Asset Pricing with and without Garbage: The Overlooked Triple-Hypothesis Problem*

Stefanos Delikouras† **George Korniotis, CEPR**‡

March 11, 2022

Abstract

Testing consumption-based asset pricing models is a triple hypothesis, which, besides considering alternative consumption measures, requires examining the assumptions for investor preferences and consumption dynamics. We formalize the triple-hypothesis problem in a GMM framework that relaxes the CRRA assumption, jointly estimates consumption growth dynamics with Euler equations, and includes the variance of the risk-free rate as a target moment. We find that using alternative consumption measures (e.g., garbage) does not address the empirical shortcomings of the canonical model with CRRA preferences and i.i.d. consumption growth. Instead, a model with Epstein-Zin preferences, non-i.i.d. consumption dynamics, and BEA consumption performs equally well.

Keywords: Cross-section of expected returns, Epstein-Zin preferences, risk-free rate, GMM, consumption growth

JEL classification: D51, D91, E21, G12

*We would like to thank Nikolaos Artavanis, Gennaro Bernile, Svetlana Brizgalova, Gregory Kadlec, Alexandros Kostakis, Alok Kumar, Tobias Mühlhofer, Brad Paye, Ngoc-Khan Tran, Konstantinos Stathopoulos, Mariano Massimiliano (Max) Croce, Alex Taylor, and seminar participants at the University of Liverpool, University of Miami, University of Manchester, Virginia Tech, and the 2019 NFA Meetings. We would also like to thank Kenneth French for providing return data and Tim Kroencke for providing consumption data. We are responsible for all remaining errors and omissions.

†Department of Finance, Miami Herbert Business School, University of Miami, 514-L Jenkins Building, Coral Gables, FL 33124, email: *sdelikouras@bus.umiami.edu*

‡Department of Finance, Miami Herbert Business School, University of Miami, 514-J Jenkins Building, Coral Gables, FL 33124, email: *gkorniotis@miami.edu*

1 Introduction

The canonical consumption capital asset pricing model (C-CAPM) of Breeden (1979) provides the underpinnings of modern asset pricing theory. Its main prediction is that in an endowment economy with complete markets, period-by-period aggregate consumption growth is the only source of systematic risk. Despite the model's straightforward theoretical prediction, its empirical performance with constant relative risk aversion (CRRA) preferences and aggregate consumption data from the Bureau of Economic Analysis (BEA) is weak, both in terms of plausibility of preference parameters (e.g., Mehra and Prescott (1985)) as well as cross-sectional accuracy (e.g., Liu et al. (2009)).

As a reaction, various extensions have been proposed. However, there are many different empirical approaches in the literature, which makes comparing the various extensions of the canonical C-CAPM impossible. For instance, the existing empirical methodologies do not test the complete set of theoretical predictions implied by the various models. Additionally, there is no uniformity in the test assets. Overall, there is a need for a general testing framework that can provide clarity as to which extensions of the C-CAPM of Breeden (1979) are the most successful. In this paper, we propose such a framework highlighting that the empirical performance of the C-CAPM depends on both consumption data and the underlying assumptions regarding investor preferences and consumption dynamics.

The proposed testing framework provides a natural generalization to the empirical approaches that replace period-by-period consumption growth based on the BEA data with alternative consumption measures. For example, Parker and Julliard (2003) propose a consumption measure defined over multiple periods while Jagannathan and Wang (2007) replace annual consumption growth with fourth-quarter to fourth-quarter consumption growth. More recently, Savov (2011) replaces BEA consumption with the quantity of municipal waste. Kroencke (2017) argues that the BEA consumption is a smoothed version of the true aggregate consumption and proposes a methodology to unfilter the BEA consumption data.

The above extensions of the C-CAPM with alternative consumption data can address the equity premium puzzle of Mehra and Prescott (1985) better than the standard BEA consumption. However, they are characterized by three important shortcomings. First, they rely exclusively on the assumption of CRRA preferences and ignore more realistic models of investor behavior towards risk (e.g., Epstein and Zin (1989)). Second, they mostly assume that consumption growth is i.i.d and ignore the fact that different consumption dynamics affect the cross-sectional performance of asset pricing tests. Last, they tend to focus on the aggregate equity risk premium and do not provide conclusive empirical evidence on whether these alternative consumption growth measures can jointly explain the cross-section of asset returns with key moments of the risk-free rate.

To address these shortcoming, we reexamine the performance of the C-CAPM with alternative consumption data within a novel empirical framework based on three elements. In our framework, we first relax the assumption of CRRA preferences and use the the non-separable discount factor of Epstein and Zin (1989). Unlike the single-parameter CRRA specification, in the Epstein-Zin model, risk aversion and intertemporal substitution are determined by two distinct coefficients. Using the Epstein-Zin specification is important because it highlights the effect of preferences on the empirical performance of alternative consumption processes within the C-CAPM framework.

Second, in addition to the cross-section of risk premia, the test moments in our empirical analysis include the mean and variance of the risk-free rate. Simultaneously fitting the cross-section of risk premia and the moments of the risk-free rate is challenging for the various consumption-based models for two reasons. Matching the mean of the risk-free rate restricts the mean of the consumption-based pricing kernel. Fitting the variance of the risk-free rate constrains the parameter that determines the elasticity of intertemporal substitution (EIS). Thus, by matching the mean and variance of the risk-free rate, the various consumption models have limited degrees of freedom to confront the cross-section of risk premia.

Third, in addition to the Euler equation for risk premia and the moments of the risk free rate, our empirical methodology jointly estimates the parameters of the consumption growth dynamics (i.e., mean, persistence, volatility) with asset pricing moments. We consider the case where consumption growth is either i.i.d or follows an AR(1) or ARMA(1,1) processes. The assumption regarding consumption dynamics affects the functional form of the Epstein-Zin pricing kernel and determines the volatility of the model-implied risk-free rate. Hence, estimating the consumption dynamics affects the overall fit of a consumption-based pricing kernel.

We implement our triple-hypothesis empirical framework using the generalized method of moments (GMM) of Hansen and Singleton (1982). The GMM procedure provides estimates of the structural parameters (i.e., risk aversion, EIS, rate of time preference) for each pricing kernel as well as fitted values for the cross-section of risk premia, the moments of the risk-free rate, and the parameters related to consumption dynamics. Based on the GMM results, we evaluate various consumption measures in terms of cross-sectional fit and plausibility of the estimated parameters similar to the existing literature on alternative consumption measure. Our primary goal however is not to find the consumption measure with the best cross-sectional fit and most plausible preference parameters. Instead, our goal is to show how different preference specifications as well as jointly fitting the variance of the risk-free rate and the moments of consumption growth affect the outcome of these tests.

In our empirical analysis, we use the traditional BEA consumption as a benchmark. We also consider the three-year measure of ultimate consumption growth (Ult) of Parker and Julliard (2003), the fourth quarter to fourth quarter (Q4) consumption growth process of Jagannathan and Wang (2007), the unfiltered consumption growth measures of Kroencke (2017), and the garbage-based measure of consumption introduced by Savov (2011). We also use the real per capita aggregate dividend growth (Div) based on the dividend process from the website of Robert Shiller. We consider the aggregate dividend growth process because in a no-trade endowment economy, consumption is entirely financed by dividends.[1] The sample period for which all these consumption series are available annually is from 1964 to 2016, which is the sample period that we adopt.

The test assets consist of the annual risk premia over the 1964 to 2016 period for the stock market and portfolios sorted on size, book-to-market, investment, and profitability. For each characteristic, we create two value-weighted portfolios with stocks in the top and bottom deciles of the characteristic. We use these portfolios because they constitute the basis for a number of return-generated factors that are commonly used in the empirical asset pricing literature, e.g., SMB, HML, CMA, and RMW factors in Fama and French (1993, 2015). Additionally, as shown in Harvey et al. (2015) and Hou et al. (2019), the above portfolios are the basis for a wide range of established patterns in the cross-section of equity returns.

In the case of CRRA preferences, our findings are consistent with the existing literature. In particular, when the volatility of the risk-free rate is not part of the target GMM moment conditions, we find that the alternative consumption measures outperform the standard BEA consumption in terms of plausibility of the estimated risk aversion coefficient. Specifically,

[1] Baker et al. (2007) document empirically that households are responsive to changes in dividends.

the unfiltered fourth quarter consumption of Kroencke (2017) and the garbage measure imply much smaller risk aversion estimates than the benchmark BEA consumption, although these estimates are still quite large from a micro-perspective (e.g., Rabin (2000)). The only process that generates a plausible risk aversion coefficient that is consistent with experimental findings (e.g., Kimball (2002)) is the aggregate dividend process. Nevertheless, the standard BEA consumption growth outperforms the alternative consumption measure it terms of cross-sectional fit.

When the volatility of the risk-free rate is included in the target moments, the CRRA model fails to explain the cross-section of risk premia across almost all consumption measures. The only notable exception is the unfiltered fourth quarter consumption of Kroencke (2017), which can fit the mean and variance of the risk-free rate while explaining almost half of the cross-sectional variation in risk premia. The overall poor performance of the CRRA model when the variance of the risk-free rate is included in the GMM moments can be explained by the fact that in our tests, the risk aversion parameter, which in the CRRA model is also the inverse of the EIS, is mainly identified by the variance of the risk-free rate and not by the cross-section of risk premia. The low estimated values for the risk aversion coefficient are able to match the variance of the risk-free rate but they cannot simultaneously explain equity risk premia.

The cross-sectional fit of the consumption-based framework improves when we consider the non-separable model of Epstein and Zin (1989). This model disentangles risk aversion from intertemporal substitution. We find that when we impose the assumption of AR(1) consumption growth within the Epstein-Zin specification, BEA consumption can explain more than 50% of the cross-sectional variation in risk premia. Further, the standard BEA consumption can fit the mean and variance of the risk-free rate with a plausible EIS coefficient and a risk aversion parameter that is much lower than that estimated in the CRRA specification.

To the contrary, the EIS coefficient implied by the unfiltered processes of Kroencke (2017) in the Epstein-Zin model with AR(1) consumption is almost zero. This value is much lower than the estimates suggested by the existing literature (e.g., Hall (1988), Vissing-Jorgensen (2002)). In other words, the unfiltered consumption processes in the Epstein-Zin model recast the equity premium puzzle as an EIS puzzle since the model requires an abnormally low EIS coefficient to fit the volatility of the risk-free rate due to the low persistence of the unfiltered consumption. Similarly, although the garbage measure and the aggregate dividend process imply the lowest risk aversion parameters in the Epstein-Zin model, both measures exhibit very poor cross-sectional fit. These findings for the Epstein-Zin pricing kernel run against the existing literature, which highlights the superior performance of the alternative consumption measures within the CRRA framework, and underscore the importance of the preferences assumption in tests of alternative consumption processes.[2]

We verify the above results with a series of robustness tests by extending the sample to include the Great Depression (1930-2016). Due to data constraints, we exclude from these tests the fourth-quarter to fourth-quarter and garbage consumption measures as well as the profitability and investment portfolios. When the Great Depression is included in the time series sample, the estimated risk aversion parameters are much lower than those from the 1964-2016 sample across all consumption processes. This is because including observations from the Great Depression dramatically increases the variability of all consumption measures.

Collectively, our results complement the empirical C-CAPM literature by proposing a novel approach for testing alternative consumption measures that takes into account investor preferences and consumption dynamics. Our triple-hypothesis framework uncovers important insights. When the volatility of the risk-free rate is part of the GMM estimation within the CRRA model, the unfiltered consumption of Kroencke (2017) outperforms the standard BEA measure in terms of plausibility of preference parameters. However, in the Epstein-Zin specification, the standard BEA consumption can explain the cross-section of risk premia at least as well as the alternative measures of consumption, while generating more plausible estimates for the risk-aversion and EIS coefficients than in the CRRA model. To the contrary, the Epstein-Zin framework, the unfiltered processes of Kroencke (2017) imply an abnormally low EIS parameter.

Put differently, we show that the ability of the alternative consumption measures (e.g., unfiltered processes) to generate more plausible prices of risk and improving the cross-sectional fit of the consumption framework is tied to the CRRA assumption. This ability vanishes when we consider models that disentangle time preferences from risk aversion (e.g., Epstein-Zin). This is because these consumption measures exhibit increased variability and almost zero persistence compared to the standard BEA consumption series. These two properties can help the CRRA discount factor to fit the volatility of the risk-free rate simultaneously

[2]We present a summary of our results in Table D.1 of the Appendix.

with the cross-section or expected returns.

Unfortunately, these near-i.i.d. consumption measures cannot fully take advantage of the flexibility of the Epstein-Zin model, which disentangles risk preferences from intertemporal substitution, due to their zero persistence. The standard BEA exhibits some moderate degree of persistence. Hence, when paired with the Epstein-Zin discount factor, which is characterized by two preference parameters (risk aversion, intertemporal substitution) instead of one, the BEA consumption can simultaneously explain the variance of the risk-fee rate and the cross-section of expected returns. A corollary of this analysis is that replacing the CRRA utility function with theoretically richer and empirically more plausible preference specifications has a greater impact on the cross-sectional accuracy of consumption-based models than using alternative measures of aggregate consumption.

Finally, we show that when testing the accuracy of alternative consumption measures, it is important to use tests that take into account the volatility of the risk-free rate and estimate the parameters that drive consumption dynamics. Fitting the variance of the risk-free rate is important for identifying the EIS. Including the parameters of consumption dynamics in the set of test moments is also important because it forces the estimation procedure to fit the cross-section of returns without generating errors in the consumption growth process (e.g., inflating the variability or the persistence of consumption growth).

Despite the general scope of this paper, we acknowledge certain limitations of our empirical analysis. For instance, we maintain the compete markets assumption of Breeden (1979) and we do not examine asset pricing models with heterogeneous agents and background risks as in Constantinides and Duffie (1996) and Kocheracota (1996). Similarly, we only consider alternative measures of aggregate consumption and we do not address the question of whether aggregate consumption is the proper input in consumption-based models given the limited stock-market participation puzzle in Malloy et al. (2009).

The rest of the paper is organized as follows. In Section 2, we present the theoretical background of our consumption-based asset pricing tests. In Section 3, we discuss the empirical methodology and in Section 4, we report the results. Finally, Section 5 presents the results from the extended sample, and Section 6 concludes.

2 Theoretical Background

In this section, we present the theoretical background of our empirical framework to address the triple-hypothesis problem of testing C-CAPM's, i.e., specifying consumption measure, choosing investor preferences, and identifying consumption dynamics. Our proposed empirical framework is based on three elements. First, we relax the time-separable CRRA assumption and consider the Epstein-Zin model for investor preferences. Second, we explicitly model and estimate consumption growth dynamics jointly with asset pricing moments. Finally, the moments of the risk-free rate are also included in the estimation.

2.1 CRRA Preferences

The starting point of our analysis is the CRRA stochastic discount factor M_t^{CRRA}. Its functional form is

$$M_t^{CRRA} = \beta\big(C_t/C_{t-1}\big)^{-\gamma}, \tag{1}$$

where γ ($\gamma > 0$) is the risk aversion coefficient and β ($\beta \in (0,1)$) is the rate of time preference. Following the arguments in Cochrane (2001), the unconditional market risk premium over the risk-free rate ($\mathbb{E}[R_{mt} - R_{ft}]$) and the implied risk aversion coefficient in the CRRA economy are respectively given by

$$\mathbb{E}\big[R_{mt} - R_{ft}\big] \approx \gamma\rho_{m,c}\sigma_m\sigma_c \quad \Leftrightarrow \quad \gamma \approx \mathbb{E}\big[R_{mt} - R_{ft}\big]/(\rho_{m,c}\sigma_m\sigma_c). \tag{2}$$

These two equalities follow from log-linearizing the aggregate investor's first-order conditions for optimal consumption and portfolio holdings. The constant $\rho_{m,c}$ above is the correlation of market excess returns to consumption growth while σ_m and σ_c are the volatilities of market excess returns and consumption growth, respectively. Using the estimates for $\mathbb{E}[R_{mt} - R_{ft}]$, $\rho_{m,c}$, σ_m, and σ_c over the 1964-2016 period, the implied risk aversion coefficient according to equation (2) is approximately 77. As emphasized by Mehra and Prescott (1985), this value is extremely large and impossible to reconcile with the results from experimental studies on risk preferences (e.g., Rabin (2000), Choi et al. (2007)) or the evidence from macroeconomic models (e.g., Lucas (1978)).

Equation (2) demonstrates how alternative consumption measures can potentially address the equity premium puzzle of Mehra and Prescott (1985) within the CRRA framework. Specifically, existing studies (e.g., Savov (2011), Kroencke (2017)) have been identified consumption data (e.g., garbage, unfiltered consumption) that preserve the correlation with the stock market and are twice as volatile as the standard BEA consumption. In this case, according to equation (2), the implied risk aversion parameter would decrease in half. Nevertheless, a risk aversion coefficient around 35 is still quite large according to the arguments in Rabin (2000).

2.2 Epstein-Zin Preferences

The first novel element of our analysis is that we move beyond the time-additive CRRA framework when assessing the empirical performance of alternative consumption measures. Specifically, we consider the Epstein-Zin model because it is probably the least controversial alternative to the CRRA function and has been extensively used in the asset pricing literature. Additionaly, the Epstein-Zin model nests the CRRA function simplifying our empirical tests. Finally, it allows us to illustrate that alternative assumptions regarding investor preferences lead to different empirical results for the various consumption data processes.

The basis of the Epstein and Zin (1989) model is its non-separable stochastic discount factor given by

$$M_t^{EZ} = \beta^{\frac{1-\gamma}{\rho}} \left(\frac{C_t}{C_{t-1}}\right)^{(1-\gamma)\frac{\rho-1}{\rho}} R_{wt}^{\frac{1-\gamma}{\rho}-1}. \tag{3}$$

The key property of the Epstein-Zin discount factor is that it disentangles time preferences from risk attitudes. Specifically, the novel parameter ρ in the Epstein-Zin specification determines the EIS (EIS $= 1/(1-\rho)$), while the coefficient γ measures risk aversion. To the contrary, in the CRRA pricing kernel of equation (1) the EIS is the inverse of the risk aversion.

The variable R_{wt} in equation (3) is the return on aggregate wealth, i.e., the return on the claim of aggregate consumption. As shown in Lustig et al. (2013), returns on aggregate wealth are quite difficult to measure. Thus, using the methodology in Delikouras (2017), we impose additional structure on the consumption growth process to express returns on aggregate wealth as a function of aggregate consumption growth. Specifically, we assume that log-consumption growth Δc_t is an autoregressive moving average process (ARMA(1,1)) with constant volatility and i.i.d. $N(0,\sigma_c^2)$ shocks ϵ_t

$$\Delta c_t = \mu_c + \phi_c \Delta c_{t-1} + \theta_c \epsilon_{t-1} + \epsilon_t, \quad \epsilon_t \sim \text{i.i.d. } N(0,\sigma_c^2). \tag{4}$$

where μ_c, σ_c^2, ϕ_c and θ_c are the unconditional mean, variance, first-order autocorrelation and moving average coefficients. We use the general ARMA(1,1) specification because it nests the i.i.d. ($\phi_c = \theta_c = 0$) and AR(1) dynamics ($\theta_c = 0$). Moreover, the ARMA(1,1) dynamics for consumption growth arise in theoretical models with limited information Croce et al. (2015).

In Appendix A, we show that when consumption growth follows an ARMA(1,1) process with constant volatility as in equation (4), the Epstein-Zin discount factor in equation (3) can be expressed as a function of consumption growth and consumption growth shocks

$$M_t^{EZ} = \tilde{\beta} e^{\left(\rho - 1 - \frac{\rho-1+\gamma}{1-\kappa_1\phi_c}\right)\Delta c_t + \frac{\rho-1+\gamma}{1-\kappa_1\phi_c}\phi_c \Delta c_{t-1} - \frac{(\rho-1+\gamma)\kappa_1\theta_c}{1-\kappa_1\phi_c}\epsilon_t + \frac{(\rho-1+\gamma)\theta_c}{1-\kappa_1\phi_c}\epsilon_{t-1}}. \tag{5}$$

The constant $\tilde{\beta}$ above, which is the effective rate of time presence, depends on preference parameters and consumption growth moments. The parameter κ_1 is a log-linearization constant that depends on the average log price-dividend ratio of the economy.[3] When the risk aversion parameter is equal to the inverse of the EIS, i.e., $\gamma = 1 - \rho$, or when consumption growth is i.i.d., i.e., $\phi_c = \theta_c = 0$, the Epstein-Zin specification reduces to the standard CRRA model. Finally, for $\theta_c = 0$, we obtain the Epstein-Zin specification for AR(1) consumption growth.

2.3 Consumption Dynamics

The second novel element of our empirical methodology is directly estimating the moments of consumption dynamics jointly with the asset pricing moments. This is important because the assumption regarding consumption dynamics affects the Epstein-Zin stochastic discount factor. In particular, the values of the persistence parameters (i.e., ϕ_c, θ_c) affect the effective risk aversion coefficients ($(\rho - 1 + \gamma)/(1 - \kappa_1\phi_c)$, $(\rho - 1 + \gamma)\kappa_1\theta_c/(1 - \kappa_1\phi_c)$). Further, in the

[3] See equation (29) in Appendix A.

ARMA(1,1) case, the pricing kernel depends on consumption growth (Δc_t) and consumption growth shocks (ϵ_t) terms. To the contrary, in the AR(1) case ($\theta_c = 0$ in equation (5)), the Epstein-Zin pricing kernel would only depend on consumption growth (Δc_t) terms.

In our empirical tests, we estimate the cross-sectional fit of the consumption-based model using various consumption measures and three alternative assumptions for consumption dynamics (i.i.d., AR(1), ARMA(1,1)). The existing empirical literature on consumption-based asset pricing either takes consumption dynamics as given or completely ignores them. Extending the literature, we propose that the moments of the consumption growth process be simultaneously estimated with the traditional Euler equations in a single GMM system for a number of reasons. First, the parameters driving consumption growth dynamics are unknown and need to be estimated. The standard errors of these parameters should also be taken into account in hypothesis testing. Second, when investor preferences are Epstein-Zin, the persistence and volatility of consumption growth heavily affect the estimation of preference parameters (risk aversion, intertemporal elasticity, etc.) and the overall fit of the model. Finally, estimating the moments in consumption dynamics is important because in a consumption framework, the moments of the model-implied risk-free rate depend on consumption growth moments.

2.4 The Risk-free Rate

The third novel element in our empirical methodology is the estimation of the various consumption-based models by augmenting the cross-section of risk premia with two key moments for the risk-free rate. These are the mean of the risk-free rate ($\mathbb{E}[R_{ft}]$) and the variance of the log-risk-free rate ($var(r_{ft})$). The mean condition has been extensively studied by the existing literature. However, the volatility condition has been largely ignored.

We include the mean of the risk-free rate in the cross-sectional tests because fitting the mean of the risk-free rate fixes the mean of the stochastic discount factor M_t at a realistic level. This prediction is implied by the unconditional Euler equation of the (conditionally) risk-free rate:

$$\mathbb{E}[R_{ft}] = \mathbb{E}[M_t]^{-1}\big(1 - Cov(R_{ft}, M_t)\big) \approx \mathbb{E}[M_t]^{-1}. \tag{6}$$

Fitting the variance of the risk-free rate is important because this moment can uniquely identify the EIS coefficient. This is particularly useful for the Epstein and Zin (1989) specification, where risk attitudes and intertemporal substitution are determined by distinct parameters. Specifically, in Appendix B, we show that based on the ARMA(1,1) assumption for log-consumption growth in equation (4), the variance of the log risk-free rate across all models used in this study (CRRA, Epstein-Zin) is equal to

$$var(r_{ft}) = (1-\rho)^2\big(\phi_c^2 \frac{1+\theta_c^2+2\phi_c\theta_c}{1-\phi_c^2}\sigma_c^2 + \theta_c^2\sigma_c^2 + 2\phi_c\theta_c\sigma_c^2\big). \tag{7}$$

Hence, we include the variance condition of equation (7) in our empirical analysis since the Epstein-Zin model cannot be estimated without it. This is because in the Epstein-Zin pricing kernel of equation (5), the effective risk aversion coefficient depends on the additive term $\gamma+\rho$. Thus, the two parameters cannot be identified unless the test assets include the variance of the risk-free rate, which only depends on the EIS coefficient ρ.

Setting $\theta_c = 0$ in equation (7), we obtain the expression for the variance of the risk-free rate in the case of AR(1) consumption growth. When both ϕ_c and θ_c are zero, i.e., consumption growth is i.i.d., the model-implied risk-free rate is constant and the moment condition for the variance of the risk-free rate is muted. This shows that the variance of the risk-free rate heavily depends on consumption growth moments. Therefore, including the variance of the risk-free rate in the set of moment conditions adds to the importance of jointly estimating consumption growth moments with Euler equations.

Including the variance of the risk-free rate in the estimation moments also highlights the tension in models with time-separable preferences (e.g., CRRA) when simultaneously fitting the high equity premium and the low volatility of the risk-free rate. Specifically, in the CRRA model, the inverse of the EIS is equal to the risk aversion coefficient. Thus, for this model we can replace $1-\rho$ in equation (7) with γ to obtain

$$var(r_{ft}) = \gamma^2\Big(\phi_c^2 \frac{1+\theta_c^2+2\phi_c\theta_c}{1-\phi_c^2} + \theta_c^2 + 2\phi_c\theta_c\Big)\sigma_c^2. \tag{8}$$

According to the above equation, the risk aversion (inverse EIS) parameter implied by the risk-free rate volatility is

$$\gamma = \frac{vol(r_{ft})}{\left(\phi_c^2 \frac{1+\theta_c^2+2\phi_c\theta_c}{1-\phi_c^2} + \theta_c^2 + 2\phi_c\theta_c\right)^{1/2}\sigma_c}. \tag{9}$$

Using the estimates over the 1964 to 2016 period for the volatility of the risk-free rate and the consumption growth moments into equation (9), the implied risk aversion parameter in the CRRA model is equal to 3.25.[4] This number is much smaller than the risk aversion coefficient ($\gamma = 77$) implied by the equity risk premium condition in equation (2).

One way to address this tension in the CRRA model is to use an alternative measure of aggregate consumption that is more volatile (i.e., large σ_c) and much less predictable (i.e., $\phi_c \approx \theta_c \approx 0$) than the standard BEA consumption. With these two properties, the implied risk aversion parameter from equation (2) decreases, while the implied inverse EIS coefficient from equation (9) increases. Ideally, the moments of the alternative consumption growth process should be such that

$$\frac{vol(r_{ft})}{\left(\phi_c^2 \frac{1+\theta_c^2+2\phi_c\theta_c}{1-\phi_c^2} + \theta_c^2 + 2\phi_c\theta_c\right)^{1/2}\sigma_c} = \frac{\mathbb{E}\left[R_{mt} - R_{ft}\right]}{\rho_{m,c}\sigma_m\sigma_c} = \gamma. \tag{10}$$

If the alternative consumption data satisfies equation (10), the CRRA model will be able to fit both the volatility of the risk-free rate and the equity risk premium with a single parameter. This is the mechanism of the alternative consumption measures proposed by the literature (e.g., Savov (2011), Kroencke (2017)).

In the Epstein-Zin model, the risk aversion and intertemporal substitution are determined by two distinct parameters. Thus, by disentangling risk attitudes from time preferences, the latter model provide another way to resolve the tension between fitting the variance of the risk-free rate and matching the cross-section of risk premia without necessarily resorting to alternative consumption measures.

[4]Stationarity of the consumption growth process, i.e., $|\phi_c| < 1$, guarantees that the square root in the denominator of equation (9) is a real number.

3 Data and Estimation Methodology

In this section, we describe the data and estimation methodology used in applying the triple-hypothesis framework for testing alternative consumption measures within the C-CAPM paradigm.

3.1 Consumption Data

For our tests, we use different measures of annual aggregate consumption growth. The benchmark aggregate consumption measure (SNonD) is per capita personal consumption expenditures (PCE) for services and non-durables from the BEA. Each component of aggregate consumption is deflated by its corresponding price index (PCE index) also from the BEA (base year 2009). The population data is from the U.S. Census Bureau. In our tests, we also use two aggregate consumption growth measures that are defined separately for services (S-K) and non-durables (NonD-K). These two measures are from Tim Kroencke's website.

For the alternative consumption measures, we follow the empirical approach in Kroencke (2017). Specifically, we consider the fourth-quarter to fourth-quarter (Q4) consumption growth process of Jagannathan and Wang (2007) and the three-year measure of ultimate consumption growth (Ult) of Parker and Julliard (2003).[5] We also use the unfiltered consumption growth measures of Kroencke (2017) for services and non-durables (SNonD-U), non-durables (NonD-U), and fourth quarter non-durables (Q4NonD-U). The data for these alternative consumption measures is from Tim Kroencke's website. As in Savov (2011), we also use municipal waste data from the U.S. Environmental Protection Agency (EPA) to calculate the growth in aggregate garbage. Finally, we consider the real per capita aggregate dividend growth (Div) using the dividend process provided Robert Shiller's website. We consider the aggregate dividend growth process because in a no-trade endowment economy, consumption is entirely financed by dividends, and thus, within then C-CAPM framework, aggregate dividends should be able to proxy for aggregate consumption growth.

Using the above consumption data, we compute annual growth rates. We use the annual

[5]Parker and Julliard (2003) consider alternative horizons (from 1 to 15 quarters) for their ultimate consumption measure. Further, the three-year ultimate consumption process should be aligned with three-year returns. However, for comparison with the remaining consumption measures, we follow Kroencke (2017) and align Parker and Julliard's three-year consumption measure with annual returns.

frequency to be consistent with existing work (e.g., Jagannathan and Wang (2007) and Parker and Julliard (2003)). Our main sample period is from 1964 to 2016. This sample has limited time-series observations. However, this is the only period for which all the alternative consumption measures and portfolio sorts are available. This sample period is also the same as in Savov (2011), Kroencke (2017) and accommodates direct comparison of our results with theirs. Hence, testing in this sample is also consistent with previous works. We also consider an extended period from 1930 to 2106. The extended sample increases the power of our statistical tests. However, a number of consumption measures (e.g., Q4, garbage) and test assets (e.g., investment and profitability portfolios) are not available in the extended 1930 - 2016 sample.

Summary statistics for the various consumption measures are reported in Table 1. Panels A and B report results for the 1964 to 2016 and 1930 to 2016 samples, respectively. According to the results in Panels A and C, the unfiltered measures of Kroencke (2017) (SnonD-U, NonD-U, Q4NonD-U), the garbage measure of Savov (2011) and the aggregate divided process are far more volatile than the standard BEA consumption growth process (SNonD). Panels A and B also reports the AR(1) and ARMA(1,1) coefficients estimated with the conditional log-likelihood methodology. As noted by Kroencke (2017), the unfiltered consumption measures are much less persistent than BEA consumption. In contrast, the Parker and Julliard (2003) consumption measure (Ult) is the most persistent consumption process since it is calculated with overlapping three-year intervals.

In Panels C and D, we report the correlation coefficients across the consumption growth measures. As expected, all cross-correlations are positive and large with the exception of the correlations with the dividend growth. This is not surprising since the dividend growth measure is not an economy-wide measure as it is based on the profitability of publicly traded firms.

3.2 Test Assets

The test assets in our empirical analysis are the risk-free asset, the aggregate stock market portfolio, and a cross-section of eight equity portfolios. These portfolios are the top and bottom decile portfolios sorted on on size, book-to-market, investment, and operating profitability. The returns for all test assets are from Kenneth French's website. Following the results in Asparouhova et al. (2013), we focus on value-weighted returns.

We use these equity portfolios because they are the building blocks for a number of return-generated factors that are commonly used in the empirical asset pricing literature, such as the Fama and French factors (1993, 2015). Moreover, as shown by Harvey et al. (2015) and Hou et al. (2019), the above portfolios are the basis for a wide range of patterns in the cross-section of equity returns. Further, these portfolios are consistent with Kroencke (2017), who uses portfolios sorted on size, book-to-market, profitability, and investment for his cross-sectional tests. Finally, we opt for a small cross-section of expected returns n, relative to the time-series dimension of our sample T to address the critic of Kleibergen and Zhan (2020). They note that inference is challenging for consumption-based models when the number of test assets (e.g., 25 size/book-to-market portfolios) is comparable to the number of time-series observations (i.e., when n and T are of the same magnitude).

Table 2 reports summary statistics for asset returns of the test assets. Panel A reports statistics for the 1964-2016 sample and Panel B reports statistics for the 1930-2016 sample. The pattern of these statistics are consistent with the patterns documented in the existing asset pricing literature. For example, the annual market return is about 7 to 8 percent, and the average returns between the top and bottom decile portfolios for each characteristics are significantly different.

3.3 Time Alignment between Consumption and Asset Returns

An important issue in empirical consumption-based asset pricing is the time alignment between annual consumption growth and asset returns. Time alignment important due to the temporal aggregation bias that affects the time-series of aggregate consumption (e.g., Breeden et al. (1989)).

One way to address the time aggregation bias is by calculating annual consumption growth using end-of-period consumption flows from December to December (e.g., Breeden et al. (1989)) or from fourth quarter to fourth quarter (e.g. Jagannathan and Wang (2007)). Kroencke (2017) imposes an additional temporal correction for his unfiltered consumption growth processes using the autoregressive approximation of Hall (1988). Further, Cochrane (1996) and Kroencke (2017) address the time aggregation bias from the perspective of asset returns. Specifically, they calculate asset returns using annual averages of monthly prices

($R_t = \sum_{k=1}^{12} P_{kt} / \sum_{k=1}^{12} P_{k,t-1}$) instead of end-of-year prices ($R_t = P_{12t}/P_{12,t-1}$).

We calculate asset returns using the traditional methodology of end-of-year prices. We do not alter the timing convention of asset returns to be consistent with the majority of the asset pricing literature, and to facilitate the replication of our results. Instead, we address the temporal aggregation bias by altering the timing convention in consumption. Specifically, for each consumption measure, we choose the timing convention that maximizes the correlation between consumption growth and stock market returns. According to equation (2), this would give each consumption measure the best chance to fit the cross-section of expected returns and the market risk premium with the smallest risk aversion estimate.

Panels A and C of Table 1 report correlation coefficients between the various consumption growth measures and the excess return on the stock market under two timing conventions. The first one is the beginning-of-period convention where consumption growth at time t is aligned with market returns at time $t-1$. The beginning of period convention has been previously used in consumption-based asset pricing tests by Campbell (2003), Yogo (2006), and Savov (2011). The second convention is the end-of-period convention where consumption growth at time t is aligned with market returns at time t.

According to the correlation estimates in Panels A and C of Table 1, the beginning-of-period convention yields higher correlation coefficients with the stock market across all annual consumption measures with the exception of the ultimate consumption process of Parker and Julliard (2003) (Ult), the fourth-quarter non-durable consumption (Q4NonD), and the unfiltered fourth-quarter consumption of Kroencke (2017) (Q4-U and Q4NonD-U). Based on this finding, for our annual tests in the 1964-2016 sample, we use the beginning-of-period convention for all consumption growth measures except for the Ult, Q4NonD, Q4-U, and Q4NonD-U measures. For our robustness tests in the 1930-2016 sample, we use the beginning-of-period convention for all consumption processes, since this is the timing convention that maximizes the correlation between consumption measures and the aggregate stock market.

3.4 Estimation Methodology: GMM System

The proposed triple-hypothesis framework for testing consumption-based models is cast within the standard GMM paradigm of Hansen and Singleton (1982), which has been extensively used in the existing literature. Specifically, we estimate the various consumption-based models with the following first-stage GMM system

$$\begin{bmatrix} \mathbb{E}[\frac{\partial l(\mu_c,\phi_c,\theta_c,\sigma_c^2;\Delta c_t,\Delta c_{t-1},\epsilon_{t-1})}{\partial \mu_c}] \\ \mathbb{E}[\frac{\partial l(\mu_c,\phi_c,\theta_c,\sigma_c^2;\Delta c_t,\Delta c_{t-1},\epsilon_{t-1})}{\partial \phi_c}] \\ \mathbb{E}[\frac{\partial l(\mu_c,\phi_c,\theta_c,\sigma_c^2;\Delta c_t,\Delta c_{t-1},\epsilon_{t-1})}{\partial \theta_c}] \\ \mathbb{E}[\frac{\partial l(\mu_c,\phi_c,\theta_c,\sigma_c^2;\Delta c_t,\Delta c_{t-1},\epsilon_{t-1})}{\partial \sigma_c^2}] \\ \mathbb{E}[(logR_{ft})^2 - \mathbb{E}[logR_{ft}]^2] - (1-\rho)^2\big(\frac{\phi_c^2(1+\theta_c^2+2\phi_c\theta_c)\sigma_c^2}{1-\phi_c^2} + \theta_c^2\sigma_c^2 + 2\theta_c\phi_c\sigma_c^2\big) \\ \mathbb{E}\big[R_{ft}M_t\big] - 1 \\ \mathbb{E}\big[\big(R_{mt} - R_{ft}\big)M_t\big] \\ \mathbb{E}\big[\big(R_{it} - R_{ft}\big)M_t\big] \quad \text{for} \quad i = 1, 2, ..., 8 \\ \mathbb{E}[-logR_{mt} + \Delta d_{mt}] - log\kappa_1. \end{bmatrix} \tag{11}$$

The gradient of the GMM objective function is calculated analytically when this is possible and numerically in all other cases.

The GMM system above includes four sets of moments. The first set consists of consumption growth moments that allow us to test the assumptions related to consumption growth dynamics. Specifically, under ARMA(1,1) dynamics, the function $l(\mu_c, \phi_c, \theta_c, \sigma_c^2; \Delta c_t, \Delta c_{t-1}, \epsilon_{t-1})$ in equation (11) is the conditional log-likelihood of the ARMA(1,1) process assuming that ϵ_0 is zero. Its partial derivatives (score vector) are numerically calculated by perturbing the corresponding parameters (μ_c, ϕ_c, θ_c) by an infinitesimally small number. For the variance of consumption shocks (σ_c^2), the partial derivative has an analytic expression that reads

$$\mathbb{E}[\frac{\partial l(\mu_c, \phi_c, \theta_c, \sigma_c^2; \Delta c_t, \Delta c_{t-1}, \epsilon_{t-1})}{\partial \sigma_c^2}] = \mathbb{E}[(\Delta c_t - \mu_c - \phi_c \Delta c_{t-1} - \theta_c \epsilon_{t-1})^2] - \sigma_c^2. \tag{12}$$

For AR(1) consumption dynamics, we impose the condition that the MA(1) coefficient θ_c is zero in the system of equation (11), and drop corresponding log-likelihood moment moment condition. In this case, the score vector has explicit solutions which are given by

$$\mathbb{E}[\frac{\partial l(\mu_c, \phi_c, \sigma_c^2; \Delta c_t, \Delta c_{t-1})}{\partial \mu_c}] = \mathbb{E}[\Delta c_t - \mu_c - \phi_c \Delta c_{t-1}] \tag{13}$$

$$\mathbb{E}[\frac{\partial l(\mu_c, \phi_c, \sigma_c^2; \Delta c_t, \Delta c_{t-1})}{\partial \theta_c}] = \mathbb{E}[(\Delta c_t - \mu_c - \phi_c \Delta c_{t-1})\Delta c_{t-1}]. \tag{14}$$

The second set of moments in equation (11) is related to the variance and mean of the risk-free rate. These moments help uniquely identify the EIS and the rate of time preference. Similar to previous work, the third set of moments is related to the cross-section of risk premia of the market portfolio and characteristics-sorted portfolios. These moments, help estimate the risk aversion coefficient and assess the cross-sectional fit of the C-CAPM. Finally, following Campbell and Shiller (1988), the last condition in equation (11) uses the aggregate log-dividend growth process (Δd_{mt}) and the log-stock market return $logR_{mt}$ to identify the log-linearization constant κ_1 of the price-dividend ratio in the Epstein-Zin model (equation (5)). This condition is not relevant for the CRRA discount factor since its analytical expression does not require any log-linearizations.

When we assume i.i.d. consumption growth (i.e., $\phi_c = \theta_c = 0$), the Epstein-Zin pricing kernel collapses to the time-separable CRRA model, and the model-implied variance of the risk-free rate is constant. Thus, for i.i.d. consumption growth, the GMM system becomes

$$\begin{bmatrix} \mathbb{E}[\Delta c_t] - \mu_c \\ \mathbb{E}[(\Delta c_t - \mu_c)^2] - \sigma_c^2 \\ \mathbb{E}\big[R_{ft} M_t\big] - 1 \\ \mathbb{E}\big[\big(R_{mt} - R_{ft}\big) M_t\big] \\ \mathbb{E}\big[\big(R_{it} - R_{ft}\big) M_t\big] \quad \text{for} \quad i = 1, 2, ..., n \end{bmatrix}. \tag{15}$$

3.5 Estimation Methodology: Weighting Matrix and Model Fit

The first-stage GMM system of equation (11) is the basis of the proposed triple-hypothesis framework. For the estimation, we use a diagonal weighting matrix. Its leading diagonal is:

$$diag(\mathbf{W}) = \begin{pmatrix} var(\Delta c_t)^{-1} \\ |cov(\Delta c_t, \Delta c_{t-1})|^{-2} \\ |cov(\Delta c_t, \Delta c_{t-1})|^{-2} \\ var(\Delta c_t)^{-2} \\ var(logR_{ft}^2)^{-2} \\ var(logR_{ft})^{-1} \\ \mathbf{1}_{(n+2)\times 1} \end{pmatrix}. \tag{16}$$

The first six elements of the weighting matrix correspond to the four moment conditions for the parameters in the ARMA(1,1) dynamics (μ_c, ϕ_c, θ_c, σ_c^2) and the two moments (variance, mean) of the risk-free rate. To determine these weights, we use higher order moments of the consumption growth and risk-free rate. The last element of weighting matrix is a column vector of ones, $\mathbf{1}_{(n+2)\times 1}$, and it corresponds to the moment conditions for the market equity premium, the cross-section of n risk premia, and the log-linearization constant κ_1.[6]

The matrix overweights the moment conditions for consumption growth dynamics and the risk-free rate. We make this choice for three reasons. First, the consumption growth variance and auto-covariance as well as the variance of the log risk-free rate are much smaller in magnitude than the risk premia. Second, by overweighting the consumption growth moments, we are not allowing the estimation procedure to fit portfolio premia at the expense of errors in the consumption growth process (e.g., inflating the variability or the persistence of consumption growth). Finally, the GMM weighting matrix overweights the moment conditions for the mean and variance of the risk-free rate because of their significance in identifying the mean of the stochastic discount factor and the EIS, respectively.

To assess the cross-sectional accuracy of the alternative consumption measures and preference specifications, we use the cross-sectional r-square (R^2), and the root-mean-square-

[6]Estimation results are virtually the same if instead of the unitary vector $\mathbf{1}_{(n+2)\times 1}$, we scale the Euler equations for the cross-section of expected returns by the diagonal of the second moments matrix ($diag(\mathbb{E}[\mathbf{R}_t'\mathbf{R}_t])$) similar to Hansen and Jagannathan (1997).

prediction error ($rmspe$), which is defined as

$$rmspe = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\mathbb{E}[R_{it} - R_{ft}]_{\text{sample}} - \mathbb{E}[R_{it} - R_{ft}]_{\text{fitted}}\right)^2}. \tag{17}$$

Based on the representative investor's Euler equation, the fitted risk premia above are given by the covariances of asset excess returns with the stochastic discount factor M_t

$$\mathbb{E}[R_{it} - R_{ft}]_{\text{fitted}} = -cov\big(R_{it} - R_{ft}, M_t\big)/\mathbb{E}[M_t], \text{ for } i = 1, ..., n. \tag{18}$$

4 Estimation Results

In this section, we present the baseline results from testing alternative consumption measures within the proposed triple-hypothesis framework. We provide a summary of the most important findings of this section in Table D.1 of the Appendix.

4.1 CRRA Preferences

We begin our empirical analysis with the estimation of the CRRA model of equation (1) under different assumptions for the consumption dynamics of the alternative consumption growth measures. We report the results in Table 3. Panel A presents the findings under the assumption of i.i.d. consumption growth and Panel B reports results under the AR(1) assumption. The results for the ARMA(1,1) process are in Table D.2 of the Appendix.

4.1.1 I.I.D. Consumption Growth

Panel A reports GMM results for the case where we assume that the alternative consumption measures are i.i.d. According to equation (8), in this case the model-implied risk-free rate is constant and the GMM system does not include the variance of the risk-free rate as a target moment. In general, the results in Panel A are consistent with the findings of the existing literature. The estimated risk aversion coefficient for the standard BEA consumption measure (SNonD) is quite large ($\gamma = 56$). This coefficient decreases substantially ($\gamma = 20 - 34$) when we consider the unfiltered consumption processes of Kroencke (SNonD-U, NonD-U, Q4-U, Q4NonD-U), the garbage measure of Savov ($\gamma = 17$), and the aggregate dividend growth ($\gamma = 7.5$). These results are consistent with the existing literature (e.g., Cochrane (2001), Parker and Julliard (2003), Savov (2011), Kroencke (2017)), and verify that the novel components our GMM approach (moment conditions, weighting matrix) are not biasing our inference.

Another important finding is related to the estimates for the discount rate parameter β. These estimates are quite large across all measures with the exemption of the unfiltered consumption, the dividend process, and garbage. This is because, as implied by equation (6), the discount rate forces the CRRA pricing kernel to fit the mean of the risk-free rate

$$\mathbb{E}[R_{f,t}] \approx \mathbb{E}[\beta e^{-\gamma \Delta c_t}]^{-1} \Leftrightarrow \beta \approx (\mathbb{E}[R_{f,t}] \times \mathbb{E}[e^{-\gamma \Delta c_t}])^{-1}. \tag{19}$$

Hence, if a large risk aversion estimate γ is required to fit the cross-section of expected returns then a large discount rate β is necessary to offset the large γ in equation (19) and match the mean of the risk-free rate.

A less studied implication of the estimation in Panel A is the cross-sectional fit of the alternative consumption measures. Specifically, the standard BEA measure (SNonD) and the one that includes the services alone (S-K) have the best cross-sectional performance in terms of R^2 and *rmspe* ($R^2 = 55\%$, $rmspe = 1.9\%$). The unfiltered process of Kroencke exhibit a marginally worse fit ($R^2 = 14\% - 50\%$), and the fit deteriorates more when we consider the garbage measure or the aggregate dividend process ($R^2 = 8\% - 27\%$).

In sum, for CRRA preferences and i.i.d. dynamics, the standard BEA consumption measure yields a large risk aversion estimate, whereas alternative consumption processes imply a lower, albeit still large, risk aversion coefficient due to their increased variance. However, this decrease in the magnitude of γ comes at the expense of worse cross-sectional fit.

4.1.2 AR(1) Consumption Growth

When we move away from the i.i.d. assumption and consider AR(1) dynamics, the model-implied risk-free rate is no longer constant and we can include its variance as a target moment in the GMM estimation. The results in Panel B of Table 3 highlight the main contribution of our paper; namely, that if we include the variance of risk-free rate and the parameter of consumption dynamics in the GMM system, then tests of alternative consumption measures yield significantly different results.

According to Panel B, when consumption is assumed to be an AR(1) process, the only consumption measure that can simultaneously explain the variance of the risk-free and the cross-section of risk premia is the unfiltered consumption of Kroencke (NonD-U). This process is characterized by high variance and low persistence such that the implied risk aversion from the cross-section of risk premia (equation (2)) and the implied inverse EIS by the volatility risk-free rate (equation (8)) are approximately the same. In contrast, the standard BEA consumption measure is much more persistent and less volatile that the unfiltered processes. Thus, according to equation (8), the inverse EIS from the risk-free rate is much lower than that required to fit the cross-section of risk premia in equation (2).

In sum, when we assume CRRA preferences, AR(1) dynamics, and include the variance of the risk-free rate in the GMM system, all consumption measures, with the exemption of the unfiltered ones, are characterized by negative cross-sectional fit and large pricing errors because they cannot simultaneously explain the cross-section of risk premia and the variance of the risk-free rate with a single parameter.[7]

To conclude, the results from Table 3 highlight the fact that including the variance of the risk-free rate and the moments of the consumption dynamics in the empirically analysis affects the empirical fit and estimated preference parameters of the alternative consumption models. Overall, under the CRRA assumption, the unfiltered consumption data of Kroencke are characterized by lower risk aversion coefficient and acceptable cross-sectional performance, especially in the case where the variance of the risk-free rate is part of the moment conditions. In this case, the remaining consumption measures have a very poor fit.

4.2 Epstein-Zin Preferences

Table 4 reports estimation results for the Epstein and Zin (1989) discount factor of equation (5). This model disentangles risk aversion from intertemporal substitution. For these tests, we only consider AR(1) and ARMA(1,1) dynamics, since according to equation (5), when consumption growth is i.i.d., the Epstein-Zin model reduces to the CRRA specification.

[7]Similar results hold for the case of CRRA preferences with ARMA(1,1) dynamics. As shown in Table D.2 of the Appendix, all the consumption measures exhibit poor cross-sectional fit with negative R^2's and large pricing errors.

4.2.1 AR(1) Consumption Growth

Panel A of Table 4 reports estimation results for the Epstein-Zin discount factor and AR(1) consumption growth. The first important finding in Table 4 is that the estimates of the consumption growth moments (i.e., μ_c, ϕ_c, σ_c^2) do not depend on the preference specification. In fact, these estimates are almost identical to the ones for the CRRA model from Panel B in Table 3 and the summary statistics in Table 1. This result verifies that the choice of the GMM weighting matrix does not allow the various consumption-based models to fit risk premia at the expense of the consumption moments.

Another important finding is that when consumption is assumed AR(1), the magnitudes of the EIS coefficient ρ in the Epstein-Zin model are very similar to the estimated values of $1-\gamma$ in the CRRA pricing kernel (Table 3, Panel B) across all consumption processes. This is because, in our empirical set-up when consumption is AR(1) and the risk-free rate is not constant, the EIS parameter ρ in the Epstein-Zin model and the risk aversion coefficient γ in the CRRA specification are both identified by the variance of the risk-free rate.

Regarding the structural risk aversion coefficients in Table 4, the standard BEA consumption process (SNonD) requires a large parameter ($\gamma \approx 35$) to fit the cross-section of risk premia. However, this value is lower than that for the CRRA case with i.i.d. assumption. The alternative consumption measures (SNonD-U, NonD-U, Q4NonD-U, Garbage) also imply relative large risk aversion coefficients for the Epstein-Zin model ($\gamma \approx 24$). Hence, contrary to the findings of the existing literature, in the Epstein-Zin specification, the differences in the implied risk-aversion parameter between the BEA and the alternative consumption measures is not as pronounced as in the CRRA case. This is because in the Epstein-Zin model, the effective risk aversion depends on the persistence of the underlying consumption process (equation (5)).

Similar to the CRRA case, the only consumption measure in which the (effective) discount rate $\tilde{\beta}$ is lower than 1 is the dividend process. The unfiltered and garbage measures imply β estimates around 1. The benchmark BEA consumption measure implies a large discount rate ($\tilde{\beta} = 1.6$), however its magnitude has substantially decreased relative to the CRRA case (Panel A in Table 3). Hence, for the benchmark consumption in addition to a lower risk aversion estimate, the Epstein-Zin model implies lower estimates for the discount rate parameter $\tilde{\beta}$.

Further, the results in Panel A of Table 4 show that the unfiltered consumption measures of Kroencke (2017) imply an almost zero EIS (EIS $= 1/(1-\rho)$), with ρ estimates ranging between -15 and -60. According to the findings in Vissing-Jorgensen (2002), the EIS parameter ranges between 0.4 and 1. The almost zero EIS for the unfiltered and garbage measures is due to the low persistence of these measures. Hence, according to equation (7), these measures require a large, in absolute magnitude, parameter ρ, or very low EIS, to align the variance of the risk-free rate with the variance and persistence of consumption growth.

The existing literature on alternative consumption is silent on the plausibility of the EIS parameters because it has ignored the variance of the risk-free rate as a target empirical moment. By including this moment in cross-sectional tests, we show that although certain alternative consumption measures imply relatively lower, albeit still quite high, risk aversion coefficients than BEA consumption, they require very large, in absolute value, EIS parameters. Overall, the findings for the risk aversion and discount rate parameters in the CRRA and Epstein-Zin models highlight the second main point of this paper, namely that when testing alternative consumption models, the preferences assumption affects the plausibility of the estimated structural parameters.

With respect to the cross-sectional fit of the various consumption measures when consumption growth is AR(1), BEA consumption yields a marginally better cross-sectional fit than the alternative consumption measures. Interesting, the cross-sectional fit of ultimate consumption, garbage, and aggregate dividends is inferior to that of the benchmark BEA consumption. Hence, similar to the CRRA case, for these consumption measures, the decrease in the estimated risk aversion comes at the cost of cross-sectional performance.

Comparing Panel A, Table 4 to Panel B, Table 3, we conclude that, when consumption is AR(1), the flexibility of Epstein-Zin preferences uniformly improves the cross-sectional fit of all consumption processes relative to the CRRA case, where risk aversion is the inverse of EIS. This improvement is more pronounced for consumption process that exhibit some degree of persistence, e.g., BEA consumption, aggregate dividend. The improvement is not as pronounced for consumption measures which are near-i.i.d. (e.g., unfiltered processes, garbage). This is because, as shown in equation (5), the effective risk aversion in the Epstein-Zin model depends on the persistence of the underlying consumption process.

4.2.2 ARMA(1,1) Consumption Growth

Our estimation for the Epstein-Zin model yields similar results when we impose the ARMA(1,1) assumption. Specifically, Panel B of Table 4 shows that the preference parameters (i.e., risk aversion and EIS) in the Epstein-Zin model with ARMA(1,1) consumption are very similar to those with AR(1) dynamics from Panel A. One notable difference are the estimates of the EIS coefficient for the near-i.i.d. processes (i.e., unfiltered consumption, garbage). These processes exhibit very low persistence, and thus fitting an ARMA(1,1) process leads to spurious results that affects the estimation of the EIS. This finding highlights the importance of selecting the right consumption dynamics and jointly estimating consumption growth moments in tests of consumption-based asset pricing models.

Comparing the Epstein-Zin results to the CRRA case with ARMA(1,1) consumption (see Table D.2), we note that the flexibility of the former specification tremendously improves the cross-sectional fit of all consumption measures. This is additional evidence on how consumption dynamics interact with preference assumptions and affect the cross-sectional performance of the various consumption measures.

Figure 1 illustrates the estimation results of Table 4 for the Epstein-Zin model, in which risk aversion and intertemporal substitution are not driven by the same parameter. The figure shows that the GMM weighting matrix forces the Epstein-Zin discount factor to perfectly fit the mean and volatility of the risk-free rate as well as consumption growth moments. Figure 1 also shows that by disentangling risk preferences from intertemporal substitution improves the fit of the consumption framework relative to the CRRA model across all measures.

4.3 Consumption Dynamics and Preference Specifications

The results of our main analysis, which are summarized in Table D.1 of the Appendix, highlight the importance of estimating the parameters in consumption dynamics jointly with asset pricing moments. This joint estimation immediately identifies whether our assumption for consumption dynamics is flawed. Specifically, when we force a persistent model (e.g., ARMA(1,1)) on i.i.d. consumption growth data the estimation results become spurious. Consider for example the ARMA(1,1) assumption across CRRA (Table D.2 in the Appendix) and Epstein-Zin preferences (Table 4). For the unfiltered consumption measures that are near-i.i.d., the GMM estimates for ϕ_c and θ_c are of the same magnitude but opposite signs. This implies that in the lag polynomial form of the ARMA(1,1) process, the autoregressive part would cancel out with the moving average polynomial.[8]

Hence, by jointly estimating the ARMA(1,1) or AR(1) processes with Euler equations, we can detect the persistence in each consumption growth process. Further, the statistical significance of the persistence estimates (e.g., t-statistics for ϕ_c, θ_c) and their magnitudes (e.g., $\phi_c = -\theta_c$) would clearly indicate which consumption measures are persistent and which are not. This is very helpful information because it could affect the choice of the stochastic discount factor used in empirical tests of alternative consumption measures. For instance, in the case of i.i.d. consumption growth, the Epstein-Zin model collapses to the standard CRRA model. When consumption growth exhibits some persistence, then the Epstein-Zin model offers additionally flexibility by disentangling risk attitudes from intertemporal substitution.

5 Extended Sample: 1930 to 2016

We conclude the empirical application of our triple-hypothesis framework with results from an extended sample that runs from 1930 through 2016 and includes the Great Depression.[9] For these tests, the fourth-quarter to fourth-quarter and garbage consumption measures as well as the investment and profitability portfolios are not available.

5.1 CRRA Preferences

Table 5 shows results for the CRRA model under two different assumptions for consumption dynamics, i.i.d. (Panel A) and AR(1) (Panel B).[10] The results are broadly consistent with the baseline tests in Table 3. Specifically, in the i.i.d. case, the unfiltered consumption measures imply lower risk aversion and discount rate parameters ($\gamma = 15$, $\beta = 1$) than the benchmark consumption ($\gamma = 23$, $\beta = 1.35$). The lowest risk aversion and discount rate coefficients ($\gamma = 4.60$, $\beta = 0.83$) correspond to the aggregate dividend process, which is the most volatile measure. Overall, the estimated risk aversion and discount rate coefficients in

[8]The lag polynomial of the ARMA(1,1) process is given by $(1 - \phi_1 L)\Delta c_t = (1 + \theta_c L)\epsilon_{c,t}$, where L is the lag operator, $Lx_t = x_{t-1}$.

[9]The first year that consumption data is available in the BEA website is 1929.

[10]We do not report the results for CRRA preferences with ARMA(1,1) consumption because they are almost identical to the ones for the 1964-2016 sample in Table D.2.

the 1930-2016 sample are much lower than those for the 1964-2016 sample due to the inclusion of the Great Recession, which significantly increases consumption growth volatility.

In terms of fit, the ultimate consumption process of Parker and Julliard (2003) can explain 95% of the cross-sectional variations of the market, small, big, high, and low portfolios, whereas the standard BEA measure can explain 75% of the cross-sectional variation. The differences in performance of the ultimate consumption across the two samples (1964 - 2016) and (1930 – 2016) can be explained by the inclusion of the Great Depression in the latter sample. During the Great Depression the time alignment of consumption growth and equity returns is distorted due to the extreme fluctuations in consumption growth. By aggregating three years of consumption growth, the ultimate consumption measure restores the time alignment between consumption growth and asset returns.[11]

When we impose the assumption of AR(1) dynamics and include the variance of the risk-free rate in the set of GMM target moment conditions, then all measures exhibit a poor cross-sectional fit. This is because the inverse EIS implied by the volatility of the risk-free rate (equation (8)) is much lower that the value required to explain the level of risk premia (equation (2)). Similar, to our baseline results, the only consumption measure that can capture the volatility of the risk free and the cross-section of risk premia is the unfiltered consumption from Kroencke (2017).

5.2 Epstein-Zin Preferences

According to the results in Table 6 for the Epstein-Zin model in the extended 1930-2016 sample, all consumption measures, not just the unfiltered ones, exhibit a good cross-sectional fit. This is due to the additional flexibility that this model offers by disentangling risk preferences from time attitudes. In terms of preference parameters, the fitted risk aversion coefficient for the standard BEA consumption is quite low ($\gamma = 18$). In fact, with the exception of the dividend measure, which implies a risk aversion coefficient of four, the remaining consumption measures (e.g., ultimate, unfiltered consumption) imply risk aversion parameters that range from 15 to 25, and are quite similar in magnitude to those of the BEA consumption.

[11]To showcase the effects of the Great Depression on the time-alignment between consumption growth and asset returns, we note that if in the 1930-2016 sample, we align time t returns with time $t-2$ consumption growth, the cross-sectional fit of the BEA consumption is almost perfect ($R^2 = 90\%$). The ultimate consumption measure includes $t-2$ consumption in its components since it captures consumption growth over 3-year horizons.

Hence, in the 1930 to 2016 sample, when preferences are assumed Epstein-Zin and consumption dynamics are assumed AR(1), the unfiltered processes of Kroencke (2017) yield similar risk aversion estimates to the standard BEA consumption. This is because, in the Epstein-Zin model, the estimated risk aversion parameter depends on both the volatility of consumption growth as well as its persistence. In the CRRA specification, risk aversion depends on consumption volatility alone. Hence, in the Epstein-Zin model, the low volatility of the standard BEA consumption is offset by its relatively high persistence. To the contrary, the unfiltered consumption measures can only rely on their increased volatility to decrease the estimated risk aversion coefficient since these processes are almost i.i.d.

Further, when consumption growth is assumed to be an AR(1) process (Panel A of Table 6), the unfiltered consumption data combined with the Epstein-Zin model imply an abnormally low EIS with very large in absolute magnitude ρ parameters ($|\rho| = 20 - 31$) compared to that the benchmark BEA consumption ($|\rho| = 3$). However, these large EIS coefficients for the unfiltered consumption decrease when we assume ARMA(1,1) consumption dynamics (Panel B of Table 6).

In terms of cross-sectional fit, all the models explain a larger portion of the cross-sectional variation in the risk premia compared tot he the 1964-2006 sample. For example, the R^2 for all models but the model with dividend growth is higher than 80%. We illustrate the high fit of the various models in Figure 2 that plots the model-implied fitted models against the sample moments.

Overall, the results from the 1930-2016 sample highlight the impact of preference specifications, consumption dynamics, and the volatility of the risk-free rate on asset-pricing tests. For CRRA preferences and AR(1) dynamics, the only consumption measures that can explain both the volatility of the risk-free rate and the cross-section of returns are the unfiltered processes of Kroencke (2017). To the contrary, when preferences are Epstein-Zin and the dynamics are AR(1), the standard BEA consumption measure yields relative lower risk aversion estimates, plausible EIS estimates, and its cross-sectional fit is comparable to the rest of the consumption measures.

6 Conclusion

This paper challenges the empirical methodology used by the existing literature in testing alternative consumption measures within the C-CAPM framework. This literature has mainly focused on the magnitude of the estimated risk aversion coefficient. It focuses on fitting the stock-market equity premium, and pays little attention to the full cross-section of expected returns. It also takes a strong stance on investor preferences by assuming a CRRA utility function. Further, it ignores the importance of fitting the volatility of the risk-free rate and it does not explicitly estimate the parameters governing consumption dynamics.

In the paper, we propose a more general empirical framework that addresses the aforementioned shortcomings within a triple-hypothesis framework. First, the proposed framework relaxes the CRRA assumption by considering the non-separable preferences of Epstein and Zin (1989). Second, we consider alternative specifications for consumption dynamics (i.i.d., AR(1), ARMA(1,1)) and estimate the unknown consumption growth parameters together with the Euler equations for risk premia in a joint GMM system. Finally, we highlight the importance of fitting the volatility of the risk-free rate as the most robust way for identifying the EIS within the Epstein-Zin framework, and for testing the effects of consumption dynamics on the moments of the model-implied risk-free rate.

Our results show that the outcomes of empirical tests of alternative consumption processes depend heavily on the assumptions for preferences specifications and consumption growth dynamics. For instance, in the case of CRRA preferences and AR(1) dynamics, only the unfiltered consumption processes of Kroencke (2017) can simultaneously explain the volatility of the risk-free rate and the cross-section of risk premia. To the contrary, when preferences are Epstein-Zin and consumption growth is AR(1), the standard BEA consumption measure can fit the moments of the risk-free rate and the cross-section of risk premia while yielding plausible estimates for the EIS and risk aversion coefficients.

Overall, our empirical analysis can serve as a tool for future tests of consumption-based models. Specifically, our findings suggest that the ability of the alternative consumption measures to improve the cross-sectional fit of C-CAPM models vanishes when we emphasize the importance of fitting the variance of the risk-free rate and relax the CRRA assumption and consider models that disentangle time preferences from risk aversion (e.g., Epstein-Zin). A corollary of results is that replacing the CRRA utility function with theoretically richer and empirically more plausible preference specifications may have a greater impact on the cross-sectional accuracy of consumption-based models than using alternative measures of aggregate consumption.

Appendix

Appendix A Explicit Solutions for the Epstein-Zin Discount Factor with ARMA(1,1) Consumption Dynamics

To derive explicit solutions for the Epstein-Zin model, we combine the linear structure of the non-separable preferences in Epstein and Zin (1989) with the ARMA(1,1) dynamics for consumption growth. The proof consists of two steps. First, we express the price-dividend ratio of the claim on aggregate consumption as a linear function of consumption growth. Second, we solve the Epstein-Zin discount factor in terms of consumption growth.

Price-dividend ratio of a claim on aggregate consumption

The representative investor chooses consumption C_t and portfolio weights $\{w_{it}\}_{i=1}^n$ to maximize lifetime utility V_t. The investor's maximization problem is given by

$$\begin{aligned} V_t \; &= \max_{C_t,\ \{w_{it}\}_{i=1}^n} \left[(1-\beta)C_t^\rho + \beta\mu_t(V_{t+1})^\rho\right]^{\frac{1}{\rho}}, \quad \text{such that} && (20) \\ W_{t+1} &= (W_t - C_t)R_{w,t+1} \\ R_{w,t+1} &= \sum_{i=1}^n w_{it}(R_{i,t+1} - R_{f,t+1}) \\ \mu_t(V_{t+1}) &= \mathbb{E}_t\left[V_{t+1}^{1-\gamma}\ \right]^{\frac{1}{1-\gamma}}. && (21) \end{aligned}$$

Above, W_t denotes aggregate wealth and $R_{w,t+1}$ is the return on aggregate wealth. Using the linear homogeneity of the objective function and the budget constraint for $\rho \neq 0$, equation (20) can be written as

$$J_t W_t = \max_{C_t,\ \{w_{i,t}\}_{i=1}^n} \left[(1-\beta)C_t^\rho + \beta(W_t - C_t)^\rho \mu_t(J_{t+1}R_{w,t+1})^\rho\right]^{\frac{1}{\rho}},$$

where J_t is marginal lifetime utility. The first-order condition for C_t reads

$$(1-\beta)C_t^{\rho-1} - \beta(W_t - C_t)^{\rho-1}\mu_t(J_{t+1}R_{w,t+1})^\rho = 0.$$

Dividing by $W_t^{\rho-1}$, we obtain

$$(1-\beta)\Big(\frac{C_t}{W_t}\Big)^{\rho-1} - \beta\Big(1 - \frac{C_t}{W_t}\Big)^{\rho-1}\mu_t(J_{t+1}R_{w,t+1})^\rho = 0. \tag{22}$$

Along an optimal consumption path, the following holds

$$J_t^\rho W_t^\rho = (1-\beta)C_t^\rho + \beta(W_t - C_t)^\rho \mu_t(J_{t+1}R_{w,t+1})^\rho.$$

Dividing by W_t^ρ, we get that

$$J_t^\rho = (1-\beta)\Big(\frac{C_t}{W_t}\Big)^\rho + \beta\Big(1-\frac{C_t}{W_t}\Big)^\rho \mu_t(J_{t+1}R_{w,t+1})^\rho. \tag{23}$$

Equations (22) and (23) imply that

$$J_t^\rho = (1-\beta)\Big(\frac{C_t}{W_t}\Big)^{\rho-1}. \tag{24}$$

We can substitute the above relation into equation (22) to get

$$(1-\beta)\Big(\frac{C_t}{W_t}\Big)^{\rho-1} - \beta(1-\beta)\Big(1-\frac{C_t}{W_t}\Big)^{\rho-1}\mu_t\Big[\Big(\frac{C_{t+1}}{W_{t+1}}\Big)^{(\rho-1)/\rho}R_{w,t+1}\Big]^\rho = 0.$$

Using the budget constraint, the first-order condition for consumption simplifies into

$$\beta\mu_t\Big[\Big(\frac{C_{t+1}}{C_t}\Big)^{(\rho-1)/\rho}R_{w,t+1}^{1/\rho}\Big]^\rho = 1. \tag{25}$$

Let $P_{c,t} = W_t - C_t$ be the price for a claim on aggregate consumption. We can use the price-dividend identity in Campbell and Shiller (1988)

$$R_{w,t+1} = \frac{C_{t+1}}{C_t}\frac{P_{c,t+1}/C_{t+1}+1}{P_{c,t}/C_t}, \tag{26}$$

to recast equation (25) as

$$\frac{1}{\beta}^{\frac{1}{\rho}}\Big(\frac{P_{c,t}}{C_t}\Big)^{\frac{1}{\rho}} = \mu_t\Big[\frac{C_{t+1}}{C_t}\Big(\frac{P_{c,t+1}}{C_{t+1}}+1\Big)^{1/\rho}\Big]. \tag{27}$$

A log-linear approximation to the price-dividend identity in equation (26) is given by

$$r_{w,t+1} = \kappa_0 + \kappa_1 pc_{t+1} - pc_t + \Delta c_{t+1}, \tag{28}$$

where $pc_t = log\frac{P_{c,t}}{C_t}$ and the parameters

$$\kappa_1 = \frac{e^{\overline{pc}}}{1+e^{\overline{pc}}} \in (0,1) \quad \text{and} \quad \kappa_0 = log(1+e^{\overline{pc}}) - \kappa_1\overline{pc}$$

are log-linearization constants with $\overline{pc} = \mathbb{E}[pc_t]$.

We conjecture that the log price-dividend ratio is linear in consumption growth and consumption growth shocks:

$$pc_t = \mu_v + \phi_v \Delta c_t + \theta_v \epsilon_t. \tag{29}$$

Using the definition of the EZ certainty equivalent from equation (21), equation (27) becomes

$$-\frac{1-\gamma}{\rho}(log\beta - pc_t) = log\mathbb{E}_t\big[e^{(1-\gamma)\Delta c_{t+1}+\frac{1-\gamma}{\rho}(\kappa_0+\kappa_1 pc_{t+1})}\big].$$

Based on the conjecture that $pc_t = \mu_v + \phi_v \Delta c_t + \theta_v \epsilon_t$, the lognormal property of consumption growth, and the ARMA(1,1) dynamics for consumption growth from equation (4), equation (27) becomes

$$\begin{aligned} &-\frac{1-\gamma}{\rho}log\beta + \frac{1-\gamma}{\rho}(\mu_v + \phi_v \Delta c_t + \theta_v \epsilon_t) = \\ &(1-\gamma)\big(\mu_c + \phi_c \Delta c_t + \theta_c \epsilon_t\big) + \frac{1-\gamma}{\rho}\kappa_0 + \frac{1-\gamma}{\rho}\kappa_1 \mu_v + \frac{1-\gamma}{\rho}\kappa_1 \phi_v \mu_c + \frac{1-\gamma}{\rho}\kappa_1 \phi_v \phi_c \Delta c_t \\ &+\frac{1-\gamma}{\rho}\kappa_1 \phi_v \theta_c \epsilon_t + 0.5\Big[\Big(\frac{1-\gamma}{\rho}\kappa_1 \phi_v + \frac{1-\gamma}{\rho}\kappa_1 \theta_v + 1 - \gamma\Big)\sigma_c\Big]^2. \end{aligned} \tag{30}$$

We can now use the method of undetermined coefficients to find the values for μ_v, ϕ_v, and θ_v. First, we collect consumption growth terms. Then, we solve for ϕ_v to get

$$\phi_v = \frac{\rho \phi_c}{1 - \kappa_1 \phi_c}. \tag{31}$$

Similarly, we collect consumption growth shock terms and use the solution for ϕ_v to get

$$\theta_v = \frac{\rho \theta_c}{1 - \kappa_1 \phi_c}. \tag{32}$$

Finally, collecting constant terms in equation (30), the solution for μ_v is given by

$$\mu_v = \frac{1}{1-\kappa_1}\Big[log\beta + \kappa_0 + (\kappa_1 \phi_v + \rho)\mu_c + 0.5\frac{1-\gamma}{\rho}(\kappa_1 \phi_v + \kappa_1 \theta_v + \rho)^2 \sigma_c^2\Big].$$

Explicit solutions for the Epstein-Zin stochastic discount factor

From Epstein and Zin (1989), the non-separable stochastic discount factor reads

$$M_{t+1} = \beta^{\frac{1-\gamma}{\rho}}\big(\frac{C_{t+1}}{C_t}\big)^{1-\gamma\frac{\rho-1}{\rho}} R_{w,t+1}^{\frac{1-\gamma}{\rho}-1}.$$

Based on the log-linearized price-dividend identity for returns on total wealth (equation (28)) and our conjecture regarding the functional form of the log-price dividend ratio (equation (29)), the stochastic discount factor can be further expressed as

$$M_{t+1} = e^{\frac{1-\gamma}{\rho}log\beta+\frac{1-\gamma}{\rho}(\rho-1)\Delta c_{t+1}+(\frac{1-\gamma}{\rho}-1)[\kappa_0+\kappa_1(\mu_v+\phi_v\Delta c_{t+1}+\theta_v\epsilon_{t+1})-(\mu_v+\phi_v\Delta c_t+\theta_v\epsilon_t)+\Delta c_{t+1}]}.$$

Using the solutions for ϕ_v, θ_v and μ_v, we conclude that

$$M_{t+1} = Exp\Big[log\beta + (\rho-1)\Delta c_{t+1} + \frac{\rho-1+\gamma}{1-\kappa_1\phi_c}\mu_c + 0.5\frac{(\rho-1+\gamma)(1-\gamma)(1+\kappa_1\theta_c)^2}{(1-\kappa_1\phi_c)^2}\sigma_c^2$$
$$-\frac{\rho-1+\gamma}{1-\kappa_1\phi_c}\Delta c_{t+1} - \frac{(\rho-1+\gamma)\kappa_1\theta_c}{1-\kappa_1\phi_c}\epsilon_{t+1} + \frac{(\rho-1+\gamma)\phi_c}{1-\kappa_1\phi_c}\Delta c_t + \frac{(\rho-1+\gamma)\theta_c}{1-\kappa_1\phi_c}\epsilon_t\Big].$$

If we let $\tilde{\beta}$ to summarize the constant terms, the Epstein-Zin discount factor reads

$$M_{t+1} = Exp\Big[log\tilde{\beta} + (\rho-1)\Delta c_{t+1} - \frac{\rho-1+\gamma}{1-\kappa_1\phi_c}\Delta c_{t+1} - \frac{(\rho-1+\gamma)\kappa_1\theta_c}{1-\kappa_1\phi_c}\epsilon_{t+1} \tag{33}$$
$$+\frac{(\rho-1+\gamma)\phi_c}{1-\kappa_1\phi_c}\Delta c_t + \frac{(\rho-1+\gamma)\theta_c}{1-\kappa_1\phi_c}\epsilon_t\Big].$$

By setting $\gamma = 1-\rho$ in the above expression, we obtain the CRRA model of equation (1). By setting θ_c equal to zero, we obtain the solution for AR(1) consumption growth. By setting both ϕ_c and θ_c equal to zero, we obtain the solution for i.i.d. consumption dynamics, which collapses to the CRRA case.

Appendix B The Volatility of the Risk-free Rate

In this section, we derive the expression for the volatility of the risk-free rate across the difference preference specifications used in this study.

Consider the conditional Euler equation for the return of the risk-free asset R_{ft} according to the Epstein-Zin specification of equation (5)

$$\mathbb{E}_t\Big[e^{log\tilde{\beta}+(\rho-1)\Delta c_{t+1}-\frac{\rho-1+\gamma}{1-\kappa_1\phi_c}\Delta c_{t+1}-\frac{(\rho-1+\gamma)\kappa_1\theta_c}{1-\kappa_1\phi_c}\epsilon_{t+1}+\frac{(\rho-1+\gamma)\phi_c}{1-\kappa_1\phi_c}\Delta c_t+\frac{(\rho-1+\gamma)\theta_c}{1-\kappa_1\phi_c}\epsilon_t}R_{f,t+1}\Big] = 0.$$

Using the ARMA(1,1) assumption for log-consumption growth from equation (4), the above relation for the log risk-free rate $r_{f,t+1}$ becomes

$$\mathbb{E}_t\Big[e^{log\tilde{\beta}+\left(\rho-1-\frac{\rho-1+\gamma}{1-\kappa_1\phi_c}\right)\mu_c+(\rho-1)(\phi_c\Delta c_t+\theta_c\epsilon_t)+\left(\rho-1-\frac{\rho-1+\gamma}{1-\kappa_1\phi_c}-\frac{(\rho-1+\gamma)\kappa_1\theta_c}{1-\kappa_1\phi_c}\right)\epsilon_{t+1}}\Big] = e^{-r_{f,t+1}}. \tag{34}$$

Based on the properties of the normal distribution for the consumption growth shocks ϵ_{t+1}, the conditional expectation from equation (34) can be written as

$$e^{log\tilde{\beta}+\left(\rho-1+\frac{1-\rho-\gamma}{1-\kappa_1\phi_c}\right)\mu_c+(\rho-1)(\phi_c\Delta c_t+\theta_c\epsilon_t)+0.5h(\rho,\gamma,\kappa_1,\phi_c,\theta_c)^2\sigma_c^2} = e^{-r_{f,t+1}},$$

where $h()$ is a function of constant terms. It immediately follows that

$$var(r_{f,t+1}) = (1-\rho)^2\big(\phi_c^2 var(\Delta c_t) + \theta_c^2 var(\epsilon_t) + 2\phi_c\theta_c covar(\Delta c_t, \epsilon_t)\big).$$

By the properties of the ARMA(1,1) model, the variance of the risk-free rate becomes

$$var(r_{f,t+1}) = (1-\rho)^2\big(\phi_c^2\frac{1+\theta_c^2+2\phi_c\theta_c}{1-\phi_c^2}\sigma_c^2 + \theta_c^2\sigma_c^2 + 2\phi_c\theta_c\sigma_c^2\big).$$

The proof for the CRRA specification is the same as above, since the CRRA discount factor is nested by the Epstein-Zin model for $\gamma = 1-\rho$. Also, by setting θ_c equal to zero, we obtain the variance of the risk-free rate for AR(1) consumption dynamics. Finally, for i.i.d. consumption growth ($\phi_c = \theta_c = 0$), the risk-free rate is constant.

Appendix C The Unconditional Variance of the ARMA(1,1) Log-consumption Growth Process

Based on equation (4) for the ARMA(1,1) log-consumption dynamics, the unconditional variance of consumption growth when the latter follows is

$$var(\Delta c_t) = var(\phi_c\Delta c_{t-1} + \theta_c\epsilon_{t-1}) + \sigma_c^2,$$

which can be written as

$$var(\Delta c_t) = \phi_c^2 var(\Delta c_t) + \theta_c^2\sigma_c^2 + 2\phi_c\theta_c\sigma_c^2 + \sigma_c^2,$$

to get

$$var(\Delta c_t) = \frac{\theta_c^2 + 2\phi_c\theta_c + 1}{1-\phi_c^2}\sigma_c^2.$$

By respectively setting $\theta_c = 0$ or $\theta_c = \phi_c = 0$, we obtain the expressions for the unconditional variance of the AR(1) and i.i.d. consumption growth processes.

Appendix D Supplemental Tables

Table D.1 Summary of Important Results

This table summarizes the results of our empirical analysis with respect to the estimated preferences parameters and cross-sectional fit across different preference specifications (CRRA in Panel A, Epstein-Zin in Panel B) and assumptions for consumption dynamics (i.i.d., AR(1), ARMA(1,1)). BEA consumption is the benchmark real aggregate consumption growth measure for services and non-durables from the Bureau of Economic Analysis. *Ultimate* is the Parker and Julliard (2003) three-year measure of ultimate consumption growth. $Q4 - Q4$ is the fourth quarter to fourth quarter consumption growth measure for non-durables and services of Jagannathan and Wang (2007). $Unfiltered$ consumption is the real aggregate consumption growth measures from Kroencke (2017). *Garbage* is the garbage-based consumption growth measure of Savov (2011). The data for the garbage measure is from the U.S. Environment Protection Agency (EPA). *Dividends* is the real per capita aggregate divided growth. Dividend data is from Shiller's website (1965-2015) and Standard and Poor's (2016-2017). γ is the risk-aversion parameter, which in the CRRA cases is also the inverse of the EIS parameter, and β is the rate of time preference. $1/(1-\rho)$ is the EIS coefficient in the Epstein-Zin model. σ_c is the volatility of the consumption growth shocks, ϕ_c is the autoregressive coefficient, and θ_c is the moving average coefficient. The sample is from 1964 to 2016, the test assets include the parameters in the consumption dynamics (mean, variance, persistence), the mean and variance of the risk-free rate, the market risk-premium, and the eight high and low portfolios from four cross-sections of value-weighted deciles independently sorted on size, book-to-market, investment, and profitability.

Panel A: CRRA Preferences

		BEA Consumption	Ultimate Consumption	Q4-Q4 Consumption	Unfiltered Consumption	Garbage	Dividends
I.I.D. (model $r_{f,t}$ is constant)	Risk aversion γ	56	61	30 - 65	20 - 35	17	7
	Discount rate β	> 1	> 1	> 1	> 1	≈ 1	< 1
	EIS $1/\gamma$	0.02	0.02	0.01 - 0.03	0.03 - 0.05	0.06	0.14
	Cons. shock volatility σ_c	1.2%	2.9%	1.4% - 2%	2.4% - 2.9%	2.9%	6.2%
	Cross-sectional fit R^2	55%	< 0	47% - 55%	15% - 51%	8%	27%
AR(1)	Risk aversion γ	4	1	4 - 6	21 - 34	18	1
	Discount rate β	≈ 1	≈ 1	≈ 1	≈ 1	≈ 1	< 1
	EIS $1/\gamma$	0.25	1	0.16 - 0.25	0.03 - 0.05	0.06	1
	Cons. shock volatility σ_c	1%	1.6%	1.3% - 2%	2.4% - 2.9%	2.9%	5.4%
	Cons. growth persistence ϕ_c	0.41	0.76	0.18 - 0.36	-0.03 - 0.03	-0.03	0.47
	Cross-sectional fit R^2	< 0	< 0	< 0	13% - 48%	5%	< 0

Panel B: Epstein-Zin Preferences

		BEA Consumption	Ultimate Consumption	Q4-Q4 Consumption	Unfiltered Consumption	Garbage	Dividends
AR(1)	Risk aversion γ	34	23	25 - 61	19 - 34	17	4
	Discount rate β	> 1	> 1	> 1	> 1	≈ 1	< 1
	EIS $1/(1-\rho)$	0.28	0.8	0.20 - 0.26	0.02 - 0.06	0.03	1.4
	Cons. shock volatility σ_c	1%	1.6%	1.3% - 2%	2.4% - 2.9%	2.9%	5.4%
	Cons. growth persistence ϕ_c	0.47	0.70	0.20 - 0.38	-0.01 - 0.04	-0.02	0.48
	cross-sectional fit R^2	53%	< 0	20% - 53%	15% - 51%	8.4%	30%
ARMA(1,1)	Risk aversion γ	38	37	30 - 74	19 - 34	20	5
	Discount rate β	> 1	> 1	> 1	> 1	≈ 1	< 1
	EIS $1/(1-\rho)$	0.28	1	0.27 - 0.38	0.06 - 0.14	0.12	1.5
	Cons. shock volatility σ_c	1%	1.6%	1.3% - 1.8%	2.4% - 2.9%	2.9%	5.3%
	Cons. growth persistence $\{\phi_c, \theta_c\}$	{0.30, 0.22}	{0.45, 0.63}	{-0.56 - 0.19, 0.23 - 0.91}	{-0.93 - 0.75, -0.73 - 0.90}	{0.65, -0.72}	{0.17, 0.42}
	Cross-sectional fit R^2	53%	< 0	21% - 55%	18% - 50%	8.5%	15%

Table D.2 CRRA Model and ARMA(1,1) Consumption Growth

This table reports GMM results for the CRRA model of equation (1) for various annual consumption measures and an ARMA(1,1) process for consumption dynamics. The test assets consist of the eight high and low portfolios from four cross-sections of value-weighted deciles independently sorted on size, book-to-market, investment, and profitability. We estimate the CRRA model using the over-identified GMM system in equation (11) that includes the risk premia for the test assets, the stock market risk premium, the consumption growth moments, and the mean and variance of the log risk-free rate. γ is the risk-aversion parameter, which in the CRRA case is also the inverse of the EIS parameter, and β is the rate of time preference. μ_c is the constant term in the ARMA(1,1) process, σ_c is the volatility of the consumption growth shocks ϵ_t, ϕ_c is the autoregressive coefficient, and θ_c is the moving average coefficient. $SNonD$ is the benchmark real aggregate consumption growth measure for services and non-durables from the Bureau of Economic Analysis. *S-K* and $NonD$-K are the real aggregate consumption growth measures for services and non-durables, respectively. Ult is the Parker and Julliard (2003) three-year measure of ultimate consumption growth. $Q4$ is the fourth quarter to fourth quarter consumption growth measure for non-durables and services of Jagannathan and Wang (2007). $Q4 - NonD$ is the fourth quarter to fourth quarter consumption growth measure for non-durables. $SNonD$-U and $NonD$-U are the unfiltered real aggregate consumption growth measures for services and non-durables, and non-durables, respectively, based on the methodology of Kroencke (2017). $Q4$-U and $Q4NonD$-U are respectively the unfiltered fourth quarter to fourth quarter consumption growth measures for non-durables and services and non-durables, respectively, also from Kroencke (2017). The data for the consumption measures *S-K*, $NonD$-K, Ult, $Q4$, $Q4 - NonD$, $SNonD$-U, $NonD$-U, $Q4$-U, and $Q4NonD$-U is from Tim Kroencke's website. Gbg is the garbage-based consumption growth measure of Savov (2011). The data for the garbage measure is from the U.S. Environment Protection Agency (EPA). Div is the real per capita aggregate divided growth. Dividend data is from Shiller's website (1965-2015) and Standard and Poor's (2016-2017). To maximize the correlation with the stock market according the estimated correlations from Table 1, we use the beginning of period alignment convention between consumption growth and asset returns for all consumption growth measures other than Ult, $Q4NonD$, $Q4$-U, and $Q4NonD$-U. χ^2_1, dof_1, and p_1 are the first-stage χ^2-test, degrees of freedom, and p-value that all moment conditions are jointly zero. χ^2_2, dof_2, and p_2 are the first-stage χ^2-test, degrees of freedom, and p-value that the moment conditions for the cross-section of the eight equity portfolios and the stock market are jointly zero. R^2 and $rmspe$ are the cross-sectional r-square and root-mean-square pricing error for the risk premia of the stock market and the cross-section of portfolios. The sample period is from 1964 to 2016.

ARMA(1,1) consumption growth, $\Delta c_t = \mu_c + \phi_c \Delta c_{t-1} + \theta_c \epsilon_{t-1} + \epsilon_t$

	SNonD	S-K	NonD-K	Ult	Q4	Q4NonD	SNonD-U	NonD-U	Q4-U	Q4NonD-U	Gbg	Div
γ	3.16	2.83	2.86	0.77	3.26	2.71	4.86	2.27	5.04	2.11	23.82	0.63
	(2.14)	(0.69)	(1.16)	(3.40)	(1.40)	(0.12)	(0.27)	(0.27)	(0.40)	(0.07)	(1.89)	(1.10)
β	1.05	1.02	1.04	1.03	1.04	1.02	1.07	1.01	1.07	1.01	0.97	0.99
	(31.39)	(20.09)	(19.44)	(79.28)	(24.71)	(3.59)	(3.98)	(10.66)	(4.91)	(2.77)	(6.90)	(111.28)
μ_c	1.43%	1.76%	0.54%	1.27%	1.35%	2.27%	0.57%	0.47%	2.72%	2.38%	0.39%	0.98%
	(1.39)	(1.20)	(0.65)	(1.79)	(1.14)	(0.39)	(0.12)	(0.07)	(0.65)	(0.32)	(2.28)	(0.86)
σ_c	1.05%	1.38%	1.00%	1.48%	1.26%	1.97%	2.32%	1.00%	2.43%	2.76%	3.30%	5.28%
	(5.02)	(4.38)	(4.52)	(5.55)	(5.59)	(3.54)	(4.83)	(5.33)	(4.62)	(2.27)	(3.39)	(1.99)
ϕ_c	0.34	-0.31	0.71	0.76	0.26	-0.68	0.65	0.61	-0.43	-0.78	0.60	0.23
	(0.73)	(-0.28)	(1.93)	(5.98)	(0.42)	(-0.15)	(0.36)	(0.25)	(-0.20)	(-0.14)	(0.05)	(0.42)
θ_c	0.21	0.81	-0.21	0.37	0.21	0.96	-0.80	-0.85	0.57	0.97	-0.62	0.36
	(0.39)	(0.51)	(-0.54)	(1.24)	(0.28)	(0.19)	(-0.43)	(-0.32)	(0.29)	(0.16)	(-0.05)	(0.41)
χ^2_1	38.43	25.23	34.26	41.88	32.40	11.96	17.27	15.11	14.62	14.34	14.69	40.65
dof_1	9	9	9	9	9	9	9	9	9	9	9	9
p_1	0	0	0	0	0	0.21	0.04	0.08	0.10	0.11	0.09	0
χ^2_2	38.43	25.23	34.26	41.88	32.40	11.96	17.27	15.11	14.62	14.34	14.70	40.65
dof_2	9	9	9	9	9	9	9	9	9	9	9	9
p_1	0	0	0	0	0	0.21	0.04	0.08	0.10	0.11	0.09	0
R^2	-634.42%	-610.21%	-657.11%	-677.91%	-634.10%	-607.97%	-495.68%	-590.57%	-514.23%	-577.73%	-163.08%	-628.89%
rmspe	7.79%	7.66%	7.91%	8.02%	7.79%	7.64%	7.01%	7.55%	7.11%	7.48%	4.64%	7.76%

References

Asparouhova, E., H. Bessembinder, and I. Kalcheva (2013). Noisy prices and inference regarding returns. *Journal of Finance 68*(2), 665–714.

Baker, M., S. Nagel, and J. Wurgler (2007). The effect of dividends on consumption. *Brookings Papers on Economic Activity 2007*(1), 231–276.

Breeden, D. T. (1979). An intertemporal asset pricing model with stochastic consumption and investment opportunities. *Journal of Financial Economics 7*(3), 265–96.

Breeden, D. T., M. Gibbons, and R. Litzenberger (1989). Empirical tests of the consumption-oriented CAPM. *Journal of Finance 44*(2), 231–62.

Campbell, J. (2003). Consumption-based asset pricing. In *Handbook of the economics of finance*, Volume 1B, pp. 801–85. Amsterdam: Elsevier.

Campbell, J. Y. and R. J. Shiller (1988). Stock prices, earnings, and expected dividends. *Journal of Finance 43*(3), 661–76.

Choi, S., R. Fisman, D. Gale, and S. Kariv (2007). Consistency and heterogeneity of individual behavior under uncertainty. *American Economic Review 97*(5), 1921–38.

Cochrane, J. (2001). *Asset pricing*. Princeton, NJ: Princeton University Press.

Cochrane, J. H. (1996). A cross-sectional test of an investment-based asset pricing model. *Journal of Political Economy 104*(3), 572?621.

Constantinides, G. M. and D. Duffie (1996). Asset pricing with heterogeneous consumers. *Journal of Political Economy 104*, 219–240.

Croce, M. M., M. Lettau, and S. C. Ludvigson (2015). Investor information, long-run risk, and the term structure of equity. *The Review of Financial Studies 28*(3), 706–742.

Delikouras, S. (2017). Where's the kink? disappointment events in consumption growth and equilibrium asset prices. *Review of Financial Studies 30*(8), 2851–2889.

Epstein, L. G. and S. E. Zin (1989). Substitution, risk aversion, and the temporal behavior of consumption and asset returns. *Econometrica 57*(4), 937–69.

Fama, E. F. and K. R. French (1993). Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics 33*(1), 3–56.

Fama, E. F. and K. R. French (2015). A five-factor asset pricing model. *Journal of Financial Economics 116*(1), 1–22.

Hall, R. (1988). Intertemporal elasticity of consumption. *Journal of Political Economy 96*(2), 339–57.

Hansen, L. P. and R. Jagannathan (1997). Assessing specification errors in stochastic discount factor models. *Journal of Finance 52*(2), 557–90.

Hansen, L. P. and K. J. Singleton (1982). Generalized instrumental variables estimation of non-linear rational expectations models. *Econometrica 50*(5), 1269–86.

Harvey, C. R., Y. Liu, and H. Zhu (2015). ... and the cross-section of expected returns. *Review of Financial Studies 29*(1), 5–68.

Hou, K., C. Xue, and L. Zhang (2019). Which factors? *Review of Finance 23*(1), 1–35.

Jagannathan, R. and Y. Wang (2007). Lazy investors, discretionary consumption, and the cross-section of stock returns. *Journal of Finance 62*(4), 540–79.

Kimball, M. S. (2002). Precautionary saving in the small and in the large. *Econometrica 58*(1), 53–73.

Kleibergen, F. and Z. Zhan (2020). Robust inference for consumption-based asset pricing. *Journal of Finance 75*(1), 507–550.

Kocheracota, N. (1996). The equity premium: It's still a puzzle. *Journal of Economic Literature 34*(1), 42–71.

Kroencke, T. A. (2017). Asset pricing without garbage. *Journal of Finance 72*(1), 47–98.

Liu, L. X., T. M. Whited, and L. Zhang (2009). Investment-based expected stock returns. *Journal of Political Economy 117*, 1105–39.

Lucas, Jr., R. E. (1978). Asset prices in an exchange economy. *Econometrica 46*(6), 1429–45.

Lustig, H., S. Van Nieuwerburgh, and A. Verdelhan (2013). The wealth-consumption ratio. *Review of Asset Pricing Studies 3*(1), 38–94.

Malloy, C. J., T. J. Moskowitz, and A. Vissing-Jorgensen (2009). Long-run stockholder consumption risk and asset returns. *Journal of Finance 64*, 2427–479.

Mehra, R. and E. C. Prescott (1985). The equity premium: A puzzle. *Journal of Monetary Economics 15*(2), 145–61.

Parker, J. A. and C. Julliard (2003). Consumption risk and the cross section of expected returns. *Journal of Political Economy 113*(1), 185–222.

Rabin, M. (2000). Risk aversion and expected-utility theory: a calibration theorem. *Econometrica 68*(5), 1281–92.

Savov, A. (2011). Asset pricing with garbage. *Journal of Finance 76*(1), 177–201.

Vissing-Jorgensen, A. (2002). Limited asset market participation and the elasticity of intertemporal substitution. *Journal of Political Economy 110*(4), 825–53.

Yogo, M. (2006). A consumption-based explanation of expected stock returns. *Journal of Finance 61*(2), 539–80.

Figures

Figure 1 Cross-Sectional Fit of Epstein-Zin Model

This figure shows sample and fitted moments for the Epstein-Zin model with ARMA(1,1) consumption dynamics across the various consumption measures. We consider the benchmark aggregate consumption growth from the BEA ($SNonD$), the unfiltered fourth-quarter non-durable ($Q4NonD - U$) consumption process from Kroencke (2017), the garbage (Gbg) measure of Savov (2011), and the aggregate dividend process (Div). The set of test moments includes the stock market risk premium ($\mathbb{E}[R_{m,t} - R_{f,t}]$), the cross section of risk premia for eight equity portfolios (high and low for size, book-to-market, investment, and profitability), the mean and volatility of the risk-free rate ($\mathbb{E}[R_{f,t}] - 1$, $vol(r_{f,t})$), as well as the mean ($\mathbb{E}[\Delta c_{t+1}]$) and volatility ($vol(\Delta c_{t+1})$) of consumption growth. The model-implied risk premia are given in equation (18), the fitted average risk-free rate is estimated according to equation (6), and the fitted volatility of the log risk-free rate (is estimated according to the square root of equation (8). Estimation results are shown in Table 4. The sample is from 1964 to 2016.

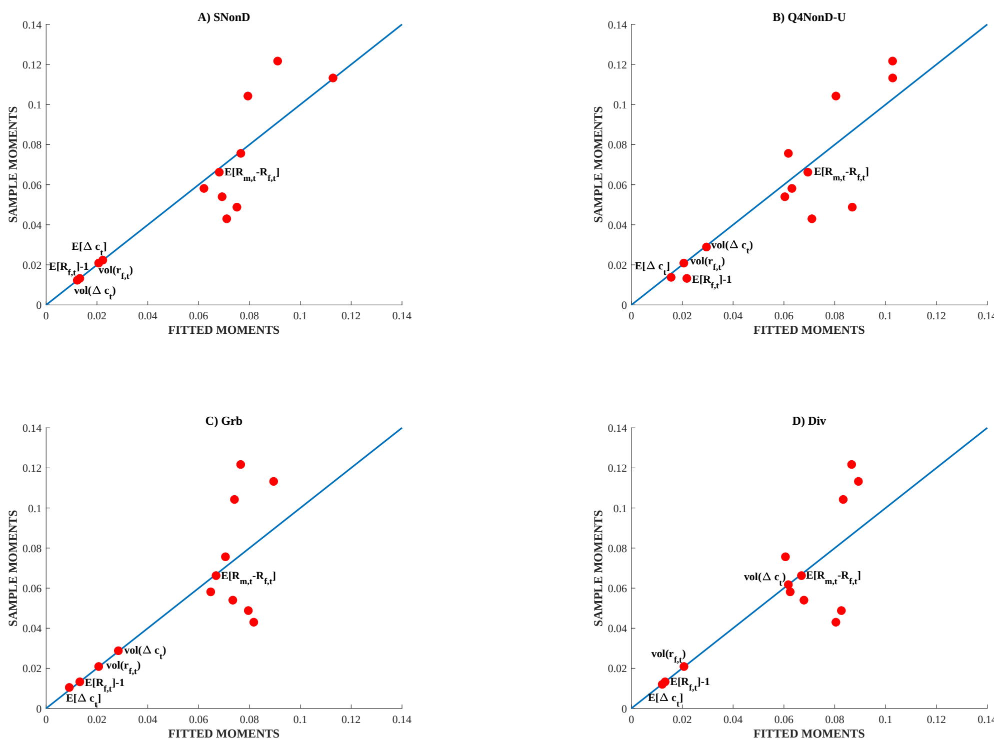
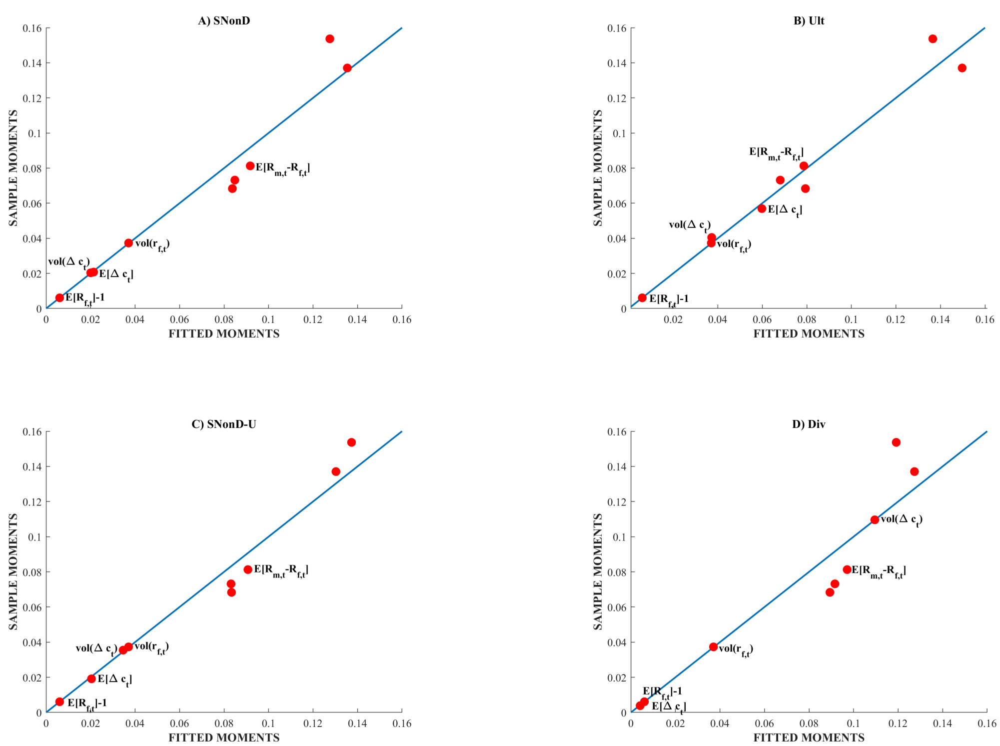


Figure 2 Cross-Sectional Fit of Epstein-Zin Model: 1930 to 2016

This figure shows sample and fitted moments for the Epstein-Zin model with ARMA(1,1) consumption dynamics across the various consumption measures. We consider the benchmark aggregate consumption growth from the BEA ($SNonD$), the unfiltered fourth-quarter non-durable ($Q4NonD - U$) consumption process from Kroencke (2017), the garbage (Gbg) measure of Savov (2011), and the aggregate dividend process (Div). The set of test moments includes the stock market risk premium ($\mathbb{E}[R_{m,t} - R_{f,t}]$), the cross section of risk premia for eight equity portfolios (high and low for size, book-to-market, investment, and profitability), the mean and volatility of the risk-free rate ($\mathbb{E}[R_{f,t}] - 1$, $vol(r_{f,t})$), as well as the mean ($\mathbb{E}[\Delta c_{t+1}]$)) and volatility ($vol(\Delta c_{t+1})$) of consumption growth. The model-implied risk premia are given in equation (18), the fitted average risk-free rate is estimated according to equation (6), and the fitted volatility of the log risk-free rate (is estimated according to the square root of equation (8). Estimation results are shown in Table 6. The sample is from 1930 to 2016



Tables

Table 1 Summary Statistics for Consumption Growth Measures

This table reports means, standard deviations, and consumption dynamics estimates, as well as cross-correlations for the various measures of real aggregate log-consumption growth. AR1 is the autoregressive coefficient in an AR(1) model for log-consumption dynamics. AR1-ARMA(1,1) and MA1-ARMA(1,1) are the autoregressive and moving average coefficients in an ARMA(1,1) model for log-consumption dynamics. The autoregressive parameters are estimated via conditional maximum-likelihood. Panels A and B show annual results for the 1965-2017 period, Panels C and D show annual results for the 1931-2017 sample. *SNonD* is the benchmark real aggregate consumption growth measure for services and non-durables from the Bureau of Economic Analysis. *S-K* and *NonD-K* are the real aggregate consumption growth measures for services and non-durables, respectively. *Ult* is the Parker and Julliard (2003) three-year measure of ultimate consumption growth. *Q4* is the fourth quarter to fourth quarter consumption growth measure for non-durables and services of Jagannathan and Wang (2007). $Q4 - NonD$ is the fourth quarter to fourth quarter consumption growth measure for non-durables. *SNonD-U* and *NonD-U* are the unfiltered real aggregate consumption growth measures for services and non-durables, and non-durables, respectively, from Kroencke (2017). *Q4-U* and *Q4NonD-U* are respectively the unfiltered fourth quarter to fourth quarter consumption growth measures for non-durables and services, and non-durables, respectively. The data for the consumption measures *S-K*, *NonD-K*, *Ult*, *Q4*, $Q4 - NonD$, *SNonD-U*, *NonD-U*, *Q4-U*, and *Q4NonD-U* is from Tim Kroencke's website. *Gbg* is the garbage-based consumption growth measure of Savov (2011). The data for the garbage measure is from the U.S. Environment Protection Agency (EPA). *Div* is the real per capita aggregate divided growth. Dividend data is from Shiller's website (1931-2015) and Standard and Poor's (2016-2017). In calculating the correlations of the various consumption growth measures with stock market excess returns, the beginning-of-period convention (beg.) aligns date t consumption growth with date $t-1$ excess stock market return. The end of the period convention (end) aligns date t consumption growth with date t excess market return.

Panel A: Consumption growth measures 1965-2017

	SNonD	S-K	NonD-K	Ult	Q4	Q4NonD	SNonD-U	NonD-U	Q4-U	Q4NonD-U	Gbg	Div
mean	2.23%	1.34%	2.12%	5.39%	1.83%	1.34%	1.79%	1.32%	1.79%	1.33%	1.04%	1.19%
st. deviation	1.24%	1.59%	1.28%	2.80%	1.40%	2.00%	2.45%	2.64%	2.34%	2.85%	2.90%	6.23%
AR1	0.47	0.39	0.55	0.81	0.39	0.20	0.02	0.04	0.04	-0.01	-0.02	0.48
AR1-ARMA (1,1)	0.33	-0.32	0.69	0.75	0.26	-0.72	0.65	0.61	-0.44	-0.81	0.82	0.23
MA1-ARMA (1,1)	0.20	0.81	-0.23	0.38	0.20	0.99	-0.80	-0.99	0.59	0.99	-0.82	0.36
market correl. (beg.)	0.39	0.47	0.25	0.15	0.32	0.33	0.43	0.42	0.21	0.20	0.58	0.48
market correl. (end)	0.04	0.16	-0.06	0.17	0.27	0.41	0.20	0.32	0.42	0.50	-0.18	0.03

Panel B: Cross-correlations of consumption growth measures 1965-2017

	SNonD	S-K	NonD-K	Ult	Q4	Q4NonD	SNonD-U	NonD-U	Q4-U	Q4NonD-U	Gbg
S-K	0.88										
NonD-K	0.88	0.68									
Ult	0.69	0.61	0.71								
Q4	0.84	0.78	0.84	0.75							
Q4NonD	0.71	0.83	0.56	0.61	0.86						
SNonD-U	0.77	0.75	0.74	0.61	0.87	0.79					
NonD-U	0.72	0.88	0.51	0.54	0.75	0.89	0.84				
Q4-U	0.56	0.54	0.58	0.61	0.88	0.81	0.83	0.67			
Q4NonD-U	0.51	0.64	0.40	0.51	0.77	0.94	0.72	0.82	0.84		
Gbg	0.57	0.52	0.49	0.32	0.40	0.30	0.40	0.41	0.18	0.15	
Div	0.22	0.29	0.03	-0.01	0.09	0.15	0.15	0.18	0.04	0.05	0.18

Panel C: Consumption growth measures 1931-2017

	SNonD	S-K	NonD-K	Ult	SNonD-U	NonD-U	Div
mean	2.07%	1.38%	2.13%	5.69%	1.91%	1.45%	0.38%
st. dev.	2.04%	2.46%	2.00%	4.07%	3.56%	3.74%	11.02%
AR1	0.44	0.32	0.56	0.69	-0.03	-0.04	0.19
AR1-ARMA(1,1)	0.48	-0.24	0.63	0.66	0.37	0.34	-0.13
MA1-ARMA(1,1)	-0.06	0.67	-0.13	0.42	-0.60	-0.54	0.41
market correl. (beg.)	0.60	0.60	0.53	0.38	0.60	0.55	0.51
market correl. (end)	0.12	0.18	0.08	0.38	0.25	0.29	-0.02

Panel D: Cross-correlations of consumption growth measures 1931-2017

	SNonD	S-K	NonD-K	Ult	SNonD-U	NonD-U
S-K	0.91					
NonD-K	0.88	0.68				
Ult	0.67	0.61	0.68			
SNonD-U	0.81	0.83	0.69	0.61		
NonD-U	0.77	0.90	0.52	0.53	0.92	
Div	0.50	0.47	0.39	0.19	0.40	0.38

Table 2 Summary Statistics for Asset Returns

This table reports summary statistics for the asset returns used in this study. The test assets consist of the risk-free asset, the aggregate stock market, and the high and low portfolios from four cross-sections of ten value-weighted portfolios sorted on size (sz), book-to-market (bm), investment (inv), and profitability (op). Asset returns are from Kenneth French's website. The risk-free rate is deflate by the growth in the PCE price index. Panel A shows summary statistics (mean and standard deviation) for asset returns in excess of the risk-free rate over the 1964-2016 period. In Panel B, the sample is from 1930 to 2016.

Panel A: Asset returns 1964-2016

	risk-free	market	sz1	sz10	bm1	bm10	inv1	inv10	op1	op10
mean	1.32%	6.62%	11.32%	5.81%	5.39%	12.17%	10.42%	4.29%	4.87%	7.56%
standard deviation	2.11%	17.62%	31.61%	17.13%	20.87%	23.39%	22.66%	24.80%	26.91%	19.02%

Panel B: Asset returns 1930-2016

	risk-free	market	sz1	sz10	bm1	bm10
mean	0.60%	8.12%	15.36%	7.31%	6.83%	13.70%
standard deviation	3.75%	20.27%	39.44%	19.10%	21.52%	33.10%

Table 3 GMM Estimation with CRRA Preferences

This table reports GMM results for the CRRA model of equation (1) for various annual consumption measures and alternative assumptions for consumption growth dynamics. In Panel A, we assume that log-consumption growth is i.i.d. and in Panel B, log-consumption growth is an AR(1) process. The test assets consist of the eight high and low portfolios from four cross-sections of value-weighted deciles independently sorted on size, book-to-market, investment, and profitability. In Panel A, we estimate the CRRA model using the over-identified GMM system in equation (15) that includes the risk premia for the test assets, the stock market risk premium, the consumption growth moments, and the mean of the risk-free rate. In Panel B, we estimate the CRRA model using the over-identified GMM system in equation (11) that also includes the variance of the log risk-free rate. γ is the risk-aversion parameter, which in the CRRA case is also the inverse of the EIS parameter, and β is the rate of time preference. μ_c is the constant term, σ_c is the volatility of the consumption growth shocks ϵ_t, ϕ_c is the autoregressive coefficient, and θ_c is the moving average coefficient. $SNonD$ is the benchmark real aggregate consumption growth measure for services and non-durables from the Bureau of Economic Analysis. S-K and $NonD$-K are the real aggregate consumption growth measures for services and non-durables, respectively. Ult is the Parker and Julliard (2003) three-year measure of ultimate consumption growth. $Q4$ is the fourth quarter to fourth quarter consumption growth measure for non-durables and services of Jagannathan and Wang (2007). $Q4 - NonD$ is the fourth quarter to fourth quarter consumption growth measure for non-durables. $SNonD$-U and $NonD$-U are the unfiltered real aggregate consumption growth measures for services and non-durables, and non-durables, respectively, based on the methodology of Kroencke (2017). $Q4$-U and $Q4NonD$-U are respectively the unfiltered fourth quarter to fourth quarter consumption growth measures for non-durables and services and non-durables, respectively, also from Kroencke (2017). The data for the consumption measures S-K, $NonD$-K, Ult, $Q4$, $Q4 - NonD$, $SNonD$-U, $NonD$-U, $Q4$-U, and $Q4NonD$-U is from Tim Kroencke's website. Gbg is the garbage-based consumption growth measure of Savov (2011). The data for the garbage measure is from the U.S. Environment Protection Agency (EPA). Div is the real per capita aggregate divided growth. Dividend data is from Shiller's website (1965-2015) and Standard and Poor's (2016-2017). To maximize the correlation with the stock market based on the estimated correlations from Table 1, we use the beginning of period alignment convention between consumption growth and asset returns for all consumption growth measures other than Ult, $Q4NonD$, $Q4$-U, and $Q4NonD$-U. χ^2_1, dof_1, and p_1 are the first-stage χ^2-test, degrees of freedom, and p-value that all moment conditions are jointly zero. χ^2_2, dof_2, and p_2 are the first-stage χ^2-test, degrees of freedom, and p-value that the moment conditions for the cross-section of the eight equity portfolios and the stock market are jointly zero. R^2 and $rmspe$ are the cross-sectional r-square and root-mean-square pricing error for the risk premia of the stock market and the cross-section of portfolios. The sample period is from 1964 to 2016.

Panel A: i.i.d. log consumption growth, $\Delta c_t = \mu_c + \epsilon_t$; GMM does not include the variance of the log risk-free rate

	SNonD	S-K	NonD-K	Ult	Q4	Q4NonD	SNonD-U	NonD-U	Q4-U	Q4NonD-U	Gbg	Div
γ	55.99	36.94	74.24	60.94	65.24	30.62	26.38	23.72	34.59	19.89	17.26	7.48
	(2.00)	(2.23)	(1.73)	(1.14)	(1.90)	(1.99)	(2.20)	(2.23)	(1.77)	(2.09)	(2.17)	(2.09)
β	2.66	1.32	3.06	5.69	2.17	1.21	1.26	1.06	1.37	1.08	1.03	0.93
	(2.23)	(7.62)	(1.96)	(1.81)	(3.30)	(9.90)	(8.69)	(10.19)	(7.97)	(13.97)	(17.83)	(18.12)
μ_c	2.23%	1.34%	2.12%	5.55%	1.83%	1.37%	1.79%	1.32%	1.88%	1.37%	1.04%	1.19%
	(13.21)	(6.20)	(12.14)	(13.96)	(9.60)	(4.95)	(5.36)	(3.67)	(5.70)	(3.47)	(2.63)	(1.41)
σ_c	1.22%	1.55%	1.26%	2.88%	1.38%	2.00%	2.43%	2.61%	2.39%	2.88%	2.86%	6.16%
	(5.13)	(4.19)	(6.03)	(5.45)	(5.13)	(4.23)	(4.71)	(3.83)	(5.74)	(4.83)	(4.08)	(2.65)
χ^2_1	6.99	17.07	7.70	6.93	14.17	24.65	15.90	16.26	15.19	21.77	17.80	20.22
dof_1	8	8	8	8	8	8	8	8	8	8	8	8
p_1	0.53	0.02	0.46	0.54	0.07	0	0.04	0.03	0.05	0	0.02	0
χ^2_2	6.99	17.07	7.70	6.93	14.17	24.65	15.90	16.26	15.19	21.77	17.80	20.22
dof_2	8	8	8	8	8	8	8	8	8	8	8	8
p_2	0.53	0.02	0.46	0.54	0.07	0	0.04	0.03	0.05	0	0.02	0
R^2	55.07%	56.62%	32.26%	-115.42%	47.39%	54.55%	14.55%	44.76%	42.11%	50.61 %	8.22%	27.17%
rmspe	1.86%	1.83%	2.29%	4.09%	2.02%	1.88%	2.57%	2.07%	2.12%	1.96%	2.67%	2.38%

Panel B: AR(1) consumption growth, $\Delta c_t = \mu_c + \phi_c \Delta c_{t-1} + \epsilon_t$

	SNonD	S-K	NonD-K	Ult	Q4	Q4NonD	SNonD-U	NonD-U	Q4-U	Q4NonD-U	Gbg	Div
γ	4.19	3.50	3.27	1.07	4.17	5.63	26.68	23.19	33.95	20.75	18.09	0.70
	(2.65)	(2.60)	(3.26)	(3.51)	(2.20)	(1.20)	(2.06)	(1.88)	(1.72)	(2.21)	(2.02)	(2.13)
β	1.08	1.03	1.05	1.04	1.06	1.05	1.26	1.06	1.37	1.08	1.03	0.99
	(27.31)	(48.43)	(45.38)	(55.92)	(28.05)	(16.46)	(8.70)	(10.10)	(7.89)	(11.36)	(11.63)	(130.32)
μ_c	1.32%	0.84%	1.02%	1.29%	1.18%	1.18%	1.69%	1.34%	1.88%	1.56%	1.19%	0.61%
	(3.81)	(3.11)	(3.47)	(2.58)	(3.63)	(3.19)	(4.01)	(2.25)	(3.20)	(1.61)	(1.22)	(0.77)
σ_c	1.05%	1.41%	1.00%	1.61%	1.26%	1.95%	2.43%	2.61%	2.39%	2.88%	2.86%	5.39%
	(4.66)	(3.85)	(5.26)	(5.77)	(5.47)	(4.57)	(4.73)	(3.85)	(5.74)	(4.29)	(3.82)	(2.02)
ϕ_c	0.41	0.37	0.51	0.76	0.36	0.18	0.03	0.03	0.02	-0.03	-0.03	0.47
	(3.38)	(3.39)	(5.01)	(10.82)	(2.69)	(1.33)	(2.22)	(2.09)	(1.72)	(-2.21)	(-1.96)	(3.12)
χ^2_1	42.77	43.19	44.86	44.94	45.64	31.36	15.88	16.25	15.28	22.79	17.50	42.23
dof_1	9	9	9	9	9	9	9	9	9	9	9	9
p_1	0	0	0	0	0	0	0.06	0.06	0.08	0	0.04	0
χ^2_2	42.77	43.19	44.86	44.94	45.64	31.36	15.88	16.25	15.28	22.79	17.50	42.23
dof_2	9	9	9	9	9	9	9	9	9	9	9	9
p_1	0	0	0	0	0	0	0.06	0.06	0.08	0	0.04	0
R^2	-615.07%	-590.80%	-651.81%	-671.67%	-617.42%	-514.90%	13.05%	42.78%	41.30%	48.44%	5.19%	-621.55%
rmspe	7.68%	7.55%	7.88%	7.99%	7.70%	7.12%	2.58%	2.09%	2.12%	1.99%	2.69%	7.72%

Table 4 GMM Estimation with Epstein-Zin Preferences

This table reports GMM results for the Epstein-Zin model of equation (5) for various annual consumption measures and alternative assumptions for consumption growth dynamics. In Panel A, we assume that log-consumption growth is an AR(1) process and in Panel B, we assume that log-consumption growth is an ARMA(1,1) process. The test assets consist of the eight high and low portfolios from four cross-sections of value-weighted decile portfolios independently sorted on size, book-to-market, investment, and profitability. In both Panels, the set of test moments from equation (11) includes the risk premia for the test assets, the stock market risk premium, the mean of the risk-free rate, the consumption growth moments, and the variance of the log risk-free rate. γ is the risk-aversion parameter, ρ is the EIS coefficient, and $\tilde{\beta}$ is the effective rate of time preference. The constant κ_1 is the log-linearization constant that depends on the long-term average of the price-dividend ratio of the stock market. μ_c is the constant term, σ_c is the volatility of the consumption growth shocks ϵ_t, ϕ_c is the autoregressive coefficient, and θ_c is the moving average coefficient. $SNonD$ is the benchmark real aggregate consumption growth measure for services and non-durables from the Bureau of Economic Analysis. S-K and $NonD$-K are the real aggregate consumption growth measures for services and non-durables, respectively. Ult is the Parker and Julliard (2003) three-year measure of ultimate consumption growth. $Q4$ is the fourth quarter to fourth quarter consumption growth measure for non-durables and services of Jagannathan and Wang (2007). $Q4 - NonD$ is the fourth quarter to fourth quarter consumption growth measure for non-durables. $SNonD$-U and $NonD$-U are the unfiltered real aggregate consumption growth measures for services and non-durables, and non-durables, respectively, based on the methodology of Kroencke (2017). $Q4$-U and $Q4NonD$-U are respectively the unfiltered fourth quarter to fourth quarter consumption growth measures for non-durables and services and non-durables, respectively, also from Kroencke (2017). The data for the consumption measures S-K, $NonD$-K, Ult, $Q4$, $Q4 - NonD$, $SNonD$-U, $NonD$-U, $Q4$-U, and $Q4NonD$-U is from Tim Kroencke's website. Gbg is the garbage-based consumption growth measure of Savov (2011). The data for the garbage measure is from the U.S. Environment Protection Agency (EPA). Div is the real per capita aggregate divided growth. Dividend data is from Shiller's website (1965-2015) and Standard and Poor's (2016-2017). To maximize the correlation with the stock market based on the estimated correlations from Table 1, we use the beginning of period alignment convention between consumption growth and asset returns for all consumption growth measures other than Ult, $Q4NonD$, $Q4$-U, and $Q4NonD$-U. χ^2_1, dof_1, and p_1 are the first-stage χ^2-test, degrees of freedom, and p-value that all moment conditions are jointly zero. χ^2_2, dof_2, and p_2 are the first-stage χ^2-test, degrees of freedom, and p-value that the moment conditions for the cross-section of the eight equity portfolios and the stock market are jointly zero. R^2 and $rmspe$ are the cross-sectional r-square and root-mean-square pricing error for the risk premia of the stock market and the cross-section of portfolios. The sample period is from 1964 to 2016.

Panel A: AR(1) consumption growth, $\Delta c_t = \mu_c + \phi_c \Delta c_{t-1} + \epsilon_t$

	SNonD	S-K	NonD-K	Ult	Q4	Q4NonD	SNonD-U	NonD-U	Q4-U	Q4NonD-U	Gbg	Div
γ	34.40	27.27	37.87	23.15	61.17	24.97	26.56	23.53	33.91	19.43	16.97	4.29
	(1.94)	(2.06)	(1.82)	(1.79)	(1.68)	(1.82)	(2.13)	(2.15)	(1.75)	(1.79)	(1.84)	(1.88)
ρ	-2.57	-2.35	-2.10	-0.26	-2.90	-3.96	-40.77	-15.84	-20.59	-59.99	-28.24	0.30
	(-2.10)	(-1.88)	(-2.29)	(-0.81)	(-1.72)	(-1.03)	(-0.16)	(-0.36)	(-0.23)	(-0.06)	(-0.14)	(0.92)
$\tilde{\beta}$	1.63	1.12	1.56	1.58	1.52	1.14	1.27	1.05	1.36	1.08	1.03	0.92
	(3.73)	(8.74)	(3.97)	(3.30)	(3.48)	(9.03)	(7.10)	(8.79)	(6.34)	(10.91)	(14.32)	(21.76)
κ_1	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96
	(48.99)	(48.99)	(48.99)	(48.99)	(48.99)	(48.99)	(48.99)	(48.99)	(48.99)	(48.99)	(48.99)	(48.99)
μ_c	1.16%	0.80%	0.97%	1.74%	1.12%	1.09%	1.75%	1.26%	1.80%	1.39%	1.07%	0.60%
	(3.58)	(3.07)	(3.51)	(3.16)	(3.64)	(2.99)	(3.72)	(2.96)	(3.98)	(2.78)	(2.22)	(0.75)
σ_c	1.05%	1.41%	1.05%	1.64%	1.26%	1.95%	2.43%	2.61%	2.39%	2.88%	2.86%	5.40%
	(4.64)	(3.87)	(5.27)	(5.32)	(5.48)	(4.59)	(4.72)	(3.88)	(5.74)	(4.80)	(4.06)	(2.03)
ϕ_c	0.47	0.39	0.53	0.70	0.38	0.20	0.02	0.04	0.03	-0.01	-0.02	0.48
	(3.96)	(3.54)	(5.35)	(8.48)	(2.90)	(1.46)	(0.16)	(0.39)	(0.24)	(-0.06)	(-0.15)	(3.12)
χ^2_1	226.37	16.48	14.19	14.50	7.64	20.37	15.41	15.97	12.57	19.74	17.43	23.78
dof_1	8	8	8	8	8	8	8	8	8	8	8	8
p_1	0	0.03	0.07	0.06	0.46	0	0.05	0.04	0.12	0.01	0.02	0
χ^2_2	184.38	16.48	14.19	14.50	7.64	20.37	15.41	15.97	12.57	19.74	17.43	23.78
dof_2	8	8	8	8	8	8	8	8	8	8	8	8
p_1	0	0.03	0.07	0.06	0.46	0	0.05	0.04	0.12	0.01	0.02	0
R^2	53.25%	47.05%	-2.18%	-121.74%	20.63%	52.89%	15.14%	44.26%	41.83%	50.49%	8.41%	30.28%
rmspe	1.90%	2.02%	2.81%	4.15%	2.48%	1.91%	2.56%	2.08%	2.12%	1.96%	2.66%	2.32%

Panel B: ARMA(1,1) consumption growth, $\Delta c_t = \mu_c + \phi_c \Delta c_{t-1} + \theta_c \epsilon_{t-1} + \epsilon_t$

	SNonD	S-K	NonD-K	Ult	Q4	Q4NonD	SNonD-U	NonD-U	Q4-U	Q4NonD-U	Gbg	Div
γ	38.10	33.06	37.66	36.74	74.11	29.51	28.66	22.88	34.28	19.47	19.65	4.70
	(2.00)	(2.22)	(1.79)	(2.15)	(1.63)	(2.02)	(1.11)	(1.43)	(1.80)	(2.14)	(1.65)	(1.93)
ρ	-2.45	-1.86	-2.25	-0.02	-2.71	-1.62	-14.60	-13.92	-9.81	-5.81	-7.05	0.35
	(-2.32)	(-1.74)	(-2.28)	(-0.12)	(-1.67)	(-1.87)	(-0.19)	(-0.20)	(-0.25)	(-0.11)	(-0.65)	(1.11)
$\tilde{\beta}$	1.77	1.18	1.55	2.18	1.64	1.18	1.25	1.01	1.37	1.08	1.03	0.93
	(3.35)	(8.42)	(3.89)	(3.02)	(2.99)	(9.85)	(7.91)	(2.95)	(7.72)	(12.86)	(12.65)	(21.76)
κ_1	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96
	(48.99)	(48.99)	(48.99)	(48.99)	(48.99)	(48.99)	(48.99)	(48.93)	(48.99)	(48.99)	(48.99)	(48.99)
μ_c	1.53%	1.39%	0.90%	3.25%	1.48%	2.13%	0.45%	0.20%	3.13%	3.02%	0.31%	1.00%
	(2.32)	(2.41)	(2.25)	(4.91)	(2.17)	(3.73)	(2.05)	(1.22)	(3.12)	(3.75)	(0.62)	(0.93)
σ_c	1.05%	1.38%	1.00%	1.64%	1.26%	1.84%	2.37%	2.59%	2.37%	2.93%	2.83%	5.26%
	(4.82)	(4.69)	(5.17)	(4.91)	(5.50)	(4.83)	(4.39)	(3.69)	(5.62)	(5.21)	(4.04)	(2.04)
ϕ_c	0.30	-0.02	0.56	0.45	0.19	-0.56	0.69	0.75	-0.68	-0.93	0.65	0.17
	(1.04)	(-0.06)	(3.11)	(3.90)	(0.56)	(-2.02)	(5.02)	(4.41)	(-1.73)	(-3.70)	(1.68)	(0.62)
θ_c	0.22	0.54	-0.06	0.63	0.23	0.91	-0.73	-0.72	0.74	0.90	-0.72	0.42
	(0.74)	(1.09)	(-0.29)	(7.28)	(0.55)	(4.86)	(-2.49)	(-2.31)	(1.44)	(1.59)	(-2.50)	(1.89)
χ^2_1	12.14	16.93	274.59	9.95	13.81	4.69×10^3	11.12	10.11	1.58×10^{11}	22.86	1.09×10^4	23.49
dof_1	8	8	8	8	8	8	8	8	8	8	8	8
p_1	0.14	0.03	0	0.26	0.08	0	0.19	0.25	0	0	0	0
χ^2_2	12.03	16.93	276.75	9.95	13.81	5.09×10^3	11.12	10.11	1.57×10^{11}	22.86	1.17×10^4	23.49
dof_2	8	8	8	8	8	8	8	8	8	8	8	8
p_1	0.14	0.03	0	0.26	0.08	0.01	0.19	0.25	0.06	0	0	0
R^2	53.08%	48.33%	-0.71%	-335.63%	21.39%	54.94%	17.65%	45.80%	42.56%	49.45%	8.53%	25.33%
rmspe	1.91%	2.00%	2.79%	5.82%	2.47%	1.87%	2.53%	2.05%	2.11%	1.98%	2.66%	2.40%

Table 5 CRRA Preferences: Extended Sample Estimation

This table reports GMM results for the CRRA model of equation (1) for various annual consumption measures and alternative assumptions for consumption growth dynamics. In Panel A, we assume that log-consumption growth is i.i.d. and in Panel B, log-consumption growth is an AR(1) process. The test assets consist of the four high and low portfolios from two cross-sections of value-weighted deciles independently sorted on size and book-to-market. In Panel A, we estimate the CRRA model using the over-identified GMM system in equation (15) that includes the risk premia for the test assets, the stock market risk premium, the consumption growth moments, and the mean of the risk-free rate. In Panel B, we estimate the CRRA model using the over-identified GMM system in equation (11) that also includes the variance of the log risk-free rate. γ is the risk-aversion parameter, which in the CRRA cases is also the inverse of the EIS parameter, and β is the rate of time preference. μ_c is the constant term, σ_c is the volatility of the consumption growth shocks ϵ_t, and ϕ_c is the autoregressive coefficient. $SNonD$ is the benchmark real aggregate consumption growth measure for services and non-durables from the Bureau of Economic Analysis. S-K and $NonD$-K are the real aggregate consumption growth measures for services and non-durables, respectively. Ult is the Parker and Julliard (2003) three-year measure of ultimate consumption growth. $SNonD$-U and $NonD$-U are the unfiltered real aggregate consumption growth measures for services and non-durables, and non-durables, respectively, based on the methodology of Kroencke (2017). The data for the consumption measures S-K, $NonD$-K, Ult, $SNonD$-U, $NonD$-U is from Tim Kroencke's website. Div is the real per capita aggregate divided growth. Dividend data is from Shiller's website (1931-2015) and Standard and Poor's (2016-2017). To maximize the correlation with the stock market based on the estimated correlations from Table 1, we use the beginning of period alignment convention between consumption growth and asset returns for all consumption growth measures. χ^2_1, dof_1, and p_1 are the first-stage χ^2-test, degrees of freedom, and p-value that all moment conditions are jointly zero. χ^2_2, dof_2, and p_2 are the first-stage χ^2-test, degrees of freedom, and p-value that the moment conditions for the cross-section of five equity portfolios are jointly zero. R^2 and $rmspe$ are the cross-sectional r-square and root-mean-square pricing error for the risk premia of the stock market and the cross-section of portfolios. The sample period is from 1930 to 2016.

Panel A: CRRA preferences and i.i.d. consumption growth, $\Delta c_t = \mu_c + \epsilon_t$

	SNonD	S-K	NonD-K	Ult	SNonD-U	NonD-U	Div
γ	23.21	21.72	25.25	13.73	14.98	15.35	4.60
	(2.98)	(2.89)	(3.05)	(3.01)	(3.00)	(2.90)	(3.14)
β	1.35	1.11	1.40	1.72	1.09	1.00	0.83
	(7.22)	(12.90)	(7.20)	(4.77)	(14.27)	(17.82)	(16.09)
μ_c	2.08%	1.38%	2.13%	5.69%	1.91%	1.45%	
(9.51)	(5.27)	(9.98)	(13.11)	(5.03)	(3.66)	(0.32)	
σ_c	2.02%	2.45%	1.97%	4.04%	3.54%	3.71%	10.96%
	(3.13)	(3.44)	(3.55)	(3.40)	(3.68)	(3.83)	(3.07)
χ^2_1	4.01	3.90	3.25	2.92	3.58	3.66	4.80
dof_1	4	4	4	4	4	4	4
p_1	0.40	0.41	0.51	0.57	0.46	0.45	0.30
χ^2_2	4.01	3.90	3.25	2.92	3.75	3.66	4.80
dof_1	4	4	4	4	4	4	4
p_2	0.40	0.41	0.51	0.57	0.46	0.45	0.30
R^2	74.96%	80.59%	75.66%	95.46%	85.86%	84.63%	62.81%
rmspe	1.77%	1.56%	1.74%	0.75%	1.33%	1.39%	2.16%

Panel B: CRRA preferences and AR(1) consumption growth, $\Delta c_t = \mu_c + \phi_c \Delta c_{t-1} + \epsilon_t$

	SNonD	S-K	NonD-K	Ult	SNonD-U	NonD-U	Div
γ	4.50	4.87	3.60	1.71	16.45	16.33	1.73
	(2.28)	(2.09)	(2.79)	(3.44)	(3.44)	(2.47)	(1.43)
β	1.08	1.05	1.06	1.09	1.07	0.98	0.97
	(22.26)	(32.82)	(30.63)	(33.41)	(12.38)	(11.40)	(28.45)
μ_c	1.28%	1.01%	1.04%	2.13%	2.29%	1.71%	0.32%
	(3.19)	(3.57)	(2.40)	(3.48)	(3.40)	(1.70)	(0.27)
σ_c	1.79%	2.28%	1.58%	2.43%	3.56%	3.73%	10.70%
	(3.35)	(3.54)	(3.98)	(3.60)	(3.58)	(3.69)	(2.85)
ϕ_c	0.41	0.31	0.53	0.66	-0.06	-0.06	0.19
	(2.87)	(2.64)	(3.49)	(7.81)	(-3.73)	(-2.34)	(1.38)
χ^2_1	16.24	13.17	17.22	14.82	3.75	3.64	7.59
dof_1	5	5	5	5	5	5	5
p_1	0	0.02	0	0.01	0.58	0.60	0.17
χ^2_2	16.24	13.17	17.22	14.82	3.75	3.64	7.59
dof_1	5	5	5	5	5	5	5
p_2	0	0.02	0	0.01	0.58	0.60	0.17
R^2	-596.64%	-542.40%	-654.03%	-684.87%	73.31%	81.63%	-460.55%
rmspe	9.44%	9.06%	9.84%	10.05%	2.05%	1.72%	8.40%

Table 6   Epstein-Zin Preferences: Extended Sample

This table reports GMM results for the Epstein-Zin model of equation (5) for various annual consumption measures and alternative assumptions for consumption growth dynamics. In Panel A, we assume that log-consumption growth is an AR(1) process and in Panel B, we assume that log-consumption growth is an ARMA(1,1) process. The test assets consist of the four high and low portfolios from two cross-sections of value-weighted decile portfolios independently sorted on size and book-to-market. We estimate the Epstein-Zin specification using the over-identified GMM system in equation (11) that includes the risk premia for the test asstes, the stock market risk premium, the consumption growth moments, and the mean and variance of the log risk-free rate. γ is the risk-aversion parameter, rho is the EIS coefficient, β is the rate of time preference in the CRRA model, and $\tilde{\beta}$ is the effective rate of time preference in the Epstein-Zin discount factor. The constant κ_1 is the log-linearization constant that depends on the long-term average of the price-dividend ratio of the stock market. μ_c is the constant term, σ_c is the volatility of the consumption growth shocks ϵ_t, ϕ_c is the autoregressive coefficient, and θ_c is the moving average coefficient. $SNonD$ is the benchmark real aggregate consumption growth measure for services and non-durables from the Bureau of Economic Analysis. S-K and $NonD$-K are the real aggregate consumption growth measures for services and non-durables, respectively. Ult is the Parker and Julliard (2003) three-year measure of ultimate consumption growth. $SNonD$-U and $NonD$-U are the unfiltered real aggregate consumption growth measures for services and non-durables, and non-durables, respectively, based on the methodology of Kroencke (2017). The data for the consumption measures S-K, $NonD$-K, Ult, $SNonD$-U, $NonD$-U is from Tim Kroencke's website. Div is the real per capita aggregate divided growth. Dividend data is from Shiller's website (1931-2015) and Standard and Poor's (2016-2017). To maximize the correlation with the stock market based on the estimated correlations from Table 1, we use the beginning of period alignment convention between consumption growth and asset returns for all consumption growth measures. χ^2_1, dof_1, and p_1 are the first-stage χ^2-test, degrees of freedom, and p-value that all moment conditions are jointly zero. χ^2_2, dof_2, and p_2 are the first-stage χ^2-test, degrees of freedom, and p-value that the moment conditions for the cross-section of five equity portfolios are jointly zero. R^2 and $rmspe$ are the cross-sectional r-square and root-mean-square pricing error for the risk premia of the stock market and the cross-section of portfolios. The sample period is from 1930 to 2016.

Panel A: Epstein-Zin preferences and AR(1) consumption growth, $\Delta c_t = \mu_c + \phi_c \Delta c_{t-1} + \epsilon_t$

	SNonD	S-K	NonD-K	Ult	SNonD-U	NonD-U	Div
γ	17.59	17.82	16.91	25.34	14.62	15.14	4.01
	(2.30)	(2.51)	(2.24)	(3.12)	(2.76)	(2.77)	(3.04)
ρ	-3.17	-3.66	-2.36	-0.63	-31.08	-20.20	-0.70
	(-1.79)	(-1.66)	(-2.01)	(-1.45)	(-0.45)	(-0.52)	(-0.59)
$\tilde{\beta}$	1.20	1.04	1.18	1.18	1.08	0.99	0.84
	(7.20)	(14.30)	(7.00)	(1.94)	(13.91)	(19.54)	(17.86)
κ_1	0.95	0.95	0.95	0.95	0.95	0.95	0.95
	(42.81)	(42.81)	(42.81)	(42.81)	(42.81)	(42.81)	(42.81)
μ_c	1.18%	0.96%	0.95%	2.03%	1.97%	1.52%	0.28%
	(3.05)	(3.48)	(2.26)	(3.96)	(5.07)	(4.14)	(0.24)
σ_c	1.79%	2.30%	1.61%	2.45%	3.54%	3.71%	10.74%
	(3.37)	(3.56)	(4.01)	(3.67)	(3.68)	(3.81)	(2.89)
ϕ_c	0.44	0.32	0.56	0.67	-0.03	-0.04	0.19
	(3.09)	(2.71)	(3.71)	(9.20)	(-0.46)	(-0.54)	(1.39)
χ^2_1	3.70	3.85	3.27	1.52	3.58	3.68	5.16
dof_1	4	4	4	4	4	4	4
p_1	0.44	0.42	0.51	0.82	0.46	0.45	0.27
χ^2_2	3.70	3.85	3.27	1.52	3.58	3.68	5.16
dof_2	4	4	4	4	4	4	4
p_1	0.44	0.42	0.51	0.82	0.46	0.45	0.27
R^2	81.32%	82.75%	86.25%	81.92%	85.65%	84.50%	68.22%
rmspe	1.53%	1.47%	1.31%	1.50%	1.34%	1.39%	1.99%

Panel B: Epstein-Zin preferences and ARMA(1,1) consumption growth, $\Delta c_t = \mu_c + \phi_c \Delta c_{t-1} + \theta_c \epsilon_{t-1} + \epsilon_t$

	SNonD	S-K	NonD-K	Ult	SNonD-U	NonD-U	Div
γ	17.55	17.89	16.55	13.91	21.81	17.92	4.19
	(2.22)	(2.39)	(2.09)	(4.39)	(2.45)	(1.56)	(3.14)
ρ	-3.20	-3.46	-2.43	-0.24	-4.28	-6.53	-0.40
	(-2.04)	(-1.82)	(-1.97)	(-0.62)	(-1.18)	(-0.47)	(-0.50)
$\tilde{\beta}$	1.20	1.04	1.17	1.30	1.24	1.03	0.83
	(6.86)	(14.06)	(6.50)	(4.13)	(7.21)	(7.06)	(16.65)
κ_1	0.95	0.95	0.95	0.95	0.95	0.95	0.95
	(42.81)	(42.81)	(42.81)	(42.81)	(42.81)	(42.81)	(42.81)
μ_c	1.15%	1.08%	0.82%	2.57%	1.25%	1.10%	0.53%
	(1.35)	(1.14)	(1.48)	(2.54)	(2.59)	(1.17)	(0.30)
σ_c	1.79%	2.28%	1.58%	2.24%	3.38%	3.65%	10.63%
	(3.38)	(3.43)	(3.89)	(4.40)	(4.52)	(4.46)	(2.86)
ϕ_c	0.46	0.23	0.62	0.57	0.38	0.27	-0.29
	(1.19)	(0.35)	(2.68)	(3.83)	(1.63)	(0.43)	(-0.86)
θ_c	-0.02	0.11	-0.10	0.51	-0.57	-0.40	0.53
	(-0.06)	(0.15)	(-0.52)	(4.21)	(-1.90)	(-0.48)	(2.02)
χ^2_1	3.77	3.93	3.18	4.37	2.59	3.42	5.32
dof_1	4	4	4	4	4	4	4
p_1	0.43	0.41	0.52	0.35	0.62	0.48	0.25
χ^2_2	3.77	3.93	3.18	4.37	2.59	3.42	5.32
dof_2	4	4	4	4	4	4	4
p_1	0.43	0.41	0.52	0.35	0.62	0.48	0.25
R^2	81.42%	82.54%	86.27%	90.27%	88.42%	85.28%	63.12%
rmspe	1.52%	1.48%	1.31%	1.10%	1.20%	1.36%	2.15%