

Technological change, campaign spending and polarization*

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Abstract

We present a model of electoral competition with endogenous platform choices and campaign spending where the division of voters between impressionable and ideological is also endogenous and depends on parties' strategic platform choices. Our approach results in a tractable model that provides interesting comparative statics on the effect of recent technological advancements. For instance, we can accommodate a new justification behind the well documented simultaneous increase in campaign spending and polarization: an increase in the effectiveness of electoral advertising or a decrease in the electorate's political awareness, surely increases polarization and may also increase campaign spending.

Keywords: *electoral competition, campaign spending, impressionable voters, semiorder lexicographic preferences*

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1 Introduction

Political campaigns are shaped by institutional and technological advancements as much as any other activity. For instance, the introduction of national TV in the United States gave place to the irruption of the advertising industry and allowed for a gradual introduction of candidate-centered professional campaigns (Hirano and Snyder, 2019). Similarly, the spread of internet gave place to the boom of social networks, which became the favorite platforms for voters to obtain information about candidates, and viceversa.¹

Coincidentally, as these technological improvements were incorporated into campaign management strategies, both campaign spending and platforms' polarization increased significantly in the U.S. For instance, polarization – steadily increasing in the liberal-conservative axis since the Second World War – recently reached levels not seen since the Reconstruction era.² Similarly, campaign spending in the US has increased since the 1960s (in real terms) but specially so since the turn of the century.³

It is often argued that technology, especially the fine targeting possibilities that come along internet and social media, is responsible for this simultaneous increase in polarization and spending.⁴ But for Hirsch (2019), spatial models of electoral competition – the most extended framework for studying polarization– typically do not support this argument (e.g., Herrera et al. 2008; Carrillo and Castanheira 2008; Ashworth and Bueno de Mesquita 2009). Hence, our main contribution is to provide a new channel for these stylized facts, with a comprehensive spatial model that depicts a causal relationship between the digitization of the economy and the empirical observations above.

We analyze the effects of technological changes on campaign spending and polarization by combining the two seminal models of Downs (1957) and Tullock (1980). In more detail, two office motivated parties first choose their electoral platforms and then decide upon the optimal level of costly electoral advertising. Voters instead are either impressionable and vote à la Tullock, or ideological and vote à la Downs as in the seminal papers by

¹According to Gottfried and Shearer (2016, 2017); Broockman and Green (2014), more than 60% of adults used internet to watch news by 2016 and one year later, adults used internet almost as often as TV to obtain news.

²According to the most used measured of polarization, DW-Nominate scores, polarization increased unambiguously since the end of the Second World War (Poole and Rosenthal, 1984). Specially so since the 1980 (McCarty et al., 2006). Moreover, the polarization levels reached by the Senators elected in 2015 is only comparable to the levels after the American Civil War (known as the Reconstruction era). See <https://legacy.voteview.com/> for the raw data. Outside the U.S., similar trends are reported for other OECD countries (Boxell et al., 2020).

³The raw data can be seen at <https://www.fec.gov/data/elections/president/2016/>, but for an explanation and additional references see Nickerson and Rogers (2014); Herrera et al. (2008).

⁴(Nickerson and Rogers, 2014) argue that the capacity for storing data is a turning point for campaign managers to turn to fine targeting of mobilization strategies and ads. Additionally, Allcott et al. (2020); Allcott and Gentzkow (2017) among others show evidence that links technology and polarization, in particular the use of social networks like Facebook.

Baron (1994); Grossman and Helpman (1996). Ideological voters support the party that proposes the platform closest to their bliss point. Hence, parties compete for a share of ideological voters as if they were competing in a Downsian model of electoral competition. On the contrary, impressionable voters are swayed towards one party or the other by costly electoral advertising. Given each party’s electoral advertising, the effectiveness of the latter determines the fraction of impressionable voters that supports each of them.

An important novelty of our approach is that we allow for the split of voters to be endogenous. The endogenous division of voters across the two types depends on the differentiation between the proposed platforms, with the fraction of ideological voters increasing in polarization. This assumption captures the idea that the more diverse platforms are, more voters vote based on their ideological preferences since platforms become “salient”. On the contrary, when parties’ platforms are similar, voters may have a “hard” time or little interest in distinguishing them, and turn to electoral advertising that determines (probabilistically) their voting behavior. Alas intuitive, one way to formalize this behavior is that voters’ preferences are described by a lexicographic semiorder (see Luce 1956; Tversky 1969; Rubinstein 1988; Leland 1994; Manzini and Mariotti 2012), consistent with the notion of the “just noticeable difference”.⁵ With such preferences, each individual chooses which party to support on the basis of platforms, but only if those are different “enough” (i.e., above a certain threshold). If the platforms are not sufficiently different, then the voter is influenced exclusively by parties’ electoral advertising.

By endogenizing voters’ types depending on the electorate’s political awareness, we maintain parties’ well established incentives to polarize and soften the advertising stage as in the literature of endogenous platforms and electoral advertising where platforms’ choice and campaign spending can be thought as horizontal and vertical differentiation models. Typically, when the marginal return of the vertical component is large, the incentives to invest in this dimension are large as well. Thus, in order to soften the vertical competition, agents differentiate more horizontally. Nevertheless, and in contrast to most of the previous contributions – that we later discuss in detail– our model predicts that technology change can simultaneously increase polarization and electoral spending. Additional differences to previous works on this topic are: i) equilibrium platforms are unique and in pure strategies when parties are symmetric, ii) we provide a tractable setting for the analysis of asymmetric parties (e.g., in their marginal cost of campaigning), and iii) we analyze three different channels in which technology may affect polarization and electoral spending.

The above sketched model encompasses the effect of technological changes on elec-

⁵In experimental psychology, the Weber-Fechner law remarks that the “just noticeable difference” is not necessarily influenced by the physiological but rather by psychological factors. The law states that the just-noticeable difference is increasing in the absolute level of the subtracts (Falmagne, 2002).

toral competition through three distinct and non-mutually exclusive channels. The first two reflect the way campaigns for impressionable -*Tullock*- voters are conducted and how technology and changes in campaign management affect a) the *effectiveness*, and b) the *cost* of electoral advertising. One could reasonably argue that recent technological advances have increased the *effectiveness of electoral advertising* since campaigns can be well targeted, and have decreased the *marginal cost of advertising* given the possibility of reaching large masses. The third channel captures how technology affects electoral competition through the electorate’s political awareness and the endogenous division of voters to impressionable and ideological. Political awareness is captured by the *conversion rate* at which impressionable voters become ideological as polarization increases. Although there is no consensus on the historical evolution of this conversion rate (or the citizen’s awareness) we show how it affects electoral competition and discuss its implications.

Consider first an increase in the *effectiveness* of electoral advertising. Since every dollar spent on campaigns leads to higher returns, parties have incentives to symmetrically increase their campaign spending (*spending effect*). To mitigate such increase in electoral advertising, parties have incentives to polarize their platforms and reduce the number of impressionable voters and hence their spending (*polarization effect*). If the spending effect dominates, campaign spending and polarization increase simultaneously and can explain the observed trends in US politics. If instead the polarization effect dominates, an increase in the electoral effectiveness is overcompensated by an increase in polarization and campaign spending decreases. Actually, the effectiveness of electoral advertising proves crucial if one wants to explain the simultaneous increase in campaign spending and polarization in terms of campaign management. A decrease in the marginal *costs* of running a campaign does not affect neither polarization nor total spending. While a decrease in the marginal cost of advertising leads to more advertising, the lower marginal cost of the latter leaves campaign spending and polarization unaffected. Finally, the *conversion rate* at which impressionable voters become ideological when polarization increases affects both campaign spending and polarization in a similar manner as a change in the effectiveness. If one wants to explain the simultaneous increase in polarization and spending exclusively through this channel, then one would require that the conversion rate has been decreasing, meaning that voters do not respond much to changes in platforms. Such low conversion rate could be attributed for example to a “media malaise”, possibly associated with mistrust of politicians and disenchantment with politics (Norris, 2000; Newton, 1999).

Luddites would argue that our results highlight the pervasive effects of technological improvements. While it is true that the adoption of new technologies alters the status quo, there are more optimistic interpretations of our results. First of all, if one party

has a comparative advantage in advertising, it chooses a more centrist policy and it is then more likely to win. That is, in expectation, in spite of party polarization, the implemented policies are more centrist. More generally, we provide a comprehensive and flexible theoretical framework that allows for a detailed understanding of the link between technology and polarization. Hence, this model would allow for theory-based empirical research and informed regulatory recommendation on controversial topics: spending caps, treatment of personal data, etc.

1.1 Related Literature

In terms of methodology and results, our work contributes to several strands of the literature.

Technology, Polarization, and Campaign Spending: In terms of research question and methodology the closest papers are by Herrera et al. (2008) and Hirsch (2019) who incorporate changes in campaign technologies in a spatial model of electoral competition and focus on polarization and spending. Contrary to us, Herrera et al. (2008) show that an improved campaign technology reduces polarization. This is because while in our model polarization softens competition in the electoral advertising stage (as the endogenous valence literature), in their model it has the exact opposite effect. Hirsch (2019) instead can also explain the simultaneous increase in polarization and campaign spending but there are several distinctive features between our and their paper. In Hirsch (2019), polarization arises “because of spending” (i.e., candidates polarize because they can compensate distance to the median with higher spending), while in our case, polarization precisely arises to reduce spending. In terms of results our papers also differ. For example, we are obtaining non-monotonicity in campaign spending, equilibrium platforms in pure strategies, and the possibility of platform convergence in equilibrium. Also the way we model and motivate technology is quite different (e.g., we consider different levels of noise in the contest compared to the all pay auction which is the extreme case of our technology we do not consider).

Prummer (2020) provides interesting recent work along the same research question but analyzed in a non-spatial setting. She focuses on changes in targeting technology and fragmentation of media networks as determinants of polarization. By characterizing candidates’ optimal policy proposals and corresponding targeting strategies, Prummer (2020) shows that recent improvements in targeting technology could have lead candidates opting for extreme policy proposals targeting a narrow subset of voters.

Salience and focusing in elections: We are contributing to a recent growing literature on behavioral political economy by thinking of our model as one of endoge-

nous valence where rather than additive separable preferences over platforms and campaigns, individuals have semiorder lexicographic preferences (see Luce 1956; Tversky 1969; Manzini and Mariotti 2012; Rubinstein 1988; Leland 1994).⁶ This assumption is in line with salience models where decision makers overweight attributes that exhibit greater differences in the available choice set (Bordalo, Gennaioli, and Shleifer 2012, 2013a,b, 2015; Bushong, Rabin, and Schwartzstein 2017; Kőszegi and Szeidl 2012; Spiegel 2014).⁷ Callander and Wilson (2006) and Nunnari and Zápala (2017) introduce related context-dependent preferences in political economy models. In the former, the utility of voting depends not only on the direct benefits of turnout but also on the context, i.e., the candidates’ polarization. In the latter, the authors provide a model of “focusing”, where voters’ attention is captured more on the issues that candidates’ proposals differ more. Similarly, Amorós and Puy (2013); Aragonés, Castanheira, and Giani (2015); Denter (forthcoming) focus on electoral competition models when parties have the ability to affect the relative salience of different issues via their strategic actions (e.g., allocation of time or effort). In our model, parties’ strategic actions affect the salience of platforms versus advertising, and hence voters in the society split among those casting votes either in an informed or uninformed manner. Therefore, although very different in nature, our model links with recent literature where some voters may be partially informed regarding parties’ policy proposals (Aragonés and Xefteris, 2017a; Eguia and Nicolò, 2019).

Endogenous Valence: In that literature (e.g., Carrillo and Castanheira (2008); Iaryczower and Mattozzi (2013) among others), voters typically have additive separable preferences over platforms and valence. In the closest work to ours (Ashworth and Bueno de Mesquita, 2009; Zakharov, 2009), platform diversification softens the competition in the valence accumulation stage.⁸ These dynamics are exactly the ones presented in our model through the endogenous division of voters to ideological and impressionable. Our model, however, proves more tractable than models with additive separable preferences. In contrast to Ashworth and Bueno de Mesquita (2009), our model admits a pure strategy equilibrium in the platform stage for most parametrizations and does

⁶To the best of our knowledge we are the first to consider an endogenous division of voters across ideological and impressionable in a long literature following Baron (1994); Grossman and Helpman (1996). This is an independent contribution of the model attributed to the different structure of voters’ preferences than those typically assumed.

⁷Semiorder lexicographic preferences can be seen as a particular case of salience in which: a) only the difference in one attribute (policy platforms) is relevant to assign the weight of each dimension, and b) individual weights on each attribute are discrete and take value 0 or 1. In aggregate terms, our assumption on the endogenous division of voters across types has a similar effect to the one of salience at the individual level: the smaller the distance between platforms, the smaller is their weight on the final outcome of the electoral competition.

⁸The opposite effect may occur when parties have ideological motives (Epstein and Nitzan, 2004; Cardona, De Freitas, and Rubí-Barceló, 2018) or there is uncertainty regarding the valence investment (Carrillo and Castanheira, 2008).

not require that voters’ ideologies are uniformly distributed. A non-uniform distribution is also permitted in Zakharov (2009) but only when focusing on local Nash equilibria. In contrast, we are able to characterize Nash equilibria in pure strategies for a general distribution of voters’ ideology (symmetric and log-concave in this paper) and perform relevant comparative statics. Also in contrast to the previous models, we are also able to solve the model and obtain results when parties have heterogeneous campaign costs, for example due to an incumbency advantage (Meirowitz, 2008; Pastine and Pastine, 2012).

Contest Theory: Finally, we also contribute to this strand of the literature because, in our model, parties compete for a share of impressionable voters as if they were competing in a Tullock contest (with the “noise” of the latter capturing the effectiveness of electoral advertising).⁹ Notice that the value of the “prize” (of the contest) allocated based on electoral advertising is endogenous and depends on platform selection. Consequently, by fixing closer platforms parties not only attract more voters from their competitor as in any Downsian model, but also increase the share of Tullock voters for which rent-dissipation arises (Tullock, 1980; Nitzan, 1994). In terms of results, the value of the prize depending on platforms provides a result in contrast to most of the contest literature where the relationship of electoral advertising and the noise of the contest success function is monotonic.

2 Model

Let two political parties $i \in \{L, R\}$ first propose (and commit to) platforms x_i in the policy space $X = [0, 1]$ and then choose the level of campaign advertising $e_i \geq 0$. Without loss of generality, we assume $x_L \leq x_R$. Let $S_i(x_L, x_R, e_L, e_R)$ be the vote share for party i and $c_i(e_i) = \mu_i e_i$ the cost of advertising, with $\mu_i > 0$ denoting the constant marginal *cost of advertising*. Without loss of generality, we assume $\mu_L \leq \mu_R$. Parties’ are office motivated with payoffs $\Pi_i = S_i(x_L, x_R, e_L, e_R) - c_i(e_i)$.¹⁰

Voters have a preferred policy x drawn from distribution $G(x)$ with corresponding density $g(x)$ symmetric and log-concave (i.e., $(\ln g(x))'' \leq 0$), with full support in X .¹¹ Independent of their ideal policy some voters are ideological and some are impressionable. The ideological citizens vote sincerely for the party whose proposed platform is closer to them and split their vote if indifferent (*à la Downs*). The utility of a voter with ideology

⁹See Corchón (2007); Konrad (2009); Serena and Corchón (2018) for surveys.

¹⁰Here each party’s objective is to maximize its vote share net of the campaign costs. Alternatively, $S_i(x_L, x_R, e_L, e_R)$ can also be interpreted as the probability of winning by assuming parties’ uncertainty on a representative voter’s preferences as in Aragonès and Xefteris (2017a).

¹¹Under symmetric campaign costs, our results can be extended to asymmetric distributions of ideology (available upon request).

x that votes for party i is $u_x(i) = -|x - x_i|$.¹²

The impressionable citizens' vote depends only on electoral advertising. In particular, we assume that given parties' advertising, the probability that an impressionable citizen votes for party i is determined *à la Tullock* and hence equal to $e_i^\eta / (e_L^\eta + e_R^\eta)$, where parameter $\eta > 0$ captures the *effectiveness of electoral advertising*.¹³ Parameter η captures how relative differences in electoral spending translate into a difference in vote shares. In other words, an increase in η makes a given difference in spending more determinant of the electoral outcome, permitting us to interpret η as the effectiveness of electoral advertising. If $\eta \rightarrow 0$, impressionable voters split equally across the two parties for any level of electoral spending. However, as η increases, the allocation of impressionable voters across parties becomes more responsive to electoral advertising. Impressionable voters voting on the basis of persuasive electoral advertising is a standard assumption in this literature (see for example the seminal papers by Baron (1994); Grossman and Helpman (1996) and a large literature thereafter). The specific proposed function is the seminal contest success function (CSF) introduced by Tullock (1980). This function is extensively used in the literature and apart from tractability also satisfies relevant axiomatic properties (Skaperdas, 1996) and can be micro-founded (see Section 5.5 of the Appendix).¹⁴

The endogenous division of voters across ideological and impressionable depends on the level of polarization. Let $y = x_R - x_L \in X$ be the platforms' polarization and $F(y)$ a continuous cumulative distribution function, log-concave (i.e., $(\ln F(y))'' \leq 0$), with corresponding density $f(y)$ with full support in X . The share of ideological voters is $F(y)$, and therefore the share of impressionable voters is $1 - F(y)$.¹⁵

It is important to stress that the above representation of our model in aggregate terms and the division of voters according to $F(y)$ is enough to understand the dynamics in our model. For presentation purposes, and to avoid further notation we use directly $F(y)$ as the foundation of our model. However, in the Appendix (Section 5.5) we detail how this division can be microfounded through voters' individual behavior. In brief,

¹²The assumption of a particular distance function is made without loss of generality.

¹³We assume $\eta > 0$ to focus on cases in which our model differs from a Downsian model. If $\eta = 0$, equilibrium spending is always zero and platform choice coincides with that of the standard Downsian model.

¹⁴The campaign stage for impressionable voters is resolved via Tullock's ratio-form CSF that facilitates the comparative statics of our model. In the symmetric case, our results would have the same qualitative features if we considered the difference-form CSF proposed by Alcalde and Dahm (2007), the tractable noise CSF proposed by Amegashie (2006) or the relative-difference CSF by Beviá and Corchón (2015) under the parameter restrictions proposed by Balart, Chowdhury, and Troumpounis (2017).

¹⁵We allow the general case where $F(0) \geq 0$ throughout the paper. However, the distinction between $F(0) = 0$ and $F(0) > 0$ is important both in interpretation and results as for example only the latter is compatible with a convergent equilibrium. Assuming that $F(0) > 0$ permits that some voters remain unaffected by electoral advertising even when the two platforms are identical and vote *à la Downs* (hence randomizing their vote). If in contrast one assumes that $F(0) = 0$, given two identical platforms all votes are driven by electoral advertising and vote *à la Tullock*.

consider voters with semiorder lexicographic preferences. Let the first attribute reflect policies and the second attribute electoral advertising. Each voter draws a preferred policy x from $G(x)$ and a level of “sensitivity” ϕ from $F_\phi(y) = Pr(\phi \leq y)$, denoted $F(y)$ for simplicity. That is, as platforms diverge, an individual voter is ideological with ex-ante probability $F(y)$, and impressionable with ex-ante probability $1 - F(y)$.¹⁶ Hence, despite the stark partition of voters into ideological and impressionable presented in the main text, an individual voter’s expected behavior *a priori* depends both on platforms and advertising. The microfoundations of individuals’ behavior (in terms of semiorder lexicographic preferences as above and in terms of a salience model) and the effect of advertising on individual voting behavior (as well as the derivation of the Tullock CSF) are detailed in Section 5.5 of the Appendix.

The timing of the game is as follows: At $t = 1$, the political parties simultaneously choose the political platforms that maximizes their payoff. At $t = 2$, having observed the platforms choices and the share of impressionable voters determined by the polarization, parties choose the advertising levels. Finally, at $t = 3$, voters vote. Given the nature of our game, we focus on subgame perfect Nash equilibria.

3 Results

Given the described game, let $\bar{x} = \frac{x_L + x_R}{2}$ be the indifferent ideological voter for $x_L \neq x_R$. Ideological voters with $x \leq \bar{x}$ vote for L , while the remaining ones vote for R . Thus, party L obtains a share $S_{Idl}^L = G(\bar{x})$ of the ideological voters and party R , $S_{Idl}^R = 1 - G(\bar{x})$. If $x_L = x_R$, then $S_{Idl}^R = S_{Idl}^L = \frac{1}{2}$. Given that the individual probability that an impressionable citizen votes for party i is $\frac{e_i^\eta}{e_L^\eta + e_R^\eta}$, the expected share of impressionable votes to party i is $S_{Imp}^i = \frac{e_i^\eta}{e_L^\eta + e_R^\eta}$ for party i . Hence, the expected vote share obtained by the parties can be then written as a weighted average of the previous two:

$$S_i(x_L, x_R, e_L, e_R) = F(y)S_{Idl}^i(x_L, x_R) + (1 - F(y))S_{Imp}^i(e_L, e_R). \quad (1)$$

This expression highlights the effect of platform choices in our game. First, platform choice affects how ideological voters split between the parties (via $S_{Idl}^i(x_L, x_R)$). Second,

¹⁶From a voter’s point of view, the realization of the value of ϕ determines if he/she is voting based on policies or advertising. If polarization is greater than ϕ , the voter is ideological. If polarization is less than ϕ the voter is impressionable. Alternatively, instead of justifying the endogenous division via semiorder lexicographic preferences, one could assume that there are two types of voters (i.e., ideological and impressionable) and types are drawn randomly where the frequency of drawn types depends on the level of polarization. These interpretations belong to the category of context depending preferences and are receiving increasing attention in Economics (Bordalo et al., 2012, 2013a,b, 2015; Bushong et al., 2017; Kőszegi and Szeidl, 2012; Spiegler, 2014).

platform choice affects the ideological-impressionable composition of the electorate (via $F(y)$). As common in Downsian type models, converging towards the opponent is beneficial due to the relocation of the indifferent voter. However, in our model platform convergence results into an increase in the share of impressionable voters, and hence a tougher competition in the (costly) advertising stage. The above trade-off makes platforms' choice a non-trivial task.

3.1 Symmetric parties

For illustrative purposes, and to highlight our main results in the simplest framework, we first pay attention to parties having identical marginal costs, i.e., $\mu_A = \mu_B = \mu$. Recall that voters' behavior is essentially parametric and hence the last stage in our backward induction reasoning is the choice of advertising. Equilibrium advertising can be solved as effort in a Tullock contest with symmetric players, in which the prize of winning equals the share of impressionable voters. The equilibrium in this stage is described in the lemma below

Lemma 1. *For all $\eta \leq 2$ there exists a unique Nash equilibrium in the campaign stage and advertising is given by $e_i^*(x_L, x_R) = (1 - F(x_R - x_L)) \frac{\eta}{4\mu}$, for all i .*

All proofs appear in the Appendix.

Our first Lemma draws from previous results in the contest theory literature. It characterizes the equilibrium advertising levels while stating a condition on the campaigns' effectiveness η such that an equilibrium in pure strategies exists. If campaigns are not effective "enough" (i.e., $\eta \leq 2$), an equilibrium in pure strategies exists and is unique with advertising being: a) increasing in the campaign effectiveness η , b) decreasing in the marginal cost μ , and c) decreasing in the platforms' polarization y (recall that $y = x_R - x_L$).¹⁷ Note that the symmetric spending in equilibrium implies that in equilibrium impressionable voters split between the two parties (i.e., $S_{Imp}^L = S_{Imp}^R = 0.5$).

Anticipating the advertising levels in the second stage, the political parties' maximization problem in the first stage is to choose the platform that maximizes their payoff.

¹⁷Restricting attention to $\eta \leq 2$ (as typical in the contest literature) does not impose a significant limitation in our paper. For $\eta > 2$ there is no equilibrium in pure strategies in the campaign stage and the characterization of the mixed strategy equilibrium results in non-tractable objects with support in countable infinite mass points as shown by Ewerhart (2015). However, and letting technicalities aside, permitting $\eta > 2$ does not render substantively new results. From Alcalde and Dahm (2010) we know that all mixed-strategy equilibria at the spending stage are payoff equivalent with parties' expected payoffs in that stage equal to zero and $E(e_i^*(x_L, x_R)) = (1 - F(x_R - x_L)) \frac{1}{2\mu}$. This equivalence result hence guarantees that platforms for $\eta > 2$ can be trivially characterized in our model by fixing $\eta = 2$.

For instance, for party L , the payoff at $t = 1$ is $\Pi_L(x_L, x_R, e_L^*(x_L, x_R), e_R^*(x_L, x_R))$, which can be written as:

$$S_L(x_L, x_R) - c_L(e_L^*(x_L, x_R)) = F(x_R - x_L)S_{Idl}^L(x_L, x_R) + (1 - F(x_R - x_L))S_{Imp}^L - \mu e_L^*(x_L, x_R)$$

where $S_{Imp}^L = \frac{1}{2}$ and $S_{Idl}^L = G(\bar{x})$ if $x_L \neq x_R$ or $S_{Idl}^L = \frac{1}{2}$ if $x_L = x_R$.

The trade-off parties face is now evident. Consider that for a given set of platforms (x_L, x_R) , the leftist party chooses to propose a less extreme platform. On the one hand, the indifferent voter is more to the right, which has a positive effect on S_{Idl}^L as in a standard Downsian model. On the other hand, it converts some ideological voters to impressionable ones, which by Lemma 1 increases the spending on advertising in the second stage of game. Similar to Tirole (1988) and Ashworth and Bueno de Mesquita (2009) divergence is a tool of softening competition in the vertical dimension (the advertising stage in our model).

The importance of each of the two forces present in the trade-off is determined by: i) the rate at which impressionable voters become ideological as polarization increases (represented by the reversed hazard rate $\frac{f(y)}{F(y)}$) and ii) the concentration of voters around the median voter $g(\frac{1}{2})$. A higher value of the reversed hazard rate increases the rate at which impressionable voters become ideological as polarization increases. Hence, it provides incentives to polarize and avert high advertising costs when competing for impressionable voters. Recall that $F(y)$ is log-concave. This implies that the relevant rate $\frac{f(y)}{F(y)}$ is decreasing in the levels of polarization y . Thus, incentives to polarize are higher the closer the two platforms are and maximal under full convergence $y = 0$. In contrast, a higher concentration of voters around the median $g(\frac{1}{2})$ increases incentives to converge at the median and hence reduces incentives to polarize. The relative importance of these two forces is moderated by the campaign effectiveness η . A higher η scales-up advertising level and campaign spending, increasing (decreasing) the relevance of $\frac{f(y)}{F(y)}$ ($g(\bar{x})$) in platforms' selection.

Let us define by \bar{y} the distance between the two platforms in the interior equilibrium (i.e., $0 < x_L^* < \frac{1}{2} < x_R^* < 1$). By solving the first order conditions, \bar{y} is implicitly defined as $\frac{f(\bar{y})}{F(\bar{y})} = \frac{2}{\eta}g(\frac{1}{2})$. As we can see and we later argue in detail, the above three forces shape this equilibrium distance. We characterize the unique equilibrium of the platform stage in the following proposition.¹⁸

Proposition 1. *Let $\eta \leq 2$. For any $\mu > 0$ there exists a unique subgame perfect Nash equilibrium.*

For $F(0) > 0$, the following equilibrium types arise:

¹⁸Strictly speaking, there is a unique equilibrium satisfying the restriction $x_L \leq x_R$. A mirror equilibrium with $x_R < x_L$ also exists.

- (Convergent equilibrium) $x_L^* = x_R^* = \frac{1}{2}$, if $\frac{f(0)}{F(0)} \leq \frac{2}{\eta}g(\frac{1}{2})$,
- (Interior equilibrium) $x_L^* = \frac{1}{2} - \bar{y}$, and $x_R^* = \frac{1}{2} + \bar{y}$, if $\frac{f(1)}{F(1)} < \frac{2}{\eta}g(\frac{1}{2}) < \frac{f(0)}{F(0)}$.
- (Extremism equilibrium) $x_L^* = 0$ and $x_R^* = 1$, if $\frac{2}{\eta}g(\frac{1}{2}) \leq \frac{f(1)}{F(1)}$.

If $F(0) = 0$, the convergent equilibrium does not exist. The extremism equilibrium arises if and only if $\frac{2}{\eta}g(\frac{1}{2}) \leq \frac{f(1)}{F(1)}$ and the interior equilibrium arises otherwise.

Electoral advertising for each of the above SPNE is uniquely characterized in Lemma 1.

First, note that in contrast to previous literature with similar dynamics –where equilibrium platforms require mixed strategies – Proposition (1) shows that there exists a unique pure strategy SPNE.¹⁹ In this unique equilibrium, the level of polarization (y^*) can be zero (convergent equilibrium), one (extremism equilibrium) or \bar{y} (interior equilibrium). The emerging type of equilibrium depends on: a) the concentration of voters around the median $g(\frac{1}{2})$, b) the rate at which impressionable voters become ideological when polarization increases $\frac{f(y)}{F(y)}$, and c) the effectiveness of electoral campaigns η .

Starting with the *effectiveness* of the electoral campaigns, a large value of η makes the competition for impressionable votes tougher, which exacerbates advertising costs in the second stage (Lemma 1). Therefore, a high value of η provides incentives to polarize platforms in the first stage in order to reduce the number of impressionable voters. If $F(0) > 0$ (i.e., there exists a share of ideological voters under convergence), as η increases, indeed we may move across types of equilibria with convergence occurring for a smaller set of parameters. But also platforms become more polarized in the interior equilibrium (i.e., \bar{y} is increasing in η). Similarly, when many voters are concentrated around the median (i.e., a high value of $g(\frac{1}{2})$), there are strong incentives to propose moderate platforms. Thus, a strong presence of moderate voters leads to equilibria of “low” polarization (again, either across equilibria types or within the interior equilibrium). If $F(0) = 0$ (i.e., no ideological voters when parties fully converge), nothing changes except that a convergent equilibrium never arises.

The *conversion rate* at which impressionable voters become ideological when polarization increases (i.e, the reversed hazard rate $\frac{f(y)}{F(y)}$) also helps understanding our result. This rate captures the incentives of increasing polarization as a way of reducing electoral advertising. By log-concavity of $F(y)$, the rate is monotonically decreasing and hence takes its maximum value at $y = 0$ and its minimum value at $y = 1$. If the maximum value

¹⁹Note here that we have restricted attention to $\eta \leq 2$ due to the mixed strategies in the campaign stage for $\eta > 2$. However, the unique pure strategy equilibrium characterized in the platform substage –which is the main difference to Ashworth and Bueno de Mesquita 2009– is also an equilibrium for $\eta > 2$ with equilibrium platforms the same as the ones characterized for $\eta = 2$. This is due to the payoff equivalence result by Alcalde and Dahm (2010).

of the conversion rate is small “enough” (i.e., $\frac{f(0)}{F(0)} \leq \frac{2}{\eta}g(\frac{1}{2})$) the original Downsian result of platform convergence emerges. Despite full convergence, the conversion rate is so low that increasing polarization does not increase the share of ideological voters enough to diminish electoral advertising in a profitable fashion. Analogously, if its minimum value is large “enough” (i.e., $\frac{2}{\eta}g(\frac{1}{2}) \leq \frac{f(1)}{F(1)}$), polarization is very effective in restraining electoral advertising and extremism emerges.

A distributional change in the function determining the distribution of voters across types also gives interesting comparative statics.

Notation 1. Let ρ parametrize the sensitivity of the conversion rate due to inputs other than polarization (e.g., awareness or interest in politics). The conversion rate $f(y; \rho)/F(y; \rho)$ satisfies the monotone likelihood ratio property (MLRP) in y if for any $\rho_1 > \rho_0$ it holds that

$$\frac{f(y; \rho_1)}{F(y; \rho_1)} > \frac{f(y; \rho_0)}{F(y; \rho_0)}$$

where $F(y; \rho)$ and $f(y; \rho)$ are differentiable in ρ .

Any increase in ρ (i.e., a change in the distribution function $F(\cdot)$ satisfying the MLRP) makes the conversion rate of impressionable voters to ideological more responsive to changes in polarization. This may move platforms across types of equilibria (favoring more polarization), while also at any interior equilibrium, polarization is increasing in ρ (it directly follows from applying implicit differentiation to the interior equilibrium condition $\frac{f(\bar{y})}{F(\bar{y})} = \frac{2}{\eta}g(\frac{1}{2})$). Thus, a change in the distribution function $F(\cdot)$ satisfying the MLRP has similar effects to an increase in electoral effectiveness.

Finally, note that platforms’ characterization does not depend on the costs of campaigning μ in any manner. Given that this cost is symmetric for the two parties, increasing or decreasing it would only rescale the equilibrium levels of advertising e_i (from Lemma 1) but will not modify the actual level of campaign spending μe_i and hence polarization.

Effects on Campaign Spending

A technological change that increases the campaign effectiveness has an ambiguous effect on campaign spending. This is apparent when we look at the relevant expression:

$$\frac{\partial \mu e_i^*(x_L^*, x_R^*)}{\partial \eta} = \overbrace{\frac{1 - F(y^*)}{4}}^{\text{Spending effect (+)}} \overbrace{-f(y^*) \frac{\partial y^*}{\partial \eta}}^{\text{Polarization effect (-)}} \frac{\eta}{4} \quad (2)$$

On the one hand, *ceteris paribus* an increase in η increases advertising (Lemma 1). On the other hand, it also increases in equilibrium level of polarization y^* (Proposition 1), which in turn, decreases the share of impressionable voters and so the levels of advertising (Lemma 1). We call the former the spending effect, while we label the latter as the polarization effect.

At the fully divergent and fully convergent equilibria there is no polarization effect, $\frac{\partial y^*}{\partial \eta} = 0$, and spending either increases monotonically with η due to the spending effect or is unaffected. At the interior equilibrium however, the polarization effect takes place and mitigates the spending effect. If polarization increases disproportionately with η , the polarization effect may even overturn the spending effect, and hence observe η and campaign spending move in opposite directions. In Lemma (2) below, we provide the conditions for a simultaneous increase in campaign spending and polarization, and we use an example to illustrate it.

Lemma 2. *If the equilibrium is interior, a technological change that increases the campaign effectiveness η – and hence polarization – also increases campaign spending due to the spending effect dominating the polarization effect if and only if effectiveness is low “enough”. Formally, if the equilibrium is interior $\frac{\partial \mu e_i^*(x_L^*, x_R^*)}{\partial \eta} \geq 0$ if and only if $\eta \leq 2g(\frac{1}{2}) \frac{1-F(\bar{y})}{f(\bar{y})} \left[\frac{F(\bar{y})}{f(\bar{y})} \right]'$.*

Example 1: $F(y)$ is uniformly distributed over $(0, 1]$ with a mass at zero $F(0) = \frac{1}{10}$. The “conversion rate” now – from impressionable to ideological voters – is proportional to polarization: $\frac{f(y)}{F(y)} = \frac{\frac{9}{10}}{\frac{1}{10} + \frac{9}{10}y}$. From Proposition (1), we are at a convergent equilibrium for $\eta \leq \frac{2}{9}g(\frac{1}{2})$, at an interior equilibrium for $\frac{2}{9}g(\frac{1}{2}) < \eta < \frac{20}{9}g(\frac{1}{2})$ and at an extremism equilibrium for $\eta \geq \frac{20}{9}g(\frac{1}{2})$. These conditions highlight how the concentration of voters around the median gives rise to different equilibrium types. If for example $g(\frac{1}{2})$ is large then extremism can be excluded as an outcome for any level of campaign effectiveness. In the interior equilibrium polarization given by $y^* = \bar{y} = \frac{\eta}{2g(\frac{1}{2})} - \frac{1}{9}$ and it is straightforward to see that polarization is increasing in η . Using the condition in Lemma (2) we can also get the non-monotone comparative statics on campaign spending in the interior equilibrium and show that campaign spending is increasing in the campaign effectiveness for η low enough (i.e., $\eta \leq \frac{10}{9}g(\frac{1}{2})$) and decreasing otherwise.

In Figure 1 we graphically represent the comparative statics of changes of η on polarization and campaign spending also assuming that $g(\frac{1}{2}) = \frac{1}{2}$. Consider first the equilibrium levels of polarization (solid line). For η lower than $\frac{1}{9}$ or greater than $\frac{10}{9}$ the convergent and extremist equilibria respectively arise. For $\frac{1}{9} < \eta < \frac{10}{9}$ the interior equilibrium arises and polarization is strictly increasing in η . Let’s now turn to campaign spending (dashed line). For $\eta \leq \frac{1}{9}$, polarization is constant and equal to zero and campaign spending is

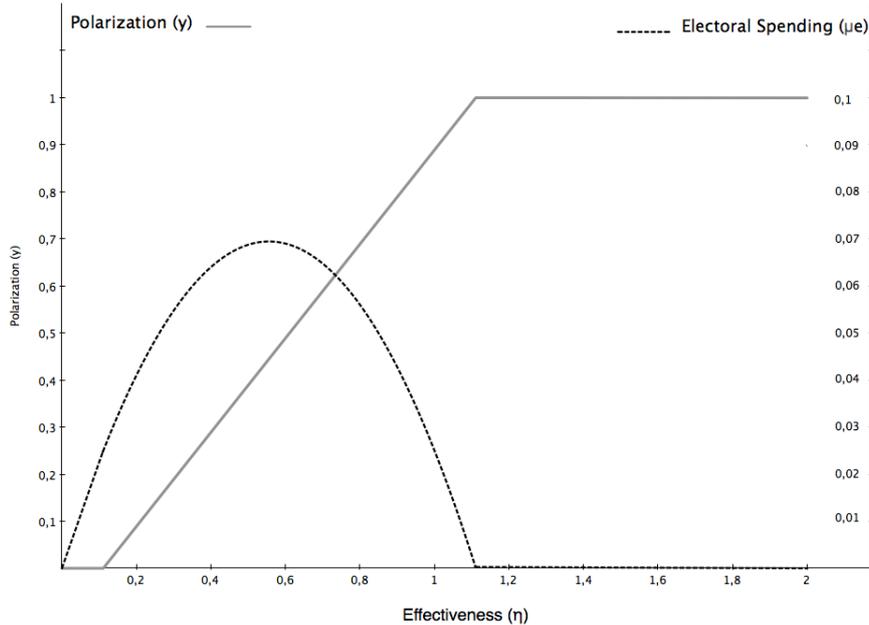


Figure 1: Comparative statics on η for campaign spending and polarization. Uniform distribution of $F(y)$ with $F(0) = \frac{1}{10}$ and $g(1/2) = 1/2$.

monotonically increasing in η (as in any standard Tullock contest). If $\eta \geq \frac{10}{9}$ the extremism equilibrium arises and due to all voters being ideological campaign spending is zero.²⁰ In the interior equilibrium interval (i.e., $\eta \in (\frac{1}{9}, \frac{10}{9})$), the non-monotonicity arises. Campaign spending increases until reaching $\eta = \frac{5}{9}$ due to the spending effect being larger than the polarization one. On the contrary, spending decreases for values of η larger than $\frac{5}{9}$ due to the polarization effect overcoming the spending effect and till extremism arises.

Polarization and campaign spending are also affected by the rate at which impressionable voters become ideological as polarization increases parametrized by ρ . A technological change that increases ρ has an ambiguous effect on campaign spending. As above, the condition comes from looking at the derivative of campaign spending with respect to ρ .

$$\frac{\partial \mu e_i^*(x_L^*, x_R^*)}{\partial \rho} = \overbrace{\frac{\partial F(y^*)}{\partial \rho} \frac{\eta}{4}}^{\text{Spending effect (+)}} \overbrace{\frac{\partial F(y^*)}{\partial y^*} \frac{\partial y^*}{\partial \rho} \frac{\eta}{4}}^{\text{Polarization effect (-)}}$$

On the one hand, an increase in ρ increases the “stock” of impressionable voters $1 - F(y)$ (this follows directly from MLRP which implies first order stochastic dominance). The spending effect then suggests that, for a given level of polarization y , an increase in ρ

²⁰ Zero spending under extremism arises because of the simplification of no mass of impressionable voters under maximal platform separation. One could trivially extend our model by including a mass of impressionable voters $0 < \delta < 1$ even under extreme polarization. In that case, in the extreme equilibrium campaign spending is positive and strictly increasing in η . Our characterization in Proposition 1 would remain unaffected and advertising levels in Lemma 1 will be simply rescaled but not qualitatively affected.

increases advertising (Lemma 1). On the other hand, an increase in ρ makes voters more responsive to polarization, by affecting the “conversion” rate $\frac{f(y)}{F(y)}$, and therefore provides incentives to increase polarization (Proposition 1). But this increased polarization in turn, decreases the share of impressionable voters and so decreases the levels of advertising (Lemma 1). As before, we label this latter effect as the polarization effect.

At the convergent equilibria there is no polarization effect since $\frac{\partial y^*}{\partial \rho} = 0$ and spending increases monotonically with ρ . That is, an increase in ρ keeping polarization constant would increase the number of impressionable voters and their weight in the parties’ maximization problem and parties would have higher incentives to increase advertising. In the interior equilibrium the polarization effect kicks in and the net effect on spending depends on the magnitude of these two effects. Recall that changes in ρ enter in the spending effect due to changes in “stock”, while they enter in the polarization effect due to changes in the “conversion”. Finally, polarization is constant in ρ once the extremism equilibrium is reached and spending is constant and equal to zero since there are no impressionable voters. The following lemma summarizes the above for the interior equilibrium.

Lemma 3. *If the equilibrium is interior, a technological change that increases ρ – and hence polarization – also increases campaign spending due to the spending effect dominating the polarization effect if and only if the effect of ρ on the “stock” of impressionable voters is “large” enough. Formally, if the equilibrium is interior $\frac{\partial \mu e_i^*(x_L^*, x_R^*)}{\partial \rho} \geq 0$ if and only if $-\frac{\partial F(\bar{y})}{\partial \rho} \geq f(\bar{y}) \frac{\partial \bar{y}}{\partial \rho}$.*

Finally, notice that in the symmetric case, the marginal cost of advertising plays no role in equilibrium: neither the platforms nor the campaign spending depend on μ . When we introduce a cost-asymmetry between the parties, the equilibrium changes, favoring the party with the lower marginal cost.

3.2 Asymmetric parties

In contrast to previous spatial models with endogenous valence, we can incorporate cost asymmetries and obtain results of similar intuition as in the symmetric case. Unbalanced contests have attracted interest in the analysis of electoral competition in general. The well documented phenomenon of incumbency advantage has been a typical example to consider asymmetric elections where one of the parties has an electoral advantage (Gelman and King, 1990; Ansolabehere and Snyder Jr, 2002).

In this section we assume, without loss of generality, that $\mu_L < \mu_R$. We first characterize campaign spending in the second stage and then show how equilibrium platforms are affected by the asymmetry. In contrast to the symmetric case, equilibrium platforms

might be in mixed strategies. For clarity of presentation, we first focus on the configurations where the unique equilibrium is in pure strategies (Propositions 2-4) and then consider the configuration where the unique equilibrium is in mixed strategies (Proposition 5).

Considering well known results from contest theory, the advertising subgame has an equilibrium in pure strategies if and only if η is restricted from above. This upper bound, $\bar{\eta}$, is implicitly defined by $\mu_L^{\bar{\eta}} + \mu_R^{\bar{\eta}} = \bar{\eta}\mu_R^{\bar{\eta}}$ (Baik, 1994; Nti, 1999). This condition implies a one-to-one relationship between cost-asymmetry $\frac{\mu_R}{\mu_L}$ and $\bar{\eta}$ (with $\bar{\eta} \rightarrow 2$ as $\frac{\mu_R}{\mu_L} \rightarrow 1$ and $\bar{\eta} \rightarrow 1$ as $\frac{\mu_R}{\mu_L} \rightarrow \infty$). Despite this restriction, the platform stage of our model can also be solved for any $\eta > \bar{\eta}$ because of payoff equivalence to the case where $\eta = \bar{\eta}$ (Alcalde and Dahm, 2010).²¹

Following Baik (1994) and Nti (1999), we can characterize equilibrium in the advertising subgame.

Lemma 4. *For $\eta \leq \bar{\eta}$ there exists a unique Nash equilibrium in the campaign stage and advertising is given by $e_i^*(x_L, x_R) = (1 - F(y)) \frac{\eta}{\mu_i} \frac{\mu_L^\eta \mu_R^\eta}{(\mu_L^\eta + \mu_R^\eta)^2}$ for all i .*

Lemma (4) shows that, in equilibrium, parties choose different levels of advertising ($e_i^*(x_L, x_R)$) although they spend equal amounts ($\mu_i e_i^*(x_L, x_R)$). The share of impressionable voters is not any longer equally split across parties, giving an advantage to the party with the lower marginal cost. This generates an asymmetry in parties' incentives to use polarization as a device to reduce campaign spending and eliminates the interior equilibria where $x_L^* + x_R^* = 1$. If the equilibrium is interior parties propose asymmetric platforms. The convergent and extremism equilibria described previously also arise. We present the conditions for the existence and uniqueness of each type of equilibrium in the following propositions.

Proposition 2. *(Convergent equilibrium) For any $\eta \leq \bar{\eta}$, $\mu_L < \mu_R$ and $F(0) > 0$, there exists a unique SPNE with $x_L^* = x_R^* = \frac{1}{2}$ if and only if $\frac{f(0)}{F(0)} \leq g(\frac{1}{2}) / \left(\frac{\mu_L^\eta \mu_R^\eta}{(\mu_L^\eta + \mu_R^\eta)^2} 2\eta + \frac{\mu_R^\eta - \mu_L^\eta}{\mu_L^\eta + \mu_R^\eta} \right)$. Campaign spending for the convergent equilibrium is uniquely characterized in Lemma 4.*

As in the symmetric case, the convergent equilibrium arises when a lot of voters are concentrated around the median (i.e., high $g(\frac{1}{2})$) and/or the conversion rate at $y = 0$ (i.e., at its maximal level $\frac{f(0)}{F(0)}$) is low. In the asymmetric case, the size of the asymmetry is also a determinant of platform convergence. If the asymmetry converges to zero, then the inequality characterizing a convergent equilibrium converges to the one of Proposition 1. As the asymmetry however increases, platform convergence becomes less likely (the denominator at the right hand side of the inequality is increasing in the asymmetry).

²¹For $\eta \in (\bar{\eta}, 2)$ the equilibrium can be characterized following Wang (2010) and Ewerhart (2017).

This is because, as the asymmetry increases, the (symmetric) convergent equilibrium becomes less attractive for the disadvantaged party that loses the competition for impressionable voters and hence has incentives to diversify and propose distinct platforms. This potentially leads to an interior equilibrium.

An equilibrium is interior if at least one of the parties chooses an interior platform. That is, there is at least one platform located between the median and the corner (either 0 or 1, depending on the party). Analogously to the symmetric case, by solving first order conditions it arises that platform distance in the interior equilibrium \bar{y} , is implicitly defined by $\frac{f(\bar{y})}{F(\bar{y})} = \frac{g(\bar{x}^*)}{2\eta} \frac{(\mu_L^\eta + \mu_R^\eta)^2}{\mu_L^\eta \mu_R^\eta}$ where $\bar{x}^* = G^{-1}(\frac{\mu_R^\eta}{\mu_L^\eta + \mu_R^\eta})$. Considering the definition of interior equilibrium, Proposition 3 describes the necessary and sufficient conditions for its existence and uniqueness.²² As we characterize in the corollary below, two type of interior equilibria can emerge depending on the cost asymmetry.

Proposition 3. (*Interior Equilibrium*) *For any $\eta \leq \bar{\eta}$ and $\mu_L < \mu_R$, there exists a unique interior SPNE if and only if one of the following conditions is satisfied*

$$\frac{F(2\bar{x}^* - 1)}{f(2\bar{x}^* - 1)} g(\bar{x}^*) < 2\eta \frac{\mu_L^\eta \mu_R^\eta}{(\mu_L^\eta + \mu_R^\eta)^2} < \frac{F(1)}{f(1)} g(0.5) + \frac{\mu_R^\eta - \mu_L^\eta}{\mu_R^\eta + \mu_L^\eta} \quad \text{and} \quad \frac{\mu_R^\eta}{\mu_L^\eta + \mu_R^\eta} < G\left(\frac{3}{4}\right) \quad (3)$$

$$\frac{F(\frac{1}{2})}{f(\frac{1}{2})} g\left(\frac{3}{4}\right) - 2G\left(\frac{3}{4}\right) + \frac{2\mu_R^\eta}{\mu_R^\eta + \mu_L^\eta} < 2\eta \frac{\mu_L^\eta \mu_R^\eta}{(\mu_L^\eta + \mu_R^\eta)^2} \quad \text{and} \quad \frac{\mu_R^\eta}{\mu_L^\eta + \mu_R^\eta} \geq G\left(\frac{3}{4}\right) \quad (4)$$

The corollary below characterizes the two types of interior equilibrium that exists, depending on which condition is satisfied. If the condition described in the first line of Proposition 3, Condition 3, holds then we find the typical interior equilibrium in which both parties play an interior strategy. On the contrary, if Condition 4 holds, then the party with a cost advantage (L) plays with an interior strategy, and the disadvantaged party plays at a corner.

Corollary 1. (*Characterization of Interior Equilibria*) *There are two types of Interior equilibria. Proposition 3 describes the necessary and sufficient conditions for existence and uniqueness. Their characterization is as follows:*

- (*Interior/Interior*) $x_L^* = \bar{x}^* - \frac{\bar{y}}{2}$ and $x_R^* = \bar{x}^* + \frac{\bar{y}}{2}$ if and only if Condition 3 and $2\eta \frac{\mu_L^\eta \mu_R^\eta}{(\mu_L^\eta + \mu_R^\eta)^2} < \frac{F(2-2\bar{x}^*)}{f(2-2\bar{x}^*)} g(\bar{x}^*)$ are satisfied.
- (*Interior/Corner*) otherwise $x_L^* < 0.5$ and $x_R^* = 1$, where x_L^* is implicitly defined by

$$2G\left(\frac{1+x_L^*}{2}\right) = \frac{F(1-x_L^*)}{f(1-x_L^*)} g\left(\frac{1+x_L^*}{2}\right) + 2\frac{\mu_R^\eta}{\mu_R^\eta + \mu_L^\eta} - 2\eta \frac{\mu_R^\eta \mu_L^\eta}{(\mu_R^\eta + \mu_L^\eta)^2}$$

Campaign spending for each of the above SPNE is uniquely characterized in Lemma 4

²²As in the symmetric case, uniqueness is subject to the constraint $x_L \leq x_R$. A mirror equilibrium $x_R < x_L$ always exists.

All interior equilibria are asymmetric. It can be that both parties propose interior platforms (*Interior/Interior*) or that the cost advantaged party proposes an interior platform while the disadvantaged party proposes an extreme platform (*Interior/Corner*). The cost asymmetry plays an important role in determining the type but also polarization level at the interior equilibrium. The cost disadvantaged party R has more incentives than the advantaged party to reduce the share of (costly) impressionable voters. Consequently, R has more incentives to separate its platform from L . Thus, in the interior equilibrium platforms are shifted towards the cost-disadvantaged party R . That is, in the (*Interior/Interior*) equilibrium the point around which parties propose equidistant platforms is to the right (i.e., $\bar{x}^* > 0.5$), while in the (*Interior/Corner*) it is the disadvantaged party that proposes an extreme platform. Note that in general, if the asymmetry is low “enough” (i.e., (3) is satisfied) the disadvantaged party may also propose an interior platform. If this asymmetry however is high “enough” (i.e., (4) is satisfied) only the (*Interior/Corner*) equilibrium arises, the disadvantaged party would like to separate more but cannot due to having reached the corner. The advantaged party best responds to that by proposing an interior platform. In that sense, “large” asymmetries never give rise to extremism equilibria either as the following proposition summarizes.

Proposition 4. (*Extremism Equilibrium*) *For any $\eta \leq \bar{\eta}$ and $\mu_L < \mu_R$, there exists a unique SPNE with $x_L^* = 0$ and $x_R^* = 1$ if and only if $g(0.5) / \left(2\eta \frac{\mu_L^\eta \mu_R^\eta}{(\mu_L^\eta + \mu_R^\eta)^2} - \frac{\mu_R^\eta - \mu_L^\eta}{\mu_L^\eta + \mu_R^\eta} \right) \leq \frac{f(1)}{F(1)}$ and campaign spending as characterized in Lemma 4.*

If the asymmetry converges to zero, the relevant inequality converges to the one of Proposition 1. As the asymmetry however increases, extremism becomes less likely (the denominator on the left hand side of the inequality is decreasing in the asymmetry). This is because, the advantaged party has no incentives to polarize as it would reduce the share of impressionable voters for whom it enjoys a great advantage. Also, as in the symmetric case, extremism is more likely when the rate at which impressionable voters become ideological is great enough under maximal platform separation (i.e., $\frac{f(1)}{F(1)}$ is high).

Finally, there exists a combination of parameters where there exists no equilibrium in pure strategies. This occurs when the advantaged party L has incentives to propose a platform to the right of the median, a situation that can not form part of any pure strategy Nash equilibrium (Lemma A.1.). In that instance we can characterize a mixed strategy equilibrium instead. In the mixed strategy equilibrium, the disadvantaged party R randomizes on two symmetric platforms around 0.5 with equal probability (see Aragonès and Xefteris 2017b for a similar characterization in an exogenous valence setting). If the disadvantaged party was playing one of these two platforms deterministically, then the advantaged party L would locate at some point between that platform and the median.

As a consequence, a better alternative would be always available to the disadvantaged party R at the other side of the median. If party R randomizes between two platforms at the two sides of the median, then the advantaged party has incentives to locate exactly at the median. If the symmetric platforms chosen by the disadvantaged party are such that its first order conditions are satisfied, then none of the parties has incentives to deviate. A formal characterization follows.

Proposition 5. (*Mixed Strategy Equilibrium*) For any $\eta \leq \bar{\eta}$ and $\mu_L < \mu_R$, such that one of the following conditions is satisfied

$$\frac{F(0)}{f(0)}g(0.5) - \frac{\mu_R^\eta - \mu_L^\eta}{\mu_L^\eta + \mu_R^\eta} < 2\eta \frac{\mu_L^\eta \mu_R^\eta}{(\mu_L^\eta + \mu_R^\eta)^2} < \frac{F(2\bar{x}^* - 1)}{f(2\bar{x}^* - 1)}g(\bar{x}^*) \quad \text{and} \quad \frac{\mu_R^\eta}{\mu_L^\eta + \mu_R^\eta} \leq G(3/4) \quad (5)$$

$$\frac{F(0)}{f(0)}g(0.5) - \frac{\mu_R^\eta - \mu_L^\eta}{\mu_L^\eta + \mu_R^\eta} < 2\eta \frac{\mu_L^\eta \mu_R^\eta}{(\mu_L^\eta + \mu_R^\eta)^2} \leq \frac{F(\frac{1}{2})}{f(\frac{1}{2})}g(\frac{3}{4}) - 2G(\frac{3}{4}) + \frac{2\mu_R^\eta}{\mu_R^\eta + \mu_L^\eta} \quad \text{and} \quad \frac{\mu_R^\eta}{\mu_L^\eta + \mu_R^\eta} > G(3/4) \quad (6)$$

There exists a mixed strategy equilibrium where $x_L^* = \frac{1}{2}$ and party R randomizes with equal probability over $x_R^* = \min\{\underline{x}_R, 1\} > \frac{1}{2}$ and $1 - x_R^*$, where \underline{x}_R is implicitly defined by

$$\left[1 - G\left(\frac{0.5 + \underline{x}_R}{2}\right)\right] = \frac{F(\underline{x}_R - 0.5)}{f(\underline{x}_R - 0.5)} \frac{g\left(\frac{0.5 + \underline{x}_R}{2}\right)}{2} + \frac{\mu_L^\eta}{\mu_R^\eta + \mu_L^\eta} - \eta \frac{\mu_R^\eta \mu_L^\eta}{(\mu_R^\eta + \mu_L^\eta)^2}$$

Campaign spending for the above SPNE is uniquely characterized in Lemma 4

The above Proposition completes the characterization of different types of equilibria across all possible parameter configurations in the asymmetric case. Compared to existing models in the endogenous valence literature that typically become intractable, we are obtaining an equilibrium in pure strategies for several configurations. And even in the case where an equilibrium in pure strategies fails to exist, the characterized mixed strategy equilibrium is very intuitive.

As with the symmetric case, the conditions determining the equilibrium type depend on the effectiveness of campaigns, the mass of ideological voters at the median and conversion rates at different points. While we have discussed the most relevant conditions, especially what concerns the cost asymmetries, in general, the incentives to polarize (or not) and move across equilibria types remain similar to the symmetric case. Proceeding with an example, we can illustrate similar intuition and comparative statics as in the symmetric case. For example, the simultaneous increase in polarization and campaign spending observed in real world politics can be reconciled also in the presence of an asymmetry.

Example 2: $F(y)$ and $G(y)$ uniformly distributed in X . In Figure 2 Panel (a) we plot the case of “low” asymmetry ($\mu_R/\mu_L = 1.2$) while in Panel (b) we plot the case of “high” asymmetry ($\mu_R/\mu_L = 2.5$). For both levels of asymmetry, we are at a convergent equilibrium only when $\eta = 0$.²³ Since impressionable voters are split equally across the two parties regardless of electoral advertising, parties do not have incentives to differentiate their platforms since the asymmetry vanishes. As long as $\eta > 0$ parties platforms are not identical. For the “low” asymmetry example, the equilibrium is always in pure strategies, while for the “high” asymmetry example, the equilibrium may be in mixed strategies. Note that in both panels, and as described in our results, the disadvantaged party R is proposing a relatively more extreme platform compared to the advantaged party L since it has incentives to increase polarization and soften the competition for impressionable voters through campaign spending.

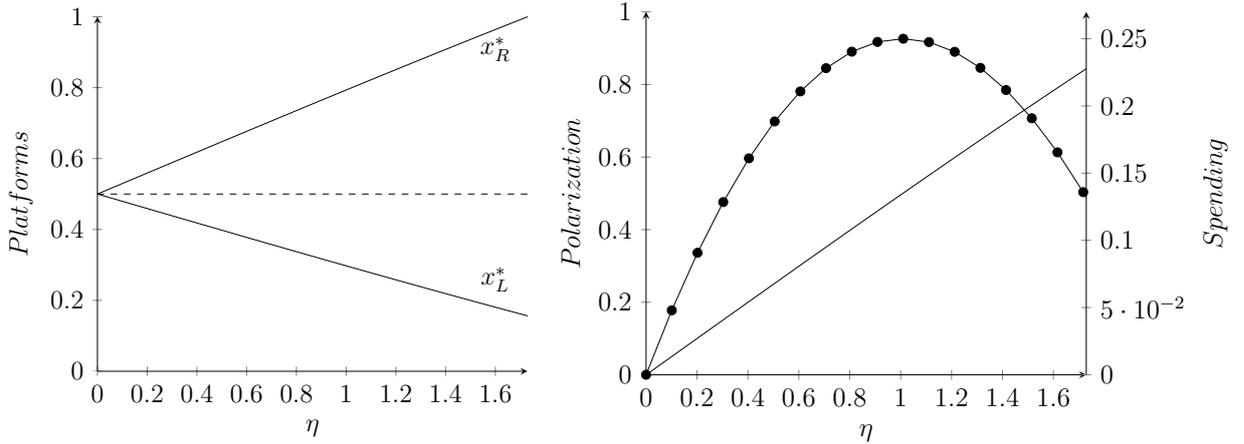
In the example of “low” asymmetry, we illustrate the divergent and asymmetric (*Interior/Interior*) equilibrium platforms as characterized in Corollary 1. In this example, platforms diverge monotonically as η increases (as in the symmetric case) and hence polarization is also increasing.

In the example of “high” asymmetry, and despite polarization being again increasing in the effectiveness of electoral advertising several differences arise. In the (*Interior/Interior*) equilibrium (i.e., $\eta \leq 0.7975$), while the disadvantaged party is always diverging from the median, the advantaged party does not respond to increases in η in a monotonic manner. Some initial divergence by L is fast vanishing till party L proposes the platform preferred by the median. For $\eta > 0.7975$ and since there can't be an equilibrium in pure strategies where $x_L^* > 0.5$, we move to the region of the (*Mixed Strategy Equilibrium*) where platforms are characterized in Proposition 5. The advantage party proposes $x_L^* = 0.5$ and the disadvantaged party proposes with equal probability two equidistant platforms x_R^* and $1 - x_R^*$.

Finally, when it comes to campaign spending, as the right panels show, we may again encounter a situation of a non-monotonic relationship between campaign spending and η (as in Figure 1 and the symmetric case). But perhaps more importantly, our results can again sustain the simultaneous increase in polarization and campaign spending due to technological changes even in the presence of asymmetries for a wide range of parameters.

²³Notice that in the formal results we have assumed the relevant cases where $\eta > 0$. In the example we permit $\eta = 0$ since this illustrates the convergent equilibrium which can not occur otherwise when $F(y)$ and $G(y)$ uniformly distributed.

Panel (a): “low” asymmetry: $\mu_R/\mu_L = 1.2$



Panel (b): “high” asymmetry: $\mu_R/\mu_L = 2.5$

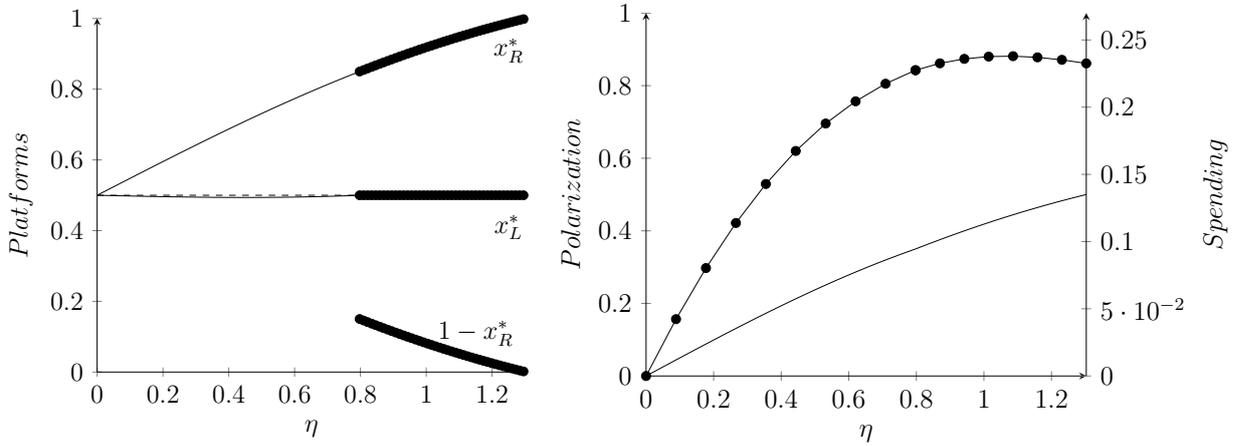


Figure 2: The two left panels depict equilibrium platforms (x_L^*, x_R^*) for different levels of asymmetry. In the top left panel the equilibrium is always in pure strategies. In the bottom left panel the equilibrium is in mixed strategies and represented by thick lines for “high” values of η . In the right panels, solid lines depict polarization ($x_R^* - x_L^*$) and dotted lines campaign spending. Graphs are plotted on the interval of η that guarantees an equilibrium in pure strategies in the advertising stage.

4 Conclusion

We suggest a tractable setting to analyze electoral competition with endogenous platforms and campaign spending. Apart from an intuitive equilibrium characterization that contributes in the corresponding literature, we provide new comparative statics on the effects of technological advances on electoral competition. For example, *“commentators have suggested that the reason for both the increased polarization and campaign spending is that skilled political operatives using sophisticated statistical tools and purchasing advertising in local markets are better able to target particular voters”* (citation from Herrera

et al. (2008, p. 502), also see NBC 2017 for a recent such example). However, their results so far were linking such technological advances with a reduction in polarization, and therefore favored alternative channels that may drive polarization such as more volatile preferences. Our results in contrast, justify the simultaneous increase in polarization and campaign spending due to recent technological changes and better targeting of the electoral campaigns. Moreover our model can address several follow-up questions.

Given that one would naturally expect further advances in campaign technology two natural questions arise: a) should we expect a further increase in polarization?, and b) what about campaign spending? Our theory would say yes, further advances in targeting will lead to further polarization. Ways to go against this trend would require policies that improve the awareness of the electorate and a shift of voters' attention from persuasive campaigns to political platforms. With regards to campaign spending, recall that our theory does not provide a monotone comparative static. As the targeting technology improves, parties have incentives to increase their campaign spending, but at the same time to polarize (which reduces campaign spending). Hence, our results are not incompatible with the observations of the 2016 presidential US election where campaign spending dropped. Furthermore, following this presidential campaign, the diffusion of plausible but false information received the name of "fake news". This notion, widely used during and after the campaigns, has direct implications on the electorate's awareness: for any given "real" platforms, the stock of ideological voters decreases with the amount of fake news. In terms of our model, fake news would increase the reversed hazard rate, causing an increase in polarization as well.

While one of our contributing messages could be the importance of the electorate's awareness as a way to affect campaign spending and polarization, political pundits have paid special attention to caps on campaign spending. In the context of symmetric costs, our model provides very clear and intuitive implications regarding the effects of this policy. If a cap is below the equilibrium campaign spending, it will completely shut down the polarization effect in our model. That is, when a cap is binding, the parties will not induce an increase in campaign spending by moving their platform towards their competitor. This eliminates one of the elements of the parties' trade-off in the choice of platform location. As a consequence, platform convergence is the only possible outcome. In other words, any cap on campaign spending smaller or equal to the equilibrium spending, will also induce convergence at the median voter. Thus our model predicts an important impact of campaign caps on party polarization. The same intuition can be sustained under asymmetric costs if the reversed hazard rate is great enough under full convergence.

5 Appendix - Proofs

This section is organized as follows. In Subsection 5.1 we show the pure strategy equilibrium in advertising for asymmetric costs. Additionally, we discuss the extension for $\eta > \bar{\eta}$ in the symmetric case. In Subsection 5.2 we write and solve the platform selection problem for the general case with asymmetric costs. Thus, taking into account the Kuhn-Tucker conditions, we solve this maximization problem and we show the first and second order conditions. Lemmata A.1, A.2 and Remark 1 in that subsection provide the conditions for existence and uniqueness of platforms that maximize the Lagrangian for the general case. Next, we show uniqueness and existence of a unique (pure) Subgame Perfect Nash Equilibrium, and we characterize the equilibrium platforms for the symmetric case (Proposition 1). Later on, we do the same for asymmetric costs. In this case, we show separately the convergent equilibrium (Proposition 2), the interior equilibrium (Proposition 3 and Corollary 1) and the extremism one (Proposition 4). Finally, we solve the mixed strategy equilibrium for the regions where the asymmetric case does not have an equilibrium in pure strategies in Subsection 5.3.

5.1 Advertising

We begin proving the more general Lemma 4, i.e., $\mu_L \neq \mu_R$, and after the proof we discuss the implications of symmetry, i.e., Lemma 1.

Proof. For any given pair (x_L, x_R) , parties simultaneously choose e_i that maximizes $(1 - F(y))\frac{e_i^\eta}{e_L^\eta + e_R^\eta} - \mu_i e_i(x_L, x_R)$. Hence, from the FOC, we obtain the first necessary condition for an interior PSNE: $e_i = \frac{1-F(y)}{\mu_i} \eta \frac{\mu_L^\eta \mu_R^\eta}{(\mu_L^\eta + \mu_R^\eta)^2}$. The equilibrium payoffs can be written as

$$(1 - F(y)) \left[\frac{\mu_i^\eta}{\mu_L^\eta + \mu_R^\eta} - \eta \frac{\mu_L^\eta \mu_R^\eta}{(\mu_L^\eta + \mu_R^\eta)^2} \right],$$

which are positive if and only if $(\mu_L^\eta + \mu_R^\eta)\mu_i^\eta - \eta\mu_L^\eta\mu_R^\eta \geq 0$. Therefore, the second necessary condition for PSNE is that $\eta \leq \bar{\eta} : \mu_R^\eta + \mu_L^\eta = \bar{\eta}\mu_R^\eta$. Moreover, for all η lower or equal than $\bar{\eta}$, the SOC holds with strict inequality, assuring not only existence but also uniqueness of the equilibrium. \square

The symmetric case. Notice that $\bar{\eta} \leq 2$ holds with strict equality if and only if $\mu_L = \mu_R$. In the symmetric case, for all $\eta \leq 2$, the conditions on the existence and uniqueness of PSNE are satisfied, and therefore $e_i = \frac{1-F(y)}{4\mu_i} \eta$. For $\eta > 2$, Alcalde and Dahm (2010) show that “the contest possesses an all-pay auction equilibrium” (Theorem 3.2 in their paper). In a symmetric contest, this theorem implies full dissipation, which

means that the expected payoff would be 0 for parties L and R , and the corresponding expected bids would be $\frac{1-F(y)}{2\mu}$.

5.2 Polarization and Spending

The objective function, existence and uniqueness

Throughout this section let $y = x_R - x_L$, $\bar{x} = \frac{x_R + x_L}{2}$, and $S_{Id}^L(x_L, x_R) = G(\bar{x}) = 1 - S_{Id}^R(x_L, x_R)$ for $x_L \neq x_R$ and $S_{Id}^L(x_L, x_R) = S_{Id}^R(x_L, x_R) = \frac{1}{2}$ otherwise. By backward induction and using the equilibrium expressions of the of advertising subgame, we can write the first-stage payoff for the political parties as follows

$$\Pi_i(x_L, x_R) = F(y)S_{Id}^i(x_L, x_R) + (1 - F(y))\frac{\mu_{-i}^\eta}{\mu_R^\eta + \mu_L^\eta} - (1 - F(y))\eta\frac{\mu_R^\eta\mu_L^\eta}{(\mu_R^\eta + \mu_L^\eta)^2}.$$

In the remaining of this subsection, we restrict attention to equilibrium in pure strategies. Since the characterized equilibrium will be unique there is no concern regarding the presence of mixed strategy equilibria. In the following subsection instead we will focus on mixed strategy equilibria for the regions where there is no equilibrium in pure strategies.

Lemma A.1. *In any equilibrium $x_L \leq 1/2$ and $x_R \geq 1/2$.*

Proof. Remind that we have assumed without loss of generality that $x_L \leq x_R$. First consider a divergent equilibrium, i.e., $x_L \neq x_R$. We proceed by contradiction. Assume an equilibrium such that $1/2 < \tilde{x}_L < \tilde{x}_R$. Then party R is strictly better off by deviating to $x_R = 2\tilde{x}_L - \tilde{x}_R$, which maintains unchanged the proportion of each type of voter and the share of impressionable votes and strictly increases the share of ideological votes of party R . Analogously we can show that party L always has a profitable deviation to $x_L = 2\tilde{x}_R - \tilde{x}_L$ when $\tilde{x}_L < \tilde{x}_R < 1/2$

Now consider a convergent equilibrium $x_L = x_R$. Given the assumption of $x_L \leq x_R$ two necessary conditions for a convergent equilibrium at $x_L = x_R = x$ are:

$$\begin{aligned} \lim_{x_L \rightarrow x^-} \Pi_L(x_L, x) &\leq \Pi_L(x, x) \\ \lim_{x_R \rightarrow x^+} \Pi_R(x, x_R) &\leq \Pi_R(x, x) \end{aligned}$$

For $x > \frac{1}{2}$, $\lim_{x_L \rightarrow x^-} G(x_L, x) > \frac{1}{2} = G(x, x)$, which implies that $\lim_{x_L \rightarrow x^-} \Pi_L(x_L, x) > \Pi_L(x, x)$ and violates the equilibrium condition for party L . A similar argument applies for party R if $x < \frac{1}{2}$. \square

Remind that when $x_L = x_R \neq 1/2$, a discontinuity can arise in the objective function, due to the discontinuity of $S_{Id}^i(x)$. However, the previous lemma restricts convergence to the case $x_L = x_R = 0.5$, in which $S_{Id}^i(x)$, and consequently $\Pi_i(x_L, x_R)$, are continuous.

Thus, by including the equilibrium constraints ($x_L \leq 1/2$ and $x_R \geq 1/2$) in the maximization problem, the Lagrangians below are continuous, and provided that the second order conditions are met, will be solved by the equilibrium platforms (due to the presence of equilibrium constraints solving the Lagrangian is a necessary but not a sufficient condition for equilibrium, as we show in Remark 1 below).

For $i \in \{L, R\}$, let $\lambda_i \geq 0$ be the multipliers associated with the feasibility constraints and $\nu_i \geq 0$ with the equilibrium constraints. The Lagrangians are:

$$\mathcal{L}_L = \Pi_L(x_L, x_R) - \lambda_L(-x_L - 0) - \nu_L(x_L - \frac{1}{2}) \quad (7)$$

$$\mathcal{L}_R = \Pi_R(x_L, x_R) - \lambda_R(x_R - 1) - \nu_R(\frac{1}{2} - x_R). \quad (8)$$

The first order conditions (FOC from now on) are:

$$\frac{\partial \mathcal{L}_L}{\partial x_L} = \frac{\partial \Pi_L(x_L, x_R)}{\partial x_L} + \lambda_L - \nu_L = 0, \quad (9)$$

$$\frac{\partial \mathcal{L}_R}{\partial x_R} = \frac{\partial \Pi_R(x_L, x_R)}{\partial x_R} - \lambda_R + \nu_R = 0. \quad (10)$$

Where,

$$\Pi'_L \equiv \frac{\partial \Pi_L(x_L, x_R)}{\partial x_L} = F(y) \frac{g(\bar{x})}{2} - f(y)G(\bar{x}) + f(y) \frac{\mu_R^\eta}{\mu_R^\eta + \mu_L^\eta} - f(y)\eta \frac{\mu_R^\eta \mu_L^\eta}{(\mu_R^\eta + \mu_L^\eta)^2}, \quad (11)$$

$$\Pi'_R \equiv \frac{\partial \Pi_R(x_L, x_R)}{\partial x_R} = -F(y) \frac{g(\bar{x})}{2} + f(y)[1 - G(\bar{x})] - f(y) \frac{\mu_L^\eta}{\mu_R^\eta + \mu_L^\eta} + f(y)\eta \frac{\mu_R^\eta \mu_L^\eta}{(\mu_R^\eta + \mu_L^\eta)^2} \quad (12)$$

Lemma A.2. *Let $F(x)$ be log-concave and $g(x)$ be symmetric and log-concave. The objective functions are strictly quasiconcave in the policy space when $x_L \leq \frac{1}{2} \leq x_R$, hence the second order conditions (SOC) are satisfied.*

The constrained optimization problem includes two linear constraints for each party, thus focusing on the quasiconcavity of the payoff functions suffices.

Proof. Without loss of generality, consider party L . Under the conditions of Lemma (A.1), continuity of Π_L is assured. Then, let us modify Equation (11) by dividing it over the densities $f(y)$ and $g(\bar{x})$:

$$\tilde{\Pi}'_L \equiv \frac{\Pi'_L}{f(y)g(\bar{x})} = \frac{F(\bar{y})}{2f(\bar{y})} - \frac{1}{g(\bar{x})} \left[G(\bar{x}) - \frac{\mu_R^\eta}{\mu_R^\eta + \mu_L^\eta} + \eta \frac{\mu_R^\eta \mu_L^\eta}{(\mu_R^\eta + \mu_L^\eta)^2} \right] \quad (13)$$

Let $\tilde{\Pi}_L$ be the primitive of $\tilde{\Pi}'_L$. $\tilde{\Pi}_L$ is strictly quasiconcave if and only if $\tilde{\Pi}'_L(x)(x' - x) > 0$ whenever $\tilde{\Pi}_L(x') > \tilde{\Pi}_L(x)$. Since strict quasiconcavity is determined by the sign of $\tilde{\Pi}'_L(x)$, which is the same of the sign of $\Pi'_L(x)$ (because $f(y)g(\bar{x})$ is strictly positive), the strict

quasiconcavity of $\tilde{\Pi}_L(x)$ guarantees the strict quasiconcavity of $\Pi_L(x)$.

Therefore, by showing the strict concavity of $\tilde{\Pi}_L$ (i.e., $\tilde{\Pi}_L'' = \frac{\partial \Pi_L'}{\partial f(y)g(\bar{x})} < 0$), we will be proving that $\Pi_L(x_L, x_R)$ is strictly quasiconcave too. Hence $\tilde{\Pi}_L''$ is

$$\frac{\partial \Pi_L'}{\partial f(y)g(\bar{x})} = -\frac{1}{2} \left[\frac{F(\bar{y})}{f(\bar{y})} \right]' - \frac{1}{2g(\bar{x})^2} \left\{ g(\bar{x})^2 - g'(\bar{x}) \left[G(\bar{x}) - \frac{\mu_R^\eta}{\mu_R^\eta + \mu_L^\eta} + \eta \frac{\mu_R^\eta \mu_L^\eta}{(\mu_R^\eta + \mu_L^\eta)^2} \right] \right\}$$

By log-concavity of $F(y)$, the term $-[\frac{F(\bar{y})}{f(\bar{y})}]'$ is negative, so we can focus on the negativity of $H = -\left\{ g(\bar{x})^2 - g'(\bar{x}) \left[G(\bar{x}) - \frac{\mu_R^\eta}{\mu_R^\eta + \mu_L^\eta} + \eta \frac{\mu_R^\eta \mu_L^\eta}{(\mu_R^\eta + \mu_L^\eta)^2} \right] \right\}$ in the expression above to guarantee strict concavity of $\tilde{\Pi}_L(x)$ (and hence strict quasi-concavity of $\Pi_L(x)$). Let us consider two cases.

- If $g'(\bar{x}) \geq 0$, then $-g'(\bar{x})[\frac{\mu_R^\eta}{\mu_R^\eta + \mu_L^\eta} - \eta \frac{\mu_R^\eta \mu_L^\eta}{(\mu_R^\eta + \mu_L^\eta)^2}]$ is negative (strictly negative for $g'(\bar{x}) > 0$) because the term in brackets is always positive given $\eta < \bar{\eta}$ (see proofs of lemmata 1 and 4). Log-concavity of $g(x)$ implies log-concavity of $G(x)$, thus $-[g(\bar{x})^2 - g'(\bar{x})G(\bar{x})]$ is negative (strictly negative for $g'(\bar{x}) = 0$). Hence H is strictly negative and $\Pi_L(x)$ strictly quasi-concave.
- If $g'(\bar{x}) < 0$, suppose there exists $\hat{x}_L : G(\frac{\hat{x}_L + x_R}{2}) = \frac{\mu_R^\eta}{\mu_R^\eta + \mu_L^\eta} - \eta \frac{\mu_R^\eta \mu_L^\eta}{(\mu_R^\eta + \mu_L^\eta)^2}$. Since $G(x)$ is increasing in x , for $x_L > \hat{x}_L$, H would be strictly negative and $\Pi_L(x)$ strictly quasi-concave. For $x_L \leq \hat{x}_L$, $G(\bar{x}) \leq \frac{\mu_R^\eta}{\mu_R^\eta + \mu_L^\eta} - \eta \frac{\mu_R^\eta \mu_L^\eta}{(\mu_R^\eta + \mu_L^\eta)^2}$ implies that $\frac{\partial \Pi_L(x_L, x_R)}{\partial x_L}$ is strictly positive which directly implies that $\Pi_L(x_L, x_R)$ is strictly quasiconcave for $x_L \in [0, \hat{x}_L]$.

We can proceed similarly to show that $\Pi_R(x_L, x_R)$ is also strictly quasiconcave (in that case we use that $g(x)$ implies that $1 - G(x)$ is log-concave). \square

Remark 1. Let (x_L, x_R) be a solution to the Lagrangians. Then, in the platforms stage, there is a divergent equilibrium ($x_L \neq x_R$) only when $\nu_L = \nu_R = 0$ and there is a convergent one ($x_L = x_R$) only when $\nu_L \geq 0$ and $\nu_R \geq 0$.

Proof. For the divergent equilibria, let $x_L \neq x_R$ be an equilibrium with $\nu_R = 0$. Suppose $\nu_L > 0$. Then $x_L = \frac{1}{2} < x_R$ and $\Pi_L'(\frac{1}{2}, x_R) > 0$. Thus, party L has incentives to deviate to a platform strictly larger than $\frac{1}{2}$ which violates Lemma (A.1). Then $x_L \neq x_R$ with $\nu_R = 0 < \nu_L$ cannot be an equilibrium. Hence, ν_L must be 0 for this type of equilibrium to exist. The same holds true for $\nu_L = 0 < \nu_R$.

For the convergent equilibrium, Lemma (A.1) implies $x_L = x_R = \frac{1}{2}$ is the only candidate to equilibrium. Hence, at the solution to the Lagrangian, $\nu_L \geq 0$ and $\nu_R \geq 0$ is true. Notice that even if $\Pi_L'(\frac{1}{2}, \frac{1}{2}) \geq 0$ and $\Pi_R'(\frac{1}{2}, \frac{1}{2}) \leq 0$, like in a standard Dawnsian game, they do not have incentives to deviate. Suppose L deviates to a platform strictly

larger than $\frac{1}{2}$, then it would become the Right party, which implies that it would have incentives to converge to $\frac{1}{2}$. \square

Proof of Proposition 1

Proof. Lemma A.1 shows that we can constraint the analysis to the case with $x_L \leq \frac{1}{2} \leq x_R$ without loss of generality. Lemma A.2 shows strict quasiconcavity of the maximization problem. Hence, the first order conditions are necessary and sufficient to characterize the equilibrium. With $\mu_L = \mu_R$ equations 11 and 12 become:

$$\Pi'_L \equiv \frac{\partial \Pi_L(x_L, x_R)}{\partial x_L} = F(y) \frac{g(\bar{x})}{2} - f(y)G(\bar{x}) + f(y)\frac{1}{2} - f(y)\eta\frac{1}{4}, \quad (14)$$

$$\Pi'_R \equiv \frac{\partial \Pi_R(x_L, x_R)}{\partial x_R} = -F(y) \frac{g(\bar{x})}{2} + f(y)[1 - G(\bar{x})] - f(y)\frac{1}{2} + f(y)\eta\frac{1}{4} \quad (15)$$

Remark 1 shows that convergence in the symmetric case implies $\Pi'_L \geq 0$ and $\Pi'_R \leq 0$. If these conditions are met, $x_L = \frac{1}{2} = x_R$ implies $y = 0$. Notice that if $F(0) = 0$, then there is no convergent equilibrium. When $F(0) > 0$, both conditions can be re-written as $\frac{f(0)}{F(0)} \leq \frac{2}{\eta}g(\frac{1}{2})$. Similarly, from Remark 1 we have full polarization when $\Pi'_L \leq 0$ and $\Pi'_R \geq 0$, evaluated at $x_L = 0, x_R = 1$. Using that $x_L = 0$ and $x_R = 1$ imply $y = 1$, both conditions above can be re-written as $\frac{f(1)}{F(1)} \geq \frac{2}{\eta}g(\frac{1}{2})$. And when there is a divergent equilibrium (without full polarization), we obtain what we call the interior/interior equilibrium when $\Pi'_L = \Pi'_R = 0$. This equality implies that polarization is implicitly defined by $\frac{f(\bar{y})}{F(\bar{y})} = \frac{2}{\eta}g(\frac{1}{2})$, when there is no convergent or extremism equilibrium, i.e., when the two inequalities above do not hold:

$$\frac{f(1)}{F(1)} < \frac{2}{\eta}g(\frac{1}{2}) < \frac{f(0)}{F(0)}.$$

\square

Proof of Proposition 2

Proof. Because of Lemma (A.2) and Remark (1), solving the Lagrangian in the case $\lambda_L = \lambda_R = 0$ and $\nu_L \geq 0, \nu_R \geq 0$ suffices for a convergent equilibrium. By Lemma A.1, a convergent equilibrium can only take place at $x_L = x_R = \frac{1}{2}$, which implies $\lambda_i = 0$ for $i = L, R$. Given that ν_L and ν_R are positive, the Kuhn-Tucker conditions for a convergent equilibrium imply $\frac{\partial \Pi_L(x, x)}{\partial x_L} \geq 0$ and $\frac{\partial \Pi_R(x, x)}{\partial x_R} \leq 0$ and using that $G(0.5) = \frac{1}{2}$ they can be written for $i = L, R$ and $-i \neq i$ as:

$$\frac{F(0)}{f(0)}g(0.5) \geq 2\eta \frac{\mu_L^\eta \mu_R^\eta}{(\mu_L^\eta + \mu_R^\eta)^2} + \frac{\mu_i^\eta - \mu_{-i}^\eta}{\mu_L^\eta + \mu_R^\eta}.$$

Given that $\mu_R \geq \mu_L$, if the equation for R is satisfied, it will also be so for L . Finally, note that by substituting $\mu_L = \mu_R$ we obtain the convergent condition for the proof of Proposition (1). For $F(0) = 0$, there is not convergent equilibrium. \square

Proof of Proposition 3 and Corollary 1

Below we provide the conditions for the existence (Proposition 3) and the characterization (Corollary 1) of the divergent equilibrium in which both parties play interior platforms. Following the classification of Corollary 1 we prove the interior/interior equilibrium in Lemma A.3 and the interior/corner equilibrium in Lemma A.4.

Lemma A.3. *Let $\bar{x}^* = G^{-1}(\frac{\mu_R^\eta}{\mu_L^\eta + \mu_R^\eta})$ and $y^* = \bar{y}$ be implicitly defined by $\frac{f(\bar{y})}{F(\bar{y})} = \frac{g(\bar{x}^*) (\mu_L^\eta + \mu_R^\eta)^2}{2\eta \mu_L^\eta \mu_R^\eta}$. Hence, there is a unique divergent interior equilibrium $x_L^* = \bar{x}^* - \frac{\bar{y}}{2}$, $x_R^* = \bar{x}^* + \frac{\bar{y}}{2}$ if and only if*

$$\frac{F(2\bar{x}^* - 1)}{f(2\bar{x}^* - 1)} g(\bar{x}^*) < 2\eta \frac{\mu_L^\eta \mu_R^\eta}{(\mu_L^\eta + \mu_R^\eta)^2} < \frac{F(2 - 2\bar{x}^*)}{f(2 - 2\bar{x}^*)} g(\bar{x}^*)$$

which only holds for $\frac{\mu_R^\eta}{\mu_R^\eta + \mu_L^\eta} < G(\frac{3}{4})$.

Proof. Because of Lemma (A.2) and Remark (1), solving the Lagrangian in the case $\lambda_i = \nu_i = 0$ for all i suffices for a divergent interior equilibrium. Using $\bar{x} = (x_L + x_R)/2$ and $y = x_R - x_L$, the pair (x_L^*, x_R^*) is uniquely defined by the pair (\bar{x}^*, y^*) with $y^* = \bar{y}$. We begin by proving that there is a unique (\bar{x}^*, \bar{y}) that solves the FOCs when $\lambda_i = 0 = \nu_i$ for all i . From $\frac{\partial \mathcal{L}_L}{\partial x_L} + \frac{\partial \mathcal{L}_R}{\partial x_R} = 0$ and $G(x)$ strictly increasing in x , we obtain the unique \bar{x}^* that solves the FOCs:

$$G(\bar{x}^*) = \frac{\mu_R^\eta}{\mu_L^\eta + \mu_R^\eta} \iff \bar{x}^* = G^{-1}\left(\frac{\mu_R^\eta}{\mu_L^\eta + \mu_R^\eta}\right).$$

Plugging \bar{x}^* in 9 or 10, we obtain $\frac{f(\bar{y})}{F(\bar{y})} = \frac{g(\bar{x}^*) (\mu_L^\eta + \mu_R^\eta)^2}{2\eta \mu_L^\eta \mu_R^\eta}$. Since the conversion rate is increasing and $g(\bar{x}^*)$ can be treated as a constant, there is a unique polarization level \bar{y} that solves the FOCs. Hence, we obtain the unique

$$x_L^* = \bar{x}^* - \frac{\bar{y}}{2}$$

and

$$x_R^* = \bar{x}^* + \frac{\bar{y}}{2}$$

that solve the FOCs. Note that for $\mu_L = \mu_R$ we immediately obtain that $G(\bar{x}^*) = \frac{1}{2}$ and by symmetry of $G(x)$ that $\bar{x}^* = \frac{1}{2}$

Finally, we have to check that the above solutions, x_L^* and x_R^* lie within the corresponding policy sub-space $x_L^* \in (0, \frac{1}{2})$ and $x_R^* \in (\frac{1}{2}, 1)$. First, $G(x)$ is increasing, so its inverse is as well. Hence, from $x_L^* \in (0, \frac{1}{2})$, we obtain that $\bar{y} > 2G^{-1}(\frac{\mu_R^\eta}{\mu_R^\eta + \mu_L^\eta}) - 1$, and from $x_R^* \in (\frac{1}{2}, 1)$, we obtain that $\bar{y} < 2 - 2G^{-1}(\frac{\mu_R^\eta}{\mu_R^\eta + \mu_L^\eta})$. Hence, it must be the case that

$$\bar{y} \in \left(2G^{-1}\left(\frac{\mu_R^\eta}{\mu_R^\eta + \mu_L^\eta}\right) - 1, 2 - 2G^{-1}\left(\frac{\mu_R^\eta}{\mu_R^\eta + \mu_L^\eta}\right) \right) \iff \bar{y} \in (2\bar{x}^* - 1, 2 - 2\bar{x}^*), \quad (16)$$

which only holds if $\frac{\mu_R^\eta}{\mu_R^\eta + \mu_L^\eta} < G(\frac{3}{4})$. Also, in Equation 16, we can use the definition of \bar{y} and that the rate $\frac{F(x)}{f(x)}$ is increasing and invertible to obtain the conditions of the equilibrium in terms of the conversion rate:

$$\begin{aligned} \frac{F\left(2G^{-1}\left(\frac{\mu_R^\eta}{\mu_R^\eta + \mu_L^\eta}\right) - 1\right)}{f\left(2G^{-1}\left(\frac{\mu_R^\eta}{\mu_R^\eta + \mu_L^\eta}\right) - 1\right)} &< \frac{F(\bar{y})}{f(\bar{y})} \frac{1}{g\left(G^{-1}\left(\frac{\mu_R^\eta}{\mu_R^\eta + \mu_L^\eta}\right)\right)} < \frac{F\left(2 - 2G^{-1}\left(\frac{\mu_R^\eta}{\mu_R^\eta + \mu_L^\eta}\right)\right)}{f\left(2 - 2G^{-1}\left(\frac{\mu_R^\eta}{\mu_R^\eta + \mu_L^\eta}\right)\right)} \\ &\iff \\ \frac{F(2\bar{x}^* - 1)}{f(2\bar{x}^* - 1)} &< \frac{F(\bar{y})}{f(\bar{y})} \frac{1}{g(\bar{x}^*)} < \frac{F(2 - 2\bar{x}^*)}{f(2 - 2\bar{x}^*)} \end{aligned}$$

Note that for the symmetric cost case, $\mu_L = \mu_R$, the above simplifies to $\frac{F(0)}{f(0)}g(\frac{1}{2}) < \frac{\eta}{2} < \frac{F(1)}{f(1)}g(\frac{1}{2})$

□

Lemma A.4. ($0 < x_L^* \leq \frac{1}{2}, x_R^* = 1$) Let $\bar{x}^* = G^{-1}(\frac{\mu_R^\eta}{\mu_L^\eta + \mu_R^\eta})$ and \bar{y} be implicitly defined by $\frac{f(\bar{y})}{F(\bar{y})} = \frac{g(\bar{x}^*)}{2\eta} \frac{(\mu_L^\eta + \mu_R^\eta)^2}{\mu_L^\eta \mu_R^\eta}$. Let x_L^* be implicitly defined by $2G(\frac{1+x_L^*}{2}) = \frac{F(1-x_L^*)}{f(1-x_L^*)}g(\frac{1+x_L^*}{2}) + 2\frac{\mu_R^\eta}{\mu_R^\eta + \mu_L^\eta} - 2\eta \frac{\mu_R^\eta \mu_L^\eta}{(\mu_R^\eta + \mu_L^\eta)^2}$.

For $\frac{\mu_R^\eta}{\mu_R^\eta + \mu_L^\eta} < G(\frac{3}{4})$, there is a unique equilibrium $(x_L^*, 1)$ if and only if

$$\frac{F(2 - 2\bar{x}^*)}{f(2 - 2\bar{x}^*)}g(\bar{x}^*) < 2\eta \frac{\mu_L^\eta \mu_R^\eta}{(\mu_L^\eta + \mu_R^\eta)^2} < \frac{F(1)}{f(1)}g(0.5) + \frac{\mu_R^\eta - \mu_L^\eta}{\mu_R^\eta + \mu_L^\eta}$$

For $\frac{\mu_R^\eta}{\mu_R^\eta + \mu_L^\eta} \geq G(\frac{3}{4})$, there is a unique equilibrium $(x_L^*, 1)$ if and only if

$$\frac{F(\frac{1}{2})}{f(\frac{1}{2})}g(\frac{3}{4}) - 2G(\frac{3}{4}) + \frac{2\mu_R^\eta}{\mu_R^\eta + \mu_L^\eta} < 2\eta \frac{\mu_L^\eta \mu_R^\eta}{(\mu_L^\eta + \mu_R^\eta)^2}$$

Proof. Because of Lemma (A.2) and Remark (1), solving the Lagrangian in the case where $\nu_L = \nu_R = \lambda_L = 0$ and $\lambda_R \geq 0$ is sufficient to find an equilibrium where $x_L^* \leq \frac{1}{2}$ and $x_R^* = 1$. Let $\bar{x} = \frac{1+x_L}{2}$ and $y = 1 - x_L$, then taking into account the conditions on

the Lagrange-multipliers:

$$\begin{aligned} \frac{\partial \Pi_L(x_L, x_R = 1)}{\partial x_L} + \frac{\partial \Pi_R(x_L, x_R = 1)}{\partial x_R} &= \lambda_R \geq 0 \\ \frac{\mu_R^\eta}{\mu_R^\eta + \mu_L^\eta} &\geq G\left(\frac{x_L + 1}{2}\right) \\ 2G^{-1}\left(\frac{\mu_R^\eta}{\mu_R^\eta + \mu_L^\eta}\right) &\geq x_L + 1 \end{aligned} \quad (17)$$

Lemma A.1 and Equation (17) imply that $x_L \leq \min\{2G^{-1}(\frac{\mu_R^\eta}{\mu_R^\eta + \mu_L^\eta}) - 1, \frac{1}{2}\}$. Taking into account that $2G^{-1}(\frac{\mu_R^\eta}{\mu_R^\eta + \mu_L^\eta}) - 1 < \frac{1}{2} \iff \frac{\mu_R^\eta}{\mu_R^\eta + \mu_L^\eta} < G(\frac{3}{4})$, we solve the following:

- If $\frac{\mu_R^\eta}{\mu_R^\eta + \mu_L^\eta} < G(\frac{3}{4})$, then $\frac{\partial \Pi_L(x_L, 1)}{\partial x_L}|_{x_L=2\bar{x}^*-1} \leq 0$ must hold in equilibrium. Plugging in $x_L = 2\bar{x}^* - 1$ and $x_R = 1$ and using $G(\bar{x}^*) = \frac{\mu_R^\eta}{\mu_R^\eta + \mu_L^\eta}$ in Equation (9) we obtain

$$\frac{F(2 - 2\bar{x}^*)}{f(2 - 2\bar{x}^*)} g(\bar{x}^*) \leq 2\eta \frac{\mu_L^\eta \mu_R^\eta}{(\mu_L^\eta + \mu_R^\eta)^2} \quad (18)$$

- If $\frac{\mu_R^\eta}{\mu_R^\eta + \mu_L^\eta} \geq G(\frac{3}{4})$, then $\frac{\partial \Pi_L(x_L, 1)}{\partial x_L}|_{x_L=0.5} \leq 0$ must hold in equilibrium. Using $x_L = 0.5$ in Equation (9) we obtain

$$\begin{aligned} \frac{F(0.5)}{f(0.5)} - \frac{2}{g(\frac{3}{4})} G(\frac{3}{4}) + \frac{2}{g(\frac{3}{4})} \frac{\mu_R^\eta}{\mu_R^\eta + \mu_L^\eta} - \frac{2}{g(\frac{3}{4})} \eta \frac{\mu_R^\eta \mu_L^\eta}{(\mu_R^\eta + \mu_L^\eta)^2} &\leq 0 \\ &\iff \\ \frac{F(0.5)}{f(0.5)} g(\frac{3}{4}) - 2G(\frac{3}{4}) + \frac{2\mu_R^\eta}{\mu_R^\eta + \mu_L^\eta} &\leq 2\eta \frac{\mu_L^\eta \mu_R^\eta}{(\mu_L^\eta + \mu_R^\eta)^2} \end{aligned} \quad (19)$$

Finally, the fully divergent equilibrium is excluded when we assure $x_L > 0$, i.e., when $2\eta \frac{\mu_L^\eta \mu_R^\eta}{(\mu_L^\eta + \mu_R^\eta)^2} < \frac{F(1)}{f(1)} g(0.5) + \frac{\mu_R^\eta - \mu_L^\eta}{\mu_R^\eta + \mu_L^\eta}$ (see Proposition 4 below). Notice that when $\frac{\mu_R^\eta}{\mu_R^\eta + \mu_L^\eta} \geq G(\frac{3}{4})$, the latter inequality always holds.²⁴ \square

Proof of Proposition 4

Proof. Because of Lemma (A.2) and Remark (1), solving the Lagrangian in the case where $\nu_i = 0$ and $\lambda_i \geq 0$ for all i is sufficient to find an equilibrium with $x_L = 0$ and

²⁴Recall that $\eta \leq \bar{\eta}$ implies that $2\eta \frac{\mu_L^\eta \mu_R^\eta}{(\mu_L^\eta + \mu_R^\eta)^2} < 2 \frac{\mu_L^\eta + \mu_R^\eta}{\mu_R^\eta} \frac{\mu_L^\eta \mu_R^\eta}{(\mu_L^\eta + \mu_R^\eta)^2} = 2 \frac{\mu_L^\eta}{\mu_L^\eta + \mu_R^\eta}$. Also $2 \frac{\mu_L^\eta}{\mu_L^\eta + \mu_R^\eta} \leq \frac{\mu_R^\eta - \mu_L^\eta}{\mu_R^\eta + \mu_L^\eta} \iff \mu_R^\eta \geq 3\mu_L^\eta$. Then, single-peakedness and symmetry of $g(x)$ imply $G(\frac{3}{4}) > \frac{3}{4}$. Finally $\mu_R^\eta \geq 3\mu_L^\eta$ follows directly from $\frac{\mu_R^\eta}{\mu_R^\eta + \mu_L^\eta} \geq G(\frac{3}{4})$

$x_R = 1$. Hence, from Equation 9 and 10,

$$\lambda_L \geq 0 \text{ iff } \frac{F(1)}{f(1)}g(0.5) + \frac{\mu_{-i}^\eta - \mu_i^\eta}{\mu_L^\eta + \mu_R^\eta} \leq 2\eta \frac{\mu_L^\eta \mu_R^\eta}{(\mu_L^\eta + \mu_R^\eta)^2}.$$

As $\mu_R \geq \mu_L$, the condition for party L is the sufficient one. Last, $\frac{F(1)}{f(1)}g(0.5) > 0$, so a necessary condition for a fully divergent equilibrium is

$$2\eta\mu_L^\eta\mu_R^\eta \geq \mu_R^{2\eta} - \mu_L^{2\eta} = (\mu_R^\eta - \mu_L^\eta)(\mu_R^\eta + \mu_L^\eta) \geq (\mu_R^\eta - \mu_L^\eta)\eta\mu_R^\eta,$$

where the last inequality follows from $\eta \leq \bar{\eta}$. And the expression above simplifies to $3\mu_L^\eta \geq \mu_R^\eta$. \square

5.3 Equilibrium in Mixed Strategies

Note that no equilibrium in pure strategies was characterized for the two following cases:

- $\frac{F(0)}{f(0)}g(0.5) - \frac{\mu_R^\eta - \mu_L^\eta}{\mu_L^\eta + \mu_R^\eta} < 2\eta \frac{\mu_L^\eta \mu_R^\eta}{(\mu_L^\eta + \mu_R^\eta)^2} < \frac{F(2\bar{x}^* - 1)}{f(2\bar{x}^* - 1)}g(\bar{x}^*)$ if $\frac{\mu_R^\eta}{\mu_L^\eta + \mu_R^\eta} \leq G(3/4)$
- $\frac{F(0)}{f(0)}g(0.5) - \frac{\mu_R^\eta - \mu_L^\eta}{\mu_L^\eta + \mu_R^\eta} < 2\eta \frac{\mu_L^\eta \mu_R^\eta}{(\mu_L^\eta + \mu_R^\eta)^2} \leq \frac{F(\frac{1}{2})}{f(\frac{1}{2})}g(\frac{3}{4}) - 2G(\frac{3}{4}) + \frac{2\mu_R^\eta}{\mu_R^\eta + \mu_L^\eta}$ if $\frac{\mu_R^\eta}{\mu_L^\eta + \mu_R^\eta} > G(3/4)$

These regions correspond to cases where party L (lower cost party) has incentives to fix a platform to the right of the median which we know by Lemma A.1 can not constitute an equilibrium in pure strategies. We can show that for these cases the following is a mixed strategy equilibrium: $x_L = \frac{1}{2}$ and party R randomizes with equal probability over $x_R^* > \frac{1}{2}$ and $1 - x_R^*$, where:

$$x_R^* = \begin{cases} 1, & \text{if } [1 - G(\frac{0.5+1}{2})] > \frac{F(0.5)}{f(0.5)} \frac{g(\frac{3}{4})}{2} + \frac{\mu_L^\eta}{\mu_R^\eta + \mu_L^\eta} - \eta \frac{\mu_R^\eta \mu_L^\eta}{(\mu_R^\eta + \mu_L^\eta)^2} \\ \underline{x}_R & \text{otherwise} \end{cases}$$

where \underline{x}_R is implicitly defined by the solution of the first order condition for party R :

$$[1 - G(\frac{0.5 + \underline{x}_R}{2})] = \frac{F(\underline{x}_R - 0.5)}{f(\underline{x}_R - 0.5)} \frac{g(\frac{0.5 + \underline{x}_R}{2})}{2} + \frac{\mu_L^\eta}{\mu_R^\eta + \mu_L^\eta} - \eta \frac{\mu_R^\eta \mu_L^\eta}{(\mu_R^\eta + \mu_L^\eta)^2}$$

To show this is an equilibrium, it suffices to show that none of the two parties has incentives to deviate:

i) If $x_R^* = \underline{x}_R$, \underline{x}_R solves the FOC for Party R . Note that since $\frac{F(0)}{f(0)}g(0.5) - \frac{\mu_R^\eta - \mu_L^\eta}{\mu_L^\eta + \mu_R^\eta} < 2\eta \frac{\mu_L^\eta \mu_R^\eta}{(\mu_L^\eta + \mu_R^\eta)^2}$ we have that that $\underline{x}_R > 1/2$ for $x_L = 0.5$ (see proof of Proposition 2). Hence, by Lemma A.2 the payoff of party R satisfies the second order condition so $\underline{x}_R > 1/2$

identifies a maximum for party R . If $x_R^* = 1$ the FOC is negative and the second order conditions are satisfied (Lemma A.2), hence R maximizes payoffs. Given that $x_L = \frac{1}{2}$, by symmetry party R must be indifferent between x_R^* and $1 - x_R^*$. Thus party R has no incentives to deviate from the proposed equilibrium strategy.

ii) Considering the mixed strategy of party R , the expected payoff for party L is: $\frac{1}{2}\{F(x_R^* - x_L)G(\frac{x_R^* + x_L}{2}) + [1 - F(x_R^* - x_L)][\frac{\mu_R^\eta}{\mu_R^\eta + \mu_L^\eta} - \eta\frac{\mu_R^\eta \mu_L^\eta}{(\mu_R^\eta + \mu_L^\eta)^2}]\}$
 $+ \frac{1}{2}\{F(x_L - 1 + x_R^*)[1 - G(\frac{1 - x_R^* + x_L}{2})] + [1 - F(x_L - 1 + x_R^*)][\frac{\mu_R^\eta}{\mu_R^\eta + \mu_L^\eta} - \eta\frac{\mu_R^\eta \mu_L^\eta}{(\mu_R^\eta + \mu_L^\eta)^2}]\}$ We can proceed as in Lemma A.2 to show that $\Pi_L(x_L, x_R)$ is quasi-concave both for $x_R = x_R^*$ and $x_R = 1 - x_R^*$ (the latter case is not covered in the proof of Lemma A.2 but can be obtained by symmetry). Then, the expected payoff for party L must be quasi-concave too and satisfies the second order conditions. The first order condition for party L is

$$-\frac{1}{2}f(x_R^* - x_L)G(\frac{x_R^* + x_L}{2}) + \frac{1}{4}F(x_R^* - x_L)g(\frac{x_R^* + x_L}{2}) + \frac{1}{2}f(x_R^* - x_L)[\frac{\mu_R^\eta}{\mu_R^\eta + \mu_L^\eta} - \eta\frac{\mu_R^\eta \mu_L^\eta}{(\mu_R^\eta + \mu_L^\eta)^2}]$$

$$+ \frac{1}{2}f(x_L - 1 + x_R^*)[1 - G(\frac{1 - x_R^* + x_L}{2})] - \frac{1}{4}F(x_L - 1 + x_R^*)g(\frac{1 - x_R^* + x_L}{2}) - \frac{1}{2}f(x_L - 1 + x_R^*)[\frac{\mu_R^\eta}{\mu_R^\eta + \mu_L^\eta} - \eta\frac{\mu_R^\eta \mu_L^\eta}{(\mu_R^\eta + \mu_L^\eta)^2}]$$

Taking into account the symmetry of $G(\cdot)$, we see that if $x_L = 0.5$ then each term in the above equation appears twice but with opposite sign. Thus the first order condition is satisfied for $x_L = \frac{1}{2}$, so party L has no incentives to deviate from the proposed equilibrium strategy.

5.4 Comparative statics

Proof of Lemma (2).

Proof. By Proposition (1), the interior equilibrium arises if $\eta \in [2g(\frac{1}{2})\frac{F(0)}{f(0)}, 2g(\frac{1}{2})\frac{F(1)}{f(1)}]$. Using the implicit function theorem we can write: $\frac{\partial \bar{y}}{\partial \eta} = \frac{1}{2g(0.5)[\frac{F(\bar{y})}{f(\bar{y})}]'}$. Then,

$$\frac{\partial \mu e_i^*(x_L^*, x_R^*)}{\partial \eta} = -f(\bar{y})\frac{\partial \bar{y}}{\partial \eta} \frac{\eta}{4} + \frac{1 - F(\bar{y})}{4} = -\frac{f(\bar{y})}{2g(0.5) \left[\frac{F(\bar{y})}{f(\bar{y})}\right]'} \frac{\eta}{4} + \frac{1 - F(\bar{y})}{4}.$$

Hence, o

$$\frac{\partial \mu e_i^*(x_L^*, x_R^*)}{\partial \eta} \geq 0 \iff 1 - F(\bar{y}) \geq \frac{f(\bar{y})}{2g(0.5) \left[\frac{F(\bar{y})}{f(\bar{y})}\right]'} \eta \iff \eta \leq \frac{1 - F(\bar{y})}{f(\bar{y})} 2g(0.5) \left[\frac{F(\bar{y})}{f(\bar{y})}\right]'$$

where $\left[\frac{F(\bar{y})}{f(\bar{y})}\right]'$ is a positive number by log-concavity of $F(y)$. □

Proof of Lemma (3)

Proof. By Proposition (1), the interior equilibrium arises if $\eta \in [2g(\frac{1}{2})\frac{F(0)}{f(0)}, 2g(\frac{1}{2})\frac{F(1)}{f(1)}]$. Then,

$$\frac{\partial \mu e_i^*(x_L^*, x_R^*)}{\partial \rho} = -\frac{\partial F(y^*)}{\partial \rho} \frac{\eta}{4} - \frac{\partial F(y^*)}{\partial y^*} \frac{\partial y^*}{\partial \rho} \frac{\eta}{4}$$

and hence $\frac{\partial \mu e_i^*(x_L^*, x_R^*)}{\partial \rho} \geq 0$ if and only if $-\frac{\partial F(\bar{y})}{\partial \rho} \geq f(\bar{y}) \frac{\partial \bar{y}}{\partial \rho}$. \square

5.5 Voters' behavior

In this section we formalize and derive two main assumptions of the model. Namely, a) the endogenous division of voters across ideological and impressionable (via semiorder lexicographic preferences or a model of salience), and b) the impressionable voters' behavior that results in a Tullock contest during the campaign stage.

In their "strict" formulation, lexicographic preferences require a tie in the first attribute to compare alternatives over a second attribute. In their "weak" formulation (or semiorder, (Tversky, 1969)), small differences between alternatives in the first attribute also lead to indifference in that attribute. In other words, small differences on the first attribute are disregarded (Fishburn, 1974). This is the exact intuition we presented in our model where as polarization decreases and platforms look more alike, individuals turn their attention to campaigns instead of platforms. In particular, semiorder preferences with binary choices (i.e., two platforms in our setup) are unrestrictive in the sense of Manzini and Mariotti (2012), and the use of semiorder lexicographic preferences microfound the aggregate behavior presented in the main text.

Assume a population of measure one, in which voters have semiorder lexicographic preferences and are heterogeneous in two dimensions. First, they draw an ideal policy $x \in [0, 1]$ from $G(x)$. Second, voters also draw a level of sensitivity, $\phi \in [0, 1]$, towards differences in the ideology space, from $F_\phi(y) = Pr(\phi \leq y)$, where ϕ is the minimal distance between the two platforms that a voter considers to be "relevant" or "distinguishable".²⁵ Voters with $\phi > x_R - x_L$ vote according to the dominated attribute (i.e., electoral advertising e_i). Voters with $\phi \leq x_R - x_L$ vote à la Downs.

Note that for the case where $x_L = x_R$, only voters with $\phi = 0$ are Downsian (and as the two platforms are identical, they vote for any of the two parties with a 50% chance). That is, if one assumes that $F(0) > 0$ then there is a mass of Downsian voters not paying attention to campaigns even though platforms are identical. If instead one assumes that $F(0) = 0$ when platforms are identical all voters vote according to campaign spending.²⁶

²⁵In terms of the experimental literature in human perception (or psychophysics), ϕ can be interpreted as the just-noticeable difference.

²⁶Note that assuming $F(0) = 0$ or $F(0) > 0$ affects the results, for instance $F(0) > 0$ is required for the existence of a convergent equilibrium. In the main text, we solve the model for the general case

Although x is ex-post irrelevant for impressionable voters, all individuals are identified by the pair (x, ϕ) . The above features can be represented in an analytical manner by adapting the semiorder lexicographic structure proposed by Luce (1978). Considering voter (x, ϕ) , we can write the voter’s evaluation of party i as:

$$\vartheta_{x,\phi}(i) = - |x - x_i| \Upsilon(\phi \leq x_R - x_L) + t^i(e_R, e_L, \theta^i)[1 - \Upsilon(\phi \leq x_R - x_L)] \quad (20)$$

where $\Upsilon(\phi \leq x_R - x_L)$ is an indicator function taking value 1 when a voter is ideological, i.e., $\phi \leq x_R - x_L = y$, and 0 when a voter is impressionable, i.e., $\phi > x_R - x_L = y$, as in the main text. Without loss of generality $- |x - x_i|$ is the utility derived from voting for party $i \in \{L, R\}$ according to ideology; and $t^i(e_R, e_L, \theta^i)$ is the impressionable voter’s utility from voting for party $i \in \{L, R\}$ according to advertisements.

Let θ^i in $t^i(e_R, e_L, \theta^i)$ be a random variable (from the candidates’ points of view) that captures how much of a party’s advertisement “leaks” to voters in the following way $t^i(e_R, e_L, \theta^i) = \log(e_i^\eta) + \theta^i$. Similarly to multinomial applications in industrial organization (Nevo, 2000) and political economy (Casas, Fawaz, and Trindade, 2016), assume θ^i to be drawn *i.i.d.* from an type I extreme-value distribution. As in McFadden (1974), the probability $Pr(t^L > t^R)$ – which in our case is the probability that an impressionable voter votes for L – becomes the contest success function $Pr(t^L > t^R) = \frac{e_L^\eta}{e_L^\eta + e_R^\eta}$. With a continuum of voters, we interpret this probability as the share of impressionable voters that vote for L . On the same lines, (Jia, 2008; Jia, Skaperdas, and Vaidya, 2013) show that for $t^i(e_R, e_L, \theta^i) = e_i \theta^i$, if θ^L and θ^R follow independent inverse exponential distributions with parameter $\eta > 0$, the probability also becomes $Pr(t^L > t^R) = \frac{e_L^\eta}{e_L^\eta + e_R^\eta}$.²⁷

For simplicity, let $F_\phi(y)$ be written as $F(y)$, which can be used to denote the proportion of ideological voters given a level of polarization y . Platform preferences and ideology sensitivity are assumed to be independent. Consequently, for a given pair of policy platforms x_L and x_R , the votes of ideological and impressionable voters can be independently aggregated (integrating over ϕ). By integrating over θ_L and θ_R under the above distributional assumptions, the impressionable vote share of party i is $S_{Imp}^i = \frac{e_i^\eta}{e_L^\eta + e_R^\eta}$. For ideological voters, by integrating over x one immediately obtains that for party L , $S_{Idl}^L = G(\bar{x})$ if $x_L \neq x_R$ and $S_{Idl}^L = \frac{1}{2}$ if $x_L = x_R$. By taking into account the above, parties’ vote shares can be immediately written as in Equation (1).

Saliency and attention

Our model can also be interpreted as an extreme case of (Bordalo et al. 2012, 2013a,b, 2015), in which “salient thinkers” give more weight to attributes that exhibit greater

$F(0) \geq 0$ and highlight this distinction.

²⁷The assumption that party shocks are identical to all individuals is made without loss of generality, as long as they are independent of ideology.

heterogeneity in the available choice set.

In the papers on salience and attention, the salient attribute is the one in which the differences are more pronounced. The less salient attribute receives less weight. For instance, in Bordalo et al. (2015) they look at price and quality: if quality is the salient attribute, the utility from consuming good k is $q_k - \omega p_k$ with ω exogenously determined and in $[0, 1]$. Instead of comparing attributes, we compare the platforms differentiation with a baseline level of polarization, ϕ , which is exogenously drawn from $F_\phi(y)$. Thus, for ϕ smaller than the equilibrium polarization, voters take platforms as the salient attribute. In particular, for $\phi \leq y$, $\omega = 0$. And for $\phi \geq y$, $1 - \omega = 0$. Thus, one can consider the rank-based weighting salience function proposed by Bordalo et al. 2012, 2013b, 2015, where:

$$v_{x,\phi}(i) = -(|x - x_i|)\omega + t^i(e_L, e_R, \theta^i)(1 - \omega)$$

The heterogeneity of “salient thinkers” is given by $F_\phi(y)$, and everything else is exactly as in the previous section.

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