Is there a Golden Parachute in Sannikov's principal–agent problem?

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- $\mathbb{P}^\alpha$: equivalent to $\mathbb{P}$, s.t. $W^\alpha_\cdot := W_\cdot - \int_0^\cdot \alpha_s \mathrm{d}s$ is $\mathbb{P}^\alpha$–Brownian motion.

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(*iii*) ξ: $\mathcal{F}_\tau$-measurable r.v., non-negative payment at retirement.

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- Cost function of Agent given by map h, increasing, strictly convex, $h(0) = 0$.
- Principal is risk-neutral.

The contracting problem

Utility criteria of Agent and Principal

$$J^{\mathrm{A}}(\mathbf{C},\alpha) := \mathbb{E}^{\mathbb{P}^{\alpha}}\left[\mathrm{e}^{-r\tau}u(\xi) + \int_0^{\tau} r\mathrm{e}^{-rs}\big(u(\pi_s) - h(\alpha_s)\big)\mathrm{d}s\right],$$

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$$V^{\mathrm{A}}(\mathbf{C}) := \sup_{\alpha\in\mathcal{A}} J^{\mathrm{A}}(\mathbf{C},\alpha), \text{ and } \widehat{\mathcal{A}}(\mathbf{C}) := \left\{\widehat{\alpha}\in\mathcal{A} : V^{\mathrm{A}}(\mathbf{C}) = J^{\mathrm{A}}(\mathbf{C},\widehat{\alpha})\right\}$$

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Principal's problem

$$V^{\mathrm{P}} := \sup_{\mathbf{C}\in\mathfrak{C}_R}\ \sup_{\widehat{\alpha}\in\widehat{\mathcal{A}}(\mathbf{C})} J^{\mathrm{P}}(\mathbf{C},\alpha), \text{ where } \mathfrak{C}_R := \big\{\mathbf{C}\in\mathfrak{C} : V^{\mathrm{A}}(\mathbf{C}) \geq R\big\}$$

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Question

How can one formulate mathematically that the model exhibits a Golden Parachute?

Dynamics of Agent's continuation utility

- By standard martingale optimality principle, value function of Agent satisfies

$$\mathrm{d}Y_t = rZ_t\mathrm{d}X_t + r\big(Y_t + u(\pi_t) + H(Z_t)\big)\mathrm{d}t, \text{ where } H(z) := \sup_{a\in A}\big\{az - h(a)\big\},$$

with optimal effort process satisfying $\hat{a}_t \in \hat{A}(Z_t) := \mathrm{argmax}\, H(Z_t)$.

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- W.l.o.g. can concentrate on contracts of the form $u^{-1}\big(Y_T^{y,Z,\pi}\big)$, with

$$Y_t^{y,Z,\pi} := y + r\int_0^t \big(Y_s^{y,Z.\pi} + u(\pi_s) + H(Z_s)\big)\mathrm{d}s + \int_0^t rZ_s\mathrm{d}X_s, \; t \in [0,T].$$

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- Starting point of Cvitanić, P. and Touzi (2018), extended to random horizon by Lin, Ren, Touzi and Yang (2020).

Reduction to a mixed control–stopping problem

By the reduction result of Lin, Ren, Touzi and Yang (2020), we have

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The dynamic programming equation is

$$\text{(DPE)} \quad v(0) = 0, \text{ and } \min\left\{v - F, \mathbf{L}v\right\} = 0, \text{ on } (0, \infty),$$

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$$\mathbf{L}v := v - \delta y v' + F^\star(\delta v') - I_0{}^+\left(\delta v', \delta v''\right),$$

$$F^\star(p) := \inf_{y \geq 0} \left\{py - F(y)\right\},$$

$$I_0(p, q) := \sup_{z \geq h'(0),\ \hat{a} \in \hat{A}(z)} \left\{\hat{a} + h(\hat{a})p + \eta z^2 q\right\}.$$

Back to Golden Parachutes

• According to Sannikov (for $\delta = 1$) previous variational ODE has a unique solution of the form

$$v(0) = 0, \ \mathbf{L}v = 0, \text{ on } [0, y_{\mathrm{gp}}], \text{ and } v = F, \text{ on } [y_{\mathrm{gp}}, +\infty),$$

where y_{gp} is a free boundary such that

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(*i*) previous definition of Golden Parachute no longer relevant;

(*ii*) requires to introduce a **face-lifted agent's utility function**;

(*iii*) contrasting some conjectures/claims by Sannikov.

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Face-lifted (inverse) utility

$$\overline{F}(y_0) := \sup_{\pi \geq 0} \sup_{T \in [0, T_0^{y_0,\pi}]} \left\{ \mathrm{e}^{-\rho T} F\big(y^{y_0,p}(T)\big) - \int_0^T \rho \mathrm{e}^{-\rho t} \pi(t) \mathrm{d}t \right\}.$$

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  - Principal reaches the upper bound of his utility.

Golden Parachute revisited

Definition

We say that the contracting model exhibits a Golden Parachute, if there exists an optimal contract $(\tau^\star, \pi^\star, \xi^\star) \in \mathfrak{C}_R$ for the relaxed formulation of Principal's problem such that $\tau^\star > 0$, and $\mathbb{P}[\xi^\star > 0] > 0$.

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From the PDE point of view

Definition

The model exhibits a Golden Parachute if Principal's relaxed value function satisfies $v = \overline{F}$ on $[y_{\rm gp}, +\infty)$ for some $y_{\rm gp} > 0$.

Some cases of NGP

Proposition

Let $\beta := h'(0)$. Then there is No Golden Parachute whenever either

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(NGP1) $\beta = 0$;

(NGP2) or $\beta > 0$, $\overline{F}''$ is non-increasing, and "β large enough";

(NGP3) or $\beta > 0$, A is an interval, h' is convex, and "β large enough".

Numerical result 1

Same parameters as in Sannikov '08: $\gamma = 2$, $\eta = 0.05$, $h = 0.5$, $\beta = 0.4$, and $\delta = 1$

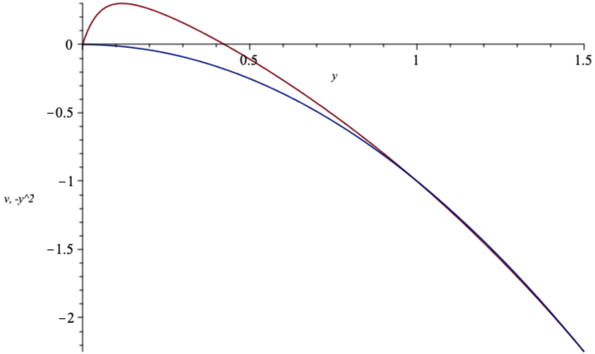

Figure: v (red), F (blue): Golden Parachute exists

Numerical result 2

$\gamma = 3/2$, $\eta = h = 1$, $\beta = 0.01$, and $\delta = 3/4$

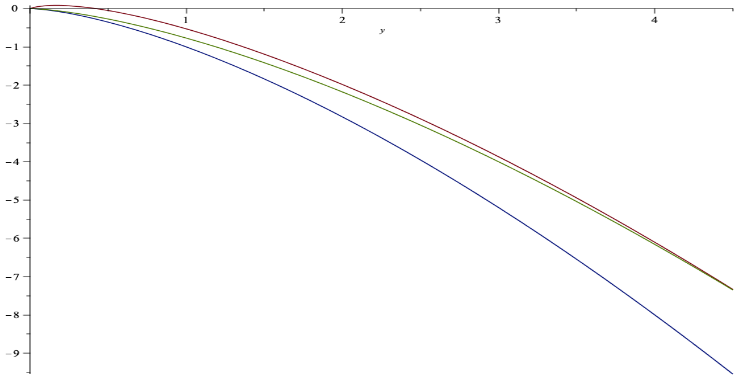

Figure: v (red), $\overline{F}$ (green), F (blue), Golden Parachute exists

Conclusions

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- When $\delta > 1$, solution must always be decreasing (meaning no informational rent), which is very different from Sannikov's message.

- Preliminary results for more general utility functions (negative powers or logarithm) seem to show that solution is very sensitive to data.

Thank you for your attention!