Binary Response Dynamic Panel Data Models with Switching State Dependence

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Abstract

This paper studies identification in binary response dynamic panel data models with switching state dependence. Departing from the standard approach of modelling binary response dynamic panel data models, where last period's choice enters as an additional regressor, this paper allows for switching dependence where current period's decision depends on whether this period's choice differs from last period's choice. This form of correlation causes inertia in individual choices and is suitable for modeling cases where individuals face some form of high "switching costs". This contemporaneous effect in choices, where the choice an individual makes in the current period directly affects current period's latent utility, results in the model being logically inconsistent, making the model both incomplete and incoherent, which might result in lack of point-identification.

Keywords: Discrete Response Models, Panel Data Models, Incompleteness, Incoherence, Partial Identification

JEL classification Numbers: C01, C23, C25.

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1 Introduction

This paper studies identification in binary response dynamic panel data models with switching dependence. In particular, departing from the standard approach of modelling binary response dynamic panel data models, where last period's choice enters as an additional regressor, this paper allows current period's decision to depend on whether this period's choice differs from last period's choice. This specification in choices, where the choice an individual makes in the current period directly affects current period's latent utility, results in the model being logically inconsistent as described in Maddala (1983), making the model both incomplete and incoherent, which might result in lack of point-identification.

In panel data settings where individuals are observed for multiple periods it is often noted that decisions are intertemporaly correlated. As pointed out by Heckman (1981) the intertemporal correlation in the decisions of individuals in panel data models comes in general through the presence of time-invariant unobservables and lagged dependent variables in the underlying functional form specification and distinguishing the exact causes of correlation in individual choices has important policy implications. Most of the literature has considered a combination of the two, assuming that the actual choices individuals made in the past affect current and future choices. In the classical binary response panel data models, point-identification is achieved only under very strong assumptions (Chamberlain (1984), Honoré (2002), Honoré and Kyriazidou (2000), Manski (1987)). Similarly, a recent growing strand of the literature studies identification in dynamic ordered response logit models, Muris et al. (2020) and Honoré et al. (2021). Semiparametric and partial identification under mild restrictions on the time-varying unobservables and the fixed effect in binary response dynamic panel data models and more general panel data models has been extensively studied by a number of authors, for example Aristodemou (2021), Chernozhukov et al. (2013), Honoré and Tamer (2006), Khan et al. (2019), Rosen (2012) and Rosen and Weidner (2013,WP).

In many settings it is also often noted that individuals who were observed making a specific choice in the past are more likely to make the same choice in the future. This form of correlation causes inertia in individual choices, which can be seen as a psychological switching cost observationally equivalent to monetary switching costs (Dubé et al. (2009))[1]. Aguirregabiria et al. (2021) and Pakes et al. (2021) explicitly examine the persistence in individuals' choices in structural dynamic panel data models. This paper examines dynamic settings where individuals choice depends on whether the choice in the current period differs from the choice of the past period. Such specification would be suitable for modeling cases where individuals face some form of high "switching costs", in terms for example of monetary costs, formation of habits and loyalty or are somehow locked-in once they made a decision and therefore staying with the same option from one period to the next has an additional effect on their utility.

Duration models have also been used in the literature modelling the duration in the current state, e.g. Lancaster (1979) and Frederiksen et al. (2007). As discussed in a series of recent papers, Gørgens and Hyslop (2018, 2019), the literature has modeled binary response panel data models either using the dynamic binary response model (DBR) where the focus is on the probability that in the current period the individual chooses one of the two options or using the multi-spell duration model (MSD) where the focus is on the probability of transitions between the options from one period to the next, and show that there is an one-to-one correspondence between the two representations. This paper departs from these specifications and models the probability of choosing a specific option in one period conditional on whether the option is different from the one chosen last period.

This paper studies identification under mild distributional assumptions on the fixed effect and the time-varying unobservables. An additional complication in this paper is that the model under consideration is incomplete and incoherent, which in general result in an inability to obtain point-identification even under full parametric assumptions. The aim is to provide new partial identification results in this kind of models, using recent developments in the treatment of incoherent and incomplete models and in semiparametric identification of fixed effects discrete response models.

The rest of the paper is structured as follows. Section 2 introduces the model and Section 3 discusses the incompleteness and incoherence of the model. Section 4 gives an outline of the

[1] A focus also in the industrial organization literature has been on passive consumers and rational inattention, see for example Clerides and Courty (2017)

identification strategy and Section 5 concludes and discusses current work-in-progress.

2 The Model of Switching Dependence

Binary response panel data models are widely used to model situations where individuals are observed over time making choices from a set that includes two alternatives, for example the choice of seeking employment or not or the choice of travelling by train or by car in a specific period. The leading example in the literature has been the static binary response model, where individuals' choices are correlated across different periods only through the presence of a time-invariant unobserved heterogeneity. This paper extends the linear index binary response static model to the linear index binary response dynamic model. This is of practical relevance because in panel data settings with repeated observations it is evident and natural to assume that individuals' past choices directly affect current and future decisions. For example, an individual's decision to seek employment in the current period is likely to be affected by his employment status last period in addition to other factors, such as potential income and years of education. This allows for correlation in choices to come through two sources, the fixed effect and the lagged dependent variable.

In this model each individual is observed for three periods, $t = \{0, 1, 2\}$, and is characterized by a set of observables $Y = (Y_0, Y_1, Y_2)$, $X = (X_1, X_2)$, and a set of unobservables (V, α), where $V = (V_1, V_2)$ and $\alpha \in \mathbb{R}$. As it is common in many discrete response models it is assumed that each individual in every period $t = 1$ and $t = 2$ chooses the option from the set of binary outcomes $Y_t \in \mathcal{Y}_t$, where $\mathcal{Y}_t = \{0, 1\}$, that maximizes their latent utility, as in McFadden (1974). The utility an individual, with covariates X_t, Y_{t-1} and unobservables V_t, α, receives from choosing a specific outcome Y_t is then given by model (1),

$$
\begin{aligned}
U_t &= X_t\beta + 1(Y_t = Y_{t-1})\gamma + \alpha + V_t \\
Y_t &= 1(U_t > 0)
\end{aligned}
\qquad (1)
$$

where $1(\cdot)$ the indicator function which equals to 1 if $(\cdot)$ is true and 0 otherwise, X_t are observed individual characteristics in period t, Y_t is the observed individual choice in period t,

α is the unobserved to the econometrician time-invariant individual fixed effect and V_t is the unobserved to the econometrician time-varying component in period t. The specification in (1) allows the utility in the current period to depend on whether the individual changes or not the option from last period, by modelling the dependence on the lagged dependent variable using a switching indicator function defined by

$$1(Y_t = Y_{t-1}) = \begin{cases} 1 & if\ Y_t = Y_{t-1} \\ 0 & otherwise \end{cases} \tag{2}$$

Such specification would be suitable in modeling cases where individuals face some form of "switching costs" such that staying with the same option from one period to the next has an additional effect on their utility, captured by the state dependence parameter γ. The important difference in this setting with the standard setting is that the lagged dependent variable does not enter as an additional regressor in the form of Y_{t-1}, like for example in Honoré and Kyriazidou (2000), where the parameter γ measures the "impact" of choosing option $Y = 1$ in period $t-1$. Nevertheless, the similarity with the common approaches in the literature is that last period's choice directly affects the decision so the choice in period $t-1$ needs to be taken into account, which creates an initial condition problem in modeling the choice in period $t = 1$, since the choice in period $t = 1$ depends on the choice in period $t = 0$. To deal with this issue, similar to Wooldridge (2005), it is assumed that the outcome in period $t = 0$, $Y_0 = y_0$, is observed, however no assumptions about its generation or its relation with the fixed effect are imposed, such that the set of conditioning covariates consists of $(x, y_0) \in \mathcal{X} \times \mathcal{Y}_0$. Section 2.1 formalizes the assumptions.

2.1 Model Assumptions

Assumption 1. *(Random Sampling) The observed data comprise a random sample of N individuals from the population. For each individual (Y, X, V, α) are defined on the probability space $(\Omega, \mathcal{F}, \mathbb{P})$, where $\mathcal{F}$ contains the Borel Sets. The support of (Y_0, X, V, α) is $(\mathcal{Y}_0 \times \mathcal{X} \times \mathcal{V} \times \mathcal{A})$ where $\mathcal{V} \subseteq \mathbb{R}^2$ and $\mathcal{A} \subseteq \mathbb{R}$.*

Assumption 2. *(Conditional Independence) X and V are stochastically independent conditional on Y_0, i.e. $V \perp X|Y_0$.*

Assumption 1 defines the underlying probability space and notation for the support of the random variables (Y, X, V, α). Under this assumption the data comprise a random sample and therefore the conditional distribution $P(y_1, y_2|x, y_0) \equiv P(Y_1 = y_1 \wedge Y_2 = y_2|X = x, Y_0 = y_0)$ is point-identified over the support of (Y_1, Y_2) for almost every $x \in \mathcal{X}$ and $y_0 \in \mathcal{Y}_0$. Assumption 2 imposes only conditional independence between X and V conditional on Y_0. Furthermore, no assumptions are imposed on α which is allowed to be correlated with both V and X in an arbitrary way. The presence of the fixed effect in the dynamic binary response model creates an endogeneity problem since the fixed effect is correlated with the switching indicator function by construction. Not imposing any assumptions on the fixed effect however creates an additional endogeneity problem between the fixed effects and the explanatory variables, X. These endogeneity problems need to be addressed for identification and consistent estimation of the parameters of interest.

For completeness define the conditional distributions of the unobservables given the observed covariates that are elements of the generic collection of conditional distributions as,

- $\{F_{V|X,Y_0}(\cdot|x, y_0) : x \in \mathcal{X}, y_0 \in \mathcal{Y}_0\}$ is an element of $\mathcal{F}_{V|X,Y_0}$
- $\{F_{\Delta V|X,Y_0}(\cdot|x, y_0) : x \in \mathcal{X}, y_0 \in \mathcal{Y}_0\}$ is an element of $\mathcal{F}_{\Delta V|X,Y_0}$, where $\Delta V = V_2 - V_1$
- $\{F_{\alpha|X,Y_0}(\cdot|x, y_0) : x \in \mathcal{X}, y_0 \in \mathcal{Y}_0\}$ is an element of $\mathcal{F}_{\alpha|X,Y_0}$
- $\{F_{(V,\alpha)|X,Y_0}(\cdot|x, y_0) : x \in \mathcal{X}, y_0 \in \mathcal{Y}_0\}$ is an element of $\mathcal{F}_{(V,\alpha)|X,Y_0}$

Finally, define a structure $S \equiv (\beta, \gamma, F_{(V,\alpha)|X,Y_0})$ as a specified collection of parameters β and γ, and joint distributions of the time-varying unobservables and the unobserved heterogeneity, $F_{(V,\alpha)|X,Y_0}$. The set of admissible structures is thus defined in Assumption 3.

Assumption 3. *(Admissible Structures) The structure S admitted by the model belongs to a collection $\mathcal{S}$ of parameters β and γ belonging to a parameter space Θ and joint distributions of the time-varying unobservables and the unobserved heterogeneity, $F_{(V,\alpha)|X,Y_0} \in \mathcal{F}_{(V,\alpha)|X,Y_0}$.*

3 Incompleteness and Incoherence

Section 2.1 imposes minimal assumptions on the fixed effect and the time-varying unobservables. As in Aristodemou (2021) identification of the parameters of interest (β, γ) will come through features of the distribution that are invariant to changes in the fixed effect, α, by considering the joint probability of the choices individuals make in periods $t = 1$ and $t = 2$, conditional on the choice in period $t = 0$.

The choice in period $t - 1$ directly affects the utility, and hence the choice in period t. Consider now the two cases where $Y_{t-1} \neq Y_t$ and $Y_{t-1} = Y_t$, when $Y_t \neq Y_{t+1}$. In the first case the utility function defined in (1) becomes,

$$\begin{aligned} U_t &= X_t\beta + \alpha + V_t \\ U_{t+1} &= X_{t+1}\beta + \alpha + V_{t+1} \end{aligned} \tag{3}$$

which do not depend on γ. In the latter case the utility function becomes,

$$\begin{aligned} U_t &= X_t\beta + \gamma + \alpha + V_t \\ U_{t+1} &= X_{t+1}\beta + \alpha + V_{t+1} \end{aligned} \tag{4}$$

and γ appears in the equation determining U_t. Notice one important feature of the utility function in (1). The underlying within period decision rule for each $y_t \in \mathcal{Y}_t$ changes given Y_{t-1} and violates the single-index restriction which is fundamental in many discrete response models. Hence, when $Y_{t-1} = 1$

$$Y_t = \begin{cases} 1 & if \quad X_t\beta + \gamma + \alpha + V_t > 0 \\ 0 & if \quad X_t\beta + \alpha + V_t < 0 \end{cases} \tag{5}$$

and when $Y_{t-1} = 0$

$$Y_t = \begin{cases} 1 & if \quad X_t\beta + \alpha + V_t > 0 \\ 0 & if \quad X_t\beta + \gamma + \alpha + V_t < 0 \end{cases} \tag{6}$$

Identification in this model comes by observing the choices individuals make in periods $t = 1$ and $t = 2$ conditional on the initial condition. The regions of unobservables (V, α),

$\mathcal{R}^{DB}_{(y_1,y_2)}(x, y_0; \beta, \gamma)$, associated with each $(Y_1, Y_2) = (y_1, y_2)$ choice when $X = x$ and $Y_0 = y_0$ are given by,

$$\begin{aligned}
\mathcal{R}^{DB}_{(0,0)}(x,0;\beta,\gamma) &= \{(V,\alpha) \in (\mathcal{V},\mathcal{A}) : x_1\beta+\gamma+\alpha+V_1 \leq 0 \textit{ and } x_2\beta+\gamma+\alpha+V_2 \leq 0\} \\
\mathcal{R}^{DB}_{(0,1)}(x,0;\beta,\gamma) &= \{(V,\alpha) \in (\mathcal{V},\mathcal{A}) : x_2\beta+\alpha+V_2 > 0 \geq x_1\beta+\gamma+\alpha+V_1\} \\
\mathcal{R}^{DB}_{(1,0)}(x,0;\beta,\gamma) &= \{(V,\alpha) \in (\mathcal{V},\mathcal{A}) : x_2\beta+\alpha+V_2 \leq 0 < x_1\beta+\alpha+V_1\} \\
\mathcal{R}^{DB}_{(1,1)}(x,0;\beta,\gamma) &= \{(V,\alpha) \in (\mathcal{V},\mathcal{A}) : x_1\beta+\alpha+V_1 \geq 0 \textit{ and } x_2\beta+\gamma+\alpha+V_2 \geq 0\} \\
\mathcal{R}^{DB}_{(0,0)}(x,1;\beta,\gamma) &= \{(V,\alpha) \in (\mathcal{V},\mathcal{A}) : x_1\beta+\alpha+V_1 \leq 0 \textit{ and } x_2\beta+\gamma+\alpha+V_2 \leq 0\} \\
\mathcal{R}^{DB}_{(0,1)}(x,1;\beta,\gamma) &= \{(V,\alpha) \in (\mathcal{V},\mathcal{A}) : x_2\beta+\alpha+V_2 > 0 \geq x_1\beta+\alpha+V_1\} \\
\mathcal{R}^{DB}_{(1,0)}(x,1;\beta,\gamma) &= \{(V,\alpha) \in (\mathcal{V},\mathcal{A}) : x_2\beta+\alpha+V_2 \leq 0 < x_1\beta+\gamma+\alpha+V_1\} \\
\mathcal{R}^{DB}_{(1,1)}(x,1;\beta,\gamma) &= \{(V,\alpha) \in (\mathcal{V},\mathcal{A}) : x_1\beta+\gamma+\alpha+V_1 > 0 \textit{ and } x_2\beta+\gamma+\alpha+V_2 > 0\} \qquad (7)
\end{aligned}$$

Figure 1 plots the regions of unobservables for the case where $Y_0 = 0$ and $\gamma < 0$,

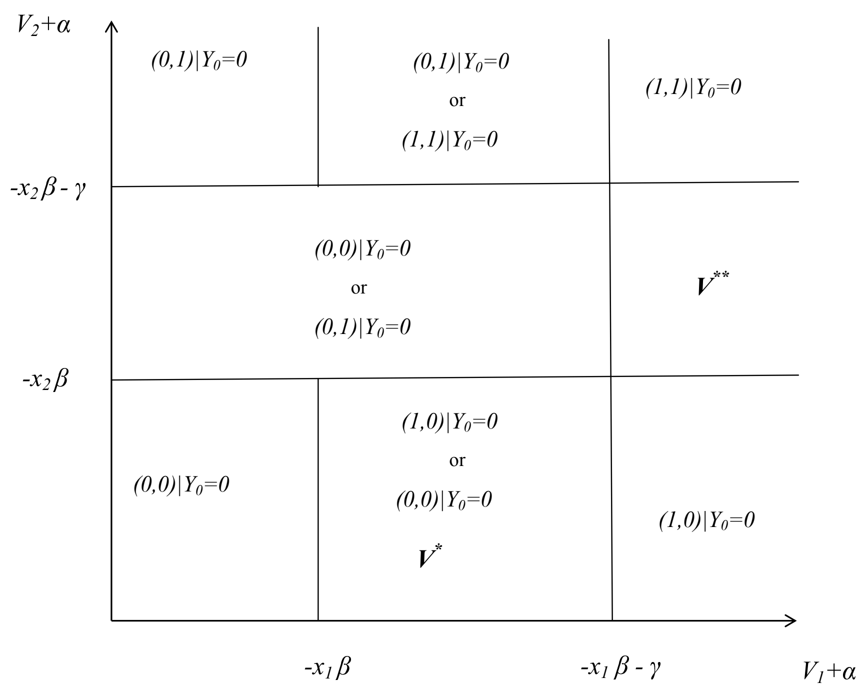

Figure 1: Regions of Unobservables for each (Y_1, Y_2) choice when $\gamma < 0$, $Y_0 = 0$ and unrestricted support of α.

Proposition 1. *From the regions defined in* (7) *and figure* (1)*, it can be seen that the dynamic model in* (1) *with* $\gamma \neq 0$ *is incomplete, in the sense that* $\exists (x, y_0) \in \mathcal{X} \times \mathcal{Y}_0$ *conditional on which there is not a unique solution to model* (1) *with probability 1.*

Proof. From (7) the regions of unobservables associated with the choice pairs $(Y_1, Y_2) = (1, 0)$ and $(Y_1, Y_2) = (0, 0)$ when $Y_0 = 0$ are defined by,

$$\begin{aligned} \mathcal{R}^{DB}_{(1,0)}(x, 0; \beta, \gamma) &= \{(V, \alpha) \in (\mathcal{V}, \mathcal{A}) : x_2\beta + \alpha + V_2 \leq 0 < x_1\beta + \alpha + V_1\} \\ \mathcal{R}^{DB}_{(0,0)}(x, 0; \beta, \gamma) &= \{(V, \alpha) \in (\mathcal{V}, \mathcal{A}) : x_1\beta + \alpha + \gamma + V_1 \leq 0 \textit{ and } x_2\beta + \gamma + \alpha + V_2 \leq 0\} \end{aligned}$$

Suppose also $\gamma < 0$ and consider any $(V, \alpha) \in \mathcal{V}^*$ such that

$$\mathcal{V}^* = \{(V, \alpha) \in (\mathcal{V}, \mathcal{A}) : -x_1\beta < V_1 + \alpha \leq -x_1\beta - \gamma \textit{ and } V_2 + \alpha \leq -x_2\beta\}$$

Then it can be shown that:

$$\begin{aligned} (V, \alpha) \in \mathcal{V}^* &\Rightarrow (V, \alpha) \in \mathcal{R}^{DB}_{(1,0)}(x, 0; \beta, \gamma) \\ (V, \alpha) \in \mathcal{V}^* &\Rightarrow (V, \alpha) \in \mathcal{R}^{DB}_{(0,0)}(x, 0; \beta, \gamma) \end{aligned}$$

This implies that the model in (1) is incomplete in the sense that there exists a pair of unobservables (V, α) such that conditional on X and Y_0 there are multiple solutions to the individual's problem. □

Proposition 2. *From the regions defined in* (7) *and figure* (1)*, it can be seen that the dynamic model in* (1) *with* $\gamma \neq 0$ *is incoherent, in the sense that* $\exists (x, y_0) \in \mathcal{X} \times \mathcal{Y}_0$ *conditional on which there is no solution to model* (1)*.*

Proof. Suppose $\gamma < 0$ and $Y_0 = 0$. Consider any $(V, \alpha) \in \mathcal{V}^{**}$, where

$$\mathcal{V}^{**} = \{(V, \alpha) \in (\mathcal{V}, \mathcal{A}) : -x_1\beta - \gamma \leq V_1 + \alpha \textit{ and } -x_2\beta < V_2 + \alpha < -x_2\beta - \gamma\}$$

If $-x_1\beta - \gamma \leq V_1 + \alpha$ then $Y_1 = 1$ and $Y_0 = 1$, which contradicts the conditioning on $Y_0 = 0$. However, $-x_1\beta - \gamma \leq V_1 + \alpha \Rightarrow -x_1\beta \leq V_1 + \alpha$. This implies that conditioning on $Y_0 = 0$, $-x_1\beta - \gamma \leq V_1 + \alpha \Rightarrow Y_1 = 1$.

For $(V, \alpha) \in \mathcal{V}^{**}$ it also means that $-x_2\beta < V_2 + \alpha < -x_2\beta - \gamma$. This can correspond to $-x_2\beta < V_2 + \alpha$ and $V_2 + \alpha < -x_2\beta - \gamma$. However, conditioning on $Y_1 = 1$ and $Y_0 = 0$ none of the two inequalities can hold. First take, $-x_2\beta < V_2 + \alpha$. This corresponds to the event $Y_2 = 1|Y_1 = 0$, which contradicts the $Y_1 = 1$ from above. Then the event $V_2 + \alpha < -x_2\beta - \gamma$ corresponds to $Y_2 = 0|Y_1 = 0$ which again contradicts $Y_1 = 1$. Therefore, $\mathcal{V}^{**} = \emptyset$ and the dynamic model in (1) is incoherent. Figure 1 shows the regions $\mathcal{V}^*$ and $\mathcal{V}^{**}$. □

Notice that no assumptions are being imposed on the support of the unobserved heterogeneiry, α. The support is unrestricted and does not depend on γ. If the support of α was allowed to depend on γ, it might be possible to find values of $\gamma \neq 0$, such that the model in (1) becomes coherent and complete.

The important feature of the model in (1) is the presence of current period's outcome in the determination of current period's utility. Unless $\gamma = 0$, the model in (1) is logically inconsistent as discussed in Maddala (1983). Such a model would resemble models of simultaneous response games like Tamer (2003) and more general binomial response models with dummy endogenous regressors as in Lewbel (2007)[2].

4 Identification

Section 3 illustrates that model (1) is both incomplete and incoherent unless $\gamma = 0$[3], which will lead us back to the binary response static panel data model. As a result, in order to model switching state dependence $\gamma \neq 0$, which implies that the identified set can not be in general characterized by a set of moment equalities and might result in an inability to obtain point-identification of the regression parameters. The aim of this paper is to study identification in binary response dynamic panel data models with switching dependence, allowing for fixed effects and not specifying a distribution for the time-varying unobservables. Identification

[2]One possible solution for the incompleteness problem is also imposing a selection mechanism among multiple potential equilibria: e.g. Bjorn and Vuong (1984), Bajari et al. (2010)

[3]Examples of papers imposing coherency and completeness conditions, include Schmidt (1981) and Blundell and Smith (1994)

thus comes from features of the distribution that are invariant to the fixed effect, by observing individuals who switch in two consecutive time periods, conditional on their initial condition. Such an approach was shown in Aristodemou (2021) to partially identify the model parameters in the case where the lagged dependent variable in the model in (1) enters as an additional regressor and therefore the model in (1) is complete and coherent.

The additional complication in the model of switching dependence is how to determine the outcome (Y_1, Y_2) when the model is not guaranteed to deliver a unique pair (y_1, y_2) or does not deliver any pair for any value of (x, v, α), which imposes extra identification challenges. For example, as is can also been seen in figure 1, $P(1,0|x,0) < F_{(V,\alpha)|X,Y_0}(R_{(1,0)}(x,0;\beta,\gamma))$. In contrast to a complete model, a model which is incomplete and/or incoherent implies that in general, the identified set for the model parameters (β, γ) can not be characterized by,

$$\Theta^0 = \left\{ \begin{array}{c} (\beta,\gamma) \in \Theta : \exists F_{(V,\alpha)|X,Y_0} \in \mathcal{F}_{(V,\alpha)|X,Y_0},\ \forall (y_1,y_2) \in (\mathcal{Y}_1 \times \mathcal{Y}_2) \\ F_{(V,\alpha)|X,Y_0}\left(\mathcal{R}_{(y_1,y_2)}(x,y_0;\beta,\gamma)\right) = P(y_1,y_2|x,y_0) \\ a.e.\ x \in \mathcal{X}\ and\ y_0 \in \mathcal{Y}_0 \end{array} \right\} \tag{8}$$

where $P(y_1, y_2|x, y_0) = P(Y_1 = y_1 \wedge Y_2 = y_2|X = x, Y_0 = y_0)$ is the observed[4]. Nevertheless, the identification strategy used in Aristodemou (2021), can still be applied, and identification bounds can still be derived that do not include the fixed effect. Theorem 1 illustrates this,

Theorem 1. *For any fixed pair x and y_0, (β, γ) satisfies*

$$\begin{aligned} P(1,0|x,0) &\leq P_{\Delta V|Y_0}[\Delta V < -\Delta x\beta | Y_0 = 0] \\ 1 - P(0,1|x,0) &\geq P_{\Delta V|Y_0}[\Delta V < -\Delta x\beta + \gamma | Y_0 = 0] \\ P(1,0|x,1) &\leq P_{\Delta V|Y_0}[\Delta V < -\Delta x\beta + \gamma | Y_0 = 1] \\ 1 - P(0,1|x,1) &\geq P_{\Delta V|Y_0}[\Delta V < -\Delta x\beta | Y_0 = 1] \end{aligned}$$

where $\Delta X = X_2 - X_1$ and $\Delta V = V_2 - V_1$.

[4]Complete and coherent models allow for the construction of a likelihood function if the model is correctly specified.

Proof. Consider event $\{Y_0 = 1 \wedge Y_1 = 0 \wedge Y_2 = 0\}$. The conditional probability for the event $(Y_1, Y_2) = (0, 1)$ conditional on $Y_0 = 0$ is,

$$\begin{aligned} P(1,0|x,0) &= P_{(V,\alpha)|X,Y_0}[\{X_1\beta + \alpha + V_1 > 0\} \wedge \{X_2\beta + \alpha + V_2 \leq 0\}|X = x, Y_0 = 0] \\ &\leq P_{V|X,Y_0}[(X_2 - X_1)\beta + V_2 - V_1 < 0|X = x, Y_0 = 0] \\ &= P_{\Delta V|X,Y_0}[\Delta V < -\Delta X\beta|X = x, Y_0 = 0] \end{aligned}$$

Applying the same argument for the rest of the sequences of events in implies that,

$$\begin{aligned} P(1,0|x,0) &\leq P_{\Delta V|Y_0}[\Delta V < -\Delta x\beta|X = x, Y_0 = 0] \\ 1 - P(0,1|x,0) &\geq P_{\Delta V|Y_0}[\Delta V < -\Delta x\beta + \gamma|X = x, Y_0 = 0] \\ P(1,0|x,1) &\leq P_{\Delta V|Y_0}[\Delta V < -\Delta x\beta + \gamma|X = x, Y_0 = 1] \\ 1 - P(0,1|x,1) &\geq P_{\Delta V|Y_0}[\Delta V < -\Delta x\beta|X = x, Y_0 = 1] \end{aligned}$$

For any fixed $X = x$ and by applying Assumption 2 , the above inequalities imply that

$$\begin{aligned} P(1,0|x,0) &\leq P_{\Delta V|Y_0}[\Delta V < -\Delta x\beta|Y_0 = 0] \\ 1 - P(0,1|x,0) &\geq P_{\Delta V|Y_0}[\Delta V < -\Delta x\beta + \gamma|Y_0 = 0] \\ P(1,0|x,1) &\leq P_{\Delta V|Y_0}[\Delta V < -\Delta x\beta + \gamma|Y_0 = 1] \\ 1 - P(0,1|x,1) &\geq P_{\Delta V|Y_0}[\Delta V < -\Delta x\beta|Y_0 = 1] \end{aligned}$$

which completes the proof. □

5 Conclusion

This paper proposes a new binary response dynamic panel data model with switching dependence, where attention is drawn to the situations when the individuals decide whether to change the option they choose from one period to the next. In many applications it is evident that individuals stay with the same choice for a number of periods, for example if they become employed they might decide to remain employed for the next periods, if the switching costs are high. The presence of the option chosen in the current period in the latent utility of the same period, results in the model being both incomplete and incoherent, however features of the distribution that do not depend on the fixed effect can still be derived and characterizing the identification bounds using these features are currently in progress. However a crucial question that needs to be answered is *how informative are these bounds* and *how to estimate the model parameters*.

Characterizing the identified set when the model is incomplete and incoherent under different approaches recently developed, such as in Beresteanu et al. (2011) and Chesher and Rosen (2012), is currently in progress. The different approaches treat the cases of multiple equilibria and no equilibria differently and application of these techniques in the fixed effects binary response dynamic panel data model with switching dependence is ongoing work-in-progress.

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