Determinacy without the Taylor Principle*

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Abstract

A long-standing issue in the theory of monetary policy is that the same path for the interest rate can be associated with multiple bounded equilibrium paths for inflation and output. We show that a small friction in memory and intertemporal coordination can remove this indeterminacy. This leaves no space for equilibrium selection by means of either the Taylor Principle or the Fiscal Theory of the Price Level. It reinforces the logical foundations of the New Keynesian model's conventional solution (a.k.a. its fundamental or MSV solution). And it liberates feedback rules to serve only one function: stabilization.

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1 Introduction

Can monetary policy regulate aggregate demand? The classic by Sargent and Wallace (1975) reminds why the answer to this question is complicated: the same path for the nominal interest rate can be consistent with multiple equilibrium paths for inflation, and thereby for output too.

To resolve this problem, the New Keynesian model lets monetary policy follow a feedback rule from inflation (or output gaps) to interest rates, requires that this rule be sufficiently steep, and shows that this pins down a unique bounded equilibrium.[1] The aforementioned requirement, known as the Taylor principle (Taylor, 1993) or as "active" monetary policy (Leeper, 1991), is often described as follows: raise interest rates aggressively in response to inflationary pressures. But this description confounds equilibrium selection with stabilization. Once these functions are separated (King, 2000), it becomes clear that the Taylor principle regards only the former—and this opens the pandora box of what the "right" approach to equilibrium selection is.

Cochrane (2011) has argued that the Taylor principle amounts to an off-equilibrium threat to "blow up" the economy, in the sense of triggering an explosion in inflation and the output gap; and he has pushed for the Fiscal Theory of the Price Level (FTPL), originally articulated by Sims (1994) and Woodford (1995), as a superior alternative. But this theory, too, can be equated to a threat to blow up the economy, now in the sense of violating the government's intertemporal budget constraint. And because off-equilibrium assumptions cannot be refuted by data, the debate is "a fundamentally religious, not scientific, issue" (Kocherlakota and Phelan, 1999, p.22)

Cochrane (2005) objects to the above interpretation of the FTPL, arguing that the government's intertemporal budget constraint must be re-read as an equilibrium condition akin to the pricing of a stock; and Bassetto (2002) offers a game-theoretic foundation that, too, bypasses the blow-up criticism. On the other hand, Atkeson, Chari, and Kehoe (2010) offer a modification of the conventional approach that allows monetary policy to achieve equilibrium selection without a reliance on a blow-up threat. So the debate never ends, it only morphs in different forms.

We offer a potential way out of this conundrum. We highlight how the possibility of multiple equilibria under "passive" policy depends on strong assumptions about memory and intertemporal coordination. Once we perturb these assumptions appropriately, all bounded equilibria unravel except for one: that known as the model's fundamental or minimum-state-variable (MSV) solution (McCallum, 1983, 2009). This is the same equilibrium as that selected by the Taylor principle, except that now there is no need for it and no room for any other selection. Determinacy obtains even with interest rate pegs.

[1] In this paper we study exclusively the local determinacy issue. A separate issue, outside this paper's scope, is the global indeterminacy issue and what policies rule out "unbounded" equilibria, i.e., self-fulfilling hyperinflation (Obstfeld and Rogoff, 1983, 2021; Cochrane, 2011) and self-fulfilling liquidity traps (Benhabib et al., 2002).

Preview of results. Our first result, Proposition 2, establishes the above lesson as follows. The economy is recast as a game among overlapping generations of players. Old players are replaced by new ones at rate $\lambda \in [0,1)$. Players' actions can be measurable in the shocks realized during their life but not those before. The standard, full-information, New Keynesian model is nested with $\lambda = 0$ and is ridden with a continuum of sunspot and backward-looking equilibria. By contrast, only the fundamental/MSV solution survives for $\lambda > 0$, even arbitrarily small.

Strictly speaking, this result precludes direct observation of past outcomes such as inflation and output. But because such outcomes are functions of the past shocks, in the limit as $\lambda \to 0$ nearly all agents become nearly perfectly informed about long histories of *both* shocks and outcomes. And yet, all sunspot and backward-looking equilibria unravel.

Our second result, Proposition 5, offers a perturbation that works even when past outcomes are observed *perfectly*. To zero in on the core issue, which is intertemporal coordination, we now assume that there is a representative agent in each period, who observes perfectly past outcomes but faces some uncertainty (or memory loss) about the shocks behind it. Such uncertainty, even if minimal, causes all sunspot and backward-looking equilibria to unravel once again.

Such unravellings are emblematic of frictions in coordination (e.g., Rubinstein, 1989; Morris and Shin, 1998; Abreu and Brunnermeier, 2003) but not a universal implication of them: the details of what is or is not common knowledge can matter greatly. Still, our results illustrate the potential fragility of the "infinite chain" that supports the aforementioned equilibria.

By this chain we mean the following: current agents are doing something against their intrinsic interest only because they expect to be rewarded appropriately by future agents, who themselves must do something against their own intrinsic interest on the basis of a similar expectation about agents further into the future, and so on. Our first result breaks this chain by precluding outcomes in the far future from responding to extrinsic impulses in the present; our second breaks it by letting tomorrow's agents be uncertain about how to "reward" today's agents.

Together, these results suggest that the distinction between "active" and "passive" monetary policy may be less consequential than previously thought: determinacy is possible even with interest rate pegs. By the same token, no space is left for equilibrium selection via the FTPL: fiscal policy can matter for inflation and output *only* insofar as it enters the model's fundamental solution.[2] Last but not least, the potential conflict between macroeconomic stabilization and equilibrium selection (e.g., Galí, 2008, p.101; Loisel, 2021) is eased: feedback rules can be used solely for replication of the optimal contingencies on shocks.[3]

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[2] E.g., this could be because the monetary authority internalizes the fiscal implications of its actions, letting in effect the the debt burden enter the intercept of Taylor rule.

[3] This conflict is avoided in some works (e.g., Atkeson, Chari, and Kehoe, 2010) by assuming that the planner a priori knows the underlying shocks, or can fully separate the policy contingencies used for equilibrium selection

Related literature and additional discussion. Although Cochrane (2011, 2017, 2018) has been the most vocal advocate of the FTPL recently, this theory and the associated debate on whether equilibrium is selected by an "active" monetary policy or a "non-Ricardian" fiscal policy go back to Leeper (1991), Sims (1994) and Woodford (1995). We refer the reader to Kocherlakota and Phelan (1999) and King (2000) and for sharp explanations of, respectively, the non-Ricardian assumption and the Taylor principle; to Atkeson, Chari, and Kehoe (2010) for monetary policies that avoid blow-up threats; and to Canzoneri, Cumby, and Diba (2010) for a review of how this debate fits in the broader context of the interaction between fiscal and monetary policy.[4] What distinguishes our contribution is the attempt to remove the need for equilibrium selection of any kind, by allowing for imperfect intertemporal coordination within the private sector.

Our first result (Proposition 2), in particular, brings to mind the literature on global games (Morris and Shin, 1998, 2003). Although the application and the formal argument are different, there is a resemblance in terms of the discontinuity of equilibria to perturbations of information and the underlying role of higher-order beliefs. Our second result (Proposition 5), on the other hand, is more closely connected to Bhaskar (1998) and Bhaskar, Mailath, and Morris (2012), which show that only Markov Perfect Equilibria survive in a class of games when a purification in payoffs is combined with finite social memory. Together, our results hint at deep connections between seemingly disparate literatures, which deserve further exploration.

A large literature has shown how informational frictions can improve the positive properties of the New Keynesian model's fundamental solution,[5] but has not addressed the determinacy issue. We add to this literature by showing that a related friction can help rule out the model's other solutions. A different literature has attempted to refine the model's solutions by requiring that they are E-stable (McCallum, 2007; Christiano et al., 2018). This approach relies on specific assumptions about what it means for an equilibrium to be "learnable" and has had mixed success on the topic of interest.[6] Still, we view this approach and ours as complementary in that they both contribute towards reinforcing the logical foundations of the model's fundamental solution.

Although we commit on REE, both the indeterminacy problem and our resolution of it extend to a larger class of solution concepts, including cognitive discounting (Gabaix, 2020), diagnostic expectations (Bordalo et al., 2018), and Bayesian equilibrium with mis-specified priors about one

from those used for stabilization. But such separation may not be possible in general.

[4]Additional contributions include Buiter (2002), Canzoneri, Cumby, and Diba (2001), Christiano (2018), McCallum (2001, 2007, 2009), McCallum and Nelson (2005), and Niepelt (2004).

[5]By letting people be inattentive to fundamentals (Mankiw and Reis, 2002; Maćkowiak and Wiederholt, 2009) and/or by arresting higher-order beliefs (Woodford, 2003; Angeletos and Lian, 2018; Angeletos and Huo, 2021).

[6]For example, sunspot equilibria can be E-stable if the interest rate rule is written as a function of expected inflation (Honkapohja and Mitra, 2004). And there is a debate on how the E-stability of backward-looking solutions depends on the observability of shocks (Cochrane, 2011; Evans and McGough, 2018).

another's knowledge or rationality (Angeletos and Sastry, 2021). Relative to REE, these concepts relax the perfect coincidence of subjective beliefs and objective distributions and, in the case of Gabaix (2020), can shrink the indeterminacy region. But they do not fully resolve the problem because they preserve a fixed-point relation between beliefs and behavior.

By contrast, Level-K Thinking (García-Schmidt and Woodford, 2019; Farhi and Werning, 2019) produces a unique solution precisely because it shuts down this fixed point relation. But whenever the environment admits multiple REE, the Level-K solution becomes infinitely sensitive to the assumed Level-0 behavior as the depth of reasoning gets larger. In this sense, this concept does not "really" resolve the indeterminacy issue; it only translates one free variable (animal spirits or equilibrium selection) to another free variable (the analysts' choice of Level-0 behavior).

2 A Simplified New Keynesian Model

In this section we introduce our version of the New Keynesian model. This contains a few simplifications that ease the exposition but do not drive the results.

From three equations to one

Time is discrete and is indexed by t. Each period, a continuum of consumers—who may or may not know the history of shocks—decide how much to spend. For the main analysis, we side-step the micro-foundations and model consumer behavior in terms of the following, ad hoc, IS curve:

$$c_t = -\sigma\left(i_t - \bar{E}_t[\pi_{t+1}]\right) + \bar{E}_t[c_{t+1}] + \sigma\varrho_t, \tag{1}$$

We similarly impose the following, also ad hoc, Phillips curve:

$$\pi_t = \kappa(c_t + \xi_t), \tag{2}$$

The notation is standard: $\bar{E}_t[\cdot] \equiv \int_{i\in[0,1]} E_{i,t}[\cdot]di$ is the *average* expectation in the population as of period t; c_t, i_t, and π_{t+1} are, respectively, aggregate spending (and output) at t, the nominal interest rate between t and $t+1$, and the rate of inflation between t and $t+1$, all expressed as log-deviations from a given steady state; $\sigma > 0$ and $\kappa \geq 0$ are fixed scalars; ϱ_t is an exogenous "demand" or "discount-rate" shock; and ξ_t is an exogenous "supply" or "cost push" shock.[7]

In the basic New Keynesian model, equation (1) follows directly from as the representative agent's Euler equation, modulo the replacement of $\bar{E}_t$ with that agent's expectation. With multiple, heterogeneously-informed agents (Angeletos and Lian, 2018; Angeletos and Huo, 2021), the

[7] These shocks can, but do not have to, be independent from one other. Also note that the scale of ξ_t is normalized so that $c_t^{\text{nat}} = -\xi_t$ corresponds to the flexible-price outcome.

appropriate IS equation is more complicated: it ties c_t to the expectations of $i_\tau - \pi_{\tau+1}$ and $c_{\tau+1}$ at all $\tau \geq t+1$, not just $\tau = t+1$, capturing the intertemporal Keynesian cross. But as explained in Section 5, our main result readily extends to such a more complicated IS curve, as well as to the standard, forward-looking, New Keynesian Phillips curve in place of (2).

As in the typical textbook treatment (Galí, 2008), we next assume that monetary policy sets interest rates according to the following Taylor rule:

$$i_t = z_t + \phi \pi_t, \tag{3}$$

where ϕ is a fixed scalar, possibly zero, and z_t is random variable, possibly correlated with ϱ_t and ξ_t.[8] Whenever $\phi \neq 0$, equation (3) defines a feedback from inflation to interest rates. We will later explain how this relates to equilibrium selection under the standard paradigm—but not under our approach. Our results will indeed apply even if $\phi = 0$, which nests interest rate pegs.

Replacing equations (3) and (2) into (1), we arrive at the following equation:

$$c_t = \theta_t + \delta \bar{E}_t \left[c_{t+1} \right] \tag{4}$$

where δ is a fixed scalar and θ_t is a random variable, defined by, respectively,

$$\delta \equiv \frac{1+\kappa\sigma}{1+\phi\kappa\sigma} \qquad \text{and} \qquad \theta_t \equiv -\frac{1}{1+\phi\kappa\sigma}\left(\sigma z_t - \sigma \varrho_t + \sigma\phi\kappa\xi_t - \sigma\kappa\bar{E}_t[\xi_{t+1}]\right).$$

Clearly, solving the whole model is the same as solving (4), which is a single first-order, forward-looking difference equation in c_t alone.

In comparison, the textbook New Keynesian model maps to a system of *two* such equations in the vector (c_t, π_t). What affords the present reduction in dimensionality is the omission of a forward-looking term in the Phillips curve. But as it will become clear in Section 5, this simplification is inconsequential for our results, just as it is for the Taylor principle. All we have done here is to reduce the standard model's determinacy question from a two-dimensional eigenvalue problem to the simpler question of whether δ is within or outside the $(-1,+1)$ interval.[9]

Sticky vs flexible prices

Note that equation (4) holds even when prices are perfectly rigid, or $\kappa = 0$. What about the opposite extreme, perfectly flexible prices?

[8] Such correlation can be feasible *conditional* on π_t either because the monetary authority knows the underlying shocks directly or because it extracts information about them from economic indicators other than inflation. Whether such information is sufficient to separate the stabilization and equilibrium selection functions of monetary policy is important for the existing literature but not for our purposes.

[9] Equation (4) is mathematically identical to a "dynamic beauty contest" of the form $a_t = \theta_t + \delta \bar{E}_t[a_{t+1}]$, where a_t is some average action. Various papers (e.g., Angeletos and Huo, 2021; Morris and Shin, 2006; Nimark, 2008) have studied such games under the restriction $|\delta| < 1$, which avoids the indeterminacy issue we are concerned with here.

As long as $\kappa > 0$, we can use the Phillips curve (2) to replace c_t with $\frac{1}{\kappa}(\pi_t - \xi_t)$ and obtain the following translation of equation (4) in terms of inflation:

$$\pi_t = \tilde{\theta}_t + \delta \bar{E}_t[\pi_{t+1}], \tag{5}$$

where δ is the same as before and $\tilde{\theta}_t \equiv \frac{\kappa}{1+\phi\kappa\sigma}\left(-\sigma\left(z_t - \varrho_t\right) + \xi_t - \bar{E}_t[\xi_{t+1}]\right)$. Needless to say, this change of variables does not affect the analysis. But it helps nest flexible prices as the limit for $\kappa \to \infty$. That is, suppose that prices are flexible, which means that the Phillips curve does not apply any more and the IS curve reduces to the Fisher equation:

$$i_t = r_t^{\text{nat}} + \bar{E}_t[\pi_{t+1}],$$

where $r_t^{\text{nat}} \equiv \varrho_t + \frac{1}{\sigma}\left(\bar{E}_t[c_{t+1}^{\text{nat}}] - c_t^{\text{nat}}\right)$ and $c_t^{\text{nat}} \equiv -\xi_t$ are the natural rates of interest and output, respectively. Combining the above equation with the Taylor rule (3) yields

$$\phi\pi_t = r_t^{\text{nat}} + \bar{E}_t[\pi_{t+1}],$$

which is the same as the limit of (5) for $\kappa \to \infty$.[10]

Together with Section 5, where we take more "seriously" the micro-foundations of the IS and Phillips curves, the above discussion illustrates that the specific form of equation (4), the short-cuts behind it, and even the nominal rigidity are inessential. What is of essence is how the average expectations, $\bar{E}_t[\cdot]$, are determined—which is what we will zero in on in the next few sections.

Fundamentals, sunspots, and the equilibrium concept

Aggregate uncertainty is of two sources: fundamentals and sunspots. The former are herein conveniently summarized in θ_t.[11] We assume that this variable is a stationary, zero-mean, Gaussian process, admitting a finite-state representation.

Assumption 1 (***Fundamentals***)**.** *The fundamental θ_t admits the following representation:*

$$\theta_t = q' x_t \qquad \text{with} \qquad x_t = R x_{t-1} + \varepsilon_t^x, \tag{6}$$

where $q \in \mathbb{R}^n$ is a vector, R is an $n \times n$ matrix of which all the eigenvalues are within the unit circle (to guarantee stationarity), $\varepsilon_t^x \sim \mathcal{N}(\mathbf{0}, \Sigma_\varepsilon)$, and Σ_ε is a positive definite matrix.

This directly nests the case in which (ϱ_t, ξ_t, z_t) follows a VARMA of any finite length. It also allows x_t to contain "news shocks," or forward guidance about future monetary policy in the sense of a signal about the future value of z_t. We henceforth refer to x_t as the *fundamental state*.

[10] Note that equation (5) is well-defined for any ϕ, including $\phi = 0$, whenever $\kappa < \infty$. But the limit as $\kappa \to \infty$ is well defined only for $\phi \neq 0$. If $\phi = 0$ and prices are *perfectly* flexible, there is no equation to pin down π_t.

[11] The fact that θ_t contains an expectation term does interfere with our results. For instance, it suffices to assume that the fundamental state x_t, introduced below, is a sufficient statistic $(z_t, \varrho_t, \xi_t, \bar{E}_t[\xi_{t+1}])$ and therefore also for θ_t.

We next introduce a sunspot variable:

Assumption 2 (***Sunspots***). *The only aggregate uncertainty other than that associated with x_t is a sunspot realized in each period. This is represented by a random variable $\eta_t \sim \mathcal{N}(0,1)$, which is independent of the fundamentals and is distributed independently and identically over time.*[12]

Let h^t denote the state of nature in period t, that is, the history of all exogenous shocks up to that point. This begs the question of whether history is finite, starting at a commonly known date ("$t = 0$"), or whether it is infinite. We opt for the latter in order to focus on stationary equilibria. That is, we let $h^t \equiv \{x_{t-k}, \eta_{t-k}\}_{k=0}^{\infty}$ and we define an equilibrium as follows:

Definition 1 (**Equilibrium**). *An equilibrium is any solution to equation (4) along which: expectations are rational, although potentially based on incomplete information about h^t; the outcome is a stationary, linear function of the underlying shocks, or*

$$c_t = \sum_{k=0}^{\infty} a_k \eta_{t-k} + \sum_{k=0}^{\infty} \gamma'_k x_{t-k} \tag{7}$$

where $a_k \in \mathbb{R}$ and $\gamma_k \in \mathbb{R}^n$ are known coefficients for all k; and the outcome is bounded in the sense that $Var(c_t)$ is finite.

This definition maintains the rational expectations hypothesis but makes room for imperfect and heterogenous knowledge of h^t. This definition also embeds three additional restrictions: stationarity, linearity, and boundedness. The stationarity restriction, which comes hand-in-hand with the assumption of infinite history, can readily be relaxed. The linearity restriction is strictly needed for tractability, but we do not have any reason to believe that it drives our results and is commonplace in the literature. The last restriction, that the variance of c_t is finite, is our version of "local determinacy" or "bounded equilibria."

This restriction implies the existence of a scalar $M > 0$ such that $|a_k| \leq M$ and $\|\gamma_k\|_1 \leq M$ for all k, where $\|\cdot\|_1$ is the L^1-norm. Our upcoming result actually uses only this weaker form of boundedness. Either way, the essence is that agents do not ever expect economy to drift far away from the steady state. How policy can accomplish this in practice—e.g., by the commitments articulated in Obstfeld and Rogoff (1983, 2021) for ruling out hyperinflation or by other means—is beyond the scope of our paper. The relevant observation, instead, is this: whereas the aforementioned restriction is *not* sufficient for pinning down a unique equilibrium in the standard paradigm, it will become so under our approach.

[12] Even though η_t is i.i.d., persistent sunspot fluctuations *are* possible, because c_t may depend on the history of η_t. And as discussed at the end of Section 4, our results are robust to letting η_t itself be persistent. The absence of persistence in η_t is merely a simplification that will help make abundantly clear how the model's backward-looking solutions and sunspot equilibria are formally related—and how they all disappear with our perturbations.

3 The Standard Paradigm

In this section we review the standard, full-information, New Keynesian model, which herein maps to a representative agent that knows the entire h^t. We first identify the model's fundamental solution; we next show how its determinacy hinges on the Taylor principle; and we finally contextualize our two, complementary departures from this benchmark.

The fundamental or MSV Solution

With a representative, fully-informed, rational agent, equation (4) rewrites

$$c_t = \theta_t + \delta \mathbb{E}_t[c_{t+1}], \tag{8}$$

where $\mathbb{E}_t[\cdot] \equiv \mathbb{E}[\cdot|h_t]$ is the rational expectation conditional on full information. Because this equation is purely forward looking and x_t is a sufficient statistic for θ_t and its expected future values, it is natural to look for a solution in which c_t is a function of x_t. Thus guess $c_t = \gamma' x_t$ for some $\gamma \in \mathbb{R}^n$; use this to compute $\mathbb{E}_t[c_{t+1}] = \gamma' R x_t$; and substitute into (8) to get $c_t = \theta_t + \delta\gamma' R x_t = [q' + \delta\gamma' R]x_t$. Clearly, the guess is verified if and only if γ' solves $\gamma' = q' + \delta\gamma' R$, which in turn is possible if and only if $I - \delta R$ is invertible (where I is the $n \times n$ identity matrix) and $\gamma' = q'(I - \delta R)^{-1}$.

To guarantee the existence of this solution, we henceforth impose the following assumption:

Assumption 3. *The matrix $I - \delta R$ is invertible.*

And we write this solution as $c_t = c_t^F$, where

$$c_t^F \equiv q' (I - \delta R)^{-1} x_t. \tag{9}$$

This is known as the model's "fundamental" or "minimum-state-variable (MSV)" solution (McCallum, 1983). It is the solution customarily used to offer a structural interpretation of the data or to guide policy. But it is *not* necessarily the model's only solution.

We will address determinacy momentarily. But let us first note the following properties. Suppose that the infinite sum $\sum_{k=0}^{\infty} \delta^k R^k$ exists. Then, $(I - \delta R)^{-1} = \sum_{k=0}^{\infty} \delta^k R^k$ and

$$c_t^F = \sum_{k=0}^{\infty} \delta^k \mathbb{E}_t[\theta_{t+k}].$$

This illustrates that c_t^F can depend on the economy's history only insofar as this pins down the current θ_t or helps forecast its future values. And it verifies that c_t^F maps to what Blanchard (1979) calls the "forward-looking solution," namely the solution of iterating (8) forward.

What if $\sum_{k=0}^{\infty} \delta^k R^k$ does not exist? In this case, c_t^F remains an REE but is no more solvable by forward induction. This relates to whether the MSV solution can feature "neo-Fisherian" effects (Cochrane, 2017; García-Schmidt and Woodford, 2019), a question that is interesting but separate from that considered here. For our purposes, the relevant quality of the MSV solution is this: along it, history matters only insofar as it is part of x_t, the fundamental state variable. This contrasts with the model's other solutions, along which history serves as a correlation device.

Determinacy and the Taylor Principle

We now turn attention to the question of whether the fundamental solution is determinate and, if not, what are the other solutions. Let us first fix language:

Definition 2 (**Taylor principle**). *The Taylor principle is satisfied if* $|\delta| < 1$.

Note that $|\delta| < 1$ if and only if $\phi > 1$ or $\phi < -1 - \frac{2}{\kappa\sigma}$. Had we restricted ϕ to be positive, we would have equated the Taylor principle to $\phi > 1$, which is more standard. Our definition shines the spotlight on $|\delta|$, which measures the absolute slope of today's average best response with respect to tomorrow's average action. Intuitively, the equilibrium is unique if and only if this kind of dynamic strategic complementarity is weak enough. This intuition is verified below.

Proposition 1 (**Full-information benchmark**). *Suppose that h^t is known to every i for all t, which means in effect that there is a representative, fully informed, agent. Then:*

(i) There always exist an equilibrium, given by the fundamental/MSV solution c_t^F, as in (9).

(ii) When the Taylor principle is satisfied $(|\delta| < 1)$, the above equilibrium is the unique one.

(iii) When this principle is violated $|\delta| > 1)$, there exist a continuum of equilibria, given by

$$c_t = (1-b)c_t^F + bc_t^B + ac_t^\eta, \tag{10}$$

where $a, b \in \mathbb{R}$ are arbitrary scalars,

$$c_t^B \equiv -\sum_{k=1}^{\infty} \delta^{-k}\theta_{t-k}, \qquad \text{and} \qquad c_t^\eta \equiv \sum_{k=0}^{\infty} \delta^{-k}\eta_{t-k}. \tag{11}$$

This result contains two lessons. The first is that, in the standard, full-information benchmark, $|\delta| < 1$ is necessary and sufficient for equilibrium uniqueness. The second is a characterization of the type of equilibria that emerge in addition to the fundamental solution once $|\delta| > 1$. We already explained the first point, so let us focus on the second.

Take equation (8), backshift it by one period, and rewrite it as follows:

$$\mathbb{E}_{t-1}[c_t] = \delta^{-1}(c_{t-1} - \theta_{t-1}). \tag{12}$$

Since η_t is unpredictable at $t-1$, the above is clearly satisfied with

$$c_t = \delta^{-1}(c_{t-1} - \theta_{t-1}) + a\eta_t, \tag{13}$$

for any $a \in \mathbb{R}$. Because $|\delta| > 1$, we can iterate backwards to obtain

$$c_t = -\sum_{k=1}^{\infty} \delta^{-k}\theta_{t-k} + a\sum_{k=0}^{\infty} \delta^{-k}\eta_{t-k}, \tag{14}$$

or equivalently $c_t = c_t^B + ac_t^\eta$. Note that this is both bounded, thanks to $|\delta| > 1$, and a solution to (12), by construction, which verifies that it constitutes an equilibrium, for any $a \in \mathbb{R}$.[13]

When there are no fundamental shocks, $c_t^B = 0$ and the solution obtained above reduces to a pure sunspot equilibrium, of arbitrary aptitude a. Along it, agents respond to the current sunspot because and only because they expect future agents to keep doing the same, in perpetuity.

In the presence of fundamental shocks, the indeterminacy takes an additional, perhaps more disturbing, form: the same path for interest rates and other fundamentals can result to different paths for aggregate spending and inflation even if we switch off the sunspots. Consider, for example, the solution given by $c_t = c_t^B$. Along it, the outcome is pinned down by past fundamentals and is invariant to both the current innovation in θ_t and any news about future fundamentals—which is the exact opposite of what happens along c_t^F, the MSV solution.

The logic behind c_t^B is basically the same as that behind sunspot equilibria: agents respond to payoff-irrelevant histories only because they expect future agents to keep doing the same, in perpetuity. This statement extends to any equilibrium of the form (10) for $b \neq 0$, and explains why all such equilibria can be thought of as both non-fundamental and backward-looking.[14]

What's next: beyond the full-information benchmark

Cleary, (13) and (14) are equivalent representations of the *same* equilibrium: one is recursive, the other is sequential. This equivalence itself hinges on perfect knowledge of past shocks and/or past outcomes. But how can we understand equilibria if such information is imperfect?

In general, such a friction can preclude simple recursive representations of REE because of the familiar infinite-regress problem (Townsend, 1983). But in this paper we are not interested in this issue. Instead, we are interested in the fact that both (13) and (14) represent an infinite chain based on "think air," a self-fulfilling prophesy that must last in perpetuity. And we want to illustrate how fragile this chain can be.

To accomplish this goal, in the rest of the paper we follow two distinct but complementary strategies. The first one, in Sections 4–5, takes off from (14); the second, in Section 6, circles back to (13). Both strategies illustrate the fragility of all non-fundamental equilibria, each one from a different perspective. And they both avoid the unwanted infinite-regress problem.

[13] Part (iii) of the Proposition adds that the same is true if we replace c_t^B with any mixture of it and the MSV solution.

[14] Blanchard (1979) refers to the analogue of c_t^B in his analysis as a "backward-looking fundamental equilibrium;" but this is not *really* fundamental, in the sense we just explained.

Parenthesis: equilibrium selection vs stabilization

Before proceeding, it is useful to clarify that ϕ, or δ, has so far played a dual role: one in shaping the MSV solution, via equation (9); and another in the argument about determinacy. The former relates to macroeconomic stabilization, the latter to equilibrium selection. But note that the monetary authority can not only pick a value of ϕ but also design the process of z_t. To the extent that the optimal policy response to the underlying demand and supply shocks can be achieved via the design of z_t, this leaves ϕ free to serve only equilibrium selection.[15]

Such a sharp separation of the stabilization and equilibrium selection functions may be hard to accomplish in practice. We will return to this point in Section 7. But let us clarify the following point about the upcoming analysis. Once we perturb the model below, the MSV solution will be the unique equilibrium regardless of ϕ, indeed even if $\phi = 0$. This will not only remove the need for equilibrium selection but also guarantee that ϕ can matter only via the MSV solution.

4 Uniqueness with Fading Memory

We now depart from the standard paradigm by relaxing the assumption of full information. As mentioned in the Introduction, this brings to mind a large literature on informational frictions, but there is a crucial difference. The existing literature bypasses the equilibrium selection issue, by explicitly or implicitly imposing the Taylor principle,[16] and focuses instead on how the MSV solution is "distorted" by removing perfect knowledge of the current value of x_t. We abstract from this familiar issue, and isolate our innovation, by introducing incomplete information *solely* about the economy's payoff-irrelevant history, namely the past values of x_t and η_t.

The main assumption and the main result

For the purposes of this and the next section, we replace the assumption of a representative, fully-informed agent with the following, incomplete-information variant:

Assumption 4 (**Memory**)**.** *In each period, a randomly selected fraction $\lambda \in (0,1]$ of agents are replaced by newborn agents. Agents know the values of the fundamental state and the sunspot variable during their lifetime but not those before: the period-t information set of an agent born s periods ago is given by $I_t^s \equiv \{(x_t,\eta_t),\ldots,(x_{t-s},\eta_{t-s})\}$.*

Three elements of this assumption are worth emphasizing. The first was already anticipated: the information set of every agent in period t contains x_t, helping guarantee, as we will verify

[15] Similar arguments can be found in King (2000), Atkeson et al. (2010) and Cochrane (2011).

[16] In Angeletos and Lian (2018), e.g., the Taylor principle is imposed once the economy exits the ZLB.

shortly, that the frictionless MSV solution remains an equilibrium. If we relax this assumption, the MSV solution itself has to be modified, but this does not interfere with our main point, which is that all other solutions disappear. The second element is that an agent's information is comprised of only the exogenous shocks and not of any endogenous past outcomes. This relates to the point made in the Introduction about whether memory of sunspots can be stored in the endogenous state variables along a recursive equilibrium. We return to this point at the end of this section. The last element to note that the OLG structure serves one and only one purpose: to remove knowledge of past shocks from an increasing fraction of the population as time passes.[17]

This form of friction in "aggregate memory" is consistent with a long, rational-expectations tradition in macroeconomics, which rules out bounded recall at the individual level but allows information to be heterogeneous across agents. That said, we welcome a behavioral reinterpretation of Assumption 4 (and of Proposition 2 below), in which agents are infinitely-lived but forget the past with probability λ.[18] Under either interpretation, λ parameterizes the rate at which memory decays, or the degree of the informational friction.

In the limit as $\lambda \to 0$, the informational friction becomes vanishingly small. One may have expected the equilibria of the full-information benchmark to be only marginally affected by such a perturbation. But this is not the case: there is a discontinuity at $\lambda = 0$, reminiscent of that shown in the global games literature (Morris and Shin, 1998, 2003).

Proposition 2 (**Determinacy without the Taylor principle**). *Suppose that memory is imperfect in the sense of Assumption 4, for any $\lambda > 0$. Regardless of ϕ, or δ, the equilibrium is unique and is given by the MSV solution, that is, by $c_t = c_t^F$ where c_t^F remains the same as in* (9).

The full result is proved in the Appendix. Here, we illustrate the main idea for the special case in which there are no fundamental disturbances, so the task reduces to checking for the existence of pure sunspot equilibria. That is, we specialize our equilibrium condition to

$$c_t = \delta \bar{E}_t[c_{t+1}]; \tag{15}$$

search for solutions of the form $c_t = \sum_{k=0}^{\infty} a_k \eta_{t-k}$; and verify that $a_k = 0$ for all k.

By Assumption 4, we have that, for all $k \geq 0$,

$$\bar{E}_t[\eta_{t-k}] = \mu_k \eta_{t-k}$$

where $\mu_k \equiv (1-\lambda)^k$ measures the fraction of the population at any given date that know, or remember, a sunspot realized k periods earlier. Future sunspots, on the other hand, are known to

[17] This is unlike Del Negro et al. (2015), Farhi and Werning (2019) and Angeletos and Huo (2021), where an OLG structure is used to modify the Keynesian cross and, thereby, the MSV solution.

[18] This suggests a possible link to the literature on bounded recall (e.g., Bordalo et al., 2020; da Silveira et al., 2020; and Afrouzi et al., 2020).

nobody. It follows that, along any candidate solution, expectations satisfy

$$\bar{E}_t[c_{t+1}] = \bar{E}_t\left[a_0\eta_{t+1} + \sum_{k=1}^{\infty} a_k\eta_{t+1-k}\right] = 0 + \sum_{k=0}^{+\infty} a_{k+1}\mu_k\eta_{t-k}.$$

and condition (15) rewrites as

$$\sum_{k=0}^{+\infty} a_k\eta_{t-k} = \delta\sum_{k=0}^{+\infty} a_{k+1}\mu_k\eta_{t-k}.$$

For this to be true for all sunspot realizations, it is necessary and sufficient that, for all $k \geq 0$,

$$a_k = \mu_k\delta a_{k+1}, \tag{16}$$

or equivalently

$$a_k = \frac{a_0}{\delta^k\mu_0\mu_1...\mu_{k-1}} = \frac{a_0}{\delta^k(1-\lambda)^{\frac{k(k-1)}{2}}}. \tag{17}$$

Unless $a_0 = 0$, this explodes to infinity as $k \to \infty$, for any δ and any $\lambda \in (0,1)$. It follows that the unique bounded solution has $a_k = 0$ for all k, which corresponds to the applicable MSV solution.

We expand on the intuition behind this argument momentarily. But first, it is useful to repeat the above argument for the knife-edge case with $\lambda = 0$, which corresponds the full-information benchmark. In this case, $\mu_k = 1$ for all k and condition (17) becomes

$$a_k = \delta^{-k}a_0.$$

When $|\delta| < 1$, this still explodes to infinity as $k \to \infty$ unless $a_0 = 0$, which means that the unique bounded solution is once again $a_k = 0$ for all k. But when $|\delta| > 1$, the above remains bounded, and indeed converges to zero as $k \to \infty$, for arbitrary a_0. This recovers the previous section's sunspot equilibria: the "normalized" sunspot equilibrium $c_t^\eta = \sum_k \delta^{-k}\eta_{t-k}$ is herein nested with $a_0 = 1$, and all its rescaling is nested by letting $a_0 = a$, for arbitrary $a \neq 0$.

Note how *both* of the above arguments, the one with $\lambda > 0$ and the one with $\lambda = 0$, use the requirement that a_k does to explode to infinity. But whereas this requirement *alone* suffices for ruling out sunspot equilibria under our perturbation ($\lambda > 0$), the standard case ($\lambda = 0$) it must be complemented by the Taylor principle. This verifies the point anticipated at the end of Section 2:

Corollary 1. *Consider the assumption that there is a known steady state and there cannot exist "unbounded" deviations from it. This is insufficient for pinning down a unique equilibrium in the standard paradigm* ($\lambda = 0$)*, but becomes sufficient under our perturbation* ($\lambda > 0$).

To put it differently, policymakers should worry about "anchoring expectations" in the sense of the above assumption, but need not worry about communicating off-equilibrium threats of the type embedded in the Taylor principle.[19]

[19]By the same token, the extensive, state-contingent, escape clauses articulated in Atkeson et al. (2010) are not

Intuition and additional remarks

The following heuristic helps build additional intuition. Let $\left\{\frac{\partial c_t}{\partial \eta_0}\right\}_{t=0}^{\infty}$ stand for the Impulse Response Function (IRF) of c_t with respect to η_0. Since $\frac{\partial c_t}{\partial \eta_0} = a_t$ for all t, we can rewrite (16) as

$$\frac{\partial c_t}{\partial \eta_0} = \mu_t \delta \frac{\partial c_{t+1}}{\partial \eta_0}.$$

This is the same condition as that characterizing the IRF of c_t to η_0 in a "twin" economy in which memory is perfect but condition (4) is modified as follows:

$$c_t = \mu_t \delta \mathbb{E}_t[c_{t+1}].$$

Under this prism, it is *as if* the degree of strategic complementary between generations t and $t+1$ has been reduced from δ to $\mu_t\delta$. Furthermore, because t large enough suffices for $|\mu_t\delta| < 1$ to hold regardless of δ, or ϕ, there is necessarily a finite period T after which c_t cannot depend on η_0. By induction then, c_t cannot depend on η_0 before T either.

This interpretation of our result brings to mind Angeletos and Lian (2018) and Angeletos and Huo (2021). These papers have emphasized how higher-order uncertainty helps attenuate GE feedbacks, especially so when over longer horizons, or longer chains of reasoning. A similar logic applies here, but there are two main differences. First, whereas these papers limit attention to how this logic influences the model's MSV solution (e.g., its response to fundamental shocks or to news about future monetary policy) under the Taylor principle, here we show how a similar logic helps rule out the model's other, non-fundamental solutions even without that principle. And second, whereas these papers allow the economy-wide information about past shocks only to improve over time (due to learning), here we let it deteriorate, at least eventually.

The see why such *eventual* deterioration is the key to our result, relax Assumption 4 in the following way: suppose that the fraction of the population that is aware of a sunspot that occurred k periods ago is given by an arbitrary, possibly non-monotonic, sequence $\{\mu_k\}_{k=0}^{\infty} \in (0,1]^{\infty}$. Such a generalization allows us to capture the following three possibilities at once: not all agents are attentive to the sunspot at the beginning ($\mu_0 < 1$); more and more agents learn about it in the short run (μ_k is initially increasing in k); and more and more agents forget it in the long run (μ_k is eventually decreasing). The aforementioned papers and the broader literature on informational frictions have focused on the first two possibilities. We instead leverage the third: our uniqueness argument goes through as long as $\lim_{k\to\infty} \mu_k = 0$, regardless of what happens in the short run. That is, the key assumption is that memory vanishes asymptotically.[20]

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needed in their entirety; what is needed is only an escape clause for "unusually large" deviations.

[20] What if $\mu_k = \bar{\mu}$ for all k, which maps to the benchmark scenario of Angeletos and Lian (2018), or more generally $\lim_{k\to\infty} \mu_k = \bar{\mu}$ for some $\bar{\mu} \in (0,1)$? In this case, the equilibrium is now unique if and only if $|\bar{\mu}\delta| < 1$, which means

Also note that the offered translation of our result in terms of strategic complementarity was conditional on η_0. This means that $\mu_t \delta$ measures the strategic complementarity between periods t and $t+1$ in a very specific sense: as perceived from agents in period 0, when they contemplate whether to react to η_0. In other words, this is about higher-order beliefs—or how current agents think, or reason, about future agents.

Let us explain. Because η_0 is payoff irrelevant in every single period, period-0 agents have an incentive to respond to it if and only if they are confident that period-1 agents will also respond to it, which can be true only if they are also confident that period-1 will themselves be confident that period-2 agents will do the same, and so on, ad infinitum. It is this kind of "infinite chain" that supports sunspot equilibria when $\lambda = 0$. And conversely, the friction we have introduced here amounts to the typical period-0 agent reasoning as follows:

> "I can see η_0. And I understand that it would make sense to react to it if I were confident that all future agents will keep conditioning their behavior on it *in perpetuity*. But I worry that future agents will fail to do so, either because they will have forgotten it, or because they may themselves worry that agents further into the future will not react to it. It therefore makes sense not to react to η_0 myself."

And the same logic rules out all sunspot and backward-looking equilibria.

Three remarks help complete the picture. First, the reasoning articulated above reminds the global games literature (Morris and Shin, 1998, 2003). Although in our context there are no dominance regions of the kind assumed in that literature, the assumption that memory vanishes asymptotically plays a similar role: it triggers a chain of contagion effects from "remote types" (uninformed agents in the far future) to "nearby types" (informed agents in the immediate future) and thereby to present behavior.

Second, the aforementioned worries don't have to be "real" (objectively true). That is, we can reinterpret Assumption 4 as follows: agents don't necessarily forget themselves but believe that others will forget. Strictly speaking, this requires a departure from REE to PBE with heterogeneous and misspecified priors about one another's knowledge, along the lines of Angeletos and Sastry (2021). But the essence is similar and highlights the role of higher-order beliefs.

Finally, our result does not rest on the sunspot η_t being uncorrelated over time. In particular, Proposition 2 readily extends to an arbitrary ARMA process of the sunspot, except for one knife-edge case: when η_t follows an AR(1) process with autocorrelation *exactly* equal to δ^{-1}. In this

that the Taylor principle is relaxed but not completely removed. This reminds Gabaix (2020) and can indeed be interpreted as a complementary foundation of that paper's approach, with $\bar{\mu}$ here playing the same role as the degree of cognitive discounting in that paper. That said, even if memory of sunspots does not vanish asymptotically, we can induce uniqueness for every δ, or ϕ, via the perturbation considered in Section 6.

case, $c_t = c_t^F + a\eta_t$ is an equilibrium for any a and is supported by knowledge of (x_t, η_t) alone. Of course, such a situation is exceedingly unlikely: it requires an exogenous variable that, by pure magic, eliminates the need of any memory. But could it be that an *endogenous* state variable, such as the past outcome, c_{t-1}, replicates this magic when agents can observe this variable?

We return to this question in Section 6. But we give a preliminary answer here:

Proposition 3 (Nearly perfect knowledge of past outcomes). *Under Assumption 4, almost all agents become arbitrarily well informed about arbitrarily long histories of c_t as $\lambda \to 0$: for any mapping from h^t to c_t as in Definition 1, any $K < \infty$ arbitrarily large but finite, and any $\epsilon, \epsilon' > 0$ arbitrarily small but positive, there exists $\hat{\lambda} > 0$ such that, whenever $\lambda \in (0, \hat{\lambda})$, $Var\left(E_t^i[c_{t-k}] - c_{t-k}\right) \leq \epsilon$ for all $k \leq K$, for at least a fraction $1 - \epsilon'$ of agents, and for every period t.*

In this sense, our uniqueness result is compatible with indirect but almost perfect knowledge of past outcomes: it is *as if* agents have received arbitrarily precise signals about $\{c_{t-1}, ..., c_{t-K}\}$, and by extension for $\{\pi_{t-1}, ..., \pi_{t-K}\}$, too, for arbitrarily large K. In Section 6, we will show that uniqueness is compatible even with perfect, direct observation of past outcomes, provided that we consider a different perturbation; and we will finally speculate on which perturbations may or may not work more generally. In the next section, we first comment on how the present approach extends to a larger class of forward-looking, rational-expectations models.

5 A Generalization

Consider the following, multi-dimensional version of equation (4):

$$y_t = Qx_t + \Delta \bar{E}_t[y_{t+1}], \tag{18}$$

where y_t is an $\ell \times 1$ vector, Q is an $\ell \times n$ matrix, and Δ is an $\ell \times \ell$ matrix. In this case, the MSV solution requires inevitability of $I - \Delta R$ (this is the analogue of Assumption 3) and is given by

$$y_t^F = Q(I - \Delta R)^{-1} x_t.$$

In the full-information benchmark, this solution is the unique bounded equilibrium if and only if all eigenvalues of Δ are within the unit circle (this is the analogue of the Taylor principle). But once we replace full information with Assumption 4, this solution becomes the unique bounded solution regardless the eigenvalues of Δ.

This hints at the broader applicability of our insights. And it underscores that the omission of a forward looking term in our Phillips curve (2) was inessential on its own right. But there was a more heroic simplification behind our IS curve (1), which we now address.

If we take our OLG, incomplete-information version of the New Keynesian model "seriously," equation (1) must be replaced by the following:

$$c_t = -\beta\omega\sigma\left\{\sum_{k=0}^{+\infty}(\beta\omega)^k \bar{E}_t\left[i_{t+k}-\pi_{t+k+1}\right]\right\}+(1-\beta\omega)\left\{\sum_{k=0}^{+\infty}(\beta\omega)^k \bar{E}_t\left[c_{t+k}\right]\right\}+\sigma\varrho_t, \tag{19}$$

where $\beta \in (0,1)$ is the subjective discount factor and $\omega = 1-\lambda \in (0,1]$ is the survival probability. This equation is basically the aggregate consumption function, combined with market clearing: the first term captures the effect of the real interest rate path, the second term captures permanent income, and $1-\beta\omega$ measures the marginal propensity to consume. A detailed derivation can be found in Angeletos and Lian (2018) and Angeletos and Huo (2021).[21] For our purposes, the key observation is that equation (19) lets c_t depend on expectations of c_{t+k}, π_{t+k}, and i_{t+k-1} for all $k \geq 1$, whereas (1) had artificially restricted this dependence to $k=1$.

Suppose, next, that we replace our ad-hoc Phillips curve (2) with the standard, NKPC:

$$\pi_t = \kappa c_t + \beta\mathbb{E}_t\left[\pi_{t+1}\right]+\kappa\xi_t. \tag{20}$$

And finally, let the Taylor rule be

$$i_t = z_t + \phi_c c_t + \phi_\pi \pi_t, \tag{21}$$

for arbitrary, possibly zero or even negative, coefficients $\phi_c, \phi_\pi \in \mathbb{R}$. Solving (20) and (21) for π_t and i_t and replacing these solutions into (19), we infer that the model reduces to the following single, forward-looking equation in aggregate spending:

$$c_t = \theta_t + \bar{E}_t\left[\sum_{k=0}^{+\infty}\delta_k c_{t+k}\right] \tag{22}$$

for some random variable θ_t and some fixed scalars δ_k.[22]

Compared to equation (4), equation (22) allows for more complex GE, or strategic, feedbacks across time. There are three such feedbacks: between income and spending; between spending and inflation; and between inflation/spending and interest rates. The first feedback is parameterized by the MPC, or $\beta\omega$; the second by κ and β; and the third by ϕ_π and ϕ_c. This explains why the δ_k coefficients are convolutions of all these parameters and suggests the following reinterpretation: for appropriate such coefficients, equation (22) can nest any linear but otherwise arbitrary "intertemporal Keynesian cross" (Auclert et al., 2018). Similarly, equation (22) can nest a more flexible Taylor rule that ties interest rates not only to current inflation and output but also

[21] That derivation requires perfect recall (equivalently, a law of iterated expectations) at the individual level. This is consistent with Assumption 4, which explains why we can import equation (19) from these works to our setting.

[22] In particular, $\theta_t \equiv \sigma\varrho_t - \beta\omega\sigma(z_t + z_t^{\text{news}}) - \phi_\pi\beta\omega\sigma\kappa\xi_t + \sigma(1-\phi_\pi\beta\omega)\kappa\xi_t^{\text{news}}$, where $z_t^{\text{news}} \equiv \sum_{k=1}^{+\infty}(\beta\omega)^k \bar{E}_t[z_{t+k}]$ and $\xi_t^{\text{news}} \equiv \sum_{k=1}^{+\infty}(\beta\omega)^k \bar{E}_t[\xi_{t+k}]$; and $\delta_k \equiv (1-\beta\omega-\beta\omega\sigma\phi_c)(\beta\omega)^k + \omega\sigma\kappa\left(-\phi_\pi\beta + (1-\omega\phi_\pi\beta)\frac{1-\omega^k}{1-\omega}\right)\beta^k$, for all k.

to the monetary authority's expectations of future inflation and future output.[23]

To guarantee the existence of the MSV solution, we now require that $I - \sum_{k=0}^{+\infty} \delta_k R^k$ is invertible; this is the analogue of Assumption 3. We can then verify that Proposition 2 goes through despite the added complexity in how current outcomes depend on expectations of the future.

Proposition 4 (Generalized result)**.** *Maintain Assumption 4 but let the equilibrium condition for c_t take the richer form of equation (22), for arbitrary $\{\delta_k\}_{k=0}^{\infty}$ such that $\sum_{k=0}^{\infty} |\delta_k|$ exists (is finite). The equilibrium remains unique and is still given by the applicable MSV solution.*

The logic is the same as in our main analysis. There, $|\delta| > 1$ supported multiple equilibria when $\lambda = 0$; but as soon as $\lambda > 0$, the multiplicity vanished, because $\lambda > 0$ meant that the *effective* degree of strategic complementarity fell below 1 eventually, in the sense that $\mu_k|\delta| < 1$ for sufficiently high k. The same properties hold here modulo the replacement of $|\delta|$ with $\sum_{k=0}^{\infty} |\delta_k|$.

It is a safe guess that, similarly Proposition 2, the above result extends to a multi-variate version of equation (22). In other words, our insights readily extend to a large class of linear, purely forward-looking, rational expectations models, like that studied by Blanchard (1979) under the full-information assumption. What is left for future work is the extension to models that add payoff-relevant state variables (e.g., capital), as in Blanchard and Kahn (1980).

6 Observing Past Outcomes

We now pay closer attention to Assumption 4, which has been the catalyst for uniqueness in the preceding analysis. Literally taken, this assumption precluded direct observation of past outcomes, such as output or inflation. But as shown in Proposition 3, such a literal interpretation could be misleading: in the limit as $\lambda \to \infty$, agents were arbitrarily well informed about arbitrarily long histories of past outcomes, albeit in an indirect way.

Under this prism, the uniqueness result of Propositions 2 and 4 seems compatible with noisy observation of past outcomes. Still, the key assumption behind this result is hard to square with a long tradition in macroeconomics that represents equilibria in recursive form, along which a small set of state variables serve as sufficient statistics for the entire history of shocks. In this section, we first illustrate how the recursive logic poses a challenge for us; we next explain why this logic itself has its own limitations; and we finally show how a different perturbation can induce basically the same result even when past outcomes are observed perfectly.

[23]This detail matters for other approaches, such as E-stability (Honkapohja and Mitra, 2004), but not for ours.

Recursive equilibria: a challenge for us, and their own limitations

Go back to the full-information, representative-agent benchmark of Section 3, let $|\delta| > 1$, shut down momentarily the fundamentals, and focus on the following, pure sunspot equilibrium:

$$c_t = c_t^\eta \equiv \sum_{k=0}^{\infty} \delta^k \eta_{t-k}.$$

This can be represented in recursive form as

$$c_t = \eta_t + \delta^{-1} c_{t-1}. \tag{23}$$

It follows that perfect knowledge of yesterday's outcome can readily substitute for perfect knowledge of the infinite history of past sunspots.

This logic extends if agents observe only a noisy, public signal of c_{t-1}.[24] But what about the more general case, in which agents' information is contaminated with idiosyncratic noise, either because of decentralized market interactions or because of rational inattention? This amounts to introducing private information, which, if we extrapolate from the literature on global games, is probably the key behind our earlier uniqueness result. But the presence of such information opens a pandora box in dynamic settings: rational expectations equilibria generally cease to be recursive on a small set of state variables, due to the infinite regress problem (Townsend, 1983).

Under this prism, one can re-read Assumption 4 as the means for illustrating the potential discontinuity of recursive, non-fundamental equilibria to incomplete information, while bypassing the infinite regress problem. Still, we find it instructive to show that the MSV solution can emerge as the unique equilibrium even with *perfect* knowledge of c_{t-1}, which is what we do next.

Breaking the infinite chain, again

The recursive equilibrium described in (23) amounts to an "infinite chain" in the following sense: current agents are conditioning their behavior on c_{t-1}, which is payoff-irrelevant from their perspective, only because they expect to be "rewarded" appropriately by future agents; but for this to be the case, future agents must themselves condition their behavior on c_t, which is payoff-irrelevant for them, on the basis of a similar expectation about the behavior of agents further into the future; and so on, ad infinitum.

This chain is based on "thin air" in the sense that there is *no* agent along this chain whose behavior is ever anchored to fundamentals. This contrasts with the dynamic GE feedbacks that

[24] In particular, suppose that all agents at t observe only the current sunspot η_t and a public signal $\tilde{c}_{t,t-1} \equiv c_{t-1} + m_t$, where m_t is zero-mean measurement error, i.i.d. across time, and independent of the sunspots. Then, the strategy $c_t = a\eta_t + \delta^{-1}\tilde{c}_{t,t-1}$, which is the same as that in (23) modulo the replacement of c_{t-1} with $\tilde{c}_{t,t-1}$, continues to be an equilibrium for any a and no matter the variance of the measurement error.

operate behind the MSV solution, which, too, work via expectations of future behavior but are ultimately triggered by intrinsic forces (or PE effects).[25] What is more, once behavior is anchored to fundamentals and fundamentals themselves are not common knowledge, thin air may loose its hold on beliefs: the aforementioned infinite chain may collapse.

To illustrate what this means, bring back the fundamental shocks and consider any of the equilibria of the form $c_t^B + ac_t^\eta$, which, recall, were obtained by "solving the model backwards." These are replicated by the following recursive strategy:

$$c_t = \delta^{-1}(c_{t-1} - \theta_{t-1}) + a\eta_t. \tag{24}$$

Contrary to that in (23), this strategy requires that the agents at t know not only c_{t-1} but also θ_{t-1}. Why is such knowledge necessary? Because this what it takes for agents at t to know how to undo the direct, intrinsic effect of θ_{t-1} on the behavior of the agents at $t-1$ and, thereby, to "reward" them to play the above strategy.

This suggests that the chain can break if the agents at t do not know what exactly it takes to reward the agents at $t-1$. To make this point crisply, we proceed as follows.

First, we introduce a new fundamental disturbance, denoted by ζ_t, which can be arbitrarily small but is not observed by future agents. This allows us to maintain knowledge of θ_{t-1} (or x_{t-1}) itself, for symmetry with the earlier analysis, and at the same time parameterize the aforementioned uncertainty by the support of ζ_t. In particular, we let ζ_t be drawn independently over time, as well as independently of any other shock in the economy, from a uniform distribution with support $[-\varepsilon, +\varepsilon]$, where ε is positive but arbitrarily small.

Second, we abstract from informational heterogeneity and think of the economy as a sequence of representative agents, or a sequence of players, one for each period. This abstracts from the question of how behavior is coordinated within a period and, instead, zeros in on the aforementioned infinite chain between current and future behavior.

Accordingly, we write the equilibrium condition at t, or equivalently the best response of the period-t representative agent, as

$$c_t = \theta_t + \zeta_t + \delta\mathbb{E}[c_{t+1}|I_t]. \tag{25}$$

where $\mathbb{E}[\cdot|I_t]$ is the rational expectation conditional on I_t, the information set of the period-t representative agent.

And finally, we require that I_t contains both the concurrent fundamentals (which explains why these are outside the expectation operator in the equation above) and the past outcome c_{t-1}

[25] Think, for example, the response of the economy at t to news about z_{t+1}, monetary policy a period later. This has a GE effect today via the expectations of future income and future inflation, which are themselves driven by the direct, PE effect of tomorrow's interest rates on spending.

(in order to accommodate how perfect knowledge of the past outcome). But we also let I_t not to nest I_{t-1}, and in particular not to contain ζ_{t-1}, even though it may contain a long history of the "main" fundamental shocks and the sunspots. Formally:

Assumption 5. *For each t, there is a representative agent whose information is given by*

$$I_t = \{\zeta_t\} \cup \{x_t ..., x_{t-K_\theta}\} \cup \{\eta_t ..., \eta_{t-K_\eta}\} \cup \{c_{t-1}, \cdots, c_{t-K_c}\}$$

for finite but possibly arbitrarily large K_η, K_c, and K_θ.

When $\varepsilon = 0$ (the ζ_t shock is absent), Assumption 5 allows replication of all sunspot and backward-looking equilibria with extremely short memory, i.e., with $K_\eta = 0$ and $K_\theta = K_c = 1$. This is precisely the recursive representation of these equilibria in the standard paradigm. But there is again a discontinuity: once $\varepsilon > 0$, all the non-fundamental equilibria unravel, no matter how long the memory of outcomes and all other shocks may be.

Proposition 5. *Suppose that Assumption 5 holds and $\varepsilon > 0$. Regardless of δ, there is unique equilibrium and is given by $c_t = c_t^F + \zeta_t$, where c_t^F is the same MSV solution as before.*

To further illustrate the logic behind this result, abstract from the θ_t shock (but of course keep the ζ_t shock) and let $I_t = \{\zeta_t, \eta_t, c_{t-1}\}$. In this case, "solving the model backwards," which literally means having the agents at $t+1$ create indifference for the agents t, requires that

$$\mathbb{E}[c_{t+1}|I_t] = \delta^{-1}(-\zeta_t + c_t).$$

Since the only "news" contained in I_{t+1} relative to I_t are η_{t+1} and ζ_{t+1}, the above is true if and only if c_{t+1} satisfies

$$c_{t+1} = a\eta_{t+1} + d\zeta_{t+1} + \delta^{-1}(-\zeta_t + c_t)$$

for some $a, d \in \mathbb{R}$. As noted before, the agents at $t+1$ may extract information about ζ_t from their knowledge of c_t. But since ζ_t is not *directly* known and c_{t+1} has to be measurable in $I_{t+1} = \{\zeta_{t+1}, \eta_{t+1}, c_t\}$, the above condition can hold only if c_t itself is measurable in ζ_t and not in any other shock, such as the sunspots realized at t or earlier. In short, because the agents at $t+1$ does not know a (small) component of the "preferences" of the agents at t, it is impossible to support the aforementioned chain of indifference. The proof in the Appendix shows verifies that an extension of this logic rules out not all equilibria but the MSV solution.[26]

[26] Noe that $c_t = c_t^F + \zeta_t$ is MSV solution of the perturbed model. This differs from c_t^F, the original MSV solution, because the relevant fundamentals now include ζ_t. But as $\varepsilon \to 0$, the new solution converges to the old one. That is, the original MSV solution is robust to the considered perturbation, while all other solutions are not. And this is the same message as that delivered in our main analysis.

Remarks

The argument given above goes through even if the ζ_t shock occurs only every, say, 10 periods rather than every single period, because once there is a chance that the chain will break at some future date the whole thing unravels. Also, the argument goes through even that the agents at $t+1$ know the current ζ_t perfectly, provided that the current agents are (incorrectly) worried that this may not be the case—which once again highlights the role of higher-order beliefs.

Earlier work by Bhaskar (1998) and Bhaskar, Mailath, and Morris (2012) has shown that only Markov Perfect Equilibria (which in our context translate to the unique MSV solution) survive in a class of games when a purification in payoffs is combined with finite social memory. Even though our environment is different, Proposition 5 is a close cousin of these earlier results, not only because these papers adopt similar informational assumptions but also because they let a single, representative player act in each per period, as we did above.[27]

An open question is how Proposition 5 and, by extension, the above literature connect to our main result in Proposition 2 or 4. There, we built a bridge to the literature on global games. Under that prism, the essence of Assumption 4 was that it removed common knowledge of the payoff-irrelevant histories, thus also breaking their potency as coordination devices. The global games literature suggests that, more generally, determinacy hinges on whether information is private versus public. The results of Mailath and Morris (2002) and Pęski (2012) seem consistent with this logic, as the former relies on "almost public monitoring" to support multiple, non-Markovian equilibria and the latter goes in the opposite direction. But the exact connections are elusive.

7 Discussion

In this section, we first discuss the applied lessons of our paper. We next comment on how both the indeterminacy issue and our approach to it extend from REE to a larger class of solution concepts, as well as how Level-K Thinking fits in this picture. We finally circle back to the role of common knowledge and the precise interpretation and robustness of our results.

Interest rate pegs, feedback rules and Ramsey optimum

Suppose that the monetary authority does not even follow a feedback rule, or $\phi = 0$. Ask then the following, classic question: is there a unique equilibrium mapping from interest rate paths to inflation and output paths? The standard answer is "no." Ours is "yes."

[27] This assumption is not innocuous; see Section 5.2 of Bhaskar, Mailath, and Morris (2012). But the results in Pęski (2012) suggest that it can be relaxed.

This is the heart of our contribution, and can be translated in terms of optimal policy is follows. Let $\{i_t^o, \pi_t^o, c_t^o\}$ denote the interest rate, inflation, and output along the *best* possible bounded equilibrium, as functions of the underlying shocks. And ask the following question: what does it take for this to be implemented as the *unique* bounded equilibrium?

The textbook answer goes as follows. If the monetary authority observes the underlying shocks, then the Ramsey optimum can be implemented with the following rule:

$$i_t = i_t^o + \phi(\pi_t - \pi_t^o),$$

where ϕ is any number that satisfies the Taylor principle.[28] Note then that the feedback from π_t to i_t serves exclusively the function of equilibrium selection: it does not affect the properties of the optimum, it merely makes sure that no other equilibrium is possible.

But what if the monetary authority does not observe the underlying shocks? Feedback rules may then be useful for the purpose of replicating the optimal contingency of interest rates on shocks, or for optimal stabilization. And at least in principle, this function could be at odds with that of equilibrium selection. See Galí (2008, p.101) for an illustration with cost-push shocks, and Loisel (2021) for a general formulation.[29]

Clearly, our results help ease this conflict: because feedback rules are no more needed for equilibrium selection, they are "free" to be used for stabilization.

On the Fiscal Theory of the Price Level

So far we have abstracted from fiscal policy, because it did not enter equations (1)-(3). Continue to assume that this is the case, but take into account the government's intertemporal budget constraint. Under the simplifying assumption that the government issues only one-period, non-contingent, nominal bonds, this constraint can be written (in levels) as follows:

$$\frac{B_{t-1}}{P_t} = PVS_t, \tag{26}$$

where B_{t-1} denotes the outstanding nominal debt, P_t denotes the nominal price level, and PVS_t denotes the present discounted value of primary surpluses. Does the incorporation of this equation make a difference for the determination of inflation and output?

While the conventional approach says no by selecting the MSV solution, the FTPL argues the opposite by selecting a different solution. To illustrate, consider a shock to tax revenue that does not enter equations (1)-(3). This shock counts as a sunspot vis-a-vis the preceding analysis and

[28] This is nested in (3) with $z_t = i_t^o - \phi\pi_t^o$, which is indeed feasible as long as the monetary authority observes the shocks that drive the Ramsey optimum.

[29] We suspect that a similar conflict may emerge under the "sophisticated" implementation of Atkeson et al. (2010), or that of Bassetto (2005), once the policymaker faces sufficient uncertainty about the underlying shocks.

does not affect P_t under the MSV solution. But there is a sunspot equilibrium that features an increase in P_t and a fall of the real debt burden, whenever the aforementioned shock is high. This sunspot equilibrium provides the fiscal authority with valuable insurance. Suppose then that the fiscal authority threatens to "blow up" its budget—i.e., violate (26)—unless this equilibrium is selected. In the face of such a "non-Ricardian" fiscal policy, the standard paradigm predicts that an equilibrium cannot exist unless the monetary authority becomes "passive" (abandons the Taylor principle) and allows this equilibrium to obtain. This is the crux of the FTPL.

This discussion puts the Taylor principle and the FTPL at equal footing: they are different but seemingly equally logical assumptions about off-equilibrium policy threats. And because such assumptions are inherently untestable, one can argue that the debate reduces to a "religious" matter (Kocherlakota and Phelan, 1999).

Cochrane (2005) disagrees with the interpretation of the non-Ricardian assumption given above. He argues that (26) must be read as an equilibrium condition, akin to the pricing of a company's stock; and that such conditions are only defined on equilibrium, so the blow-up logic does not apply to the FTPL even though it applies to the Taylor principle. This translates the debate in another form: that about the exact meaning of (26).

Atkeson, Chari, and Kehoe (2010), on the other hand, consider a class of "sophisticated" monetary policies that allow the monetary authority to select its preferred equilibrium without a reliance on either the Taylor principle or any other blow-up threat. This represents an improvement over the conventional approach, but does not really address the debate with the FTPL: Bassetto (2002, 2005) effectively shows that the fiscal authority could also select a possibly very different equilibrium by engaging in similarly sophisticated strategies.

Our paper helps avoid these conundrums. Under our perturbations, the fundamental/MSV solution emerges as the only possible equilibrium regardless of whether monetary policy is "active" or "passive." By the same token, there is no space for equilibrium selection by means of a "non-Ricardian" fiscal policy; no meaningful connection between the FTPL and Ricardian equivalence (Barro, 1974); and no reason to debate whether (26) is a real constraint or merely an equilibrium condition.

But of course there is room for fiscal considerations—such as seigniorage or the real debt burden—to enter the monetary authority's choice of $\{z_t\}$ and thereby the MSV solution. In other words, this solution itself is logically consistent with the "unpleasant arithmetic" of Sargent and Wallace (1981), the evidence in Sargent (1982), and the Ramsey literature on how monetary policy can substitute for fiscal policy and/or ease tax distortions (e.g.,Chari et al., 1994; Benigno and Woodford, 2003; Correia et al., 2008).

Discounted Euler equations

Suppose we replace our IS equation (1) with the following variant:

$$c_t = -m_i i_t + m_\pi \bar{E}_t[\pi_{t+1}] + m_c \bar{E}_t[c_{t+1}] + \varrho_t, \tag{27}$$

for some positive scalars m_i, m_π, m_c. When $m_c < 1$, this nests the "discounted" Euler equations generated by liquidity constraints in McKay et al. (2017) and by cognitive discounting in Gabaix (2020). The opposite case, $m_c > 1$, is consistent with the broader HANK literature (Werning, 2015; Bilbiie, 2020), as well as with over-extrapolation or "cognitive hyperopia". Finally, $m_i \neq m_\pi$ could capture differential attention to (or salience of) nominal interest rates and inflation.

With these modifications, the entire analysis goes through modulo the following adjustment in the definition of δ :

$$\delta = \frac{m_\pi \sigma\kappa + m_c}{1 + m_i \sigma\phi\kappa}$$

The Taylor principle is still the same in the δ space, but of course changes in the ϕ space: we now have that $|\delta| < 1$ if and only if $\phi \in (-\infty, \underline{\phi}) \cup (\overline{\phi}, +\infty)$, where

$$\underline{\phi} \equiv -\frac{m_\pi}{m_i} - \frac{1+m_c}{\sigma\kappa m_i} \qquad \text{and} \qquad \overline{\phi} \equiv \frac{m_\pi}{m_i} + \frac{m_c - 1}{\sigma\kappa m_i}$$

Depending on the m's, these thresholds can be either smaller or larger than the ones in the main analysis. In this sense, the model's region of indeterminacy may either shrink or expand by the above modifications. For instance, Gabaix (2020) assumes $m_i = m_\pi$ and $m_c < 1$, obtains $\overline{\phi} < 1$, and uses this to argue that cognitive discounting relaxes the Taylor principle and, thereby, eases the potential conflict between the stabilization and equilibrium selection functions of monetary policy. From this perspective, that paper and ours are complements. But none of these enrichments changes the fact that indeterminacy remains for sufficiently "passive" monetary policy, and this is where our approach offers a potential way out.

Alternative Solution Concepts

Throughout, we have preserved Rational Expectations Equilibrium (REE), relaxing only the assumption of perfect information about the past. REE is defined by the requirement that the agents' subjective model of the economy *exactly* coincides with the true model generated by their behavior. One can capture bounded rationality by allowing a discrepancy between the former and the latter. But as long as one allows for a two-way feedback between them, the kind of indeterminacy we have studied here remains possible, and so does our resolution to it.

This circles back our earlier discussion of Gabaix (2020): the solution concept in that paper allows the objective model to feed into the subjective model, albeit with a distortion relative to

REE. The same is true for Diagnostic Expectations (Bordalo et al., 2018); for Perfect Bayesian Equilibrium with mis-specified priors (Angeletos and Sastry, 2021); and for Woodford (2019)'s model of "finite planning horizons," at least once learning is allowed (Xie, 2019). All these concepts are close cousins of REE in the sense that they preserve the two-way feedback between beliefs and outcomes, thus also preserving the indeterminacy problem we have addressed in this paper.

Contrast this class of concepts with Level-K Thinking (García-Schmidt and Woodford, 2019; Farhi and Werning, 2019). The latter pins down a unique solution by shutting down the feedback from objective truth to subjective beliefs. But this begs the question of how agents adjust their behavior over time, in the light of repeated, systematic discrepancies between what they expect to happen and what actually happens. Accordingly, we believe that Level-K Thinking is more appropriate for unprecedented experiences (e.g., the recent ZLB experience) than for the kind of stationary environments we are concerned with in this paper.

Furthermore, one may argue that Level-K Thinking does not "really" resolve the indeterminacy problem and, instead, only translates it to a different dimension: whenever $|\delta| > 1$, the level-k outcome becomes *infinitely* sensitive to the arbitrary level-0 outcome as $k \to \infty$. In this sense, one free variable (the sunspot) is replaced by another free variable (the level-0 outcome).[30] By contrast, our approach leaves neither kind of freedom in specifying beliefs.

Determinacy and common knowledge

Our results do not mean that sunspot and backward-looking equilibria are fragile to all kinds of "noise." We gave a counter-example with noisy but public observation of c_{t-1} in the previous section. Furthermore, the results of Weinstein and Yildiz (2007) underscore that, although multiple equilibria may be "degenerate" in an appropriate topology, this statement by itself can be vacuous: with enough freedom in choosing priors and information structures, one can recast equilibrium indeterminacy as strategic uncertainty along a unique equilibrium.

Under this prism, a key task for theory is to understand how a model's determinacy and its predictions more generally depend on common knowledge. The global games literature has accomplished this task quite comprehensively for static coordination games, offering, inter alia, a

[30] To clarify this point, consider what Level-K Thinking means in our setting. First, level-0 behavior is exogenously specified, by a random process $\{c_t^0\}$. Level-1 behavior is then defined as the best response to the belief that others play according to level-0 behavior, that is, $c_t^1 \equiv \theta_t + \delta \mathbb{E}_t[c_{t+1}^0]$,where $\mathbb{E}_t$ is the full-information expectation operator. This amounts to using the "wrong" beliefs about what other players do but the "correct" beliefs about the random variables θ_t and c_{t+1}^0. Iterating K times, for any finite K, gives the level-K outcome as $c_t^K \equiv \sum_{k=0}^K \delta^k \mathbb{E}_t[\theta_{t+k}] + \delta^K \mathbb{E}_t[c_{t+K}^0]$. The solution concept says that actual behavior is given by $c_t = c_t^K$ for all periods and states of nature, where both K and $\{c_t^0\}$ are free variables for the modeler to choose. Clearly, $\{c_t^K\}$ is uniquely determined for any given K and any given $\{c_t^0\}$. But because $\{c_t^0\}$ is a free variable, the original indeterminacy issue is effectively transformed to the modeler's (or the reader's) uncertainty about $\{c_t^0\}$. Furthermore, the bite of this uncertainty is most severe precisely when the indeterminacy issue is present: whenever $|\delta| > 1$, the sensitivity of $\{c_t^K\}$ to $\{c_t^0\}$ explodes to infinity as $K \to \infty$.

sharp understanding of how determinacy in such games depends on assumptions about private versus public information. An analogue for dynamic games, or dynamic macroeconomic models, is missing. But our results have illustrated the ramifications of this research agenda for the specific context of interest.

To sum up, we invite the following reading of our results: not as an unequivocal resolution of the New Keynesian model's indeterminacy issue, but rather as a sharp illustration of how fragile its sunspot and backward-looking solutions can be to appropriate relaxations of common knowledge and, therefore, as a rationale for paying less attention to, if not entirely dispensing with, these solutions.

8 Conclusion

In this paper we revisited the local indeterminacy issue of the New Keynesian model. We highlighted how all sunspot and backward-looking equilibria hinge on an infinite chain between current and future behavior. And we showed how "easy" it can be to break this chain by relaxing the model's assumptions about memory and intertemporal coordination.

To keep the analysis tractable, we followed two stark but complementary approaches: our first result (Proposition 2) allowed rich information heterogeneity within each period at the expense of abstracting from endogenous learning; our second result (Proposition 5) considered the opposite extreme. We discussed the limits of these approaches; we suggested links to the literatures on global games and the fragility of non-Markovian equilibria; and we speculated that, in general, determinacy is likely to depend on the subtler question of how much common knowledge is afforded both within and across time.

Notwithstanding the last point, our results lend support to the practice of focusing on its fundamental/MSV solution regardless of whether monetary policy is "active" or "passive." To put it differently, our results provided a rationale for why equilibrium can be determinate even with interest rate pegs—or why monetary policy may be able to regulate aggregate demand without a strict reliance on off-equilibrium threats of the kind embedded in the Taylor principle.

At the same time, our findings shined a new spotlight on the familiar boundedness assumption. This assumption is customarily justified by an implicit policy commitment to kill a self-fulling hyperinflation or any other large deviation from a given steady state. But whereas in the standard paradigm such a commitment is insufficient for pinning down a unique equilibrium in the neighborhood of any given steady state, it becomes sufficient in our context. Under this prism, the notion of "anchoring expectations" takes a new meaning: there is no need to engage in the kind of off-equilibrium threats that are needed for equilibrium selection in the standard paradigm; it suffices to anchor expectations in the sense of establishing common knowledge of a steady state and of the aforementioned commitment. How this is accomplished in practice is beyond the scope of our paper. But it is an integral part of our contribution to shift attention from the Taylor principle, or the debate with the FTPL, to exactly this question.

Let us close with a comment on the strategic interaction between the monetary and the fiscal authority. Specifying a proper game between the two authorities requires a unique mapping from those player's actions—interest rates and government deficits, respectively—to their payoffs. Such a mapping is missing in the standard paradigm, because the same paths for interest rates and government deficits can be associated with multiple equilibria within the private sector. By providing such a mapping, our paper opens a new way for studying this interaction.

Appendix: Proofs

As discussed after Definition 1, our proofs use a weaker boundedness criterion than the requirement of a finite $Var(c_t)$. The next lemma verifies that that the latter implies the former. The rest of the Appendix provides the proofs for all the results.

Lemma 1. *Consider any candidate equilibrium, defined as in Definition 1. There exist a finite scalar $M > 0$ such that $|a_k| \leq M$ and $\|\gamma_k\|_1 \leq M$ for all k, where $\|\cdot\|_1$ is the L^1-norm.*

Substituting (6) into (7), we have that any candidate equilibrium can be rewritten as

$$c_t = \sum_{k=0}^{\infty} a_k \eta_{t-k} + \sum_{k=0}^{\infty} \Gamma'_k \varepsilon^x_{t-k}, \tag{28}$$

where, for all $k \geq 0$,

$$\Gamma'_k \equiv \sum_{l=0}^{k} \gamma'_{k-l} R^l. \tag{29}$$

Since η_t and ε^x_t are independent of each other as well as independent over time, we have

$$Var(c_t) = \sum_{k=0}^{\infty} \left(a_k^2 + \Gamma'_k \Sigma_\varepsilon \Gamma_k \right).$$

This can be finite only if $\lim_{k\to+\infty} |a_k| = 0$ and $\lim_{k\to+\infty} \|\Gamma_k\|_1 = 0$.[31] From (29), $\gamma'_k = \Gamma'_k - \Gamma'_{k-1} R$ for all $k \geq 1$. It follows that $\lim_{k\to+\infty} \|\gamma_k\|_1 = 0$ as well. We conclude that there exist a scalar $M > 0$, large enough but finite, such that $|a_k| \leq M$ and $\|\gamma_k\|_1 \leq M$ for all k.

Proof of Proposition 1

Suppose that memory is perfect ($\lambda = 0$). The equilibrium condition (8) becomes

$$c_t = \theta_t + \delta \mathbb{E}_t[c_{t+1}], \tag{30}$$

where $\mathbb{E}_t[\cdot] \equiv \mathbb{E}[\cdot|h^t]$ is the full-information rational expectation operator.

Part (i) follows directly from the fact that $c^F_t \equiv q' (I - \delta R)^{-1} x_t$ satisfies (30).

Consider part (ii). Let $\{c_t\}$ be any equilibrium and define $\hat{c}_t = c_t - c^F_t$. From (30),

$$\hat{c}_t = \delta \mathbb{E}_t[\hat{c}_{t+1}]. \tag{31}$$

From Definition 1,

$$\hat{c}_t = \sum_{k=0}^{\infty} \hat{a}_k \eta_{t-k} + \sum_{k=0}^{\infty} \hat{\gamma}'_k x_{t-k},$$

[31] To prove the latter statement, note that, because Σ_ε is positive definite, there exists an invertible L such that $\Sigma_\varepsilon = L'L$ by Cholesky decomposition. The finiteness of $Var(c_t)$ then implies $\lim_{k\to+\infty} \|L\Gamma_k\|_1 = 0$, which implies $\lim_{k\to+\infty} \|\Gamma_k\|_1 = 0$.

with $|\hat{a}_k| \le \hat{M}$ and $\|\hat{\gamma}'_k\|_1 \le \hat{M}$ for all k, for some finite $\hat{M} > 0$. From Assumptions 1–2, we have

$$\mathbb{E}_t[\hat{c}_{t+1}] = \sum_{k=0}^{+\infty} \hat{a}_{k+1}\eta_{t-k} + \sum_{k=0}^{\infty} \hat{\gamma}'_{k+1} x_{t-k} + \hat{\gamma}'_0 R x_t.$$

The equilibrium condition (31) can thus be rewritten as

$$\sum_{k=0}^{\infty} \hat{a}_k \eta_{t-k} + \sum_{k=0}^{\infty} \hat{\gamma}'_k x_{t-k} = \delta\left(\sum_{k=0}^{+\infty} \hat{a}_{k+1}\eta_{t-k} + \sum_{k=0}^{\infty} \hat{\gamma}'_{k+1} x_{t-k} + \hat{\gamma}'_0 R x_t\right).$$

For this to be true for all t and all states of nature, the following restrictions on coefficients are necessary and sufficient:

$$\hat{a}_k = \delta \hat{a}_{k+1} \; \forall k \ge 0, \qquad \hat{\gamma}'_0 = \delta\hat{\gamma}'_1 + \delta\hat{\gamma}'_0 R \qquad \text{and} \qquad \hat{\gamma}'_k = \delta\hat{\gamma}'_{k+1} \; \forall k \ge 1.$$

When the Taylor principle is satisfied ($|\delta| < 1$), $\hat{a}_k$ and $\hat{\gamma}_k$ explodes unless $\hat{a}_0 = 0$ and $\hat{\gamma}'_1 = 0$. Since $I - \delta R$ is invertible from Assumption 3, $\hat{\gamma}'_0 = 0$ too. We know that the only bounded solution of (31) is $\hat{c}_t = 0$. As a result, c_t^F is the unique equilibrium.

Finally, consider part (iii). $c_t^B \equiv -\sum_{k=1}^{\infty} \delta^{-k}\theta_{t-k}$ and $c_t^\eta \equiv \sum_{k=0}^{\infty} \delta^{-k}\eta_{t-k}$ are bounded (the infinite sums converge) when the Taylor principle is violated ($|\delta| > 1$). c_t^B satisfies (30). So does $c_t = (1-b)c_t^F + bc_t^B + ac_t^\eta$ for arbitrary $b, a \in \mathbb{R}$.

Proof of Proposition 2

Since the sunspots $\{\eta_{t-k}\}_{k=0}^{\infty}$ are orthogonal to the fundamental states $\{x_{t-k}\}_{k=0}^{\infty}$, the argument in the main text proves that $a_k = 0$ for all k. We can thus focus on solutions of the following form:

$$c_t = \sum_{k=0}^{\infty} \gamma'_k x_{t-k}. \tag{32}$$

And the remaining task is to show that $\gamma'_0 = q'(I - \delta R)^{-1}$ and $\gamma'_k = 0$ for all $k \ge 1$, which is to say that only the MSV solution survives.

To start with, note that, since x_t is a stationary Gaussian vector given by (6), the following projections apply for all $k \ge s \ge 0$:

$$\mathbb{E}\left[x_{t-k} | I_t^s\right] = W_{k,s} x_{t-s},$$

where $I_t^s \equiv \{x_t, ..., x_{t-s}\}$ is the period-t information set of an agent born s periods before and

$$W_{k,s} = \mathbb{E}\left[x_{t-k} x'_{t-s}\right] \mathbb{E}\left[x_t x'_t\right]^{-1} = \mathbb{E}\left[x_t x'_t\right] \left(R'\right)^{k-s} \mathbb{E}\left[x_t x'_t\right]^{-1}$$

is an $n \times n$ matrix capturing the relevant projection coefficients.

Next, note that

$$\|W_{k,s}\|_1 \le \|\mathbb{E}\left[x_t x'_t\right]\|_1 \|\left(R'\right)^{k-s}\|_1 \|\mathbb{E}\left[x_t x'_t\right]^{-1}\|_1, \tag{33}$$

where $\|\cdot\|_1$ is the 1-norm. Since all the eigenvalues of R are within the unit circle, we know the spectral radius $\rho(R) = \rho(R') < 1$. From Gelfand's formula, we know that there exists $\bar{\Lambda} \in (0,1)$ and $M_1 > 0$ such that

$$\| (R')^{k-s} \|_1 \leq M_1 \bar{\Lambda}^{k-s},$$

for all $k \geq s \geq 0$. Together with the fact that $E[x_t x_t']$ is invertible (because Σ_ε is positive definite and $\rho(R) < 1$), we know that there exists $M_2 > 0$ such that

$$\|W_{k,s}\|_1 \leq M_2 \bar{\Lambda}^{k-s}. \tag{34}$$

Now, from Assumption 4, we know

$$\bar{E}_t[x_{t-k}] = (1-\lambda)^k x_{t-k} + \sum_{s=0}^{k-1} \lambda (1-\lambda)^s \mathbb{E}[x_{t-k} | I_t^s] \equiv \sum_{s=0}^{k} V_{k,s} x_{t-s}, \tag{35}$$

where, for all $k \geq s \geq 0$,

$$V_{k,k} = (1-\lambda)^k I_{n\times n} \quad \text{and} \quad V_{k,s} = \lambda (1-\lambda)^s W_{k,s}.$$

Together with (34), we know that there exits $M_3 > 0$ and $\Lambda = \max\{1-\lambda, \bar{\Lambda}\} \in (0,1)$ such that for all $k \geq s \geq 0$,

$$\|V_{k,s}\|_1 \leq M_3 \Lambda^k. \tag{36}$$

Now consider an equilibrium in the form of (32). From equilibrium condition (8), we know

$$\begin{aligned}
\sum_{k=0}^{+\infty} \gamma_k' x_{t-k} &= \theta_t + \delta \bar{E}_t \left[\sum_{k=0}^{+\infty} \gamma_k' x_{t+1-k} \right] \\
&= (q' + \delta \gamma_0' R + \delta \gamma_1') x_t + \delta \bar{E}_t \left[\sum_{k=1}^{+\infty} \gamma_{k+1}' x_{t-k} \right] \\
&= (q' + \delta \gamma_0' R + \delta \gamma_1') x_t + \delta \sum_{k=1}^{+\infty} \gamma_{k+1}' \left(\sum_{s=0}^{k} V_{k,s} x_{t-s} \right),
\end{aligned}$$

where we use the fact that all agents at t know the values of the fundamental state x_t.

For this to be true for all states of nature, we can compare coefficients on each x_{t-k}, we have

$$\begin{aligned}
\gamma_0' &= q' + \delta \gamma_0' R + \delta \gamma_1' \\
\gamma_k' &= \delta \sum_{l=k}^{+\infty} \gamma_{l+1}' V_{l,k} \quad \forall k \geq 1.
\end{aligned} \tag{37}$$

From Definition 1, we know that there is a scalar $M > 0$ such that $\|\gamma_k'\|_1 \leq M$ for all $k \geq 0$, where $\|\cdot\|_1$ is the 1-norm. From (36) and (37), we know that, for all $k \geq 1$,

$$\|\gamma_k'\|_1 \leq \delta \sum_{l=k}^{+\infty} \|V_{l,k}\|_1 M \leq \delta M_3 \frac{\Lambda^k}{1-\Lambda} M. \tag{38}$$

Because $\lim_{k\to\infty}\Lambda^k = 0$, there necessarily exists an $\hat{k}$ finite but large enough $\delta M_3 \frac{\Lambda^{\hat{k}}}{1-\Lambda} < 1$. So we know that, for all $k \geq \hat{k}$,

$$\|\gamma_k'\|_1 \leq \delta M_3 \frac{\Lambda^{\hat{k}}}{1-\Lambda} M.$$

Now, we can use the above formula and (37) to provide a tighter bound of $\|\gamma_k'\|_1$: for all $k \geq \hat{k}$,

$$\|\gamma_k'\|_1 \leq \left(\delta M_3 \frac{\Lambda^{\hat{k}}}{1-\Lambda}\right)^2 M.$$

We can keep iterating. For for all $k \geq \hat{k}$ and $l \geq 0$,

$$\|\gamma_k'\|_1 \leq \left(\delta M_3 \frac{\Lambda^{\hat{k}}}{1-\Lambda}\right)^l M.$$

Since $\delta M_3 \frac{\Lambda^{\hat{k}}}{1-\Lambda} < 1$, we then have $\gamma_k^{'} = 0$ for all $k \geq \hat{k}$. Using (37) and doing backward induction, we then know $\gamma_k' = 0$ for all $k \geq 1$ and

$$\gamma_0' = q' + \delta\gamma_0' R,$$

which means $\gamma_0' = q'(I - \delta R)^{-1}$. Together, this means that the equilibrium is unique and is given by $c_t = c_t^F$, where c_t^F is defined in (9).

Proof of Proposition 3

Consider a candidate equilibrium c_t in Definition 1. We first notice that, for the period-t agent born s periods ago, her information set I_t^s in Assumption 4 can be written equivalently as

$$I_t^s = \left\{\eta_{t-s}, ..., \eta_t, x_{t-s}, \varepsilon_{t-s+1}^x, \cdots, \varepsilon_t^x\right\},$$

where ε_t^x is the innovation in x_t in Assumption 1. From (28) in the proof of Lemma 1, we know that c_t can be written as

$$c_t = \sum_{k=0}^{\infty} a_k \eta_{t-k} + \sum_{k=0}^{\infty} \Gamma_k' \varepsilon_{t-k}^x,$$

where Γ_k' is given by (29). From the law of total variances, we have

$$Var\left(E_t\left[c_t | I_t^s\right] - c_t\right) \leq Var\left(\sum_{k=s+1}^{\infty} a_k \eta_{t-k} + \sum_{k=s}^{\infty} \Gamma_k' \varepsilon_{t-k}^x\right).$$

Since η_t and ε_t^x are independent of each other as well as independent over time, the finiteness of $Var(c_t)$ implies that

$$\lim_{s\to+\infty} Var\left(\sum_{k=s+1}^{\infty} a_k \eta_{t-k} + \sum_{k=s}^{\infty} \Gamma_k' \varepsilon_{t-k}^x\right) = 0.$$

As a result, for any $\epsilon > 0$ arbitrarily small but positive, there exists $\hat{s}_0$, such that

$$Var\left(E_t\left[c_t|I_t^s\right] - c_t\right) \leq \epsilon$$

for all $s \geq \hat{s}_0$ and every t. Similarly, for each $k \leq K$, there exists $\hat{s}_k$, such that

$$Var\left(E_t\left[c_{t-k}|I_t^s\right] - c_{t-k}\right) \leq \epsilon$$

for all $s \geq \hat{s}_k$ and every t. Now, for any $\epsilon' > 0$ arbitrarily small but positive, we can find $\hat{\lambda} > 0$ such that $\left(1-\hat{\lambda}\right)^{\hat{s}_k} \geq 1-\epsilon'$ for all $k \in \{0, \cdots, K\}$. Together, this means that whenever $\lambda \in (0, \hat{\lambda})$, $Var\left(E_t^i\left[c_{t-k}\right] - c_{t-k}\right) \leq \epsilon$ for all $k \leq K$, for at least a fraction $1-\epsilon'$ of agents, and for every period t.

Proof of Proposition 4

We first find the MSV solution $c_t^F = \gamma' x_t$ for some $\gamma \in \mathbb{R}^n$. From (4), we have

$$\gamma' = q' + \gamma'\left(\sum_{k=0}^{+\infty} \delta R^k\right).$$

Since $I - \sum_{k=0}^{+\infty} \delta_k R^k$ is invertible, the unique solution is $\gamma' = q'(I - \delta \sum_{k=0}^{+\infty} R^k)^{-1}$. We henceforth denote this solution as

$$c_t^F \equiv q'(I - \delta \sum_{k=0}^{+\infty} R^k)^{-1} x_t. \tag{39}$$

Consider an equilibrium taking the form of (7). We use (4):

$$\sum_{l=0}^{+\infty} a_l \eta_{t-l} + \sum_{l=0}^{\infty} \gamma_l' x_{t-l} = q' x_t + \bar{E}_t\left[\sum_{k=0}^{+\infty} \delta_k \left(\sum_{l=0}^{+\infty} a_l \eta_{t+k-l} + \sum_{l=0}^{\infty} \gamma_l' x_{t+k-l}\right)\right]. \tag{40}$$

We know

$$\bar{E}_t[\eta_{t-l}] = \begin{cases} \mu_l \eta_{t-l} & \text{if } l \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

where $\mu_l = (1-\lambda)^l$ is the measure of agents who remember a sunspot realized l periods earlier as in the proof of Proposition 2. Comparing coefficient in front of η_{t-l} and using the facts that each sunspot is orthogonal to all fundamentals:

$$a_l = \mu_l \sum_{k=0}^{+\infty} \delta_k a_{k+l} \quad \forall l \geq 0. \tag{41}$$

Because $\lim_{l\to\infty} \mu_l = 0$, there necessarily exists an $\hat{l}$ finite but large enough $\mu_{\hat{l}} \sum_{k=0}^{\infty} |\delta_k| < 1$.

Since we are focusing bounded equilibria as in Definition 1, there exists a scalar $M > 0$, arbitrarily large but finite, such that $|a_l| \leq M$ for all l. From (41), we then know that, for all $l \geq \hat{l}$,

$$|a_l| \leq \mu_{\hat{l}} M \sum_{k=0}^{+\infty} |\delta_k|, \tag{42}$$

where we also use the fact that the sequence $\{\mu_l\}_{l=0}^{\infty}$ is decreasing. Now, we can use (41) and (42) to provide a tigehter bound of $|a_l|$. That is, for all $l \geq \hat{l}$,

$$|a_l| \leq \left(\mu_{\hat{l}} \sum_{k=0}^{\infty} |\delta_k|\right)^2 M.$$

We can keep iterating. Since $\mu_{\hat{l}} \sum_{k=0}^{\infty} |\delta_k| < 1$, we then have $a_l = 0$ for all $l \geq \hat{l}$. Using (41) and doing backward induction, we then know $a_l = 0$ for all l.

Now, (40) can be simplified as

$$\begin{aligned} \sum_{l=0}^{\infty} \gamma_l' x_{t-l} &= q' x_t + \bar{E}_t \left[\sum_{k=0}^{+\infty} \delta_k \sum_{l=0}^{\infty} \gamma_l' x_{t+k-l} \right]. \\ &= q' x_t + \sum_{k=0}^{+\infty} \delta_k \sum_{l=0}^{k} \gamma_l' R^{k-l} x_t + \bar{E}_t \left[\sum_{l=1}^{+\infty} \left(\sum_{k=0}^{+\infty} \delta_k \beta_{k+l}' \right) x_{t-l} \right]. \end{aligned} \tag{43}$$

For this to be true for all states of nature, we can compare coefficients on each x_{t-l}:

$$\gamma_0' = q' + \sum_{k=0}^{+\infty} \delta_k \sum_{l=0}^{k} \gamma_l' R^{k-l} \tag{44}$$

$$\gamma_l' = \sum_{s=l}^{+\infty} \left(\sum_{k=0}^{+\infty} \delta_k \gamma_{k+s}' \right) V_{s,l} \quad \forall l \geq 1, \tag{45}$$

where $V_{s,l}$ is defined in (35).

From Definition 1, we know that there is a scalar $M > 0$ such that $\|\gamma_l'\|_1 \leq M$ for all $l \geq 0$, where $\|\cdot\|_1$ is the 1-norm. From (45), we know, for all $l \geq 1$

$$\|\gamma_l'\|_1 \leq \left(\sum_{k=0}^{+\infty} |\delta_k| \right) \left(\sum_{s=l}^{+\infty} \|V_{s,l}\|_1 \right) M \leq \left(\sum_{k=0}^{+\infty} |\delta_k| \right) M_3 \frac{\Lambda^l}{1-\Lambda} M, \tag{46}$$

where M_3 and Λ are defined in (35) Because $\lim_{l \to \infty} \Lambda^l = 0$, there necessarily exists an $\hat{l}$ finite but large enough such that $\left(\sum_{k=0}^{+\infty} |\delta_k|\right) M_3 \frac{\Lambda^{\hat{l}}}{1-\Lambda} < 1$. So we know that, for all $l \geq \hat{l}$,

$$\|\gamma_l'\|_1 \leq \left(\sum_{k=0}^{+\infty} |\delta_k| \right) M_3 \frac{\Lambda^{\hat{l}}}{1-\Lambda} M.$$

Now, we can use the above formula and (45) to provide a tighter bound of $\|\gamma_l'\|_1$: for all $l \geq \hat{l}$,

$$\|\gamma_l'\|_1 \leq \left(\left(\sum_{k=0}^{+\infty} |\delta_k| \right) M_3 \frac{\Lambda^{\hat{l}}}{1-\Lambda} \right)^2 M.$$

We can keep iterating. Since $\left(\sum_{k=0}^{+\infty} |\delta_k|\right) M_3 \frac{\Lambda^{\hat{l}}}{1-\Lambda} < 1$, we then have $\gamma_l' = 0$ for all $l \geq \hat{l}$. Using (45) and doing backward induction, we then know $\gamma_l' = 0$ for all $l \geq 1$ and, from (44),

$$\gamma_0' = q' + \gamma_0' \left(\sum_{k=0}^{+\infty} \delta_k R^k \right),$$

which means $\gamma_0' = q'\left(I - \delta\sum_{k=0}^{+\infty}\delta_k R^k\right)^{-1}$. Together, this means that the equilibrium is unique and is given by $c_t = c_t^F$, where c_t^F is defined in (39). This proves the Proposition.

Proof of Proposition 5

Given Assumption 5, an possible equilibrium takes the form of

$$c_t = \sum_{k=0}^{K_\eta} a_k \eta_{t-k} + \sum_{k=1}^{K_\beta} \beta_k c_{t-k} + \sum_{k=0}^{K_\theta} \gamma_k' x_{t-k} + \chi\zeta_t.$$

From (25), we have that

$$\begin{aligned}
\sum_{k=0}^{K_\eta} a_k \eta_{t-k} + \sum_{k=1}^{K_\beta} \beta_k c_{t-k} + \sum_{k=0}^{K_\theta} \gamma_k' x_{t-k} + \chi\zeta_t &= \theta_t + \zeta_t + \delta\mathbb{E}[\sum_{k=0}^{K_\eta-1} a_{k+1}\eta_{t-k} + \sum_{k=0}^{K_{\beta-1}} \beta_{k+1} c_{t-k} + \sum_{k=0}^{K_\theta-1} \gamma_{k+1}' x_{t-k} | I_t] \\
&= q' x_t + \zeta_t + \delta\left[\sum_{k=0}^{K_\eta-1} a_{k+1}\eta_{t-k} + \sum_{k=1}^{K_{\beta-1}} \beta_{k+1} c_{t-k} + \sum_{k=0}^{K_\theta-1} \gamma_{k+1}' x_{t-k} + \gamma_0' R x_t\right] \\
&\quad + \delta\beta_1\left[\sum_{k=0}^{K_\eta} a_k \eta_{t-k} + \sum_{k=1}^{K_\beta} \beta_k c_{t-k} + \sum_{k=0}^{K_\theta} \gamma_k' x_{t-k} + \chi\zeta_t\right]
\end{aligned}$$

where we use Assumptions 1–2 and the fact that ζ_t is drawn independently over time. For this to be true for all states of nature, we can compare coefficients:

$$a_k = \delta a_{k+1} + \delta\beta_1 a_k \quad \forall k \in \{0, \cdots, K_\eta - 1\} \quad \text{and} \quad a_{K_\eta} = \delta\beta_1 a_{K_\eta} \tag{47}$$

$$\beta_k = \delta\beta_{k+1} + \delta\beta_1\beta_k \quad \forall k \in \{1, \cdots, K_\beta - 1\} \quad \text{and} \quad \beta_{K_\beta} = \delta\beta_1\beta_{K_\beta} \tag{48}$$

$$\gamma_k' = \delta\gamma_{k+1}' + \delta\beta_1\gamma_k' \quad \forall k \in \{1, \cdots, K_\theta - 1\} \quad \text{and} \quad \gamma_{K_\theta}' = \delta\beta_1\gamma_{K_\theta}' \tag{49}$$

$$\gamma_0' = q' + \delta\gamma_1' + \delta\beta_1\gamma_0' + \gamma_0' R \quad \text{and} \quad \chi = 1 + \delta\beta_1\chi. \tag{50}$$

First, from the second equation in (50), we know $\delta\beta_1 \neq 1$. Then, from the second parts of (47)–(49), we know $a_{K_\eta} = 0$, $\beta_{K_\beta} = 0$, and $\gamma_{K_\theta}' = 0$. From backward induction on (47)–(50), we know that all a, b, γ are zero except for the following:

$$\gamma_0' = q' + \gamma_0' R,$$

which means $\gamma_0' = q'(I - R)^{-1}$. We also know that $\chi = 1$. We conclude that the unique solution is

$$c_t = c_t^F + \zeta_t,$$

where c_t^F is given by (9).

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