Forecasting realized volatility with wavelet decomposition

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Abstract

Forecasting Realized Volatility (RV) is of paramount importance for both academics and practitioners. During recent decades, academic literature has made substantial progress both in terms of methods and predictors under consideration. Despite the popularity of technical indicators, there has been only scarce reference to the effectiveness of this group of predictors in forecasting RV. This paper examines the out-of-sample forecasting performance of technical indicators for S&P500 RV relative to macroeconomic predictors. Our main contribution is to demonstrate that these sets of predictors impact volatility at different frequencies and thus are complementary. Specifically, technical indicators perform especially strongly for forecasting the short frequency component which complements macroeconomic variables which perform strongly at longer frequencies. We demonstrate that by generating economically motivated amalgamation forecasts from these predictors that takes into account the frequency dimension leads to substantial improvements in forecast accuracy. Moreover, we examine timing effects and assess the economic significance of our forecasts.

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1 Introduction

The implications of realized volatility (RV) in the fields of risk management, asset allocation and asset pricing has attracted academic interest during recent decades. Schwert (1989a,b) showed that RV makes countercyclical movements, but finds no link between RV and economic variables. Lately, there has been substantial enhancements in forecasting RV. Recently the debate on the role of economic determinants as predictors of volatility has been reignited. However, it demonstrates contrasting results (in several asset classes, see among others, Wang et al. (2018), Nonejad (2017), Chao (2016), Conrad and Loch (2015), Mittnik et al. (2015), Christiansen et al. (2012), Paye (2012) and Mele (2007)). Papers that provide a positive perspective of the problem at hand, claim that including exogenous variables adds predictive power. Nevertheless, there are groups of predictors that have been largely overlooked by this literature, so far. Second, the literature ignores the potential ability of predictors to capture information at different frequencies. In applications thus far, the predictors are tested towards their performance on the original series rather than testing their ability to capture different dynamics. Last, the economic significance of RV forecasts is rarely assessed.

A separate strand of literature demonstrates there is complementarity between technical indicators and macroeconomic predictors in different asset classes such as equity markets (Lin, 2018; Neely et al., 2014), exchange rates (Panopoulou and Souropanis, 2019; Buncic and Piras, 2016) and bond risk premia (Goh et al., 2013) but not in the RV context thus far. Furthermore, this literature does not yet provide a complete understanding of why technical indicators and macroeconomic predictors are complementary. In this paper, we examine one new plausible explanation for this complementarity, that they convey information about different frequencies of the predicted series. Specifically, by their very nature technical indicators are short-term focused and thus could contain more information for the higher frequency component of RV. In contrast, macro variables tend to evolve much more gradually and are thus more likely contain information about the lower frequency component of RV. For example, Engle et al. (2013) and Mittnik et al. (2015) demonstrate that the long-term volatility component is captured by macroeconomic variables. Hence, the complementarity of these predictors could be due to them each capturing a different part of the frequency domain. This would lead directly to improvements in forecast performance since if one forecasts high frequency movements and the other forecasts low frequency movements then aggregating these together should enhance the overall predictability of RV.

This paper will fill the following gaps. First, we study technical indicators as a new candidate group of predictors for RV forecasting, which have received very limited coverage in the academic dialogue, so far. Second and most importantly, we will explore a new explanation for why different groups of predictors could be complementary; specifically, that different groups of predictors could, due to their very nature, forecast differing frequency components of RV. Third,
we will examine whether an economically motivated approach or a purely statistical approach is better able to enhance the predictive power of RV forecasts given that each frequency component is modelled. This links to an ongoing debate in the literature focuses over how you can best amalgamate forecasts. Is it better to enable the data to speak or can economic restrictions help to generate superior performance? Advances in data science suggest that advanced techniques benefit the end-user (e.g. Kelly and Pruitt, 2013; Bianchi et al., 2021) while another school of thought proposes simple restrictions motivated by economics add value (see e.g. Campbell and Thompson, 2008; Pettenuzzo et al., 2014).

Technical indicators generate signals based on patterns and trends, instead of calculating an intrinsic value that a particular asset should have according to a model. Despite the fact that technical indicators are a major tool for practitioners and are used for a significant period of time, this particular group of predictors has been neglected for a long time by academia as atheoretical1. Nevertheless, recent literature on returns forecasting has pointed out solid arguments in favour of including them in the ongoing academic dialogue (Han et al., 2016; Menkhoff, 2010; Menkhoff and Taylor, 2007). There are numerous publications qualifying them as promising candidate predictors that have impact on returns forecasting, in several asset classes.2 The intuition accompanying the impact of technical rules on volatility is that the buy/sell signals generated by the rules should play a vital role in future volatility via well-established transmission channels. For this purpose, we evaluate the out-of-sample performance of a few simple but very popular rules, against an autoregressive process. The very nature of these technical indicators are short-term oriented, which suggests they should be more informative about the near future rather than longer-term movements.

Next, we highlight that the proposed frequency decomposition setup is connected to and builds on the approach followed by a handful of papers and has strong intuition underlying it. Lin (2018) decompose total stock market returns into three component a) the expected return, b) the discount rate news and c) cash flow news; the author argues that technical rules forecasting ability is driven by the fact that they primarily capture future cash flow news. Neely et al. (2014) argue that the enhanced predictability can be attributed to the fact that the two groups capture different parts of the business cycle, as well as, to the ability of technical rules to provide better forecasts in different sentiment periods (especially during recessions). On the other hand, Conrad and Loch (2015) employ a GARCH-MIDAS to decompose stock returns into short-run and long-run components and examine the long-run (conditional) volatility component using economic variables. Yi et al. (2019) claim that the long-cycle economic trend, as derived from a 12-month moving average of the economic predictor, and its difference with the actual data of the

1The only empirical work around technical rules forecasting of RV has been done contemporaneously by Liu and Pan (2020).

2See among others Panopoulou and Souropanis, 2019; Jamali and Yamani, 2019; Lin, 2018; Zarrabi et al., 2017; Li and Tsiakas, 2017; Hsu et al. 2016; Buncic and Piras, 2016; Neely et al., 2014; Goh et al., 2013; Neely and Weller, 2011; Neely et al., 2009; De Zwart et al., 2009; Park and Irwin, 2007; Gehrig and Menkhoff, 2006; Olson, 2004; Cheung and Chinn, 2001.
predictor reflect the existence of two separate information sets of the current economic state. Ferreira and Santa-Clara (2011) employ the sum-of-the-parts (SOM) method to decompose time series into frequencies and take advantage of the difference in the persistence among the components. The aforementioned papers reveal the interest of academia on understanding the additional and distinct layers of information within the time series.

A tool that provides an in-depth overview of time series components is wavelet decomposition. This methodology decompose series to obtain the time-varying and frequency-varying characteristics of a series just by adjusting the window size. Hence, the different frequency features of the time series can be derived and analyzed independently. Considering this, Faria and Verona (2018a) argue that the historical average is able to solely capture low frequencies, rather than abrupt jumps in the series. Similarly, Asgharian et al. (2016) employ wavelet decomposition on stock and bond volatility in order to study the effects of macroeconomic and financial variables on the long-run frequency component. Czudaj (2019) argues that wavelet decomposition mimics the heterogeneity of agents. Rua and Nunes (2009) employ wavelet decomposition to distinguish between a long-term and a short-term investor. As a result, different frequencies can represent agents with different kinds of behavior.

Our extension of the existing academic dialogue is to investigate a series of research questions. The first question we address in this paper is “should we consider technical indicators as candidate predictors in the RV forecasting?” . The second and most interesting research question is “why do technical indicators contain complementary information to macroeconomic predictors? Is it due to capturing information at different frequencies?”. The third research question is “how can we best amalgamate information at different frequencies? Is a purely statistical approach sufficient or does imposing economically motivated restrictions lead to superior results?”.

We perform the forecasting analysis on the monthly RV of S&P500 spanning from January 1950 until December 2018. The forecasts are calculated recursively and the out-of-sample period begins in January 1966. With respect to technical indicators, in order to avoid loss of information, we generate technical indicators as the intra-month average signals for a moving average, momentum, relative strength index and exponential moving average rule. Regarding macroeconomic variables, we use the popular dataset proposed by Goyal and Welch (2008). The respective dataset has been used in recent wavelet-based research, for example Faria and Verona (2020) and Yi et al. (2019). Due to the counter-cyclical movements of RV, we follow the existing literature and compare the performance of the rival model against an autoregressive model. The results are evaluated with the use of the $R^2_{OOS}$ metric and the MSFE-adjusted

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3Rua (2011) argues that the method has relatively been overlooked in forecasting setup. Nevertheless, among others see, Caraiani (2017) has employed the method in exchange rate forecasting, Faria and Verona (2018a,b; 2020) in equity markets, Risse (2019) in gold returns. Complementary, Zhang et al. (2017) demonstrate improvements in a set of autoregressive processes by augmenting them with a wavelet-based multiresolution analysis.

4The dataset has been downloaded from Amit Goyal’s website.
We initially create forecasts according to the standard linear framework to horserace the candidate predictors. Next, we employ the wavelet multiresolution decomposition to shed light on the time and frequency properties of the RV series. Despite the potentially insightful information wavelets provide, to our knowledge, they have rarely been used in the context of RV forecasting. The tool allows us to forecast each frequency separately and compare the performance of our predictors. As a first step, we follow the approach of Faria and Verona (2020), and decompose the dependent variable into different frequencies. Specifically, the original RV series is transformed into orthogonal short, medium and long-run frequencies, which are then each forecast separately. Given the aforementioned arguments, as well as the discussed progressive academic literature on identifying distinct information within the time series. We anticipate that different predictors are able to forecast different frequencies, subject to their nature, namely fast/trend moving predictors will be able to forecast more rapidly changing frequencies and so on. We move one step further by allowing the forecaster to align the relevant predictors with jump, seasonality and the trend components of the RV series. Hence, in the first step, we are able to examine the impact of the candidate predictors at various frequency dimensions and in the second step we tailor a superior overall forecast that exploits information from both types of predictors.

Our results provide evidence that a set of predictors based on technical indicators should be included for RV forecasting. By themselves, technical indicators outperform the benchmark by as much as 2.52% which is better than any macroeconomic variable. However, there are also interesting frequency effects. As a pattern-based signalling predictor, it is doing well in short and medium-run signals. Remarkably, macroeconomic predictors perform equally well in forecasting the medium and long frequency component. Furthermore, our findings suggest that aggregating information by taking into account both types of predictors can enhance the performance significantly. We employ few forecast combination methods and principal components to take into account the entire information set. To our knowledge, this paper is the first to consider such an approach. The results provide strong evidence supporting the complementarity of the two groups of predictors. Moreover, by generating forecast series after imposing economically motivated constraints, we improve further the forecasting performance up to 2.99%. In brief, we sum the forecast the short frequency component with technical indicators, the forecast of the long frequency component with macroeconomic predictors and the average of those two groups for the medium frequency component. Moreover, we see that timing is important in the conducted experiment, since the predictors demonstrate gains during crises and high-sentiment periods. The proposed model is more resilient towards timing effects, providing further evidence that the two groups work complementarily. Last, we move outside the norm of the RV

statistic proposed by Clark and West (2007).

Please note Faria and Verona (2018 a,b) decompose the predictor variables, in contrast we decompose the dependent variable.
forecasting literature and examine the economic performance of our results by employing the volatility managed framework.

Our contribution to the ongoing debate around RV forecasting is based on the following premises. First, we propose a new group of predictors in the RV forecasting literature that has not received much attention yet. In addition, we present a theoretical framework that allows us to understand the transmission channels of the rules over volatility. Second, we use wavelet decomposition in the RV forecasting, a method that has not been very common in the forecasting literature, so far. Third, we provide evidence that macroeconomic predictors and technical indicators capture distinct information at different frequencies. Fourth, we argue that modeling RV forecasting by aligning each frequency to be forecasted by a different predictor group and then summing the forecasted frequencies; this provides substantial gains both in terms of forecast accuracy and economic value. Last, we identify the sources of performance.

The remainder of the paper is organized as follows. Section 2 presents the theoretical framework supporting the implications of technical signals and macroeconomic predictors in future volatility. Section 3 elaborates on the Data and the calculations of technical indicators. In Section 4, we demonstrate the methodology used. In Sections 5, we discuss our findings. In Section 6, we summarize the economic performance of the models. In the last part, section 7, we present the main conclusions.

2 Theoretical framework

2.1 Theoretical Framework for Technical Indicators

The predictability of RV by technical rules can be attributed to transmission channels that link technical indicators with RV. As a matter of fact, the literature around technical indicators has built a theoretical framework to support the predictability of the predictors on returns (among others see Menkhoff and Taylor, 2007). However, research documenting their implications on volatility is sparse.

Frankel and Froot (1990) claim that in highly liquid markets “much trading is based on noise rather than news, and leads to excessive volatility”. Allen and Taylor (1992) argue that fundamentalists are pointing towards a long-term equilibrium. On the other hand, technical analysis is a short-run investment decision toolkit that produces noise around the long-term fundamental path. Among others, De Long et al. (1990) and, more recently, Lee et al. (2002) and Zhu and Zhou (2009) argue, first, that the existence of non-fundamental traders has implications on volatility in the market and, second, the profitability of technical rules can be attributed to the existence of irrational traders that introduce noise in the market.

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6 The authors are referring to exchange rates.
7 In an earlier study, Allen and Taylor (1990) encompass chartists among the noisy traders. Fiess and MacDonald (2002) claim that High, Low and Close prices contain information that can forecast volatility, attributed to their properties as extreme value estimators.
Building on the above, we identify four transmission channels of technical indicators with RV. Firstly, buy signals attract more technical traders that introduce more noise in the market and increase volatility. Traders are not always rational, with such behavior being documented in the very early literature (see Daniel et al., 1998). Rules of thumbs, herding behavior and cognitive biases have been identified as some of the decision making criteria implemented by traders. In line with the above, Brown and Cliff (2004) claim that the acquisition of information is costly and not all investors can access it. Hence, less informed investors tend to follow the herd and create momentum in the returns, provoking deviations in the market from the equilibrium level and generate positive returns. Consequently, this first body of literature proposes a positive relationship between technical trading activity and volatility.

Secondly, negative signals are adjusted to more quickly, leading to larger absolute price changes and thus higher volatility (Chen and Ghysels, 2011). This effect can be further strengthened by these additional channels. Firstly, short-sale constraints can lead to negative signals being incorporated more rapidly during market declines; this can occur because information from bearish investors is hidden by the short sales constraints (Hong and Stein, 2003). Secondly, volatility feedback effects can lead to positive signals being more muted whereas negative signals can be amplified (Campbell and Hentschel, 1991); this again leads to higher volatility following negative signals.

Thirdly, there is the high sentiment period channel. Smith et al. (2016) show that the superiority of technical rules during high-sentiment seasons can be attributed to the fact that markets exhibit trends. The uninformed traders are considered to be sentiment traders, since their investment decisions are based on trends and other tools rather than models supported by theory. Lee et al. (2002) show that upward trends in sentiment result in reduced volatility and higher returns. Shu and Chang (2015) provide two ways through which higher sentiment leads to lower volatility. First they demonstrate that higher sentiment can be associated with lower risk aversion which leads to lower volatility. Secondly they demonstrate that higher sentiment can be associated with investors becoming more patient which again leads to lower volatility. This third body of literature also predicts that technical trading activity is negatively correlated with volatility.

2.2 Theoretical Framework for Macroeconomic Predictors

Although the initial purpose of the paper is to propose technical indicators as a novel and effective group of candidate predictors for RV, we acknowledge that they could plausibly be best implemented in association with the established macroeconomic predictors. Therefore, we briefly review the work on macroeconomic predictors and especially how this relates to frequency since we propose this as the source of the complementarity between the two groups of predictors. The seminal paper of Schwert (1989b) has challenges the link between volatility and macroeconomic activity, however, newer work provide positives evidence in terms of the
existing relationship. The relationship is mainly established on the uncertainty regarding the existing state of the economy and/or the expected stock returns which generate volatility in the ex-post return. The economic drivers of volatility can be summarized and labelled as i) the business cycle, ii) liquidity effects, iii) learning effects, iv) uncertainty about fundamentals and v) country characteristics.

The first channel linking volatility with macroeconomic predictors is based on the business cycle. A stylised fact is that equity market volatility increases during recessions (Schwert, 1989b; Engle and Rangel, 2008; and Brandt and Kang, 2004). Schwert (1989b) claims that there are asymmetries in this relationship, since higher fluctuations take place during periods of recession. Engle and Rangel (2008) suggest macroeconomic variables can be linked to lower frequency volatility (in their model unconditional volatility). Mele (2007) finds that risk premia change asymmetrically when the economic state changes; specifically, when the economy enters a negative state then risk premia increase and thus generates volatility. On the other hand, the positive state of the economy does not impact discount rates much and thus does not lead to volatility. This has been the primary channel investigated in the literature so far. However, several further channels have been proposed.

The second channel linking volatility with macroeconomics is liquidity. Shortage of liquidity in the market creates forces in the market that increase trading volumes (Schwert, 1989b). Leveraged asset holders face pressure to short sale, in order to maintain liquidity (Brunnermeier and Pedersen, 2009). Hence, prices drop further, spiralling the problem of liquidity in the market and generating further return volatility. Mittnik et al. (2015) argue that illiquidity in the market leads to higher uncertainty, as a result stock volatility is pushed upwards. Thus, liquidity and volatility have negative relation. This has two implications for our research. Firstly, it suggests that variables that impact financial intermediaries such as default spread and term spread should be included. Secondly, it suggests that macroeconomic variables could impact equity volatility at a medium or even short frequency via this liquidity channel.

The third channel attributes the link to the existence of the so called learning effect. Timmermann (1993) argues that agents follow a learning curve over time. The investor adjusts the parameters of their subjective dividend growth model. Hence, at the beginning of the learning process, the formulated priors are not solid and investors tend to overreact to news. Veronesi (1999) shows that volatility tends to be higher during periods of high uncertainty about the true state of the economy. Brennan and Xia (2001) provide a different perspective by modelling the expected dividends as a stochastic process. This non-observability leads to a learning process. On the other hand, Adam et al. (2016) focus on the effects of subjective beliefs and learning about stock price behavior (rather than dividends) reshape the learning process argument, by suggesting to focus on the stock price behavior rather than the dividends; their model can generate the level of market volatility observed in the data.

The fourth channel is intersected with the other channels. The literature has pointed out the
important impact of uncertainty around fundamentals upon market volatility. In an early study, Officer (1973) argues that volatility derived from macroeconomic variables and market volatility are associated. Bansal and Yaron (2004) claim that news influences investors’ perceptions regarding future growth rates of stocks. David and Veronesi (2013) provide evidence that inflation and earnings uncertainty may enhance stock market volatility, since investors are not sure what is the upcoming state of the economy. This can lead to fluctuations in equity valuations due to fluctuations in estimated cashflows, for example. Engle and Rangel (2008) consider a number of variables that have an effect on uncertainty about interest rates (as a result, risk premia, as well) and discount rates. In general terms, the intuition behind this relationship is that macroeconomic variables, such as monetary related variables, reveal information regarding future uncertainty regarding risk premia and discount rates.

The last channel is related to country specific characteristics. The most developed countries are less prone to extreme events and are also better equipped to moderate the economic impact of these given their institutions tend to be more mature. Interestingly, market liberalisations could plausibly have either a positive or negative effect on volatility as foreign speculators could either help stabilise the local market or destabilise since substantial capital outflows could occur at short notice; Bekaert and Harvey (2000) find there is an insignificant relationship. Aggarwal et al. (1999) find that large increases in volatility in emerging markets are generally local events such as currency crises, hyperinflation or political issues, the only global event they find is the Black Monday crash of October 1987.

In this part, we briefly discuss the main channels that link macroeconomic variables with volatility. The literature on the topic is extensive and we thus focus on research that most closely links to our questions of interest.

3 Data Description

This section discusses the dataset at hand and the technical indicators used in our experiment.

3.1 Data

For the purposes of our analysis, we follow the main stream of the literature and use S&P500 as the world leading equity market index in order to test our hypothesis. We collect daily spot prices from Yahoo Finance in order to generate technical indicators and realized volatility (RV). We collect the data for macroeconomic predictors from the website of Amit Goyal. The dataset spans from January 1950 to December 2018, giving a total of 828 observations. Following Neely et al. (2014), the in-sample period ends in December 1965, which enables sufficient observations for parameter estimation.
3.2 Realized Volatility

We construct RV as the sum of squared daily returns (among others see Wang et al., 2018; Christiansen et al., 2012; Paye, 2012; Engle et al., 2013; Schwert, 1989b) as the monthly proxy for the variance of stock returns at the monthly frequency \(^8\), such as:

\[
RV_t = \ln \left( \sqrt{\frac{1}{T} \sum_{i=1}^{T} r_{i,t}^2} \right)
\]

where \(T\) denotes the number of trading days within a month \(t\) and \(r\) is the logarithmic daily return of the trading day \(i\).

Figure 1 shows the path of RV through the entire sample period. In general, we observe a few peaks of increased volatility during the 70-year period, but the main events standing out correspond to the two major crises, the first one known as the Black Monday of 1987 and the recent financial turmoil of 2008. The existence of such events can be seen in the substantially high positive kurtosis, as shown in Table 1.

[FIGURE 1 AROUND HERE]

3.3 Technical Indicators

We employ 14 widely used technical indicators based on Moving Average (\(MA\)), Momentum (\(MOM\)), Relative Strength Index (\(RSI\)) an Exponential Moving Averages (\(EMA\)) rules, in order to test whether technical indicators are efficient predictors of RV. By construction, technical indicators are looking for existing trends in order to generate buy-sell signals for each trading day of the month, namely a 1 or 0 value, respectively. Theory supports that such patterns are considered to take place regularly, otherwise such techniques wouldn’t be so popular, and last long enough to be recognised and be profitable (Menkhoff and Taylor, 2007).

The initial scepticism over this tool, due to its controversy with the Efficient Market Hypothesis, was followed by a growing academic literature trying to explain irrationalities in the market or deviations from the efficient level. It has been pointed out that such rules provide effective forecasts for several asset classes (for exchange rates see among others Allen and Taylor, 1990; for equity markets see Neely et al., 2014; for bond premia see Goh et al. 2013; for hedge funds see Smith et al., 2016; for oil see Czudaj, 2019).

3.3.1 Moving Average

Following the most recent literature, (see among others, Panopoulou and Souropanis, 2019; Lin, 2018; Zarrabi et al., 2017; Marshall et al., 2017; Buncic and Piras, 2016; Han et al., 2016; We winsorize the upper and bottom 5% of RV in order to secure that our results are not driven by few extreme values.

\(^8\)We winsorize the upper and bottom 5% of RV in order to secure that our results are not driven by few extreme values.
Baetje and Menkhoff, 2016; Neely et al., 2014; and, relatively early publications such as Acar and Satchell, 1997; and Brock et al., 1992), we consider Moving Average based rules as the first group of technical indicators. These rules are based on the investor receiving a buy (sell) signal when the fast changing short MA crosses the slow moving long MA, pointing to a shift in the trend. Breaks in the trend or merges of new ones are expressed when the moving average of spot prices over a short period of time \((s)\) is greater than the moving average over a longer period of time \((l)\), such as:

\[
x_{i,t} = \begin{cases} 
1 & \text{if } MA_{s,t} \geq MA_{l,t} \\
0 & \text{if } MA_{s,t} < MA_{l,t} 
\end{cases},
\]

where \(P_t\) is the price level at day \(t\) and \(s, l\) denote the short and long-term period, respectively. We consider \(s\) equal to \([1,2,3]\) days and \(l\) equal to \([9,12]\) days and denote the related rule by \(MA(s,l)\).

### 3.3.2 Momentum

The second group of predictors taken into account are Momentum rules (MOM). MOM is another member of the extended family of trend following rules. It is simple to construct and quite common in academic literature (see among others, Panopoulou and Souropanis, 2019; Jamali and Yamani, 2019; Lin, 2018; Zarrabi et al., 2017; Buncic and Piras, 2016; Baetje and Menkhoff, 2016; Neely et al., 2014; Goh et al., 2013). Marshall et al. (2017) argue that Momentum rules and MAs are closely linked, despite the fact that there are significant differences in the timing of identifying trends, since MA rules move faster than MOM.

The rule is based on the theoretical framework that movements of asset prices tend to persist. Momentum trading strategies have been detected in other markets as well. For example, Grinblatt et al. (1995) claim that mutual funds accommodate momentum strategies when investing. The trend following component is captured by comparing the spot price at time \(t\) with a past spot price at time \(t-k\). An indication of buy (sell) signal is given, when the current prices are higher (lower) than \(k\) periods before.

The signal is generated according to the following mathematical expression:

\[
S_{i,t} = \begin{cases} 
1 & \text{if } P_t \geq P_{t-k} \\
0 & \text{if } P_t < P_{t-k} 
\end{cases},
\]

where \(k\) is the daily lag set to \([9,12]\) and denote the related predictors by \(MOM(k)\).

### 3.3.3 Relative Strength Index

A less popular group of rules in the academic literature are the oscillators. This group indicates the existence of rapid oversold or overbought movements that are subject to market corrections
(early references of the rule are made by Levy, 1967; and Wilder, 1978; and for more recent studies see Panopoulou and Souropanis, 2019; Jamali and Yamani; Czudaj, 2019; Zarrabi et al., 2017; Buncic and Piras, 2016). RSI is a contrarian indicator that detects overbought or oversold conditions in the price of an asset. The values of the rule lie between 0 and 100, such as:

\[
RSI_{i,t} = 100 - \frac{100}{1 + \frac{MA(n)(dc_t)}{MA(n)(uc_t)}}
\]

where \(MA(n)\) denotes the n-period Moving Average of upclose (uc) or downclose (dc) measures, defined as:

\[
uc_t = \begin{cases} \Delta P_t & \text{if } \Delta P_t > 0 \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad dc_t = \begin{cases} -\Delta P_t & \text{if } \Delta P_t < 0 \\ 0 & \text{otherwise} \end{cases}
\]

the sell (buy) signals are generated when the RSI value at day \(t\) exceeds (less than) 50, such as:

\[
S_{i,t} = \begin{cases} 1 & \text{if } RSI_t < 50 \\ 0 & \text{if } RSI_t \geq 50 \end{cases}
\]

We employ two versions of the index accounting for \(n\) equals to 7 and 14 days.

### 3.3.4 Exponential Moving Average

The last type of predictors considered belongs to the group of Exponential Moving Averages (EMA). This type is less common in the literature, but is useful due to the fact that weights higher the most recent observations than the simple MA, as discussed above (among others see Panopoulou and Souropanis, 2019; Zarrabi et al., 2017). The weights on past observations decrease exponentially. Hence, it much more adaptable to new trends than the former two momentum type of predictors. Similar to MA rules, the buy/sell signal process is generated when the short EMA exceeds the long EMA, so that:

\[
S_{i,t} = \begin{cases} 1 & \text{if } EMA_{s,t} \geq EMA_{l,t} \\ 0 & \text{if } EMA_{s,t} < EMA_{l,t} \end{cases}, \quad EMA_t = (S_t - EMA_{t-1}) * m + EMA_{t-1}
\]

where \(m\) is a weighting multiplier, or else an accelerator, given by \(m = \frac{2}{j+1}\) where \(j = s, l\). The \(EMA(s,l)\) rule we employ sets \(s = [3, 5]\) and \(l = [9, 12]\) days.

Following a similar setup, as proposed by Gao et al. (2018), after constructing the rules on

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\(^9\)The authors define a buy signal when RSI is lower than 30 and a sell signal when it is over 70. They assume that the respective signal is maintained until the RSI hits the contrary threshold value.
a daily basis, we create the monthly signals as the monthly average. Hence, instead of using the end-of-month signal, we use the average of intra month signals as a proxy of the monthly signals. Thus, the predictors’ values range from 0 to 1. The investor instead receives a more informative signal showing depth of the buy/sell signals within a month, answering the question how far long/short did technical rules advise an investor to go on a particular month?

3.4 Macroeconomic Predictors

We include the macro variables dataset introduced by Goyal and Welch (2008), which is widely used in the return forecasting literature.\textsuperscript{10} The variables used in our framework as presented below:

1. Dividend Price Ratio (DP): difference between the log of dividends and the log of prices.
2. Dividend Yield (DY): difference between the log of dividends and the log of lagged stock prices.
7. Net Equity Expansion (NTIS): the ratio of 12-month moving sums of net issues by NYSE listed stocks divided by the total market capitalization of NYSE stocks.
8. Treasury Bill Rate (TBL): interest rate on a three-month U.S. Treasury bill.
11. Term Spread (TMS): difference between the long term yield on government bonds and the T-Bill.

\textsuperscript{10}Several of the variables have been used in the literature either on their own, for example Engle et al. (2013) use the inflation rate, or as part of a dataset, for example, Paye (2012) considers a slightly different dataset and provides a theoretical intuition regarding the link between macroeconomic variables and realized volatility. Complementary to the aforementioned paper, Christiansen et al. (2012) and Mittnik et al.(2015) use a large set of variables, many of which are included among the Goyal and Welch (2008) dataset.

14. Inflation (INFL): inflation is the Consumer Price Index (CPI). Following the general practice, we lag inflation for an extra month in order to account for delay in releases of the CPI.

3.5 Summary Statistics

In Table 1, we report the summary statistics for the realized volatility, technical indicators and macroeconomic predictors at hand. The descriptive statistics for RV and candidate predictors are the mean, standard deviation, median, kurtosis and skewness.

Panel A of the Table shows the respective statistics for technical indicators. We see that most predictors have similar characteristics, since the values do not vary significantly across each candidate predictor, irrespective of the group they belong. RSI(k) and EMA(s,l) display a different type of characteristics, since their mean, median and standard deviation is slightly higher than MA(s,l), apart from RSI(7). Also, these predictors are more negatively skewed than their rivals.

In Panel B of the Table, we observe that all variables display leptokurtotic properties implying the advanced likelihood of taking extreme values, such as during financial crisis. Apart from DFY and DE, all predictors are only slightly skewed, either positively or negatively, indicating that the variables are approximately normally distributed.

4 Methodology

In this section, we discuss the main methodological approaches implemented in our experiments. In brief, we describe the forecasting process of realized volatility and the main models used with the aim of enhancing the forecasting ability of our predictors. We apply simple combination forecasts models and principal components to the individual forecasts at hand. The reason is bifold, first, each set of predictors is able to capture different type of information, and the second is based on the premise that the forecaster is unable to have a perfect insight on the true model beforehand. At the end of the section, we report the evaluation methods of our forecasts.

4.1 Predictive Regression

Following the literature, we begin our analysis by using simple bivariate predictive regressions to test whether technical indicators or macroeconomic variables are effective predictors of realized volatility. Given that RV time series exhibits persistence and experience strong counter-cyclical
moves (see among others Schwert, 1989b; and Cenesizoglu and Timmermann, 2012), we include the autoregressive terms. The predictive regression considered in our framework takes the following form:

$$RV_t = a_j + \sum_{i=1}^{p} b_{j,i}RV_{t-i} + \beta_j X_{j,t-1} + u_t$$

(2)

where $X$ is the candidate predictor ($j = 1,\ldots,28$) under consideration and $u$ is the zero-mean error term. The optimal number of lags $p$ is selected according to the $R^2_{adj}$ criterion among a maximum number of 6 lags. The coefficients of the regression are estimated via OLS method. Hence, the forecaster takes into account all available information from July 1950 up to time December 1965 in order to generate the one-month-ahead forecast.\textsuperscript{11} For the second forecast, the investor updates her information set by adding one more observation. The process is iterated until the end of the out-of-sample period. We test the performance of our predictors over a substantial period of time, starting from January 1966. The forecasting regression is given by:

$$\hat{RV}_{j,t} = \hat{a}_j + \sum_{i=1}^{p} \hat{b}_{j,i}RV_{t-i} + \hat{\beta}_j X_{j,t-1}$$

(3)

We select an AR($p$) process as the benchmark model in order to compare our out-of-sample results, such as:

$$\hat{RV}_t = \hat{a} + \sum_{i=1}^{p} \hat{b}_i RV_{t-i}$$

(4)

4.2 RV Signal Decomposition

Wavelet analysis is a powerful tool to decompose time-series data into orthogonal components with different frequencies. Following Caraiani (2017) and Faria and Verona (2020), to our dependent variable we apply wavelet decomposition analysis with the use of maximal overlap discrete wavelet transform (MODWT) and Haar wavelet filter with reflecting boundary conditions.\textsuperscript{12,13} Thus, we concentrate on isolating the information of the candidate predictors on the frequency components. We also apply 6 levels of Multiresolution Analysis (MRA). Hence, we break the initial series into 6 wavelet details, plus the smooth component. The new series capture gradually more and more abrupt changes as the level increases, while the smooth detail works more as a long trend/memory series, such as:

$$RV_t = RV_t^{L1} + RV_t^{L2} + RV_t^{L3} + RV_t^{L4} + RV_t^{L5} + RV_t^{L6} + RV_t^S$$

(5)

\textsuperscript{11} The first 6 observations are used for the selection of the autoregressive term.

\textsuperscript{12} Risse (2019), Faria and Verona (2018a, 2018b) use wavelet decomposition on the predictor variables.

\textsuperscript{13} Berger (2016) argues that Haar filtering is optimal for such applications, since it is not contaminated with future information.
where \( L \) denotes the level; lower levels capture lower frequencies and \( S \) is the smoothing detail. We split the frequency into 3 aggregated frequencies, short, medium and long. The short signal (SS) is calculated as \( RV_t^{SS} = RV_t^{L1} + RV_t^{L2} \), the medium (MS) as \( RV_t^{MS} = RV_t^{L3} + RV_t^{L4} + RV_t^{L5} \), and finally, the long signal (LS) as \( RV_t^{LS} = RV_t^{L6} + RV_t^S \). In order to remedy the process from forward looking bias, we form the components per iteration \( t \). The forecaster generates signals takes into account only the information available at time \( t \) in order to generate \( \hat{RV}_t^{SS} \), \( \hat{RV}_t^{MS} \) and \( \hat{RV}_t^{LS} \), respectively.\(^{14}\)

\[
RV_t^f = \sum_{k}^K RV_t^{Lk}
\]  

where \( f \) denotes the frequency of the signals, i.e. short, medium, long, \( k \) and \( K \) show the lowest and highest levels' number that have been taken into account per signal (\( K = 7 \), 6 levels plus the smooth component \( S \)), as described above.\(^{15}\)

To provide a further insight regarding the frequency series over the entire sample period, we plot the three components. Figure 2 allows the reader to visualise the path of each signal, \( RV_t^f \). We see that the abrupt changes become smoother as the signal becomes longer. Whereas, the more abrupt jumps are clearer in terms of clustering and magnitude during crisis periods.

\[\text{FIGURE 2 AROUND HERE}\]

4.3 In-Sample Estimates

We use the univariate prediction regression, as shown in equation (??) in order to test whether technical indicators and macroeconomic predictors are able to provide significant in-sample estimates and increase \( R^2 \) relative to the benchmark. In-sample analysis provides evidence of goodness of fit. We use Newey-West t-statistics that are robust to heteroskedasticity and autocorrelation.\(^{16}\) Nevertheless, the large sample size is sufficient to substantially ameliorate any size-related bias.

4.4 Forecasts Combinations and Principal Components

Out-of-sample analysis tests the performance of a model or predictor by using the information set prior to the point of time that will be forecasted. This performance is of major importance for financial applications. Moreover, in-sample estimates can suffer from overfitting or structural breaks within the used time span. We illustrate the method for RV below and we follow the same approach for the frequency component forecasts, when aggregating information.

\(^{14}\)To shed more light on the forecasting process in favour of replicability, the forecaster uses the information set from July 1950 until December 1965 in order to forecast the actual signal for January 1966. The latter signal, January 1966, is calculated after accounting all available information from July 1950 to January 1966.

\(^{15}\)In a relatively similar setup, Faria and Verona (2020) apply weights on the forecasted frequency components.

\(^{16}\)We do not employ the Inoue and Kilian (2004) recommendation due to the fact that we do not impose any sign constraints on the predictors.
4.4.1 Forecast Combinations

Forecast combination is a simple but popular technique in forecasting literature (see among others Jordan et al., 2017 & 2014; Rapach et al., 2010; Rapach and Strauss, 2010 & 2008; Timmermann, 2018 & 2008) and, particularly, in realized volatility forecasting (e.g. Paye, 2012). It has been argued that this method improves forecasts by aggregating information from different type of predictors, rather than solely trusting the performance of individual predictors. We begin by using naive forecast combinations by equally weighting the forecasts of each predictor, which often perform well when the weights have estimation error, such as:

\[ \hat{RV}_{POOL,t} = \sum_{j=1}^{N} w_j \hat{RV}_{j,t} \] (7)

where \( \hat{RV}_{POOL,t} \) is the combined forecast for a set of predictors \( q = TECH, MACRO, ALL \), the ex ante allocated weight on the forecast of each rule is calculated as \( w_j = \frac{1}{N} \), \( N \) is equal to 14 candidate predictors per group \( q \) and \( \hat{RV}_{j,t} \) is the generated forecast from predictor \( j \). We refer to these forecasts as \( RV - POOL \).

4.4.2 Median

Following Li et al. (2015) and Cuaresma et al. (2018), we consider an additional simple rule of combining forecasts by using the median of the forecasted values for each time \( t \), so that:

\[ \hat{RV}^q_{MEDIAN,t} = \text{median}[\hat{RV}_{j,t}] \]

where \( \hat{RV}_t \) is the 1xN vector containing the entire set of forecasts for the individual predictors \( j = 1, \ldots, 14 \) corresponding to group \( q \) at time \( t \).

4.4.3 Trimmed Combination Forecasts

The last combination forecasts method employed is the Trimmed mean (TRIM) combination forecasts (among others see Cuaresma et al., 2018; and Della Corte & Tsiakas, 2012). Initially, we sort all forecasts, for each time period \( t \), and discard the upper and lower 3 forecasts. In this way, we exclude the extreme values that might have a severe impact on naively combined forecasts. Afterwards, we take a simple average of the remaining individual forecasts, so that:

\[ \hat{RV}^q_{TRIM,t} = \frac{1}{N-n} \sum_{j=n+1}^{N} \hat{RV}_{j,t} \] (8)

where \( \hat{RV}_{j,t} \) the \( n + 1 \) up to the \( N - n \) predictor of group \( q \), after being sorted according to the returns per time \( t \), and \( n = 3 \).

### 4.4.4 Principal Components

Despite the fact that we do not account for an extremely large number of predictors, it would still be very useful to extract only the useful information from the entire set by applying principal components (for a similar setup see among others Panopoulou and Souropanis, 2019; Buncic and Tischhauser, 2017; Çakmakli and van Dijk, 2016; Neely et al. 2014; Ludvigson and Ng, 2007). Principal components are able to exploit comovements within a large set of predictors by creating new uncorrelated predictors. By construction, the initial components filter out most noise and every new component contains less and less useful information. The forecasts are generated such as:

\[
\hat{RV}^{q}_{PCA,t} = \hat{a} + \sum_{i=1}^{p} \hat{b}_{c,i} RV_{t-i} + \hat{\beta}_c \sum_{c=1}^{C} \hat{F}^{q}_{c,t-1}
\]

(9)

where \( \hat{F}^{q}_{c,t} \) is the optimal number of the first \( k \) principal components corresponding to group \( q \). In order to select the optimal number of components, we employ the AIC Information Criterion after considering the performance of the first four components, namely \( C \) ranges between 1 and 4 components.

### 4.5 Statistical Evaluation

We assess the forecasting performance by comparing the predictive model OOS forecasts (i.e. the forecasts generated from equations (??) and (??)-(??)) against those of the AR benchmark (equation (??)). For these purposes, we employ the \( R^2_{OOS} \) statistic of Campbell and Thompson (2008). Positive values indicate that the forecasts of the rival models greater forecasting accuracy than the compared benchmark, namely technical indicators able to improve the forecasting performance of a simple autoregressive process.

Moreover, the competing models are both linear and nested. Hence, we use the adjusted Mean Squared Error proposed by Clark and West (2007; denoted as CW onwards) to test the null hypothesis that the benchmark’s MSFE is less or equal than the rival’s one, against the one-sided alternative hypothesis that the benchmark’s MSE is greater than the rival’s, such as:

\[
MSE_{adj} = \frac{1}{F} \sum_{t=M+1}^{T-1} \{(RV_t - \hat{RV}_{t,b})^2 - (RV_t - \hat{RV}_{t,j})^2 + (\hat{RV}_{t,b} - \hat{RV}_{t,j})^2\}
\]

The t-statistic of \( MSE_{adj} \) is obtained by regressing

\[
\hat{f}_t = (RV_t - \hat{RV}_{t,b})^2 - (RV_t - \hat{RV}_{t,j})^2 + (\hat{RV}_{t,b} - \hat{RV}_{t,j})^2
\]
on a constant. The CW compares the forecast accuracy after correcting for the bias introduced from the additional parameters included in the regression, which might not reflect better forecasting performance, necessarily. We denote the forecast series generated by the benchmark with $b$ and those by the rival with $j$.

5 Empirical Results

The purpose of the section is twofold. First, we provide evidence on the sources of the contribution of technical indicators in RV forecasting and compare it to macroeconomic predictors. Second, we implement a model that exploits information at different time frequencies from both types of predictors to examine whether this can generate superior forecasts.

We report our empirical findings based on the in-sample and out-of-sample performance. For this purpose, we decompose RV by frequency in order to identify the signals that play most significant role in the forecasting process. Some frequency components might prove to be more easily predictable than others due to the theoretical principles, on which we derive technical indicators, as short-term trend following predictors. On the other hand, macroeconomic predictors tend to be slower moving and may capture cyclical behavior at longer frequencies. The last part of our exercise is based on the timing of the forecasts. It has been pointed out in the literature that predictors tend to change forecasting performance during different time periods, different economic states and different sub-samples. To this end, we check the forecasting performance against different sub-samples, economic conditions and levels of volatility.

5.1 In-Sample Results

In-sample analysis might suffer from drawbacks, as briefly discussed in previous subsection, but they are still useful in validating the out-of-sample results and the use of the model at hand. They also provide an initial view of the relationship of interest and importantly are informative about the sign of the relationship. In our experiment, the estimation sample spans from July 1950 to December 2018.\footnote{We use the first 6 observations in order to calculate and select the autoregressive terms.} With respect to the in-sample estimates as reported in Table 2, we see that most technical indicators have a negative impact on RV, namely positive signals tend to reduce volatility in returns.

The sign of the coefficient is in line with the existing literature that points out the asymmetric nature of volatility, namely its tendency to decrease during upward movements in returns. The results advocate a negative relationship which is consistent with the sentiment channel (Smith et al., 2016), the market crash channel (Hong and Stein, 2003) and the business cycle channel (Hamilton and Lin, 1996). In addition, it is clear that the autoregressive terms play a substantial role. However, it is also clear that the $R^2$ is significantly improved by adding a technical indicator.
With respect to macroeconomic predictors as we see in Panel B of Table 2, the autoregressive terms demonstrate similar behavior in terms of magnitude and statistical significance of the coefficients. Moreover, we observe that more than half of the coefficients are significant. From the latter, most predictors have a negative sign. Overall, the interpretative power of macroeconomics is lower than that of technical indicators, as it can be seen in the last column of the table.

5.2 Out-Of-Sample Results

Our objective is to comprehensively study the forecasting performance of technical indicators and macro variables for realized volatility. For the empirical analysis, we use the initial 185 monthly observations, by additionally accounting for the RV lags, we use 15 years of data\textsuperscript{19}, for training the forecasting model and then we recursively add one more observation until the end of the sample. Hence, we allow our model to use only information up to time time \( t \) in order to generate the forecast for \( t + 1 \).

We replicate the experiment calculating the forecasts using a rolling window of 185 observations in order to understand whether the results are specification free.\textsuperscript{20} Rolling regressions, by construction, might lose relevant information from the beginning of sample but are more efficient in exploiting abrupt shifts in the information set. Table 3 illustrates the results for both, recursive and rolling windows.

We observe in Table 3 that technical indicators are able to outperform the benchmark and improve the forecasts by as much as 2.52\% and 1.99\% for the recursive window and 2.45\% and 2.56\% for the rolling window. However, we see that the EMA and RSI(14) predictors do not provide significant improvement in terms of \( R^2_{OOS} \) nor gains in terms of the CW test.

From the fourth column of Table 3 onwards, we present the respective results for macroeconomic predictors. We observe that macroeconomic predictors are not able to demonstrate improvements in the forecasting performance above 1\%, except for DFY that generates gains of 1.05\% against the benchmark. The combination forecasts for the macro variables provide slightly stronger out-of-sample \( R^2_{OOS} \) compared to technical indicators. Nevertheless, even given the positivity of the results, we still need to examine whether this forecasting performance can be further enhanced and also firstly how consistently gains are generated over time.

\textsuperscript{19}The time span is in line with Neely et al., 2014
\textsuperscript{20}There is a series of studies, among the most recent see Baetje and Menkhoff (2016) and Wei et al. (2019), that are concerned regarding the impact of the information set’s window of the information set. To this end, Baetje and Menkhoff (2016) introduce a rolling-recursive setting. However, Rossi (2013), focusing on exchange rate forecasting, argues that the selection of window forecasting scheme does not seem to play a vital role in the forecasting performance of a model.
Next, we illustrate the path of the forecasting performance over the entire out-of-sample period for both TECH and MACRO to see how this evolves over time. To do this, we plot the Scaled Net Cumulative Squared Errors (SNCSE):

\[
SNCSE = \frac{\sum_{1}^{t}(RV_{t} - \hat{RV}_{b,t})^2 - \sum_{1}^{t}(RV_{t} - \hat{RV}_{j,t})^2}{\sum_{1}^{P}(RV_{t} - \hat{RV}_{b,t})^2}, \text{ where } t = 1, ..., P
\]  

(10)

Figure 3 shows that technical indicators are a relatively stable predictor of realized volatility. We see that all predictors are able to generate gains relative to the benchmark during turbulent periods. The first half of 2000 is also a good period for technical indicators. Actually, these periods provide the majority of their gains, rather than following a stable upward trend. Other than this, we can conclude that it is a group with stable performance over time, demonstrating marginal losses in terms of performance during some periods.

[FIGURE 3 AROUND HERE]

From Figure 4, we observe that macroeconomic predictors demonstrate more volatile performance over the entire out-of-sample periods. In general, there are particular periods of time that have an impact on the entire group of predictors. For example, the 1973-1975 period benefits the rivals against the benchmark. This is even more clear in the events of Black Monday, in 1987. Regarding the subprime mortgage crisis of 2008, there a few candidates that demonstrate superior performance compared to the benchmark. Regarding the performance of POOL, Median and Trim3 forecasts, we observe that they benefit significantly at the beginning of the sample and stabilize their performance afterwards. On the other hand, PCA demonstrate more volatilite behavior by generating large gains in some months but also substantial losses in other.

[FIGURE 4 AROUND HERE]

5.3 Sources of Performance

In this section, we will try to understand and identify the components that play a vital role in the forecasting process. We implement wavelet decomposition, a novel method in RV forecasting, in order to shed light on the ability of technical indicators and macroeconomic predictors to forecast the intrinsic frequency signals of RV, the dependent variable. This enables us to examine whether forecasting gains are driven by specific frequency components of volatility. Moreover, we gain further insight on whether RV modelling can be enhanced by aggregating the forecasts of separate frequency components by the two types of predictors. Last, we provide evidence in favor of economically informed modelling rather than a purely statistical approach when aggregating forecasts.
5.3.1 RV Signal Decomposition

This technique will help us further understand the sources of performance for both groups of predictors. The methodology will shed light on the frequency components of RV that technical indicators and macroeconomic predictors are able to capture better. By construction, technical indicators are trend following rules that abide by short-term signals. Hence, we would expect to see that they do better with short and medium signals than long ones. Motivated by Engle et al. (2013)\textsuperscript{21}, we anticipate that macroeconomic predictors will be able to forecast the long-term component of the series better than the shorter term component. Hence, intuitively the two sets of predictors provide complementary information at different frequencies and thus combining them together should lead to enhanced performance.

The forecasting process for SS, MS and LS is given by:

\begin{equation}
\hat{RV}_{t} = \hat{a}_{j} + \sum_{i=1}^{p} \hat{b}_{j,i} RV_{t-i} + \hat{\beta}_{j} X_{j,t-1}
\end{equation}

Faria and Verona (2018a) build on the work of Ferreira and Santa-Clara (2011) by proposing the sum of some of the forecasted decomposed parts. We follow a similar approach to Faria and Verona (2018a) but implement it in a novel application to RV. Given the equation (11), the sum of forecasts of decomposed parts should approximate the actual RV series, such as:

\begin{equation}
\hat{RV}_{t,j} = \hat{RV}^{SS}_{t,j} + \hat{RV}^{MS}_{t,j} + \hat{RV}^{LS}_{t,j}
\end{equation}

We impose rational constraints on both groups of predictors that allow technical indicators to capture information from the short and medium frequency components and the long term component is captured by an AR(p) process. The purpose of this constraint is that we do not anticipate technical indicators to contain information about the long term signal, and secondly, if we model the long frequency component being determined by technical rules then we will simply obtain the same results as without the decomposition.\textsuperscript{22}

The results of equations (11) and (12) are summarised in Table 4, below. We observe a significant improvement in the forecasting performance both in terms of $R^2_{OOS}$ and CW statistic. Almost all predictors are statistically significant at 1% and 5% level and improve significantly the forecasting performance against the benchmark (equation (11)) up to 5.04%. We find the forecasting performance is much stronger in terms of magnitude for the short signal than for the medium signal. This makes intuitive sense given that technical analysis by its very nature is looking at the near term and consequently it is anticipated that its effects would be more

\textsuperscript{21}With the use of spline GARCH and MIDAS filters, Engle et al. (2013) argue that the long-term volatility component is captured by macroeconomic variables, such as inflation and industrial production growth.

\textsuperscript{22}We note that by regressing each level $y$ on some variable $x$, and, afterwards, summing all the slope coefficients, we get the same coefficient as if we regressed the original $y$ on $x$. This is the underlying reason that we anticipate POOL and Median to deliver the same results after summing the forecasts of the decomposed series as those prior to the decomposition.
In the case of macroeconomic predictors, we impose rational constraints, as we did with technical indicators. Hence, we allow macroeconomic predictors to forecast the medium and long signal whereas the short signal is captured by an AR(p) process. Intuitively, macroeconomic predictors capture long term dynamics of the RV series. It is interesting to see that macroeconomic predictors are able to forecast the medium and more emphatically the long frequency component of the RV series. Comparing the results with those of Panel A of Table 4, we see that the two sets of predictors demonstrate a complementary type of behavior in forecasting the inner dynamics of the original series.

We see that the short signal is the most volatile, as expected due to its construction, which means that it will have the greatest influence on the amalgamated forecast. Technical indicators are able to capture this respective information effectively. However, the AR(p) process as shown in the first column of Panel B, seems unable to benefit the forecasts of Macroeconomic aggregates, dragging the results downwards. With respect to the long frequency, it does not affect the performance of technical indicators too much, since the long signal of RV fluctuates gently, as can be seen from Figure 2. Thus the aggregate forecast is mainly determined by the medium signal and, especially, by the short signal. In addition, the benchmark does relatively well on the amalgamation forecast, since its signal is opposite to that of the medium frequency and reduces the magnitude of the summed errors.

5.4 Aggregating Information

5.4.1 Aggregating Information; statistical modelling approach

In this part of the paper, we provide further evidence in favor of RV forecasting by aggregating information from the entire data set, i.e. including both technical indicators and macroeconomic predictors. We model RV forecasting by considering that both types of predictors capture different type of information, isolated in a frequency component. As discussed in a previous part of the paper, intuitively, technical indicators and macroeconomic predictors should be able to provide complementary information regarding the frequency components of RV. Here, we are taking a purely statistical approach and investigating whether the methods are able to exploit the information contained at different frequencies by the set of predictors. Hence, by aggregating the respective forecasts, we should be able to benefit from improving the accuracy of our forecasts.
We are following a similar two-step approach as in section 5.3.1, but instead, we are including all the predictors to forecast each frequency using all of the data for each of the combination techniques. In the next step, we sum together the forecasts for each of the frequency components.

In the first column of Table 5, we see that the results demonstrate a deterioration in the forecasting ability of all methods apart from pooled forecasts. The latter increase to 1.56% from 1.17% and 1.28%, when considering TECH and MACRO groups, respectively, as shown in Table 3. Relatively similar behavior has been documented in the return forecasting literature.\textsuperscript{23,24}

We observe the same type of increase in the predictive ability of PCA improves the rival PCA generated but any of the two types of predictors, showing that the two groups incorporate different type of information. The second most important result is linked to the frequency component aggregates, since the forecasting performance is superior to the benchmark by 2.57%, as shown in the last column of Table 5. The aforementioned result provides strong evidence that the frequency components contain different types of information that can be captured by each set of predictors.\textsuperscript{25}

\textbf{5.4.2 Aggregating Information; economically motivated approach}

In this part of the paper, we provide further evidence on RV forecasting from both types of predictors. In contrast to the previous section where we employed a purely statistical approach, in this section we impose economically motivated restriction in the spirit of Campbell and Thompson (2008). As discussed in a previous part of the paper, intuitively, technical indicators and macroeconomic predictors should be able to provide complementary information regarding the frequency components of RV. Here we impose that the short signal is based on technical indicators and the long signal is based on macro-financial predictors with the medium component based on both series. By following this approach of considering the frequency nature of the predictor, we should be able to enhance our forecasts.

The results demonstrate clear improvement for all methods under consideration. All forecasts are statistically significant at 1% level. Moreover, there is an overall improvement not only against the respective results, as shown in Tables 3 and 4, but mainly, they are able to outperform those generated with the use of principal components as discussed in the previous section, in Table 5. For example, PCA boosts its performance with an OOS gain of 2.96% increased from 0.08% under the statistical approach (and less than 2.30% in Tables 3 and 4). Qualitatively, we observe similar performance for the remaining three combination forecasting methodologies by

\textsuperscript{23}For instance, see Panopoulou and Souropanis (2019)
\textsuperscript{24}To our knowledge, this is the first study considering both types of predictors in RV forecasting.
\textsuperscript{25}There is a remarkable difference between the performance of the predictors per frequency and the respective aggregate forecast, especially for PCA for the long frequency. This can be attributed to a few outliers, during crisis periods, especially during the recent global financial turbulence, that generate substantial errors in the forecasts. These errors are alleviated when aggregating.
comparing their performance against the one displayed in earlier tables. Thus, overall there is strong support for the use of amalgamating forecasts using the economic approach.

[TABLE 6 AROUND HERE]

To give further insight on the behavior of the economically motivated forecasted series, we present the respective SNSCE. We see that all predictors exhibit a clear upward path over time demonstrating substantial gains over the benchmark. Despite the fact that the crisis tend to favored the rival models, the subprime mortgage crisis of 2008 did not aid their performance. Overall, all methods generally show a gradual positive performance over time, with strong gains around crash periods (Cold War political turbulances of early 1970s, 1987 crash; dot-com bubble; to some extent the recent financial crisis).

[FIGURE 5 AROUND HERE]

5.5 Do Financial Conditions Matter?

5.5.1 Performance during Recessions and Expansions

In this section, we check the validity of our model during expansionary and recessionary periods, the importance of which has been outlined by the literature. There is almost uniformity in the conclusions supporting better forecasting ability during recessions.26

To further understand the impact of expansions and recessions, we employ the modified $R^2_{OOS}$, so that

$$R_c^2 = 1 - \frac{\sum_1^P (RV_t - \hat{RV}_t)^2 I_c}{\sum_1^P (RV_t - \hat{RV}_{b,t})I_c^2}, \quad c = \text{expansion, recession}$$

where $I_c^r$ is a dummy variable that takes values 1 for expansion periods and 0 for recessions. We use the business cycle dates reported by National Bureau of Economic Research (NBER). Hence, we measure the predictors’ performance on the period the forecast outcome occurs.

Overall, the CW statistic is significant for both periods, as reported in the first and second column of Table 7, offer some support to the prevailing argument that forecasting performance of technical indicators is higher during expansion periods, an outcome attributed to the trends created during these periods. Moreover, the results show a change in the magnitude of the performance across expansions and recessions for both technical indicators and macroeconomic predictors.

Moreover, we observe that by aggregating information in Panels C to E, there is a significant improvement in the performance during recessions. Last, it is clear that the overall performance of our proposed economically motivated model outperforms all rivals, especially during recessions, leading to gains as high as 18.52% for PCA.

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26For example see Neely et al. (2014).
5.5.2 Performance during High and Low VIX periods

In order to further understand the performance of technical indicators, we use the Chicago Board Options Exchange Market Volatility Index (VIX) to measure the performance of the predictors during periods of high/low financial stress, as indicated by VIX (“fear index”). The Index has been used as an alternative sentiment metric (e.g. Smith et al., 2016; Barunik et al., 2016). An increase in VIX implies that the market is expecting higher future volatility.\(^\text{27}\)

However, the Index has only recently been developed, so there are not enough observations to support the experiment for the entire sample period. Thus, we use the available data spanning from January 1986 to December 2018.

Following Smith et al. (2016), the classification of high- and low-VIX periods is made according to the value of the index at time \(t\) compared to the sample median, such as:

\[
VIX_{-\text{period}_t} = \begin{cases} 
\text{high if } VIX_t \geq \text{median}(VIX_{1:t}) \\
\text{low if } VIX_t < \text{median}(VIX_{1:t})
\end{cases}
\] \hspace{1cm} (14)

We use the observations up to December 1994 in order to identify High and Low VIX periods, as calculated from equation (14). Hence, the out-of-sample period starts in January 1995, namely 289 observations. The results are reported in columns 3 and 4 of Table 7.

Technical indicators are able to highly improve their performance against the benchmark during periods of low stress, whereas macroeconomic predictors fail to demonstrate any gains in terms of CW statistic in any of the two periods. The results for technical indicators contradict those provided by Smith et al. (2016), since the authors show that technical traders enjoy better market-timing during high-sentiment periods. The amalgamation results in panels C to E indicate consistent outperformance of the benchmark during high VIX states. We also note that our proposed model with the economically motivated forecasts (Panel E) generally perform better than the statistically motivated forecasts. In addition, the timing of higher gains shifts from low to high VIX periods.

5.5.3 Performance during Alternative OOS Periods

Investigating further the effect of timing in the performance of the predictors, we demonstrate their effectiveness during different out-of-sample periods (1983:1-2018:12 and 2000:1-2018:12). In columns five and six of Table 7, we illustrate the results for the candidate predictors by starting the out-of-sample period from 1983 and 2000. Chen and Vincent (2016) argue that there are several turning points in the S&P 500 Index that have different amount of duration. Hence, in order to see whether these turning points have an actual impact on the forecasts, we begin the out-of-sample period from two different points in time that incorporate some of these

\(^{27}\)For an alternative investor sentiment index see Baker and Wurgler (2007).
turning points.\textsuperscript{28,29}

Technical indicators outperform the benchmark in both sub-periods. However, macroeconomic predictors have weaker performance in the sub-periods. Nevertheless, amalgamation forecasts consistently generate substantial gains. Our proposed method (panel E) based on economic constraints generally outperforms the other methods. Most importantly, it is able to maintain their leading position and increase it even further, when beginning the out-of-sample period from 2000. This is important for two reasons. First, it alleviates the issue of model selection since the models are almost equally well performing during the different sub-samples. Second, it clearly demonstrates that the good performance is robust over time rather than being driven by particular time periods.

[TABLE 7 AROUND HERE]

5.6 h-month-ahead forecasts

In this section we test the robustness of our approach to alternative forecast horizons. Thus, we seek to ascertain whether RV can be forecast at horizons greater than one step ahead. Specifically, we examine their performance over h-month ahead forecasts for 3 months, 6 months and 12 months. For example, a six-month-ahead forecast of the RV for 1966M1 is made using data available up to 1965M07. This will enable us to verify the approach we propose which is based on frequency decomposition of RV works consistently for different forecast horizons.

In Table 8, we observe that the $R^2_{OOS}$ for technical indicators is negative, irrespective of which period is forecasted, which is in line with the fact that technical indicators provide information around short-term movements. On the other hand, macroeconomic variables demonstrate more robust performance in forecasting longer horizons, as well as they prove to be winners in all three specifications, against their rivals. Overall, most methods do better when forecasting 6 months ahead.

However, the results of the economic constraint, as reported in the last column of the Table, are qualitatively similar to its rivals in six-month horizon, but fails to systematically generate superior forecasts, but remains statistically significant in most cases. This provides additional evidence regarding our proposed model. Taking into account both types of predictors can generate not only gains in short-term forecasting but also at longer horizons.

[TABLE 8 AROUND HERE]

\textsuperscript{28}Chen and Vincent (2016) argue that these points are Feb. 1966 (8 months), Nov. 1968 (18 months), Jan. 1973 (21 months), Sep. 1976 (18 months), Nov. 1980 (21 months), Aug. 1987 (4 months), Jul. 1990 (3 months), Mar. 2000 (31 months) and Oct. 2007 (17 months).

\textsuperscript{29}The theoretical basis of this evolution is the Adaptive Market Hypothesis, proposed by Andrew Lo (2004), according to which, market allowed for excess returns for a particular period of time (until 1996 according to Sullivan et al., 1999), but adjusted later on. For relative literature, see among others, Neely et al (2009), Taylor (2014), Park and Irwin (2007) and Sullivan et al. (1999).
5.7 Leverage Effect

In this part of the paper, we test the so called “leverage effect” on the forecasting performance of the proposed approach. In brief, the literature has established a negative relationship between market returns and volatility, as it has been described in previous section. Our objective is to identify whether the forecasts of the proposed methods contain distinct information from market movements. For this purpose, we employ the following two-step process. First, we generate the forecasts per frequency $f$ and then we aggregate them, as described in section 5.3.1, 5.4.1 and 5.4.2.\(^{30}\)

\[
\hat{RV}_t^f = \hat{a}_j + \sum_{i=1}^{p} \hat{b}_{j,i}RV_{t-i} + \hat{\beta}_jX_{j,t-1} + \hat{\gamma}EP_{t-1}
\]

where $EP$ denotes the equity premium as control variable.

The results reported in Table 9, below, show an improvement in the forecasting performance compared to the respective results in Tables 4, 5 and 6. Moreover, the results remain qualitatively the same, i.e. we do not observe any significant improvement in the performance of any of the specifications under consideration. There is strong evidence that the equity premium contains separate information from the predictors under consideration. Thus, the positive performance of technical indicators cannot be attributed to the leverage effect

[Table 9 around here]

6 Economic Performance

In this part of the paper we will evaluate the volatility forecasts in terms of economic performance. The applicability of the proposed RV forecasting setup has economic interest, as well. Hence, testing for forecasting accuracy and economic significance work complementary to each other. For this purpose, we isolate the volatility effects and formulate the portfolios based on the volatility managed portfolios framework. Given the large number predictors and specifications, we report merely the aggregate forecasts ($POOL, TRIM3, MEDIAN, PCA$) of sole technical indicators, macroeconomic predictors, all considered together, frequency aggregates of technical indicators and macroeconomics, as well as, the economically motivated approach.

We consider a US based investor that rebalances her portfolio at the end of each month $t$ between the risky asset and the risk free. We assess the economic performance of the volatility

---

\(^{30}\)In brief, the forecasts of the first column are generated after employing technical indicators to forecast the short and medium signals (we use the AR($p$) process to forecast the long signal), those of the second column are generated after employing macroeconomic predictors to forecast the medium and long signals (we use the AR($p$) process to forecast the short signal), the forecasts in the third column are generated by employing both groups of predictors without decomposing RV, the forecasts of the next column are generated by using all predictors to forecast all three signals and amalgamate the forecasts afterwards, and last, we employ the economically motivated approach.
forecasts by employing the most common metrics, namely the Certainty Equivalent Return (CER) and Sharpe Ratio (SR), as well as, the end-of-period portfolio return. The CER is defined as:

\[
CER = \hat{r}_p - \frac{\gamma \sigma^2_p}{2}
\]  

(16)

where \( \hat{r}_p \) and \( \sigma^2_p \) is the mean and variance of the portfolio return over the out-of-sample period and \( \gamma \) is the level of risk aversion, we use \( \gamma = 5 \). CER can be interpreted as a performance fee that our investor is eager to pay, in order to switch the portfolio from the risk free asset to the risky asset.

Given that estimating the economic value is not standard in the RV forecasting literature, our analysis in this part is inspired by Moreira and Muir (2017). We construct volatility-managed portfolios, since they take directly RV into account when allocating weights to the risky asset rather than adopting the standard mean-variance framework.

Our investor adjusts her weights at the end of every time \( t \), by accounting for the relative riskiness of the asset. The main benefit of this strategy, with respect to our experiment focus, is that the portfolio weights does not depend on the forecasts of returns. We compare the utility and other metrics of our portfolio against those of a buy and hold strategy.

The investor allocates weights such as:

\[
w_{t+1} = \frac{\hat{RV}_{1:t}}{RV_{t+1}}
\]  

(17)

where \( \hat{RV}_{t+1} \) is the forecast of each specification or the RV historical average, namely \( \bar{RV}_{1:t} \), when employing the buy-and-hold strategy. Thus, in periods of high forecasted RV, the investor reduces the exposure to the risky asset, and vice versa.

The results, presented in Table 10, show clearly a definite increase in every metric.31 We could also argue that the differences among the competing models is not remarkable and there are only marginal and sporadic differences. Moreover, PCA in Panels D and E deliver the lowest returns, but remarkably the less volatile portfolios. Last, we see that every specification beats the buy-and-hold strategy of around 160 and 90 basis points, without and after accounting for transaction costs, respectively. The CER of the former is equal to 4.89%. 32 Despite the modest results against the AR(p) portfolio returns, the PCA method of technical indicators, as well as, of the economically motivated approach stands out, by providing gains equal to 29 basis points.

The overall performance remains qualitatively the same when the portfolio is rebalanced

31 The results remain qualitatively the same after accounting \( \gamma = 3 \), as in Campbell & Thompson (2008).  
32 We should point out that the benchmark delivers a higher CER value than the most common benchmark in asset allocation exercises, namely the variance of a 60-months rolling window, as in Neely et al. (2014).
net of transactions costs and with transaction costs. We find evidence that the results of EA models are driven mainly by the reduced variance of the portfolio returns, rather than higher returns. PCA-EA generate the lowest variance, but also the lowest average returns per annum. On the other hand, macroeconomic predictors and technical indicators (Panels B and A) are able to deliver higher end-of-period cumulative returns, depending on accounting transaction costs or not, but macroeconomic predictors compromise the variance of the portfolio returns.

The fact that most portfolios outperform the benchmark provides evidence there is valuable information contained by the predictors rather than a single chance discovery.

7 Conclusions
Forecasting realized volatility has become very topical in academic literature during recent years. Researchers have employed various models and predictors in order to answer the question at hand. Since the seminal work of Schwert (1989b), there has been significant progress towards enhancing predictability. Our first contribution to this dialogue is to provide further insight and show the role that technical indicators play in this context. The importance of technical rules has been outlined several times by the literature, but little attention has been given to their impact on realized volatility. Our second contribution is to provide an explanation for why technical indicators and macro predictors complement each other. Namely that they provide information for RV at different time frequencies which is established via a frequency decomposition of RV. We establish strong evidence that macroeconomic predictors capture longer-term information and technical indicators capture shorter-term information about RV. Our third contribution is to demonstrate how the information contained at different frequencies can be best amalgamated into a statistical approach or can economic constraints further improve accuracy. We clearly find that using economically motivated constraints substantially enhances forecasting performance of RV by explicitly recognizing the short-term nature of technical indicators and the longer-term nature of macroeconomic variables.

In order to assess the predictive performance of technical indicators on S&P500 index RV, we employ fourteen widely used rules for a substantial period of time, starting in January 1990 until December 2018. Additionally, we use fourteen common macroeconomic predictors for RV. Furthermore, both in-sample and out-of-sample technical indicators have explanatory power for RV; the importance of technical indicators and macroeconomic series should be complementary which we establish via wavelet decomposition of RV into different frequencies. Specifically, technical rules capture shorter-term signals whereas macroeconomic predictors capture longer-term information. Our findings suggest that there are three factors accounting for the positive performance of predictors. First, the forecasts are generated by discarding noisy the economically motivated constraint. Second, technical rules have explanatory power for RV; the importance of technical indicators and macroeconomic series should be complementary which we establish via wavelet decomposition of RV into different frequencies. Specifically, technical rules capture shorter-term signals whereas macroeconomic predictors capture longer-term information. Our findings suggest that there are three factors accounting for the positive performance of predictors. First, the forecasts are generated by discarding noisy
components and take advantage merely of the frequencies that each group can theoretically have access on. Second, aggregating information from a large set of predictors, allows the model to extract information from multiple channels. In terms of timing, the positive performance is stronger during crisis periods. In addition, our results show that technical indicators do better during recessions, as suggested by the literature, and low sentiment periods, whereas the proposed amalgamation model demonstrates a smoother performance and is less affected by sporadic turbulence in sentiment or the economy.

Moreover, we assess the economic gains delivered by our RV forecasts. We construct portfolios under the volatility-managed portfolios framework. The results provide strong evidence that our framework is able to deliver significantly superior economic gains against a simple buy-and-hold strategy. Moreover, the portfolios generate higher annual returns and are less volatile. When our returns are compared with an AR(p) benchmark, we observe marginal gains in terms of utility gains.

Despite the fact that technical indicators are vastly used in the forecasting literature of returns, very little attention has been paid to the implications of technical analysis on future volatility. We anticipate that future literature on this topic will clarify more aspects of the research question at hand. We find that new tools are able to generate significant improvements in the forecasting exercise and provide better insight on different segments that indicate such a relationship. The frequency dimension of predictive variables should be considered in future applications where there is strong economic motivation since this can enhance substantially forecast power.
References


<table>
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<tr>
<th>Variables</th>
<th>Mean</th>
<th>St.Dev.</th>
<th>Median</th>
<th>Kurtosis</th>
<th>Skewness</th>
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Panel A: Technical Indicators

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<th>Kurtosis</th>
<th>Skewness</th>
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<td>0.5789</td>
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Panel B: Macroeconomic Predictors

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Notes: The table illustrates the Mean, standard deviation, median, kurtosis and skewness of realized volatility and predictors for the entire sample period starting from January 1950 to December 2018.
### Table 2: In-Sample Estimates

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<th>AR(1)</th>
<th>AR(2)</th>
<th>AR(3)</th>
<th>AR(4)</th>
<th>AR(5)</th>
<th>( R^2 )</th>
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<tbody>
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<td>-0.3961***</td>
<td>0.1376***</td>
<td>-0.0193</td>
<td>0.0983**</td>
<td>0.1308***</td>
<td>39.96</td>
<td></td>
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#### Panel A: Technical Indicators

| MA(1,9)  | -0.5088*** | 0.3800*** | 0.1559*** | -0.0266 | 0.0874** | 0.1339*** | 46.40 |
| MA(2,9)  | -0.4909*** | 0.3795*** | 0.1528*** | -0.0237 | 0.0885** | 0.1376*** | 46.14 |
| MA(3,9)  | -0.4688*** | 0.3792*** | 0.1537*** | -0.0212 | 0.0870** | 0.1342*** | 45.66 |
| MA(1,12) | -0.4446*** | 0.3777*** | 0.1543*** | -0.0223 | 0.0871** | 0.1360*** | 46.16 |
| MA(2,12) | -0.4201*** | 0.3770*** | 0.1566*** | -0.0190 | 0.0851** | 0.1351*** | 45.63 |
| MA(3,12) | -0.4153*** | 0.3757*** | 0.1553*** | -0.0193 | 0.0860** | 0.1332*** | 45.58 |

#### Panel B: Macroeconomic Predictors

<table>
<thead>
<tr>
<th>Variables</th>
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<th>AR(2)</th>
<th>AR(3)</th>
<th>AR(4)</th>
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<tbody>
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Notes: The Table reports the regression coefficients for in-sample predictive regression over the entire sample period, namely January 1950 to December 2018. The in-sample predictive regression is given by: \( RV_t = \alpha_j + \sum_{i=1}^{p} b_{j,i} RV_{t-i} + \beta X_{j,t-1} + u_t \), where RV is the Realized Volatility, \( p \) is the lagged RV and is selects the optimal number of lags, ranging from 1 to 6, via the \( R^2 \), and X is the predictor under consideration. The seventh column reports the \( R^2 \) for the full sample. The asterisks "**", "***" and "****" indicate the statistical significance at 10 %, 5 % and 1% level, respectively.
Table 3: Out-of-Sample Forecasts

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<th>$R^2_{OOS,roll}$</th>
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<th>$R^2_{OOS,rec}$</th>
<th>$R^2_{OOS,roll}$</th>
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<td>BM</td>
<td>0.17***</td>
<td>0.09**</td>
</tr>
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<td>MOM(9)</td>
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<td>-1.81</td>
</tr>
<tr>
<td>MOM(12)</td>
<td>0.68**</td>
<td>-0.24*</td>
<td>TBL</td>
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<td>-1.29**</td>
</tr>
<tr>
<td>RSI(7)</td>
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<td>-0.41*</td>
<td>LTY</td>
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<td>0.33***</td>
</tr>
<tr>
<td>RSI(14)</td>
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<td>LTR</td>
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<td>-0.50*</td>
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<td>EMA(3,9)</td>
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<td>-0.90</td>
<td>TMS</td>
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<td>-0.78</td>
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<tr>
<td>EMA(5,9)</td>
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<td>-0.89</td>
<td>DFY</td>
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<tr>
<td>EMA(3,12)</td>
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<td>-1.07</td>
<td>DFR</td>
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<td>-0.15*</td>
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<tr>
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<td>2.56***</td>
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<td>1.87***</td>
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<td>0.00**</td>
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<td>2.45***</td>
</tr>
<tr>
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<td>0.43***</td>
<td>PCA</td>
<td>0.75***</td>
<td>-1.12***</td>
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</tbody>
</table>

Notes: The table reports the $R^2_{OOS}$, which measures the reduction in MSFE, relative to the MSFE of the benchmark, $AR(p)$ model. The first and third columns denote the out-of-sample results calculated with a recursive window, and the second and fourth columns show the results for the forecasts calculated with a rolling window. We apply the CW-statistic, which tests the null that the benchmark forecast MSFE is less or equal to the regressor’s forecast MSFE against the one-sided alternative that the RW’s forecast MSFE is greater to the MSFE of its rival. "***", "**" or "*" indicate significance at the level of 1%, 5% and 10%, respectively, of the MSFE-adjusted statistic.
Table 4: Out-of-Sample Forecasts for Decomposed RV

<table>
<thead>
<tr>
<th>Panel A: Technical Indicators</th>
<th>Panel B: Macroeconomic Predictors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Var.</td>
<td>$RV^{SS}$</td>
</tr>
<tr>
<td>MA(1,9)</td>
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</tr>
<tr>
<td>MA(2,9)</td>
<td>4.43***</td>
</tr>
<tr>
<td>MA(3,9)</td>
<td>3.95***</td>
</tr>
<tr>
<td>MA(1,12)</td>
<td>4.09***</td>
</tr>
<tr>
<td>MA(2,12)</td>
<td>3.35***</td>
</tr>
<tr>
<td>MA(3,12)</td>
<td>3.15***</td>
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<td>MOM(9)</td>
<td>3.92***</td>
</tr>
<tr>
<td>MOM(12)</td>
<td>2.80***</td>
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<tr>
<td>RSI(7)</td>
<td>3.47***</td>
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<tr>
<td>RSI(14)</td>
<td>1.58***</td>
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<tr>
<td>EMA(3,9)</td>
<td>0.87***</td>
</tr>
<tr>
<td>EMA(5,9)</td>
<td>0.31**</td>
</tr>
<tr>
<td>EMA(3,12)</td>
<td>0.32**</td>
</tr>
<tr>
<td>EMA(5,12)</td>
<td>0.57</td>
</tr>
<tr>
<td>POOL</td>
<td>3.26***</td>
</tr>
<tr>
<td>MEDIAN</td>
<td>3.69***</td>
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<tr>
<td>TRIM3</td>
<td>3.45***</td>
</tr>
<tr>
<td>PCA</td>
<td>4.08***</td>
</tr>
</tbody>
</table>

Notes: The Table presents the out-of-sample performance of the candidate predictors on different frequency signals, in terms of $R^2_{OOS}$ and the CW test. The forecasts are generated as $\hat{RV}_f = \hat{a}_j + \sum_{i=1}^{p} \hat{b}_{ij} RV_{t-i} + \hat{\beta}_j X_{j,t-1}$. For the long frequency signal, we restrict the model to an AR(p) process, since we assume that the economically relationship between technical indicators and RV will be weak in long frequencies. Similarly, we constrain the short signal for the macroeconomic predictors. Also, see notes Table 3.

Table 5: Aggregating Information

<table>
<thead>
<tr>
<th>Method</th>
<th>ALL</th>
<th>$\hat{RV}^{SA}_{SS}$</th>
<th>$\hat{RV}^{SA}_{MS}$</th>
<th>$\hat{RV}^{SA}_{LS}$</th>
<th>$\sum_{f=1}^{3} \hat{RV}^{SA}_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>POOL</td>
<td>1.56***</td>
<td>2.31***</td>
<td>3.66***</td>
<td>9.60***</td>
<td>1.56***</td>
</tr>
<tr>
<td>MEDIAN</td>
<td>0.93***</td>
<td>1.08***</td>
<td>1.68***</td>
<td>3.31***</td>
<td>0.07</td>
</tr>
<tr>
<td>TRIM3</td>
<td>1.36***</td>
<td>2.00***</td>
<td>3.13***</td>
<td>9.61***</td>
<td>1.36***</td>
</tr>
<tr>
<td>PCA</td>
<td>0.08***</td>
<td>2.78***</td>
<td>13.02***</td>
<td>35.64***</td>
<td>2.57***</td>
</tr>
</tbody>
</table>

Notes: In the Table above we generate the forecasts by using both types of predictors, in terms of $R^2_{OOS}$ and the CW test. In the first column we take into account all predictors together to forecast RV. In columns two to four, we present the results under the statistically motivated approach (SA), i.e. we take all predictors into account to forecast each frequency component separately. In the last column, we aggregate the forecasts of the statistical constraint. For further details, see Tables 3 and 4.
Table 6: Aggregating Information

<table>
<thead>
<tr>
<th>Method</th>
<th>$\sum_{f=1}^{3} \hat{RV}_{f}^{EA}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>POOL</td>
<td>2.88***</td>
</tr>
<tr>
<td>MEDIAN</td>
<td>2.83***</td>
</tr>
<tr>
<td>TRIM3</td>
<td>2.99***</td>
</tr>
<tr>
<td>PCA</td>
<td>2.96***</td>
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</tbody>
</table>

Notes: Table 6 reports the results of the economically motivated approach (EA), in terms of $R_{OOS}^2$ and the CW test. The forecasts are generated by forecasting the short component with the use of technical indicators, the long component with the use of macroeconomic predictors and the medium component by averaging the respective forecasts of each of the two groups of predictors. For further details, see Tables 3 and 4.
Table 7: Forecasts during different timing in Financial Conditions

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Technical Indicators</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>POOL</td>
<td>0.83**</td>
<td>2.60***</td>
<td>1.92**</td>
<td>2.12**</td>
<td>1.29***</td>
<td>2.01***</td>
</tr>
<tr>
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<td>0.87**</td>
<td>2.69***</td>
<td>2.21**</td>
<td>2.21**</td>
<td>1.39***</td>
<td>2.26***</td>
</tr>
<tr>
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<td>0.78**</td>
<td>2.93***</td>
<td>2.07**</td>
<td>2.15**</td>
<td>1.30***</td>
<td>2.18***</td>
</tr>
<tr>
<td>PCA</td>
<td>1.60***</td>
<td>3.66***</td>
<td>1.24*</td>
<td>4.50***</td>
<td>2.29***</td>
<td>3.84***</td>
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<tr>
<td><strong>Panel B: Macroeconomic Predictors</strong></td>
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<td></td>
</tr>
<tr>
<td>POOL</td>
<td>0.64**</td>
<td>5.86***</td>
<td>0.36</td>
<td>-0.25</td>
<td>0.15</td>
<td>0.36**</td>
</tr>
<tr>
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<td>0.68***</td>
<td>4.26***</td>
<td>0.36</td>
<td>-0.32</td>
<td>0.21</td>
<td>0.17</td>
</tr>
<tr>
<td>TRIM3</td>
<td>0.62**</td>
<td>7.22***</td>
<td>0.52</td>
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<td>-0.17</td>
<td>-0.37</td>
</tr>
<tr>
<td>PCA</td>
<td>-1.76**</td>
<td>13.34***</td>
<td>-1.89</td>
<td>-3.50</td>
<td>-1.35</td>
<td>0.69*</td>
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<tr>
<td><strong>Panel C: All predictors Taken Together</strong></td>
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<td></td>
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</tr>
<tr>
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<td>0.97***</td>
<td>4.51***</td>
<td>1.28**</td>
<td>1.18**</td>
<td>0.89***</td>
<td>1.36***</td>
</tr>
<tr>
<td>MEDIAN</td>
<td>0.62***</td>
<td>2.51***</td>
<td>1.09**</td>
<td>0.47</td>
<td>0.72***</td>
<td>1.26***</td>
</tr>
<tr>
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<td>4.35***</td>
<td>1.07**</td>
<td>0.71</td>
<td>0.64**</td>
<td>1.08***</td>
</tr>
<tr>
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<td>-0.21</td>
<td>-0.39</td>
<td>1.58**</td>
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<tr>
<td><strong>Panel D: Statistically Motivated Approach</strong></td>
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<td></td>
</tr>
<tr>
<td>POOL</td>
<td>0.97***</td>
<td>4.51***</td>
<td>1.28**</td>
<td>1.18**</td>
<td>0.89***</td>
<td>1.36***</td>
</tr>
<tr>
<td>MEDIAN</td>
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<td>0.94**</td>
<td>0.87**</td>
<td>0.38</td>
<td>0.38**</td>
<td>0.61**</td>
</tr>
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<td>TRIM3</td>
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<td>4.35***</td>
<td>1.07**</td>
<td>0.71</td>
<td>0.64**</td>
<td>1.08***</td>
</tr>
<tr>
<td>PCA</td>
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<td>17.21***</td>
<td>4.62***</td>
<td>3.40***</td>
<td>2.71***</td>
<td>3.78***</td>
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<tr>
<td><strong>Panel E: Economically Motivated Approach</strong></td>
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<td>9.12***</td>
<td>2.37**</td>
<td>1.64*</td>
<td>1.45***</td>
<td>2.48***</td>
</tr>
<tr>
<td>MEDIAN</td>
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<td>7.22***</td>
<td>2.48**</td>
<td>2.08**</td>
<td>1.58***</td>
<td>2.59***</td>
</tr>
<tr>
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<td>10.31***</td>
<td>2.56**</td>
<td>1.55**</td>
<td>1.34***</td>
<td>2.04***</td>
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<td>3.24***</td>
<td>4.92***</td>
</tr>
</tbody>
</table>

Notes: The Table above present timing effects of the methods under consideration, in terms of $R_{OOS}^2$ and the CW test. The first and second column demonstrates the results for expansionary and recession periods, denoted as “Exp” and “Rec”. Columns third and fourth show the results for high- and low-VIX periods, denoted as “H-VIX” and “L-VIX”. The next three columns illustrate the results for the alternative out-of-sample periods, starting in 1983, 2000 and 2010, denoted as “OOS1983” and “OOS2000”, respectively. Also, see notes Table 3 and 4.
### Table 8: h-month-ahead

<table>
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<tr>
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<th>ALL</th>
<th>SE</th>
<th>EA</th>
</tr>
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<tbody>
<tr>
<td><strong>Panel A: h = 3</strong></td>
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<td></td>
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<tr>
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<td>0.95***</td>
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<td>0.18</td>
<td>0.11*</td>
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<tr>
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<td>0.80***</td>
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<td>-0.39</td>
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<tr>
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<td>1.07***</td>
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<td>0.23</td>
<td>0.00**</td>
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<td>0.82***</td>
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<td>-0.75***</td>
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<td><strong>Panel B: h = 6</strong></td>
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<td>1.42***</td>
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<tr>
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<td>-0.44**</td>
<td>-1.03***</td>
<td>1.25***</td>
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<tr>
<td><strong>Panel C: h = 12</strong></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td>0.22*</td>
<td>0.22*</td>
<td>0.16*</td>
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<tr>
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<td>0.47***</td>
<td>0.02</td>
<td>-0.17</td>
<td>-0.39</td>
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<tr>
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<td>1.31***</td>
<td>0.35**</td>
<td>0.35**</td>
<td>0.81***</td>
</tr>
<tr>
<td>PCA</td>
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<td>0.61***</td>
<td>-0.24**</td>
<td>-1.29**</td>
<td>0.59***</td>
</tr>
</tbody>
</table>

Notes: The Table illustrates the h-month-ahead forecasting performance of the candidate predictors, where \( h = [3, 6, 12] \), in terms of \( R_{OOS}^2 \) and the CW test. The first column, is generated with the use of technical rules. For the results of the second column we employ macroeconomic predictors. Column three presents the results after taking into account all predictors together without employing frequency decomposition. The fourth column is calculated after forecasting the decomposed series of RV and aggregating the frequency forecasts. The last column is given after employing the proposed economic approach. Also, see notes Tables 3 and 4.

### Table 9: Leverage Effect

<table>
<thead>
<tr>
<th>Predictor</th>
<th>TECH</th>
<th>MACRO</th>
<th>ALL</th>
<th>SA</th>
<th>EA</th>
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</thead>
<tbody>
<tr>
<td>POOL</td>
<td>5.39***</td>
<td>2.81***</td>
<td>5.63***</td>
<td>6.19***</td>
<td>6.92***</td>
</tr>
<tr>
<td>MEDIAN</td>
<td>5.20***</td>
<td>2.45***</td>
<td>5.58***</td>
<td>5.51***</td>
<td>6.40***</td>
</tr>
<tr>
<td>TRIM3</td>
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<td>3.01***</td>
<td>5.59***</td>
<td>6.31***</td>
<td>6.97***</td>
</tr>
<tr>
<td>PCA</td>
<td>5.40***</td>
<td>3.72***</td>
<td>3.75***</td>
<td>6.30***</td>
<td>6.60***</td>
</tr>
</tbody>
</table>

Notes: The Table reports the forecasting performance after using the equity premium (EP) as control variable, such as: \( \tilde{R}_{V_t} = \tilde{a}_j + \sum_{i=1}^{p} \tilde{b}_{ji} RV_{t-1} + \tilde{\beta}_j X_{j,t-1} + \tilde{\gamma} EP_{t-1} \). Also, see notes in Tables 4.5 and 6.
Table 10: Volatility-Managed Portfolio

<table>
<thead>
<tr>
<th></th>
<th>CER</th>
<th>cumRet</th>
<th>SR(ann.)</th>
<th>Rp(ann.)</th>
<th>σp(ann.)</th>
<th>CER</th>
<th>cumRet</th>
<th>SR(ann.)</th>
<th>Rp(ann.)</th>
<th>σp(ann.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RV_b=Buy &amp; Hold</td>
<td>4.89</td>
<td>137.23</td>
<td>0.38</td>
<td>5.69</td>
<td>14.93</td>
<td>4.89</td>
<td>137.23</td>
<td>0.38</td>
<td>5.69</td>
<td>14.93</td>
</tr>
<tr>
<td>RV_b=60m roll.</td>
<td>5.15</td>
<td>47.36</td>
<td>0.30</td>
<td>3.11</td>
<td>10.41</td>
<td>5.06</td>
<td>45.39</td>
<td>0.29</td>
<td>3.03</td>
<td>10.42</td>
</tr>
<tr>
<td>RV_b=AR(p)</td>
<td>6.00</td>
<td>132.44</td>
<td>0.45</td>
<td>5.17</td>
<td>11.54</td>
<td>5.81</td>
<td>87.71</td>
<td>0.38</td>
<td>4.39</td>
<td>11.57</td>
</tr>
</tbody>
</table>

Panel A: Technical Indicators

<table>
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<tr>
<th></th>
<th>CER</th>
<th>cumRet</th>
<th>SR(ann.)</th>
<th>Rp(ann.)</th>
<th>σp(ann.)</th>
<th>CER</th>
<th>cumRet</th>
<th>SR(ann.)</th>
<th>Rp(ann.)</th>
<th>σp(ann.)</th>
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</thead>
<tbody>
<tr>
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<td>0.45</td>
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<td>11.59</td>
<td>5.83</td>
<td>89.77</td>
<td>0.38</td>
<td>4.44</td>
<td>11.61</td>
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<tr>
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Panel B: Macroeconomic Predictors

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Panel C: All Predictors Taken Together

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Panel D: Statistical Approach, Frequency Aggregates

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Panel E: Economically Motivated Approach, Frequency Aggregates

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Notes: The Table presents the Certainty Equivalent Return (CER), cumulative return, annualized Sharpe Ratio (SR), annualized portfolio return (Rp), annualized portfolio volatility (σp). The benchmarks are a buy and hold strategy, the volatility of equity premium over a 60 month rolling window and the AR(p), as discussed in previous sections. The weights of the rival models on the risky asset are a ratio between the volatility forecast and the historical volatility. The five Panels (A-E) of the table show the economic performance of different specifications of RV forecasting, as discussed in previous sections. The first five columns show the results of economic performance without accounting any transaction cost, whereas the last five columns show the results after accounting for transaction costs equal to 30 basis points. Last, we assume that the investor’s level of risk aversion is γ = 5.
Figure 1: Monthly RV for S&P500

Notes: Figure 1 presents the time series of the monthly realized volatility for S&P 500 from January 1950 to December 2018.
Figure 2: Frequency Decomposed Signals for RV for S&P500

Notes: Figure 2 presents the time series of frequency signals of the monthly realized volatility for S&P 500 from January 1950 to December 2018.
Notes: Figure 3 presents predictive performance of technical indicators against the benchmark, in terms of the Scaled Net CSE, where \( SNCSE = \frac{\sum_t (RV_t - \hat{RV}_{b,t})^2 - \sum_t (RV_t - \hat{RV}_{j,t})^2}{\sum_t (RV_t - \hat{RV}_{b,t})^2} \). The out-of-sample period spans from January 1966 to December 2018.
Figure 4: SNCSE for macroeconomic predictors

Notes: Figure 4 presents predictive performance of macroeconomic predictors against the benchmark, in terms of the Scaled Net CSE, where $SNCSE = \frac{\sum_t (RV_t - \hat{RV}_{b,t})^2 - \sum_t (RV_t - \hat{RV}_{j,t})^2}{\sum_t (RV_t - \hat{RV}_{b,t})^2}$. The out-of-sample period spans from January 1966 to December 2018.
Figure 5: SNSCE for Frequency Aggregates

Notes: Figure 5 presents predictive performance of the economically motivated frequency aggregate forecast against the benchmark, in terms of the Scaled Net CSE, where 

\[ SNCSE = \frac{\sum_t (RV_t - \hat{RV}_{b,t})^2 - \sum_t (RV_t - \hat{RV}_{j,t})^2}{\sum_t (RV_t - \hat{RV}_{b,t})^2}. \]

The out-of-sample period spans from January 1966 to December 2018. The forecast time series are generated by summing the forecasts of the separate frequency series. The forecasts of the short frequency signal is predicted with the use of technical indicators, the long frequency is predicted with the use of macroeconomic predictors and the medium signal is an average of forecasts from both types of predictors.