \omega_Y}{\bar{Y} \sigma \lambda_i} \epsilon_{t,G} \quad (17)$$

which relates \mathcal{D}_t to current and past shocks to the value of the multiplier.

What do these shocks capture? Notice that since (real) debt can either be financed through distortionary taxes or through distortionary inflation, ours is a standard model of optimal policy under incomplete markets (e.g. Aiyagari et al (2002), Schmitt-Grohé and Uribe (2004), Faraglia et al (2013), FMOS among others). As is well known, in these models shocks to the economy translate to changes in the excess burden of distortions, the multiplier ψ_{gov} that measures the magnitude of these distortions behaves like a random walk, since the planner wants to spread evenly the costs across periods.⁸ In the presence of long term debt ($\delta > 0$) the sequence of shocks $\{\epsilon_{t-l,G}\}_{l=0}^{\infty}$ influences interest rates because all the lags of these variables enter into the state vector, as the FONC reveal.

To clarify further the role played by the sequence $\{\epsilon_{t-l,G}\}_{l=0}^{\infty}$ we iterate forward on constraint (5) to get:

$$E_t \sum_{j=0}^{\infty} \beta^j \bar{S} \hat{S}_{t+j} = \frac{\bar{b}_\delta}{1 - \beta \delta} \hat{b}_{t-1,\delta} + \bar{b}_\delta \sum_{j=0}^{\infty} \beta^j \delta^j E_t \left[-\sigma \left(\frac{\bar{Y}}{\bar{C}} \hat{Y}_{t+j} - \frac{\bar{G}}{\bar{C}} \hat{G}_{t+j} \right) - \sum_{l=0}^j \hat{\pi}_{t+l} + \hat{\xi}_{t+j} \right] \quad (18)$$

where $\bar{S} \hat{S}_t \equiv \left[-\bar{G} \left(\hat{G}_t (1 + \sigma \frac{\bar{G}}{\bar{C}}) - \sigma \frac{\bar{Y}}{\bar{C}} \hat{Y}_t + \hat{\xi}_t \right) + \frac{\bar{\tau} \bar{Y} (1 + \eta)}{\eta} \left((1 + \gamma_h) \hat{Y}_t + \frac{\hat{\tau}_t}{1 - \bar{\tau}} + \hat{\xi}_t \right) \right]$ is the surplus of the government multiplied by marginal utility.

(18) is the *intertemporal consolidated budget constraint* linking the present discounted value of the fiscal surplus to the real value of debt outstanding in t . Notice also that (18) is equivalent to (5) in terms of the Ramsey policy.⁹ Consider the impact of a shock which lowers the LHS of (18)

⁸ To be accurate, even in the case of lump sum taxation ψ_{gov} evolves as a random walk when $\mathcal{I}_P = 0$, since the planner does not control taxes. However, in the case where $\mathcal{I}_P = 1$, where the planner influences taxes through debt, assuming non distortionary taxation would imply $\psi_{gov,t} = 0$ for all t . To see this note that in this case (11) can be written as:

$$\frac{\beta \bar{b}_\delta}{1 - \beta \delta} \left(\psi_{gov,t} - E_t \psi_{gov,t+1} \right) + \beta \phi_{\tau,b} (1 - \rho_\tau) E_t \left(\sum_{j=0}^{\infty} (\rho_\tau \beta)^j \left(\frac{\bar{\tau} \bar{Y} (1 + \eta)}{\eta (1 - \bar{\tau})} \psi_{gov,t+1+j} \right) \right) = 0,$$

since lump sum taxes will not show up in the Phillips curve, we have $\kappa_2 = 0$. The above difference equation gives $\psi_{gov,t} = 0$ when $\phi_{\tau,b} > 0$.

In the case where inflation is not distortionary (i.e. the model does not admit a Phillips curve) it is also easy to show that $\psi_{gov,t} = 0$.

⁹See for example Aiyagari et al (2002).

relative to the RHS. This may, for example, occur following a shock which lowers taxes. In response to this shock the constraint tightens, the value of the multiplier ψ_{gov} increases. Therefore, $\epsilon_{t,G} > 0$. To satisfy the constraint, the monetary authority needs to engineer a drop in the real payout of debt (the RHS of (18)) either through increasing future inflation and /or increasing future output when $\sigma > 0$. Note that under commitment it is feasible to make such promises about the future course of economic variables. The terms that enter in \mathcal{D}_t in equation (17) are essentially the promises made by the planner to manipulate inflation and output, and hence also the interest rate, in response to shocks to the consolidated budget which have occurred in the past. Following FMOS, we label this impact an 'interest rate twisting' effect of optimal policy.

2.3 Assuming No Debt Concerns

We now consider an alternative setup in which the planner maximizes (7) subject to (2), (3) and (4) and leaves the consolidated budget (5) outside the optimal policy program. (5) continues to hold but is satisfied in equilibrium given optimal policies and the tax rule (6). In this version of the model the multiplier $\psi_{gov,t}$ is obviously dropped from the list of model variables.

Letting superscript *NDC* denote the equilibrium under "no debt concerns", the first order conditions¹⁰ become:

$$\begin{aligned} -\hat{\pi}_t^{NDC} + \Delta\psi_{\pi,t}^{NDC} - \frac{\psi_{i,t-1}^{NDC}}{\beta} &= 0 \\ -\lambda_Y \hat{Y}_t^{NDC} - \psi_{\pi,t}^{NDC} \kappa_1 + \sigma \frac{\bar{Y}}{C} (\psi_{i,t}^{NDC} - \frac{\psi_{i,t-1}^{NDC}}{\beta}) &= 0 \\ -\lambda_i \hat{i}_t^{NDC} + \psi_{i,t}^{NDC} + \psi_{ZLB,t}^{NDC} &= 0 \end{aligned}$$

The above equations give rise to the following interest rate rule:

Proposition 2. *Assume that the planner does not account for the consolidated budget in optimization. The optimal interest rate rule is given by*

$$\hat{i}_t^{NDC} = \max\{\mathcal{T}_t^{NDC} + \mathcal{Z}_t^{NDC}, -i^*\} \quad (19)$$

$$\mathcal{T}_t^{NDC} \equiv \phi_\pi \hat{\pi}_t^{NDC} + \phi_Y \Delta \hat{Y}_t^{NDC} + \phi_i \hat{i}_{t-1}^{NDC} + \frac{1}{\beta} \Delta \hat{i}_{t-1}^{NDC}$$

and

$$\mathcal{Z}_t^{NDC} = -\frac{1}{\lambda_i} \left(1 + \frac{1}{\beta} + \frac{\kappa_1 \bar{C}}{\sigma \beta \bar{Y}}\right) \psi_{ZLB,t-1}^{NDC} + \frac{1}{\beta \lambda_i} \psi_{ZLB,t-2}^{NDC}.$$

Notice that the expressions for components \mathcal{T}_t^{NDC} and \mathcal{Z}_t^{NDC} in (19) are essentially the same as in the optimal rule (13) of the benchmark model. The difference between the two policy rules emerges because (19) omits the term \mathcal{D}_t which appears in the baseline 'debt concerns' version of optimal policy.

2.4 Discussion

As we have seen, \mathcal{Z}_t and \mathcal{D}_t are functions of lagged state variables. These objects therefore summarize changes in interest rates that have been revealed in the past and apply to policy in t . The two policy rules in Propositions 1 and 2 are thus similar to equation (1), the interest rate rule employed by the recent DSGE literature and in which 'forward guidance shocks' make interest rates deviate from the Taylor rule component. The difference is that forward guidance in (1) is exogenous, whereas the state

¹⁰See online appendix for a detailed description of the planner's program.

variables which enter in \mathcal{Z}_t and \mathcal{D}_t are endogenous functions of fundamental shocks to preferences, taxes, spending etc. We refer to \mathcal{Z}_t and \mathcal{D}_t as *endogenous forward guidance*.

As discussed previously, several papers have studied optimal monetary policies using a framework broadly similar to ours, also studying the interactions between monetary and fiscal policies. Cochrane (2001) sets up Ramsey program where the planner minimizes the volatility of inflation subject to the intertemporal budget constraint (as is the case in our debt concerns model). Eggertsson and Woodford (2003, 2006) analyze optimal interest rate policies at the ZLB. Faraglia et al (2013) and Lustig et al (2009) extend the analysis of jointly optimal monetary and fiscal policy of Schmitt-Grohé and Uribe (2004), by introducing long term debt; Bouakez et al (2018) assume an occasionally binding ZLB. ’

Are \mathcal{Z}_t and \mathcal{D}_t really new? The above mentioned papers also give rise to endogenous FG and therefore key forces of our model, that we will analyze below, are also present in these models. The model of Eggertsson and Woodford (2003) (for example) features FG \mathcal{Z}_t ; moreover, in the presence of long term debt in Faraglia et al (2013) and Lustig et al (2009), \mathcal{D}_t is part of optimal policy.

Our approach has certain advantages: First, because the focus of these papers is mainly theoretical, building on nonlinear models, they do not derive simple interest rate rules which can be easily embedded in a DSGE framework. The optimal policies which emerge from our model are comparable to the DSGE literature, and since \mathcal{Z}_t and \mathcal{D}_t enter additively in the policy function we are also able to separately identify their effects (more on this below). Second, since Faraglia et al (2013) and Lustig et al (2009) bring together monetary and fiscal policies under one authority, in these papers \mathcal{D}_t does not only summarize promises made by the planner to manipulate future inflation and interest rates, as it is the case here, but also promises to manipulate future taxes. This makes difficult to evaluate which part of \mathcal{D}_t in these models is relevant for monetary policy and which is not.¹¹ Through assuming that tax policies are exogenous to the planning program, our analysis can isolate the effects of \mathcal{D}_t on optimal inflation and interest rates.

In the following sections we study the impact of forward guidance in our model. We first focus on the properties of the model in the neighborhood of the steady state (in Section 3) thus focusing on the role played by \mathcal{D}_t . We establish a link between the policies in the debt concerns and no debt concerns models and Leeper’s (1991) famous analysis on the monetary/fiscal policy interactions. Our key result is that in the case of debt concerns, monetary policy becomes subservient to fiscal policy, and behaves similarly to the ’passive’ monetary policy models defined in Leeper’s work. Fiscal policy is ’active’ and does not respond to government debt. The opposite holds under no debt concerns.

3 Monetary and Fiscal Policy Interactions

3.1 Determinacy Under Optimal Policies

3.1.1 Fiscal Policy

Consider the case where the ZLB constraint is not binding. Since ours is a linear model approximated around a non-stochastic steady state with a positive nominal interest rate, equilibria with non-binding ZLB constraints can occur if shocks are not too big to drive the economy far away from steady state. We have: $\hat{i}_t = \mathcal{T}_t + \mathcal{D}_t$ and $\hat{i}_t^{NDC} = \overline{\mathcal{T}_t}^{NDC}$. To derive analytical results we first consider a simplistic setup setting $\lambda_Y = \sigma = \delta = \rho_\tau = \overline{G} = 0$. We further assume that tax shocks are the only source of uncertainty in the economy.¹² Under these assumptions and using equations (9) and (10) to

¹¹A key result of the literature on jointly optimal moterary/fiscal policies under commitment is that optimizing governments want to use taxes in order to absorb fiscal shocks. Inflation is not volatile in these models (see also Schmitt-Grohé and Uribe (2004)). This implies interest rate twisting in these models is mainly a fiscal phenomenon.

¹²Our findings in this section do not hinge on the nature of shocks that hit the economy. We assume only tax shocks and set $\overline{G} = 0$ to simplify the algebra; otherwise we could assume more than one disturbance in the model and all results described below would carry through.

substitute out $\psi_{\pi,t}$ and $\psi_{i,t}$ in the debt concerns model we get:

$$-\hat{\pi}_t - \frac{\lambda_i \hat{i}_{t-1}}{\beta} + \bar{b}_\delta \tilde{\eta} \Delta \psi_{gov,t} = 0 \quad (20)$$

where $\tilde{\eta} = \left(1 + \frac{(1-\beta)(1+\gamma_h)}{\kappa_1}\right)$.¹³ $\psi_{gov,t}$ is again a martingale and so $E_t \Delta \psi_{gov,t+1} = 0$. From the Euler equation we have $\hat{i}_t = E_t \hat{\pi}_{t+1}$.

Using the Phillips curve to substitute $\hat{Y}_t = \frac{\hat{\pi}_t - \beta E_t \hat{\pi}_{t+1} - \kappa_2 \hat{\tau}_t}{\kappa_1}$ the consolidated budget constraint can be written as follows:

$$\beta \bar{b}_\delta \hat{b}_{t,\delta} - \beta \bar{b}_\delta \tilde{\eta} E_t \hat{\pi}_{t+1} + (1-\beta) \bar{b}_\delta \left(\frac{1}{1-\bar{\tau}} - \frac{\kappa_2}{\kappa_1} (1+\gamma_h) \right) \hat{\tau}_t = \bar{b}_\delta \hat{b}_{t-1,\delta} - \bar{b}_\delta \tilde{\eta} \hat{\pi}_t \quad (21)$$

Finally, using $\frac{\kappa_2}{\kappa_1} = \frac{\bar{\tau}}{(1-\bar{\tau})\gamma_h}$ and the tax rule (6) we have:

$$\hat{b}_{t,\delta} - \tilde{\eta} E_t \hat{\pi}_{t+1} = \underbrace{\frac{1}{\beta} \left[1 - \frac{(1-\beta)\phi_{\tau,b}}{(1-\bar{\tau})\gamma_h} (\gamma_h - \bar{\tau}(1+\gamma_h)) \right]}_{\tilde{\epsilon}} \hat{b}_{t-1,\delta} - \frac{\tilde{\eta}}{\beta} \hat{\pi}_t - \underbrace{\frac{1}{\beta} \frac{(1-\beta)}{(1-\bar{\tau})\gamma_h} (\gamma_h - \bar{\tau}(1+\gamma_h))}_{A} \epsilon_{\tau,t} \quad (22)$$

Coefficient $\tilde{\epsilon}$ is a key object in determining the dynamics of government debt in our model. In the case where $\tilde{\epsilon} > 1$ government debt becomes an explosive process, whereas if $\tilde{\epsilon} < 1$ debt is a stationary process. Notice that the magnitude of $\tilde{\epsilon}$ hinges crucially on the size of the feedback effect of lagged debt on taxes, $\phi_{\tau,b}$, and also on the steady state level of taxes and the Frisch elasticity $\frac{1}{\gamma_h}$ since these quantities influence the responsiveness of aggregate hours and tax revenues to shocks when taxes are distortionary.

To identify which values for $\tilde{\epsilon}$ give rise to a unique rational expectations equilibrium use the Euler equation together with (20) to write

$$\frac{\lambda_i \hat{i}_t}{\beta} = -E_t \hat{\pi}_{t+1} + \bar{b}_\delta \tilde{\eta} E_t \Delta \psi_{gov,t+1} = -\hat{i}_t \quad (23)$$

where the last equality makes use of the martingale property of $\psi_{gov,t+1}$. From (23) we get $\hat{i}_t = 0$ when $\lambda_i \geq 0$. Therefore, we have $\hat{\pi}_t = \bar{b}_\delta \tilde{\eta} \Delta \psi_{gov,t}$ and with this we can write (22) as

$$\hat{b}_{t,\delta} = \tilde{\epsilon} \hat{b}_{t-1,\delta} - \frac{\bar{b}_\delta \tilde{\eta}^2}{\beta} \Delta \psi_{gov,t} - A \epsilon_{\tau,t} \quad (24)$$

Equation (24) together with the martingale property $E_t \Delta \psi_{gov,t+1} = 0$ pin down the values of $\hat{b}_{t,\delta}$ and $\Delta \psi_{gov,t}$.

It is now easy to show that the solution to this system of equations is unique only when $\tilde{\epsilon} > 1$. To see this assume that $\tilde{\epsilon}$ is less than one and solve equation (24) backwards to obtain:

$$\hat{b}_{t,\delta} = - \sum_{j=0}^{\infty} \tilde{\epsilon}^j \left(\frac{\bar{b}_\delta \tilde{\eta}^2}{\beta} \Delta \psi_{gov,t-j} + A \epsilon_{\tau,t-j} \right) \quad (25)$$

which determines the debt level at t as function of the lagged shocks $\Delta \psi_{gov,t-j}$ and $\epsilon_{\tau,t-j}$. Notice that the value of $\Delta \psi_{gov,t-j}$ is not pinned down in this model; the martingale property $E_{t-j-1} \Delta \psi_{gov,t-j} = 0$ does not determine a unique value for this object.

In the unique rational expectations equilibrium with $\tilde{\epsilon} > 1$, (24) is solved forward giving

$$\hat{b}_{t-1,\delta} = E_t \sum_{j=0}^{\infty} \frac{1}{\tilde{\epsilon}^{j+1}} \left(\frac{\bar{b}_\delta \tilde{\eta}^2}{\beta} \Delta \psi_{gov,t+j} + A \epsilon_{\tau,t+j} \right) = \frac{1}{\tilde{\epsilon}} \left(\frac{\bar{b}_\delta \tilde{\eta}^2}{\beta} \Delta \psi_{gov,t} + A \epsilon_{\tau,t} \right) \quad (26)$$

¹³Under $\sigma = \bar{G} = 0$ we have $\omega_Y = \frac{\bar{\tau}(1+\eta)}{\eta} \bar{Y} (1+\gamma_h)$. From the budget constraint we get $\frac{\bar{\tau}(1+\eta)}{\eta} \bar{Y} = (1-\beta) \bar{b}_\delta$.

From (26) the equilibrium satisfies $\Delta\psi_{gov,t} = -\frac{\beta}{b_\delta\bar{\eta}^2}A\epsilon_{\tau,t}$ and therefore $\hat{b}_{t,\delta} = 0$ for all t ¹⁴

Now consider the case of no debt concerns. We have $\psi_{gov,t} = 0$ for all t and the consolidated budget constraint can be written as:

$$\hat{b}_{t,\delta}^{NDC} = \tilde{\epsilon}\hat{b}_{t-1,\delta}^{NDC} - A\epsilon_{\tau,t} \quad (27)$$

A unique equilibrium can be found when $\tilde{\epsilon} < 1$ so that (27) is solved backwards. We summarize the above findings in the following Proposition.

Proposition 3. *Assume that parameters λ_Y , \bar{G} , δ and σ equal zero. Assume further that the planner takes into account the consolidated budget constraint. Determinacy of the equilibrium requires:*

$$\tilde{\epsilon} \equiv \frac{1}{\beta} \left[1 - \frac{(1-\beta)\phi_{\tau,b}}{(1-\bar{\tau})\gamma_h} \left(\gamma_h - \bar{\tau}(1+\gamma_h) \right) \right] > 1$$

In contrast in the “no debt concerns case”, where the planner does not account for the consolidated budget, determinacy requires $\tilde{\epsilon} < 1$.

The above condition extends the analysis of Leeper (1991) to the case of distortionary taxes and in a model where monetary policy is optimal. In the case where $\tilde{\epsilon} < 1$ (the no debt concerns model) fiscal policy is sufficiently responsive to debt levels, and so taxes adjust to guarantee the sustainability of debt. In the sense of Leeper (1991), fiscal policy is “passive”. In contrast, in the baseline model with debt concerns, fiscal policy needs to be “active” for the rational expectations equilibrium to be unique or, to put it differently, taxes should not respond aggressively to deviations of debt from its steady state value.

The above findings, that determinacy requires an explosive debt process in the debt concerns model, and a mean reverting process under no debt concerns, do not hinge on the assumptions made in Proposition 3. They hold more generally, for positive values of λ_Y , σ , δ , and \bar{G} , and therefore, also holds when monetary policy follows the rules derived in Propositions 1 and 2.

3.1.2 Monetary Policy: an example affirming Leeper.

What do these optimal interest rate rules tell us about whether monetary policy is active/passive? Leeper’s classification hinges on the response of interest rates on inflation (and the output gap). When interest rates respond strongly to inflation monetary policy is active and conversely, when they do not respond strongly, it is passive. Admittedly, Leeper’s analysis is not easy to map into our optimal policy framework: Even though we can show that \mathcal{T}^{NDC} defines an active monetary policy, it is not obvious how we could show that $\hat{i}_t = \mathcal{T} + \mathcal{D}$ defines a passive policy, especially when $\mathcal{T}^{NDC} = \mathcal{T}$, as was previously illustrated. If anything, since \mathcal{D} is a moving average of mean zero innovations to the Lagrange multiplier, it would seem that interest rates in the debt concerns model will on average be equal to \mathcal{T} and therefore also equal to \mathcal{T}^{NDC} .

This is however not the case. The innovations in \mathcal{D} are not orthogonal to inflation; they are functions of the same fundamental shocks (in preferences, spending, taxes etc) which drive inflation and so they are correlated with inflation (and with the remaining variables in \mathcal{T}). In principle we can express \mathcal{D} as a function of inflation, lagged values of interest rates etc. We can thus map the optimal policy into Leeper’s analysis.

In the next subsection we use the numerical solution of the model to approximate \mathcal{D} as a function of the variables in \mathcal{T} . To derive an analytical solution we assume $\lambda_Y = \sigma = \lambda_i = 0$, as in the previous paragraph, however, now letting $\delta > 0$. Note that under these assumptions we cannot derive an

¹⁴In other words, debt does not explode if it equals zero for all t .

Notice that if this fails we have: $\Delta\psi_{gov,t} = \frac{\beta}{b_\delta\bar{\eta}^2} \left(\tilde{\epsilon}\hat{b}_{t-1,\delta} - A\epsilon_{\tau,t} \right)$ and therefore, $E_{t-1}\Delta\psi_{gov,t} = \frac{\beta}{b_\delta\bar{\eta}^2} \tilde{\epsilon}\hat{b}_{t-1,\delta} \neq 0$. Thus $\hat{b}_{t,\delta} = 0$ is the only path consistent with the random walk.

optimal policy rule directly from the first order conditions; however, we can find a Taylor rule that implements the allocation under optimal policy.

Assume that the policy rule is of the form:

$$\hat{i}_t = \tilde{\phi}_\pi \hat{\pi}_t \quad (28)$$

Consider first the no debt concerns case, assuming again, for simplicity, that preference shocks are absent so that in equilibrium $\hat{i}_t = \hat{\pi}_t = 0$. Combining (28) with the Euler equation $\hat{i}_t = E_t \hat{\pi}_{t+1}$ we get the following difference equation in inflation:

$$\hat{\pi}_t \tilde{\phi}_\pi = E_t \hat{\pi}_{t+1}$$

Standard results yield that the equilibrium $\hat{i}_t = \hat{\pi}_t = 0$ is unique if and only if $\tilde{\phi}_\pi > 1$.

Now consider the case of debt concerns. The equilibrium under optimal policy satisfies:

$$\hat{\pi}_t = \frac{\omega_Y}{\kappa_1} \Delta \psi_{gov,t} + \frac{\bar{b}_\delta}{1 - \beta \delta} \sum_{l=0}^{\infty} \delta^l \Delta \psi_{gov,t-l} \quad \hat{i}_t = \frac{\bar{b}_\delta \delta}{1 - \beta \delta} \sum_{l=0}^{\infty} \delta^l \Delta \psi_{gov,t-l} \quad (29)$$

Assuming that parameter values are such that $\frac{\omega_Y}{\kappa_1} \approx 0$ and considering a rule of the form (28) that can implement this allocation we have:

$$\tilde{\phi}_\pi \frac{\bar{b}_\delta}{1 - \beta \delta} \sum_{l=0}^{\infty} \delta^l \Delta \psi_{gov,t-l} = \delta \frac{\bar{b}_\delta}{1 - \beta \delta} \sum_{l=0}^{\infty} \delta^l \Delta \psi_{gov,t-l} \quad (30)$$

hence $\tilde{\phi}_\pi = \delta < 1$.

This simple example shows that whereas the no debt concerns model requires a coefficient $\tilde{\phi}_\pi$ which exceeds unity (the standard condition for 'active' monetary policy), the debt concerns model features 'passive' monetary policy under an interest rate rule of the form (28) which does not include the interest rate twisting term $-\frac{\bar{b}_\delta}{1 - \beta \delta} \sum_{l=0}^{\infty} \delta^l \Delta \psi_{gov,t-l}$.¹⁵

We summarize the above in the following proposition:

Proposition 4. *Assume $\lambda_Y = \sigma = \lambda_i = 0$. The optimal policy is a rule of the form (28). In the case of no debt concerns, $\tilde{\phi}_\pi \geq 1$, and monetary policy is 'active'. Under debt concerns, $\tilde{\phi}_\pi = \delta < 1$, and monetary policy is 'passive'.*

3.1.3 Monetary Policy: Numerical Examples

For more plausible calibrations of the model the above properties can be demonstrated using numerical simulations. In Table 1 we show the coefficients we obtain from Taylor rule approximations of optimal policy under debt concerns model under various model specifications. More precisely, we approximate the optimal policies using rules of the form:

$$\hat{i}_t = \tilde{\phi}_\pi \hat{\pi}_t + \tilde{\phi}_Y \hat{Y}_t + \tilde{\phi}_i \hat{i}_{t-1} + \tilde{\phi}_{\Delta i} \Delta \hat{i}_{t-1} \quad (32)$$

which do not include object \mathcal{D}_t .¹⁶ The coefficients $\tilde{\phi}$ are such that a model with rule (32) produces impulse responses to shocks close to the analogous objects of the optimal policy model.¹⁷

¹⁵ If this term were included in rule (28) we would have

$$\hat{i}_t = \tilde{\phi}_\pi \hat{\pi}_t - \frac{\bar{b}_\delta}{1 - \beta \delta} \sum_{l=0}^{\infty} \delta^l \Delta \psi_{gov,t-l} \quad (31)$$

and the optimal policy sets $\tilde{\phi}_\pi = 1 + \delta > 1$.

¹⁶Equivalently we approximate \mathcal{D}_t as a function of the RHS variables in (32).

¹⁷Computing coefficients through OLS regressions does not (generally) provide a good fit. It may also give us coefficients which are not consistent with a unique equilibrium. This pattern is consistent with the findings of Cochrane (2011) and Schmitt-Grohé and Uribe (2004).

Obviously, the impulse responses of the model under (32) do not perfectly match the responses under optimal policy. Under $\delta > 0$ we would need several lags of interest rates (possibly also lags of inflation and output) to produce a perfect fit. We minimize the distance between the two models. We obtain a perfect fit when we set with $\delta = 0$.

We set $\tilde{\phi}_Y = 0$ (setting also $\lambda_Y = 0$) in the top panel of Table 1 to isolate our focus on coefficient $\tilde{\phi}_\pi$. Each of the columns of the table corresponds to a different calibration of parameters λ_i and σ . Consider the first of these columns which sets $\lambda_i = 0.5$ and $\sigma = 1$. For each parameter ($\tilde{\phi}_\pi$, $\tilde{\phi}_i$ etc) we report two numbers: The top number corresponds to the estimates of (32). The bottom number (reported in parenthesis) corresponds to the coefficient of component \mathcal{T} derived in Proposition 1. Notice that the estimates of (32) yield a much weaker response of interest rates to inflation ($\tilde{\phi}_\pi = 0.104$ vs. 0.537 in the analytic solution) and also the coefficients on \hat{i}_{t-1} and $\Delta\hat{i}_{t-1}$ are much lower than their analytic solution counterparts. (32) is basically a 'passive money' rule.

The same pattern emerges in the remaining model specifications. For each of the calibrations considered in Columns 2 to 4 the estimates of $\tilde{\phi}_\pi$, $\tilde{\phi}_i$ and $\tilde{\phi}_{\Delta i}$ are lower. The bottom panel of the table considers the case $\lambda_Y > 0$ and reports the estimates of $\tilde{\phi}_Y$. The response to inflation in (32) continues to be weaker than in the exact solution of the model. The estimated output coefficient $\tilde{\phi}_Y$ is also weaker.

The above patterns hold for many alternative calibrations of the model which for brevity we leave outside the table.

[Table 1 About Here]

3.2 The effects of shocks

The previous subsection showed that in the debt concerns model fiscal policy is 'active' and so debt becomes a non-stationary process. Monetary policy is subservient to fiscal policy and as we showed, it can be represented (in reduced form) with a Taylor rule which has the standard features of 'passive money' models. The opposite holds under no debt concerns: the policy mix becomes 'active monetary/passive fiscal'.

These findings are crucial to understand the working of the model and the effects of economic shocks which we now study. A standard implication of 'passive money' models is that they magnify macroeconomic volatility. Shocks which hit the economy lead to larger fluctuations in inflation and output (see for example Bianchi and Ilut (2018)). In our optimal policy model this will also be the case. When optimization is subject to the consolidated budget the planner needs to partially give up on the goal of stabilizing inflation, output and interest rates to satisfy the constraint.

To show this clearly we first derive the impulse responses analytically, in a simplified setup assuming, as previously, $\lambda_Y = \lambda_i = \sigma = 0$ but now let $\bar{G}, \delta, \rho_\tau > 0$. We consider shocks at date 0 which change the values of parameters $\{G_0, \xi_0, \tau_0\}$ assuming that after $t = 0$ there are no further shocks to the economy. Under these assumptions we derive in the appendix analytically the optimal paths of inflation and interest rates and the path of the multiplier $\psi_{gov,t}$. We set $\phi_{\tau,b} = 0$, assuming that taxes do not respond to debt, as Bianchi and Melosi (2018) do.

The appendix arrives to the following expressions for equilibrium inflation and interest rates under debt concerns:

$$\hat{\pi}_t = \frac{\omega_Y}{\kappa_1} \Delta\psi_{gov,0} \mathcal{I}_{t=0} + \frac{\bar{b}_\delta \delta^t}{1 - \beta\delta} \Delta\psi_{gov,0} \quad (33)$$

$$\hat{i}_t = -\rho_\xi^t (\rho_\xi - 1) \hat{\xi}_0 + \frac{\bar{b}_\delta \delta^{t+1}}{1 - \beta\delta} \Delta\psi_{gov,0} \quad (34)$$

for $t = 0, 1, 2, \dots$ and where $\mathcal{I}_{t=0}$ takes the value 1 at $t = 0$ and 0 otherwise. Moreover, $\Delta\psi_{gov,0}$ is a linear function of $\{\hat{G}_0, \hat{\xi}_0, \hat{\tau}_0\}$. We have:

$$v_{gov} \Delta\psi_{gov,0} = v_G \hat{G}_0 + v_\tau \hat{\tau}_0 + v_\xi \hat{\xi}_0 \quad (35)$$

where $v_{gov}, v_G, v_\xi > 0$ and $v_\tau < 0$ and therefore the partial derivatives satisfy $\frac{\partial \Delta\psi_{gov,0}}{\partial \hat{G}_0} > 0$, $\frac{\partial \Delta\psi_{gov,0}}{\partial \hat{\tau}_0} < 0$ and $\frac{\partial \Delta\psi_{gov,0}}{\partial \hat{\xi}_0} > 0$.

We also show in the appendix that in the no debt concerns model $\hat{i}_t = -\rho_\xi^t (\rho_\xi - 1) \hat{\xi}_0$ and $\hat{\pi}_t = 0$ for all t .

3.2.1 Preference Shocks

We consider first the impact of a preference shock which lowers the value $\hat{\xi}_0$. The previous derivations show that in the no debt concerns equilibrium this shock has not effect on inflation. This result is standard: Since the planner does not want to smooth the nominal interest rate, i.e. $\lambda_i = 0$, \hat{i}_t drops one for one with the real rate thus fully stabilizing inflation.

Now consider the case of the 'debt concerns' model. Since $\frac{\partial \Delta \psi_{gov,0}}{\partial \xi_0} > 0$, a drop in $\hat{\xi}_0$ lowers $\Delta \psi_{gov,0}$. From (33) and (34) we see that inflation turns negative after the shock and the nominal interest rate drops below real rate, $-\rho_\xi^t (\rho_\xi - 1) \hat{\xi}_0$. What is going on? Notice that a preference shock has two impacts on the consolidated budget: First, it increases real bond prices and hence increases the real payout of government debt; second, it increases the present value of surpluses which finance the debt. Since the planner has to satisfy the intertemporal constraint she will either use inflation or use deflation to adjust the real payout of debt so that the constraint is satisfied with equality. If the first effect is stronger, and real debt increases more than the surpluses, then inflation is optimal. In contrast, if the second effect dominates, then it is optimal to make inflation negative. In our analytic solution the second effect is stronger for all values $0 < \delta < 1$ and so inflation becomes negative in response to the preference shock.¹⁸

Figure 1 shows the above responses over 40 periods.¹⁹ The left column of the figure shows the case of the preference shock; the solid line represents the case of the debt concerns model and the dashed line shows 'no debt concerns'. The top two panels show the responses of inflation and interest rates (which are of course consistent with the analytic solution) the bottom two panels show output and the market value of debt.

[Figure 1 About Here]

Equilibrium output under debt concerns is given by:

$$\hat{Y}_t = \frac{1}{\kappa_1} \left(\frac{\omega_Y}{\kappa_1} \Delta \psi_{gov,0} \mathcal{I}_{t=0} + \bar{b}_\delta \delta^t \Delta \psi_{gov,0} - \kappa_2 \rho_\tau^t \hat{\tau}_0 \right)$$

(see appendix). Output drops in response to the preference shock because $\Delta \psi_{gov,0}$ turns negative. Under no debt concerns, output remains roughly constant after the shock.²⁰

3.2.2 Fiscal Shocks

The middle and right columns of Figure 1 study the responses to a positive spending shock and a negative tax shock respectively. In both cases the consolidated budget constraint tightens because the present value of the government's surplus falls and so $\Delta \psi_{gov,0} > 0$. From (33) and (34), interest rates rise in response to the shocks in the debt concerns model and inflation rises above its steady state value. Positive inflation ensures the solvency of the consolidated budget. In contrast, under no debt concerns inflation does not respond to these shocks because debt rises and taxes eventually adjust to make the surplus positive.

[Table 2 About Here]

¹⁸When $\delta = 1$, long bonds are consols, and preference shocks have zero impact on the consolidated budget.

¹⁹The values we assign to the model's parameters are presented in Table 2. The top panel of the table reports values for parameters which are common across the two versions of the model, the bottom panel reports the value that we assign to $\phi_{\tau,b}$ in each version separately. In the debt concerns model we set $\phi_{\tau,b} = 0.0$ and in the case where the Fed has "no debt concerns" we set $\phi_{\tau,b} = 0.07$. $\phi_{\tau,b} = 0.07$ is chosen so that debt has a near unit root, consistent with the empirical evidence on US government debt (see e.g. Marcet and Scott (2009) among others).

The value of $\phi_{b,\tau}$ which gives $\bar{\epsilon} = 1$, the cutoff for determinacy, is equal to 0.065.

²⁰In the NDC scenario output changes are driven by taxes. From the Phillips curve we get

$$\hat{Y}_t^{NDC} = -\frac{\kappa_2}{\kappa_1} \hat{\tau}_t^{NDC} \quad (36)$$

where $\kappa_1 \equiv -\frac{(1+\eta)\bar{Y}}{\theta} \gamma_h$, $\kappa_2 \equiv -\frac{(1+\eta)\bar{Y}}{\theta} \frac{\bar{\tau}}{(1-\bar{\tau})}$. Notice that the negative shock in preferences makes taxes drop. Even though the market value to GDP increases, fewer bonds need to be issued and so $\hat{b}_{t,\delta}$ is lower along the optimal path.

3.2.3 Responses under $\lambda_i, \lambda_Y, \sigma > 0$

In Figure 2 we assume $\lambda_i = \lambda_Y = 0.5$ and $\sigma = 1$. Now inflation and output change in response to the shocks in the no debt concerns model: Because the planner wants to smooth interest rates, following a preference shock, \hat{i}_t does not drop one for one with the real rate to fully stabilize inflation; moreover, since the planner also wants to smooth output, she optimally adjusts inflation in order to partially offset fiscal shocks.

Contrasting the responses of the two models, it is clear that the qualitative patterns outlined previously remain. In the debt concerns model inflation becomes negative in response to a preference shock, and positive in response to spending and tax shocks. The (numerical) partial derivatives $\frac{\partial \Delta \psi_{gov,0}}{\partial \hat{G}_0}$, $\frac{\partial \Delta \psi_{gov,0}}{\partial \hat{\tau}_0}$, $\frac{\partial \Delta \psi_{gov,0}}{\partial \hat{\xi}_0}$ have the same sign as before. This again implies that interest rates drop more in the debt concerns model, following a negative shock to preferences and they rise (relative to the no debt concerns model) in response to shocks to the fiscal variables.

[Figure 2 About Here]

The responses shown in Figures 1 and 2 reveal, that shocks have a larger impact on macroeconomic variables under debt concerns than in the case of no debt concerns. As discussed previously, higher volatility is a standard prediction of models where monetary policy is 'passive'. In our optimal policy model higher volatility derives from the fact that the planner needs to partially give up on the objective to stabilize macroeconomic variables in order to satisfy the consolidated budget.

4 Optimal Policies at the ZLB

We now study the dynamics of the economy when interest rates hit the ZLB. As it is standard we consider a shock which lowers the value of $\hat{\xi}_0$ sufficiently so that the ZLB binds. Standard results imply that this shock puts downward pressure on prices and output, leading to deflation and to a recession during the LT. Monetary policy can then commit to keep interest rates low for a long period, and stabilize inflation (e.g. Eggertsson and Woodford (2003)). We show in this section that in the case of debt concerns the above well known result does not apply. Committing to keep interest rates low for a long period, does not stabilize inflation during the LT (and may even lead to deflation). In contrast, in the case of debt concerns, FG is very effective in stabilizing inflation.

4.1 FG in the LT: An analytical example

We first demonstrate optimal policy using the simplified version of the model, setting $\sigma = \lambda_Y = \lambda_i = 0$. Recall that in this model optimal monetary policy is equivalent to a simple interest rate rule of the form (28). In the case of debt concerns we need to set $\tilde{\phi}_\pi = \delta < 1$ for the equilibrium to be uniquely defined, whereas under no debt concerns we have $\tilde{\phi}_\pi > 1$. In the case where the ZLB may bind we can augment (28) with the endogenous FG shock \mathcal{Z}_t and write:

$$\hat{i}_t = \tilde{\phi}_\pi \hat{\pi}_t + \mathcal{Z}_t \quad (37)$$

Our goal is to characterize the path of \mathcal{Z}_t under the two versions of the model.

4.1.1 FG and stepping on a rake

Before showing the solution for \mathcal{Z}_t under optimal policy it is worthwhile to first describe the impact of FG under arbitrary sequences \mathcal{Z}_t when monetary policy is given (37). Note that while \mathcal{Z}_t in our model equals 0 if the ZLB had not binded in $t - 1$, we consider here \mathcal{Z}_t to be a standard (exogenous) FG shock, as in the DGSE literature discussed previously. This us enables to build intuition for the endogenous FG case we will study next. For the moment, we also assume no preference shocks.

Suppose that the sequence $\{\mathcal{Z}_t\}_{t \geq 0}$ is revealed to private agents in period 0. In the case where $\tilde{\phi}_\pi > 1$, combining (37) with the Euler equation and iterating forward on the resulting difference equation gives:

$$\hat{\pi}_t = - \sum_{j \geq 0} \left(\frac{1}{\tilde{\phi}_\pi} \right)^{j+1} \mathcal{Z}_{t+j}. \quad (38)$$

According to (38) a shock $\mathcal{Z}_{t+\bar{t}} < 0$, $\bar{t} > 0$ lowering the nominal interest rate increases the value of inflation in periods $t, t+1, \dots, t+\bar{t}$. The magnitude of the response is governed by coefficient $\tilde{\phi}_\pi$.

In contrast, in the case where $\tilde{\phi}_\pi = \delta < 1$ equilibrium inflation satisfies:

$$\hat{\pi}_t = \sum_{j \geq 0}^{t-1} \delta^j \mathcal{Z}_{t-j-1} + \delta^t \hat{\pi}_0. \quad (39)$$

which implies that a shock $\mathcal{Z}_{t+\bar{t}} < 0$ will lower the value of $\hat{\pi}_{t+\bar{t}-1}, \hat{\pi}_{t+\bar{t}}, \hat{\pi}_{t+\bar{t}+1}, \dots$. In the debt concerns model therefore, FG has the opposite impact, promising to lower the interest rate reduces inflation, starting one period before the shock arrives.

What is going on? Though (38) fully characterizes the path of inflation in response to an anticipated FG shock in $t+\bar{t}$ (and this response conforms with the common intuition that lower rates produce higher inflation), in contrast, in the debt concerns model, (39) does not tell us how inflation will behave, because we do not have the initial condition $\hat{\pi}_0$. To fully characterize inflation we need to make use of the intertemporal consolidated budget constraint at $t=0$.²¹ Under the assumptions made in this section ($\sigma=0$ and $\omega_Y \approx 0$ —the latter amounts to assuming that steady state taxes are low) we can show that the government surpluses are constant, in the absence of shocks. We therefore have:

$$-\frac{\bar{b}_\delta}{1-\beta\delta} \sum_{t \geq 0} (\beta\delta)^t \hat{\pi}_t \approx 0 \quad (40)$$

which basically says that the drop in inflation in $t+\bar{t}-1$ (and onwards) needs to be compensated by a rise in inflation between periods 0 and $t+\bar{t}-2$. When the monetary authority announces $\mathcal{Z}_{t+\bar{t}} < 0$, $\hat{\pi}_0$ increases to satisfy (40) and since $\hat{i}_0 = \delta\hat{\pi}_0$, the nominal interest rate will also increase. This will lead to a further increase in prices in period 1 and positive interest rates subsequently, until inflation switches sign in period $t+\bar{t}-1$.

This pattern is essentially the stepping on a rake of Sims (2011) and Cochrane (2018). When monetary policy is passive a drop in the interest rate leads to a rise in inflation first (the conventional effect), only to reverse the sign of inflation after a few periods. This property is key in our debt concerns model.

4.1.2 Optimal FG policies

We now can interpret optimal policy in response to a shock to preferences which drives the economy to the LT. We assume that a preference shock occurs in period 0, lowering the value of $\hat{\xi}_0$. This shock is i.i.d. and moreover, for simplicity, we assume that after period 0 there are no further shocks to the economy. We set $\hat{\xi}_0 < -\hat{i}^*$. The shock is large enough for the ZLB to bind.

The appendix shows that the optimal paths of inflation and interest rates are given by:

$$\hat{\pi}_t = \begin{cases} \frac{\bar{b}_\delta}{1-\beta\delta} \Delta\psi_{gov,0} & t=0 \\ -\hat{i}^* - \hat{\xi}_0 & t=1 \\ \max\{-\hat{i}^*, \frac{\bar{b}_\delta \delta^t}{1-\beta\delta} \Delta\psi_{gov,0}\} & t \geq 2 \end{cases} \quad \hat{i}_t = \begin{cases} -\hat{i}^* & t=0 \\ \max\{-\hat{i}^*, \frac{\bar{b}_\delta \delta^{t+1}}{1-\beta\delta} \Delta\psi_{gov,0}\} & t \geq 1 \end{cases} \quad (41)$$

²¹Since we assume perfect foresight the date 0 constraint is sufficient. The intertemporal constraints at $t \geq 1$ will be satisfied since $\hat{b}_{t \geq 0}$ can be chosen as a residual (see FMOS).

in the case of the debt concerns model and

$$\hat{\pi}_t^{NDC} = \begin{cases} 0 & t = 0 \text{ and } t \geq 2 \\ -\hat{i}^* - \hat{\xi}_0 & t = 1 \end{cases} \quad \hat{i}_t^{NDC} = \begin{cases} -\hat{i}^* & t = 0 \\ 0 & t \geq 1 \end{cases} \quad (42)$$

in the no debt concerns model. Notice that according to (41) and (42) the two models predict positive inflation $t = 1$, (both equal to $-\hat{i}^* - \hat{\xi}_0$, trivially otherwise the ZLB would be violated) and different inflation levels at $t = 0$ and $t \geq 2$. In the case of no debt concerns, inflation returns to its steady state value from period 2 onwards. Under debt concerns inflation and interest rates continue being different from zero, the sign of the responses hinges on the sign of the term $\Delta\psi_{gov,0}$ which measures the impact of the LT shock on the consolidated budget. In the case where $\Delta\psi_{gov,0} < 0$, following a negative shock to preferences, the inflation rate becomes negative. In contrast, if $\Delta\psi_{gov,0} > 0$, inflation turns positive after the shock.

What does endogenous FG do in this model? Using (42) and the rule (37) we can show that the optimal path for \mathcal{Z}^{NDC} is:

$$\mathcal{Z}_1^{NDC} = \tilde{\phi}_\pi(\hat{i}^* + \hat{\xi}_0) < 0 \quad \text{and} \quad \mathcal{Z}_t^{NDC} = 0, \quad t \neq 1.$$

In other words, FG keeps the interest rate low in period 1, which enables inflation to rise in that period. This is the standard result of Eggertsson and Woodford (2003).

In the case of debt concerns model, using (41) and $\tilde{\phi}_\pi = \delta$, we have:

$$\mathcal{Z}_t = \begin{cases} \max\{-\hat{i}^*, \frac{\bar{b}_\delta \delta^{t+1}}{1-\beta\delta} \Delta\psi_{gov,0}\} + \delta(\hat{\xi}_0 + \hat{i}^*) < 0 & t = 1 \\ \max\{-\hat{i}^*, \frac{\bar{b}_\delta \delta^{t+1}}{1-\beta\delta} \Delta\psi_{gov,0}\} + \delta\hat{i}^* < 0 & t > 1, i_{t-1} = -\hat{i}^* \\ 0 & \text{others} \end{cases} \quad (43)$$

Let us consider the case where $\Delta\psi_{gov,0} < 0$ as in the previous section. For the sake of the exposition let us also focus on a case where $\frac{\bar{b}_\delta \delta}{1-\beta\delta} \Delta\psi_{gov,0} < -\hat{i}^* < \frac{\bar{b}_\delta \delta^2}{1-\beta\delta} \Delta\psi_{gov,0}$. Under this condition the shock is large enough to make the ZLB bind in period 0, but from period 1 onwards the interest rate is given by $\frac{\bar{b}_\delta \delta^{t+1}}{1-\beta\delta} \Delta\psi_{gov,0}$.

From (43) it is clear that \mathcal{Z}_1 is negative. Moreover, it holds that $\mathcal{Z}_{\geq 2} = 0$. Therefore, FG lowers the interest rate in period 1 only. We can show that $\hat{\pi}_t$ is given by:

$$\hat{\pi}_t = \delta^{t-1} \underbrace{(-\hat{i}^* - \hat{\xi}_0)}_{\hat{\pi}_1} + \delta^{t-2} \mathcal{Z}_1, \quad t = 2, 3, \dots$$

Since $\mathcal{Z}_1 < 0$ this equation suggests that impact of \mathcal{Z}_1 is to lower inflation after $t = 1$. Clearly, once we set $\mathcal{Z}_1 = 0$ we have that $\hat{\pi}_{t \geq 2} = \delta^{t-1} \hat{\pi}_1 > 0$. In this example, FG under debt concerns has exactly opposite impact from FG under no debt concerns.

4.2 A Numerical Example

The example of the previous subsection, is of course a very particular case; assuming $\sigma = 0$ means that output growth exerts no influence on the real rate. Inflation in $t = 1$ has to equal $-(\hat{i}^* + \hat{\xi}_0)$ under both debt concerns and no debt concerns, otherwise the ZLB is violated. In the case where $\sigma > 0$ a LT trap can lead to deflation; since output drops at the onset of the episode, positive expected output growth will increase the real rate, 'allowing' inflation to turn negative without violating the ZLB. It is thus important to contrast the properties of the two models under more plausible calibrations when $\sigma, \lambda_Y, \lambda_i > 0$, to see whether the findings of the previous paragraph generalize.

This is done in Figure 3 which shows the responses of inflation, output, interest rates and debt under the two versions of the model assuming the parameter values of Table 2. The solid (blue)

line shows responses in the debt concerns model. The dashed line shows responses under no debt concerns.

Qualitatively, the pattern of adjustment of inflation and interest rates resembles the analytical paths derived previously. (LTs last longer now because we assume a persistent shock). Under no debt concerns inflation turns positive during the period when the interest rate is at the ZLB and gradually goes back to zero after the economy escapes from the LT, as interest rates also gradually return to their steady state value.²² When the shock hits aggregate output drops, and the planner promises to gradually increase output and ultimately engineer a boom as the economy escapes from the LT. These are standard properties of the NK model's response to the LT shock (see Eggertsson and Woodford (2003)).

[Figure 3 About Here]

In the case of debt concerns, we see that following the preference shock in period 0, inflation drops sharply. Inflation turns positive for few periods, but subsequently becomes negative again. As the bottom left panel shows interest rates in the debt concerns model remain longer at the ZLB. This leads to a drop in the inflation rate below its steady state value, rather than a rise, consistent with our analytical results in the previous subsection.

To investigate the impact of FG, in Figure 4 we show the responses of interest rates, output and inflation setting $Z = 0$. The two models are now solved using the analytical solution derived in Propositions 1 and 2, therefore we set $\hat{i} = \mathcal{T} + \mathcal{D}$ in the debt concerns model and $\hat{i}^{NDC} = \mathcal{T}^{NDC}$ in the case of no debt concerns. Notice that following the shock to preferences, under no debt concerns inflation drops a lot more (than in Figure 3), interest rates stay at the ZLB for roughly half the time and output losses are substantial. Standard intuition applies: FG keeps interest rates low in the recovery and this mitigates the impact of the LT in the macroeconomy. Under debt concerns, however, the responses shown in Figure 4 are virtually unchanged relative to the optimal policies shown in Figure 3. Inflation becomes slightly less negative at long horizons and interest rates remain at the ZLB for fewer periods, but these effects are not substantial. In the case of debt concerns FG Z has only a minor impact on the macroeconomy.

[Figure 4 About Here]

4.3 Can inflation be positive under debt concerns?

Our analytical results in this section were derived under the assumption that following a shock to preferences which lowers the discount factor driving the economy to a LT, the present value of government surpluses increases more than the market value of debt. We had $\Delta\psi_{gov,0} < 0$. This was also the case in the numerical example shown in subsection 4.2.

The reader may be asking herself whether it holds more generally that LTs lead to persistent deflation and interest rates which stay for a long period at the ZLB. Is it possible to have $\Delta\psi_{gov,0} > 0$? Since LTs lead to a sharp drop in output and since ours is a model with distortionary taxes it is possible (under some calibrations) that the sign of $\Delta\psi_{gov,0}$ becomes positive. If we had assumed $\Delta\psi_{gov,0} > 0$ in our analytical model in this section, then (according to (41)) interest rates would stay for one period at the ZLB, and then rise above steady state level. Inflation would be positive along the transition path, since raising the nominal rate pushes prices upwards under debt concerns.

In the quantitative model of the next section we will allow for several features which will tend to make the government's surplus drop when the US economy enters in the LT. Besides distortionary

²²The fact that the adjustment of inflation is gradual is due to the persistent shock and also due to the assumption $\sigma > 0, \lambda_Y > 0$.

In the analytical example of the previous subsection the planner lowered interest rates only in $t = 1$, in the no debt concerns model, and this led to a rise in inflation in that period. This path was optimal, as opposed to extending lower rates beyond period 1, because inflation beyond period 1 is not useful to satisfy the ZLB. This path implies a sharp rise in \hat{Y}_1 and then $\hat{Y}_{t \geq 2} = 0$.

When the planner wants to smooth output she commits to keep interest rates at the ZLB for longer, generating high inflation today through expectations of high inflation tomorrow. This mitigates the output rise in period 1.

taxation we will estimate a model with automatic stabilizers in fiscal policy and multiple shocks (to spending, taxes etc) will hit the US economy at the moment when the interest rate hits the ZLB. In our simulations of the debt concerns model, inflation will indeed be positive but interest rates will escape from the ZLB very rapidly so that this model will not display persistence of interest rates at the ZLB; this is at odds with the US data. When we will shut down fiscal shocks, the model will predict persistently low rates and also predict negative inflation.

5 Quantitative Analysis

In this section we fit an augmented version of the theoretical framework developed in the previous sections to US data. The model essentially augments the previous setup with habit formation, a trend in TFP, and shocks to markups, TFP and to the consolidated budget. The fiscal block of the model closely follows Bianchi and Ilut (2017); we assume that besides spending, governments have to finance transfers to the private sector, which we model as a stochastic process with two components (a trend and a cyclical component).

We first provide an overview of the quantitative model and describe our estimation approach and the output we get from this exercise. We then make use of the estimated output to perform a predictive analysis of macro variables during the Great Recession. Our aim is to evaluate which one of the two versions of our model, the debt concerns or the no debt concerns, is better able to match key aspects of the joint evolution of inflation, interest rates and output in the United states during the Great Recession.

5.1 The model

5.1.1 Households

We assume that household preferences are of the following form:

$$E_0 \sum_{t=0}^{\infty} \beta^t \xi_t \left(\log(C_t - \Omega C_{t-1}^a) - \chi \frac{h_t^{1+\gamma_h}}{1+\gamma_h} \right)$$

where C_t denote the consumption of the household, ΩC_{t-1}^a is an external habit stock, where $0 < \Omega < 1$ and C_{t-1}^a denotes the average level of consumption in $t-1$. The household derives disutility from exerting labor effort h_t . Parameters χ and γ_h govern the household's preferences over leisure. As previously, ξ_t is the preference shifter which impacts the relative discounting of current and future utility flows.

The household maximizes utility subject to the flow budget constraint:

$$P_t C_t + P_{t,L} B_{t,\delta} + P_{t,S} B_{t,S} = (1 - \tau_t) W_t h_t + P_t T_t + B_{t-1,S} + (1 + \delta P_{t,L}) B_{t,\delta} + P_t D_t$$

$B_{t,\delta}$ is a long government bond which pays decaying coupons to the household. $P_{t,L}$ is the price of the asset. $B_{t,S}$ denotes the quantity of short term (one-period) government debt purchased by the household. We assume that short debt is in zero net supply. W_t denotes the nominal wage and P_t is the price level. Finally, D_t is real dividends paid by monopolistically competitive firms and T_t denotes lump-sum transfers given to the household by the fiscal authority.

The first order conditions from the household's program are:

$$P_{t,L} = \beta E_t \frac{u_c(C_{t+1} - \Omega C_t^a)}{u_c(C_t - \Omega C_{t-1}^a)} \frac{\xi_{t+1}}{\xi_t} \frac{P_t}{P_{t+1}} (1 + \delta P_{L,t+1})$$

$$\chi \frac{h_t^{\gamma_h}}{u_c(C_t - \Omega C_{t-1}^a)} = w_t (1 - \tau_t) \quad \text{and}$$

$$P_{t,S} = \beta E_t \frac{u_c(C_{t+1} - \Omega C_t^a)}{u_c(C_t - \Omega C_{t-1}^a)} \frac{P_t}{P_{t+1}} \frac{\xi_{t+1}}{\xi_t}$$

where u_c denotes the marginal utility of the consumption.

5.1.2 Firms

We assume that output is produced by a continuum of monopolistically competitive firms which operate technologies with labor as the sole input. Aggregate output is produced by a representative, perfectly competitive, final-good producer, which aggregates the intermediate products of firms to form aggregate output according to $Y_t = \left(\int_0^1 Y_t(j)^{\frac{1+\eta_t}{\eta_t}} dj \right)^{\frac{\eta_t}{1+\eta_t}}$. η_t is a (time varying) parameter that governs the elasticity of substitution across differentiated products. Profit maximization for final good producers gives the following demand of intermediate goods: $Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{\eta_t} Y_t$.

The production function of the generic intermediate good firm j is $Y_t(j) = A_t h_t(j)^{1-\alpha}$, where A_t denotes the level of TFP in the economy.

We further assume that intermediate goods firms face price adjustment costs as in Rotemberg (1982). The cost function of firm j is the following: $AC_t(j) = \frac{\theta}{2} \left(\frac{P_t(j)}{P_{t-1}(j)} - \pi \right)^2 Y_t$. $\theta \geq 0$ again governs the degree of price stickiness. π is the steady state level of gross inflation.

Intermediate-good producers seek to maximize profits subject to the constraints imposed by the above equations. The first-order conditions (stated formally in the online appendix) give us the (non-linear) New-Keynesian Phillips Curve:

$$\theta(\pi_t - \bar{\pi})\pi_t = (1 + \eta_t) \left(1 - \frac{MC_t}{P_t} \right) + \beta \theta E_t \frac{C_t - \Omega C_{t-1}^a}{C_{t+1} - \Omega C_t^a} \frac{Y_{t+1}}{Y_t} (\pi_{t+1} - \bar{\pi}) \pi_{t+1}$$

where MC_t denotes marginal costs of production.

Finally, we assume that the log growth rate of TFP evolves according to the following stochastic process

$$\ln \left(\frac{A_t}{A_{t-1}} \right) \equiv a_t = (1 - \rho_a)\gamma + \rho_a a_{t-1} + \epsilon_{a,t}$$

The parameter γ denotes the steady-state growth rate of the economy.

5.1.3 Government

The government levies distortionary taxes to finance spending G_t and transfers T_t . Imposing that short debt is in zero net supply we write the flow budget constraint as:

$$P_{t,L} B_{t,\delta} = (1 + \delta P_{t,L}) B_{t-1,\delta} + P_t (G_t + T_t) - \tau_t W_t h_t + \Lambda_t$$

$G_t - w_t h_t \tau_t - T_t$ is the real primary surplus. As Bianchi and Melosi (2017), we augment the flow budget with an exogenous shock variable Λ_t capturing features that we have left outside the model. These could derive from changes in the maturity of debt or the term premium, but also (more crucially) from variation in revenues and spending from sources that we do not model explicitly here (e.g. tax revenues from capital income or consumption taxation, public investment, etc).

As in Section 2 we assume that taxes follow an exogenous rule which relates the current tax rate to the lagged value of debt. In the next subsection we define this rule in the log-linear version of the model.

5.1.4 Log-Linear Model

Since productivity grows over time in our model we rescale model variables and linearize the model equations around the deterministic steady state. For brevity we relegate all derivations and the description of the log-linear equations characterizing the competitive equilibrium of the economy to the online appendix. Here we describe the functional forms we adopt for the fiscal policy variables (taxes and transfers) and the stochastic processes for the exogenous shocks.

We assume the following feedback rule for taxes:

$$\hat{\tau}_t = \rho_\tau \hat{\tau}_{t-1} + (1 - \rho_\tau) \left[\phi_{\tau,b} \hat{b}_{\delta,t-1} + \phi_{\tau,y} (\hat{Y}_t - \hat{Y}_t^n) + \phi_{\tau,g} (g^{-1} \hat{g}_t + \hat{t}r_t^*) \right] + \epsilon_{\tau,t} \quad (44)$$

Notice that we now allow aggregate output (in deviation from its natural level \hat{Y}_t^n)²³ and the level of government expenditures (composed of government spending and transfers) to impact directly the tax rate in period t . The variable \hat{tr}_t^* is defined as the long-term component in government transfers, and as in Bianchi and Ilut (2017) evolves exogenously following an AR(1) process:

$$\hat{tr}_t^* = \rho_{tr^*} \hat{tr}_{t-1}^* + \epsilon_{tr^*,t} \quad (45)$$

The deviation of transfers \hat{tr}_t from their long-run trend \hat{tr}_t^* responds to its first order lag and to the output gap. We have:

$$\hat{tr}_t - \hat{tr}_t^* = \rho_{tr} (\hat{tr}_{t-1} - \hat{tr}_{t-1}^*) + (1 - \rho_{tr}) \phi_{tr,y} (\hat{Y}_t - \hat{Y}_t^n) + \epsilon_{tr,t} \quad (46)$$

We assume that $\epsilon_{tr,t}$ and $\epsilon_{tr^*,t}$ are i.i.d. The stochastic processes of the remaining exogenous variables are assumed to follow:

$$\hat{x}_t = \rho_x \hat{x}_{t-1} + \epsilon_{x,t}$$

for $\hat{x} \in \{\hat{G}, \hat{a}, \hat{\Lambda}, \hat{\xi}, \hat{\eta}\}$ and where $\epsilon_{x,t}$ is an i.i.d shock to variable x . Thus, exogenous shocks to spending, TFP, $\hat{\Lambda}$, markups, and preferences follow first order autoregressive processes.

5.1.5 The planner's objective

For the quantitative model of this section we adopt the following objective function:

$$-\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[\hat{\pi}_t^2 + \lambda_Y \left(\hat{Y}_t - \hat{Y}_t^n \right)^2 + \lambda_i \left(\hat{i}_t - \hat{i}_{t-1} \right)^2 \right] \quad (47)$$

assuming that the planner seeks to minimize the deviation of inflation in t from the steady state level $\bar{\pi}$, the deviation of output from its natural level \hat{Y}_t^n , and the change in value of the nominal interest rate relative to the value in the previous period.

Note that (47) is commonly used in estimated DSGE models of optimal monetary policy (see for example Debortoli and Lakdawala (2016), among others). We thus follow the recent literature in our choice of the objective. Under (47) the optimal policy model continues to admit a closed form expression for the interest rate rule which, as in section 2, features components \mathcal{T} , \mathcal{D} , and \mathcal{Z} . For brevity we do not derive here the interest rate rule.

5.2 Estimation

Our quantitative analysis will be carried out in two steps. First, we will estimate the model using data until the fourth quarter of 2008, ignoring the ZLB constraint. Second, we will use the estimates of model parameters to investigate the performance of the model in the Great Recession accounting for the ZLB. In this section we describe our approach in model estimation and report the estimates of the structural parameters.

5.2.1 Selecting the ‘‘Correct’’ Model to Estimate

The task of estimating the model is complicated because we have two model versions we can choose from: the version where monetary policy takes into account the consolidated budget and the version where monetary policy has no debt concerns. Which of these models do we want to estimate? Should we estimate both?

If we estimate both versions the estimates of the structural parameters will vary across model versions. This would imply that potential differences in terms of the performance of the models in

²³We define the natural level of output as the flexible price level. When the planner chooses optimal policies we assume she does not account for their impact on \hat{Y}_t^n . For brevity, we discuss the details in the online appendix.

the Great Recession will be partly driven by the different values of the structural parameters, not by the models' endogenous propagation mechanisms which we would like to analyse. We thus choose to estimate one model only. To choose which one, we have to decide whether assuming no debt concerns describes more accurately US monetary policy since the 1980s when our sample begins, or if over this period, monetary policy was subservient to fiscal policy as in the debt concerns model. Bianchi and Ilut (2018) estimate a DSGE model allowing for the monetary/fiscal policy mix to vary through time. They find that from the 1980s and until 2008 monetary policy was 'active' and fiscal policy passive, or, in other words, monetary policy was not affected by the debt aggregate. Since this corresponds closely to the no debt concerns model we estimate our structural optimal policy framework assuming that the Fed chooses allocations without taking into account the consolidated budget.

5.2.2 Sample and Variables

We fit the model to US observations using data from the period 1980Q1-2008Q4. The macroeconomic aggregates that we employ in estimation are: output, inflation, the federal funds rate, tax revenues, total government expenditures, government spending, and the market value of government debt. We express the last four series as a fraction of GDP. The details on the sources and construction of these variables, together with the measurement equations we employ to link the series to model variables, are spelled out in the online appendix.

5.2.3 Priors and Calibrated Parameters

As is typical in the literature, we proceed with the estimation of the model by first selecting prior distributions for the parameters we wish to estimate and picking values for parameters that we want to fix in estimation. Table 3 summarizes the calibrated values of the parameters that we fix and the right side of Table 4 reports our choice of prior distributions for the parameters we estimate with Bayesian techniques. The priors are in line with previous papers in the literature (see e.g. Bianchi and Ilut (2017)) and are relatively loose.

[Tables 3 and 4 About Here]

We fix the values of the labor share, α , the elasticity of labor supply, $\frac{1}{\gamma_h}$, the demand elasticity parameter, η , the decay factor of the long term bond, δ . We assume $\alpha = 0.66$ and $\gamma_h = 1$. The decay factor δ is set to 0.95 which gives us an average maturity of 5 years, consistent with US data. The steady state value of η is such that markups are 15 percent. Finally, we normalize the steady-state value of output to unity.

5.2.4 Posterior Distributions and the Impact of Shocks

The left side of Table 4 reports the posterior estimates of the model parameter distributions. According to the values reported in the table, the Phillips curve is relatively flat (the mean estimate of κ is 0.017) and the distribution of the habit parameter Ω is centered around roughly 0.5. Moreover, the estimated response of taxes to the lagged value of debt is low ($\phi_{\tau,b} = 0.068$ at the mean). These values are close to the analogous objects reported in Bianchi and Ilut (2018).

Notice also that our model's estimates of the coefficients λ_i and λ_Y are in range of recent estimates of optimal Ramsey models with US data (see Debortoli and Lakdawala (2016) and references therein).

[Figures 5 and 6 About Here]

To gain insights into how these parameter estimates determine the model's behavior in Figures 5 and 6 we show the responses of inflation, interest rates, output and debt-to-GDP to each of the shock processes considered in the model. We assume a one standard deviation innovation to each process, based on the estimates reported in Table 4. Figure 5 considers the case of a preference

shock (left panel), a spending shock (middle left panel), a tax shock (middle right panel), and a markup shock (right panel). Figure 6 shows the responses to a shock to the government budget, a shock to TFP, and shocks to the business cycle and long-term components of transfers (from left to right). The dashed red line is the no debt concerns model estimated in this section. For the sake of comparison we have added the responses under debt concerns with the solid (blue) line. Notice that the impact effects of each of the shocks on the model variables are now measured in percentage points. Therefore, 1 is a 1 percent increase of a variable relative to the balanced growth path, 0.1 is a 0.1 percent increase, etc.

From the Figures we see that under no debt concerns, inflation is basically unaffected by shocks to preferences, spending, taxes, transfers, and shocks to the budget. Shocks to TFP exert a small influence on inflation, but mainly markup shocks are the key driving force behind inflation variability. This model property can be explained as follows: First, preference and spending shocks exert only a minor influence because these shocks are persistent and also because monetary policy is optimal. Due to high persistence these shocks do not provoke large movements to the real rate. Given welfare losses derive from the volatility of interest rate growth in (47), the planner can adjust permanently the nominal interest rate by a few basis points in response to the shocks, without impinging substantial welfare losses. Tax shocks on the other hand, under no debt concerns, affect the inflation output tradeoff through their influence on the Phillips curve, but our estimates in Table 4 suggest that this effect is not large. This also applies to shocks to the consolidated budget which can lead to changes in taxes.

Notice that the finding that markup shocks are a key driving force behind inflation dynamics is not out of line with the rest of the literature. Several studies have reached a similar conclusion (e.g. Fratto and Uhlig (2014), Hall (2011), Michallat and Saez (2014)). Some authors have suggested that this property hints at a failure of the NK model in (endogenously) explaining inflation. It is important, however, to stress that our optimal policy model offers a different perspective: Inflation is driven by shocks to markups only, because monetary policy is very effective in stabilizing inflation against other types of shocks.

This however, does not hold for the equilibrium Ramsey policy under debt concerns. As can be seen from Figures 5 and 6, in this case inflation responds strongly to all shocks, including to shocks in fiscal variables. Debt concerns implies higher macroeconomic volatility.

5.3 Quantitative Analysis: Optimal Policies in the Great Recession

In this section we evaluate our model’s properties in the Great Recession. Our main goal is to identify the forces behind the Fed’s forward guidance policies, through evaluating the ability of each of the two versions of the model to match the behavior of macroeconomic variables during the recession. To this end we perform a ‘forecasting’ exercise, whereby we recover the shocks which, in the first quarter of 2009, brought the US economy to the LT, and trace their impacts on the behavior of macroeconomic variables throughout the Great Recession. We thus trace our model’s ability to capture: i) the persistence of interest rates we observe in the data, ii) the lack of deflation observed, and iii) the dynamics of output growth and government debt throughout the sample. A similar exercise is considered in Bianchi and Melosi (2017), Del Negro et al (2015) and others.

We do this (essentially impulse response analysis) separately for the debt concerns and no debt concerns models in order to assess their relative fit. Since we assumed that up to the fourth quarter of 2008 monetary policy did not take into account the evolution of debt aggregates, when we consider the debt concerns equilibrium post 2009, we essentially assume that there was a shift in policy at the onset of the recession. For our experiment we make the shift permanent assuming an unexpected permanent shock which changed the structure of policy. We contrast the properties of the model when the shift occurs, with the properties when it does not occur and monetary policy adheres to its objective to stabilize inflation and the output gap.

5.3.1 Constructing Paths of Macroeconomic Variables

We first describe formally our approach in constructing the paths of macroeconomic variables. Essentially, our methodology consists of recovering the initial values of the state variables and the shocks which drove the US economy to the LT in the first quarter of 2009, then we solve the model forward to trace the effects of the shocks on key variables and in each of the two versions of the model.

We proceed in two steps. First, we recover the smoothed shocks and initial conditions of the estimated (no debt concerns) model using data for the period 1980Q1-2009Q1. We make use of a standard Kalman filter augmented to account for piecewise linear solutions, allowing us to deal with an occasionally binding ZLB constraint.²⁴ Using this methodology we obtain the piecewise linear solution of the model:

$$X_t = \mathcal{G}_t + \mathcal{E}_t X_{t-1} + \mathcal{F}_t \epsilon_t, \quad (48)$$

$t = 1, \dots, T^*$ where $t = 1$ corresponds to 1980Q1 and $t = T^*$ corresponds to 2009Q1. X_t and ϵ_t are vectors containing, respectively, all model variables and exogenous shocks, and \mathcal{E}_t and \mathcal{F}_t are state-space matrices of appropriate size. We recover the filtered variables $\{X_t\}_{t=0}^{T^*}$ and shocks $\{\epsilon_t\}_{t=1}^{T^*}$.

In the second step we create ‘forecasts’ of macro variables during the Great Recession. More precisely, we assume that after 2009Q1 $\epsilon_t = 0$ for $t > T^*$ (no more shocks hit) and let the model run for many periods to recover the paths of macroeconomic variables, accounting for the binding ZLB. We do this separately for each of the two versions of the model, debt concerns and no debt concerns. Under no debt concerns, we simply make use of (48) to recover the macroeconomic variables. In the case of debt concerns we assume a one-off unanticipated and permanent switch in policy assuming that the central bank begins to take into account the consolidated budget at the end of 2009Q1 (after the vector ϵ_{T^*} has been realized). We solve the model forward using standard techniques. A more formal description of the solution is provided in the online appendix.

5.3.2 Model Evaluation: Optimal Policies in the Great Recession

Figure 7 plots the paths of inflation (top left), interest rate (top right), debt and output growth (bottom left and right respectively). The solid lines show again the case of debt concerns, the dashed lines show the no debt concerns model. The dotted (black) line is the filtered data observations.

[Figure 7 About Here]

Forecasts of Macro Variables: No Debt Concerns

Consider first the performance of the no debt concerns model and its predicted paths for the macroeconomic variables after 2009:Q1. There are several noteworthy features: First, note that the model generates a positive inflation rate during the Great Recession; the inflation rate drops to around 1 per cent in the beginning of 2009, and very fast –after a couple of quarters– reaches a level, which exceeds the steady state level of inflation (roughly 2 per cent). Inflation in the model is thus higher than in the data (where we observe slightly negative inflation for a couple of quarters in 2009) and displays high persistence. Second, the model predicts that interest rates remain at the ZLB for several quarters after 2009. In particular, interest rates are at the ZLB until the 2011:Q3 and then gradually return to steady state. Third, the model performs very well in matching the behavior of the growth rate of output. As can be seen from the middle right panel, the model predicts that output growth recovers rapidly in 2009 and subsequently is stabilized close to the steady state rate of TFP growth. Finally, the model underperforms in matching the dynamics of government debt: Public debt increases only slightly in the model whereas the rise in the data is considerable.

As it is well known, both the lack of deflation during the Great Recession and the high persistence of interest rates at the ZLB are facts which are difficult to explain through a NK model’s endogenous

²⁴This is needed because the interest rate hit the ZLB in 2009Q1.

The algorithm we employ is similar to the one used in Kollmann et al. (2016). See the online appendix for a formal description.

propagation mechanisms. NK models typically imply negative inflation rates at the ZLB but also that interest rates recover fast and move away from the ZLB (see for example Del Negro et al (2015), Fratto and Uhlig (2014) among numerous others). The no debt concerns model has little difficulty in generating persistently low rates and also generates positive inflation at the ZLB, though inflation is higher than in the data.

To understand these findings note that the model impinges two key channels through which keeping interest rates at the ZLB for a long period of time is an optimal policy outcome: persistent preference shocks and endogenous FG. Both of these matter for the persistence of interest rates. However, preference shocks make inflation negative; the model's prediction that inflation rates are positive even at the onset of the LT can thus be fully accounted by the Fed's endogenous FG policies. This is consistent with the findings of Eggertsson and Woodford (2003): Under full commitment, Ramsey policy can nearly fully offset the negative effects of a LT shock on inflation through promising to keep the policy rate low and thus generate high inflation when the economy escapes from the LT. Higher future expected inflation translates into high current inflation because of the forward looking Phillips curve. This also explains why inflation remains higher than the steady state value throughout the entire simulated path, thus overshooting the data.

Is it crucial that the model cannot match the behavior of debt? Not at all. Recall that under no debt concerns, optimal inflation is not impacted by the evolution of the debt level. Therefore, the fact that the debt to GDP ratio does not rise sufficiently in response to the shocks which hit the economy in 2009:Q1 is inconsequential for the path of inflation (see also below).

Forecasts of Macro Variables: Debt Concerns

Consider now the responses in the debt concerns model shown in the Figure. Following the switch in optimal policy towards debt concerns, inflation increases considerably reaching around 12 percent in 2009, and subsequently decays monotonically towards the 2 percent value which is the steady state level. Interest rates are at the ZLB for only one period and output growth rises sharply in early 2009 before it drops again later for a few quarters. This model cannot track well the behavior of macroeconomic aggregates.

Recall from the analysis of Section 4 that a model with debt concerns can give rise to high inflation rates during a LT in the case where large fiscal shocks tighten the intertemporal consolidated budget constraint. This is essentially what happens here. When the economy switches towards debt concerns in 2009Q1, the accumulated level of government debt is above its steady-state value and the primary deficit is also above its long-run average. When the planner begins to take into account the sustainability of debt, the high initial debt level makes inflating away public debt optimal.²⁵

High debt levels are not the only source of inflation. In 2009Q1 the primary deficit in the data is also above its long-run average, this is due to the high spending and transfers shocks experienced by the US economy in that period. This is shown in the bottom right panel of the Figure. Since high deficits reduce the present discounted value of the government's surplus, inflation must increase to stabilize the budget.

To show how much fiscal variables and high debt contribute towards the substantial inflation witnessed in this model, we ran a counterfactual simulation shutting down fiscal shocks and assuming that debt is at steady state when the economy enters the LT. The results are shown in Figure 8. Note that in this case the debt concerns model predicts deflation and interest rates stay at the ZLB for many quarters. This pattern is consistent with the analysis of Section 4. The LT shock slackens the intertemporal constraint and it is optimal to lower prices. To accomplish this interest rates must remain at the ZLB for several periods.

[Figure 8 About Here]

²⁵ Essentially the planner solves an initial allocation program, since when the switch occurs, the multiplier on the consolidated budget constraint is 0. It is a standard feature of Ramsey models that at the beginning of the planning horizon inflation is optimal.

For completeness the dashed (red) lines in Figure 8 demonstrate what happens under no debt concerns and in the absence of fiscal shocks. Unsurprisingly, shutting down fiscal variables in this model has only a negligible impact on macroeconomic variables.

To sum up, the debt concerns model predicts high inflation at the onset of the LT, and high interest rates in response to the shocks experienced by the US economy in 2009. These observations are at odds with the US data. When we remove fiscal shocks, the model can generate a persistent LT, however inflation becomes negative, in line with our theoretical findings in Section 4.

5.3.3 Debt Concerns at QE 2

In the previous paragraph we investigated the effects of a switch in policy towards debt concerns, imposing that the switch occurs in 2009Q1, the time when the Fed conducted the first round of quantitative easing. Given the assumption made in the debt concerns model that the Fed sets $\hat{b}_{t,\delta}$, the net debt in the hands of private agents, this timing seems appropriate. Moreover, as was discussed in the introduction, it is widely thought that QE marked a change in the conduct of monetary policy, whereby the Fed crossed the boundary to fiscal policy and begun to respond to the debt aggregate.

As we have seen our optimal policy under debt concerns does not provide a good fit to the US data; it produces incredibly high inflation rates and interest rates which display no persistence at the ZLB. In this and the next paragraph we discuss this finding, to provide an answer to the question 'Did the Fed partially give up its inflation and output stabilization goals, and become subservient to fiscal policy'? We first show that our results are robust towards alternative timing assumptions, showing that the debt concerns model continues to produce high inflation and high interest rates, when we assume that the switch occurs after 2009Q1. Then, in section 5.3.4, we explain why some of our assumptions have been crucial to get the results we showed, explaining also how alternative modelling assumptions would change the relative successes and failures of the debt concerns and no debt concerns models.

[Figure 9 About Here]

We have run the experiment of section 5.3.2 many times assuming different dates for the switch in 2009, 2010 and 2011 and our previous findings remain. For brevity we only show here the case where the switch occurs in the fourth quarter of 2010, when the second round of quantitative easing was announced. The results are displayed in Figure 9. As can be seen from the Figure, inflation increases even further under debt concerns, and interest rates rise dramatically to above 15 percent after the switch occurs. As is well known, the US economy during this period experienced dramatic shifts in fiscal variables, with spending levels increasing rapidly and taxes dropping significantly, following the enactment of the American Investment and Recovery Act by the US Congress. These fiscal shocks put the government budget in strain, and a monetary authority that has to ensure debt sustainability finds optimal to let inflation rise. In the debt concerns model high inflation is accompanied by high interest rates.

The no debt concerns model portrayed with the dashed red lines in Figure 9 continues to provide a good fit of the data.

5.3.4 Discussion: Are debt concerns un-pragmatic?

Should the above results be taken to mean that monetary policy in the United States did not respond at all to debt aggregates during the Great Recession? We have made several assumptions concerning the conduct of policy, assuming from the outset that the planner can fully commit to future sequences of interest rates and inflation, and also assuming a permanent switch in monetary policy in 2009Q1 towards debt concerns. These assumptions are of course not innocuous for our results.

Consider, for the sake of the argument, an alternative scenario whereby the switch is temporary, monetary policy reverts to debt concerns in 2009Q1 but for a finite number of periods. In this case, the debt concerns model will produce essentially the same results as the no debt concerns model. To see this note that the multiplier ψ_{gov} is the state variable driving the difference between the two

models; under a temporary switch the private sector will expect ψ_{gov} to equal zero eventually; by the random walk property we will have $\psi_{govt} = \mathcal{D} = 0$ for all t .

Of course this is also an extreme scenario. A more plausible assumption is that private agents have own estimates about the probability that the Fed responds to the debt aggregate, optimal policy switches from one regime to the other and then switches back. In such a Markov switching model, the debt concerns and no debt concerns models will produce different outcomes. But the differences will not be as large as in the case of the permanent switch considered here. An extension of the theory presented in this paper which allows for Markov switches would also allow us to estimate the likelihood of debt concerns directly from the data.

The assumption of perfect commitment also has a bearing on our results. It is well known that under no commitment LTs lead to deflation and large recessions. Under debt concerns, however, having a high debt level leads to high and persistent inflation²⁶ Debt concerns will thus have an advantage in matching the inflation data.

To put an equation behind this, note that in the case of no commitment, the optimal policy rule which emerges from the model of Section 2 is of the form:

$$\hat{i}_t = \underbrace{\phi_\pi \hat{\pi}_t + \phi_Y \hat{Y}_t}_{\mathcal{T}^{nc}} + \omega_{gov} \psi_{govt}^{nc} \quad (49)$$

where nc denotes 'no commitment' and the last term in (49) is relevant only under debt concerns (ψ_{gov}^{nc} will continue to follow a random walk in this case). Rules of the form \mathcal{T}^{nc} typically lead to deflation during a LT. There is no (endogenous) FG in this model. However, under debt concerns ψ_{gov}^{nc} is a function of debt and for as long as debt is high, this term will make monetary policy deviate from \mathcal{T}^{nc} to produce high inflation.

A fruitful extension of the framework presented in this paper will allow for partial commitment, either allowing the Fed to commit for a finite number of periods (as in Lanteri and Clymo (2018)) or to re-optimize randomly and renege on some of its commitments (as in Debortoli and Nunes (2010) and Debortoli and Lakwadala (2017)). It would also be interesting to use such a model to estimate the degree of commitment of the Fed during the Great Recession through extending the methodology of Debortoli and Lakwadala (2017) to allow for an occasionally binding ZLB. This is however technically challenging and beyond the scope of this paper. We leave it to future work.

6 Conclusion

We offer a tractable framework of endogenous forward guidance which allows us to investigate the key forces that determined the behavior of interest rates in the Great Recession. Our model nests both the case where the Fed's policies reflect debt sustainability concerns and the case where they do not. A Ramsey planner (the Fed) sets allocations under commitment to minimize the deviations of inflation, output and interest rates from their respective target levels. If the planner has debt sustainability concerns then optimization is subject to the consolidated budget constraint; otherwise it is not.

The optimal policy rule we obtain from our model, endogenizes the 'forward guidance' shocks assumed in the recent DSGE literature. The short-term rate in our model is expressed as the sum of a Taylor rule component (a function of inflation and output growth), a component, which represents commitment to keep interest rates low at the exit from a LT episode, and, in the case the planner has debt concerns, an additional component which captures the impact of past shocks to the consolidated budget constraint. These additional components which make interest rates deviate from the Taylor rule represent endogenous forward guidance.

We show that in the presence of debt concerns, monetary policy becomes subservient to fiscal policy. Taxes do not increase in response to a high debt level and inflation adjusts to make the budget solvent. As a result, inflation, output and interest rates become more volatile responding to

²⁶See for example Faraglia et al (2013).

fiscal shocks. LT episodes are longer, however 'keeping interest rates low' for a long period, does not result in positive inflation rates at the onset of a LT episode. In contrast, in the absence of debt concerns, the impact of commitments to keep interest rates low at the exit from the LT, on inflation and output is substantial. In this case monetary policy accomplishes to turn inflation positive at the onset of the episode, through promising higher inflation rates in future periods.

We embed our optimal policy framework in a medium-scale DGSE model and estimate it with US data. Our quantitative findings suggest that interest rate policies in the Great Recession reflected mainly the Fed's commitment to stabilize inflation and the output gap.

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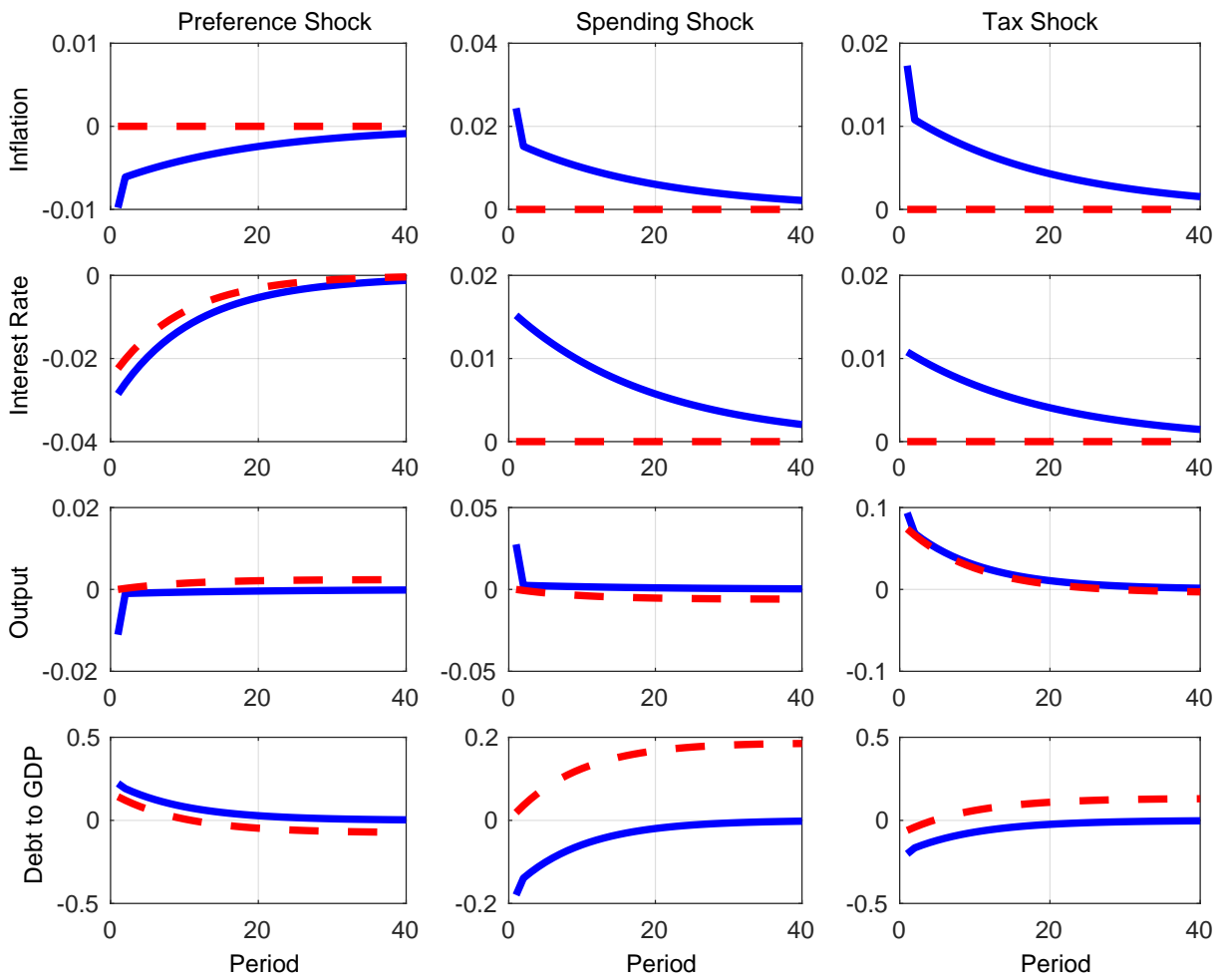
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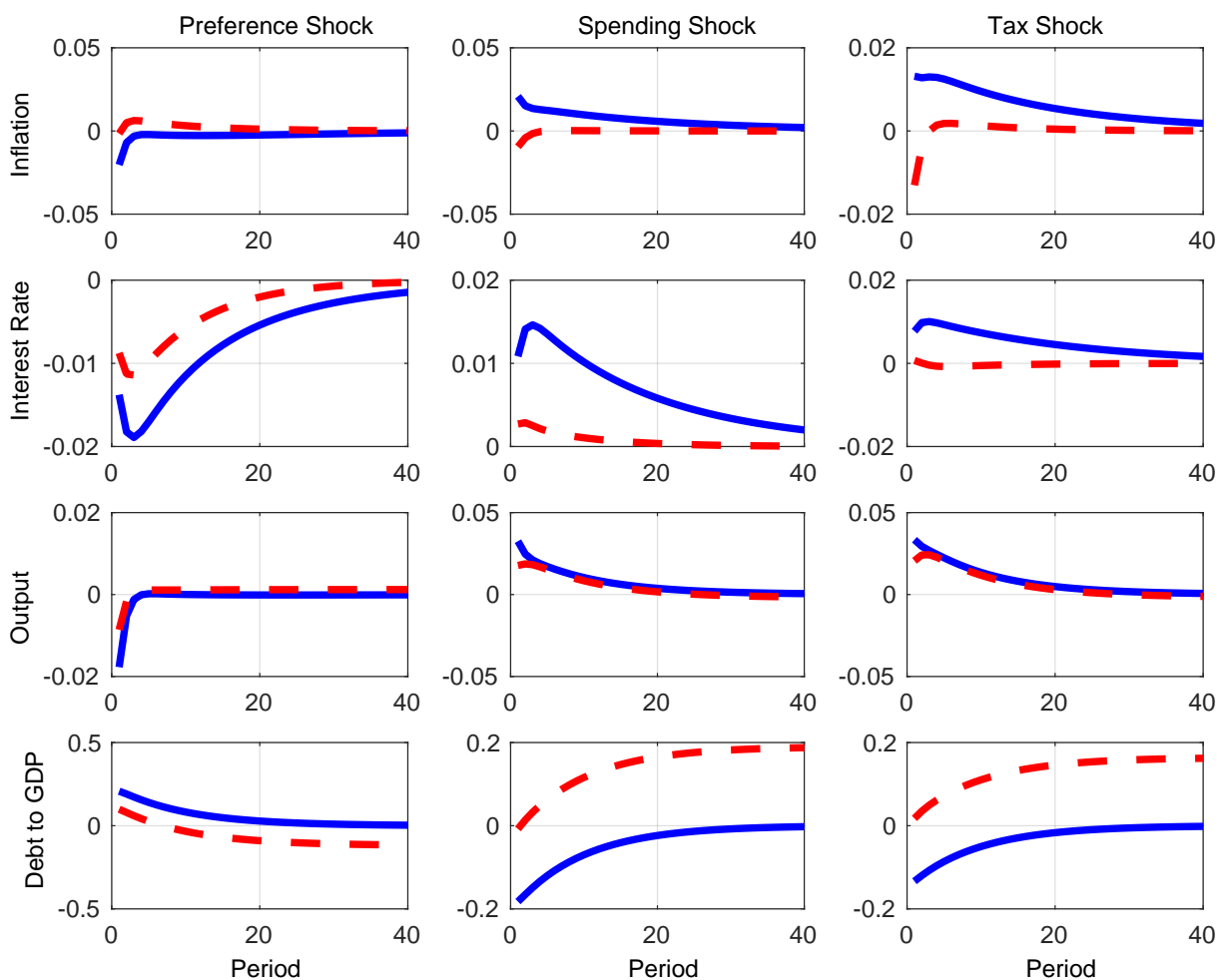
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Figure 1: **Optimal Policies away from the ZLB:** $\lambda_Y = \lambda_i = 0$.



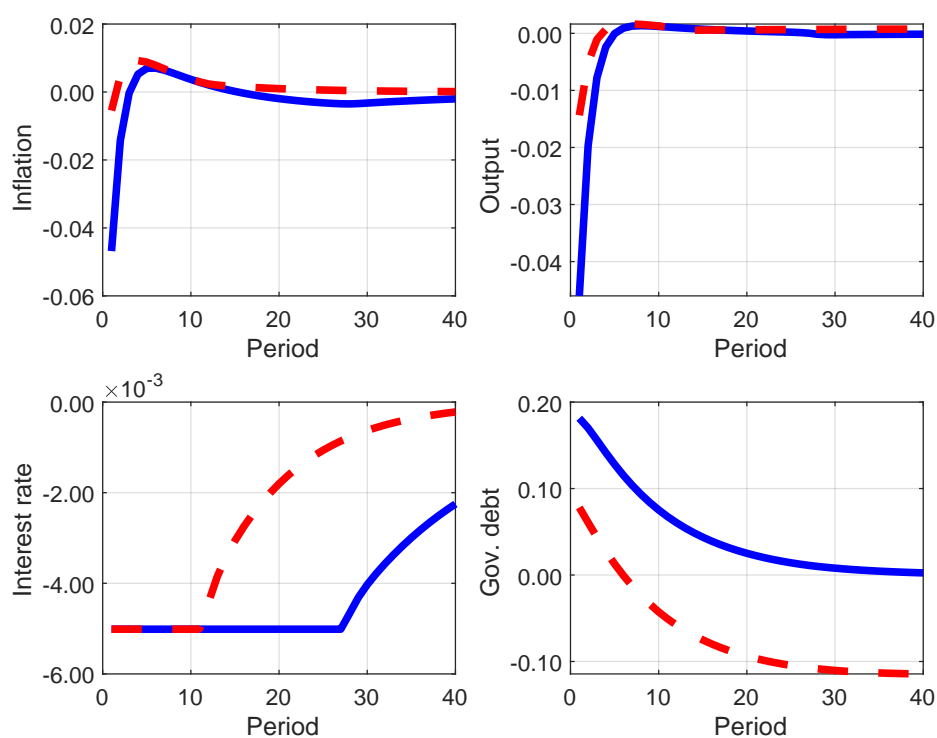
Notes: The figure plots the response of model variables to a shock in preferences (left panels), spending (middle panels) and taxes (right panels). The solid (blue) line represents the debt concerns model and the dashed (red) line the no debt concerns model. We set $\lambda_Y = \lambda_i = 0$ in the planner's objective. See Section 3.2 for details.

Figure 2: **Optimal Policies away from the ZLB:** $\lambda_Y = \lambda_i = 0.5$, $\sigma = 1$.



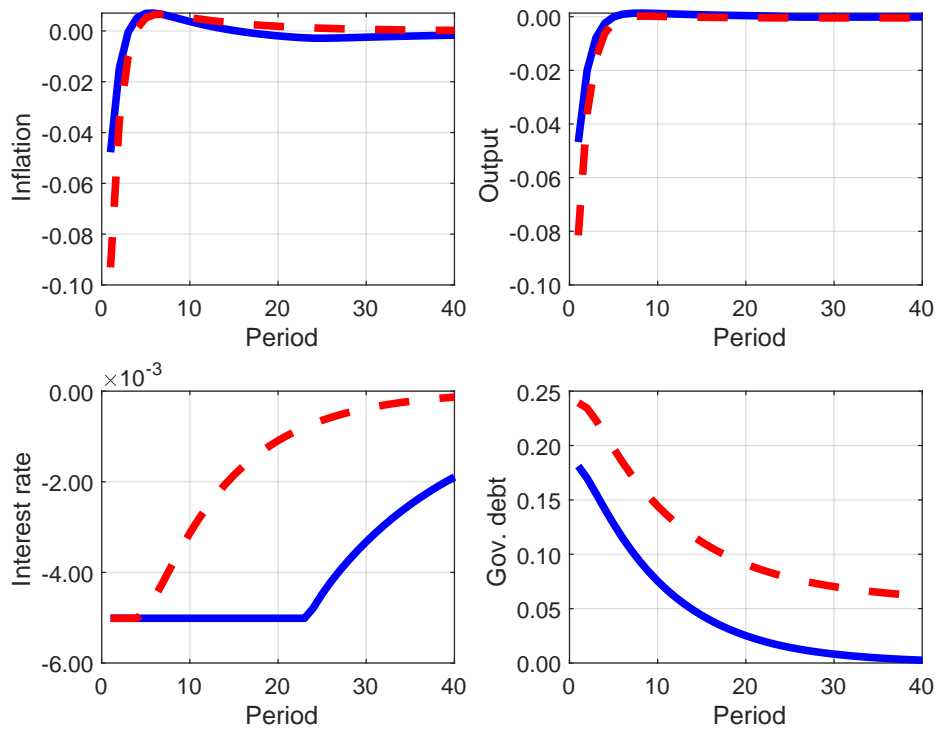
Notes: The figure plots the response of model variables to a shock in preferences (left panel), spending (middle panels) and taxes (right panels). The solid (blue) line represents the debt concerns model and the dashed (red) line the no debt concerns model. We set $\lambda_Y = \lambda_i = 0.5$ in the planner's objective. See Section 3.2 for details.

Figure 3: Optimal Policies at the ZLB.



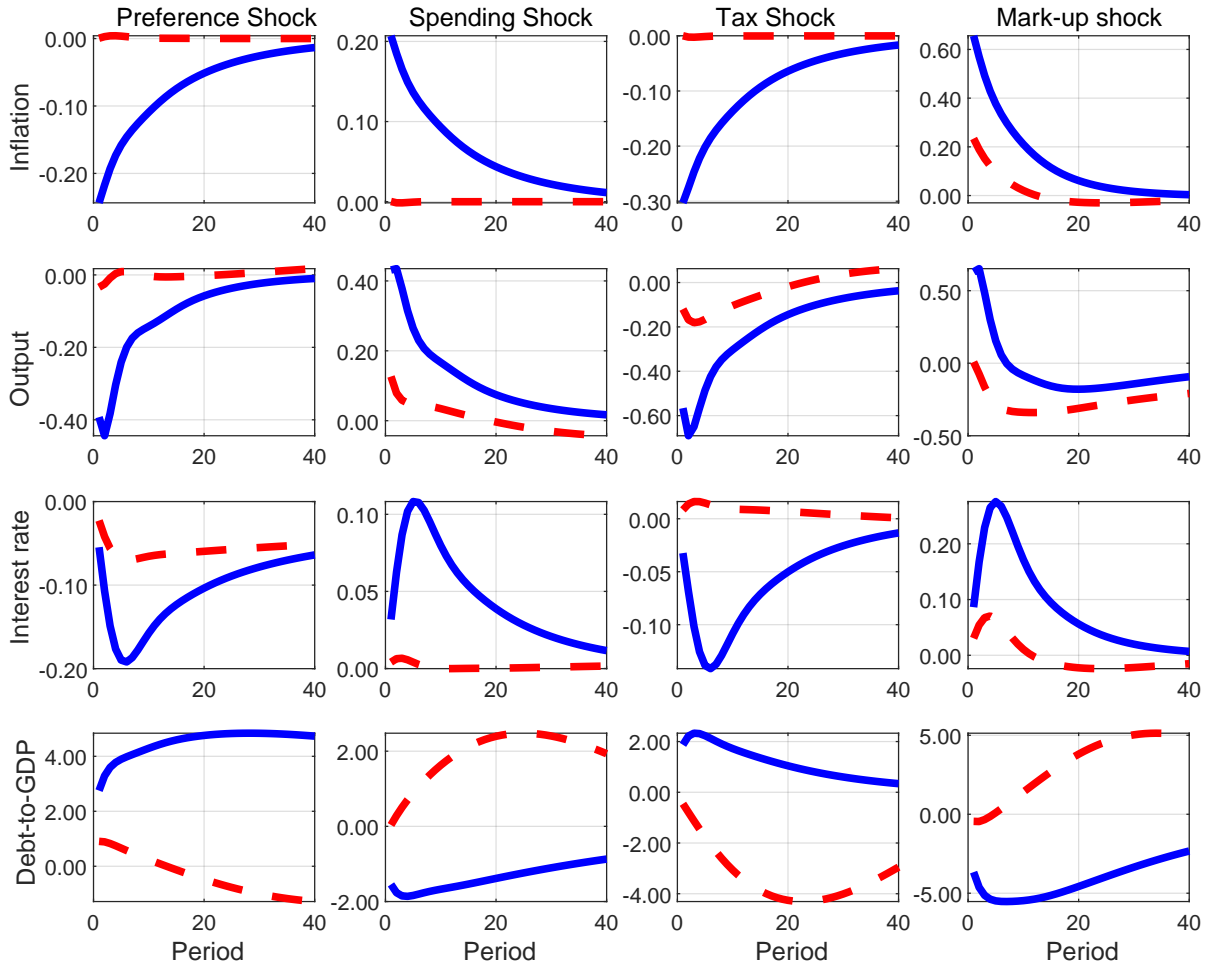
Notes: The figure plots the response of model variables to a preference shock which drives the economy to the LT. The solid (blue) line represents the debt concerns model and the dashed (red) line the no debt concerns model. We set $\lambda_Y = \lambda_i = 0.5$ in the planner's objective.

Figure 4: Responses to a LT shock when $\mathcal{Z} = 0$



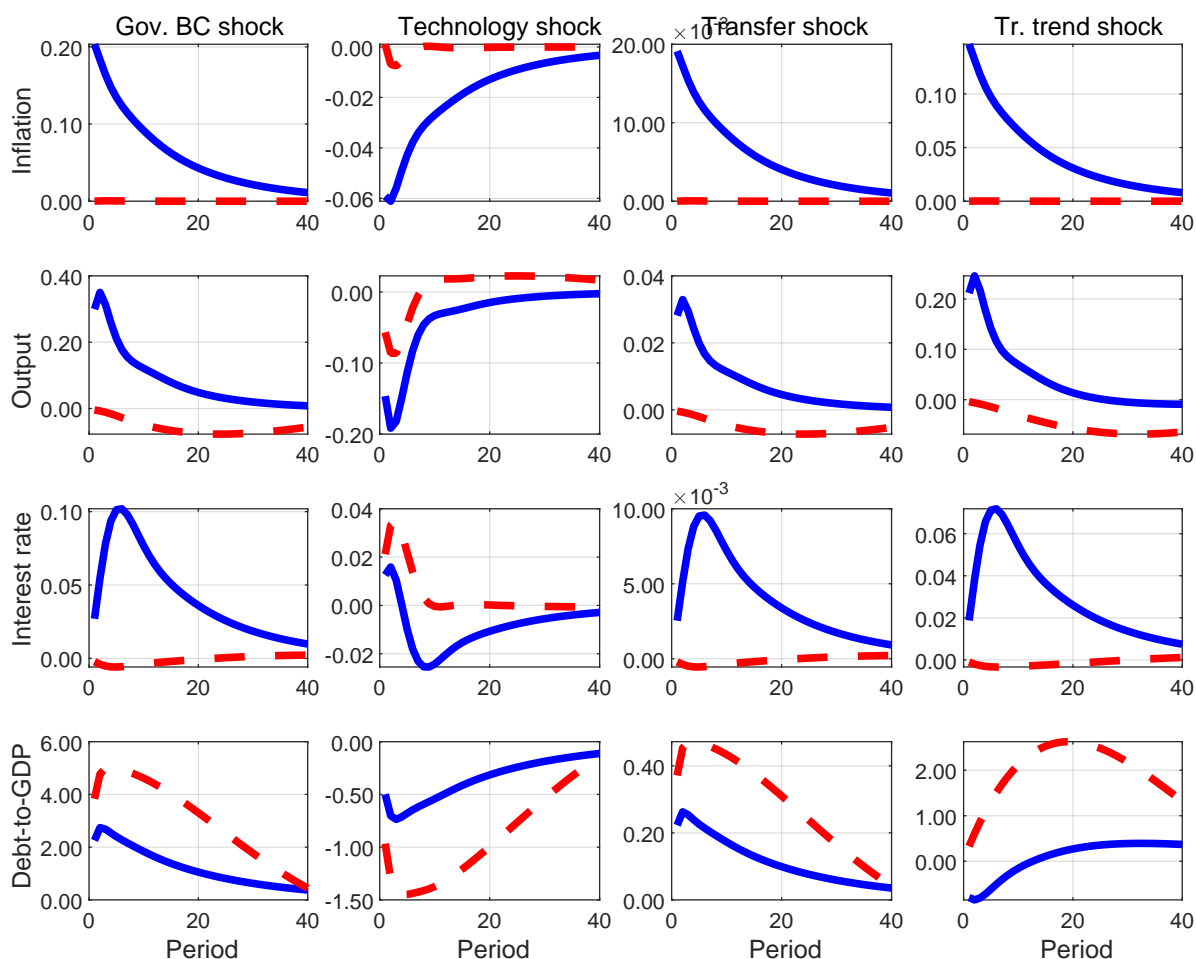
Notes: The figure plots the response of model variables to a preference shock which drives the economy to the LT and setting $\mathcal{Z} = 0$. The parameter values are the same as the ones assumed in Figure 3.

Figure 5: Responses to Shocks: Quantitative Model



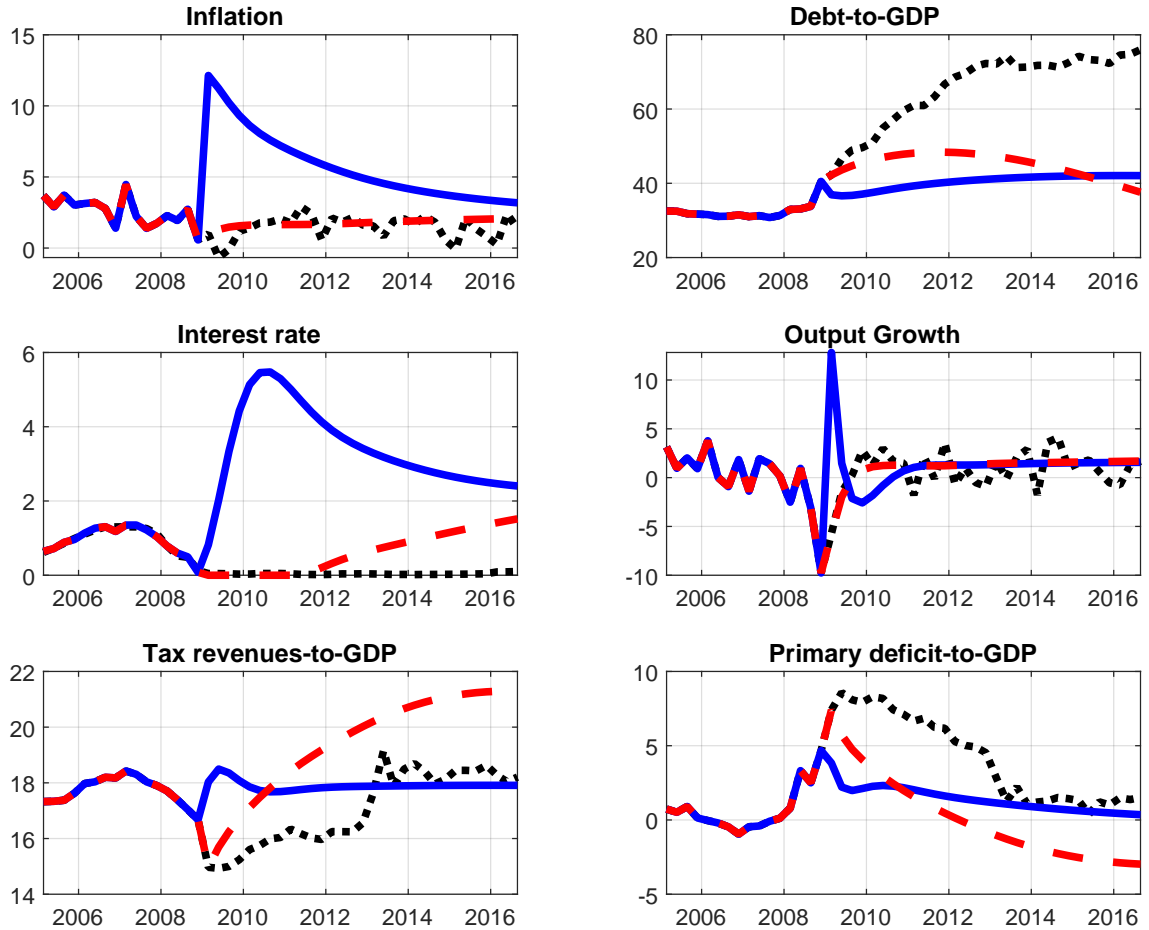
Notes: The figure plots the responses of model variables to economic shocks in the quantitative model of Section 5. To construct the impulse response functions we apply the estimates of the model reported in this Section. The size of each shock is 1 standard deviation from its posterior distribution estimate. The left panels show the responses to a preference shock, the middle left panels the responses to a spending shock, the middle right panels the responses to a tax shock, and the right panels the responses to a markup shock. The solid (blue) line represents the debt concerns model and the dashed (red) line the no debt concerns model.

Figure 6: Responses to Shocks: Quantitative Model



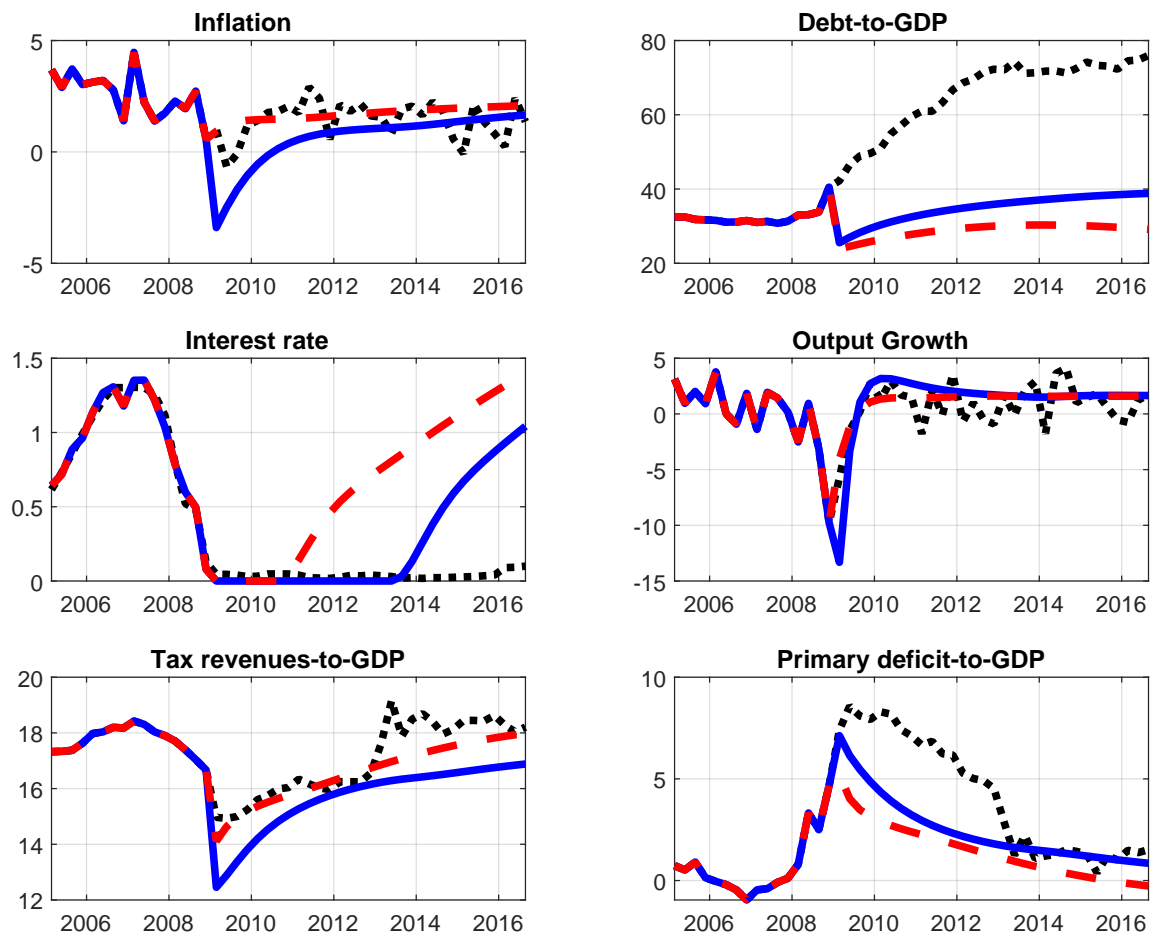
Notes: The figure plots the responses of model variables to economic shocks in the quantitative model of Section 5. To construct the impulse response functions we apply the estimates of the model reported in this Section. The size of each shock is 1 standard deviation from its posterior distribution estimate. The left panels show the responses to a shock to the government budget constraint, the middle left panels the responses to a TFP shock, the middle right panels the responses to a transfer shock, and the right panels the responses to the long-run components of transfers. The solid (blue) line represents the debt concerns model and the dashed (red) line the no debt concerns model.

Figure 7: **Baseline Forecasts in the Great Recession**



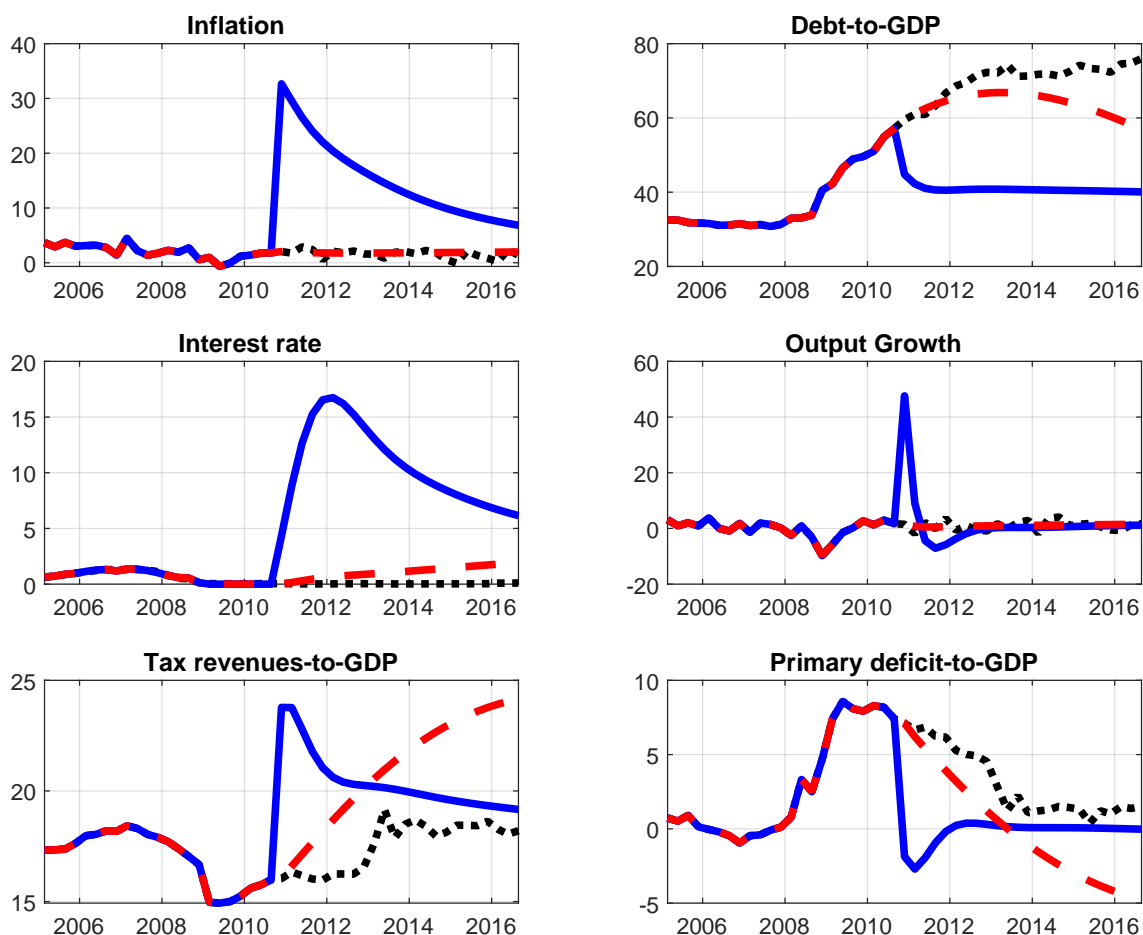
Notes: The figure plots forecasts of inflation (top left), market value of debt-to-GDP (top right), interest rates (middle left), output growth (middle right), tax revenues-to-GDP (bottom left) and the primary deficit-to-GDP ratio (bottom right) in the models. The solid (blue) line represents the model with debt concerns. The dashed (red) line is the no debt concerns model. The model series are obtained using the Kalman filter and accounting for the piecewise linear solution of the model under the ZLB. The data represented with dashed, grey lines correspond to the time series used in estimation (see Section 5).

Figure 8: Forecasts in the Great Recession: the Role of Fiscal Variables



Notes: The figure plots forecasts of inflation (top left), market value of debt-to-GDP (top right), interest rates (middle left), output growth (middle right), tax revenues-to-GDP (bottom left) and the primary deficit-to-GDP ratio (bottom right) in the models, in the counter-factual case where all fiscal variables (government debt, government spending, taxes and transfers) are set to their steady-state value at the end of period 2009Q1. The solid (blue) line represents the debt concerns model. The dashed (red) line is the no debt concerns model. The data is the same as in Figure 7.

Figure 9: Switching at Quantitative Easing 2



Notes: The figure plots forecasts of model variables when we assume that monetary policy switches to debt concerns in the 4th quarter of 2010. The construction of variables and data follows the procedure described in Figure 7.

Table 1: Model Implied Taylor Rules

| Parameter | Coefficients of Taylor Rule | | | |
|---------------------------|-----------------------------|------------------|-------------------|-------------------|
| | $\sigma = 1$ | | $\sigma = 2$ | |
| | $\lambda_i = 0.5$ | $\lambda_i = 1$ | $\lambda_i = 0.5$ | $\lambda_i = 1$ |
| $\tilde{\phi}_\pi$ | 0.104 (0.537) | 0.070 (0.268) | 0.080 (0.268) | 0.055 (0.134) |
| $\tilde{\phi}_i$ | 0.870 (1.270) | 0.905 (1.270) | 0.897 (1.135) | 0.923 (1.135) |
| $\tilde{\phi}_{\Delta i}$ | 0.509 (1.005) | 0.528 (1.005) | 0.585 (1.005) | 0.599 (1.005) |
| <hr/> | | | | |
| $\tilde{\phi}_\pi$ | 0.271 (0.537) | 0.209 (0.268) | -0.039 (0.268) | -0.024 (0.134) |
| $\tilde{\phi}_Y$ | 0.052 (0.800) | 0.003 (0.800) | 0.2305 (0.200) | 0.156 (0.200) |
| $\tilde{\phi}_i$ | 0.629 (1.270) | 0.750 (1.270) | 0.705 (1.135) | 0.765 (1.135) |
| $\tilde{\phi}_{\Delta i}$ | 0.0904 (1.005) | 0.252 (1.005) | -0.272 (1.005) | -0.232 (1.005) |

Notes: The table reports model implied Taylor rules $\hat{i}_t = \tilde{\phi}_\pi \hat{\pi}_t + \tilde{\phi}_Y \Delta \hat{Y}_t + \tilde{\phi}_i \Delta \hat{i}_{t-1} + \tilde{\phi}_{\Delta i} \Delta \hat{i}_{t-1}$ under debt concerns. (See Section 3). The top panel assumes $\lambda_Y = 0$ the bottom panel sets $\lambda_Y = 0.5$. Each of the columns reports the values of parameters ($\tilde{\phi}_\pi, \tilde{\phi}_Y, \tilde{\phi}_i, \tilde{\phi}_{\Delta i}$) under alternative calibrations of λ_i and σ . The numbers in parentheses report the values we obtain from the analytical solution of the model (object \mathcal{T}).

Table 2: Calibration

| Parameters Common Across Models | | |
|---------------------------------|----------|---|
| Parameter | Value | Label |
| β | 0.995 | Discount factor |
| λ_Y | {0, 0.5} | Loss function - weight on output |
| λ_i | {0, 0.5} | Loss function - weight on interest rate |
| θ | 17.5 | Price Stickiness |
| η | -6.88 | Elasticity of Demand |
| σ | {0, 1} | Inverse of IES |
| γ_h | 1 | Inverse of Frisch Elasticity |
| ρ_τ | 0.9 | Persistence of Taxes |
| ρ_ξ | 0.9 | Persistence of ξ |
| ρ_G | 0.9 | Persistence of Spending |
| \bar{b}_δ | 0.1321 | Debt Level |
| $\bar{\tau}$ | 0.2545 | Tax Rate |
| \bar{Y} | 1 | Output |
| \bar{G} | 0.1 | Spending |

| Parameters Not Common Across Models | | |
|-------------------------------------|--------------|-----------------------|
| Parameter | Value | Label |
| $\phi_{\tau,b}$ | 0.00 0.07 | Tax rule coefficients |

Notes: The table reports the values of model parameters assumed in the numerical experiments in Section 3. β notes the discount factor chosen to target a steady state real interest rate of 2 percent. λ_Y and λ_i are the weights on output and interest rates in the objective of the planner. Parameter η is calibrated to target markups of 17 percent in steady state. θ is calibrated as in Schmitt-Grohé and Uribe (2004). Finally, the steady state level of debt is assumed equal to 60 percent of GDP (at annual horizon), and the level of public spending is 10 percent of aggregate output which is normalized to unity in steady state.

Finally, the values $\{\lambda_Y, \lambda_i, \sigma\} = \{0, 0, 0\}$ correspond to the impulse responses shown in Figure 1. The values $\{\lambda_Y, \lambda_i, \sigma\} = \{0.5, 0.5, 1\}$ correspond to Figure 2.

The bottom panel of the Table reports the value of the coefficient $\phi_{\tau,b}$ in the tax policy rule (6). As discussed in text we set $\phi_{\tau,b} = 0.07$ in the no debt concerns model to have a determinate equilibrium. In the debt concerns case we set $\phi_{\tau,b} = 0.00$ to find a unique equilibrium. See text for further details.

Table 3: **Calibrated parameters**

| | Parameter | Value |
|--------------|-------------------------------------|-------|
| y | steady state output (normalization) | 1 |
| $1 - \alpha$ | labor share | 0.66 |
| δ | decaying rate of coupon bonds | 0.95 |
| η | demand Elasticity | -7.66 |
| γ_h | inverse of Frisch elasticity | 1 |

Notes: The table reports model parameters whose values we fix in estimation. See text for details.

Table 4: **Estimated parameters**

| | Parameter | Posterior | | distrib | Prior | |
|------------------------------------|------------------------|-----------|------------------|---------|-------|--------|
| | | mean | 90 % interval | | par A | par B |
| Quarterly trends | | | | | | |
| 100γ | growth rate | 0.397 | [0.318 ; 0.469] | G | 0.4 | 0.05 |
| $100(\beta^{-1} - 1)$ | discount rate | 0.226 | [0.089 ; 0.363] | G | 0.25 | 0.1 |
| Households and firms | | | | | | |
| $100 \log \pi$ | inflation | 0.586 | [0.517 ; 0.656] | G | 0.5 | 0.05 |
| g | g-to-GDP | 1.066 | [1.055 ; 1.077] | N | 1.06 | 0.04 |
| $b_L/4$ | debt-to-GDP | 0.241 | [0.177 ; 0.301] | N | 0.25 | 0.05 |
| tax | taxes-to-GDP | 0.044 | [0.042 ; 0.047] | N | 0.045 | 0.0025 |
| Ω | habits | 0.456 | [0.359 ; 0.554] | B | 0.7 | 0.1 |
| κ | slope NKPC | 0.017 | [0.003 ; 0.028] | G | 0.3 | 0.15 |
| Central bank preferences | | | | | | |
| λ_i | i.r smoothing | 0.167 | [0.08 ; 0.253] | G | 0.25 | 0.1 |
| λ_Y | y smoothing | 1.218 | [0.875 ; 1.557] | G | 1 | 0.2 |
| Fiscal rules | | | | | | |
| $\phi_{\tau,b}$ | τ response to b | 0.068 | [0.039 ; 0.097] | G | 0.07 | 0.02 |
| $\phi_{\tau,y}$ | τ response to y | 0.283 | [-0.033 ; 0.622] | N | 0.4 | 0.2 |
| $\phi_{\tau,g}$ | τ response to g | 0.458 | [0.037 ; 0.856] | N | 0.5 | 0.25 |
| $\phi_{tr,y}$ | tr response to y | -0.141 | [-0.356 ; 0.078] | N | -0.4 | 0.2 |
| Shocks, persistence | | | | | | |
| ρ_ξ | preference | 0.993 | [0.989 ; 0.996] | B | 0.5 | 0.2 |
| ρ_η | markup | 0.95 | [0.902 ; 0.996] | B | 0.5 | 0.2 |
| ρ_a | tfp | 0.477 | [0.4 ; 0.552] | B | 0.5 | 0.2 |
| ρ_g | gov. spending | 0.976 | [0.96 ; 0.993] | B | 0.5 | 0.2 |
| ρ_τ | tax rate | 0.948 | [0.914 ; 0.983] | B | 0.5 | 0.2 |
| ρ_{tr} | transfers | 0.224 | [0.133 ; 0.311] | B | 0.2 | 0.05 |
| ρ_{tr^*} | transfers trend | 0.95 | [0.912 ; 0.988] | B | 0.5 | 0.2 |
| ρ_λ | government b.c | 0.247 | [0.096 ; 0.389] | B | 0.5 | 0.2 |
| Shocks, standard deviations | | | | | | |
| σ_τ | tax rate | 9.365 | [6.826 ; 11.963] | IG | 10 | 2 |
| σ_g | gov. spending | 0.045 | [0.031 ; 0.058] | IG | 0.1 | 2 |
| σ_η | markup | 0.825 | [0.703 ; 0.944] | IG | 1 | 1 |
| σ_ξ | preference | 0.036 | [0.03 ; 0.041] | IG | 0.1 | 2 |
| σ_a | tfp | 0.226 | [0.202 ; 0.251] | IG | 1 | 1 |
| σ_m | int. rate, m.e | 0.484 | [0.429 ; 0.535] | IG | 2 | 2 |
| σ_λ | government b.c | 0.367 | [0.318 ; 0.413] | IG | 2 | 2 |
| σ_{tr} | transfers | 0.303 | [0.248 ; 0.356] | IG | 1 | 1 |
| σ_{tr^*} | transfers trend | 3.793 | [3.328 ; 4.228] | IG | 1 | 1 |

Notes: The table reports the prior and posterior distributions of the estimated parameters. The first column reports the mean of the posterior of each parameter, obtained from Monte-Carlo simulations of the posterior distribution using the MH algorithm. The second column reports the 90% HPD intervals obtained from the same draws. The third column indicates the assumed prior distribution (B: beta, G: gamma, IG: inverse gamma, N: normal). The fourth and fifth columns report the first and second moments of the priors.

Proof of Proposition 1.

From the first order conditions of the planner's program (equations (8) to (12)) we have:

$$\begin{aligned} \kappa_1 \Delta \psi_{\pi,t} &= -\lambda_Y \Delta \hat{Y}_t + \sigma \frac{\bar{Y}}{\bar{C}} \left(\lambda_i \Delta \hat{i}_t + \Delta \psi_{ZLB,t} - \frac{\lambda_i \Delta \hat{i}_{t-1} + \Delta \psi_{ZLB,t-1}}{\beta} \right) \\ &\quad + \sigma \frac{\bar{Y}}{\bar{C}} \bar{b}_\delta \sum_{l=0}^{\infty} \delta^l (\Delta \psi_{gov,t-l} - \Delta \psi_{gov,t-l-1}) + \omega_Y \Delta \psi_{gov,t} \end{aligned}$$

and so (8) becomes

$$-\hat{\pi}_t + \frac{1}{\kappa_1} \left[-\lambda_Y \Delta \hat{Y}_t + \sigma \frac{\bar{Y}}{\bar{C}} \left(\lambda_i \Delta \hat{i}_t + \Delta \psi_{ZLB,t} - \frac{\lambda_i \Delta \hat{i}_{t-1} + \Delta \psi_{ZLB,t-1}}{\beta} \right) \right] \quad (50)$$

$$+ \sigma \frac{\bar{Y}}{\bar{C}} \bar{b}_\delta \sum_{l=0}^{\infty} \delta^l (\Delta \psi_{gov,t-l} - \Delta \psi_{gov,t-l-1}) + \omega_Y \Delta \psi_{gov,t} \left] - \frac{\lambda_i \hat{i}_{t-1} + \psi_{ZLB,t-1}}{\beta} + \frac{\bar{b}_\delta}{1 - \beta \delta} \sum_{l=0}^{\infty} \delta^l \Delta \psi_{gov,t-l} = 0 \quad (51)$$

$$(52)$$

Rearranging (51) we get:

$$\frac{\sigma \lambda_i \bar{Y}}{\kappa_1 \bar{C}} \hat{i}_t = \hat{\pi}_t + \frac{\lambda_Y \Delta \hat{Y}_t}{\kappa_1} + \left(\frac{\sigma \lambda_i \bar{Y}}{\kappa_1 \bar{C}} + \frac{\lambda_i}{\beta} \right) \hat{i}_{t-1} + \frac{\sigma \lambda_i \bar{Y}}{\kappa_1 \beta} \Delta \hat{i}_{t-1} - \frac{\bar{Y}}{\bar{C}} \frac{\sigma}{\kappa_1} \psi_{ZLB,t} + \frac{\sigma \bar{Y}}{\kappa_1 \bar{C}} \left(1 + \frac{1}{\beta} + \frac{\bar{C}}{\bar{Y}} \frac{\kappa_1}{\beta \sigma} \right) \psi_{ZLB,t-1} \quad (53)$$

$$- \frac{\sigma \bar{Y}}{\kappa_1 \bar{C}} \bar{b}_\delta \sum_{l=0}^{\infty} \delta^l (\Delta \psi_{gov,t-l} - \Delta \psi_{gov,t-l-1}) + \frac{\omega_Y}{\kappa_1} \Delta \psi_{gov,t} + \frac{\bar{b}_\delta}{(1 - \beta \delta)} \sum_{l=0}^{\infty} \delta^l \Delta \psi_{gov,t-l} \quad (54)$$

We have two cases. 1. $\psi_{ZLB,t} > 0$ so that the ZLB binds. In this case it follows trivially that $\hat{i}_t = -i^*$. 2. $\psi_{ZLB,t} = 0$. In this case rearranging (54) we can easily get (13) in Proposition 1. ■

Proof of Proposition 2.

The proof is trivial: Following the steps in the proof of Proposition 1 and setting $\psi_{gov,t} = 0$ for all t we can arrive to the optimal policy in the no debt concerns model. ■

Derivations for the analytical model of subsections 2.5.2 and 2.6

Assuming $\lambda_Y = \lambda_i = \sigma = 0$ and focusing on the case where the ZLB does not bind, the first order conditions become:

$$\begin{aligned} -\hat{\pi}_t + \Delta \psi_{\pi,t} + \frac{\bar{b}_\delta}{1 - \beta \delta} \sum_{l=0}^{\infty} \delta^l \Delta \psi_{gov,t-l} &= 0 \\ -\psi_{\pi,t} \kappa_1 + \omega_Y \psi_{gov,t} &= 0 \end{aligned}$$

Rearranging we get

$$\hat{\pi}_t = \frac{\omega_Y}{\kappa_1} \Delta \psi_{gov,t} + \frac{\bar{b}_\delta}{1 - \beta \delta} \sum_{l=0}^{\infty} \delta^l \Delta \psi_{gov,t-l} \quad (55)$$

the optimal inflation path in this model.

Under perfect foresight (from period 0 onwards) we can drop conditional expectations and the multiplier ψ_{gov} satisfies $\psi_{gov,t} = \psi_{gov,t+1}$. Therefore, $\Delta\psi_{gov,t} = 0$ for $t \geq 1$ and $\Delta\psi_{gov,0} \neq 0$ since a shock hits in period 0. Therefore, (55) becomes:

$$\hat{\pi}_t = \frac{\omega_Y}{\kappa_1} \Delta\psi_{gov,0} \mathcal{I}_{t=0} + \frac{\bar{b}_\delta}{1 - \beta\delta} \delta^t \Delta\psi_{gov,0} \quad (56)$$

as is claimed in text. Moreover, from the Euler equation we have

$$\hat{i}_t = \hat{\pi}_{t+1} - \hat{\xi}_{t+1} + \hat{\xi}_t = \frac{\bar{b}_\delta}{1 - \beta\delta} \delta^{t+1} \Delta\psi_{gov,0} - \rho_\xi^t (\rho_\xi - 1) \hat{\xi}_0$$

From the Phillips curve we obtain the following expression for \hat{Y}_t :

$$\hat{Y}_t = \frac{1}{\kappa_1} \left[\hat{\pi}_t - \beta \hat{\pi}_{t+1} - \kappa_2 \hat{\tau}_t + \kappa_3 \hat{G}_t \right] \quad (57)$$

which, after noting that taxes dont respond to lagged debt in this version of the model and $\kappa_3 = 0$, becomes:

$$\hat{Y}_t = \frac{1}{\kappa_1} \left[\frac{\omega_Y}{\kappa_1} \Delta\psi_{gov,0} \mathcal{I}_{t=0} + \bar{b}_\delta \delta^t \Delta\psi_{gov,0} - \kappa_2 \rho_\tau^t \hat{\tau}_0 \right] \quad (58)$$

Now lets turn to the intertemporal budget constraint to find an analytical expression for $\Delta\psi_{gov,0}$ as a function of the shocks. Notice that since there is perfect certainty after period 0, the date 0 intertemporal constraint (18) is sufficient for the equilibrium (see FMOS). Given the parameterization of the model and assuming for simplicity $\hat{b}_{-1,\delta} = 0$ we have

$$\sum_{t=0}^{\infty} \beta^t \left[-\bar{G} \left(\hat{G}_t + \hat{\xi}_t \right) + \frac{\bar{\tau}\bar{Y}(1+\eta)}{\eta} \left((1 + \gamma_h) \hat{Y}_t + \frac{\hat{\tau}_t}{1 - \bar{\tau}} + \hat{\xi}_t \right) \right] = \bar{b}_\delta \sum_{t=0}^{\infty} \beta^t \delta^t E_t \left[\hat{\xi}_t - \sum_{l=0}^t \hat{\pi}_l \right] \quad (59)$$

Substituting out the expressions for output and inflation, the LHS of the above equation becomes:

$$\begin{aligned} & -\hat{G}_0 \bar{G} \sum_{t=0}^{\infty} \beta^t \rho_G^t + \hat{\tau}_0 \frac{\bar{\tau}\bar{Y}(1+\eta)}{\eta} \left(\frac{1}{1 - \bar{\tau}} - (1 + \gamma_h) \frac{\kappa_2}{\kappa_1} \right) \sum_{t=0}^{\infty} \beta^t \rho_\tau^t \\ & + \frac{\bar{\tau}\bar{Y}(1+\eta)}{\eta} \frac{1}{\kappa_1} \sum_{t=0}^{\infty} \beta^t \left[\frac{\omega_Y}{\kappa_1} \Delta\psi_{gov,0} \mathcal{I}_{t=0} + \bar{b}_\delta \delta^t \Delta\psi_{gov,0} \right] + \left(-\bar{G} + \frac{\bar{\tau}\bar{Y}(1+\eta)}{\eta} \right) \sum_{t=0}^{\infty} \beta^t \rho_\xi^t \hat{\xi}_0 = \\ & \bar{b}_\delta \sum_{t=0}^{\infty} \beta^t \delta^t \rho_\xi^t \hat{\xi}_0 - \bar{b}_\delta \sum_{t=0}^{\infty} \beta^t \delta^t \left(\frac{\omega_Y}{\kappa_1} \Delta\psi_{gov,0} \mathcal{I}_{t=0} + \sum_{l=0}^t \frac{\bar{b}_\delta}{1 - \beta\delta} \delta^l \Delta\psi_{gov,0} \right) \end{aligned}$$

Rearranging all $\Delta\psi_{gov,0}$ terms to the LHS of the above equation we get:

$$\underbrace{\left[\frac{\bar{\tau}\bar{Y}(1+\eta)}{\eta} \frac{1}{\kappa_1} \left(\frac{\omega_Y}{\kappa_1} \sum_{t=0}^{\infty} \beta^t + \bar{b}_\delta \delta^t \right) + \bar{b}_\delta \sum_{t=0}^{\infty} \beta^t \delta^t \left(\frac{\omega_Y}{\kappa_1} + \sum_{l=0}^t \frac{\bar{b}_\delta}{1 - \beta\delta} \delta^l \right) \right]}_{v_{gov}} \Delta\psi_{gov,0}$$

on the LHS where $v_{gov} > 0$. On the RHS we have:

$$\underbrace{\frac{\bar{G}}{1 - \beta\rho_G}}_{v_G} \hat{G}_0 - \underbrace{\frac{1}{1 - \beta\rho_\tau} \frac{\bar{\tau}\bar{Y}(1+\eta)}{\eta} \left(\frac{1}{1 - \bar{\tau}} - (1 + \gamma_h) \frac{\kappa_2}{\kappa_1} \right)}_{v_\tau} \hat{\tau}_0 + \underbrace{\bar{b}_\delta \frac{\beta(1 - \rho_\xi)(1 - \delta)}{(1 - \beta\rho_\xi)(1 - \beta\delta)(1 - \beta\rho_\xi\delta)}}_{v_\xi} \hat{\xi}_0$$

The coefficient v_ξ is positive and becomes zero when $\delta = 1$. Coefficient v_G is positive.

Derivations of the analytical model in Section 4.

It is straightforward to show that the FONC give:

$$-\hat{\pi}_t + \frac{\omega_Y}{\kappa_1} \Delta \psi_{gov,0} \mathcal{I}_{t=0} + \frac{\bar{b}_\delta}{1 - \beta\delta} \delta^t \Delta \psi_{gov,0} - \frac{\lambda_i \hat{i}_{t-1}}{\beta} + \frac{\psi_{ZLB,t-1}}{\beta} = 0 \quad (60)$$

in the debt concerns model and

$$-\hat{\pi}_t^{NDC} - \frac{\lambda_i \hat{i}_{t-1}^{NDC}}{\beta} + \frac{\psi_{ZLB,t-1}^{NDC}}{\beta} = 0 \quad (61)$$

in the no debt concerns model.

Focus first on the NDC equilibrium. According to (61) $\hat{\pi}_0^{NDC} = 0$ clearly. Moreover, since the shock is large we have: $\hat{i}_0^{NDC} = \hat{\pi}_1^{NDC} - \hat{\xi}_0 = -\hat{i}^*$. This gives the value of $\hat{\pi}_1^{NDC}$. From $t = 1$ onwards, lets guess (and then verify) that $\hat{i}_t^{NDC} = \hat{\pi}_{t+1}^{NDC} > -\hat{i}^*$. From (61) we have $-\hat{\pi}_{t+1}^{NDC} - \frac{\lambda_i \hat{\pi}_{t+1}^{NDC}}{\beta} = 0$ and so inflation equals zero from period 2 onwards. Clearly this gives $\hat{i}_t^{NDC} = 0$ for $t \geq 0$ consistent with the guess.

Now consider the debt concerns model. From (60) we have: $\hat{\pi}_0 = \frac{\omega_Y}{\kappa_1} \Delta \psi_{gov,0} \mathcal{I}_{t=0} + \frac{\bar{b}_\delta}{1 - \beta\delta} \Delta \psi_{gov,0}$. Moreover, from the binding ZLB $\hat{i}_0 = -\hat{i}^*$, we have $\hat{\pi}_1 = -\hat{i}^* - \hat{\xi}_0$. For $t > 1$ it must be $\hat{\pi}_t = -\hat{i}^*$ if $\hat{i}_{t-1} = -\hat{i}^*$, however,

$$-\hat{\pi}_t + \frac{\bar{b}_\delta}{1 - \beta\delta} \delta^t \Delta \psi_{gov,0} - \frac{\lambda_i \hat{\pi}_t}{\beta} = 0$$

if $\hat{i}_{t-1} > -\hat{i}^*$. Setting $\lambda_i = 0$ gives equation (41) in text. Given $\hat{i}_t = \hat{\pi}_{t+1}$ it is simple to obtain the sequence of interest rates. Finally, from $\mathcal{Z}_t = \hat{i}_t - \tilde{\phi}_\pi \hat{\pi}_t$ when $\hat{i}_{t-1} = -\hat{i}^*$ it is simple to derive the path of \mathcal{Z}_t .