Vertical Integration and Bargaining: Linear vs Two-part tariffs

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Abstract

We examine the implications of different contractual forms for welfare as well as for firms' profits in a framework in which a vertically integrated firm sells its good to an independent downstream firm. Under downstream Bertrand competition, the standard result of the desirability of two-part tariffs over linear contracts in terms of welfare may be reversed. We obtain that the linear contract can generate higher consumer surplus and welfare than the two-part tariff when the independent downstream firm is rather powerful in determining the contract terms. In that case, the fixed fee is negative and the integrated firm makes more profits under a linear contract than under a two-part tariff. These results do not remain robust under downstream Cournot competition. Irrespective of the mode of downstream competition, the preferred contract type of the integrated firm is always welfare superior.

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1. Introduction

An important feature of vertical trading, that is, trading among vertically related (upstream and downstream) firms,[1] is the contract type employed. In reality, vertical contracts range from simple linear to more complex non-linear tariffs. The vertical contractual form is a crucial determinant of both market outcomes and market efficiency. The literature on vertical contracting has investigated the desirability of different contractual forms from the viewpoint of consumers and society, as well as from the viewpoint of involved firms, focusing on vertically separated markets. That is, markets where upstream firms reach final consumers only through independent downstream firms. In this paper, we examine the desirability of different contractual forms by introducing vertical integration in the vertical chain.

In many industries, vertically integrated firms, that is, firms that are present in both the upstream and the downstream stage(s) of the production chain, exist. These firms, besides selling directly to consumers, also sell their products to independent downstream firms. For instance, the existence of dual channels of distribution is nowadays a widespread phenomenon: manufacturers sell their products directly to final consumers through their websites (the "internet" channel) and indirectly via independent retail stores (the "traditional" channel).[2] Moreover, in the petroleum/oil industry, vertically integrated oil refiners have long supplied petroleum products (e.g, gasoline) to company-owned as well as to independent competing retail stations.[3]

In this paper, we develop a framework in which a vertically integrated firm sells its good to an independent downstream firm and examine the implications of the contractual form for welfare as well as for firms' profits. We consider two commonly used contract types: a linear contract, that specifies only a per-unit input price, and a two-part tariff, that specifies a per-unit input price and a fixed fee. Under each contract type, the integrated firm and its downstream rival engage first in Nash bargaining over the contract terms, and then compete in the final-goods market.

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[1]The vertical relationship might take place between input producers and final-good manufacturers and/or wholesalers and retailers.

[2]Real-world examples abide: clothing manufacturers, book publishers, record labels, and many more companies, sell their products both directly to consumers through their websites and indirectly via independent retail stores. A survey report reveals that about 42% of the top suppliers in a variety of industries such as IBM, Hewlett–Packard, Nike, Pioneer Electronics, Mattel, Estee Lauder, the former Compaq, Dell, and Cisco System are selling directly to consumers through the "internet" channel (Dan, Xu, & Liu, 2012).

[3]For more examples, see also Moresi and Schwartz (2017).

Under downstream Bertrand competition, we find that the standard result of the desirability of the two-part tariffs over the linear contracts in terms of welfare may be reversed. In particular, the linear contract can generate higher consumer surplus and welfare than the two-part tariff when the independent downstream firm is powerful in negotiating contract terms.

In order to delve into the intuition of this result let us start with some important observations. Under a linear contract, the two standard externalities are at work: the vertical externality (double-margin), where prices are too high from the industry's point of view, and the horizontal externality (downstream competition), where prices are too low from industry's point of view. Interestingly, these two externalities are still present under a two-part tariff contract as the latter cannot achieve the maximum industry profit (Moresi and Schwartz, 2017). Moreover, unlike the independent firm, the integrated firm has an outside option: if there is negotiation-breakdown, its disagreement profit is equal to the integrated monopoly profit.

In the absence of a fixed fee, the independent downstream firm pushes for a lower per-unit input price which weakens the vertical externality but exacerbates the horizontal one. In the presence of a fixed fee, its incentive to push for a lower per-unit input price is less intense as it now internalizes the horizontal externality that the firms exert upon each other. Consequently, when the independent downstream firm is powerful in determining contract terms, the per-unit input price tends to be lower under a linear contract. Yet, whether the per-unit input price is indeed lower under a linear contract ultimately depends on integrated firm's participation constraint: the integrated firm must earn more than its outside option. In particular, the per-unit input price will be lower under a linear than under a two-part tariff if and only if the participation constraint of the integrated firm is stricter under the latter than under the former, that is, if and only if the fixed fee is negative. Indeed, when the independent downstream firm is powerful, the fixed fee is negative and the per-unit input price is lower under a linear contract than under a two-part tariff. A lower per-unit input price under a linear contract implies lower final-good prices and higher final-good outputs for both firms.

A negative fixed fee implies that the integrated firm makes a fixed payment to its downstream rival. Under downstream Bertrand competition and linear demand, the equilibrium gross profits of the integrated firm are higher than its outside option. Hence, the integrated firm ends up paying a fixed fee to its downstream rival when the latter is powerful in bargaining contract terms, and still gets a share of the industry profit that is higher than its outside option.

Exactly when the fixed fee is negative, and the per-unit input price is higher under the two-part tariff, the integrated firm prefers the linear contract. Industry profits are higher under a two-part tariff than under a linear contract. When the fixed fee is negative, the integrated firm gets a small fraction of the surplus generated under a two-part tariff. A switch from a two-part tariff to a linear contract, increases the slice of the pie that goes to the integrated firm and compensates it for the reduction in the size of the pie. Consequently, the independent downstream firm prefers the two-part tariff in that case, as it obtains a larger slice of the larger pie. Analogous arguments hold when the integrated firm is more powerful in determining contract terms: the fixed fee is positive, and the integrated firm prefers the two-part tariff as it obtains a larger slice of the larger pie, whereas the independent downstream firm prefers the linear tariff as it obtains a larger slice of the smaller pie (which compensates for the reduction in the size of the pie). The two firms' preferences over the contract type always collide.

The result that, under downstream Bertrand, the per-unit input price can be lower under a linear contract when the independent downstream rival is powerful in bargaining terms does not remain robust under downstream Cournot. As in the case of Bertrand, the independent downstream firm's incentive to push for a lower per-unit input price is more intense under a linear contract than under a two-part tariff, but whether the pre-unit input price ends up being lower under a linear contract depends on the integrated firm's participation constraint. Unlike Bertrand competition, the fixed fee under Cournot is always positive since the equilibrium gross profits of the integrated firm are always lower than its outside option. Therefore, under Cournot, the integrated firm's participation constraint is always stricter under a linear contract, and the per-unit input price is always lower under a two-part tariff, implying that the latter is always welfare superior. Moreover, the integrated firm always prefers the two-part tariff contract.

It should be emphasized here that under either mode of downstream competition, the preferred contract type of the integrated firm is always welfare superior: the incentives of the integrated firm and the society as a whole on which contract type should be employed are aligned. Even though we do not formally endogenize the contract-type decision, our analysis suggests that, irrespective of the mode of downstream competition as well as irrespective of the distribution of the bargaining weight over contract terms, the society is always better off when the integrated firm gets to decide the contract type.

A number of theoretical studies consider vertically integrated firm(s) dealing with downstream rival(s), e.g., Chen, 2001; Chemla, 2003; Ordover and Shaffer, 2007; Arya et al., 2008; Höffler and Schmidt, 2008; Bourreau et al., 2011; Reisinger and Tarantino, 2015; Moresi and Schwartz, 2017; Milliou and Petrakis, 2019. However, none of these studies examine the desirability of two-part tariffs vs. linear tariffs from the viewpoint of consumers and society, as well as from the viewpoint of involved firms.

The desirability of different contractual forms has been investigated by a number of studies that focus on vertically separated markets, i.e., markets where upstream firm(s) reach final consumers only through independent downstream firm(s), e.g., Gal-Or, 1991; Rey and Stiglitz, 1995; Milliou and Petrakis, 2007; Milliou et al., 2012; Reisinger and Schnitzer, 2012; Milliou and Pavlou, 2013; Milliou and Petrakis, 2020; Constantatos and Pinopoulos, 2021.

Regarding efficiency, it has been widely recognized that two-part tariffs are more efficient than linear tariffs, that is, two-part tariffs generate higher consumer surplus and total welfare than linear tariffs. This is indeed the case when the number of downstream firms is exogenously given and it is the same under both contractual forms. Reisinger and Schnitzer (2012) and Milliou and Petrakis (2020) explore the implications of linear and two-part tariff contracts under free entry. The former considers a successive oligopoly model with simultaneous free entry in both up- and downstream market, whereas the latter considers an upstream supplier and free entry in the downstream market. Both studies show that linear tariffs can generate higher consumer surplus and total welfare due to higher market entry. In the present paper, this qualitative result is obtained by abstracting from free entry and by introducing a vertically integrated firm; hence the driving mechanism behind our results is different.

Regarding profitability, Milliou and Petrakis (2007) has shown that an upstream supplier may prefer to deal with downstream firms through a linear rather than a two-part tariff contract. They consider interim observable contracts[4] with no free entry.[5] The upstream supplier prefers to trade through linear tariffs when its bargaining power vis-à-vis the downstream firms is low.[6] Still,

[4]Interim observable contracts mean that contract terms are initially unobservable (during negotiations) but they become observable before downstream competition takes place.

[5]In their model, there are two, exogenously given, downstream firms competing á la Cournot, and the upstream monopolist is the result of an upstream merger.

[6]Intuitively, the upstream supplier suffers less from its commitment problem since the downstream firms know that in the absence of fixed fees it cannot behave too opportunistically. The fact that an upstream supplier may prefer to

welfare would take its highest value if the upstream supplier traded through two-part tariff contracts. In this paper, we assume that the supplier is integrated with one of the downstream firms, and find that the vertically integrated supplier prefers to trade with a linear contract under Bertrand competition when its bargaining weight vis-à-vis the downstream rival's is low and/or final goods are close substitutes. A switch from a two-part tariff to a linear contract increases the slice of the pie that goes to the integrated supplier and compensates for the reduction in the size of the pie.

The rest of the paper is organized as follows. Section 2 describes the baseline model. Section 3 presents and compares the equilibrium analysis of the two alternative contract types under a general demand function and downstream Bertrand competition. The linear demand case is examined in Section 4, while Section 5 extends the baseline model by considering the case of downstream Cournot competition. Section 6 concludes the paper. All proofs are relegated to the Appendix.

2. The model

We consider a vertically related industry consisting of two firms, an integrated firm, I, and an independent downstream firm, D. The upstream division of the integrated firm is the sole producer of an essential input for the production of the final good. The downstream division of the integrated firm obtains the input internally from its upstream partner at marginal cost, while D procures the input from I. Each firm transforms one unit of input into one unit of the final good. Without loss of generality, both upstream and downstream constant marginal costs are normalized to zero; firm D faces only the cost of sourcing the input from I, which depends on the contract type. We consider two contract types: a *linear contract* (L) that specifies a per-unit input price w, and a *two-part tariff* (T), consisting of (w, F) where F is a fixed fee.

We assume a general demand function, $q_i(p_i, p_j),\ i = I, D,\ i \neq j$, which is twice differentiable with $\partial q_i/\partial p_i < 0$ and $\partial q_i/\partial p_j > 0$: demand is downward sloping and the cross effects are positive. We further assume that own effects are larger than cross effects, i.e., $|\partial q_i/\partial p_i| > \partial p_i/\partial q_j$, which implies that final-goods are imperfect substitutes.

trade through linear contracts has also been shown by Milliou and Pavlou (2013) in a context where the supplier has all bargaining power and undertakes R&D investments.

The timing of the game, under each contract type, is as follows. In the *first* stage, I and D engage in Nash bargaining over contract terms. The bargaining weights (i.e., exogenous Nash parameters) of I and D are β and $1-\beta$ respectively, with $\beta \in (0,1)$. In the *second* stage, firms I and D choose, simultaneously and separately, their final-good prices, i.e., they engage in Bertrand competition.

We make the following standard assumptions: (*i*) second order conditions are satisfied, (*ii*) there is strategic complementarity: firms' best-response functions in the downstream market are upward sloping, $dp_i/dp_j > 0$, (*iii*) best-response functions are well-behaved and have slope less than one, $dp_i/dp_j < 1$, and thus there exist unique and stable Bertrand equilibria.

3. Equilibrium analysis

The equilibrium analysis in the downstream market is the same under both contract types. Firm I's and firm D's (gross) profits are:

$$\pi_I = p_I q_I(p_I, p_D) + w q_D(p_I, p_D), \qquad \pi_D = (p_D - w) q_D(p_I, p_D). \tag{1}$$

Industry profits are:

$$\pi_{ind} = p_I q_I(p_I, p_D) + p_D q_D(p_I, p_D). \tag{2}$$

Using (1) and (2), we can then rewrite each firm's (gross) profits as function of industry profits:

$$\pi_I = \pi_{ind} + w q_D(p_I, p_D) - p_D q_D(p_I, p_D), \qquad \pi_D = \pi_{ind} - w q_D(p_I, p_D) - p_I q_I(p_I, p_D).$$

Firm I chooses its price p_I, taking p_D as given, to maximize its profit π_I. The first order condition is:

$$\frac{\partial \pi_{ind}}{\partial p_I} + w \underbrace{\frac{\partial q_D(p_I, p_D)}{\partial p_I}}_{\substack{accommodation \\ effect}} - p_D \underbrace{\frac{\partial q_D(p_I, p_D)}{\partial p_I}}_{\substack{horizontal \\ externality}} = 0. \tag{3}$$

On the one hand, the final-good price p_I will be distorted away from that which would induce the maximum industry profit due to the *horizontal externality* that I exerts upon D. Intuitively, the final-good price tends to be *lower* than that which would induce the maximum industry profit. On the other hand, the horizontal externality is mitigated by the *accommodation* effect: the integrated firm realizes that any customer lost in the downstream market can be recovered via the upstream market providing it with a credible commitment to relax downstream competition (Church, 2008).[7] Due to the accommodation effect, final-good price tends to be *higher* than that which would induce the maximum industry profit.

The above F.O.C can be rewritten as:

$$\frac{\partial \pi_{ind}}{\partial p_I} - \underbrace{(p_D - w)\frac{\partial q_D(p_I, p_D)}{\partial p_I}}_{\substack{net\ horizontal \\ externality}} = 0. \tag{4}$$

Since the accommodation effect only serves as to mitigate the horizontal externality, we shall call the horizontal externality exerted by firm I as the *net* horizontal externality effect.

Firm D chooses its price p_D, taking p_I as given, to maximize its profit π_D. The first order condition is:

$$\frac{\partial \pi_{ind}}{\partial p_D} - \underbrace{w\frac{\partial q_D(p_I, p_D)}{\partial p_D}}_{\substack{vertical \\ externality}} - \underbrace{p_I\frac{\partial q_I(p_I, p_D)}{\partial p_D}}_{\substack{horizontal \\ externality}} = 0. \tag{5}$$

On the one hand, the final-good price p_D will be distorted away from that which would induce the maximum industry profit due to the *horizontal externality* that D exerts upon I. Intuitively, the final-good price tends to be *lower* than that which would induce the maximum industry profit. On the other hand, the final-good price tends to be *higher* than that which would induce the maximum industry profit due to the *vertical externality*.

Solving together (4) and (5), we obtain the last-stage subgame equilibrium final-good prices as functions of the input price, $p_i(w), i = I, D$, with $\partial p_i / \partial w > 0$. We denote each firm's gross profits as functions of the input price:

$$\pi_I(w) = p_I(w)q_I(w) + wq_D(w), \qquad \pi_D(w) = (p_D(w) - w)q_D(w), \tag{6}$$

[7] This effect is also known as the *collusive* effect (Chen, 2001) or the *softening* effect (Bourreau *et al*., 2011).

where $q_I(w) = q_I(p_I(w), p_D(w)), q_D(w) = q_D(p_I(w), p_D(w))$. We assume that profit functions have the usual properties, i.e., $\partial\pi_D(w)/\partial w < 0$ and $\partial\pi_I(w)/\partial w > 0$ up to some maximum.

Next, we proceed to the upstream stage, i.e., we determine the equilibrium contract terms, first under a two-part tariff contract, and then under a linear contract.

3.1. Two-part tariff contract

Firms I and D choose w and F to maximize the following generalized Nash product:

$$[\pi_I(w) + F - d]^{\beta} [\pi_D(w) - F]^{1-\beta} . \tag{7}$$

The disagreement payoff of D is zero since it has no alternative trading partner. However, I's disagreement payoff, d, is not zero since in the case of an unsuccessful negotiation with D, it can still sell its good directly to consumers. More specifically, if an agreement between I and D is not reached, the I's disagreement payoff is $d = \pi_I^{mon}$, with π_I^{mon} denoting I's profit when it is the only firm present in the market, i.e. the integrated monopoly profit:

$$\pi_I^{mon} \equiv p_I q_I(p_I)$$

Note that $\pi_I(w) + F > \pi_I^{mon}$ must hold: the negotiated per-unit input price must be such that firm I earns more than its outside option; otherwise it would not have an incentive to bargain with D in the first place.[8]

Maximizing (7) with respect to F we obtain:

$$F(w) = \beta\pi_D(w) - (1-\beta)[\pi_I(w) - \pi_I^{mon}]. \tag{8}$$

Using the above, the requirement $\pi_I(w) + F > \pi_I^{mon}$ reduces to $\pi_{ind}(w) > \pi_I^{mon}$: the negotiated per-unit input price must be such that the industry profit, when firm D is active, is higher than the integrated monopoly profit.

Substituting (8) into (7), the generalized Nash product reduces to an expression proportional to industry profits: w is chosen to maximize industry profits minus I's disagreement payoff,

[8] Recall that $\beta \in (0,1)$.

$$[\pi_D(w) + \pi_I(w) - \pi_I^{mon}].$$

Since the disagreement payoff does not depend on w, the two firms essentially choose w so as to maximize the industry profit, with the F.O.C being:

$$\frac{\partial \pi_{ind}(w)}{\partial w} = 0. \tag{9}$$

After some straightforward but tedious calculations, the above can be rewritten as:

$$\underbrace{\left[(p_D - w)\frac{\partial q_D(p_I, p_D)}{\partial p_I}\right]}_{A}\underbrace{\frac{\partial p_I}{\partial w}}_{+} + \underbrace{\left[p_I \frac{\partial q_I(p_I, p_D)}{\partial p_D} + w\frac{\partial q_D(p_I, p_D)}{\partial p_D}\right]}_{B}\underbrace{\frac{\partial p_D}{\partial w}}_{+} = 0. \tag{10}$$

The solution of (10) gives the equilibrium marginal input price w^T, which is independent of β. Note that A above encompasses the net horizontal externality exerted by firm I (see (4)). Note also that B encompasses the vertical externality, as well as the horizontal externality exerted by firm D (see (5)). Note that in the extreme case where final-goods are totally differentiated, i.e., final-goods are independent in demand, the two-part tariff can achieve the maximum industry profit, denoted by π_{ind}^m. To see this, it is straightforward that for $\partial q_i/\partial p_j = 0$, $A = 0$, and for (10) to be satisfied it must hold $B = 0$ and hence $w = 0$: when final-goods are totally differentiated, the only externality at play is the vertical one, and it is eliminated by setting the per-unit input price equal to upstream marginal cost (normalized to zero here). However, when final-goods are imperfect substitutes ($\partial q_i/\partial p_j > 0$), a two-part tariff contract *cannot* achieve the maximum industry profit, as pointed out by Moresi and Schwartz (2017) since the two externalities do not vanish: in that case (10) holds when A and B are non-zero and have opposite signs; it is straightforward that $A > 0$ and $B < 0$. The latter implies:

$$w^T > -p_I^T \frac{\partial q_I(p_I, p_D)/\partial p_D}{\partial q_D(p_I, p_D)/\partial p_D} > 0,$$

where $p_I^T = p_I(w^T)$. Even though firms cannot achieve the maximum industry profit, they still try to mitigate the horizontal externality and thus they set the per-unit input price above marginal cost.[9]

Each firm's net profits are:

$$\pi_I^T(\beta) = \pi_I(w^T) + F(w^T) = \beta[\pi_{ind}^T - \pi_I^{mon}] + \pi_I^{mon}$$
$$\pi_D^T(\beta) = \pi_D(w^T) - F(w^T) = (1-\beta)[\pi_{ind}^T - \pi_I^{mon}] \tag{11}$$

Note that both firms make positive net profits whenever $\pi_{ind}(w^T) = \pi_{ind}^T > \pi_I^{mon}$.

3.2. Linear contract

Firms I and D choose w to maximize the following generalized Nash product:

$$[\pi_I(w) - d]^\beta [\pi_D(w)]^{1-\beta}\ . \tag{12}$$

As in the case of a two-part tariff, the disagreement payoff of D is zero and the disagreement payoff of I is π_I^{mon}. Note that $\pi_I(w) > \pi_I^{mon}$ must hold: the negotiated input price must be such that firm I earns more than its outside option; otherwise it would not have an incentive to bargain with D in the first place. The F.O.C of (12) is:

$$\left[\frac{\beta}{\pi_I(w) - \pi_I^{mon}}\right]\frac{\partial \pi_I(w)}{\partial w} + \left[\frac{1-\beta}{\pi_D(w)}\right]\frac{\partial \pi_D(w)}{\partial w} = 0. \tag{13}$$

Using the fact that $\pi_I(w) = \pi_{ind}(w) - \pi_D(w)$, and after some straightforward calculations, (13) can be rewritten as:

$$\frac{\partial \pi_{ind}(w)}{\partial w} = \frac{\partial \pi_D(w)}{\partial w}\frac{[\beta\pi_D(w) - (1-\beta)[\pi_I(w) - \pi_I^{mon}]]}{\beta\pi_D(w)}. \tag{14}$$

[9]The upstream division of the integrated firm does not charge a per-unit input price to its own downstream division, or even if it charged, this would be irrelevant since the integrated firm maximizes its joint profits from both divisions.

The solution of (14) gives the equilibrium input price $w^L(\beta)$ which depends on β. It must hold $\pi_I(w^L(\beta)) = \pi_I^L(\beta) > \pi_I^{mon}$.

3.3. Comparison of per-unit input prices

Under a general demand, it is not a priori clear whether the per-unit input price is higher under a two-part tariff than under a linear tariff. We need to compare (9) to (14). If we insert w^T into (14) and obtain that the RHS of (14) is zero, we conclude that $w^L = w^T$ since (14) replicates (9). If we insert w^T into (14) and obtain that the RHS of (14) is positive, then we conclude that $w^L < w^T$ while if we obtain that the RHS of (14) is negative we conclude that $w^L > w^T$. Since $\partial\pi_D(w)/\partial w < 0$, the sign of the RHS of (14) depends on the sign of $\beta\pi_D(w) - (1-\beta)[\pi_I(w) - \pi_I^{mon}]$. By evaluating the latter on w^T, and from (8), we have:

$$\beta\pi_D(w^T) - (1-\beta)[\pi_I(w^T) - \pi_I^{mon}] = F(w^T)$$

If $\pi_I(w^T) < \pi_I^{mon}$, it is straightforward that $F(w^T) > 0$: if the equilibrium gross profits of firm I are lower than its outside option, then it always receives a fixed payment from D. In that case, $w^L > w^T$, that is, the equilibrium per-unit input price is always lower under a two-part tariff.

If $\pi_I(w^T) > \pi_I^{mon}$, then the fixed fee can be negative and the per-unit input price under a linear contract can be lower than the per-unit price under a two-part tariff contract. In particular, we have that $\partial F(w^T)/\partial\beta = [\pi_{ind}(w^T) - \pi_I^{mon}] > 0$ and

$$F(w^T) = \pi_D(w^T) > 0, \qquad F(w^T) = -[\pi_I(w^T) - \pi_I^{mon}] < 0,$$

which lead to the following Lemma.

Lemma 1. *Suppose that* $\pi_I(w^T) > \pi_I^{mon}$ *holds. Then, under Bertrand downstream competition and general demand, there exists a unique* $\hat{\beta} \in (0,1)$ *such that: (i) for* $\beta < \hat{\beta}$ *it holds* $F(w^T) < 0$ *and* $w^L < w^T$*, (ii) for* $\beta > \hat{\beta}$ *it holds* $F(w^T) > 0$ *and* $w^L > w^T$.

If the equilibrium gross profits of firm I are higher than its outside option, then when β is very low, i.e., when D is very powerful in determining contract terms, firm I is willing to make a fixed payment to D. Note that if $\pi_I(w^T) > \pi_I^{mon}$ then $\pi_I(w^T) + \pi_D(w^T) = \pi_{ind}(w^T) = \pi_{ind}^T > \pi_I^{mon}$, with the latter implying positive net profit for firm I (see (11)): even when I is rather weak in negotiating terms, it is willing to pay a F to D and still get a share of π_{ind}^T that is higher than its outside option.

Recall from the earlier analysis (Eqs. (4) and (5)) that the two standard externalities are at work: the *vertical* and the *horizontal* externality, with the horizontal externality exerted by firm I on firm D being mitigated by the accommodation effect. Consider first the perspective of I. In the absence of a fixed fee, firm I always pushes for a higher w and in doing so it ignores the negative impact on D. A higher w weakens the horizontal externality but at the same time exacerbates the vertical externality. In the presence of a fixed fee, however, the incentive of firm I to push for a higher w is less intense as I internalizes the impact of a higher w on firm D. Hence, we expect that when firm I is powerful in determining contract terms, the per-unit input price will be lower under a two-part tariff contract. Indeed, in the limit as $\beta \to 1$, the RHS of (14) is negative which implies $w^L > w^T$.

Consider now the perspective of firm D. In the absence of a fixed fee, firm D always pushes for a lower w which weakens the vertical externality but exacerbates the horizontal externality. In the presence of a fixed fee, the incentive of D to push for a lower w is less intense as it now internalizes the horizontal externality that firms exert upon each other. Hence, when firm D is powerful in determining contract terms, the per-unit input price tends to be lower under a linear contract. However, whether the per-unit input price is indeed lower under a linear contract depends on firm I's participation constraint: I must earn more than its outside option. Under a linear tariff, I's participation constraint is $\pi_I(w) > \pi_I^{mon}$, whereas under a two-part tariff, it is $\pi_I(w) + F(w) > \pi_I^{mon}$. Hence, the per-unit input price will be lower under a linear contract than under a two-part tariff if and only if the participation constraint of I is stricter under the latter than under the former: $w^L < w^T$ if and only if $F(w^T) < 0$. A necessary condition for the latter is that $\pi_I(w^T) > \pi_I^{mon}$.

In the next Section, we turn our attention to the linear demand. There, we find that $\pi_I(w^T) > \pi_I^{mon}$ and obtain an explicit formula for $\hat{\beta}$, where for $\beta < \hat{\beta}$ the equilibrium fixed fee is negative and the per-unit input price is higher under a two-part tariff and vice-versa.

4. Linear demand

In this section, we focus on linear demand and investigate (*i*) under which contract type the per-unit input price is higher, and (*ii*) the desirability of the contract types. Consumers' direct demands for the final-goods are (Singh and Vives, 1984):

$$q_i(p_i, p_j) = \frac{a(1-\theta) - p_i + \theta p_j}{1-\theta^2}, \qquad i,j = I, D, \qquad i \neq j \tag{15}$$

where p_i, denotes the final-good price for i's good, q_i denotes i's final-good quantity and $\theta \in (0,1)$ reflects the degree of substitutability among firms' final-goods, with higher values of θ reflecting greater inter-brand substitution. Since we apply the linear demand function in the game described and analyzed in Sections 2 and 3, we relegate the detailed equilibrium analysis under each contract type, as well as the proofs of Lemmas and Propositions, to the Appendix.

The following Lemma verifies the common intuition that as long as final-goods are imperfect substitutes, the integrated firm, under either contract type, does not fully foreclose its rival; in other words, it always engages in bargaining with the unintegrated downstream firm.

Lemma 2. *Under downstream Bertrand competition and linear demand, the integrated firm never forecloses its downstream rival under either contract type*.

Lemma 2 holds for any given distribution of the exogenous Nash bargaining parameter β. That is, even when β is low, which implies that firm I is weak in negotiating terms, it can still earn more than in the case where it forecloses D.

Under a two-part tariff contract, the equilibrium marginal input price and fixed fee are:

$$w^T = \frac{a(2+\theta)^2\theta}{2(4+5\theta^2)}, \qquad F(\beta) = \frac{a^2(1-\theta)[\beta(4+5\theta^2)(1+\theta^2) - \theta^2(5+4\theta^2)]}{(1+\theta)(4+5\theta^2)^2}. \tag{16}$$

The optimal marginal input price does not depend on β. Moreover, unless final-goods are totally differentiated (independent in demand), the input price is higher than upstream marginal cost. The bargaining weight β affects the distribution of industry profit to be shared between the two firms via the fixed fee F. From the second expression in (16), we obtain the following result.

Proposition 1. *Under downstream Bertrand competition, linear demand, and a two-part tariff, the equilibrium fixed fee is positive (negative) when* $\beta > (<) \hat{\beta}(\theta)$, *with*

$$\hat{\beta}(\theta) \equiv \frac{\theta^2(5+4\theta^2)}{(4+5\theta^2)(1+\theta^2)},$$

and $\partial\hat{\beta}/\partial\theta > 0$.

As discussed in Section 3, when the equilibrium gross profits of the integrated firm are higher than its outside option, the fixed fee can be negative. This is indeed the case under a linear demand,

$$\pi_I(w^T) - \pi_I^{mon} = \pi_I(w^T) - \frac{a^2}{4} = \frac{a^2(1-\theta)\theta^2(5+4\theta^2)}{(1+\theta)(4+5\theta^2)^2} > 0.$$

When firm I is sufficiently weaker in bargaining-weight terms, it ends up paying a fixed amount to firm D. As noted in Section 3, even when the fixed fee is negative, I is better off than in the case where it forecloses firm D. The total industry profit is always higher than I's outside option, $\pi_{ind}^T > a^2/4 = \pi_I^{mon}$, and I always makes positive net profits; even when I is rather weak in negotiating terms, it is willing to pay a F to D and still get a share of π_{ind}^T that is higher than its outside option. Note also that $\partial\hat{\beta}/\partial\theta > 0$, implying that a negative fixed fee is more likely the closer substitutes final-goods are. Recall that integrated firm I benefits from the accommodation effect, that is, from a credible commitment to relax downstream competition. The higher is θ, the stronger is that effect and hence the more likely is that firm I will pay a F to firm D.

Under a linear contract, the equilibrium input price is:

$$w^L(\beta) = \frac{a(2+\theta)[2(1+\theta)(4-2\theta+\theta^2)+\beta(2-\theta)(2+\theta^2)-(2-\theta)K]}{4(1+\theta)(8+\theta^2)}, \qquad (17)$$

with $K = \sqrt{16(1-\theta^2)(1-\beta)+\beta^2(2+\theta^2)^2}$.

In the absence of a fixed fee, firms do not maximize jointly the industry profits and the optimal input price depends on β. It can be checked from (17) that $\partial w^L(\beta)/\partial\beta > 0$, verifying the common intuition that the stronger is firm I (resp., firm D), the higher (resp., lower) is the input price.

Regarding the efficiency of the two contract types we obtain the following result.

Proposition 2. *Under downstream Bertrand competition and linear demand (i)* $w^T > w^L, p_i^T > p_i^L$, $q_i^T < q_i^L$, $i = I, D$, *and (ii)* $CS^T < CS^L$, $TW^T < TW^L$, *if and only if* $\beta < \hat{\beta}(\theta)$, *where* $\hat{\beta}(\theta)$ *is given in Proposition 1*.

As discussed in Section 3, when the equilibrium fixed fee is negative then the equilibrium per-unit input price is higher under a two-part tariff than under a linear contract. Proposition 2 verifies that, indeed, for all values of β that the fixed fee is negative, i.e., $\beta < \hat{\beta}(\theta)$, the optimal per-unit input price is higher under a two-part tariff than under a linear tariff. A lower per-unit input price under a linear contract than under a two-part tariff implies lower final-good prices as well as higher final-good quantities for both firms; consumer surplus and total welfare are higher under the linear tariff.

Now that we have established the desirability of the two contractual forms from the consumer and/or social point of view, we investigate their desirability from the firms' point of view. The following Proposition states which contract type each firm prefers.

Proposition 3. *Under downstream Bertrand competition and linear demand:*
(i) for $\beta > \hat{\beta}(\theta)$, it holds $\pi_I^T > \pi_I^L$ and $\pi_D^T < \pi_D^L$,
(ii) for $\beta < \hat{\beta}(\theta)$, it holds $\pi_I^T < \pi_I^L$ and $\pi_D^T > \pi_D^L$,
with $\hat{\beta}(\theta)$ given in Proposition 1.

Proposition 3 reveals that the two firms have divergent preferences regarding the contract types. Firm I prefers the two-part tariff when β is relatively high, while it prefers the linear contract when β is relatively low. Exactly the opposite holds true for the independent downstream firm D.

The industry profit is larger under T than under L, i.e., $\pi_{ind}^L < \pi_{ind}^T$. This is straightforward from (9) and (13), where the industry profit is maximized under T but not under L. Nevertheless, besides the *size* of the 'pie' what also matters is the *distribution* of the pie. After evaluating each firm's share of the profits, we have that

$$\frac{\pi_I^L}{\pi_{ind}^L} > \frac{\pi_I^T}{\pi_{ind}^T}, \tag{18}$$

if and only if $\beta < \hat{\beta}(\theta)$. Recall from Proposition 1 that when $\beta < \hat{\beta}(\theta)$ I pays firm D a fixed amount. Also recall that $\partial\hat{\beta}/\partial\theta > 0$. Thus, when β is relatively low (D is relatively stronger in bargaining-weight terms) and/or θ is relatively high (final goods are close substitutes), firm I prefers the linear contract: a switch from a two-part tariff to a linear contract increases the slice of the pie that goes to firm I, and compensates for the reduction in the size of the pie. Clearly, firm D prefers the two-part tariff since it can capture a larger share of the larger pie.

For $\hat{\beta}(\theta) < \beta$, the inequality in (18) is reversed so that when β is relatively high (I is stronger in bargaining-weight terms) and/or θ is low (final-goods are very differentiated), a switch from a two-part tariff to a linear contract increases the slice of the pie that goes to the downstream firm D and compensates for the reduction in the size of the pie. Clearly, firm I prefers the two-part tariff since it can capture a larger share of the larger pie.

From Propositions 2 and 3, we immediately obtain the following.

Corollary 1. *Under downstream Bertrand competition, the preferred contract type for firm I is always welfare superior, whereas the preferred contract type for firm D is always welfare inferior.*

Hence, the incentives of firm I and the society on which contract type should be employed are aligned.

5. Downstream Cournot competition

In this section, we check the robustness of our results when there is downstream Cournot instead of Bertrand competition. As in the Bertrand case, we start our analysis under a general demand.

5.1. General demand

We assume an inverse demand function, $p_i(q_i, q_j),\ i = I, D,\ i \neq j$, that is twice differentiable with $\partial p_i / \partial q_i < 0$ and $\partial p_i / \partial q_j < 0$: demand is downward sloping and the cross effects are negative. Own effects are larger than cross effects, i.e., $|\partial p_i / \partial q_i| > |\partial p_i / \partial q_j|$, which implies that final-goods are imperfect substitutes. We make the following assumptions: (*i*) the second order conditions are satisfied, (*ii*) there is strategic substitutability: firms' best-responses in the downstream market are downward sloping, $dq_i / dq_j < 0$, (*iii*) best-response functions are well-behaved and have slope less than one, $|dq_i / dq_j| < 1$, and thus there exist unique and stable Cournot equilibria.

The equilibrium analysis in the downstream market is the same under both contract types. Firm I's and firm D's (gross) profits are:

$$\tilde{\pi}_I = p_I(q_I, q_D) q_I + w q_D, \qquad \tilde{\pi}_D = (p_D(q_I, q_D) - w) q_D. \tag{19}$$

Industry profits are:

$$\tilde{\pi}_{ind} = p_I(q_I, q_D)q_I + p_D(q_I, q_D)q_D. \tag{20}$$

Using (19) and (20), we can rewrite each firm's (gross) profits as function of industry profits:

$$\tilde{\pi}_I = \tilde{\pi}_{ind} + wq_D - p_D(q_I, q_D)q_D, \qquad \tilde{\pi}_D = \tilde{\pi}_{ind} - wq_D - p_I(q_I, q_D)q_I.$$

Firm I chooses its quantity q_I, taking q_D as given, to maximize $\tilde{\pi}_I$. The first order condition is:

$$\frac{\partial \tilde{\pi}_{ind}}{\partial q_I} - \underbrace{q_D \frac{\partial p_D(q_I, q_D)}{\partial q_I}}_{\substack{horizontal \\ externality}} = 0. \tag{21}$$

As is well-known, the accommodation effect present under downstream Bertrand, is not present under downstream Cournot: when firms set quantities rather than prices, the integrated firm takes the rival's output and hence the demand for its input as given, and consequently it does not perceive that variations in its own output will affect its upstream profit (e.g., Chen, 2001; Arya et al., 2008; Church, 2008).[10]

Firm D chooses its quantity q_D, taking q_I as given, to maximize $\tilde{\pi}_D$. The first order condition is:

$$\frac{\partial \tilde{\pi}_{ind}}{\partial q_D} - \underbrace{w}_{\substack{vertical \\ externality}} - \underbrace{q_I \frac{\partial p_I(q_I, q_D)}{\partial q_D}}_{\substack{horizontal \\ externality}} = 0. \tag{22}$$

Solving together (21) and (22), we obtain the last-stage subgame equilibrium final-good outputs as functions of the input price, $\tilde{q}_i(w), i = I, D$. We denote each firm's gross profits as functions of the input price:

[10]Two exceptions, where the accommodation effect is present under downstream quantity competition, are Gans (2007) and Constantatos and Pinopoulos (2019). In Gans (2007), the standard order between up- and downstream decisions is reversed, that is, downstream decisions precede upstream ones. In Constantatos and Pinopoulos (2019), upstream decisions precede downstream ones but the integrated firm chooses its input quantity instead of input price.

$$\tilde{\pi}_I(w) = \tilde{p}_I(w)\tilde{q}_I(w) + w\tilde{q}_D(w), \qquad \tilde{\pi}_D(w) = (\tilde{p}_D(w) - w)\tilde{q}_D(w), \tag{23}$$

where $\tilde{p}_I(w) = p_I(\tilde{q}_I(w), \tilde{q}_D(w)), \tilde{p}_D(w) = p_D(\tilde{q}_I(w), \tilde{q}_D(w))$. We assume that profit functions have the usual properties, i.e., $\partial\tilde{\pi}_D(w)/\partial w < 0$ and $\partial\tilde{\pi}_I(w)/\partial w > 0$ up to some maximum.

Under a two-part tariff contract, firms I and D choose w and F to maximize:

$$[\tilde{\pi}_I(w) + F - d]^\beta [\tilde{\pi}_D(w) - F]^{1-\beta},$$

where I's disagreement payoff is $d = \pi_I^{mon}$. Following the same steps as in the Bertrand case, the two firms essentially choose w to maximize the industry profit, with the F.O.C being:

$$\frac{\partial\tilde{\pi}_{ind}(w)}{\partial w} = 0. \tag{24}$$

The solution of (24) gives the equilibrium marginal input price $\widetilde{w}^T$, which is independent of β.

Under a linear contract, firms I and D choose w to maximize:

$$[\tilde{\pi}_I(w) - d]^\beta [\tilde{\pi}_D(w)]^{1-\beta} ,$$

where I's disagreement payoff is $d = \pi_I^{mon}$. Following the same steps as in the Bertrand case, the F.O.C, after some straightforward but tedious calculations, can be written as:

$$\frac{\partial\tilde{\pi}_{ind}(w)}{\partial w} = \frac{\partial\tilde{\pi}_D(w)}{\partial w}\frac{\beta\tilde{\pi}_D(w) - (1-\beta)[\tilde{\pi}_I(w) - \pi_I^{mon}]}{\beta\tilde{\pi}_D(w)}. \tag{25}$$

The solution of (25) gives the equilibrium input price $\widetilde{w}^L(\beta)$ which depends on β.

Again, under a general demand, it is not a priori clear whether the per-unit input price is higher under a two-part tariff than under a linear tariff. Since $\partial\tilde{\pi}_D(w)/\partial w < 0$ and $\beta\tilde{\pi}_D(w) > 0$, the RHS of (25) takes the sign of the expression $\beta\tilde{\pi}_D(w) - (1-\beta)[\tilde{\pi}_I(w) - \pi_I^{mon}]$.

When evaluated at $\widetilde{w}^T$, $\beta\tilde{\pi}_D(w) - (1-\beta)[\tilde{\pi}_I(w) - \pi_I^{mon}]$ is written as:

$$\beta\tilde{\pi}_D(\widetilde{w}^T) - (1-\beta)[\tilde{\pi}_I(\widetilde{w}^T) - \pi_I^{mon}] = F(\widetilde{w}^T).$$

If $\tilde{\pi}_I(\widetilde{w}^T) < \pi_I^{mon}$ then $F(\widetilde{w}^T) > 0$: if the equilibrium gross profits of firm I are lower than its outside option, then it always receives a fixed payment from D, the RHS of (25) is negative and, hence, $\widetilde{w}^L > \widetilde{w}^T$. In the next subsection, we turn our attention to the linear demand, and find that $\widetilde{\pi}_I(\widetilde{w}^T) < \pi_I^{mon}$ implying that the equilibrium fixed fee is always positive and the per-unit input price is always lower under a two-part tariff.

5.2. Linear demand

By inverting the system of demands in Section 4, we obtain inverse demands as:

$$p_i(q_i,q_j) = a - q_i - \theta q_j, \qquad i,j = I,D, \qquad i \neq j.$$

Since the steps followed for the equilibrium outcomes are the same as in the case of Bertrand competition, we relegate the complete analysis of the Cournot case, as well as all proofs of Lemmas and Propositions, in the Appendix. We present here our main results.

The following Lemma verifies that, as in the case of Bertrand competition, the integrated firm, under either contract type, does not foreclose its rival under Cournot competition, that is, it always engages in bargaining with the unintegrated downstream firm.

Lemma 3. *Under downstream Cournot competition and linear demand, the integrated firm never forecloses its downstream rival under either contract type*.

Under a two-part tariff, the equilibrium marginal input price and fixed fee are:

$$\widetilde{w}^T = \frac{a(2-\theta)^2\theta}{2(4-3\theta^2)}, \qquad \tilde{F}(\beta) = \frac{a^2(1-\theta)^2(4\beta + 3(1-\beta)\theta^2)}{(4-3\theta^2)^2}. \tag{26}$$

As in the case of Bertrand competition, the equilibrium marginal input price is independent of β, and unless final-goods are totally differentiated (independent in demand), the equilibrium input price is higher than upstream marginal cost. Unlike Bertrand competition, however, the fixed fee cannot be negative under Cournot: it is straightforward from the second expression in (26).

It can be checked from (16) and (26) that the marginal input price is higher under Bertrand than under Cournot. Firms use w as a way to mitigate the negative horizontal externality that exert upon each other; the fiercer is downstream competition, the more urgent it is for parties to increase the marginal input price. Even though the accommodation effect that relaxes competition downstream is present only under Bertrand, the horizontal externality is still more pronounced under Bertrand than under Cournot, and hence the marginal input price is higher under the former than under the latter. A lower input price under Cournot implies a higher fixed fee that is always positive: I never ends up paying a F to D.[11] As discussed in subsection 5.1., the fixed fee is always positive when the equilibrium gross profits of the integrated firm are lower than its outside option. This is indeed the case under a linear demand,

$$\tilde{\pi}_I(\widetilde{w}^T) - \pi_I^{mon} = \tilde{\pi}_I(\widetilde{w}^T) - \frac{a^2}{4} = -\frac{3a^2(1-\theta)^2\theta^2}{(4-3\theta^2)^2} < 0.$$

Under a linear contract, the equilibrium input price is:

$$\widetilde{w}^L(\beta) = \frac{a(2-\theta)[\theta(4+\theta) + 2\beta(2-\theta-\theta^2)]}{2(8-3\theta^2)}. \quad (27)$$

The optimal input price depends on β; it can be easily checked from (27) that $\partial\widetilde{w}^L(\beta)/\partial\beta > 0$. By comparing (17) and (27), it can be checked that the input price is higher (lower) under Bertrand than under Cournot when $\beta > (<)\, \theta^2/(4-\theta^2)$.[12]

Under a linear tariff, irrespective of the mode of downstream competition, firm I always pushes for a higher w. The accommodation effect is present under Bertrand but not under Cournot, hence it is more urgent for I to push for a higher w under Bertrand than under Cournot. Put it differently, I's concession costs when bargaining, i.e., accepting a lower input price, are higher under Bertrand due to the accommodation effect. At the same time, D always pushes for a lower w. Under Bertrand competition, the accommodation effect increases firm D's demand for the input – firm I's credible commitment to set a higher final-good price raises D's final-good price but to a lesser extent, and thus also increases D's final-good output. Because the accommodation effect is not present under Cournot, it is more urgent for D to push for a lower w under Bertrand than under Cournot. Put it differently, D's concession costs when bargaining, i.e., agreeing on a higher input price, are higher under Bertrand than under Cournot. Consequently, when β is high, i.e., I is powerful, input price

[11] The fact that the accommodation effect is not present under downstream Cournot implies that the integrated firm does not perceive that variations in its own output will affect its upstream profit and, thus, a higher per-unit input price under two-part tariff cannot relax downstream competition too much (compared to downstream Bertrand) so as to increase the profits of the integrated firm at that level that it can compensate D via a negative fixed fee.

[12] Arya et al. (2008) prove that the input price under linear contracting is higher under Bertrand competition compared to Cournot competition for =1.

is higher under Bertrand than under Cournot, while when β is low, i.e., D is powerful, input price is lower under Bertrand.

Regarding the efficiency of the two contract types under Cournot competition we obtain the following result.

Proposition 4. *Under downstream Cournot competition and linear demand:*
(i) $\tilde{w}^T < \tilde{w}^L$ *and* $\tilde{p}_i^T < \tilde{p}_i^L, i = I, D$.
(ii) $\widetilde{CS}^T > \widetilde{CS}^L$ *and* $\widetilde{TW}^T > \widetilde{TW}^L$.

Proposition 4 verifies the standard conclusions regarding the desirability of two-part tariff over linear contract: the former generates higher consumer surplus (CS) and total welfare (TW) than the latter. From Propositions 2 and 4, we obtain immediately the following.

Corollary 2. *Unlike downstream Bertrand competition, the linear tariff cannot be welfare superior under downstream Cournot competition.*

Recall that under Bertrand, the per-unit input price is lower under a linear than under a two-part tariff, and hence the former is welfare superior, when β is low. Under Cournot, the per-unit input price is never lower under a linear tariff and hence the latter is always welfare inferior.

As in the case of Bertrand competition, D's incentive to push for a lower w is more intense under a linear contract than under a two-part tariff since under the latter it internalizes the horizontal externality that firms exert upon each other, but whether w ends up being lower under a linear contract when D is powerful (β is low) ultimately depends on I's participation constraint. Unlike Bertrand competition, the fixed fee under Cournot competition is always positive, implying that I's participation constraint is always stricter under a linear contract, and thus $w^L > w^T$.

Regarding the desirability of the two contract types from the perspective of the firms, we obtain the following result.

Proposition 5. *Under downstream Cournot competition and linear demand:*
(i) $\tilde{\pi}_I^T > \tilde{\pi}_I^L$,
(ii) $\tilde{\pi}_D^T > (<) \tilde{\pi}_D^L$ *whenever* $\beta < (>) \bar{\beta}(\theta) \equiv 3\theta^2/4$.

Proposition 5 reveals that the two firms have divergent preferences regarding the contract types only when β is relatively high: in that case, firm I prefers the two-part tariff whereas firm D prefers the linear contract. When β is relatively low, both firms prefer the two-part tariff.

As in the case of Bertrand competition, the industry profit is larger under T than under L, i.e., $\tilde{\pi}_{ind}^{L} < \tilde{\pi}_{ind}^{T}$. After evaluating each firm's share of the profits, we have that

$$\frac{\tilde{\pi}_I^L}{\tilde{\pi}_{ind}^L} > \frac{\tilde{\pi}_I^T}{\tilde{\pi}_{ind}^T}, \tag{28}$$

if and only if

$$\beta < \underline{\beta}(\theta) \equiv \frac{(3 - 4\theta + 2\theta^2) - \sqrt{9 - 24\theta + 25\theta^2 - 10\theta^3 + \theta^4}}{2(1 - \theta)^2}.$$

According to Proposition 5, the integrated firm always prefers a two-part tariff. For $\beta > \underline{\beta}(\theta)$, T guarantees I a larger share of higher overall industry profits, thus I unequivocally prefers T. For $\beta < \underline{\beta}(\theta)$, I faces a harder decision between a larger share of a smaller pie under L, and the opposite under T. As it turns out, I is better off under T, implying that the two-part tariff is preferred for any value of the parameters β and θ.

From Propositions 3 and 5, we obtain the following.

Corollary 3. *Unlike downstream Bertrand competition, the integrated firm never prefers the linear tariff under Cournot competition*.

Recall that under Bertrand competition, when β is low, the fixed fee is negative: firm I makes a fixed payment to firm D. Whenever the fixed fee is negative, firm I prefers the linear tariff. With Cournot competition, the fixed fee is always positive, i.e., firm I always receives a fixed payment from D and thus never prefers the linear tariff.

Unlike firm I, D does not always prefer the two-part tariff; its preferred contract depends on the distribution of bargaining weight and the degree of product substitutability. In particular, D prefers L when its bargaining weight vis-à-vis the integrated firm is relatively low and/or final goods are not too close substitutes; otherwise it prefers T. It can be easily verified that $\underline{\beta}(\theta) < \bar{\beta}(\theta)$. For $\beta < \underline{\beta}(\theta)$, T gives D a larger share of higher industry profits, therefore D unambiguously prefers T. For

$\beta > \underline{\beta}(\theta)$, D faces a trade-off: a larger share of a smaller pie under L, and the opposite under T. For $\underline{\beta}(\theta) < \beta < \bar{\beta}(\theta)$ firm D prefers T whereas for $\bar{\beta}(\theta) < \beta$, it prefers L.

From Propositions 4 and 5, we obtain the following.

Corollary 4. *Under downstream Cournot competition, the preferred contract type for firm I is always welfare superior, whereas the preferred contract type for firm D is welfare superior when D is relatively more powerful in bargaining.*

7. Conclusions

Considering a framework in which an integrated firm sells its input to a downstream rival while engaging first in Nash bargaining over the contract terms, and then competing in the final-goods market, we have shown that the standard result of the desirability of two-part tariffs over linear contracts in terms of welfare may be reversed. Under downstream Bertrand competition, when the independent downstream firm is rather powerful in determining the contract terms, the per-unit input price under two-part tariff is set above the per-unit input price under linear tariff in order to relax downstream competition while the independent firm is compensated via the fixed fee. Here, the integrated firm prefers to supply the independent retailer with its input and pay a fixed fee to it (negative fixed fee) rather than foreclosure it.

Under downstream Cournot competition, the aforementioned reversed result does not prevail since the accommodation effect is not present under Cournot, that is, the integrated firm does not perceive that variations in its own output will affect its upstream profit. Regarding profitability, we show that, under either mode of downstream competition, the preferred contract type for the integrated firm is always the welfare superior one.

Our paper contributes to the literature on vertical contracting by considering the desirability in terms of welfare and profits of the two standard contractual forms under vertical integration and bargaining. We emphasize the important role of the distribution of the bargaining weights among firms in a vertical chain for the implications on consumer's surplus, profits and welfare.

Appendix

A. Linear demand with Bertrand competition

We first characterize the equilibrium outcomes, and then provide all relevant proofs. We solve the model backwards, starting from the second stage of the game. Irrespective of the contract type, from (4) and (5), we obtain each firm's best-response functions in the downstream market:

$$p_I(p_D, w) = \frac{a(1-\theta) + \theta w + \theta p_D}{2}, \qquad p_D(p_I, w) = \frac{a(1-\theta) + w + \theta p_I}{2}. \qquad (A1)$$

Solving the system of best-responses, we obtain final-good prices and final-good quantities, for given input price:

$$p_I(w) = \frac{a(2+\theta)(1-\theta) + 3\theta w}{4-\theta^2}, \quad p_D(w) = \frac{a(2+\theta)(1-\theta) + (2+\theta^2)w}{4-\theta^2}.$$

$$(A2)$$

$$q_I(w) = \frac{a(2+\theta) - \theta(1+\theta)w}{(1+\theta)(4-\theta^2)}, \quad q_D(w) = \frac{a(2+\theta) - 2(1+\theta)w}{(1+\theta)(4-\theta^2)}.$$

Each firm's gross profits are:

$$\pi_I(w) = \frac{a^2(1-\theta)(2+\theta)^2 + a(8+8\theta+\theta^3+\theta^4)w - (1+\theta)(8+\theta^2)w^2}{(1+\theta)(4-\theta^2)^2},$$

$$(A3)$$

$$\pi_D(w) = \frac{(1-\theta)[a(2+\theta) - 2(1+\theta)w]^2}{(1+\theta)(4-\theta^2)^2}.$$

Next, we solve the first stage, that is, we determine the equilibrium contract terms under a two-part tariff contract and under a linear contract. In both cases, the disagreement payoff of D is zero since it has no alternative trading partner. However, I's disagreement payoff, d, is not zero since in the case of an unsuccessful negotiation with D, it can still sell its good directly to consumers. More specifically, if an agreement between I and D is not reached, then I's disagreement payoff is $d = a^2/4$, i.e., the integrated monopoly profit π_I^{mon}.

Two-part tariff contract. Using (A3), we obtain from (10) the equilibrium marginal input price in (16). Substituting the latter back into (A3) and using (8) we obtain the equilibrium fixed fee in (16). The equilibrium net profits, obtained from (11), are:

$$\pi_I^T = \frac{a^2\left(4\beta(1-\theta)(1+\theta^2) + (1+\theta)(4+5\theta^2)\right)}{4(1+\theta)(4+5\theta^2)},$$

$$(A4)$$

$$\pi_D^T = \frac{a^2(1-\beta)(1-\theta)(1+\theta^2)}{(1+\theta)(4+5\theta^2)},$$

Linear contract. Using (A3), we obtain from (14) the optimal input price in (17). The equilibrium net profits are:

$$\pi_I^L = \frac{a^2[2\beta(2+\theta^2)(K-\beta(2+\theta^2)) + 16\beta(1-\theta^2) + 4(8+\theta^2)(1+\theta)^2]}{16(1+\theta)^2(8+\theta^2)},$$

$$(A5)$$

$$\pi_D^L = \frac{(1-\theta)a^2[(2+\theta^2)(2-\beta)+K]}{4(1+\theta)(8+\theta^2)^2}.$$

with $K = \sqrt{16(1-\theta^2)(1-\beta) + \beta^2(2+\theta^2)^2}$.

Proof of Lemma 2. Substituting the per-unit input prices from (16) and (17) into the last expression in (A2), we obtain:

$$q_D^T = \frac{a(2+\theta^2)}{(1+\theta)(4+5\theta^2)} > 0, \qquad q_D^L = \frac{[a(2-\beta)(2+\theta^2)+K]}{2(1+\theta)(8+\theta^2)} > 0.$$

Moreover, from the first expressions in (A4) – (A5), we obtain $\pi_I^T > a^2/4$ and $\pi_I^L > a^2/4$: the integrated firm always makes more profits when supplying firm D than when it fully forecloses D.

Proof of Proposition 1. It is straightforward from the second expression in (16) that $F(\beta) > (<) 0$ when $\beta > (<) \hat{\beta}(\theta)$, with $\hat{\beta}(\theta) \equiv [\theta^2(5+4\theta^2)]/[(4+5\theta^2)(1+\theta^2)]$.

Proof of Proposition 2. Substituting the per-unit input prices from (16) – (17) into the expressions in (A2), we obtain the equilibrium final-good prices and outputs under the two contract types. We calculate consumer surplus and total welfare by using $CS = [q_I^2 + 2\theta q_I q_D + q_D^2]/2$ and $TW = CS + \pi_I + \pi_D$. By direct comparisons of per-unit input prices, final-good prices, consumer surplus and total welfare under the two alternative contract types, we obtain Proposition 2.

Proof of Proposition 3. By comparing expressions in (A4) and (A5), we obtain Proposition 3.

B. Linear demand with Cournot competition

We first characterize the equilibrium outcomes, and then provide all relevant proofs. We solve the model backwards, starting from the second stage of the game. Irrespective of the contract type, from (21) and (22), we obtain each firm's best-response functions in the downstream market:

$$q_I(q_D) = \frac{a - \theta q_D}{2}, \qquad q_D(q_I, w) = \frac{a - w - \theta q_I}{2}. \tag{B1}$$

Solving the system of best-responses, we obtain final-good quantities and final-good prices, for given input price:

$$\tilde{q}_I(w) = \frac{a(2-\theta) + \theta w}{4 - \theta^2}, \quad \tilde{q}_D(w) = \frac{a(2-\theta) - 2w}{4 - \theta^2}.$$

$$(B2)$$

$$\tilde{p}_I(w) = \frac{a(2-\theta) + \theta w}{4 - \theta^2}, \quad \tilde{p}_D(w) = \frac{a(2-\theta) + (2-\theta^2)w}{4 - \theta^2}.$$

Each firm's gross profits are:

$$\tilde{\pi}_I(w) = \frac{[a(2-\theta) + \theta w]^2 + w[a(2-\theta) - 2w](4-\theta^2)}{(4-\theta^2)^2},$$

$$(B3)$$

$$\tilde{\pi}_D(w) = \left[\frac{a(2-\theta) - 2w}{4 - \theta^2}\right]^2.$$

Next, we solve the first stage, that is, we determine the equilibrium contract terms under a two-part tariff contract and under a linear contract given that I's disagreement payoff is $d = a^2/4$, i.e., the integrated monopoly profit π_I^{mon}.

Two-part tariff contract. Firms I and D choose w and F to maximize:

$$\left[\tilde{\pi}_I(w) + F - \frac{a^2}{4}\right]^\beta [\tilde{\pi}_D(w) - F]^{1-\beta} . \qquad (B4)$$

Maximizing (B4) with respect to F we have:

$$\tilde{F}(w) = \beta\tilde{\pi}_D(w) - (1-\beta)\left[\tilde{\pi}_I(w) - \frac{a^2}{4}\right]. \qquad (B5)$$

Substituting (B5) into (B4), the generalized Nash product reduces to an expression proportional to industry profits: w is chosen to maximize industry profits minus I's disagreement payoff. Since the disagreement payoff does not depend on w, the two firms essentially choose w so as to maximize the industry profit. By substituting (B3) into (24), we obtain the equilibrium per-unit input price in (26). Substituting the latter back into (B3), and using (B5), we obtain the equilibrium fixed fee in (26). The equilibrium net profits, obtained from (B3), are:

$$\tilde{\pi}_I^T = \frac{a^2(16 - 3(6-\theta)(2-\theta)\theta^2)}{4(4-3\theta^2)^2}, \qquad \tilde{\pi}_D^T = \frac{4a^2(1-\theta)^2}{(4-3\theta^2)^2}. \qquad (B6)$$

Linear contract. Firms I and D choose w to maximize:

$$[\tilde{\pi}_I(w) - d]^\beta [\tilde{\pi}_D(w)]^{1-\beta}.$$

By substituting (B3) into (25), we obtain the optimal input price in (27). The equilibrium net profits are:

$$\tilde{\pi}_I^L = \frac{a^2(8 + 8\beta(1-\theta)^2 + 4\beta^2(1-\theta)^2 - 3\theta^2)}{4(8-3\theta^2)},$$

$$\tilde{\pi}_D^L = \frac{4a^2(2-\beta)^2(1-\theta)^2}{(8-3\theta^2)^2}. \qquad (B7)$$

Proof of Lemma 3. Using (26), (27) and (B2), we obtain:

$$\tilde{q}_D^T = \frac{2a(1-\theta)}{4-3\theta^2} > 0, \qquad \tilde{q}_D^L = \frac{2a(2-\beta)(1-\theta)}{8-3\theta^2} > 0.$$

Moreover, from the first expressions in (B6) – (B7), we obtain $\tilde{\pi}_I^T > a^2/4$ and $\tilde{\pi}_I^L > a^2/4$: the integrated firm always makes more profits when supplying firm D than when it fully forecloses D.

Proof of Proposition 4. Substituting the per-unit input prices from (26) – (27) into the expressions in (B2), we obtain the equilibrium final-good prices and outputs under the two contract types. We calculate consumer surplus and total welfare by using $CS = [q_I^2 + 2\theta q_I q_D + q_D^2]/2$ and $TW = CS + \pi_I + \pi_D$. By direct comparisons of per-unit input prices, final-good prices, consumer surplus and total welfare under the two alternative contract types, we obtain Proposition 4.

Proof of Proposition 5. By comparing expressions in (B6) and (B7), we obtain Proposition 5.

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