Systematic Risk and Exchange-rate exposure of pair arbitrage portfolios

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PRELIMINARY VERSION (14 June 2022)

Abstract

We analyse the exchange-rate exposure (exposure) and systematic risk (beta) of individual stocks and pair arbitrage portfolios of Bertrand-type competing firms in an international duopoly. As the currency appreciates the beta of the firm from the depreciating (appreciating) country, decreases (increases). As the home (foreign) market demand uncertainty decreases the exposure for the home (foreign) firm increases (decreases). The addition of a domestic competitor does not alter the results. The exposure of the pair arbitrage portfolio is positive and increases with decreasing home or foreign demand uncertainty, while the portfolio beta decreases with the exchange rate. A number of calibrating examples are investigated.

Keywords: Exchange-rate Exposure; Systematic risk; Pair arbitrage portfolios

JEL Classification: F23, L13, D21

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1. Introduction

Systematic risk or beta, holds a fundamental place in financial theory and practice. The literature includes highly influential studies like Subrahamanyam and Thomadakis (1980), Campbell and Vuolteenaho (2004), and Andersen *et al.* (2005) among others. Financial managers use betas in capital budgeting and investors in managing their portfolios. The importance of the topic leads to the objective of this article, which is to study the beta of individual stocks and pair arbitrage portfolios in an international duopoly and examine its relation with exchange rates.

In the consumable goods category, the Detergent market is dominated by the British Unilever (UN) and the US Procter & Gamble (P&G), and it is an example of an international duopoly. In the durable goods category, the Consumer Desktop Computer Microprocessor market dominated by two US firms, Intel and Advanced Micro Devices and the large jet airliner market dominated by Airbus and Boeing since the1990s, are examples of international duopolies.

In this article, we study exchange-rate exposure and systematic risk of individual stocks and pair arbitrage portfolios in an international duopoly of firms offering differentiated goods and there is demand uncertainty in both home and foreign market. Initially, we find that an increase in the exchange rate, that is, a currency depreciation in the home country, leads to a price increase by the home firm and a price decrease by the foreign firm. The home increases its price by taking advantage of the local currency depreciation and the foreign firm decreases its price to restore competitiveness. Additionally, as uncertainty decreases, the home and foreign equilibrium prices increase, namely, price competition is relaxed.

Concerning the exchange rate exposure and systematic risk of the individual stocks of each firm in each country we obtain the following results. As the currency appreciates the systematic risk of the firm from the currency depreciating (appreciating) country, decreases (increases). Moreover, as the home (foreign) market demand uncertainty decreases the exposure for the home (foreign) firm increases (decreases). The addition of a domestic competitor does not alter the results.

Concerning the pair arbitrage portfolio, we obtain that the exchange-rate exposure of the pair arbitrage portfolio is positive and increases when the home or foreign demand uncertainty decreases, and its systematic risk is decreasing in the exchange rate.[1] A series of calibrating examples are presented.

The rest of the article is organized as follows. Section 2 presents the related literature. In Section 3, we analyze the equilibrium outcome of the international duopoly and examine the exposure and beta of the individual stocks, whereas in Section 4 we examine the exposure and beta of the pair arbitrage portfolio. Section 5 concludes.

2. Related literature

Our work contributes to three strands of literature. The first one integrates the theory of the firm in product and financial markets (see O'Brien, 2011; Subrahmanyam and Thomadakis, 1980; Thomadakis, 1976). These studies use the Capital Asset Pricing Model (CAPM) to derive the relationship between product market characteristics and the firm's systematic risk. However, to our knowledge there is no study in this literature studying the impact of exchange rates on the firm's systematic risk.

The second strand studies arbitrage portfolios. A long/short equity is an investment strategy which entails buying (going long) equities that are likely to increase in value and selling short equities that are likely to decrease in value (Jacobs, et al., 1999). In an arbitrage portfolio the money from the short sales is used in the purchase of the long positions, so the portfolio costs nothing (Simon and Blume, 1994, p.119).

[1] A pair arbitrage portfolio is a long/short portfolio that costs nothing and includes two stocks of firms that produce close substitutes. See Section 2.

A pair portfolio contains two stocks of firms that produce close substitutes (see Gatev, et al., 2006). Gatev, et al., (2006) find that common factor exposures of stocks in the same industry make more likely to form pair portfolios of stocks. Hence, a pair arbitrage portfolio is a long/short portfolio that costs nothing and includes two stocks of firms that produce close substitutes. However, to our knowledge there is no study in the literature studying the impact of exchange rates on systematic risk and exchange-rate exposure (exposure) of pair arbitrage portfolios.

The third relevant strand of literature studies international oligopolies. The earliest competition models between domestic and foreign firms assume perfect substitutability: Krugman (1987), Froot and Klemperer (1989), and Yang (1997), while more recent models assume imperfect substitutability (Bodnar et al, 2002; Bartram et al., 2010). Moreover, most of the previous literature looks at exchange-rate pass-through (pass-through), the impact of the exchange rate on prices, not on profits; we study the impact on profits, firm values and arbitrage portfolios.

Bodnar et al. (2002) study pass-through and exposure in a model where the exporting firm cannot sell in its own market and the domestic firm cannot produce abroad. Bartram et al., (2010) extend the Bodnar et al. model by deriving pass-through in competitive industries where firms produce and compete in foreign and domestic markets. In their model exposure depends on market share, product substitutability, and pass-through.

Marston (2001) and Floden et al. (2008) emphasize the importance of market structure on the firms' pass-through and exposure. Marston (2001) also studies the case of exposure under uncertainty, where the uncertainty comes from the exchange rate only in the case of a monopoly, while we study demand uncertainty in the case of an oligopoly.

Froot and Klemperer (1989) develop a two period oligopoly model, where future exchange rates affect current market shares, and current international pricing. Their model has one foreign and one domestic firm competing in the domestic (U.S.) market with homogenous goods.

Gross and Schmitt (2000) use the switching cost model of Froot and Klemperer. Here too, there are two foreign firms serving a market with no home production under a Bertrand setting studying pass-through, but not exposure.

Finally, Andrikopoulos and Dassiou examine exposure in an international "Rule of Three" market structure that allows both within and between countries competition. They conclude that the addition of a domestic competitor increases the exposure of both international competitors relative to duopoly unless the pass-through of one of its rivals is elastic.

3. International duopoly

We examine an international Bertrand duopoly of firms with differentiated goods and linear demands. $\tilde{k}, (\tilde{u})$ is the demand uncertainty in the home (foreign) country.[2] As the foreign (home) currency appreciates (depreciates) the exchange rate (S) increases. The demand functions for the home (h) and the foreign firm (f) are:

$$\begin{bmatrix} \tilde{q}_h(P_h, P_f; S, \tilde{k}) \\ \tilde{q}_f(P_h, P_f; S, \tilde{u}) \end{bmatrix} = \begin{bmatrix} (1+\tilde{k})\theta_0 \\ (1+\tilde{u})\lambda_0 \end{bmatrix} + \begin{bmatrix} \theta_h & \theta_f S \\ \lambda_h \frac{1}{S} & \lambda_f \end{bmatrix} \begin{bmatrix} P_h \\ P_f \end{bmatrix} \tag{1}$$

with $E(\tilde{k}) = 0, Var(\tilde{k}) = \sigma_k^2, E(\tilde{u}) = 0, Var(\tilde{u}) = \sigma_u^2, Cov(\tilde{k}, \tilde{u}) = \rho_{k,u}\sigma_k\sigma_u$ and $\theta_h, \lambda_f < 0, 0 < \theta_f < |\theta_h|, 0 < \lambda_h < |\lambda_f|$, that is, the own effect in the demand functions (in absolute terms) is higher than the cross effect (of the other firm). Profits are given by:

$$\tilde{\Pi}_h = (P_h - c_h)\tilde{q}_h(P_h, P_f; S, \tilde{k}), \tilde{\Pi}_f = (P_f - c_f)\tilde{q}_f(P_h, P_f; S, \tilde{u}) \tag{2}$$

[2] Uncertainty at the constant term of the demand function (demand shocks) has been studied in Industrial Organization; see, Rey and Tirole (1986), Staiger and Wolak (1992), Reynolds and Wilson (2000).

where c_h and c_f are the constant marginal costs of h and f, respectively, with $0 < c_h < \theta_0$ and $0 < c_f < \lambda_0$. The random rate of return on firms' stock is:

$$\tilde{r}_h = \frac{\tilde{\Pi}_h}{v_h} - 1, \tilde{r}_f = \frac{\tilde{\Pi}_f}{v_f} - 1 \tag{3}$$

where the value of each firm v_h, v_f is the present value of the end of period profits. From the CAPM, we have:

$$\begin{aligned} E(\tilde{r}_h) &= RF_h + MP_h Cov(\tilde{r}_h, \tilde{r}_{mh}) \\ E(\tilde{r}_f) &= RF_f + MP_f Cov(\tilde{r}_f, \tilde{r}_{mf}) \end{aligned} \tag{4}$$

RF_h, RF_f are the risk-free rates and $MP_h \equiv \frac{E(\tilde{r}_{mh}) - RF_h}{Var(\tilde{r}_{mh})}$, $MP_f \equiv \frac{E(\tilde{r}_{mf}) - RF_f}{Var(\tilde{r}_{mf})}$ is the market price of risk in the home and foreign country, respectively.[3] $\tilde{r}_{mh}, \tilde{r}_{mf}$ is the random rate of return on the home and foreign market portfolio, respectively. We define $K \equiv 1 - MP_h Cov(\tilde{k}, \tilde{r}_{mh})$ and $U \equiv 1 - MP_f Cov(\tilde{u}, \tilde{r}_{mf})$ as the certainty equivalents of 1+$\tilde{k}$ and $1 + \tilde{u}$ respectively. In addition, throughout the paper, we assume $Cov(\tilde{k}, \tilde{r}_{mh}) > 0$ and $Cov(\tilde{u}, \tilde{r}_{mf}) > 0$, hence K and U are between zero and one.[4] As $MP_h(MP_f)$ increases, the certainty equivalent $K(U)$ decreases.

3.1 Equilibrium analysis

From (1), (2) and (3), we obtain:

$$\begin{aligned} Cov(\tilde{r}_h, \tilde{r}_{mh}) &= \frac{(P_h - c_h)\theta_0 Cov(\tilde{k}, \tilde{r}_{mh})}{v_h} \\ Cov(\tilde{r}_f, \tilde{r}_{mf}) &= \frac{(P_f - c_f)\lambda_0 Cov(\tilde{u}, \tilde{r}_{mf})}{v_f} \end{aligned} \tag{5}$$

[3] The Uncovered Interest rate Parity (UIP) is an equilibrium condition based on the absence of arbitrage. However, UIP requires perfect competition in traded goods, deep financial markets and free capital flows (see for instance Engel, 2016). UIP states that (see Feenstra, and Taylor, 2008): $(1 + RF_f) = \frac{S_{t+1}}{S_t}(1 + RF_h)$, which in an one year context collapses to: $RF_f = RF_h$. In what follows, we allow the two risk free rates to be different. In the calibration subsection we make these two equal to study the effects of a change in the exchange rate and in uncertainty.

[4] This is equivalent to assume that each firm has a positive beta; see expression (10).

The value of each firm v_h, v_f, by (1), (2), (3), (4) and (5) becomes:

$$v_h = \frac{(P_h - c_h)(\theta_0 K + \theta_h P_h + \theta_f S P_f)}{1 + RF_h}$$

$$v_f = \frac{\left(P_f - c_f\right)(\lambda_0 U + \lambda_h \frac{1}{S} P_h + \lambda_f P_f)}{1 + RF_f} \tag{6}$$

Each firm $i, i = h, f$ maximizes v_i with respect to (*wrt*) p_i, which gives the first order conditions (*FOCs*):

$$\begin{bmatrix}\theta_0 K \\ \lambda_0 U\end{bmatrix} + \begin{bmatrix}\theta_h & \theta_f S \\ \lambda_h \frac{1}{S} & \lambda_f\end{bmatrix}\begin{bmatrix}P_h \\ P_f\end{bmatrix} + \begin{bmatrix}\theta_h (P_h - c_h) \\ \lambda_f \left(P_f - c_f\right)\end{bmatrix} = \begin{bmatrix}0 \\ 0\end{bmatrix}$$

By solving *FOCs*, we obtain the equilibrium prices at this Bertrand game, P_h^D, P_f^D (the formulas are available upon request). By differentiation of P_h^D, P_f^D *wrt* S and by $0 < \theta_f < |\theta_h|, 0 < \lambda_h < \left|\lambda_f\right|, \lambda_f < 0, \theta_h < 0$ we obtain $dP_h^D/dS > 0, dP_f^D/dS < 0$.[5] An increase in *S* leads to a price increase (decrease) by firm *h* (*f*). Firm *h* increases its price by taking advantage of the local currency depreciation and firm *f* decreases its price to restore competitiveness. Pass-through refers to the firm's price elasticity *wrt S*, i.e. $\varepsilon_{P_i,S} \equiv \frac{dP_i}{dS}\frac{S}{P_i}$ and in equilibrium:[6]

$$0 < \varepsilon_{P_h^D,S} < 1, -1 < \varepsilon_{P_f^D,S} < 0 \tag{7}$$

The positive (negative) sign of pass-through of the home (foreign) firm is because in an international duopoly, there is only *between* (countries) competition. The sensitivity of the equilibrium prices in both countries to *S* is (in absolute terms for firm *f*) lower than one; hence pass-through is not elastic.

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[5] The proof is immediate.
[6] The proof is immediate.

Moreover, as uncertainty decreases, i.e. as K and U increase, the home and foreign equilibrium prices increase, namely, $dP_h^D/dK > 0, dP_f^D/dK > 0, dP_h^D/dU > 0, dP_f^D/dU > 0$.[7] In other, words, as demand uncertainty decreases, price competition is relaxed.

3.2 Exchange-rate exposure

We now examine the impact of S on the equilibrium values of the firms i.e. *exchange-rate exposure*. Inserting the equilibrium prices P_h^D, P_f^D into (6), we take the equilibrium values of each firm v_h^D, v_f^D and by differentiation of v_h^D, v_f^D *wrt S*, we obtain the exchange-rate exposures:

$$\frac{dv_h^D}{dS} = \frac{(P_h^D - c_h)}{1+RF_h}\theta_f P_f^D(1+\varepsilon_{P_f,S}) \tag{8a}$$
$$\frac{dv_f^D}{dS} = \frac{\left(P_f^D - c_f\right)}{(1+RF_f)S^2}\lambda_h P_h^D(\varepsilon_{P_h,S} - 1)$$

(9a) can be written as:

$$\frac{dv_h^D}{dS} = \frac{1}{1+RF_h}\frac{d\Pi_h^D}{dS} \tag{8b}$$
$$\frac{dv_f^D}{dS} = \frac{1}{1+RF_f}\frac{d\Pi_f^D}{dS}$$

where $\Pi_h^D = (P_h^D - c_h)(K\theta_0 + \theta_h P_h^D + \theta_f S P_f^D), \Pi_f^D = \left(P_f^D - c_f\right)(U\lambda_0 + \lambda_h \frac{1}{S} P_h^D + \lambda_f P_f^D)$

and $\frac{d\Pi_h^D}{dS}(\frac{d\Pi_f^D}{dS})$ measures the profit exposure for the home (foreign firm) under K and U. According to (8b), the value exposure of the firm (domestic or foreign) is the discounted profit exposure at the risk-free rate (domestic or foreign).

Lemma 1. Depreciation in the home country, leads to an increase (decrease) in the value of the home (foreign) firm, that is, $\frac{dv_h^D}{dS} > 0, \frac{dv_f^D}{dS} < 0$. Exchange-rate exposure is positive (negative) for the firm in the country where depreciation (appreciation) occurs.

Proof. Since $\varepsilon_{P_h^D,S} < 1$ and $|\varepsilon_{P_f^D,S}| < 1$, by (7) we derive the result in Lemma 1.■

[7] The proofs are immediate.

Under home country depreciation, firm f reduces its price but this reduction does not offset the increase in S (since $|\varepsilon_{P_f^D,S}| < 1$) leading to an increase in the value of firm h, i.e. positive $\frac{dv_h^D}{dS}$. On the other hand, firm h increases its price but at a level that does not restore its competitiveness gains via the increase in S (since $\varepsilon_{P_h^D,S} < 1$), thus, $\frac{dv_f^D}{dS}$ is negative. In other words, as the home currency depreciates, the home firm will gain at the expense of her overseas rival.[8]

The above results also cover the cases where one of the two firms is a monopoly in its home market and competes in the foreign market with the domestic firm there (with both products to be served in the foreign market), i.e. $\theta_f = 0$ and $\lambda_h > 0$, in this case, from (9a), $\frac{dv_h^D}{dS} = 0$ and $\frac{dv_f^D}{dS} \neq 0$, the exposure of the domestic (foreign) firm is (non) zero, or $\theta_f > 0$ and $\lambda_h = 0$, where, $\frac{dv_h^D}{dS} \neq 0$ and $\frac{dv_f^D}{dS} = 0$, the exposure of the foreign (domestic) firm is (non) zero.

Lemma

2a. As the home (foreign) market demand uncertainty decreases, the exchange-rate exposure for firm h (f) increases (decreases) that is, $\frac{d^2v_h^D}{dSdK} > 0, \left(\frac{d^2v_f^D}{dSdU} < 0\right)$.

2b. As the foreign (home) market demand uncertainty decreases, the exchange-rate exposure for firm h (f) increases (decreases) that is, $\frac{d^2v_h^D}{dSdU} > 0, (\frac{d^2v_f^D}{dSdK} < 0)$.

Proof. 2a. By differentiation of $\frac{dv_h^D}{dS}$ in (9a) *wrt K* and of $\frac{dv_f^D}{dS}$ in (9a) *wrt U* and by the use of (8), we have $\frac{d^2v_h^D}{dSdK} = \frac{4\theta_0\theta_f\theta_h\lambda_f^D\left(U\lambda_0 - \lambda_f^D c_f\right)}{(4\theta_h\lambda_f^D - \theta_f\lambda_h)^2(1+RF_h)} > 0$ and $\frac{d^2v_f^D}{dSdU} = -\frac{4\lambda_0\theta_h\lambda_f^D\lambda_h(K\theta_0 - \theta_h c_h)}{S^2(4\theta_h\lambda_f^D - \theta_f\lambda_h)^2(1+RF_f)} < 0$ since $\theta_h < 0,\ \lambda_f^D < 0, \theta_f > 0, \lambda_h > 0$. ■

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[8] Alternatively, $\frac{dv_f^D}{d(\frac{1}{S})} > 0$. As the foreign currency depreciates, the foreign firm will gain.

2b. By differentiation of $\frac{dv_h^D}{dS}$ *wrt* U and of $\frac{dv_f^D}{dS}$ in (8) *wrt* K, we have $\frac{d^2v_h^D}{dSdU} = \left(\frac{\theta_f}{1+RF_h}\right)\left(\frac{dP_h^D}{dU}P_f^D + (P_h^D - c_h)\frac{dP_f^D}{dU})(1+\varepsilon_{P_f,S}) + (P_h^D - c_h)P_f^D\frac{d\varepsilon_{P_f,S}}{dU}\right) > 0$ and $\frac{d^2v_f^D}{dSdK} = \left(\frac{\lambda_h}{(1+RF_f)S^2}\right)\left(\frac{dP_f^D}{dK}P_h^D + \left(P_f^D - c_f\right)\frac{dP_h^D}{dK})(\varepsilon_{P_h,S} - 1) + \left(P_f^D - c_f\right)P_h^D\frac{d\varepsilon_{P_h,S}}{dK}\right) < 0$ since $\frac{dP_h^D}{dU}, \frac{dP_f^D}{dU}, \frac{dP_f^D}{dK}, \frac{dP_h^D}{dK}, \frac{d\varepsilon_{P_f,S}}{dU} > 0, \frac{d\varepsilon_{P_h,S}}{dK} < 0$ and by (8): $1 + \varepsilon_{P_f,S} > 0$ and $\varepsilon_{P_h,S} - 1 < 0$. ■

Lemma 1 states that a home currency depreciation, increases (decreases) the value of the home (foreign) firm, while Lemma 2 states that this mechanism is more intense when uncertainty in the home (foreign) country is lower.

3.3 Systematic risk

Beta coefficient measures systematic risk, and it is defined as, $\beta_h \equiv \frac{Cov(\tilde{r}_h,\tilde{r}_{mh})}{Var(\tilde{r}_{mh})}, \beta_f \equiv \frac{Cov(\tilde{r}_f,\tilde{r}_{mf})}{Var(\tilde{r}_{mf})}$ for firm *h* and *f*, respectively. Using (1), (2), (6) and the *FOCs*, we obtain:

$$\beta_h^D = -\frac{(1+RF_h)\theta_0 Cov\left(\tilde{k},\tilde{r}_{mh}\right)}{m_h^D\theta_h Var(\tilde{r}_{mh})} > 0$$

$$\beta_f^D = -\frac{(1+RF_f)\lambda_0 Cov\left(\tilde{u},\tilde{r}_{mf}\right)}{m_f^D\lambda_f^D Var\left(\tilde{r}_{mf}\right)} > 0 \tag{9a}$$

or, by the definitions of K, U, MP_h, MP_f:

$$\beta_h^D = -\frac{(1+RF_h)}{E(\tilde{r}_{mh}) - RF_h}\frac{\theta_0(1-K)}{\theta_h m_h^D}$$

$$\beta_f^D = -\frac{(1+RF_f)}{E\left(\tilde{r}_{mf}\right) - RF_f}\frac{\lambda_0(1-U)}{\lambda_f m_f^D} \tag{9b}$$

with both beta coefficients to be positive. $m_i^D \equiv (P_i^D - c_i)$ is the equilibrium price-cost margin of firm i.

Proposition 1. The impact of the exchange rate on the beta of firm from the currency depreciating (appreciating) country is negative (positive) and given by (10):

$$\frac{d\beta_h^D}{dS} = (1+RF_h)\frac{\theta_0}{\theta_h}\frac{Cov(\tilde{k},\tilde{r}_{mh})}{Var(\tilde{r}_{mh})}\left(\frac{\varepsilon_{P_h,S}}{Sm_h^D L_h}\right) = \frac{(1+RF_h)(1-K)\theta_0}{(E(\tilde{r}_{mh})-RF_h)\theta_h}\frac{\varepsilon_{P_h,S}}{Sm_h^D L_h} < 0$$

$$\frac{d\beta_f^D}{dS} = (1+RF_f)\frac{\lambda_0}{\lambda_f}\frac{Cov(\tilde{u},\tilde{r}_{mf})}{Var(\tilde{r}_{mf})}\left(\frac{\varepsilon_{P_f,S}}{Sm_f^D L_f}\right) = \frac{(1+RF_f)(1-U)\lambda_0}{(E(\tilde{r}_{mf})-RF_f)\lambda_f}\frac{\varepsilon_{P_f,S}}{Sm_f^D L_f} > 0 \qquad (10)$$

where $L_\mathrm{i} \equiv \frac{P_i^D - c_i}{P_i^D}$ is the Lerner index of firm i.

Proof. By differentiation of (8) *wrt S* and by the definitions for $\varepsilon_{P_i,S}, m_i^D$ and L_i and by (7), we conclude to Proposition 1. ■

Note that *S* affects β_i^D via P_i^D. Hence, as the home currency depreciates, the ability of firm *h* to price above its marginal cost increases ($dm_h^D/dS > 0$, since $dP_h^D/dS > 0$) and this leads to a lower beta for firm *h* (see (9)). On the other hand, a home currency depreciation reduces the ability of firm *f* to price above its marginal cost ($dm_f^D/dS < 0$, since $dP_f^D/dS < 0$) and this leads to a higher beta for firm *f*. Therefore, Proposition 1 states that as the home currency depreciates, the optimal level of systematic risk of the home (foreign) firm decreases (increases).[9] The effect of *S* on beta depends on the sensitivity of prices in *S* and the ability of the firm to price above its marginal cost, i.e. a measure of market power of the firm.

The question is why beta falls when the price-cost margin increases in the home country? The intuition is that an increase in *S*, increases the price-cost margin and hence, the firm has more market power in the product market. Consequently, the firm can offer a lower return to investors in order to attract them to contribute capital.[10] In contrast, its rival loses

[9] We can prove that as uncertainty decreases (*K* and *U* increase) the home and foreign equilibrium levels of beta decrease, namely, $\frac{d\beta_h^D}{dK} < 0, \frac{d\beta_h^D}{dU} < 0, \frac{d\beta_f^D}{d\mathrm{K}} < 0, \frac{d\beta_f^D}{d\mathrm{U}} < 0$, the proofs are available upon request.

[10] Similar to a decrease in a firm's production cost, which makes the firm more competitive in the product market. Consequently, the firm can offer a lower return to investors to attract them to contribute capital (Wong; 1994).

competitiveness so that investors are willing to supply capital only at higher costs. Given the CAPM, stock returns and beta are directly linearly related.[11]

4. Portfolio analysis

Suppose an investor forms a portfolio consisting the two stocks offered by firms who are competitors in the duopoly under discussion, one stock from the home and one from the foreign country, (w_{h}, w_{f}), like in the Detergent market, UN and P&G. Note that a positive (negative) weight w_i indicates a long (short) position. The value of the portfolio, its beta and its beta derivative are:

$$\begin{gathered} v_{pg} = w_h v_{hg} + w_f v_{fg} \\ \beta_{pg} = w_h \beta_{hg} + w_f \beta_{fg} \\ \frac{d\beta_{pg}}{dS} = w_h \frac{d\beta_{hg}}{dS} + w_f \frac{d\beta_{fg}}{dS} \end{gathered} \tag{11}$$

for a constantly reweighted portfolio that maintains constant portfolio weights, such as an index fund and since $\frac{d\beta_{hg}}{dS} < 0, \frac{d\beta_{fg}}{dS} > 0$ by Proposition 1, $\frac{d\beta_{pg}}{dS}$ is zero when $w_h = \frac{d\beta_{fg}/dS}{(d\beta_{fg}/dS)-(d\beta_{hg}/dS)} > 0$, where g stands for global, for simplicity we omit g, however, all the betas are global throughout. [12]

4.1 Arbitrage portfolios

We form a pair arbitrage portfolio (Simon and Blume, 1994, p.119) (henceforth portfolio) with one stock from the home and one from the foreign country and investigate its behavior *wrt* *S*. The portfolio is the nonzero tuple (w_{h}, w_{f}), where $w_{\mathrm{h}} + w_{\mathrm{f}} = 0$.[13]

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[11] We have also studied the case of two home and one foreign firm; the addition of a domestic competitor does not alter the qualitative results in Subsection 3.2 and 3.3. The proof is available upon request.

[12] "constant portfolio weights" (varying shares).

[13] Zero cost portfolio with a long (short) position at the stock from the home (foreign) country.

Inserting the equilibrium prices, derived in the Bertrand Nash equilibrium at Section 3, into (6), we obtain the equilibrium value of the portfolio, $v_p^D = v_h^D - v_f^D$. Then, we calculate its exposure, $\frac{dv_p^D}{dS} = \frac{dv_h^D}{dS} - \frac{dv_f^D}{dS}$, its beta $\beta_p^D = \beta_h^D - \beta_f^D$ and its beta derivative $\frac{d\beta_p^D}{dS} = \frac{d\beta_h^D}{dS} - \frac{d\beta_f^D}{dS}$:

$$\frac{dv_p^D}{dS} = \frac{(U\lambda_0 - \lambda_f c_f)2\theta_f(2K\theta_0\lambda_f - \theta_f\lambda_h c_h + 2\theta_h\lambda_f c_h - SU\lambda_0\theta_f + S\theta_f\lambda_f c_f)\theta_h}{(\theta_f\lambda_h - 4\theta_h\lambda_f)^2(1+RF_h)} - \frac{(K\theta_0 - \theta_h c_h)2\lambda_h(K\theta_0\lambda_h - \theta_h\lambda_h c_h - 2SU\lambda_0\theta_h + S\theta_f\lambda_h c_f - 2S\theta_h\lambda_f c_f)\lambda_f}{S^3(\theta_f\lambda_h - 4\theta_h\lambda_f)^2(1+RF_f)}$$

$$\beta_p^D = \frac{1+RF_f}{E(\tilde{r}_{mf}) - RF_f}\frac{\lambda_0(1-U)S(4\theta_h\lambda_f - \theta_f\lambda_h)}{\lambda_f(\lambda_h(K\theta_0 - \theta_h c_h) + Sc_f(\theta_f\lambda_h - 2\theta_h\lambda_f) - 2SU\lambda_0\theta_h)} - \frac{1+RF_h}{E(\tilde{r}_{mh}) - RF_h}\frac{\theta_0(1-K)S(4\theta_h\lambda_f - \theta_f\lambda_h)}{\theta_h(S\theta_f(U\lambda_0 - \lambda_f c_f) + c_h(\theta_f\lambda_h - 2\theta_h\lambda_f) - 2K\theta_0\lambda_f)} \tag{12}$$

$$\frac{d\beta_p^D}{dS} = \frac{(1+RF_h)\theta_0(1-K)}{(E(\tilde{r}_{mh}) - RF_h)\theta_h}\frac{-\theta_f(\theta_f\lambda_h - 4\theta_h\lambda_f)(U\lambda_0 - \lambda_f c_f)}{(2K\theta_0\lambda_f - \theta_f\lambda_h c_h + 2\theta_h\lambda_f c_h - SU\lambda_0\theta_f + S\theta_f\lambda_f c_f)^2} - \frac{(1+RF_f)\lambda_0(1-U)}{(E(\tilde{r}_{mf}) - RF_f)\lambda_f}\frac{\lambda_h(\theta_f\lambda_h - 4\theta_h\lambda_f)(K\theta_0 - \theta_h c_h)}{(K\theta_0\lambda_h - \theta_h\lambda_h c_h - 2SU\lambda_0\theta_h + S\theta_f\lambda_h c_f - 2S\theta_h\lambda_f c_f)^2}$$

The sign of these expressions depends on the demand and cost parameters, the risk-free rates and the exchange rate. Even for symmetric demand and cost parameters and risk-free rates, the difference $v_h - v_f$ is not zero since there is asymmetry in the two markets due to the existence of S (with $S \neq 1$). Nevertheless, by Lemma 1 and Proposition 1 we conclude the following.

Proposition 2. The exchange-rate exposure of the pair arbitrage portfolio is positive and increases when the home or foreign demand uncertainty decreases (*K* or *U* increase). The beta of the pair arbitrage portfolio is decreasing in the exchange rate.

Proof. By Lemma 1: $\frac{dv_h^D}{dS} - \frac{dv_f^D}{dS} > 0$. By Lemma 2: $\frac{d^2v_h^D}{dSdK} - \frac{d^2v_f^D}{dSdK} > 0$ and $\frac{d^2v_h^D}{dSdU} - \frac{d^2v_f^D}{dSdU} > 0$. By Proposition 1: $\frac{d\beta_h^D}{dS} - \frac{d\beta_f^D}{dS} < 0$. ■ Proposition 2 is discussed in the numerical examples that follow.

4.2 Numerical examples

Since the expressions in (12) are too complex, we offer some numerical examples where we calibrate some of the parameters under symmetry to illustrate the application and practical importance for portfolio management. [14]

We consider a market with $\theta_h = \lambda_f = -1, \theta_f = \lambda_h = 0.5, c_h = c_f = 0.3, \theta_0 = \lambda_0 = 1, RF_f = RF_h = 0.01$, $E(\tilde{r}_{mh}) = E(\tilde{r}_{mf}) = 0.05$ and three different scenarios of home and foreign demand uncertainty, which give nine different combinations of home and foreign demand certainty equivalents (*K*,*U*): (0.3,0.3),(0.3,0.6),(0.3,0.9),(0.6,0.3),(0.6,0.6),(0.6,0.9),(0.9,0.3),(0.9,0.6),(0.9,0.9), and correspond to high (0.3), medium (0.6) and low (0.9) levels of uncertainty. [15] Figure 1 presents the exposure, $\frac{dv_p^D}{dS}$, the beta β_p^D, and the beta derivative $\frac{d\beta_p^D}{dS}$ of the portfolio under these scenarios.

Each graph depicts the relationship between *S* (the *horizontal* axis) and the exposure (Panel A), beta (Panel B) or beta derivative *wrt S* (Panel C) of the portfolio (the *vertical* axis) at that level of *S* for different *K* and *U*. The top (bottom) figures are for three different levels of home (foreign) demand uncertainty *K* (*U*), 0.3, 0.6 and 0.9, respectively. In each top (bottom) figure, we have drawn three lines that correspond to different levels of foreign (home) demand uncertainty, *U(K)*=0.3, solid line, *U(K)*=0.6, bold line and *U(K)*=0.9, dash line, respectively.

[14] We assume symmetric demand and cost parameters, as well as, risk-free rates between the two countries, to focus on the different level of demand uncertainty and the exchange rate.
[15] When *K* (*U*) is high, uncertainty is low and vice versa.

Figure 1, Panel A shows that the portfolio exposure is positive and increases when the home or foreign demand uncertainty decreases, i.e. increase in *K* and *U* (Proposition 2). Moreover, the portfolio exposure is increasing in *S* for high values of *S* and otherwise decreasing. For the top left graph this *S* threshold is 1.3863, 1.4312 and 1.3866 for *U*=0.3, *U*=0.6 and *U*=0.9, respectively. Note that for all graphs these thresholds need to be higher than 1, meaning that the portfolio exposure is increasing in *S* when the foreign is more expensive than the home currency.

[Figure 1]

Figure 1, Panel B shows that the portfolio beta is positive or negative depending on *S*, *K*, *U* values, nevertheless, it is always decreasing in *S* (Proposition 2). For example, in the top left graph and symmetric certainty equivalents, i.e. *K*=*U*=0.3, the portfolio beta is negative for *S*>1, hence a more severe home currency depreciation turns the foreign stock beta to be higher than the home stock beta. Moreover, the portfolio beta is increasing in foreign but decreasing in home certainty equivalents for intermediary or high values of *K* and *U*.

Finally, Figure 1, Panel C shows that the impact of *S* on the portfolio beta is negative (Proposition 2) and increasing in *U* and *K*. The beta derivative *wrt S* is also increasing in *S* with a diminishing rate.

Take for instance UN and P&G, with UN being the home and P&G the foreign, the portfolio is long in UN and short in P&G. As the dollar appreciates, the portfolio exposure increases while its beta decreases and decreases faster for high levels of foreign market (US) uncertainty (low *U*). The beta derivative *wrt S* is also increasing with an appreciating dollar.

5. Conclusions

We study exchange-rate exposure and systematic risk of individual stocks and pair arbitrage portfolios in an international duopoly of firms offering differentiated goods. As the currency appreciates the systematic risk of the firm from the currency depreciating (appreciating) country, decreases (increases). Moreover, as the home (foreign) market demand uncertainty decreases the exposure for the home (foreign) firm increases (decreases). The addition of a domestic competitor does not alter the results.

The exchange-rate exposure of the pair arbitrage portfolio is positive and increases when the home or foreign demand uncertainty decreases, and its systematic risk is decreasing in the exchange rate. In a series of calibrating examples, we find that the portfolio exposure is positive and increasing in the exchange rate for high values of the exchange rate and otherwise decreasing. The systematic risk of the pair arbitrage portfolio is decreasing in the exchange rate, increasing in foreign but decreasing in home demand uncertainty for intermediary or low values of home and foreign uncertainty. Finally, the impact of the exchange rate on the systematic risk of the pair arbitrage portfolio is negative, decreasing in home and foreign demand uncertainty and increasing in the exchange rate.

Our model emphasizes the mode of competition in international markets and its impact on the link between exposure and beta, the addition of a distinction between high and low performance firms is an interesting future direction.

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Figures

Figure 1. Panel A. Exchange-rate exposure of pair arbitrage portfolios for different levels of home and foreign demand uncertainty.

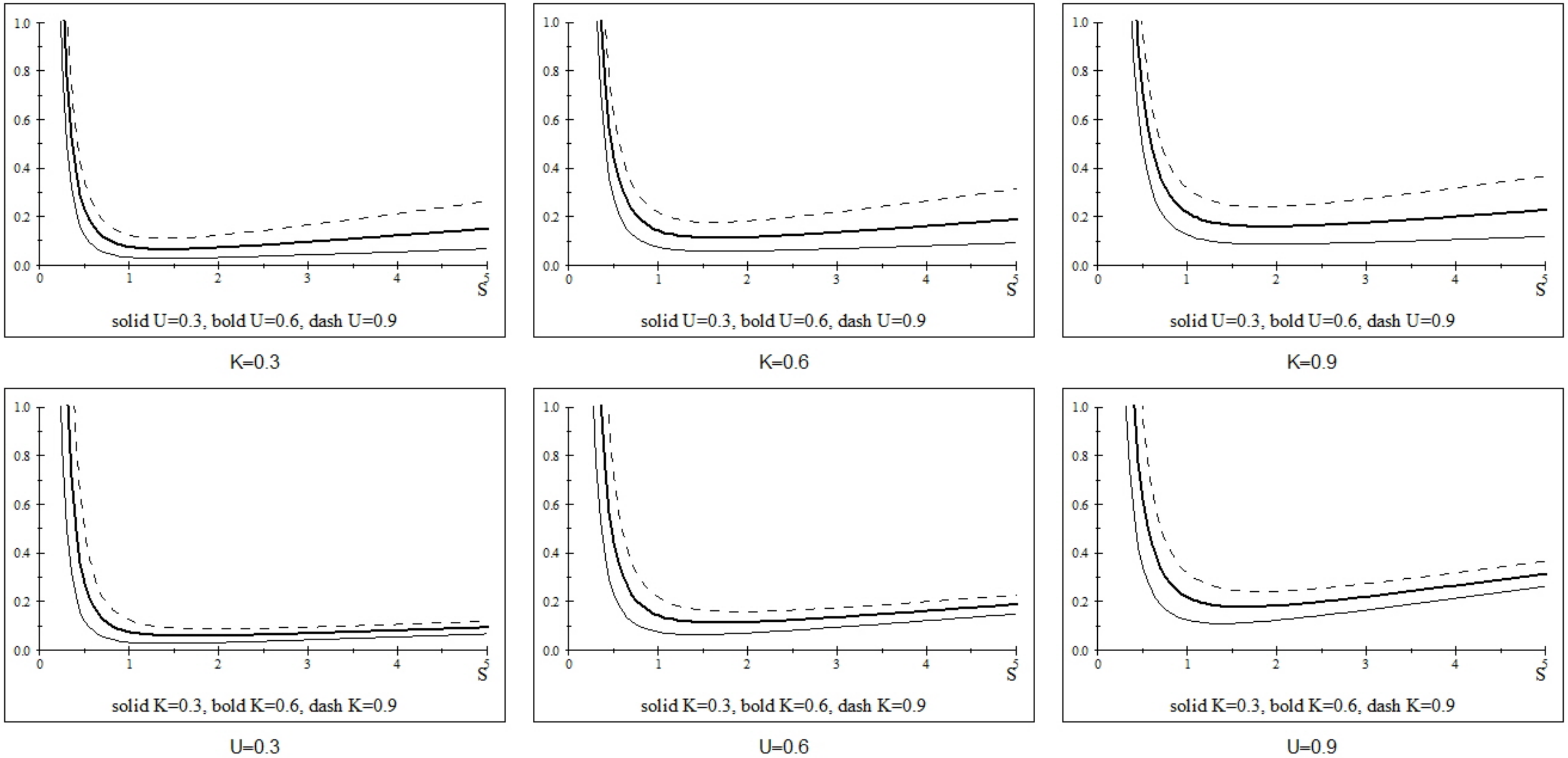


Figure 1. Panel B. Systematic risk of pair arbitrage portfolios for different levels of home and foreign demand uncertainty.

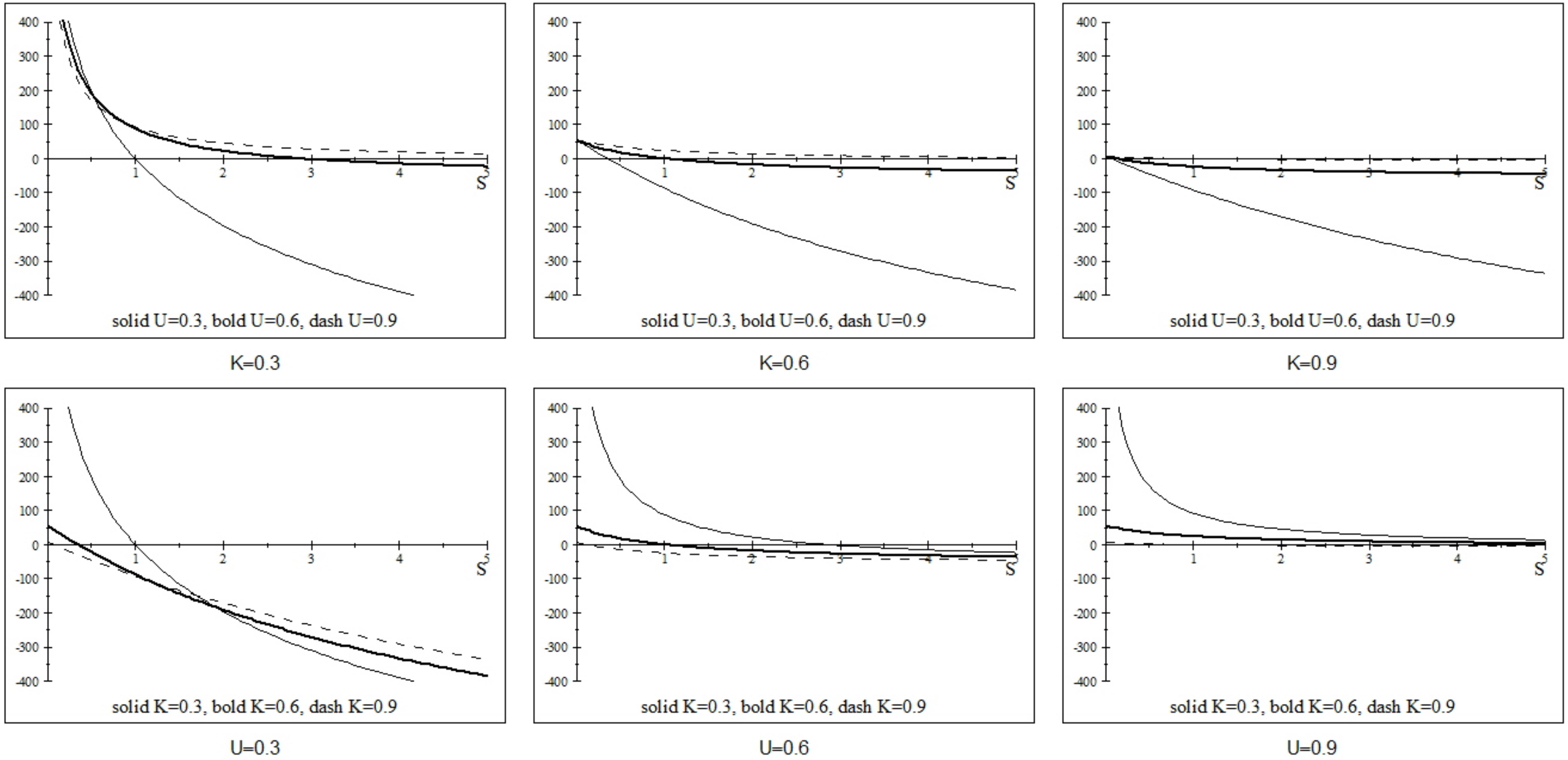
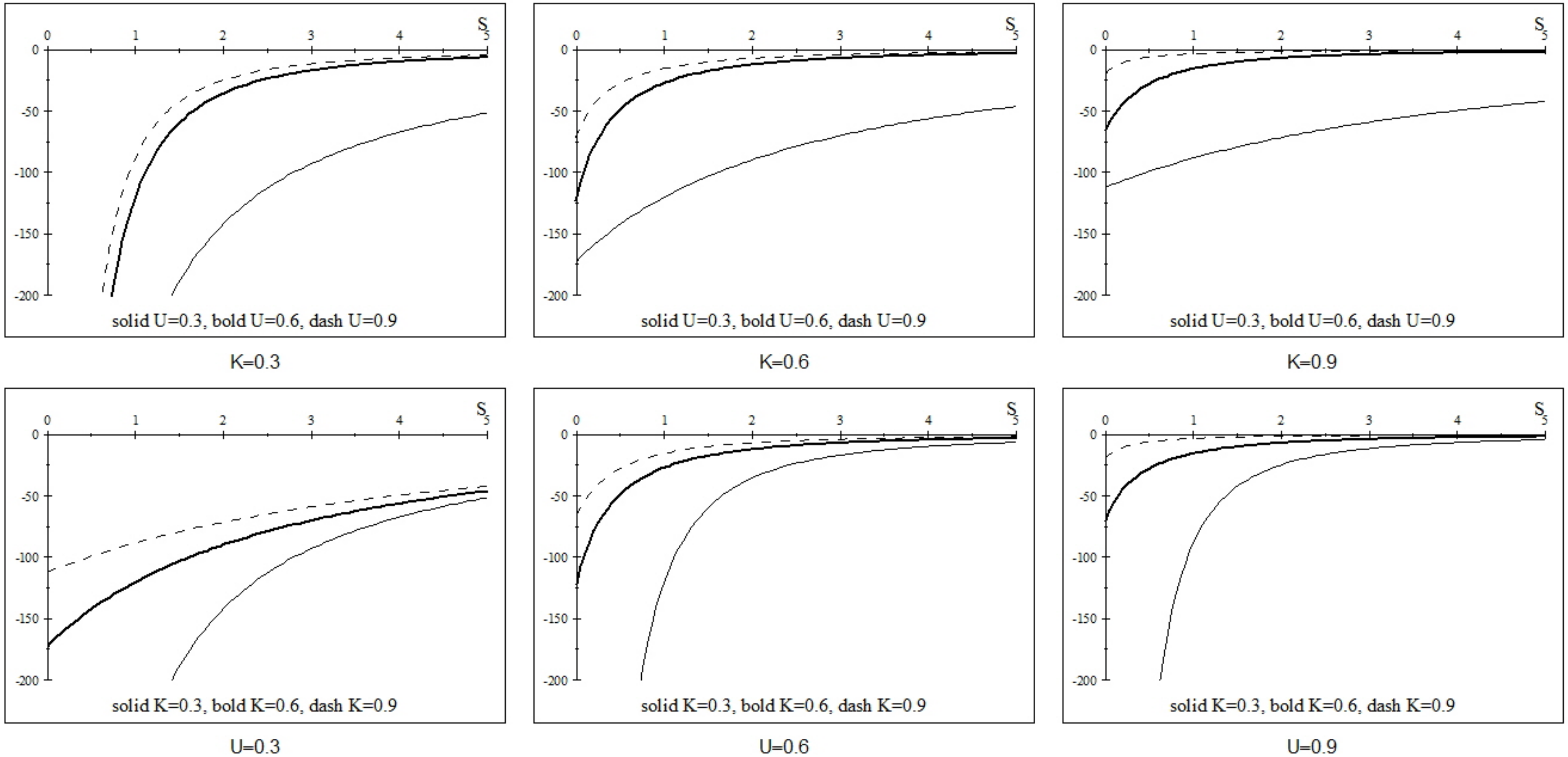


Figure 1. Panel C. Changes in systematic risk of pair arbitrage portfolios for different levels of home and foreign demand uncertainty.



Notes: The figures in Panels A, B and C were produced calculating $\frac{dv_p^D}{dS}$, β_p^D, and $\frac{d\beta_p^D}{dS}$ respectively, for $\theta_h = \lambda_f = -1, \theta_f = \lambda_h = 0.5, c_h = c_f = 0.3, \theta_0 = \lambda_0 = 1, RF_f = RF_h = 0.01$, $E(\tilde{r}_{mh}) = E(\tilde{r}_{mf}) = 0.05$ and three different levels of home (*K*) and foreign (*U*) demand certainty equivalents, which correspond to high, medium and low levels of uncertainty in the home and abroad, respectively, since when the certainty equivalents are high, the uncertainty is low and vice versa. Each graph depicts the relationship between the exchange rate *S* (the horizontal axis) and the exchange-rate exposure, $\frac{dv_p^D}{dS}$, (Panel A), beta, β_p^D, (Panel B) or the change in systematic risk (beta derivative *wrt S*) (Panel C) of the pair arbitrage portfolio (the vertical axis) at that level of *S* for different levels of demand uncertainty. In each panel, the top figures are for three levels of home (*K*) demand uncertainty, 0.3, 0.6 and 0.9, while the bottom figures are for three levels of foreign (*U*) demand uncertainty, 0.3, 0.6 and 0.9. In each figure at the top (bottom), we have drawn three lines that correspond to different levels of foreign (home) demand uncertainty, *U (K)*=0.3, solid line, *U (K)*=0.6, bold line and *U*

(K)=0.9, dash line, respectively. θ_h, λ_f^D are the own effects and θ_f, λ_h are the cross effects in the demand functions, θ_0 and λ_0 are the constant terms in the home and foreign country respectively, that embody the effects of all factors other than price that affect demand, c_h and c_f are the constant marginal costs of *home* and *f*oreign firm, RF_h, RF_f are the risk-free rates in the home and foreign country and $E(\tilde{r}_{mh})$ and $E(\tilde{r}_{mf})$ are the expected rates of return on the home and foreign market portfolios, respectively.