# Population Density and the Local Economy pre- and post-Pandemic Breakout

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**Abstract**

This paper presents a spatial equilibrium search model that explains skill premium and unemployment rate disparities between high and low population density locations. This framework features a knowledge-based agglomeration economy that draws high-skilled workers together. With the benefits of this concentration being reflected in high-skilled workers' salaries and with these workers' increased consumption of local goods and services, more densely populated locations end up with higher wage premium and lower unemployment rates, matching the findings in the data. Moreover, given the post-pandemic trend for increasing remote work (RW), the model can explain the pattern reversal in unemployment rate during 2020 with the help of a relocation exercise where 6.6% of high-skilled workers relocate from the high to low population density regions. Conditional on the permanence of these remote working conditions, the model's results call for a revisiting of place-based policies that focus on attracting high-skilled jobs in an area.



*Keywords*: Spatial equilibrium, Skill premium, Unemployment, Agglomeration, COVID-19

*JEL Classification*: J23, J24, J31, J64, O31, R12, R23

# 1 Introduction

Agglomeration economies, despite having been studied since considerable time, are still a prevalent topic in economics that inspires a wide range of questions. Agglomeration is based on the idea that physical proximity facilitates knowledge spillover, which is usually considered to be greater among individuals with higher human capital, and enhances productivity (Marshall, 1890; Jacobs, 1969; Lucas Jr, 1988; Glaeser, 1999; Glaeser and Resseger, 2010). More recent evidence can be also found in Abel et al. (2012) who use US metropolitan statistical areas data to study the relation between urban productivity and density. They show that productivity increases with population density and that this increase rises in human capital.

The main goal of this paper is to introduce a model to, firstly, demonstrate how agglomeration effects can explain disparate unemployment rate and wage premium[1]patterns between regions with different population density and, secondly, provide a rationale behind the post COVID-19 outbreak change in these patterns' trajectories, specifically in the case of unemployment rate in 2020. More precisely, in Figure 1[2], we can see that the average unemployment rate of the least densely populated counties has been almost consistently lower than its counterpart of the most densely populated counties. However, this is no longer a fact for the year 2020 where the situation is reversed. As for the wage premium, in Figure 3[3], we look at the ratio of the average wage of those employed in the tradable sector over the average wage of those employed in the non-tradable sector over the years 2005-2020. In this figure, it is clearly shown that the wage premium is higher in counties with greater population density for almost all the 16 years examined.

Why is the wage premium in Figure 3 based on the distinction between the tradable and the non-tradable sector? The reason is that during the last decade, starting mainly after the paper of Moretti (2010) on local multipliers, there has

[1]In the model, this, essentially, tradable sector wage premium will be equivalent to the skill-premium since, as in Davis and Dingel (2019), I assume that skill enters the production function only in the tradable sector.

[2]Industry categorization as tradable or non-tradable is according to Hlatshwayo and Spence (2014). More details on the data and calculations used to produce Figure 1 are in section 10.

[3]Details on the data and calculations performed to produce Figure 3 are in section 10.

been a growing interest in the significance of the tradable sector in the local job creation (De Blasio and Menon, 2011; Moretti and Thulin, 2013; Fleming and Measham, 2014; Van Dijk, 2016; Frocrain and Giraud, 2017; Kemeny and Osman, 2018). The main channel through which the job multiplier operates is the increased demand for local goods and services by the well-paid, high-skilled workers in the tradable sector. How are their results connected to agglomeration? While, when estimating the multiplier in most of these papers, agglomeration is not directly taken into consideration, a large multiplier value, such as the one in Moretti and Thulin (2013), is in line with the presence of agglomeration economies.

In this paper I provide a theoretical framework that is in line with and rationalizes the findings in Figures 1 and 3, by combining agglomeration with an increased demand for local goods and services by those employed in the tradable sector. This is accomplished through a two-sector spatial general equilibrium search model that features agglomeration, heterogeneous agents and non-homothetic preferences. In the model, the agglomeration force rests in the exchange of ideas among skilled individuals. This draws together in the same location higher-skilled agents who expect to find a job and be compensated according to the heightened value of their skill in a knowledge-intensive local economy. The greater payment of these agents implies an increase in the demand for non-tradable goods in these locations which in turn raises the expected wage of the non-tradable sector's workers, resulting in lower unemployment and attracting more people in this region. At the same time, due to the tradable sector's wage increasing both in a worker's ability and in a region's level of idea exchange in a way that overcompensates for the higher prices in densely populated locations, the ratio of average wages in the two sectors turns to be higher in these locations.

In terms of the model's theoretical structure, this paper draws inspiration from other spatial equilibrium models focused on skill premia such as Behrens et al. (2014) and Davis and Dingel (2019). In particular, the functional form of the idea exchange environment and the setup of tradable and non-tradable sectors' production are borrowed directly from Davis and Dingel (2019). However, in their paper, the equilibrium is frictionless and, hence, the model cannot explain spatial disparities in US unemployment rate such as those reported in Kline and Moretti (2013) or those appearing in Figure 3. Moreover, in contrast to their model in which all agents consume a fixed, exogenously given amount of non-tradable good, this model assumes preferences that result in an individual's consumption share of the non-tradable good being increasing in income. This implies additional demand for the non-tradable good, since now the non-tradable good's consumption does not only depend on the mass of people living in denser locations but also on the incomes of these people. In this way, the present model incorporates a fact that has been long taken into account by local policies, i.e. that higher-salaried people tend to spend more on non-traded services (e.g., Moretti and Thulin, 2013).

The elements brought together by the present model also allow for a theoretical exercise that can elucidate empirical observations for the initial period after the pandemic's onset. In particular, the observation refers to the reversal of the unemployment pattern for the year 2020 in Figure 1, which was of unepredented magnitude as shown in Figure 2. For the same period, Chetty et al. (2020) find, among others, that the reduction of spending, especially spending on non-tradable goods and services, was particularly high in more affluent ZIP-codes. They also find that the fall in job postings was much sharper in affluent areas. In the current model, these empirical facts are rationalized by means of a relocation exercise in which a part of people in the tradable sector, i.e. the higher earning individuals of the economy, are able to perform their occupational tasks remotely and move to the lower population density region. The empirical base for such a rationalization is quite strong given the existence of a wide literature on telecommuting, the growth of which took off during the current COVID-19 era (e.g. Althoff et al., 2020; Barrero et al., 2021; Bartik et al., 2020; Bick et al., 2020; Liu and Su, 2020). To be clear, it is not claimed that relocation is the sole reason behind this change in unemployment rate but rather that it is one of the main reasons and the focus of the present theoretical exercise. As it will be shown, consistent with the fact that a part of people who were able to work from home (WFH) continued living in the same location, the model can produce the data moments given only a small percentage of relocated workers (6.6%).

The paper's structure is the following: section 2 presents the model outline. In sections 3 and 4, the equilibrium description follows together with the analysis of how the relative unemployent rate and tradable sector wage premium look like for the high and low population density regions. The corresponding analysis for these two measures in the context of the relocation exercise comes after, in section 5. In the next section, namely section 6, the equilibrium and the relocation exercise are solved in order to match a set of empirical moments. Finally, section 7 concludes.

# 2 The Model

I consider an economy in which there is a number of locations $C$ and a continuum of individuals of mass $L$ per $C$ unit areas[4]. These individuals are heterogeneous with respect to skill/ability $z$. This skill is distributed in the population according to a continuous probability density function $\phi(z)$ over the compact support, $Z$, on the interval $[zmin, zmax] \subset \mathbb{R}_+$. This distribution is common knowledge to everyone in the economy. In each location[5,6] there are two sectors: the tradable and the non-tradable. Agents choose both in which location to live and in which sector to look for a job in order to maximize their utility. In the non-tradable sector, all agents are equal in terms of their marginal product, irrespectively of their ability, while in the tradable sector, the ability of an agent is a defining factor of his/her marginal product[7]. For every location unit, I assume that there is a large mass of potential firms in each sector from which the endogenous mass of firms that will post vacancies will originate. All the decisions - comsumption, occupational, locational and position posting decisions - are met simultaneously and before the employer-employee matches take place.

[4]The unit used to measure the area could be anything such as square mile/kilometer.

[5]From now on, for convenience purposes, the terms location, region and place will be used interchangeably to refer to the same concept.

[6]Henceforth, all the analysis will be made on a per unit land area base. Namely, whenever there is a reference to location, region or place I will mean location, region or place unit area. Therefore, each location's population will essentially be the location's population density and it will be referred to as such.

[7]While this is an abstraction from the fact that skilled agents exist in both sectors, I follow Davis and Dingel (2019) on this because I want to focus on the tradable sector's multiplier effect (see section 1). Since, in this model, the high population density regions will essentially be "loci of knowledge" and attention is given to wage premia and unemployment, this simplification is perfectly in line with empirical evidence which shows that skilled tradable services drive the superstar economies and cities and that they can explain changes in the return to skill almost single-handedly (Eckert et al., 2020).

## 2.1 Agents' utility maximization

Individuals split their income, $I$, between the tradable and the non-tradable good in order to maximize their utility. Their preferences are assumed to be of the price independent generalized linearity (PIGL) type proposed by Muellbauer (1975, 1976) and in particular of the subclass appearing in Boppart (2014). It is also assumed that higher population density constitutes a disutility for the agent and it can be represented by a price $p_{hc}$ which is paid by every dweller in location $c$. I take $p_{hc}$ to be an increasing function of $L_c$, namely $p_{hc} = p_h(L_c)$, $\frac{\partial p_h(L_c)}{\partial L_c} > 0$, where $L_c$ is area's $c$ population density. The indirect utility in line with these assumptions is:

$$v(p_{nc}, p_{hc}, I) = \frac{1}{\epsilon}\frac{(I^{\epsilon} - r)}{p_{nc}^{\epsilon}} + \frac{r-1}{\epsilon} - p_{hc} \tag{1}$$

Here, the tradable good is considered to be the numéraire and $p_{nc}$ is the price of the non-tradable good in area $c$. For the parameters $r$ and $\epsilon$ of the PIGL utility, it holds that $0 < \epsilon < 1$ and $r > 0$. The resulting expenditure on the tradable good is $rI^{1-\epsilon}$ while on the non-tradable good is $I - rI^{1-\epsilon}$. I assume that everyone in the economy is initially allocated an endowment equal to $r^{\frac{1}{\epsilon}}$ so as to avoid negative consumption. From the demand functions, we can see that the way consumption changes with income differs for each good: the tradable good's consumption is a concave function of income while the non-tradable good's consumption is a convex one.

Hence, when meeting their decisions as regards where to live and in which industry to look for a job, individuals maximize (1). Agents are perfectly mobile with respect to their locational choice, however, they should decide on only one location to reside in and apply for a vacancy. As for the job applications in one's chosen location, an agent can only apply for one position.

## 2.2 Intra-location sorting

In each area, firms in both sectors may choose to enter the market by posting a vacancy and setting the wage attached to this vacancy. Each area's dwellers direct their search accordingly by deciding for which vacancy to apply. In every location, there is one search market in the non-tradable sector and several search submarkets in the tradable sector, since in the latter case, a worker's type enters production resulting in different submarkets with different posted wages and market tightness. The matches that take place in each of the markets are given by a matching function, $m(v, u)$, that takes the vacancies, $v$, and the unemployed applying for the positions, $u$, as inputs. Function $m$ has the standard properties, i.e. it is increasing and concave in both terms as well as has constant returns to scale. Moreover, with $q = \frac{v}{u}$ standing for market tightness, the probability that an unemployed worker finds a job, $m(q, 1)$, is increasing in $q$ while the probability that a vacancy is filled, $\frac{m(q,1)}{q}$, is decreasing in $q$. In the following, I assume a Cobb-Douglas form for the matching function with $\eta$ and $1 - \eta$ being the elasticity of $m(v, u)$ with respect to vacancies and unemployed workers queueing for the positions respectively and $B$ being the matching efficiency parameter:

$$m(v, u) = Bv^{\eta}u^{1-\eta}$$

A non-tradable good producer in location $c$ solves the following problem:

$$\max_{q_{nc}, w_{nc}} Bq_{nc}^{\eta-1}(p_{nc} - w_{nc}) - k_n \quad s.t. \quad U_c = Bq_{nc}^{\eta}w_{nc} \tag{2}$$

The terms $w_{nc}$, $U_c$ and $k_n$ represent the wage attached to a vacancy in the non-tradable sector of location $c$, the reservation wage which is equal to the expected income of an individual that applies for a position in the non-tradable sector in $c$, and the cost of posting a vacancy in the non-tradable sector respectively. It is implied that in the non-tradable sector an employer-employee match produces output of value equal to $p_{nc}$. By substituting for $w_{nc}$ in the objective function in $(2)$

using the constraint, firms are left to maximize with respect to $q_{nc}$. The optimal market tightness in the non-tradable sector in area $c$, $q^*_{nc}$, is:

$$q^*_{nc} = \left(\frac{U_c}{B(1-\eta)p_{nc}}\right)^{\frac{1}{\eta}}$$

(3)

Condition (3) implies that the optimal wage posted in the non-tradable sector ($(1-\eta)p_{nc}$) in location $c$ should be equal to the product of the output value and the elasticity of the matching function with respect to $u$. Thus, given the reservation wage ($U_c$), a higher price, which suggests a higher posted wage, means a lower market tightness since more people will apply for the position due to the higher posted wage.

For the tradable good producer in location $c$, the corresponding maximization problem is:

$$\max_{q_{tc},w_{tc}} Bq_{tc}(z,Z_c)^{\eta-1}(\tilde{z}(z,Z_c)-w_{tc}(z,Z_c))-k_t \quad s.t. \quad U_c(z,Z_c) = Bq^{\eta}_{tc}(z,Z_c)w_{tc}(z,Z_c)$$

(4)

In (4), $w_{tc}(z,Z_c)$, $U_c(z,Z_c)$ and $k_t$, have similar meanings to their counterpart symbols for the non-tradable sector. However, in this sector, when an employee-employer match takes place, the value of production is not equal to the price alone, as in the non-tradable sector, but to $\tilde{z}(z,Z_c)$. More specifically, $Z_c$ is the level of the local idea exchanges and is defined as in Davis and Dingel (2019), i.e.

$$Z_c = \left(1 - e^{\left(-\nu L\int_{t,c}(1-\beta)\phi(z)\,dz\right)}\right)\int_{t,c}\frac{z}{\int_{t,c}\phi(z)\,dz}\phi(z)\,dz$$

Here, $\beta$ is a parameter for which it holds that $0 < \beta < 1$ and $\nu > 0$. Furthermore, $\int_{t,c}$ stands for integrating over $z$ for the $z$ types in the tradable sector of location $c$. According to this function of $Z_c$, knowledge spillover is considered to depend both on the average ability of people involved in the knowledge exchange, i.e. the second term in the product, and on the mass of the potential exchange partners[8], i.e. $L \int_{t,c} (1-\beta)\phi(z)\, dz$. In their numerical example, Davis and Dingel (2019) consider a value for $\nu$ that makes the exponential term very small ($\nu = 50$). Here, by extending this from an example to the whole model in general, I assume high values of $\nu$ which means that $Z_c$ is approximately:

$$Z_c = \int_{t,c} \frac{z}{\int_{t,c} \phi(z)\, dz} \phi(z)\, dz \tag{5}$$

Hence, output $\tilde{z}(z, Z_c)$ is a function of a worker's type and of the knowledge exchange. Moreover, it is assumed to have some specific properties.

**Assumption 1.** $\tilde{z}(z, Z_c)$ is a twice-differentiable function, strictly increasing in all its terms. It is also strictly supermodular on $Z \times S$, where $S$ is the codomain of $Z_c$.

Based on assumption 1, the increase in production when hiring a more able individual is bigger when the local level of knowledge exchange is higher. Bearing this in mind and returning to the problem in (4), by substituting for $w_{tc}$ using the constraint, firms maximize with respect to the tightness of the market in which they post their positions. This time, the optimal $q^*_{tc}(z, Z_c)$ is:

$$q^*_{tc}(z, Z_c) = \left( \frac{U_c(z, Z_c)}{B(1-\eta)\tilde{z}(z, Z_c)} \right)^{\frac{1}{\eta}} \tag{6}$$

Using (3) and (6) in the zero-profit conditions for the two sectors, we get that:

$$U_c(z, Z_c) = \left( \frac{\eta}{k_t} \right)^{\frac{\eta}{1-\eta}} B^{\frac{1}{1-\eta}} (1-\eta) \tilde{z}(z, Z_c)^{\frac{1}{1-\eta}} \quad \text{and} \quad U_c = \left( \frac{\eta}{k_n} \right)^{\frac{\eta}{1-\eta}} B^{\frac{1}{1-\eta}} (1-\eta) p_{nc}^{\frac{1}{1-\eta}}$$

[8]More details on the intuition behind $Z_c$'s functional form can be found in Davis and Dingel (2019).

$$(7)$$

Considering these results, what is the intra-location sorting that will take place? Among the people who choose to settle in place $c$, the ones with higher $z$ will apply for positions in the tradable sector. The reason is that it is in this sector where one's higher skill enters the production function. Therefore, individuals of higher skill have the comparative advantage in the tradable sector in which they are compensated adequately to their skills[9].

## 2.3 Inter-location sorting

Intra-location, we have just seen that the set of lower skilled individuals living in an area chooses the nontradable sector while the respective set of higher skilled individuals chooses the tradable sector. However, when it comes to the whole ability distribution, the question of how agents sort into different regions remains to be answered.

**Lemma 1.** There is a critical ability level, $z_\delta$, that partitions the ability distribution into two parts: everyone with $z > z_\delta$ looks for a job in the tradable sector of the location they choose to settle in, and everyone with $z < z_\delta$ looks for a job in the non-tradable sector of their respective location[10].

In a way, lemma 1, extends the intra-location sorting pattern, i.e. that, within a given location, higher ability individuals sort into the local tradable sector, to the whole economy. The relevant threshold ability level $z_\delta$ satisfies:

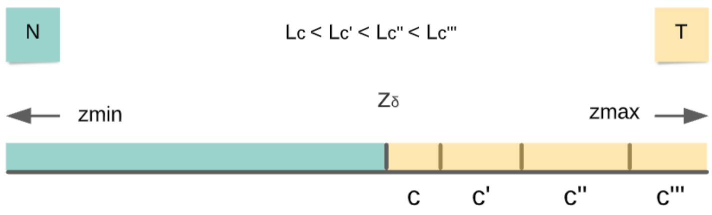
$$U_c(z_\delta, Z_c) = U_c$$

[9]Practically, this can be seen when looking at $(7)$. There, it becomes clear that $U_c(z, Z_c)$ shares the properties of $\tilde{z}(z, Z_c)$ and, thus, is strictly increasing in $z$ and has a positive cross-derivative. Hence, by assuming that a location $c$ resident with ability $z'$ optimally searches for work in the tradable sector while the opposite holds for any other location $c$ resident with higher ability $z''$, we end up with $U_c(z', Z_c) > U_c(z'', Z_c)$. Given what Assumption 1 implies for $U_c(z, Z_c)$, this inequality cannot be true.

[10]See section 8 for the proof.

$$(8)$$

**Lemma 2.** For any locations $c, c'$, if $L_c < L_{c'}$, then $p_{nc} < p_{nc'}$ and $p_{hc} < p_{hc'}$ [11].




Lemma 1 and lemma 2 imply an equilibrium distribution that will look like the figure below lemma 2. In this figure, N and T stand for the individuals who applied for jobs in the non-tradable and tradable sectors respectively. Higher skilled individuals will tend to collocate. This is due to the complementarity between the local level of idea exchange and individual skill and due to the former being increasing in the average skill of the tradable sector workers. Given this concentration of people and the increased demand for the non-tradable good, locations with higher population density and local level of knowledge spillover (e.g., $L_{c'} > L_c$ and $Z_{c'} > Z_c$) will also have greater prices ($p_{nc'} > p_{nc}$). Since, based on one's skill, not everyone is compensated enough to bear the higher costs in a densely populated location, people of lower skill opt for less densely populated locations. Hence, the area $c$ for which $(8)$ holds is the area with the lowest population density, $L_c$. As is apparent in the figure, the lowest $z$ tradable sector potential workers in the economy, i.e. type $z_\delta$ workers, will be located in the place with the lowest population density, since, in the other locations, they cannot benefit as much from the higher levels of idea exchange as their more able counterparts and at the same time they have to deal with higher prices.

[11] See section 8 for the proof.

# 3 Equilibrium

Given the prior analysis, we already know that, in equilibrium[12], firms maximize their expected profits, zero-profit conditions in (7) as well as the indifference condition in (8) hold and agents make the optimal occupational and locational choices. Moreover, the following conditions are also true:

$$L = \sum_{c=1}^{C} L_c$$

(9)

$$L_c = L_{tc} + L_{nc} \quad where \quad L_{tc} = L \int_{t,c} \phi(z)\, dz, \quad L_{nc} = L \int_{n,c} \phi(z)\, dz$$

(10)

$$U_c \int_{n,c} \phi(z)\, dz = U_c \int_{n,c} \phi(z)\, dz + \int_{t,c} U_c(z, Z_c)\phi(z)\, dz - r \left( U_c^{1-\epsilon} \int_{n,c} \phi(z)\, dz + \int_{t,c} U_c(z, Z_c)^{1-\epsilon}\phi(z)\, dz \right)$$

(11)

$$L \int_{t,c} q^*_{tc}(z, Z_c)\phi(z)\, dz = Q_{ct} \quad and \quad L \int_{n,c} q^*_{nc}\phi(z)\, dz = Q_{cn}$$

(12)

$$\frac{1}{\epsilon}\frac{(U_c^\epsilon - r)}{p^\epsilon_{nc}} - p_{hc} = \frac{1}{\epsilon}\frac{(U_{c'}^\epsilon - r)}{p^\epsilon_{nc'}} - p_{hc'}$$

(13)

$$\frac{1}{\epsilon}\frac{(U_c^\epsilon(z_{cc'}, Z_c) - r)}{p^\epsilon_{nc}} - p_{hc} = \frac{1}{\epsilon}\frac{(U_{c'}^\epsilon(z_{cc'}, Z_{c'}) - r)}{p^\epsilon_{nc'}} - p_{hc'}$$

[12]For further analysis regarding equal and unequal sized two-location equilibria see 8.2.

$$(14)$$

where $c'$ is the next, population-wise, denser location than $c$ and $z_{cc'} \in (\, z_\delta, zmax]\,$. Relations $(9)$ and $(10)$ are aggregate conditions which ensure that the sum of different regions' populations per unit is equal to total population per $c$ units and that each region's population can be divided into the agents in the tradable sector ($L_{tc}$) and those in the non-tradable sector ($L_{nc}$). Relation $(11)$ refers to the non-tradable good's market clearing condition in location $c$. The equalities in $(12)$ say that the job postings implied by the profit maximization in the two sectors are equal to the actual job postings in the economy. Symbols $Q_{ct}$ and $Q_{cn}$ stand for the region $c$ endogenous mass of firms that post a position in the tradable and the non-tradable sector respectively. In $(13)$, we see that in equilibrium the utilities of the non-tradable sector potential workers should be equal across different locations. Finally, $(14)$ gives the condition based on which we determine the cutoff ability level that divides the tradable sector's potential workers' ability interval for the locations $c$ and $c'$[13].

# 4 Unemployment and Skill Premium

After stating the conditions that describe the equilibrium, I look at how unemployment rate and wage premium differ across locations with different population density. This will be examined in a two-location setting with $L_2 > L_1$ and, hence, $Z_2 > Z_1$. Unemployment in a region $c$ can be written as:

$$\frac{L_c - Bq_{nc}^{*\eta} L_{nc} - BL \int_{t,c} q_{tc}^{*\eta}(z, Z_c)\phi(z)\, dz}{L_c}$$

Unemployment rate is the ratio of the individuals in a region that did not match in any sector's labor market, non-tradable or tradable, over the total population of the region.

**Proposition 1.** In an economy where assumption 1 holds, for locations 1 and 2

[13]If $L_c < L_{c'}$, we are referring to locations for which it applies that $L_{c'} - L_c < L_{c''} - L_c$ where $c''$ represents any other region in the economy that has $L_{c''} > L_c$ other than $c'$.

with $L_1 < L_2$, the following conditions are jointly sufficient for the unemployment rate to be lower in the more densely populated location: [14]

(a) $\phi^{'}(z) \leq 0$

(b) $\tilde{z}$ is log-convex in its first input

(c) $\tilde{z}$ is log-supermodular

(d) $r \leq 1$

Let us see what the conditions in proposition 1 imply. Conditions (b) and (c) refer to the $z$-elasticity of $\tilde{z}$ and how it increases with both $\tilde{z}$'s inputs, $z$ and $Z_c$. In practice, conditions (b), (c) and (d) ensure that the $z$-elasticities of a tradable worker's demands for the non-tradable and tradable good are non-decreasing in his/her skill $z$ and in the local level of idea exchange $Z_c$. As already discussed in section 2, the demand for both goods is increasing in income, thus, in a location with a higher population density the overall non-tradable and tradable good demands will be greater. What is added here is that, the two demands' percentage increases do not decrease as we go from lower to higher $z$ and in locations with higher $Z_c$. As a result, if we consider the different $z$-types of agents that make up the residents of a location based on the equilibrium in section 2, in densely populated regions, the workers that need to be hired in order to satisfy the local consumption demands are relatively more. Condition (a), which states that the ability probability density is decreasing, essentially ensures the previous result when we consider the skill distribution in the population. In the proof, it can be seen that condition (a), which arises from an implicitly defined matching function beteen the tradable sector workers[15], ensures that the ability interval of the tradable sector workers on location 2 is greater than the same interval in location 1. This result is intuitive in the sense that we would expect to find a wider range of workers in terms of skills in a high population density region when compared to the range of workers in a less densely populated region.

Regarding the parameter condition, i.e., condition (d), it is a sufficient condition that essentially states that $r$ shall not take very high values. This is reasonable,

[14]See section 8 for the proof.

[15]Defined as in the proof of proposition 2 in the appendix of Davis and Dingel (2019).

since for very large $r$, the concavity and convexity of the tradable and non-tradable goods expenditures respectively disappear.

Moving on to the skill premium, the measure is given by:

$$\frac{\int_{t,c} U_c(z, Z_c)\phi(z)\,dz}{U_c \int_{t,c} \phi(z)\,dz}$$

With this ratio I measure the skill premium while also taking into consideration the possibility of getting no salary due to unemployment in each sector[16]. Except for the case in which we have two sectors with the same level of unemployment rate, this measure will in general return different results than other usual skill premium measures that consider only the wages of those employed in a sector. Here, except for the wage itself, namely the posted wages $w_{nc}$ and $w_{tc}$ in the model, the probability of finding a job also comes into play when looking at the average wage in a sector.

**Proposition 2.** In an economy as the one arising from proposition 1, for any locations 2 and 1 with $L_1 < L_2$, it will be that the skill premium is greater in the more densely populated location.[17]

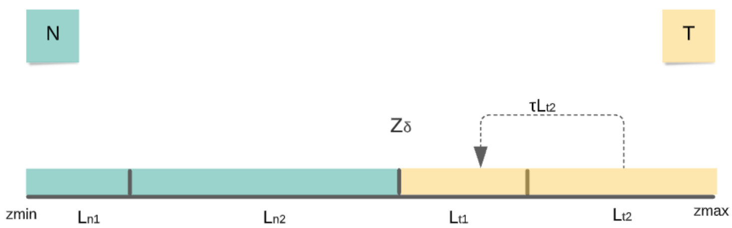
Proposition 2 arises directly from the assumptions developed in proposition 1.They imply that the $z$-elasticity of the tradable sector workers' reservation wage is also non-decreasing in $z$ and $Z_c$ but to an even greater degree than the $z$-elasticity of their non-tradable good demand. This is to be expected since part of the income increase is also translated into higher tradable good consumption. Hence, the increased average wage in location's 2 tradable sector is only partially translated into greater average wage in the non-tradable sector of the same location. As a result, while location 1 scores lower in the average wage of both sectors, location 2 has a higher skill premium.

[16]Adjusting the skill premium for the incidence of unemployment can be important also in other other contexts such as in Eggert et al. (2010).

[17]See section 8 for the proof.

# 5 Relocation Exercise

Given the state of affairs post-COVID-19 outbreak, the phenomenon of telecommuting has abruptly come to the foreground and it seems that it is here to stay[18]. This increase in WFH has in turn given rise to many other questions[19]. In the following, I present a theoretical exercise in which, after the initial equilibrium analysed earlier has been established, a percentage of individuals in the tradable sector[20] now works remotely and relocates. I check how this affects relative skill premium and unemployment rate in locations with different population density.




What happens in this exercise is illustrated in the figure above. Starting from the equilibrium described in section 3, a percentage, $\tau$, of tradable sector agents works from home and relocates. Because this group of individuals will now be able to perform its occupational tasks from any location, the incentive to move pertains solely to the individuals of the tradable sector in the location with the higher prices. In the two locations setting with initial population densities $L_1$, $L_2$ with $L_1 < L_2$, for $\tau$ such that the latter inequality is preserved even after the relocation of agents from area 2 to area 1, the new $L_1^{'}$ and $L_2^{'}$ are:

[18]See, for example, Althoff et al. (2020), Barrero et al. (2021), Bick et al. (2020) and Davis et al. (2021).

[19]For example questions regarding remote working and gender inequality (e.g. Adams-Prassl et al., 2020; Alon et al., 2020)

[20]The tradable sector choice depends mostly on empirical findings such as the ones appearing in Figure 4 of Papanikolaou and Schmidt (2020) or Table 3 in Dingel and Neiman (2020). In both papers, the group of industries in which there is the lowest possibility for the industry-related jobs to be done remotely is dominated by non-tradable industries.

$$L_1^{'} = L_{t1} + L_{n1} + \tau L_{t2} \quad and \quad L_2^{'} = (1-\tau)L_{t2} + L_{n2}$$

$$(15)$$

Whilst there are restrictions with respect to mobility, i.e. nobody except for the $\tau L_{t2}$ can move to a different location, the goods markets will still clear and prices will alter due to the demand change for the non-tradable good in each location. The new conditions for non-tradable goods in locations 1 and 2 are:

$$U_1 L_{n1} = \left(U_1 - rU_1^{1-\epsilon}\right) L_{n1} + \int_{t,1} \left(U_1(z, Z_1) - rU_1(z, Z_1)^{1-\epsilon}\right) \phi(z)\, dz + \tau \int_{t,2} \left(U_2(z, Z_2) - rU_2(z, Z_2)^{1-\epsilon}\right) \phi(z)\, dz$$

$$U_2 L_{n2} = \left(U_2 - rU_2^{1-\epsilon}\right) L_{n2} + (1-\tau) \int_{t,2} \left(U_2(z, Z_2) - rU_2(z, Z_2)^{1-\epsilon}\right) \phi(z)\, dz$$

$$(16)$$

In $(16)$ the non-tradable sector's total income in each location is equal to the total expenditure on this good from this location's residents. By comparing $(16)$ and $(11)$, we see that there is additional demand for the non-tradable good in region 1 while the equivalent demand in region 2 is diminished. Thus, we can conclude that, after relocation, $q_{n1}^{'*} > q_{n1}^{*}$ and $w_{n1}^{\prime} > w_{n1}$ while $q_{n2}^{'*} < q_{n2}^{*}$ and $w_{n2}^{\prime} < w_{n1}$. Assuming that, short-term, given this specific population movement the rest agents are constrained with respect to their location choice, let us explore what will happen to the unemployment rate and the relative average wages post-relocation.

Starting with the unemployment rate, how we express the new unemployment rate will depend on whether the group of tradable sector workers that relocated will be considered based on the location they live in or on the location where their occupation is. For example, during the first year of the pandemic crisis, 2020, the group of people that relocated may not have been adequately recorded in some administrative or survey data. This could be due to the uncertainty of the situation since it was not clear, and to this day it still is not fully clear, whether that was a permanent or a temporary condition. To anticipate any of these two cases, I discuss two measures of unemployment rate. Concerning the case in which people are classified according to their occupation's location, the unemployment rate will be:

$$\frac{L_c - Bq_{nc}^{'*\eta}L_{nc} - BL\int_{t,c} q_{tc}^{*\eta}(z, Z_c)\phi(z)\,dz}{L_c} \quad for \quad c \in \{1, 2\}$$

Comparing this unemployment rate measure with the one of the initial equilibrium, they seem identical. In reality, most parts are indeed identical with the exception of the new non-tradable good's labor market tightness in each city, $q_{nc}^{'*}$. As we saw before, because of the additional non-tradable demand in location 1, $q_{n1}^{'*}$ has increased. The opposite happens in location 2. Hence, since, in both regions, only the employment levels of the non-tradable sectors alter, we see that, after the relocation, unemployment rate in the less densely populated area drecreases whereas unemployment rate in the more densely populated area increases. This occurs through the change in the non-tradable sector's employment rate:

$$\frac{Bq_{n1}^{'*\eta}L_{n1}}{L_1} > \frac{Bq_{n1}^{*\eta}L_{n1}}{L_1} \quad and \quad \frac{Bq_{n2}^{'*\eta}L_{n2}}{L_2} < \frac{Bq_{n2}^{*\eta}L_{n2}}{L_2}$$

In the case in which every individual is classified based on his/her place of residence, unemployment rate, for locations 1 and 2 respectively, will be given by:

$$\frac{L_1^{'} - Bq_{n1}^{'*\eta}L_{n1} - BL\left(\int_{t,1} q_{t1}^{*\eta}(z, Z_1)\phi(z)\,dz + \tau\int_{t,2} q_{t2}^{*\eta}(z, Z_2)\phi(z)\,dz +\right)}{L_1^{'}} \quad and$$

$$\frac{L_2^{'} - Bq_{n2}^{'*\eta}L_{n2} - BL(1-\tau)\int_{t,2} q_{t2}^{*\eta}(z, Z_2)\phi(z)\,dz}{L_2^{'}}$$

**Proposition 3.** In a two-location economy as described from proposition 1 and proposition 2, with $L_1 < L_2$, $\frac{\eta}{1-\epsilon} > 1$ is a sufficient condition for the unemployment rate in location 1 to decrease after the relocation of a percentage of location's 2 tradable sector agents from region 2 to region 1. The opposite is true for location's 2 unemployment rate[21].

[21] See section 8 for the proof.

Proposition 3-and the reasoning behind its proof-essentially says that, as a rule, due to the non-tradable good's demand transfer from the densely populated location to the lower population density location, the unemployment rate will decrease in location 1 and will increase in location 2. The only potential situation in which the model could lead to a different prediction, is when the elasticity of the non-tradable good's produced quantity with respect to its demand from the tradable sector workers ($\frac{\eta}{1-\epsilon}$) in the area is very low.

Moving on to the ratio of average wages in the tradable and non-tradable sectors, for the case in which individuals are classified based on the place of their employment, it is easy to see that for the destination location, the one to which people relocated, the ratio decreases while for the origin location, the one from which people relocated, the ratio increases[22]. This is to be expected, since with this measure, the only wage premium term that changes is the denominator, i.e. the average non-tradable sector wage, which increases for location 1 and decreases for location 2.

If we additionally consider what will change if we use the definition according to which people are classified based on where they reside, the answer for location 2 is easily derived. Comparing the before and after wages ratio for this area, we see that:

$$\frac{(1-\tau)\int_{t,2} U_2(z,Z_2)\phi(z)\,dz}{U_2'(1-\tau)\int_{t,2}\phi(z)\,dz} > \frac{\int_{t,2} U_2(z,Z_2)\phi(z)\,dz}{U_2\int_{t,2}\phi(z)\,dz}$$

where, again, the only term that plays role is the decreased average non-tradable wage. What happens with location 1 is proved in proposition 4 below.

**Proposition 4.** In a two-location economy as described from proposition 1 and proposition 2, with $L_1 < L_2$, there exists a value $M < 1$ and when it holds that $\tau \leq M$, the average wages ratio in location 1 increases after the relocation of a percentage $\tau$ of location's 2 tradable sector agents from region 2 to region 1.

We need proposition 4 because in the case of location 1 the pre- and post-relocation

[22] $\frac{\int_{t,1} U_1(z,Z_1)\phi(z)\,dz}{U_1'\int_{t,1}\phi(z)\,dz} < \frac{\int_{t,1} U_1(z,Z_1)\phi(z)\,dz}{U_1\int_{t,1}\phi(z)\,dz}$ and $\frac{\int_{t,2} U_2(z,Z_2)\phi(z)\,dz}{U_2'\int_{t,2}\phi(z)\,dz} > \frac{\int_{t,2} U_2(z,Z_2)\phi(z)\,dz}{U_2\int_{t,2}\phi(z)\,dz}$ where the first term compares the pre- and post-relocation skill premia for region 1 and the second term shows the same measures for region 2.

wage premium inequality is not as obvious as before. Hence, the usefulness of proposition 4 lies in showing that there exists a range of values that are smaller than 1, for which it holds that when $\tau$ lies in this range the skill premium increases after relocation.

# 6 Quantitative analysis

For the quantitative analysis of the model, I find a set of parameter values that solves the equilibrium in section 3 while matching the empirical moments of interest. These moments are: the unemployment rate in high population density counties, the unemployment rate in low population density counties, the ratio of the tradable sector average wage to the non-tradable sector average wage in the higher density counties, the same ratio for the lower population density counties, the ratio of the non-tradable sector average wage in higher population density counties to the non-tradable sector average wage in lower population density counties and the population densities ratio for the high and low population density counties.

The data sources used to obtain these empirical moments are the ones mentioned in section 9. However, this time instead of having a repeated cross-section of counties for the period 2005-2020, I create a panel, i.e. I keep the counties that are present in all 16 years of the current population survey. Based on the time average of each county's population density, the counties are then divided into two groups where: all members of one group have higher population density than all members of the other group and the ratio between the average population density of each group is equal to a certain number. This ratio will be targeted in the analysis ($\frac{L_2}{L_1}$). As targets for the model's unemployment rate, skill premium and average non-tradable sector wages ratio for the high and low densiy locations measures, I use the time average (2019-2016, 2008-2005)[23] of the across-counties weighted average of the same measures in the data. The weights are the counties' population density.

[23]The years for which the average unemployment was close to the natural level of unemployment plus 2019, the year before the pandemic's onset.

The empirical moments resulting from this process are given in the second column of Table 1 ("Data US"). The elements of this table's fifth column ("Data PA") were obtained using the same data processing for the counties of the panel that belong to the state of Pennsylvania. The purpose of this is to additionally check whether the model can adapt and satisfactorily reproduce empirical moments that may reveal the effect of within state, across counties relocations[24]. As for the choice of Pennsylvania, it was not random in the sense that a) it belongs to the group of states for which there are more than ten counties in the panel and b) this state's counties can be divided in two groups which have a population density ratio close to the one we get when we use the counties of the entire panel.

Congestion cost is assumed to have the form $p_{hc} = \theta L_c^{\gamma}$. Regarding production $\tilde{z}$ in the tradable sector, its chosen form is similar to the form proposed in Davis and Dingel (2019) and it satisfies assumtpion 1 as well as proposition's 1 conditions on $\tilde{z}$. It is:

$$\tilde{z}(z_n, Z_c) = \beta z_n + (1 - \beta) A z_n^2 Z_c$$

Here, $z_n = \frac{z - zmin}{zmax - zmin}$. The $z$ distribution is a truncated distribution based on the truncation of the values of a normal distribution with mean 0 and standard deviation 0.6. This distribution satisfies condition (a) of proposition 1 with strict inequality. In my solutions, already from the start, I truncate the values between 0 and 1 [25]. Moreover, I also solve the model equilibrium for a uniform distribution which, again, satisfies condition (a) of proposition 1 but this time with equality. In sum, I solve the quilibrium four times, i.e. two distributions (truncated normal, uniform) $\times$ two sets of targeted moments (US and Pennsylvania (PA)). In Table 1, we see that in all cases, the five out of six empirical moments are hit either exactly or very close. Only the skill premium for the low density counties seems to be somewhat underestimated by the model although it fares well for the PA data (around 8-9% below the target). The parameter values that return these results are given in Table 2. All the parameters take on feasible values, with $r$ being in line

[24] Within state, across county relocation is taking place within USA as can be seen in e.g. HireAHelper data.

[25] A graphical example can be found in section 10.

with the inequality in (d) of proposition 1.

Keeping the same set of parameter values (Table 2) with the exception of $B$, I then find the $\tau$ and the $B$ values for which the model's post-relocation unemployment rate matches the unemployment rate in the high and low population density groups for the year 2020. As stated in proposition 3, were we to allow only for the effect of $\tau$, after the relocation, we would end up with a lower unemployment rate in low population density counties and with a higher rate in high population density counties compared to the respective equilibrium outcomes. However, because COVID-19 had an overall negative effect, I let this show up in the model by allowing both $\tau$ and $B$ to change in order to match the unemployment rate in the data. The data and model moments are presented in Table 3 and coincide completely.

In Table 4, we see that this is achieved through a relocation of a $6.7\%$ of the tradable sector's agents and a reduction in $B$ from $1.75$ to $1.66$ for the US model with a uniform distribution. For the model with the normal distribution, $\tau$ is $6.6\%$ and $B$ falls from $1.4$ down to $1.34$. In order to showcase how $B$ and $\tau$ operate for locations of different density, in Figures 5(a) and 5(b), there are unemployment rate indifference curves, when we consider the normal skill distribution, for the US low and high population density locations respectively. For the low population density location in order to stay at the same level of unemployment rate at a higher $\tau$, which would reduce unemployment rate, we need to lower $B$. The opposite holds for the greater population density location. Figures 6(a) and 6(b) show the same indifference curves but for the case in which we have a uniform skill distribution model.

Table 1: Equilibrium moments

| Moments | Data US | Normal, US | Uniform, US | Data PA | Normal, PA | Uniform, PA |
|---|---|---|---|---|---|---|
| HD/LD | 3 | 3 | 3 | 3 | 3 | 3 |
| URHD | 4% | 4% | 4% | 3% | 3% | 3% |
| URLD | 4.3% | 4.3% | 4.3% | 3.7% | 3.7% | 3.7% |
| TWHD/NWHD | 1.59 | 1.59 | 1.59 | 1.51 | 1.55 | 1.51 |
| TWLD/NWLD | 1.5 | 1.21 | 1.23 | 1.33 | 1.23 | 1.21 |
| NWHD/NWLD | 1.13 | 1.13 | 1.15 | 1.24 | 1.24 | 1.24 |

**Table 1 notes**: HD= population density of high population density location, LD= population density of low population density location, URHD =unemployment rate in high population density location, URHD =unemployment rate in low population density location, TWHD=average tradable sector wage in high population density location, TWLD=average tradable sector wage in low population density location, NWHD=average non-tradable sector wage in high population density location, NWLD=average non-tradable sector wage in low population density location. In the data columns, the low and high density refer to the counties groups created based on the process described in section 6.

Table 2: Parameters (Equilibrium)

| Parameters | Normal, US | Uniform, US | Normal, PA | Uniform, PA |
|---|---|---|---|---|
| B | 1.4 | 1.75 | 1.4 | 1.75 |
| $\eta$ | 0.38 | 0.35 | 0.38 | 0.35 |
| $\beta$ | 0.72 | 0.72 | 0.72 | 0.72 |
| $\epsilon$ | 0.1 | 0.1 | 0.1 | 0.1 |
| $k_t$ | 0.54 | 1.2 | 1.08 | 1.2 |
| $k_n$ | 0.89 | 2.2 | 2.24 | 2.3 |
| $A$ | 1.29 | 1.96 | 2.35 | 2 |
| $\theta L^\gamma$ | 1.16 | 0.98 | 0.56 | 0.36 |
| $r$ | 0.35 | 0.5 | 0.35 | 0.52 |
| $\gamma$ | 0.1 | 0.1 | 0.13 | 0.08 |

Table 3: Relocation exercise moments

| Moments | Data US | Uniform, US | Normal, US |
|---|---|---|---|
| URLD | 8.4% | 8.4% | 8.4% |
| URHD | 10% | 10% | 10% |

Table 4: Relocation exercise parameters

| Parameter | Uniform, US | Normal, US |
|---|---|---|
| $\tau$ | 6.7% | 6.6% |
| $B$ | 1.66 | 1.34 |

# 7 Conclusions

In this paper, I document regional unemployment rate and tradable sector wage premium disparities among counties of different population densities. In the spatial equilibrium literature, these two economic variables have been so far examined using models structured to study them separately. For this reason, I develop a model that jointly rationalizes these regional heterogeneities and, thus, provides a unified framework that sheds light on their interconnection.

This is accomplished through the introduction of a two-sector spatial general equilibrium economy that features labor market search frictions and heterogeneous agents. The role of the agglomeration force in the model is played by the local level of idea exchange, which, in turn, causes congestion and price externalities. Moreover, in line with real-world policies which take into consideration the fact that high-salaried workers spend a significant amount of money on local non-tradable services, the model's preferences are non-homothetic in order to allow the non-tradable good's consumption share to be increasing in income. Specifically, the models' main mechanisms work as following: higher skilled individuals congregate together by applying for jobs in the tradable sector of the same location. This is because, in the model, this sector makes use of the available idea exchange environment, an environment that is enhanced by the greater skill of the agents that exchange ideas. As a result of their own higher skill and the greater knowledge spillover, these individuals are expected to have the highest earnings in the economy and to consume more of the non-tradable good. This leads to an increase in the non-tradable good's demand that lowers unemployment and increases the expected wage of people in the non-tradable sector, attracting more non-tradable sector potential workers in the region. Due to congestion costs and higher non-tradable good prices, not everyone finds it beneficial to move in that region and, hence, the rest look for a job in another less densely populated region. Despite the lower unemployment rate in the location with higher population density, the wage premium is greater because the most skilled individuals' congregation, overcompensates these agents beyond the higher prices that prevail in this location.

Making further use of the model's potential, I perform a theoretical exercise using this framework in order to explain the fact that unemployment rate became quite more acute in counties with higher population density during the pandemic's onset in 2020. The focus in this exercise is on the relocation, especially, of a subset of high-skilled individuals who are able to telecommute, a phenomenon that gained marked importance in our lives post-COVID-19 outbreak in 2020. By allowing a small percentage, around 6.6% based on the model's findings, of agents active in the denser area's tradable sector to perform their occupational tasks while residing in the less dense area, we are able to match the unemployment rate pattern in 2020.

This outcome is important especially in the following sense: the literature around local job multipliers is, and justifiably so, very broad. Until recently, its results were mostly pointing to, usually place-based, policies relating to the enticement of high-skilled industries that would have a beneficial impact on the general regional economy through job creation. However, the findings of this paper manifest the potential precariousness of the new jobs created in less high-skilled industries in the face of a strong health shock. The latter has the potential to reshuffle the cards for regional economies through the phenonmenon of relocation, at least to the extent that it restricts the demand for local goods from high earning workers who would otherwise reside in the place they work.

However, adapting to a possible new normality, place-based policies could now add to their goals the attraction of high-skilled digital nomads. The present model offers itself as a benchmark framework which can be used to study such policies. The specific ways in which this can be studied are many and need to be examined both theoretically and empirically, something that will definitely be part of this paper's author's future research agenda.

# 8 Appendix

## 8.1 Lemmata, Propositions

**Lemma 1**
Let $\Theta_c$ be the set of a location's $c$ inhabitants' abilities. Assume that $z \in \Theta_{ntc}$, $z' \in \Theta_{tc}$, $z'' \in \Theta_{ntc'}$, $z < z' < z''$, where $t$ and $nt$ denote the subsets referring to the abilities of the individuals active in the tradable and non-tradable sector, respectively. Since $z$ and $z''$ are both looking for a job in the non-tradable sector, in equilibrium, it should hold that their utilities should be equalized across their respective locations $c$ and $c'$. This means that:

$$\frac{1}{\epsilon}\frac{(U_c^\epsilon - r)}{p_{nc}^\epsilon} - p_{hc} = \frac{1}{\epsilon}\frac{(U_{c'}^\epsilon - r)}{p_{nc'}^\epsilon} - p_{hc'}$$

At the same time, for $z'$ that is looking for a job in the tradable sector in $c$, it holds that:

$$\frac{1}{\epsilon}\frac{(U_c^\epsilon(z', Z_c) - r)}{p_{nc}^\epsilon} - p_{hc} \geqslant \frac{1}{\epsilon}\frac{(U_c^\epsilon - r)}{p_{nc}^\epsilon} - p_{hc}$$

Since $z'' > z'$, given Assumption 1,

$$\frac{1}{\epsilon}\frac{(U_c^\epsilon(z'', Z_c) - r)}{p_{nc}^\epsilon} - p_{hc} > \frac{1}{\epsilon}\frac{(U_c^\epsilon(z', Z_c) - r)}{p_{nc}^\epsilon} - p_{hc}$$

and $z''$ would have a higher utility as tradable sector job seeker in location $c$ than as a non-tradable sector job seeker in location $c'$. Hence, it cannot be the case that any non-tradable sector job seeker has higher ability than any tradable sector job seeker in the economy. In fact, there is an ability threshold, $z_\delta$, which is pinned down by the equality

$$\frac{1}{\epsilon}\frac{(U_c^\epsilon(z_\delta, Z_c) - r)}{p_{nc}^\epsilon} - p_{hc} = \frac{1}{\epsilon}\frac{(U_c^\epsilon - r)}{p_{nc}^\epsilon} - p_{hc}$$

This equality essentially boils down to $U_c(z_\delta, Z_c) = U_c$.

**Lemma 2**

Let us assume that $L_c < L_{c'}$, and, hence, $p_{hc} < p_{hc'}$, but that $p_{nc} > p_{nc'}$. Then, in equilibrium, from the utility equalization among non-tradable sector workers and from $(7)$, it should hold that:

$$\frac{(U_c^\epsilon - r)}{p_{nc}^\epsilon} < \frac{(U_{c'}^\epsilon - r)}{p_{nc'}^\epsilon} \Rightarrow$$

$$\frac{\left(\left(\left(\frac{\eta}{k_n}\right)^{\frac{\eta}{1-\eta}} B^{\frac{1}{1-\eta}}(1-\eta)p_{nc}^{\frac{1}{1-\eta}}\right)^\epsilon - r\right)}{p_{nc}^\epsilon} < \frac{\left(\left(\left(\frac{\eta}{k_n}\right)^{\frac{\eta}{1-\eta}} B^{\frac{1}{1-\eta}}(1-\eta)p_{nc'}^{\frac{1}{1-\eta}}\right)^\epsilon - r\right)}{p_{nc'}^\epsilon} \Rightarrow$$

$$\left(\left(\frac{\eta}{k_n}\right)^{\frac{\eta}{1-\eta}} B^{\frac{1}{1-\eta}}(1-\eta)\right)^\epsilon p_{nc}^{\frac{\eta\epsilon}{1-\eta}} - \frac{r}{p_{nc}^\epsilon} < \left(\left(\frac{\eta}{k_n}\right)^{\frac{\eta}{1-\eta}} B^{\frac{1}{1-\eta}}(1-\eta)\right)^\epsilon p_{nc'}^{\frac{\eta\epsilon}{1-\eta}} - \frac{r}{p_{nc'}^\epsilon} \Rightarrow$$

$$\left(\left(\frac{\eta}{k_n}\right)^{\frac{\eta}{1-\eta}} B^{\frac{1}{1-\eta}}(1-\eta)\right)^\epsilon p_{nc}^{\frac{\eta\epsilon}{1-\eta}} - \left(\left(\frac{\eta}{k_n}\right)^{\frac{\eta}{1-\eta}} B^{\frac{1}{1-\eta}}(1-\eta)\right)^\epsilon p_{nc'}^{\frac{\eta\epsilon}{1-\eta}} < \frac{r}{p_{nc}^\epsilon} - \frac{r}{p_{nc'}^\epsilon}$$

Since we assumed that $p_{nc'}^\epsilon < p_{nc}^\epsilon$, the inequality cannot hold.

**Proposition 1**

If we use $(3)$, $(6)$ and $(7)$, we get that employment in the non-tradable and tradable sectors are respectively equal to:

$$\frac{L\left(\frac{\eta}{k_n}\right)^{\frac{\eta}{1-\eta}} B^{\frac{1}{1-\eta}} p_{nc}^{\frac{\eta}{1-\eta}} \int_{n,c} \phi(z)\,dz}{L_c} \quad and \quad \frac{L\left(\frac{\eta}{k_t}\right)^{\frac{\eta}{1-\eta}} B^{\frac{1}{1-\eta}} \int_{t,c} \tilde{z}(z, Z_c)^{\frac{\eta}{1-\eta}} \phi(z)\,dz}{L_c}$$

In order for the lower population area to have a lower level of non-tradable employment, it should hold that:

$$\frac{L\left(\frac{\eta}{k_n}\right)^{\frac{\eta}{1-\eta}} B^{\frac{1}{1-\eta}} p_{n1}^{\frac{\eta}{1-\eta}} \int_{n,1} \phi(z)\,dz}{L_1} < \frac{L\left(\frac{\eta}{k_n}\right)^{\frac{\eta}{1-\eta}} B^{\frac{1}{1-\eta}} p_{n2}^{\frac{\eta}{1-\eta}} \int_{n,2} \phi(z)\,dz}{L_2} \Rightarrow$$

$$\left(\frac{p_{n1}}{p_{n2}}\right)^{\frac{\eta}{1-\eta}} < \frac{L_1 \int_{n,2} \phi(z)\, dz}{L_2 \int_{n,1} \phi(z)\, dz} = \frac{L_1 U_1^{1-\epsilon} \int_{t,2} \left(U_2(z, Z_2) - rU_2(z, Z_2)^{1-\epsilon}\right) \phi(z)\, dz}{L_2 U_2^{1-\epsilon} \int_{t,1} \left(U_1(z, Z_1) - rU_1(z, Z_1)^{1-\epsilon}\right) \phi(z)\, dz}$$

The last implication is derived when we substitute for $\frac{\int_{n,2} \phi(z)\, dz}{\int_{n,1} \phi(z)\, dz}$ using (12) and the zero profit conditions. Let us first see when the following holds:

$$\frac{\int_{t,1} \left(U_1(z, Z_1) - rU_1(z, Z_1)^{1-\epsilon}\right) \phi(z)\, dz}{U_1^{1-\epsilon}} < \frac{\int_{t,2} \left(U_2(z, Z_2) - rU_2(z, Z_2)^{1-\epsilon}\right) \phi(z)\, dz}{U_2^{1-\epsilon}}$$

$$(A.1)$$

Let us denote the ability intervals for the workers that look for a job in the tradable sectors of the two locations as $(z_\delta, z_{12})$ and $(z_{12}, z_2)$. At this point, as Davis & Dingel (2019), I will implicitly define a function $f(z)$ with $f'(z) = \frac{L_2 \phi(z)}{L_1 \phi(f(z))}$ and the endpoint $f(z_\delta) = z_{12}$. We see that $f(z)$ is an increasing function and that, given the endpoint, $f(z) \geq z$.

At $z_\delta$, $U_1 = U_1(z_\delta, Z_1) < U_2 < U_2(f(z_\delta), Z_2)$ and

$$\frac{U_2(f(z_\delta), Z_2) - U_2^{1-\epsilon}(f(z_\delta), Z_2)}{U_2^{1-\epsilon}} > U_1^\epsilon - 1$$

This implies that $\frac{U_2(f(z_\delta), Z_2)}{U_2^{1-\epsilon}} - U_1^\epsilon > r\left(\frac{U_2^{1-\epsilon}(f(z_\delta), Z_2)}{U_2^{1-\epsilon}} - 1\right)$ and , thus,

$$\Rightarrow r < \frac{\frac{U_2(f(z_\delta), Z_2)}{U_2^{1-\epsilon}} - U_1^\epsilon}{\frac{U_2^{1-\epsilon}(f(z_\delta), Z_2)}{U_2^{1-\epsilon}} - 1}$$

If we assume that $r \leq 1$, the inequality above is guaranteed to hold .
However, we want that $\frac{U_1(z, Z_1) - rU_1^{1-\epsilon}(z, Z_1)}{U_1^{1-\epsilon}} < \frac{U_2(f(z), Z_2) - rU_2^{1-\epsilon}(f(z), Z_2)}{U_2^{1-\epsilon}}$ holds $\forall z \in (z_\delta, z_{12})$. The inequality can be equivalently expressed as:

$$\frac{\tilde{z}(z,Z_1)^{\frac{1}{1-\eta}} - rC_2^{-\epsilon}\tilde{z}(z,Z_1)^{\frac{1-\epsilon}{1-\eta}}}{U_1^{1-\epsilon}} < \frac{\tilde{z}(f(z),Z_2)^{\frac{1}{1-\eta}} - rC_2^{-\epsilon}\tilde{z}(f(z),Z_2)^{\frac{1-\epsilon}{1-\eta}}}{U_2^{1-\epsilon}}$$

$$(A.2)$$

where $C_2 = \left(\frac{\eta}{k_t}\right)^{\frac{\eta}{1-\eta}} B^{\frac{1}{1-\eta}}(1-\eta)$. If we show that:

$$\frac{\frac{\partial\left(\tilde{z}(z,Z_1)^{\frac{1}{1-\eta}} - rC_2^{-\epsilon}\tilde{z}(z,Z_1)^{\frac{1-\epsilon}{1-\eta}}\right)}{\partial z}}{\tilde{z}(z,Z_1)^{\frac{1}{1-\eta}} - rC_2^{-\epsilon}\tilde{z}(z,Z_1)^{\frac{1-\epsilon}{1-\eta}}} < \frac{\frac{\partial\left(\tilde{z}(z,Z_1)^{\frac{1}{1-\eta}} - rC_2^{-\epsilon}\tilde{z}(z,Z_1)^{\frac{1-\epsilon}{1-\eta}}\right)}{\partial z}}{\tilde{z}(z,Z_1)^{\frac{1}{1-\eta}} - rC_2^{-\epsilon}\tilde{z}(z,Z_2)^{\frac{1-\epsilon}{1-\eta}}}$$

The following conditions guarantee that this inequality is true:

**1.** $\phi^{'}(z) \leq 0$, which, from the definition of $f$, implies that $f'(z) > 1$ and that $f''(z) > 0$.

**2.**

$$\frac{\partial^2 log\left(\tilde{z}(z,Z_c)\right)}{\partial z \partial z} \leq 0$$

namely $\tilde{z}$ is log-convex in its first input.

**3.**

$$\frac{\partial^2 log\left(\tilde{z}(z,Z_c)\right)}{\partial z \partial Z_c} \leq 0$$

namely $\tilde{z}$ is log-supermodular.

Hence,

$$\frac{U_1(z,Z_1) - rU_1^{1-\epsilon}(z,Z_1)}{U_1^{1-\epsilon}} < \frac{U_2(f(z),Z_2) - rU_2^{1-\epsilon}(f(z),Z_2)}{U_2^{1-\epsilon}} \quad \forall z \in (z_\delta, z_{12})$$

If we multiply both sides by $\phi(z)$ and integrate from $z_\delta$ to $z_{12}$, it follows that:

$$\int_{z_\delta}^{z_{12}} \left(U_2(f(z), Z_2) - rU_2(f(z), Z_2)^{1-\epsilon}\right) f'(z)\phi(f(z))\frac{L_1U_1^{1-\epsilon}}{L_2U_2^{1-\epsilon}}\,dz > \int_{z_\delta}^{z_{12}} \left(U_1(z, Z_1) - rU_1(z, Z_1)^{1-\epsilon}\right)\phi(z)\,dz$$

$$\Leftrightarrow \frac{\int_{z_{12}}^{z_2} \left(U_2(z, Z_2) - rU_2(z, Z_2)^{1-\epsilon}\right)\phi(z)\,dz}{L_2U_2^{1-\epsilon}} > \frac{\int_{z_\delta}^{z_{12}} \left(U_1(z, Z_1) - rU_1(z, Z_1)^{1-\epsilon}\right)\phi(z)\,dz}{L_1U_1^{1-\epsilon}}$$

As we saw before, this suggests that $\frac{\int_{n,1}\phi(z)\,dz}{\int_{n,2}\phi(z)\,dz} < \frac{L_1}{L_2}$ and, hence,

$$\frac{L_1\int_{n,2}\phi(z)\,dz}{L_2\int_{n,1}\phi(z)\,dz} > 1 > \left(\frac{p_{n1}}{p_{n2}}\right)^{\frac{\eta}{1-\eta}}$$

Following the same logic, it can also be demonstrated that:

$$\frac{\int_{z_{12}}^{z_2} \tilde{z}(z, Z_2)^{\frac{\eta}{1-\eta}}\phi(z)\,dz}{L_2} > \frac{\int_{z_\delta}^{z_{12}} \tilde{z}(z, Z_1)^{\frac{\eta}{1-\eta}}\phi(z)\,dz}{L_1}$$

$$(A.3)$$

Inequality $(A.3)$ establishes that also the tradable sector employment in location $1$ is less than the tradable sector employment in location $2$.

**Proposition 2**

In a similar way as in Proposition 1, it can be shown that:

$$\frac{\int_{z_{12}}^{z_2} \tilde{z}(z, Z_2)^{\frac{1}{1-\eta}}\phi(z)\,dz}{L_2U_2} > \frac{\int_{z_\delta}^{z_{12}} \tilde{z}(z, Z_1)^{\frac{1}{1-\eta}}\phi(z)\,dz}{L_1U_1}$$

Hence, if we demonstrate that $\frac{\int_{z_\delta}^{z_{12}}\phi(z)\,dz}{\int_{z_{12}}^{z_2}\phi(z)\,dz} > \frac{L_1}{L_2}$, it will be enough to establish the desired result:

$$\frac{\int_{z_{12}}^{z_2} \tilde{z}(z, Z_2)^{\frac{1}{1-\eta}}\phi(z)\,dz}{U_2\int_{z_{12}}^{z_2}\phi(z)\,dz} > \frac{\int_{z_\delta}^{z_{12}} \tilde{z}(z, Z_1)^{\frac{1}{1-\eta}}\phi(z)\,dz}{U_1\int_{z_\delta}^{z_{12}}\phi(z)\,dz}$$

The inequality $\frac{\int_{z_\delta}^{z_{12}} \phi(z)\,dz}{\int_{z_{12}}^{z_2} \phi(z)\,dz} \geqslant \frac{L_1}{L_2}$ holds, as implied in Proposition 1.

**Proposition 3**

Starting from the non-tradable sector's employment rate in location $1$, to show that it is higher after the relocation, it is sufficient to prove that:

$$\left(\frac{p'_{n1}}{p_{n1}}\right)^{\frac{\eta}{1-\eta}} > 1+\tau\frac{L_{t2}}{L_1} \Rightarrow \left(1+\tau\frac{\int_{z_{12}}^{z_2} U_2(z,Z_2) - rU_2^{1-\epsilon}(z,Z_2)\phi(z)}{\int_{z_\delta}^{z_{12}} U_1(z,Z_1) - rU_1^{1-\epsilon}(z,Z_1)\phi(z)}\right)^{\frac{\eta}{1-\epsilon}} > 1+\tau\frac{L_{t2}}{L_1}$$

Through the findings of Proposition 1, we can already see that:

$$\frac{\int_{z_{12}}^{z_2} U_2(z,Z_2) - rU_2^{1-\epsilon}(z,Z_2)\phi(z)}{\int_{z_\delta}^{z_{12}} U_1(z,Z_1) - rU_1^{1-\epsilon}(z,Z_1)\phi(z)} > \frac{L_{t2}}{L_1}$$

This implies that $\frac{\eta}{1-\epsilon}$ should be quite low in order for the initial inequality not to hold. An explicit restriction on the parameters in order to avoid that would be $\eta+\epsilon \geq 1$. Thus, the employment rate in location 1 is now higher than before. The employment rate among the residents of location $1$ who are in the tradable sector of any location is also higher, since, as it was shown in the proof of Proposition 1, it is true that:

$$\frac{L_{t2}}{L_1} < \frac{\int_{z_{12}}^{z_2} \tilde{z}(z,Z_2)^{\frac{\eta}{1-\eta}}\phi(z)\,dz}{\int_{z_\delta}^{z_{12}} \tilde{z}(z,Z_1)^{\frac{\eta}{1-\eta}}\phi(z)\,dz}$$

Following a similar line of reasoning, employment in the non-tradable sector of location $2$, increases only if:

$$\left(\frac{p'_{n2}}{p_{n2}}\right)^{\frac{\eta}{1-\eta}} > 1-\tau\frac{L_{t2}}{L_2} \Rightarrow (1-\tau)^{\frac{\eta}{1-\epsilon}} + \tau\frac{L_{t2}}{L_2} > 1$$

an inequality that is not true and, hence, employment in the non-tradable sector in location $2$ is lower after the relocation. As for the tradable sector of the same location, in order to have lower employment than before, it should be that:

$$-\tau < -\tau \frac{L_{t2}}{L_2}$$

which is true.

**Proposition 4**

The after relocation average wages ratio for location $1$ is:

$$\frac{\left(\int_{z_{12}}^{z_\delta} U_1(z, Z_1)\phi(z)\,dz + \tau \int_{z_{12}}^{z_2} U_2(z, Z_2)\phi(z)\,dz\right)}{U_1'\left(L_{t1} + \tau L_{t2}\right)}$$

If we substitute for $U_1'$ in the new ratio and for $U_1$ in the old ratio using $(17)$ and $(12)$ respectively, the former will be more than the latter if:

$$\left(1 + \tau \frac{\int_{z_{12}}^{z_2} \left(U_2(z, Z_2) - rU_2^{1-\epsilon}(z, Z_2)\right)\phi(z)\,dz}{\int_{z_\delta}^{z_{12}} \left(U_1(z, Z_1) - rU_1^{1-\epsilon}(z, Z_1)\right)\phi(z)\,dz}\right)^{\frac{1}{1-\epsilon}} < \frac{L_{t1}(\int_{z_\delta}^{z_{12}} U_1(z, Z_1)\phi(z)\,dz + \tau \int_{z_{12}}^{z_2} U_2(z, Z_2)\phi(z)\,dz)}{(L_{t1} + \tau L_{t2}) \int_{z_\delta}^{z_{12}} U_1(z, Z_1)\phi(z)\,dz}$$

It would be sufficient to show that:

$$1 + \tau \frac{\int_{z_{12}}^{z_2} \left(U_2(z, Z_2) - rU_2^{1-\epsilon}(z, Z_2)\right)\phi(z)\,dz}{\int_{z_\delta}^{z_{12}} \left(U_1(z, Z_1) - rU_1^{1-\epsilon}(z, Z_1)\right)\phi(z)\,dz} < \frac{L_{t1}(\int_{t,1} U_1(z, Z_1)\phi(z)\,dz + \tau \int_{z_{12}}^{z_2} U_2(z, Z_2)\phi(z)\,dz)}{(L_{t1} + \tau L_{t2}) \int_{z_\delta}^{z_{12}} U_1(z, Z_1)\phi(z)\,dz} \Rightarrow$$

By rearranging the terms, dividing them with $L_{t2} \int_{z_\delta}^{z_{12}} U_1(z, Z_1)\phi(z)\,dz$ and passing them to the left side, we have:

$$\tau \left(\int_{z_{12}}^{z_2} U_2(z, Z_2)\phi(z)\,dz\right) MM - r\left(\int_{z_{12}}^{z_2} U_2(z, Z_2)^{1-\epsilon}\phi(z)\,dz\right) MMM <$$

$$\tau \left(\int_{z_\delta}^{z_{12}} \left(rU_1^{1-\epsilon}(z, Z_1) - U_1(z, Z_1)\right)\phi(z)\,dz\right)$$

where

$$MM = r\frac{L_{t1}\int_{z_\delta}^{z_{12}} U_1(z,Z_1)^{1-\epsilon}\phi(z)\,dz}{L_{t2}\int_{z_\delta}^{z_{12}} U_1(z,Z_1)\phi(z)\,dz} + \tau$$

$$MMM = \tau + \frac{L_{t1}}{L_{t2}}$$

When $\tau$ is 0, the inequality is true. Moreover, there exists a value M equal to:

$$r\frac{\int_{z_{12}}^{z_2} U_2(z,Z_2)^{1-\epsilon}\phi(z)\,dz}{\left(\int_{z_\delta}^{z_{12}}\left(U_1(z,Z_1) - rU_1^{1-\epsilon}(z,Z_1)\right)\phi(z)\,dz\right) + \int_{z_{12}}^{z_2} U_2(z,Z_2)\phi(z)\,dz}$$

which is smaller than 1 and it holds that when $\tau \leq M$, it is guaranteed that the skill premium measure that takes into consideration the people that relocated to this area is higher after the relocation.

## 8.2 Unequal sized and equal sized 2-location equilibria

Let us start with the equal sized two-city equilibrium, i.e. $L_1 = L_2 = \frac{L}{2}$. We know that, in equilibrium, the utilities across the non-tradable sector potential workers should be equal. For equal sized cities, this implies that $p_{n1} = p_{n2}$. What remains to check is whether there is this $z$ type, let it be $z_{12}$, which divides the ability interval of tradable sector potential workers into two parts, one for each city. This type should be indifferent between either location. With equal populations and prices, this cannot be the case if there is inequality of any direction between the knowledge exchange level, i.e. if $Z_1 \gtrless Z_2$. In this case, $\tilde{z}(z_{12}, Z_c) > \tilde{z}(z_{12}, Z_{c'})$ where $Z_c > Z_{c'}$, whether $Z_c$ is $Z_1$ or $Z_2$. From the definition of $Z_c$ in $(5)$, if the tradable sector workers with $z \in (z_{12}, zmax]$ are in $c$, we know that $Z_c > Z_{c'}$, hence, we would not reach an equal sized cities equilibrium. For the unequal-sized cities, $L_1 < L_2$, let us consider:

$$\frac{1}{\epsilon}\frac{(U_2^\epsilon(z_{12}, Z_1) - r)}{p_{n2}^\epsilon} - p_{h2} - \frac{1}{\epsilon}\frac{(U_1^\epsilon(z_{12}, Z_2) - r)}{p_{n1}^\epsilon} - p_{h1}$$

If we use $(13), (11)$ and $(8)$, then we can write the expression above as:

$$\frac{1}{\epsilon}C_1^{\epsilon}\left(\frac{C_2}{C_1}\right)^{\eta\epsilon}\tilde{z}(z_\delta, Z_1)^{\frac{\eta\epsilon}{1-\eta}}\left(1-\Omega^{\frac{\eta\epsilon}{1-\eta}}\right)+\frac{C_2^{\epsilon\eta}C_1^{\epsilon-\eta\epsilon}}{\epsilon\tilde{z}(z_\delta, Z_1)^{\epsilon}}\left(\frac{\tilde{z}(z_{12}, Z_2)^{\frac{\epsilon}{1-\eta}}}{\Omega^{\frac{\epsilon(1-\eta)}{1-\epsilon}}}-\tilde{z}(z_{12}, Z_1)^{\frac{\epsilon}{1-\eta}}\right)$$

$$(A.4)$$

where $L_1 = \int_{zmin}^{z_1}\phi(z)dz + \int_{z_\delta}^{z_{12}}\phi(z)dz$, $\Omega = \frac{\int_{zmin}^{z_1}\phi(z)dz\int_{z_{12}}^{zmax}\left(\tilde{z}(z,Z_2)-r\tilde{z}(z,Z_2)^{1-\epsilon}\right)}{\int_{z_1}^{z_\delta}\phi(z)dz\int_{z_\delta}^{z_{12}}(\tilde{z}(z,Z_1)-r\tilde{z}(z,Z_1)^{1-\epsilon})}$. As shown, at $L_1 = \frac{L}{2}$, $(A.4)$ is positive and $\lim_{L_1\to 0}(A.4) < 0$. Since $(A.4)$ is also continuous at $z_1, z_\delta$ and $z_{12}$, there is existence of unequal-sized equilibrium.

# 9 Empirical Analysis

For the stylized facts, I used US individual data from the Integrated Public Use Microdata Series-Current Population Survey[26] (IPUMS-CPS) and land area as well as county population data from the US Census Bureau. In order to estimate the population density of each county, two files are used, one file with counties' population estimate (var:POPESTIMATE) and one file with land area data. For all the years' counties densities, the land area[27] is drawn from the same year, i.e. year 2010 (var:LND110210D). Moving on to CPS, it is used to estimate unemployment and skill premium for each county. The years used are from 2020 to 2005. Before continuing to the figures, two points should be noted: first, the wage data employed in the construction of skill-premium are referring to the wage and salary income for the previous calendar year and this is the reason why I used the files from 2006 to 2021. Second, for each year, I work with the counties that overlap between CPS and the population density file I created based on the Census Bureau data. The latter one is much more complete in terms of number of counties included. Since this happens for each year individually, we end up with a sample of repeated cross-sections in terms of counties.

For both unemployment rate and skill premium, I consider individuals that

[26]Flood et al. (2021).

[27]Land area is measured in square miles.

are between 23 and 67 years old, belong to the labor force, do not work for the government (federal, state or local), are not in the arm forces and are not unpaid family workers. For each county's unemployment for every year, the ratio of people who are unemployed over the total population is estimated, taking into account the CPS given population weights. Afterwards, the counties are ranked based on their population densities. Based on the latter, I select the top and bottom 10% of counties in the list and I create two groups. These groups have as their average unemployment rate the weighted average of the unemployment rate of the counties of their group with the weights being the population densities. The results are drawn in Figure 1. In Figure 2, the difference between the two unemployment rate bars for each year of Figure 1 is drawn.

For the skill premium, the logic behind the process is similar, but this time there is the need to define tradable and non-tradable sectors. The categorization is along the lines of the industry splits in Hlatshwayo and Spence (2014). The only two differences are that: first, I split the several industries into tradable and non-tradable only if each of the industry has at least 30% in each category. For example, in their paper, administrative and waste services are categorized as 89.8 percent non-tradable and 10.2 percent tradable. I define them only as non-tradable. Second, the Information and Communications Technology (ICT) industries as well as the educational services that do not belong to elementary and secondary schools were included in the tradable sectors.

The categories split in the script is done in the following way: First, I make use of the industry variable (var:ind) to identify an industry in CPS. A sample for each category - tradable and non-tradable - is created based on keeping the observations that state an industry that either belongs completely to the category or has at least 30% in this category. As soon as the two samples are created, the weights of the observations that have an industry that is split between categories are multiplied by the industry's percentage in this category. For instance, in the non-tradable category sample, the weights of the individuals that report an industry that is part of professional services are multiplied by $0.392$. After adjusting the weights in this way, a wage weighted average is calculated for each sample, county and year. Then, for every county-year pair the two average wages from the two samples are taken to create the ratio (skill-premium). Taking again the weighted average for the top and bottom 10% of counties, we get Figure 3. It includes also the unemployed people in a category, e.g. people who are looking for a job in an industry of the non-tradable sector but are currently unemployed and have wage equal to zero.

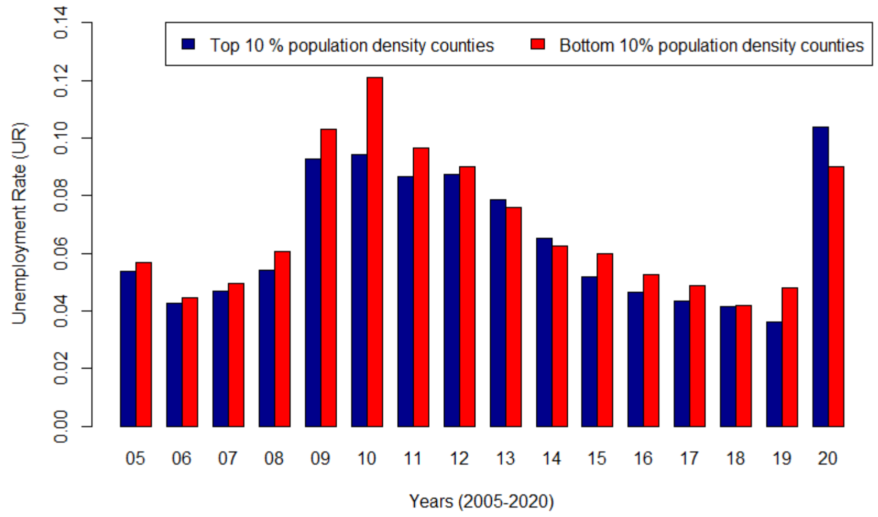



Figure 1: Unemployment Rate for the most (blue) and least (red) densely populated counties in US for the years 2005-2020

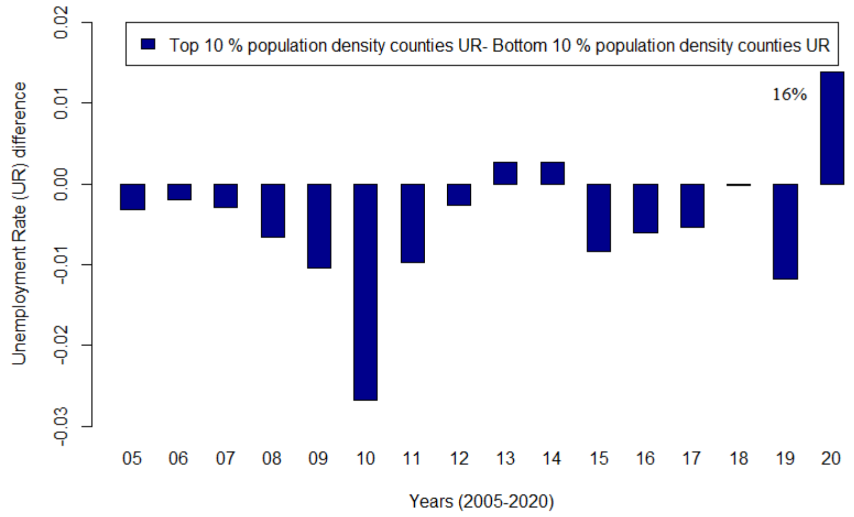



Figure 2: Unemployment Rate difference between the most and least densely populated counties in US for the years 2005-2020

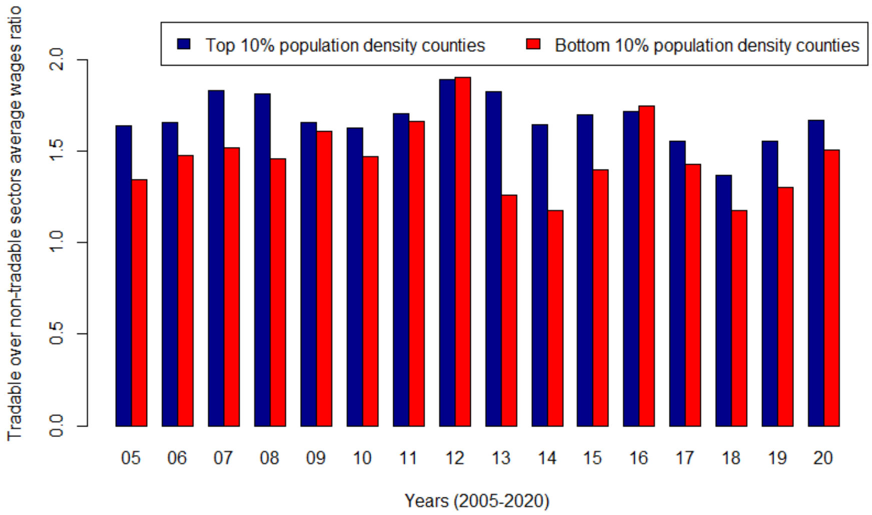



Figure 3: Tradable over non-tradable sector average wage for the top 10% and bottom 10%

# 10 Distributions example

In this part, I provide an example to illustrate how the $z$-ability distribution is created in section 6. Let us say that we have a normal distribution with mean equal to 0 and standard deviation equal to 3. In Figure 4, its probability density is shown to where the "Normal Distribution (ND)" arrow points. If $zmax = 7$ and $zmin = 2$, the truncated nomal distribution between these two values is given below the "Truncated Normal Distribution (TND)" arrow. Its derivative, which is negative and, thus, satisfies Proposition's 1 (a), is the blue line "Derivative of TND". By normalizing z, I end up with the "Normalized Truncated Normal Distribution (NTND)", which has as its derivative the curve appearing next to the "Derivative of NTND" arrow.

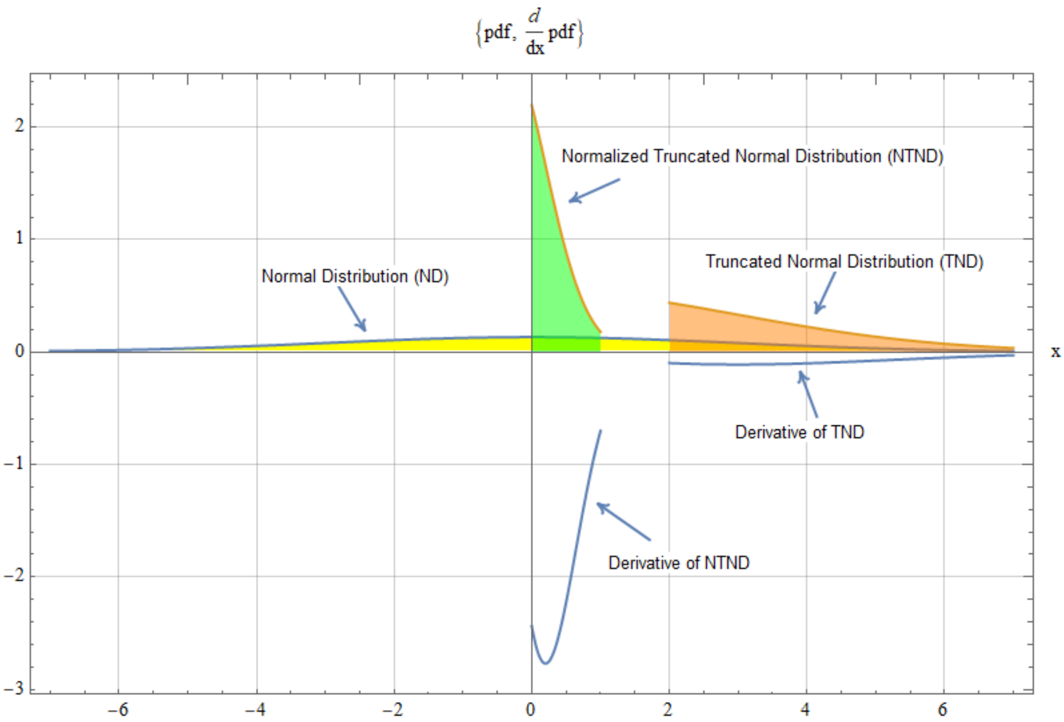
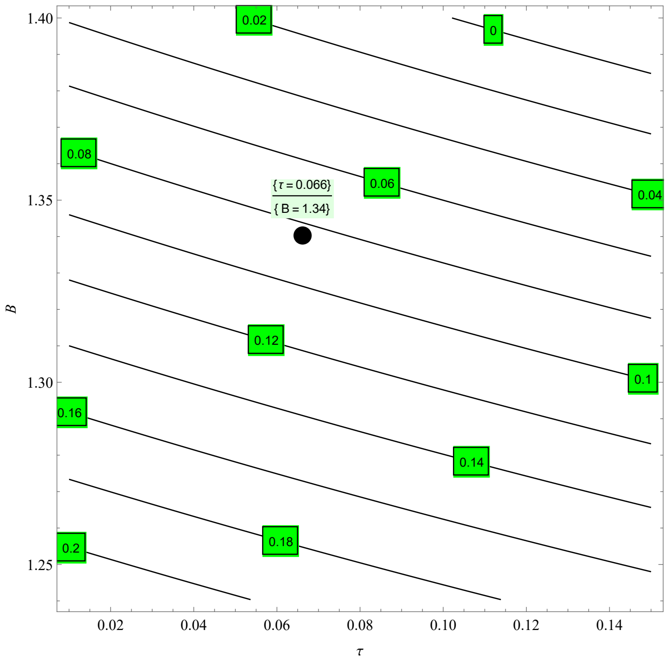
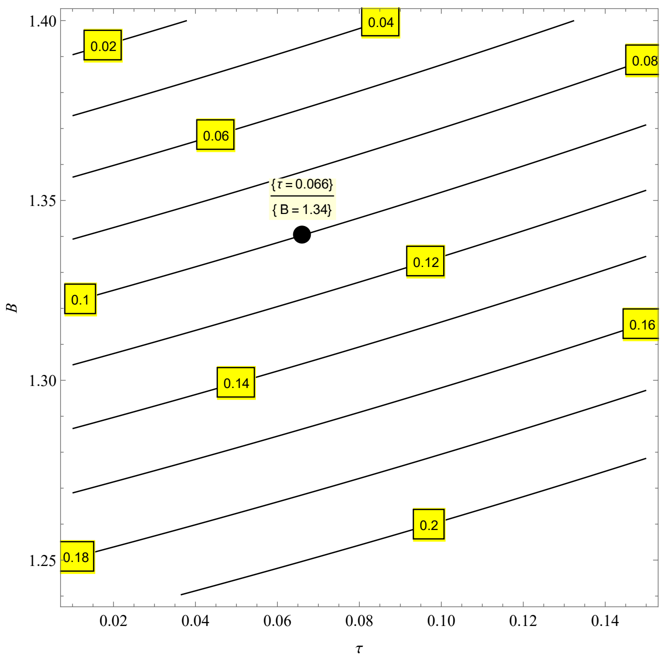



Figure 4

# 11 Post-relocation results: unemployment rate indifference curves

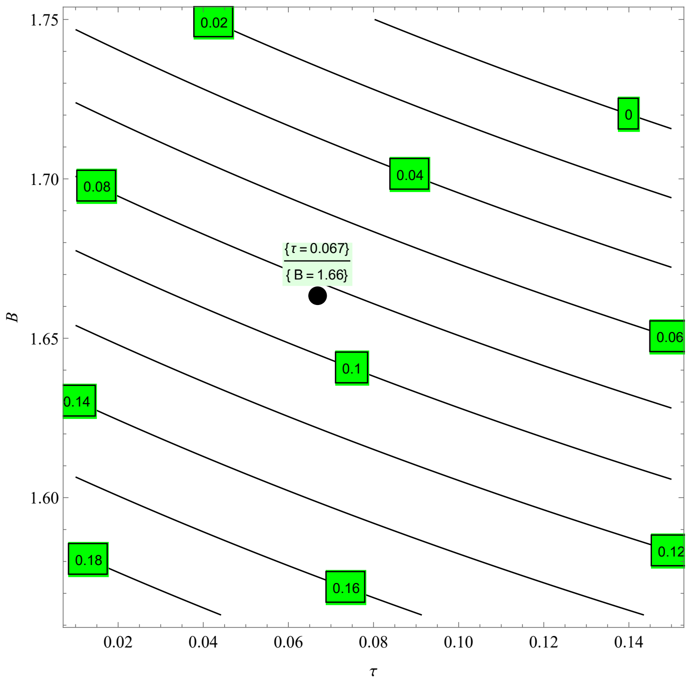



(a)

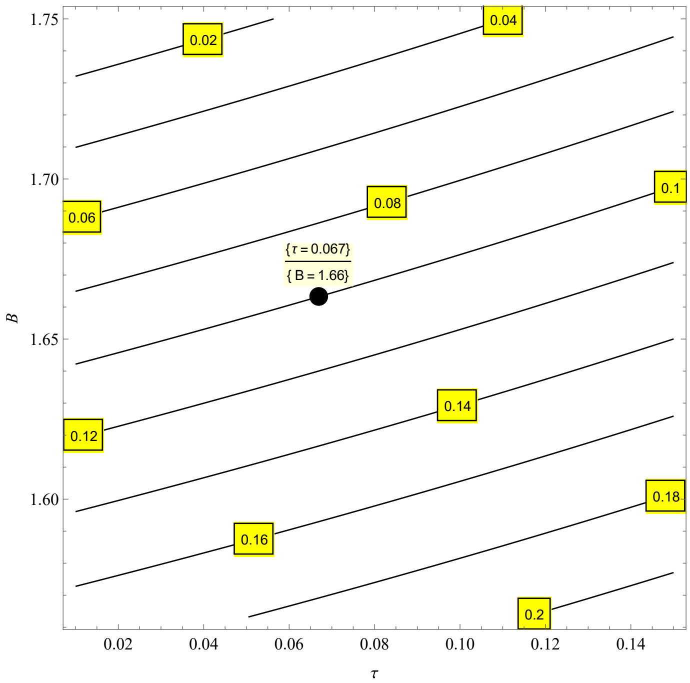



(b)

Figure 5: Normal distribution (a) URLD indifference curves (a) URHD indifference curves




(a)




(b)

Figure 6: Uniform distribution (a) URLD indifference curves (a) URHD indifference curves

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