Beyond Cobb-Douglas: Flexibly Estimating Matching Functions with Unobserved Matching Efficiency

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Why Care About the Matching Function?

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  - a convenient modeling device of costly trading process
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- Welfare calculations depend on the matching function.
  - E.g. Hosios (1990) condition: Elasticity of m wrt to v equal to bargaining coefficient.
- Dynamics of labor market depend on matching function:
  - e.g. how quickly demand stimulus translates into employment depends on the elasticity of the matching function with respect to vacancies.

Identification Challenges: Functional Form

Consider a CRTS Matching Function with Time-Varying Matching Efficiency:

$$M_t\left(V_t, E_t U_t\right) = A_t M\left(V_t, E_t U_t\right)$$

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Literature generally assumes Cobb-Douglas for $M(V, EU)$

- restricts the elasticity with respect to (V, EU)
- restrictions are welfare relevant (cf. Hosios) and have positive implications for business cycle dynamics.

Identification Challenges: Endogeneity

$$M_t = A_t V_t^{\gamma} (E_t U_t)^{1-\gamma}$$

Estimating equation (OLS):

$$ln\left(\frac{M_t}{U_t}\right) = \gamma ln\left(\frac{V_t}{U_t}\right) + \underbrace{a_t + (1-\gamma)\, e_t}_{\text{"search efficacy"}}$$

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- but observed search might respond to variation in a_t:
  - $cov\left(a_t, \frac{V_t}{U_t}\right) \neq 0$ (Borowczyk-Martins, Jolivet, Postel-Vinay, 2013).

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- also unobserved search effort e_t can respond to market tightness:
  - $cov\left(e_t, \frac{V_t}{U_t}\right) \neq 0$ (Hornstein and Kudlyak, 2016).

Contributions

1. Non-parametric identification (Matzkin 2003; Bajari and Benkard 2005; Brancaccio, Kalouptsidi, Papageorgiou 2020):
   - allow for unobserved matching efficacy; check cyclical properties
   - multiple types of job-seekers
   - based on an independence assumption (testable)
2. Empirically estimate the non-parametric matching function and
   - decompose changes in hiring over 2001-2017
   - estimate elasticity of matching function 2001-2017.

- Recurring Theme: Trade-off between functional form and independence.

Related Literature

- Multiple Types of Job Seekers
  - Blanchard and Diamond (1989), Burgess (1993), Faberman, Gomme and Lkhagvasuren (2015), Kroft, Lange, Notowidigdo, and Katz (2016), Sedlacek (2016), Kudlyak and Lange (2017), Kroft, Lange, Notowidigdo, and Tudball (2019), Faberman, Mueller, Sahin and Topa (2021)
- Search Effort
  - Hornstein and Kudlyak (2016), Mukoyama, Patterson, and Sahin (2018)
- Vacancies
  - Davis, Faberman and Haltiwanger (2013), Gavazza, Mongey, and Violante (2018).
- Matching Function Estimation
  - Petrongolo and Pissarides (2001), Borowcyk-Martins, Jolivet and Postel-Vinay (2013), Elsby, Michaels and Ratner (2015), Hornstein and Kudlyak (2016).

Outline

1. Theory:
   1.1 Non-parametric Identification (simpler case)
   1.2 Expanded Model (multiple types)
2. Empirics:
   2.1 Data
   2.2 Estimation Results

Non-Parametric Identification: Basic Case

Basic Setup

- Assume search effort s_t, is unobserved
- <u>**Goal:**</u> estimate $m(v_t, s_t)$ nonparametrically *and* recover unobserved search effort s_t!

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- Assume search effort s_t, is unobserved
- **Goal:** estimate $m(v_t, s_t)$ nonparametrically *and* recover unobserved search effort s_t!
- Use literature on nonparametric identification (Matzkin 2003)
- **Assumptions**
  - $m(.)$ strictly increasing in s
  - $m(.)$ is homogeneous of degree one
  - independence of v, s (testable)

Basic Setup-Intuition

$$\underbrace{m_t}_{\text{matches}} = \underbrace{m\left(v_t\,, s_t\right)}_{m(\text{vacancies, search effort})}$$

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- Once we identify $m(.,.)$, we recover s_t by inverting $m_t = m\left(v_t, s_t\right)$.

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- Once we identify $m(.,.)$, we recover s_t by inverting $m_t = m\left(v_t, s_t\right)$.
- We assume, but do not impose independence of v_t and s_t!

Basic Setup Identification

Let $G(m_t|v_t)$ denote observed distribution of M given V at point (m_t, v_t):

$$
\begin{aligned}
G\left(m_t|v_t\right) &= \Pr\left(m\left(V,S\right) \leq m_t|V = v_t\right) \\
\text{monotonicity} &= \Pr\left(S \leq m^{-1}\left(V, m_t\right)|V = v_t\right) \\
\text{independence} &= \Pr\left(S \leq m^{-1}\left(v_t, m_t\right)\right) \\
&= F_s\left(m^{-1}\left(v_t, m_t\right)\right)
\end{aligned}
$$

Recover the Distribution of S: F_s

$$F_s(s) = \underbrace{G(m|v)}_{\text{data}}$$

- Pick point (m_0, v_0, s_0) to normalize s_0.
    - Then $F(s_0) = G(m_0|v_0)$ is observed.

- Now, because of homogeneity $m(\lambda v, \lambda s) = \lambda m$

$$F_s(\lambda s_0) = G(\lambda m_0 | v = \lambda v_0)$$

and let λ vary to recover $F_s(.)$

Back out s and $m(.)$

$$\underbrace{F_s(s)}_{\text{just recovered}} = \underbrace{G_{m|v}(m|v)}_{\text{data}}$$

- For a given (v, m) we can use above equation to back out s

$$s = F_s^{-1}\left(G_{m|v}(m|v)\right)$$

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- For a given (v, m) we can use above equation to back out s

$$s = F_s^{-1}\left(G_{m|v}(m|v)\right)$$

- We can also obtain the matching function by inverting:

$$m(v, s) = G_{m|v}^{-1}(F_s(s)\,|v)$$

Non-Parametric Identification: Expanded Model (Multiple Types)

Expanded Model – Overview

- Allow for multiple types of workers, each with their own search efficiency, e.g.

$$S_t = s_t^u u_t + s_t^n n_t + s_t^e e_t$$

- Matches given by

$$m_t = m\left(V_t, A_t S_t\right)$$

Expanded Model – Overview

$$S_t = s_t^u u_t + s_t^n n_t + s_t^e e_t$$

- Since

$$\Pr(E|i,t) = \frac{m_t}{S_t} s_t^i$$

can recover relative search effort from relative job finding probabilities

$$\frac{s_t^i}{s_t^u} = \frac{P(E|i,t)}{P(E|u,t)}$$

  - only need to identify search efficiency of one group of workers, e.g. s_t^u

- Assume CRTS and independence of V_t, A_t conditional on S_t (testable)

▸ Monte Carlo Results for MP model

- Can back out matching efficiency, A_t and matching function, $m(.)$

Empirics

Overview - Empirics

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2. Use CPS / JOLTS to:
    2.1 estimate the matching function
    2.2 test the identifying independence assumption

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1. Use measure of Mukoyama et al. (2018) to pin down search of unemployed, s_t^u
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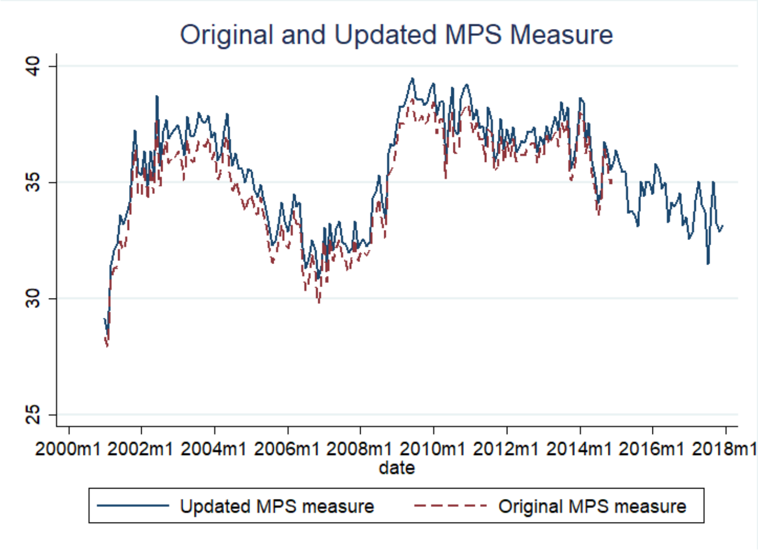
   - allow for changes in recruiting effort using Davis et al. (2013) index
2. Use CPS / JOLTS to:
   2.1 estimate the matching function
   2.2 test the identifying independence assumption
3. Applications 2001-2017:
   3.1 Measure changes in matching efficiency, A_t
   3.2 Decompose changes in hiring to different factors
   3.3 How did responsiveness of matching function wrt vacancies change?
   3.4 Elasticity estimates: non-parametric vs Cobb-Douglas

Data for Estimation

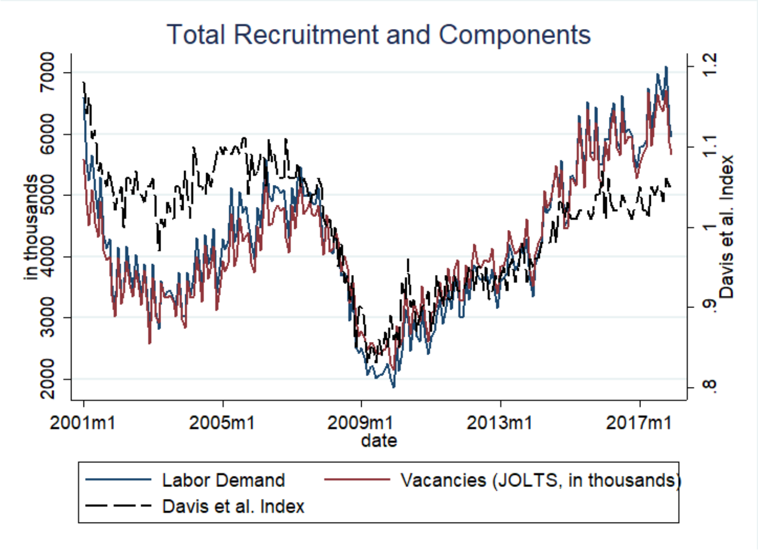
- Job Openings and Labor Turnover Survey (JOLTS)
  - monthly matches and vacancies from Dec 2001 through July 2017
- Current Population Survey (CPS)
  - stocks of unemployed, employed, out of the labor force
  - monthly transition probabilities to (new) employment by group

Matches JOLTS

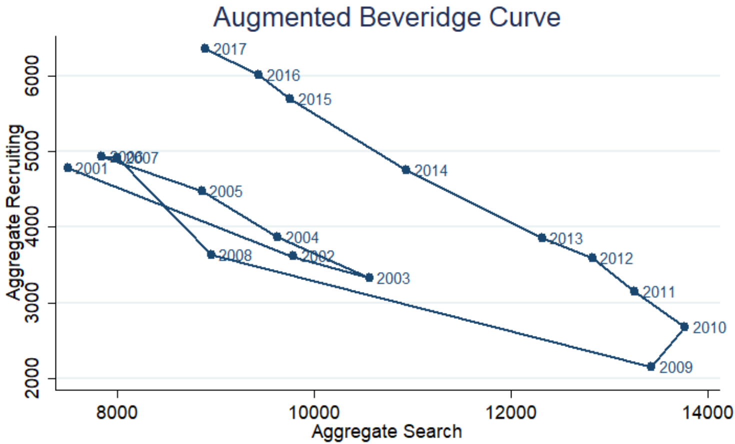
Mukoyama et al. Unemployed Search



Vacancy Recruiting Intensity Index (DFH)



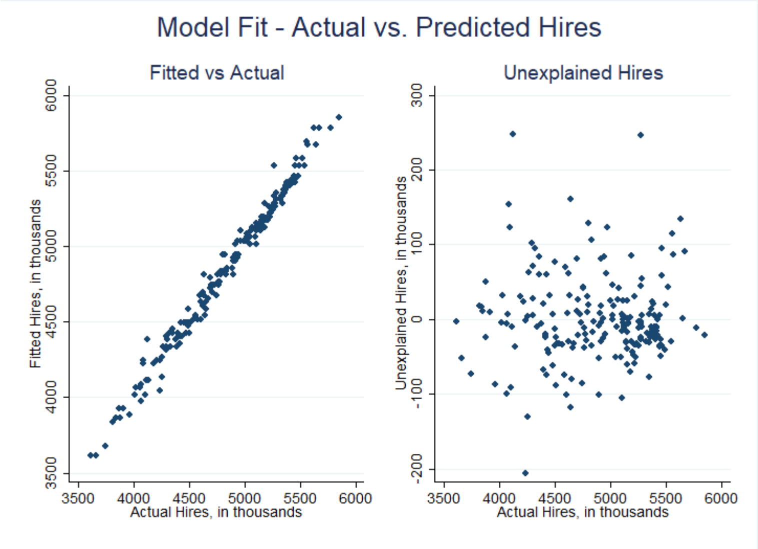
Augmented Beveridge Curve



Aggregate Search and Recruiting are annual averages measured in thousands of unemployed and vacancy equivalents. These account for composition, for observed recruiting intensity (using the Davis index) and observed search using the MPS index. They also include unobserved search effort imputed using the method outlined in this paper.

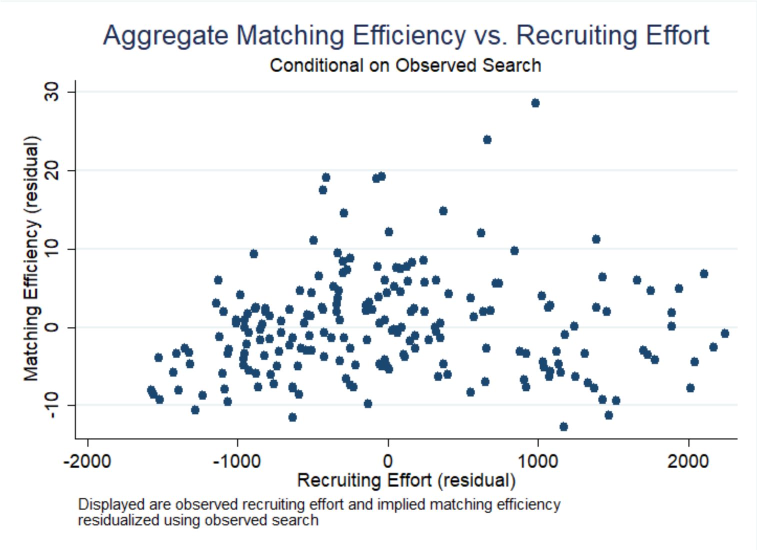
Empirical Implementation: Main Results

Model Fit



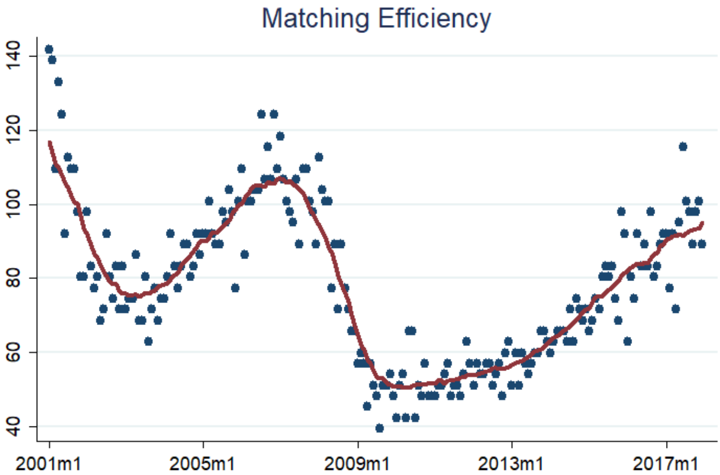
Model Fit - Actual vs. Predicted Hires
Fitted vs Actual
Fitted Hires, in thousands
3500
4000
4500
5000
5500
6000
Actual Hires, in thousands
3500
4000
4500
5000
5500
6000
Unexplained Hires
Unexplained Hires, in thousands
-200
-100
0
100
200
300
Actual Hires, in thousands
3500
4000
4500
5000
5500
6000

Independence of V_t and A_t conditional on S_t



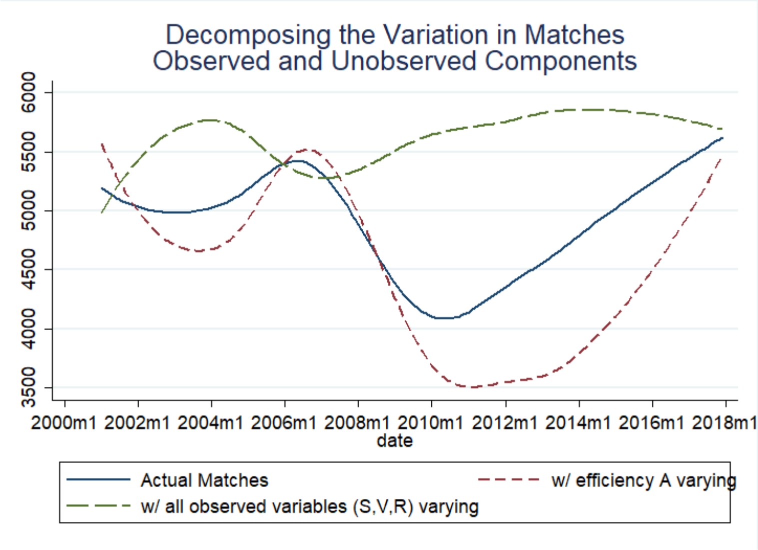
- correlation = 0.16

Matching Efficiency

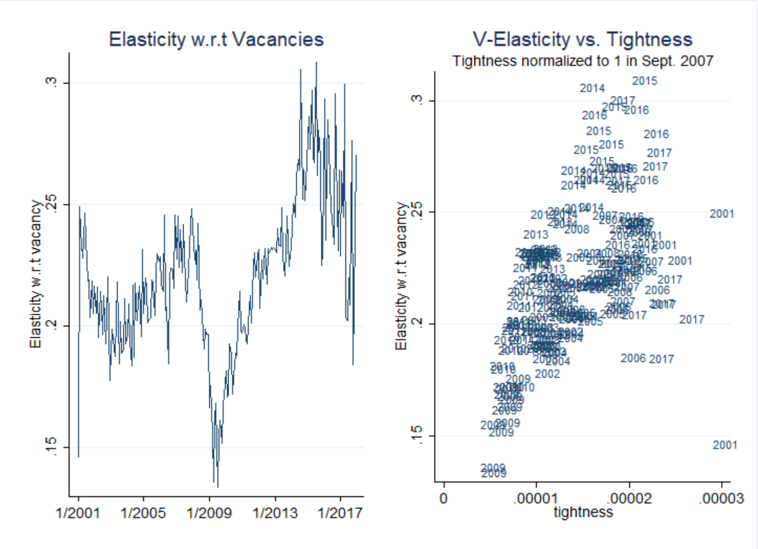


The scatter are the monthly estimates and the smooth line a 18 months moving average of matching efficiency

Decomposition of Hires



Elasticity of Matching Function wrt Vacancies



Elasticity with respect to Vacancies - Comparison

Table 1: Cobb-Douglas Specifications

	Using Observed Search S	
Log(Obs. Recruiting Effort) (V)	0.617 (0.015)**	0.195 (0.005)**
Log(Obs. Search) (S)	0.384 (0.015)**	
Log(Matching Efficiency*Search) (A*S)		0.805 (0.005)**
Constant	-4.189 (0.174)**	-12.474 (0.085)**
N	204	204

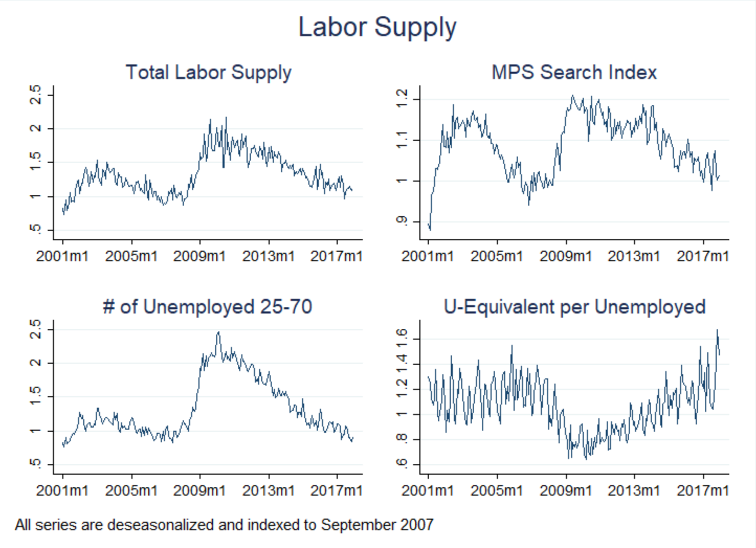
* $p < 0.05$; ** $p < 0.01$

The table reports linear regressions of log hiring (JOLTS) on log search and recruiting effort. Recruitment effort equals the number of vacancies adjusted using the Davis et al. index. Both specifications impose that the coefficients sum to 1.

Conclusion

- Matching function estimation
  - relax strong parametric restrictions
  - allow for multiple job seekers
  - back out matching efficiency that responds to market conditions
- Estimated elasticity wrt vacancies around 0.25, well below lit estimates
  - increases when markets are tight
- Matching efficiency is strongly procyclical
  - accounts for most of the decline in hires during Great Recession

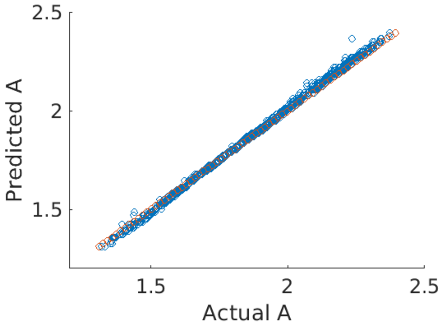
Components of Search

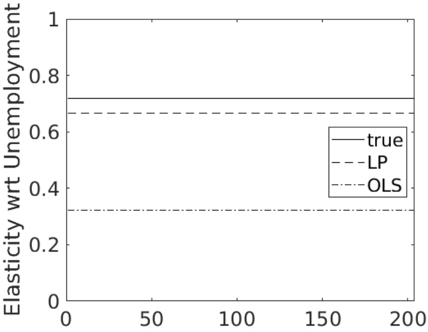


Application in MP Setup

- Follow Shimer's (2005) approach but now allow matching efficiency, A_t, to vary
  - use Shimer's (2005) calibration
  - calibrate to match implied volatility of A_t in Cobb-Douglas specification in Shimer (2005)
- Simulate MP model under two scenarios:
  - A_t and p_t are independently distributed
  - A_t and p_t are perfectly positively correlated

A and p are independent





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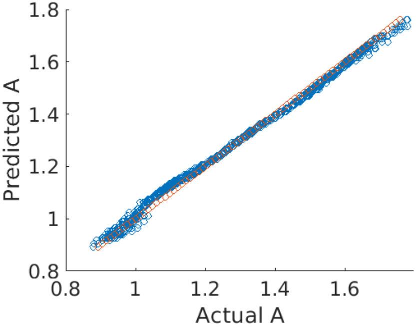

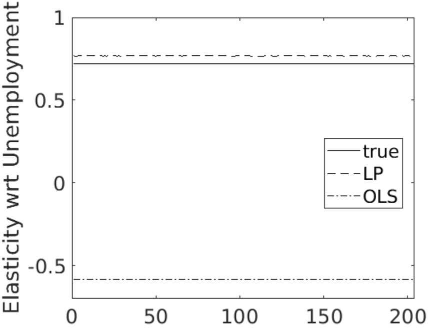

Go Back

Instrumental Variable Approach

$$v_t = h\left(z_t, \eta_t\right)$$

where z_t is independent from s_t and η_t. E.g.

$$v_t = \beta_0 + \beta_1 z_t + \eta_t$$

- Now v_t and s_t are independent, *conditional* on η_t:
  - v_t now only depends on z_t which is independent of s_t.

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- Now v_t and s_t are independent, *conditional* on η_t:
  - v_t now only depends on z_t which is independent of s_t.
- Following same steps as before leads to

$$G_{m|v,\eta}(m_t|v_t, \eta_t) = F_{s|\eta}(s_t|\eta_t)$$

  and can back out s_t and the matching function.

Instrumental Variable Approach

$$v_t = h(z_t, \eta_t)$$

- Can now test

$$\hat{s}_t \perp z_t$$

- Carries through in the case of multiple worker types.
    - now need instrument independent of search effort of *one* group of workers, not all.
- Potential instrument: innovation in vacancies (conditional on observed search). Back

Cobb-Douglas: Exploiting the Functional Form

Regression Basics

$$M_t = AV_t^{\gamma} S_t^{1-\gamma}$$

$$m_t = a + \gamma v_t + (1-\gamma) s_t$$

- In a regression framework, identify $\{\gamma, \{s_t\}_t\}$ by
  - assuming $S_t \perp V_t$
  - imposing a normalization (say $S_0 = 1$) .

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- In a regression framework, identify $\{\gamma, \{s_t\}_t\}$ by
  - assuming $S_t \perp V_t$
  - imposing a normalization (say $S_0 = 1$) .
- Do we really need this much?

Relaxing $S_t \perp V_t$ using Cobb-Douglas

$$M_t = AV_t^{\gamma} S_t^{1-\gamma}$$

- Normalize at some point (V_0, M_0):

$$M_0 = AV_0^{\gamma} S_0^{1-\gamma}$$

- Given observed V_0 solve for

$$Prob(S < S_0 | V_0) = Prob(M < M_0 | V_0) = \pi_0$$

Relaxing $S_t \perp V_t$ using Cobb-Douglas

Take some other point $V_1 = \lambda V_0$. At this point:

$$M(S, V_1) = A(\lambda V_0)^{\gamma} S^{1-\gamma} = \lambda^{\gamma} A V_0^{\gamma} S^{1-\gamma}$$

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$$M(S, V_1) = A(\lambda V_0)^\gamma S^{1-\gamma} = \lambda^\gamma A V_0^\gamma S^{1-\gamma}$$

Assume that

$$\pi_0 = Pr(S < S_0 | V_0) = Pr(S < S_0 | V_1)$$

Relaxing $S_t \perp V_t$ using Cobb-Douglas

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Assume that

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Then

$$\begin{aligned} Pr(M < \lambda^\gamma M_0 | V_1) &= Pr\left(V_1^\gamma S^{1-\gamma} < \lambda^\gamma V_0^\gamma S_0^{1-\gamma} | V_1\right) \\ &= Pr\left(\lambda^\gamma V_0^\gamma S^{1-\gamma} < \lambda^\gamma V_0^\gamma S_0^{1-\gamma} | V_1\right) \\ &= Pr\left(S^{1-\gamma} < S_0^{1-\gamma} | V_1\right) = \pi_0 \end{aligned}$$

Relaxing $S_t \perp V_t$ using Cobb-Douglas

Take some other point $V_1 = \lambda V_0$. At this point:

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Thus, γ is identified from

$$Pr(M < \lambda^\gamma M_0 | V_1) \equiv \pi_0$$

Point?

1. Identification requires that $\exists(V_0, V_1)$ and S_0 s.t:

$$Pr(S < S_0|V_0) = Pr(S < S_0|V_1)$$

   1.1 Much weaker than $V \perp S$.

Point?

1. Identification requires that $\exists(V_0, V_1)$ and S_0 s.t:

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   1.1 Much weaker than $V \perp S$.

2. With this assumption:

   2.1 back out γ and S_t for all observations (V_t, M_t)

   2.2 and test $S \perp V$.

Expanded Model Identification

- Following same steps as in baseline case can obtain

$$F_{s^u|\widetilde{u}}\left(s^u|\widetilde{u}\right) = G_{m|\widetilde{u},v}\left(m|\widetilde{u},v\right)$$

where

  - $F_{s^u|\widetilde{u}}(.)$ is the distribution of s^u conditional on $\widetilde{u}$ and
  - $G_{m|\widetilde{u},v}(.)$ is the distribution of m conditional on $\widetilde{u}$ and v

- Details:

$$\begin{aligned}
G_{m|\widetilde{u},v}\left(m\left(s\widetilde{u},v\right)|\widetilde{u},v\right) &= Pr\left(M < m\left(s^u\widetilde{u},v\right)|\widetilde{u},v\right) \\
\text{monotonicity} &= Pr\left(s^u < \frac{m^{-1}\left(M,v\right)}{\widetilde{u}}|\widetilde{u},v\right) \\
\text{independence} &= Pr\left(s^u < \frac{m^{-1}\left(M,v\right)}{\widetilde{u}}|\widetilde{u}\right) \\
&= F_{s^u|\widetilde{u}}\left(s^u|\widetilde{u}\right)
\end{aligned}$$

Back

Recover $F_{s^u|\widetilde{u}}$

$$F_{s^u|\widetilde{u}}\left(s^u|\widetilde{u}\right) = \underbrace{G_{m|\widetilde{u},v}\left(m|\widetilde{u},v\right)}_{\text{data}}$$

- As before, normalize

$$m_0 = m\left(s_0^u \widetilde{u}_0, v_0\right)$$

  for some $\left(m_0, s_0^u, \widetilde{u}_0, v_0\right)$

- Multiply through by λ to obtain

$$F_{s^u|\lambda\widetilde{u}_0}\left(s_0^u|\lambda\widetilde{u}_0\right) = G_{m|\lambda\widetilde{u}_0,\lambda v_0}\left(\lambda m_0|\lambda\widetilde{u}_0, \lambda v_0\right)$$

Thus for every $\lambda\widetilde{u}_0$ can back out one point in distribution of $F_{s^u|\lambda\widetilde{u}_0}(.)$

Recover $F_{s^u|\widetilde{u}}$

$$F_{s^u|\lambda\widetilde{u}_0}(s_0^u|\lambda\widetilde{u}_0) = \underbrace{G_{m|\lambda\widetilde{u}_0,\lambda v_0}(\lambda m_0|\lambda\widetilde{u}_0, \lambda v_0)}_{\text{data}}$$

- Let

$$\begin{aligned} \widetilde{u}_1 &= \lambda\widetilde{u}_0 \\ v_1 &= \lambda v_0 \\ m_1 &= \lambda m_0 \end{aligned}$$

- From before

$$F_{s^u|\widetilde{u}_1}(s_0^u|\widetilde{u}_1) = G_{m|\widetilde{u}_1,v_1}(m_1|\widetilde{u}_1, v_1)$$

Recover $F_{s^u|\widetilde{u}}$

$$F_{s^u|\lambda\widetilde{u}_0}(s_0^u|\lambda\widetilde{u}_0) = \underbrace{G_{m|\lambda\widetilde{u}_0,\lambda v_0}(\lambda m_0|\lambda\widetilde{u}_0, \lambda v_0)}_{\text{data}}$$

- Let

$$\begin{aligned} \widetilde{u}_1 &= \lambda\widetilde{u}_0 \\ v_1 &= \lambda v_0 \\ m_1 &= \lambda m_0 \end{aligned}$$

- From before

$$F_{s^u|\widetilde{u}_1}(s_0^u|\widetilde{u}_1) = G_{m|\widetilde{u}_1,v_1}(m_1|\widetilde{u}_1, v_1)$$

- Multiply through by θ (this time multiplying s rather than $\widetilde{u}$) to obtain

$$F_{s^u|\widetilde{u}_1}(\theta s_0^u|\widetilde{u}_1) = G_{m|\widetilde{u}_1,\theta v_1}(\theta m_1|\widetilde{u}_1, \theta v_1)$$

  Now for every $\widetilde{u}_1$ can trace out distribution of $F_{s^u|\widetilde{u}_1}$

Back

Back out s^u and $m(.)$

- Go back to our key equation

$$\underbrace{F_{s^u|\widetilde{u}}\left(s^u|\widetilde{u}\right)}_{\text{just recovered}} = \underbrace{G_{m|\widetilde{u},v}\left(m|\widetilde{u},v\right)}_{\text{data}}$$

- For a given $\widetilde{u}$, v, m, we can use above equation to back out ss

$$s^u = F^{-1}_{s^u|\widetilde{u}}\left(G_{m|\widetilde{u},v}\left(m|\widetilde{u},v\right)|\widetilde{u}\right)$$

- Similarly we can obtain the matching function, $m(.)$:
    - the number of matches, m for any $(s^u, \widetilde{u}, v)$ is given by

$$m\left(s^u\widetilde{u},v\right) = G^{-1}_{m|\widetilde{u},v}\left(F_{s^u|\widetilde{u}}\left(s^u|\widetilde{u}\right)|\widetilde{u},v\right)$$

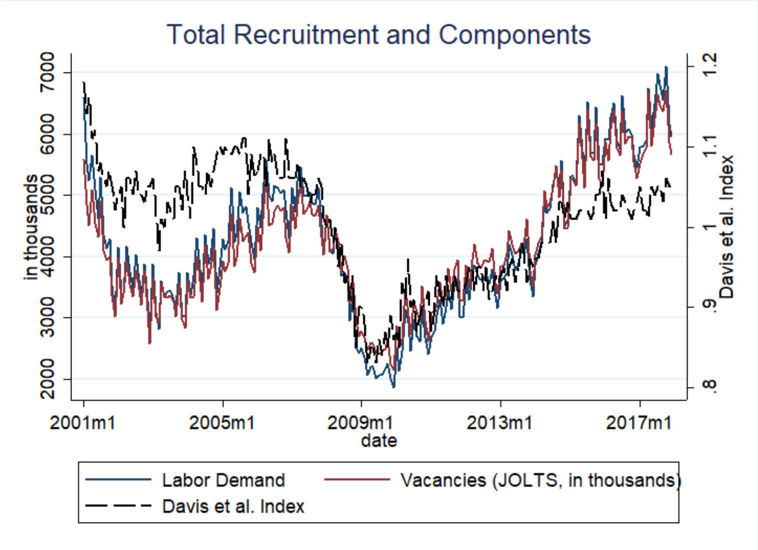
▸ Back

Allow for Vacancy Recruiting Intensity

- Use Index of Recruiting Intensity per Vacancy
  - Davis, Faberman and Haltiwanger (2013)
- Assume elasticity of job-filling rate wrt hires rate is the same in the aggregate as in the micro data
  - no assumptions on matching function/labor supply
- Now effective labor demand is

$$\text{Recruiting Intensity per Vacancy } \times v$$

Vacancy Recruiting Intensity Index (DFH)



Expanded Model Setup

- **Labor Demand:** vacancies, v
- **Labor Supply:**
  - 3 types of job seekers:
    - unemployed (u),
    - out of the labor force (n)
    - employed (e)
  - Each job seeker of type i exerts search effort s_t^i, $i \in \{u, n, e\}$
  - Economy wide search effort is

$$S_t = s_t^u u_t + s_t^n n_t + s_t^e e_t$$

Expanded Model Setup

$$S_t = s_t^u u_t + s_t^n n_t + s_t^e e_t$$

- **Matching Function:**

$$m_t = m(v_t, S_t)$$

and for type i probability of finding (new) job is given by

$$\Pr(E|i,t) = \frac{m_t}{S_t} s_t^i$$

Expanded Model Setup

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- **Goal:** Using data on m_t, v_t, u_t, n_t, e_t and $\Pr(E|i)$, recover:
  - $m(.)$ non-parametrically and
  - $s_t^i \ \forall i \in \{u, n, e\}$

Discussion: Multiple Types

$$S_t = s_t^u u_t + s_t^n n_t + s_t^e e_t$$

- We can allow for arbitrary # of different types of job seekers.

- s_t^i summarizes search efficiency which can be due to
  - variation in search effort
  - type-specific time-varying match efficiency

$$s_t^i = A_t^i \times \text{search effort}_t^i$$

Expanded Model Identification

- Since

$$\Pr(E|i,t) = \frac{m_t}{S_t} s_t^i$$

can recover relative search effort from relative job finding probabilities

$$\frac{s_t^i}{s_t^u} = \frac{P(E|i,t)}{P(E|u,t)}$$

Expanded Model Identification

- Since
$$\Pr(E|i,t) = \frac{m_t}{S_t} s_t^i$$
can recover relative search effort from relative job finding probabilities
$$\frac{s_t^i}{s_t^u} = \frac{P(E|i,t)}{P(E|u,t)}$$

- Rewrite
$$S_t = s_t^u \underbrace{\left(u_t + \frac{s_t^n}{s_t^u} n_t + \frac{s_t^e}{s_t^u} e_t\right)}_{\widetilde{u}_t \text{ (data)}} = s_t^u \widetilde{u}_t$$
$$m_t = m\left(v_t, s_t^u \widetilde{u}_t\right)$$
where $\widetilde{u}_t$ is measurable. Need to recover s_t^u and $m(.)$

Assumptions

$$m = m\left(v, s^u \widetilde{u}\right)$$

- $m(.)$ strictly increasing in s^u
- $m\left(.\right)$ is homogeneous of degree one
- $s^u \perp v|\widetilde{u}$
- Note: We require $s^u \perp v|\widetilde{u}$ for only one type of job seeker, not all!
- could also use an instrument

Expanded Model Identification: As before

Following same steps as in baseline case we can

1. obtain the distribution of s^u
2. back out the s^u up to a normalization,
3. get the matching function m(.)

▸ Details ▸ Go Back

Existing Estimates: Endogeneity of Tightness Θ

$$\frac{M(S_t, V_t)}{S_t} = A_t \Theta_t^{\alpha}$$

- Endogeneity of job seekers: $log\,(A_t) \not\perp log(\Theta_t)$ induces bias in regressions.

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- Dynamic Approaches following Borowcyk-Martins, Jolivet and Postel-Vinay (2013) assume
  - a dynamic specification on A_t with $\Theta_{t-k} \perp A_t$ and instrumenting for Θ_t using Θ_{t-k}
  - e.g. Cahuc et al. (2016) instrument with last quarter tightness for current tightness

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  - e.g. Cahuc et al. (2016) instrument with last quarter tightness for current tightness
- Problem if A_t = search intensity of job seekers and A_t itself depends on Θ_t

$\Rightarrow$ then any instrument for Θ_t that has "power" will also be endogenous

Existing Estimates I: Early Literature

- Common Assumption: Matching function is CRS

$$\frac{M_t}{U_t} = A_t \Theta_t^{\alpha}$$

where $\Theta_t = \frac{V_t}{U_t}$.

Existing Estimates I: Early Literature

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$$\frac{M_t}{U_t} = A_t \Theta_t^{\alpha}$$

where $\Theta_t = \frac{V_t}{U_t}$.

- Early literature (see Petrongolo and Pissarides (2000))
  - typically assumed $A_t \perp \Theta_t$
  - ignore other types of job seekers
  - estimate with OLS in logs

Existing Estimates II: Later Literature

- Try to incorporate multiple types of job search

$$M(S_t, V_t) = A_t S_t^{1-\alpha} V_t^{\alpha}$$

with for example

$$S_t = U_t + s_{E,t} E_t + s_{N,t} N_t$$

where $(s_{E,t}, S_{N,t})$ fraction searching on-the-job or from OLF

  - Broersma & van Ours 1999; Mumford and Smith 1999; Fallick & Fleischman 2001; Sunde 2007

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    - Borowcyk-Martins, Jolivet and Postel-Vinay 2013; Cahuc et al. 2016.
  - additional problem if A_t responds to Θ_t, maybe because A_t includes search effort of job seekers.