

# Active and Passive (Un)conventional Monetary and Fiscal Policies for Debt Stability

Gleb Kurovskiy, Luisa Lambertini, Stylianos Tsiaras  
Ecole Polytechnique Fédérale de Lausanne

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## Abstract

Since COVID-19 pandemic there is an unprecedented increase in social insurance transfers both in the EA and the US. In this paper, we explore different fiscal and monetary strategies in times of large debt accumulation. We build a New Keynesian DSGE model with household heterogeneity, financial frictions, nominal rigidities, and an unconstrained central bank that can purchase bonds in exchange of reserves. We identify QE as a potential tool for debt stabilization. Profits earned from the bonds-reserves spread can be remitted from the central bank to the treasury and can be a substantial fiscal revenue. We analyse and compare QE as a debt stabilization tool versus government spending and taxation changes under an active (conventional) monetary policy and a passive monetary policy framework.

*Keywords:* Fiscal Policy; Monetary Policy; Quantitative Easing ; DSGE;

# 1. Introduction

Since our world was hit by the COVID-19 pandemic, there was unprecedented increase in social insurance transfers both in the EA and the US. In the US the total legislative actions due to the pandemic event had a budgetary cost of more than \$5 trillion.<sup>1</sup> As [Romer \(2021\)](#) points out, this is about four times the amount spent on the 2009 American Recovery and Reinvestment Act aimed in helping the US economy to recover from the financial crisis. There were extraordinary in level explicit or implicit transfers to households and businesses that dramatically increased the US debt to GDP and posed questions on its stability. At the same time the US monetary policy reduced its rates to zero and started an extensive Quantitative Easing programme that has accounted for asset purchases of more than \$4 trillion.

In this paper we ask how this new debt is going to be repaid and we study whether QE can provide sufficient fiscal revenues for debt stability. The fiscal revenues from QE are generated by the profits the central bank generates on the spread on the purchased bonds' return and the interest rate on reserves paid back to the financial institutions. This, given back to the Treasury as remittances can be a substantial fiscal revenue. Furthermore we study two other possible avenues for debt stabilization: classical fiscal adjustments, that is government spending and taxation that respond inversely to changes in the debt-to-GDP ratio to keep debt bounded and a passive monetary policy that accommodates higher inflation to inflate out the new debt.

To shed more light on the fiscal revenues generated from QE, [Figure 1](#) shows the central bank remittances to the treasury and the debt service-to-gdp ratio. As can be seen, during the years of extensive QE, the remittances could account for over a third of the total fiscal revenues needed for the interest payments on the debt. QE profits are, and can potentially be even more, a substantial fiscal revenue for debt stability. Importantly, QE revenues are high in times of crisis, when the interest rate on reserves is low thus widening the spread between bond returns and reserve payments.

To study those questions we develop a Two Agents New Keynesian (TANK) DSGE model calibrated to the US with nominal rigidities, financial intermediaries, a rich fiscal sector and a central bank that can purchase assets from the banks by issuing reserves. Furthermore, to accommodate a passive monetary policy framework we have partially unfunded debt similarly to [Bianchi, Faccini, and Melosi \(2020\)](#). The QE framework follows [Gertler and Karadi \(2013\)](#) but a formal representation of reserves and the asset swap mechanism induced by the QE is developed. The real economy part of the model follows closely [Bianchi et al. \(2020\)](#) and banks are modelled similarly to [Gertler and Kiyotaki \(2010\)](#): a costly enforcement problem creates a leverage constraint on the intermediaries. QE induces more lending through the relaxation of the leverage constraints of the financial intermediaries, similarly to [Sims and Wu \(2021\)](#), by the exchange of banks' government bonds with reserves and thus stimulates aggregate demand.

A setting is developed where central bank purchases of government bonds financed

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<sup>1</sup>Committee for a Responsible Federal Budget (CRFB), "COVID Money Tracker," <https://www.covidmoneytracker.org/>.

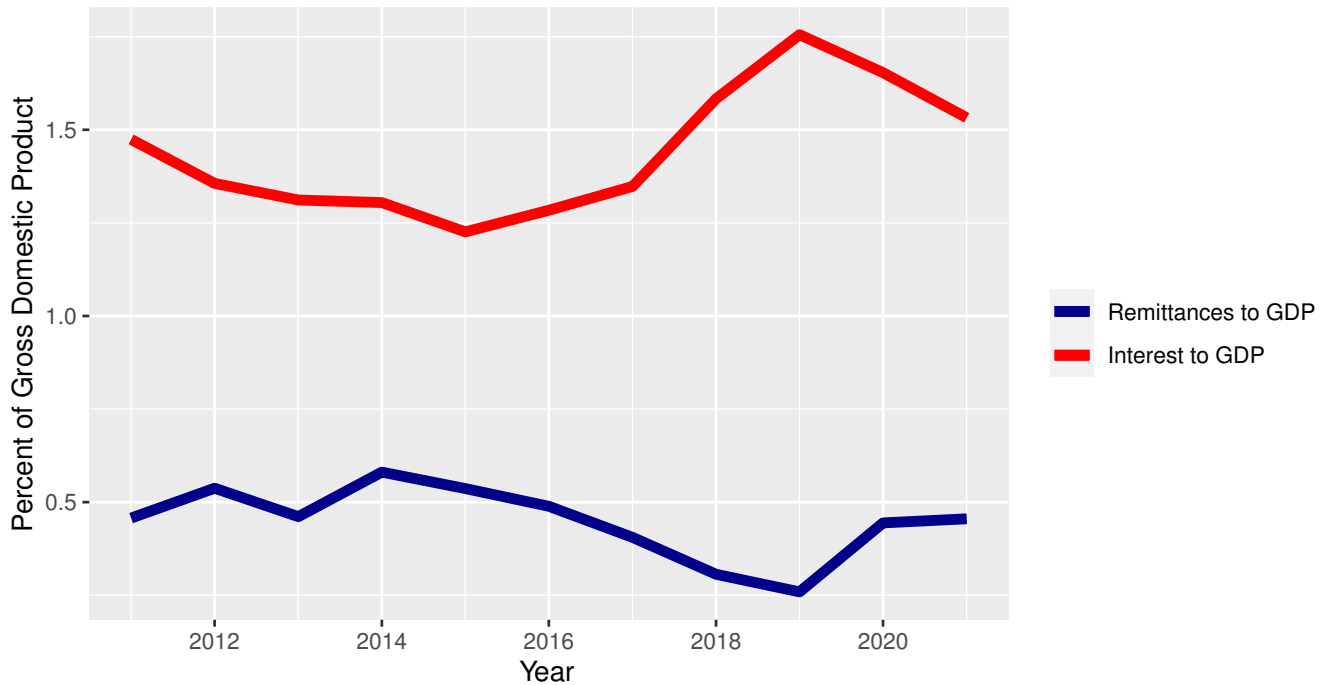


Fig. 1. Central Bank to Treasury Remittances and Federal Interest Expenses. Source: Board of Governors of the Federal Reserve System (US) and U.S. Bureau of Economic Analysis

by reserves create profits for the central bank due to the positive spread. These profits are remitted to the treasury and can be used as fiscal revenue for debt repayment together with other fiscal tools, as in Reis (2017). QE in the model works as a credit-stimulating mechanism to the real economy. QE-induced effects are possible due to the existence of the bank’s leverage constraint, which eliminates the perfect substitutability of assets and leads to money non-neutrality and thus profits for the central bank. QE also stimulates the economy indirectly thanks to the remittances to the treasury and the reduced need for fiscal adjustment.

For the first two cases of our analysis, namely debt stabilization through QE revenues and fiscal adjustment via an increase in revenues, monetary policy satisfies the Taylor principle and the fiscal authority is in control of debt stabilization. This is in words of Leeper (1991) active monetary policy, which pays no attention to the state of government debt and is free to set its control variable as it sees fit, and a passive fiscal policy which responds to government debt shocks. The third case for debt stabilization, a passive monetary policy framework is implemented by the introduction of unfunded transfer shocks, similarly to Bianchi et al. (2020). These are transfers that are not backed by future fiscal adjustments, making a share of the overall government debt unfunded. The central bank accommodates the increase in inflation necessary

to stabilize the unfunded amount of debt. As a result, these shocks trigger persistent movements in inflation and a decline in real interest rates, leading to a fiscal theory of trend inflation.

We show that after a large transfer to the households, the use of QE does not only reduce spreads and stimulate the economy but also provides debt stabilization even without any use of the classic fiscal adjustment tools or a passive monetary policy. QE as a debt stabilization tool, works by the revenue generated from the bond-reserves spread receivable by the central bank. It does not distort the economy by increasing consumption, capital or labour taxation or by decreasing aggregate demand as in the case of a government spending reduction.

Passive monetary policy, which accommodates the inflation needed for debt stabilization, is the most efficient way for debt stability in our exercises. By sacrificing an increase in inflation, a reduction of the debt-to-gdp ratio leads to ample fiscal space for tax reduction and an increase to government spending and transfers to households that increase aggregate demand. Nevertheless, due to institutional reasons related with a high inflation target, we think of this scenario as possibly unrealistic.

The outline of the paper is as follows. Section 2 presents some stylized facts that relate remittances from the Federal Reserve to the Treasury with revenue from QE operations. Section 3 presents the TANK model with banking that we use. The following sections present the main results of the paper and the last section concludes.

## 2. Stylized facts of QE contribution to the Treasury

In this section we present stylized facts that relate remittances from the Federal Reserve to the Treasury with revenue from QE operations. Firstly, we document that the Federal Reserve purchased long-term bonds by issuing reserves to the banks. QE revenue can thus be projected as the yield gap, the difference between the yield paid on long-term bonds and interest paid on reserves. Secondly, we document that the yield gap becomes larger during recessions rather than during the period of monetary policy normalization. As a result, QE is more profitable during an economic downturn. Long-term yields usually respond less to short-term Fed funds interest rate fluctuations and are, instead, determined by the potential output and secular trends. Thirdly, in a zero lower bound environment, buying long-term bonds decreases their yield and makes QE less profitable. It results in purchasing bonds being profitable only during the crisis and for a limited period of time. Finally, purchase of long-term bonds accounts for up to 60% of total remittances from the Federal Reserve to the Treasury, which makes it an important factor for debt stabilization during recession.

### 2.1. *QE leads to a large issuance of new reserves in the banking system*

The Federal Reserve started QE during the financial crisis of 2007-2008. Since then, the QE program has included the purchase of mortgage-backed securities and long-term bonds. These assets are usually purchased from the banks that, in return,

receive newly issued reserves [McLeay, Radia, and Thomas \(2014\)](#). Before the Global Financial crisis, banks kept limited excess reserves, but these grew rapidly since the start of QE programs. It also led to an increase in the monetary base that consists of cash and banks' reserves. Figure 2 shows that in 2022 the Federal Reserve holdings of the US Treasury almost reached \$6 trillion, while banks' reserves topped \$4 trillion.

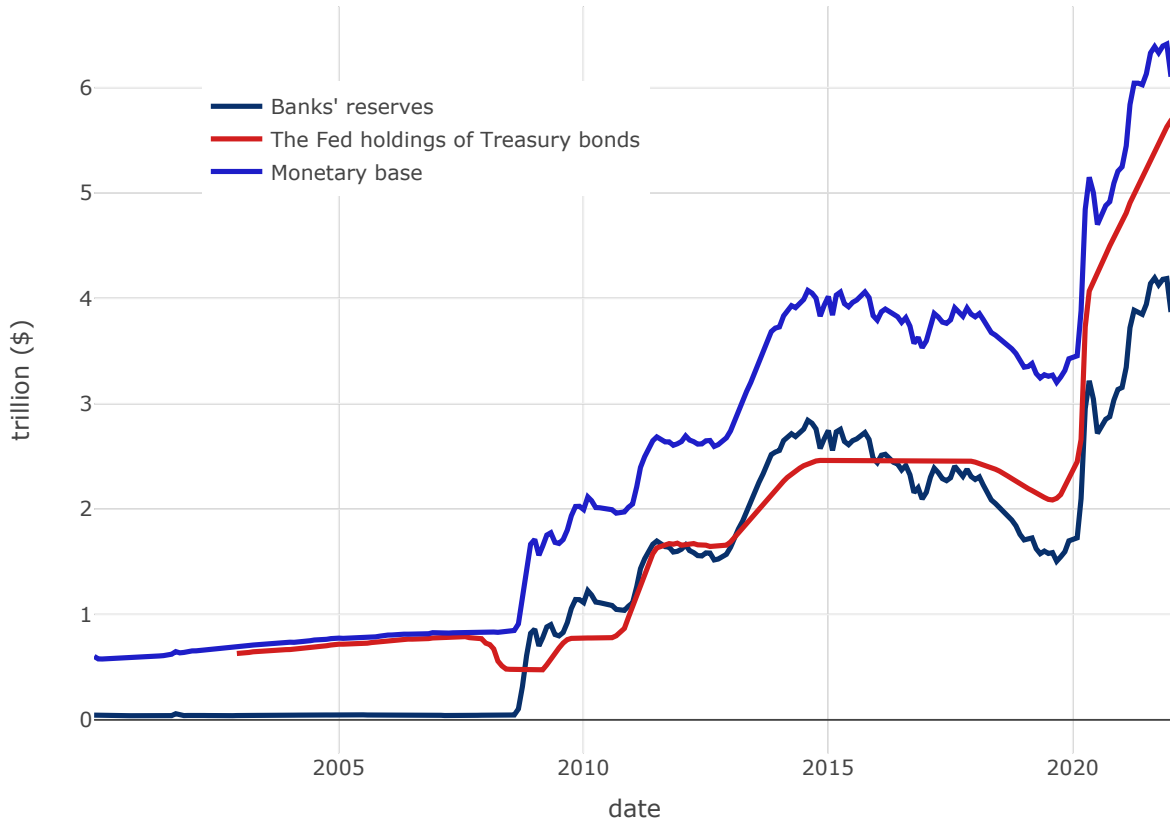


Fig. 2. The Federal Reserve holdings of US treasury, total reserves owned by the banks and the US monetary base

The Federal Reserve pays excess reserves interest rate on bank's reserves holdings and receives yield for holding long-term bonds. Since the latter is usually higher than the former, the Federal Reserve receives revenue that is (after costs) remitted back to the Treasury. This source of revenue is not constant and depends on the yield gap. We further investigate the yield gap driving factors that are primarily the state of the economy and the number of purchased long-term bonds.

## 2.2. *The yield gap is lower during monetary policy normalization*

The interest rate on excess reserves is usually the lowest rate in the economy. The longer bonds yields inherit term risk premia and are higher than the reserves inter-

est rates. Figure 4 shows that the yield gap, however, fluctuated depending on the economic cycle. Short-term yields are the first to decline in response to expansionary monetary policy, while long-term yields respond less to short-term interest rate fluctuations.

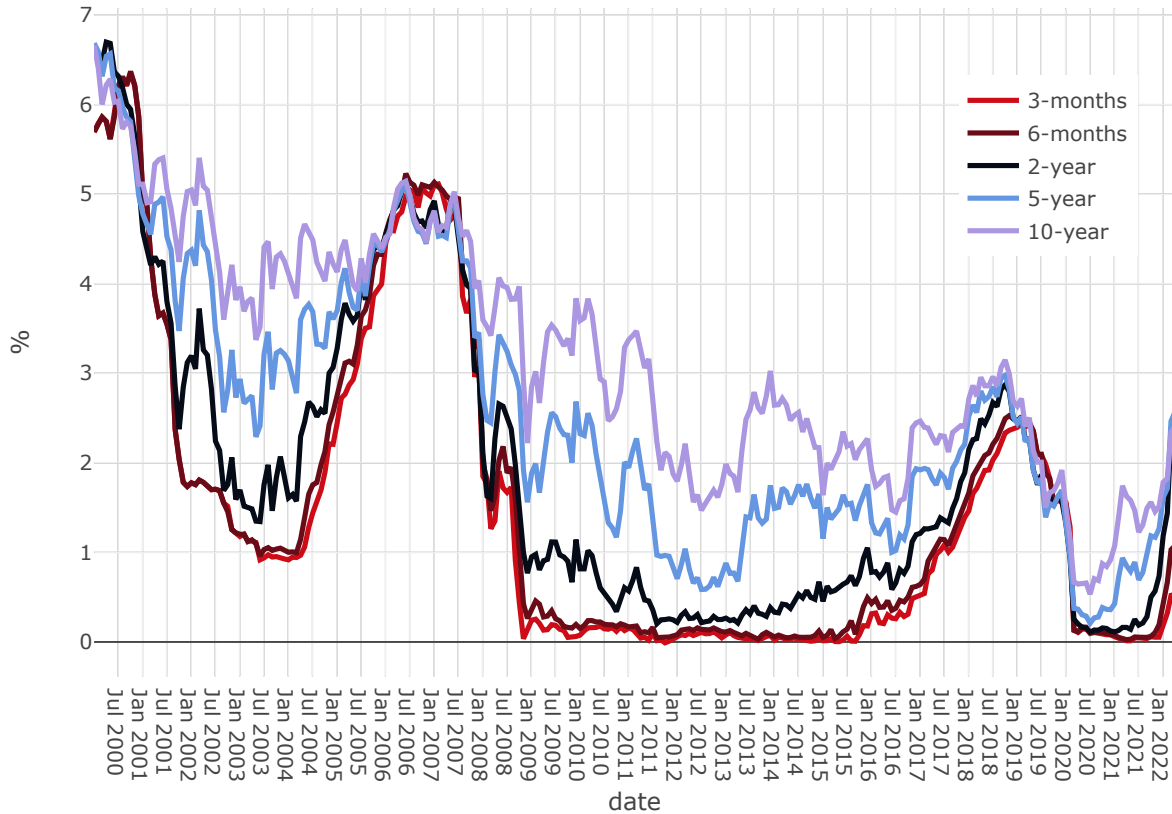


Fig. 3. Bond yields at different maturities

To further investigate how the Fed funds rate effect yields at different maturities we conduct an empirical analysis. Table 2.2 shows that the Fed funds rates is the best at explaining shorter rather than longer yields. Long-term yields are less volatile and are mainly driven by secular trends. It makes harder to decrease these rates with conventional monetary policy tools. Stability of long-term yield makes QE revenues higher during the crisis times and smaller during monetary policy normalization.

### 2.3. *Purchasing of long-term bonds decreases the yield gap*

Another important factor for remittances is that purchases of bonds by themselves decrease yields. Table 2.3 shows that QE purchases decrease both two and ten-year yields. To verify this result, we collect data on the Federal Reserve bonds purchases

	<i>Dependent variable:</i>					
	3 month yield (1)	6 month yield (2)	2 year yield (3)	5 year yield (4)	10 year yield (5)	30 year yield (6)
Fed funds rate	0.944*** (0.009)	0.947*** (0.010)	0.871*** (0.017)	0.711*** (0.024)	0.558*** (0.029)	0.413*** (0.031)
Constant	-0.021 (0.022)	0.073*** (0.024)	0.528*** (0.042)	1.406*** (0.058)	2.273*** (0.070)	3.135*** (0.076)
Observations	268	268	268	268	268	268
Adjusted R <sup>2</sup>	0.976	0.973	0.904	0.769	0.585	0.396

Note:

\*p<0.1; \*\*p<0.05; \*\*\*p<0.01

Table 1: How does the Fed funds interest rate affects bond yields

split by maturity, date on government yields and episodes of different QE policies. Results show negative impact of bond purchases on their yields, which is in line with [Bernanke \(2020\)](#), [Ihrig, Klee, Li, Wei, and Kachovec \(2018\)](#). From 2009 to 2016, we can see in figure 3 that short rates were at zero lower bound while long-term yields were steadily decreasing. Expansion of long-term bond purchases increased the stock of bonds owned by the Federal Reserve and simultaneously decreased revenue per dollar value of bonds.

	<i>Dependent variable:</i>	
	10 year yield (1)	2 year yield (2)
3 month yield	0.515*** (0.027)	0.912*** (0.014)
Purchase of 5-10 year bonds	-0.006* (0.004)	
Purchase of 6 month-2 year bonds		-0.008*** (0.003)
Time effects	YES	YES
Observations	267	267
Adjusted R <sup>2</sup>	0.739	0.946

Note:

\*p<0.1; \*\*p<0.05; \*\*\*p<0.01

Table 2: How does the Federal Reserve bond purchases affect bond yields

Bond purchases have two different effects on QE revenue. From the one side, they increase the stock of owned bonds and, therefore, the revenue from buying bonds. From the other side, long-term bond purchases lead to yield convergence between the interest rate on reserves and yield from owing bonds. However, the opposite is also true. During the monetary policy normalization and QT (quantitative tightening) the yield gap increases and this will make some extra profits to the Treasury.

## 2.4. *Purchasing of long-term bonds accounts for up to 60% of the Federal Reserve remittances to the Treasury*

Figure 4 shows the total revenue from purchasing and owning long-term bonds as a percentage of GDP. At its peak in 2014, revenue reached 60% of the total Federal reserves remittance to the Treasury. However, we can also see that the bond purchases during the COVID-19 pandemic have become much less effective. The reason stands for the lowering of long-term yields. By this time, the Federal Reserve has already owned a big stock of bonds, and extra purchases decreased the yield for all long-term bonds.

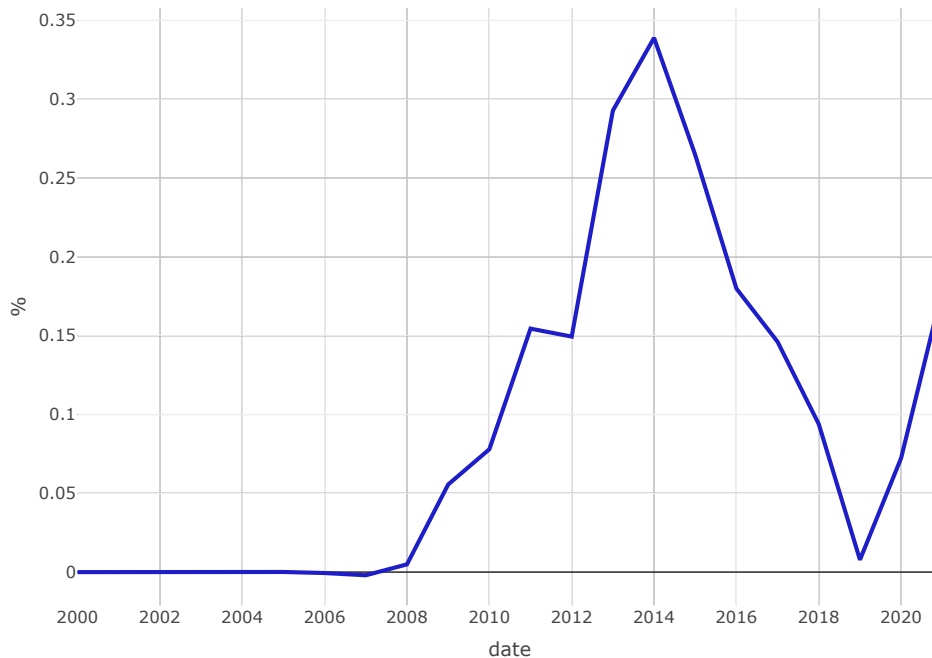


Fig. 4. QE revenue from bonds' purchases as % of GDP

## 3. The Model

The economy is populated by two types of households: savers and hand to mouth households that differ in their ability to participate in the assets market. A continuum of firms and financial intermediaries owned by the savers, labour wide unions that set the wages, capital goods producers and retailers, a monetary authority and the treasury complete the model economy.

There is a moral hazard problem between the savers and the banks. Banks can steal a fraction of their funds and return them to their families. This problem introduces an incentive constraint to the model to be followed by the banks. Furthermore, the central bank performs its (active or passive) conventional monetary policy under a Taylor rule,



but can also engage in asset purchases and pay the investors back the same value in newly created reserves. Lastly, there is a rich fiscal sector that aims for debt stabilization.

### 3.1. Households

All households are assumed to have identical preferences, given by

$$U^j = \log (C_t^j - \chi C_{t-1}^j) - \psi \frac{(L_t^j)^{1+\eta}}{1+\eta}. \quad (1)$$

$C_t^j$  denotes the per capita consumption of the household members and  $L_t^j$  the supply of labour. The super-index  $j \in [S, N]$  specifies the household type ( $S$  for “savers” or  $N$  for “hand to mouth”).  $\beta \in [0, 1]$  is the discount factor. Due to the stochastic setting, households make expectations for the future based on what they know in time  $t$  and  $\mathbb{E}_t$  is the expectation operator at time  $t$ . Finally,  $\epsilon$  is the inverse Frisch elasticity of labour supply and  $\chi$  is the relative utility weight of labour.

**Savers.** Optimizers account to a measure of  $(1 - \lambda)$  of the economy’s population. They allocate their funds in consumption  $C_t^S$  and short-term deposits  $D_t^S$  which are remunerated at the risk-free rate of the economy  $R_t$ . Their income is made deposit returns, lump sum transfers  $Z_t^S$ , after tax labour income and the profits from firms and banks that they own  $\Pi_t$ .

Savers’ budget constraint then is:

$$\begin{aligned} P_t(1 - \tau_t^C)C_t^S + R_t^{-1}D_t &= D_{t-1} + (1 - \tau_t^L)W_tL_t^S + P_tZ_t^S + \Pi_t \end{aligned} \quad (2)$$

where  $W_t$  denotes the wage rate that applies to all household members, and  $\tau_t^C, \tau_t^L, \tau_t^K$  denote the tax rates on consumption, labour income and capital respectively. Their optimality conditions are:

$$\begin{aligned} \text{Euler Deposits} : 1 &= R_t \mathbb{E}_t [\Lambda_{t,t+1}^S] \\ \text{Euler Long term debt} : 1 &= \mathbb{E}_t \left[ \Lambda_{t,t+1}^S \frac{(1 + \rho_B P_{t+1}^B)}{P_t^B} \right] \\ \text{Stochastic Discount Factor} : \Lambda_{t,t+1}^S &\equiv \beta \frac{\lambda_{t+1}^S}{\lambda_t^S} \\ \text{where} : \lambda_t^S &= \frac{1}{C_t^S - \chi C_{t-1}^S} - \beta \chi \frac{1}{C_{t+1}^S - \chi C_t^S} \\ \text{Labour Supply} : -\frac{U_{L,t}^S}{\lambda_t^S(1 - \tau^L)} &= -W_t \end{aligned}$$

**Hand to Mouth.** Hand-to-mouth households consume all of their disposable, after-

tax income every period. Their income is made of their labour income and lump sum government transfers. Profits are earned only by the savers that own the firms. It is assumed that the hand-to-mouth households supply differentiated labour services, and set their wage to be equal to the average wage that is optimally chosen by the savers, therefore there is a common wage level  $W_t$  for both groups, Their budget constraint is as follows:

$$P_t(1 + \tau_t^C)C_t^N = (1 - \tau_t^L)W_tL_t^N + P_tZ_t^N. \quad (3)$$

Their optimality conditions are:

$$\begin{aligned} \text{Labour Supply : } -\frac{U_{L,t}^N}{\lambda_t^N(1 - \tau_t^L)} &= -W_t \\ \text{where : } \lambda_t^N &= \frac{\frac{1}{C_t^N - \chi C_{t-1}^N} - \beta\chi\frac{1}{C_{t+1}^N - \chi C_t^N}}{1 - \tau_t^C} \end{aligned}$$

### 3.2. Banks

Banks are funded with deposits from the saver households, extend credit to non-financial firms and buy long-term bonds from the government. At QE they exchange the asset purchased by the central bank with reserves. Each bank  $j$  allocates its funds to buying a quantity  $s_{j,t}$  of financial claims on non-financial firms at price  $Q_t$  and long term government bonds  $b_{j,t+1}^B$  at price  $P_t^B$ . Banks' liabilities are made up from households' deposits  $d_{j,t+1}^H$ . When the central bank proceeds in securities' purchases ( $Q_t s_t$  or  $P_t^B$ ) it pays back the bank with an equivalent value of reserves  $m_{j,t}$ . Finally,  $n_{j,t+1}$  is the capital equity accumulated. Formally, the bank's balance sheet is:

$$Q_t s_{j,t}^B + P_t^B b_{j,t}^B + m_{j,t}^B = n_{j,t} + d_{j,t}^H. \quad (4)$$

To limit bankers' ability to save and eventually overcome their financial constraint by using own funds, following [Gertler and Kiyotaki \(2010\)](#) we assume the following: Each period, a fraction  $1 - \sigma_B$  of bankers, exit and give retain earnings to their household. An equal number of new bankers enter at the same time. They begin with a start up fund of  $\xi$  given to them by their household.

The bank's net worth evolves as the difference between interest gains on assets and interest payments on liabilities.

$$n_{j,t+1} = R_{k,t}Q_{t-1}s_{j,t-1}^B + R_{b,t}q_{t-1}b_{j,t-1}^B + R_{m,t}m_{j,t}^B - R_t d_{j,t}^H.$$

Following [Woodford \(2001\)](#) long-term government bond  $B_t$  with a maturity decaying at a constant rate  $\rho \in [0, 1]$  and duration  $(1 - \beta\rho)^{-1}$  can be purchased at price  $P_t^B$ .

Therefore realized return that the bank earns from the maturity bond is:

$$R_{b,t+1} = \frac{1 + \rho_B P_{t+1}^B}{P_t^B}$$

Let  $Z_t$  be the net period income flow to the bank from a loan that is financing to a firm and  $\delta$  the depreciation rate of capital being financed. Then the rate of return to the bank on the loan,  $R_{k,t+1}$ , is given by:

$$R_{k,t+1} = \frac{Z_t + (1 - \delta)Q_{t+1}}{Q_t}. \quad (5)$$

The bond return Central bank reserves bear a zero weight in the banks' constraint and, as it will be shown momentarily, have a gross return  $R_{m,t}$  equal to the risk-free rate  $R_t$ . It follows that banks have no incentive to hold reserves in equilibrium.

The bankers' objective at the end of period  $t$ , is to maximize the expected present value of future dividends. Since the banks are owned by the optimizing households, their stochastic discount factor  $\Lambda_{t+1}$  is used as the discounting measure.

$$V_{j,t} = \mathbb{E}_t \sum_{j=1}^{\infty} (1 - \sigma_B) \sigma_B^{j-1} \Lambda_{t+1} n_{j,t+1}. \quad (6)$$

To motivate a limit on the banks' ability to obtain deposits, a costly enforcement constraint is introduced in the same fashion as in [Gertler and Kiyotaki \(2010\)](#).

A banker can abscond a fraction of her assets and transfer them back to her household members, depositors can force the bank into bankruptcy and get the remaining fraction of assets. It is assumed that the banker can divert loans easier than diverting bonds and reserves.

The depositors continue providing funds to the bank as long as the following incentive constraint is not violated:

$$V_{j,t} \geq \theta [Q_t s_{j,t}^B + \Delta P_t^B b_{j,t}^B + \omega m_{j,t}^B]. \quad (7)$$

where  $\theta$  is the fraction of assets that the banker may divert and  $\Delta \in (0, 1)$  and  $\omega \in (0, 1)$  are the ratios of how many bonds and how much reserves the banker can divert. On the left of (7) is the franchise value of the banker, which is what the banker would lose from diverting, while on the right are the banker's gains from diverting, which is a fraction  $\theta$  of her assets.

The maximum adjusted leverage ratio of the bank after the solution of the bank's problem (see Appendix) yields:

$$\phi_t = \frac{\mathbb{E}_t \Lambda_{t,t+1} R_{t+1}}{\theta - \mathbb{E}_t \Lambda_{t,t+1} (R_{k,t+1} - R_{t+1})}. \quad (8)$$

Maximum adjusted leverage ratio depends positively on the marginal cost of the de-

posits and on the excess value of bank assets. As the credit spread increases, banks' franchise value  $V_t$  increases and the probability of a bank diverting its funds declines. On the other hand, as the proportion of assets that a bank can divert,  $\theta$  increases, the constraint binds more.

Importantly, the maximum adjusted leverage ratio does not depend on any individual bank characteristics, therefore the heterogeneity in the bankers' holdings and net worth, does not affect aggregate dynamics. Hence, it is straightforward to express individual financial sector variables in aggregate form.

**Aggregation.** Let  $S_t^B$  be the total quantity of loans that banks intermediate,  $B_t^B$  the total number of government bonds they hold,  $M_t^B$  the total quantity of reserves and  $N_t$  their total net worth. Furthermore, by definition, total deposits acquired by the households  $D_t$  are equal with the total deposits of the banking sector. Using capital letters for the aggregate variables, the banks' aggregate balance sheet becomes

$$Q_t S_t^B + P_t^B B_t^B + M_t^B = N_t + D_t^H. \quad (9)$$

Since the leverage ratio (8) does not depend on factors associated with an individual bank's characteristics we can sum up across banks and get the aggregate bank constraint in terms of the total net worth in the economy:

$$Q_t S_t^B + \Delta P_t^B B_t^B + \omega M_t^B = \phi_t N_t. \quad (10)$$

The above equation gives the overall demand for loans  $Q_t S_t$ . When the incentive constraint is binding, the demand for assets is constrained by the net worth of the bank adjusted by the leverage. We can get some intuition here for what changes in the bank's constraint during the QE. No matter the security the central bank purchases, since their weights are higher than the weight of reserves ( $1 > \Delta > \omega$ ), the exchange of securities with reserves relaxes the constraint and stimulates lending to the non-financial sector. If the constraint does not bind, then all three types of assets would have the same returns, equal to the deposit rate and QE would be ineffective.

Aggregate net worth is the sum of the new bankers' and the existing bankers' equity:  $N_{t+1} = N_{y,t+1} + N_{o,t+1}$ . Young bankers' net worth is the earnings from loans multiplied by  $\xi_B$  which is the fraction of asset gains that being transferred from households to the new bankers

$$N_{y,t+1} = \xi [R_{k,t} Q_{t-1} S_{t-1}^B + R_{b,t} P_{t-1}^B B_{t-1}^B + R_{m,t} M_{t-1}^B]$$

and the net worth of the old is the probability of survival for an existing banker multiplied by the net earnings from assets and liabilities

$$N_{o,t+1} = \sigma [R_{k,t} Q_{t-1} S_{t-1}^B + R_{b,t} P_{t-1}^B B_{t-1}^B + R_{m,t} M_{t-1}^B - R_t D_t^H].$$

### 3.3. Conventional Monetary Policy and Asset Purchases

The central bank uses two policy tools. Firstly, it adjusts the policy rate according to the Taylor rule specified here below. Secondly, it engages in asset purchases with the

banks.

*Asset Purchases.* When balance sheet constraints are tight, excess returns rise. Central bank purchases relax the incentive constraint of the banks and increase aggregate demand, thus driving up asset prices.<sup>2</sup>

Under a QE operation, the central bank buys bonds from banks. It does this by paying the assets purchased by their respective price  $P_t^B$ . To finance those purchases it creates electronically reserves  $M_t$  that pay back purchases from households and banks:

$$P_t^B B_t^G = M_t. \quad (11)$$

It is assumed that the central bank turns over any profits to the treasury and receives transfers to cover any losses. The central bank's budget constraint is:

$$T_t^{CB} + R_t M_{t-1} + P_t^B B_t^G = R_{b,t} P_{t-1}^B B_{t-1}^G + M_t$$

where  $T_t^{CB}$  are transfers of the central bank to the treasury.

Given (11) we can write the total remittances from the central bank to the treasury as:

$$T_t^{CB} = (R_{b,t} - R_t) P_t^B B_t^G. \quad (12)$$

Therefore, the central bank remittances stem from the spread between the bonds purchases and the interest on reserves that is paid to the banks for the same value as the bonds purchases as seen by (11). As long the spread is positive and the value of bonds purchased non zero, central bank will earn profits from QE.

The total quantity of bonds in the economy can be decomposed as:

$$B_t = B_t^B + B_t^G. \quad (13)$$

If we combine these identities and insert them into the balance sheet constraint of the banks we have:

$$Q_t S_t \leq \phi N_t + \Delta(q_t B_t^G - q_t B_t) \quad (14)$$

The above constraint implies that when government purchase bonds it relaxes the balance sheet constraint of the banking sector. This can, in financial stress periods, reduce the excess returns and stimulate the economy. When this constraint does not bind and the inequality holds, bond purchases made by the government are neutral. This happens due to frictionless arbitrage that characterizes the economy when the banks has no binding constraint. [Wallace \(1981\)](#) in his seminal paper has made use of that assumption for the neutrality theorem of the open market operations.

The share of the total assets that is purchased by the government follows a rule that responds to debt-to-gdp deviations similarly to the fiscal rules presented momentarily. This setting makes QE a fiscal stabilization tool. When debt is high, the central bank buys more bonds thus generating extra fiscal revenue for the treasury. The QE rule reads as follows:

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<sup>2</sup>See [Araújo, Schommer, and Woodford \(2015\)](#) for a same intuition under a different setting.

$$\hat{b}_t^G = \rho_{b^G} \hat{b}_{t-1}^G - (1 - \rho_{b^G}) \gamma_{b^G} \hat{b}_{t-1}^G + \zeta_{b^G,t} \quad (15)$$

where  $b_t = \frac{P_t^B B_t}{P_t Y_t}$  is the debt-to-gdp ratio and the variables in  $\hat{x}_t$  denote the percentage deviation from their own steady state.  $\gamma_{b^G}$  is the magnitude of the asset purchases relative to the debt deviations. A higher (lower) value for  $\gamma_{b^G}$  provides debt stability (instability). The  $\zeta_{b^G,t}$  is a QE shock following second order stochastic process.

**Conventional Monetary Policy.** The central bank also sets the nominal interest rate in response to movements in inflation originating by funded fiscal shocks but also it accommodates the inflation necessary to stabilize the unfunded portion of debt. Following [Bianchi et al. \(2020\)](#) the latter can be captured by a rule in which the central bank reacts to deviations of inflation from the level of inflation needed to stabilize the unfunded share of debt. This level of inflation,  $\hat{\pi}_t^F$ , is tolerated by the central bank, or the fiscal trend inflation. Conventional monetary policy is active when it responds to deviations of inflation,  $\hat{\pi}_t$ , from the inflation needed to accommodate to stabilize the unfunded share of debt,  $\hat{\pi}_t^F$ .

The Taylor rule is as follows:

$$\hat{R}_{n,t} = \rho_r \hat{R}_{n,t-1} + (1 - \rho_r) [\phi_\pi (\hat{\pi}_t - \hat{\pi}_t^F) + \phi_y \hat{y}_t] + \epsilon_{MPS} \quad (16)$$

where  $\epsilon_{MPS}$  is a monetary policy shock.

Utilising this, it provides a tractable way to have active and passive conventional monetary policy in the same framework. The central bank responds differently depending on whether the increase in debt is funded (back by fiscal tools or QE) or undunded.

### 3.4. Fiscal Policy

The government budget constraint is:

$$P_t^B B_t + \tau_t^K R_t^K K_t + \tau_t^L W_t L_t + \tau_t^C P_t C_t + P_t T_t^{CB} = (1 + \rho P_t^B) B_{t-1} + P_t G_t + P_t Z_t \quad (17)$$

where  $C_t = \mu_t C_t^N + (1 - \mu_t) C_t^S$  denotes aggregate consumption and  $Z_t = Z_t^N = Z_t^S$  meaning that every household receives the same government transfer regardless of its type. The government finances government expenditures and transfers and the roll-over of expiring long term debt by raising taxes and issuing new long term debt.

The budget constraint can also be written in terms of debt-to-gdp ratio as follows. Let  $b_t = \frac{P_t^B B_t}{P_t Y_t}$  be the debt-to-GDP ratio.

$$b_t = \frac{b_{t-1} R_{b,t-1}}{\frac{Y_t}{Y_{t-1}}} - (\tau_t^K r K_t K_{t-1} + \tau_t^C C_t + \tau_t^L W_t L_t) / Y_t - T_t^{CB} / Y_t + G_t / Y_t + Z_t / Y_t + \epsilon_t^b \quad (18)$$

where  $R_{b,t} = \frac{1 + \rho_B P_{t+1}^B}{P_t^B}$  is the realized return from the maturity bond and  $\epsilon_t^b$  is a debt shock.

The fiscal authority adjusts government spending  $G_t$ , transfers  $Z_t$ , and tax rates on capital income, labour income, and consumption  $\tau^J$ ,  $J \in K, C, L$  as a function of the debt-to-gdp deviations from its steady state value. Note that government consumption and transfers move pro-cyclically with the debt-to-gdp deviations, where taxes counter-cyclically. For each variable  $x_t$ , we use  $\hat{x}_t$  to denote the percentage deviation from its own steady state.

$$\hat{g}_t = \rho_G \hat{g}_{t-1} - (1 - \rho_G) \gamma_G \hat{b}_{t-1}^M + \zeta_{g,t} \quad (19)$$

$$\hat{z}_t = \rho_Z \hat{z}_{t-1} - (1 - \rho_Z) \gamma_Z \hat{b}_{t-1}^M + \zeta_{Z,t}^F + \zeta_{Z,t}^M \quad (20)$$

$$\hat{\tau}_t^J = \rho_J \hat{\tau}_{t-1}^J + (1 - \rho_J) \gamma_J \hat{b}_{t-1}^M. \quad (21)$$

$\hat{b}_{t-1}^M$  is the portion of the debt to gdp ratio that the government is committed to stabilize with fiscal adjustments. This, is possible given that the elasticities of the government rules  $\gamma_G, \gamma_J, \gamma_Z > 0$  are large enough to guarantee that  $\hat{b}_{t-1}^M$  remains on the stable path. The fiscal authority does not make fiscal adjustments in response to the remaining, unfunded, portion of debt  $\hat{b}_{t-1}^F$ .

The variables  $\zeta_{Z,t}^F$  and  $\zeta_{Z,t}^M$  denote shocks to transfers that are respectively unfunded and funded. The shocks  $\zeta_{Z,t}^F$ ,  $\zeta_{Z,t}^M$  and  $\zeta_{g,t}$  follow AR(1) Gaussian stochastic processes.

## 4. Quantitative Analysis

### 4.1. Model Parametrization

We calibrate the model to the US data. For the real economy we follow [Bianchi et al. \(2020\)](#) where they estimate a very similar model for the US data. They do a thorough estimation of the taxation parameters in the model and also for the shock processes. We take these parameters as given. For the financial sector, we take the values from [Gertler and Karadi \(2013\)](#) where they do calibrate their model for the US banking sector.

Financial parameter values are chosen in order to match specific US banking characteristics namely the banks' average leverage, lending spread and planning horizon. There are three parameters that characterise the behaviour of the financial sector in the model. This is the absconding rate  $\theta$ , the fraction of entering bankers initial capital fund  $\xi_B$ , and the steady-state value of the survival rate,  $\sigma_B$ . We calibrate these parameters to match certain steady-state moments following the moments reported in [Gertler and Karadi \(2013\)](#). The steady-state leverage of the banks is set equal to 6, which corresponds to the average asset-over-equity ratio of monetary and other financial institutions as well as non-financial corporations, with weights equal to their share of assets in total assets between 1999Q1 and 2014Q4. Second, the steady-state spread of the lending rate over the risk-free rate,  $R^k - R$  is set to 100 basis points at the steady state. We set the fraction of bonds that can be absconded  $\Delta$  to 50% targeting a steady state bond spread half to the lending spread, at 50 basis points. The absconding rate of reserves  $\omega$  is set to zero. Similarly to [Sims and Wu \(2021\)](#) we assume that they are fully

recoverable by depositors in the event of bankruptcy. Since they are in essence central bank money, the central bank has full control on them.

The bond purchase shock is modelled as an AR(2) process. The AR(2) process in contrast with an AR(1) captures the expectation of the further expansion of central bank purchases in the future, which is the case in the US bond purchase programs. To illustrate this pattern, the first AR coefficient is chosen to 1.700 and the second being -0.730 while the initial shock is chosen to 0.015.

#### 4.2. *Transfer Shocks: Taxes vs Government Spending vs QE as Debt Stabiliser*

We show responses of our model economy to a one time transfer to households. This captures in a crude way the Covid-19 transfers took place. The question we answer is in what way the increase in debt will be stabilized. We provide three different tools where we use them interchangeably. The first is an increase in taxes, the second a reduction of government spending, and the third, our novelty, an increase in the assets purchased by the central bank in order to generate revenues for the treasury that will stabilise the debt.

Figure 5 shows the responses of some important macro variables to a transfer shock (as can be seen in the bottom row of the graph) and the subsequent response of the three tools that follow the processes defined in sections 3.4 for the fiscal rules and 3.3 for the asset purchase rule. All the three tools manage to provide debt stability after a transfer shock. This heavily depends on the magnitude of the parameters  $\gamma_{QE}$ ,  $\gamma_G$  and  $\gamma_T$  which define how strong the variables respond to changes of the debt-to-gdp ratio from its steady state.

Given that this is a transfer shock, and benefits both types of households, their consumption increases on impact. Then, the shock becomes contractionary for the cases of taxes and government spending adjustments, since agents need either to pay for this transfer by higher taxes, or aggregate demand decreases due to the lower government spending. The case of QE as a fiscal stabilization tool shows that while it is also contractionary, the economy rebounds much faster than in the other two cases.



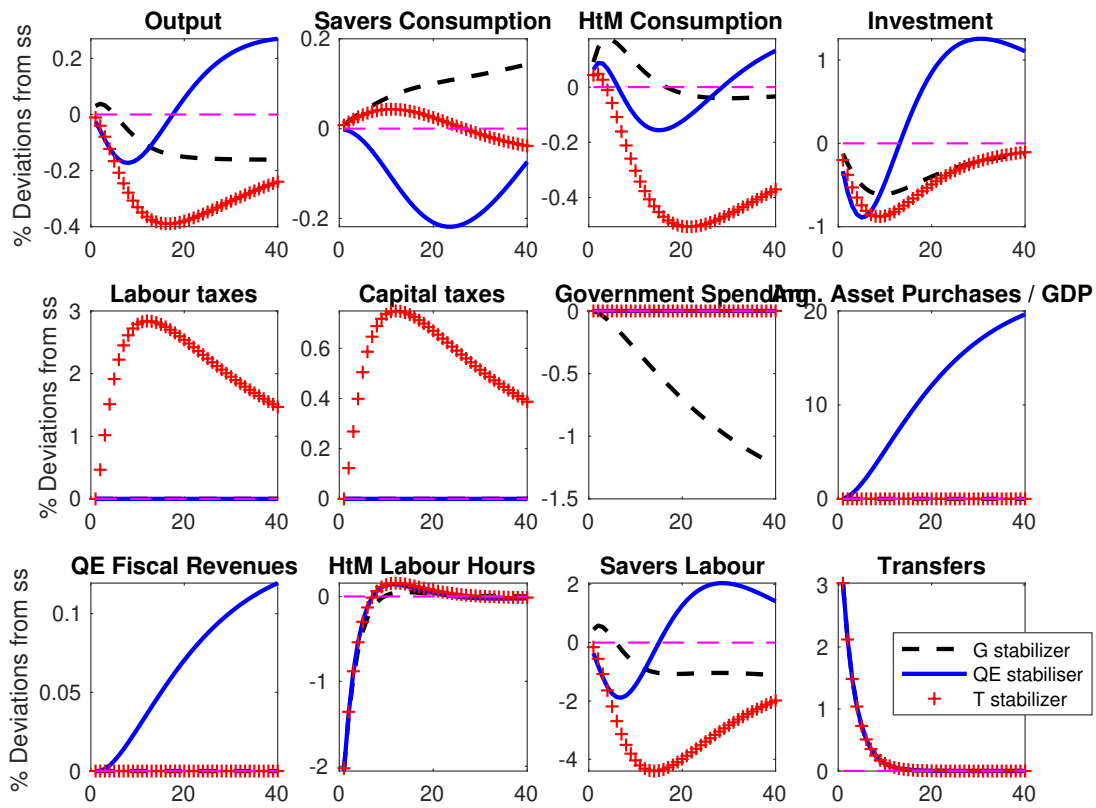


Fig. 5. A Transfer Shock under Different Debt Stabilizers

Figure 6 shows the responses of inflation, the real interest rate and the debt-to-gdp ratio. A combination of a transfer shock and QE seems to be deflationary, similarly to the recent experience of Covid-19. In terms of debt to gdp ratio, the QE use as a fiscal tool requires the issuance of more new debt by the treasury and the purchase of this debt by the central bank in order to be able to repay only with those revenues the extra fiscal needs. The taxes provide a fast reduction of the debt-to-gdp ratio but at the cost of a higher contraction in output.

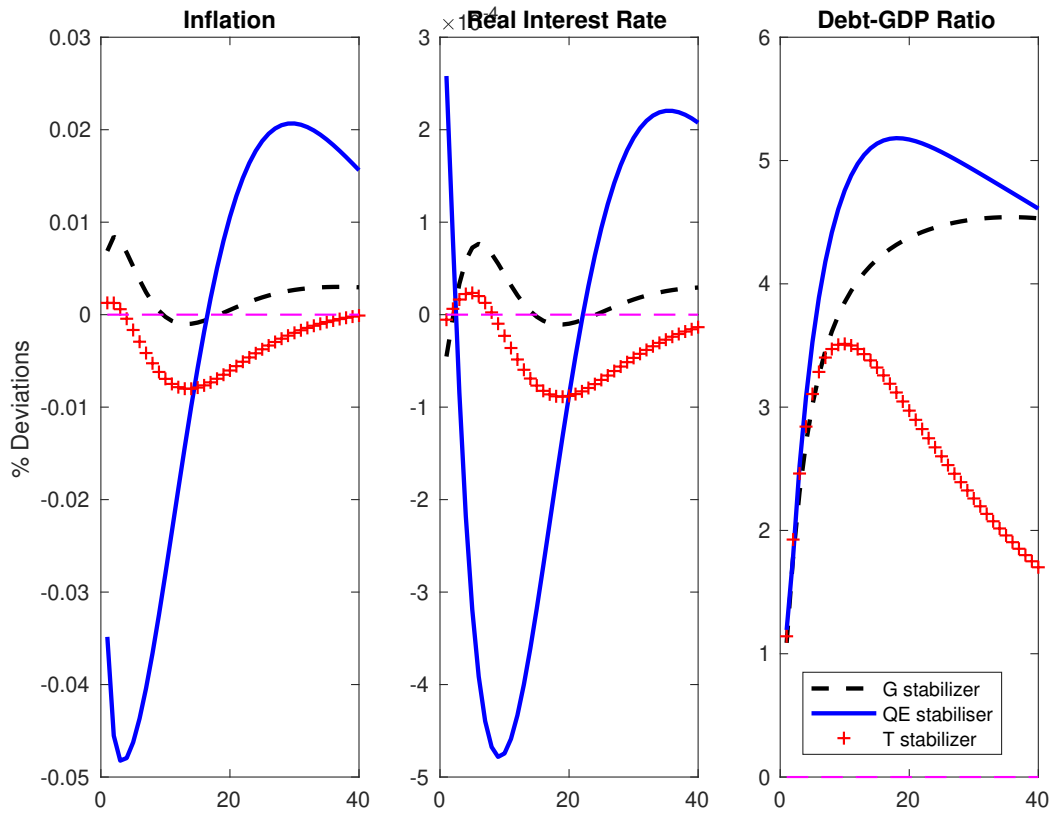


Fig. 6. A Transfer Shock under Different Debt Stabilizers

### 4.3. Transfers with a Passive Monetary Policy

We test passive monetary policy as an alternative form of debt stabilization when the economy is hit by a transfer shock identical to the one analysed above. We now assume that the central bank is ready to accommodate a level of inflation necessary to provide debt stability, therefore conducting a passive monetary policy. We then compare this policy with the classical fiscal adjustments (both a reduction of government spending and an increase in taxes) and fiscal adjustments together with QE.

Figures 7 and 8 show the result of this exercise. There are now two policy regimes: One where monetary policy is passive and two where monetary policy is active and fiscal policy is passive together with QE since both help in debt stabilization. We note as funded' the active monetary policy regime, meaning that this increase in debt due to the transfer shock is funded by taxation, QE or any other form of primary surplus.

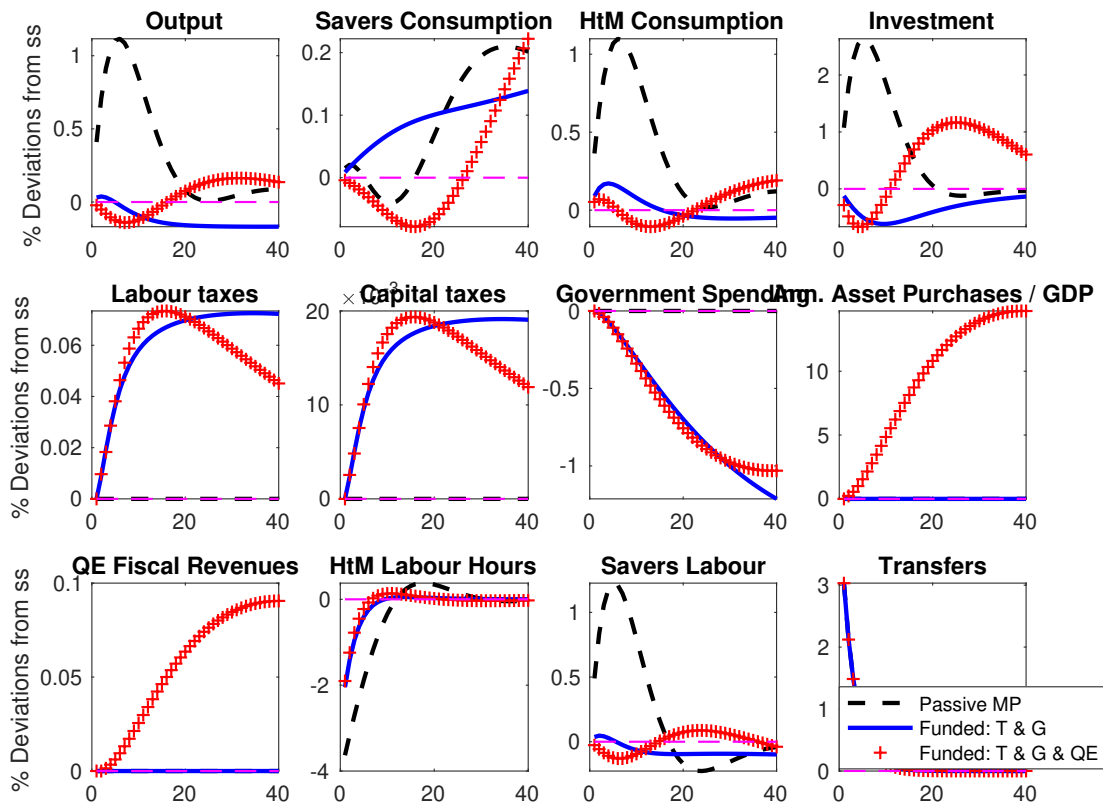


Fig. 7. A Transfer Shock under with funded and unfunded debt

In Figure 8 is evident the sharp difference of the passive monetary policy and the other measures that are along with an active monetary policy. A passive monetary policy leads to a fall in the real interest rate, as the fiscal and the monetary authorities coordinate to let inflation rise to stabilize the increase in transfers as it is clear from the figure. Moving to Figure 7, this has an different impact on the macroeconomy. While the active monetary policy regime is contractionary, the passive regime increases output, consumption and investment. An increase in output and the lower financing costs lead to a reduction in the debt-to-gdp ratio despite the higher spending by the government. Note that in the passive monetary policy regime no taxes or government spending are adjusted to increase the primary balance.

Comparing the passive monetary policy regime with the fiscal adjustments and the QE as a fiscal stabilizers yields two main points. Firstly, the passive monetary policy is the most efficient in terms of debt stability; providing even a reduction in debt after a transfer shock. This is achieved by an increase in inflation that is accommodated by the central bank. Secondly, the second best policy mix of those compared is fiscal adjustments with a parallel use of QE. This manages to keep taxes, government spend-

ing reduction and the recession at a much lower level than in the case where only the classical fiscal adjustment tools are employed.

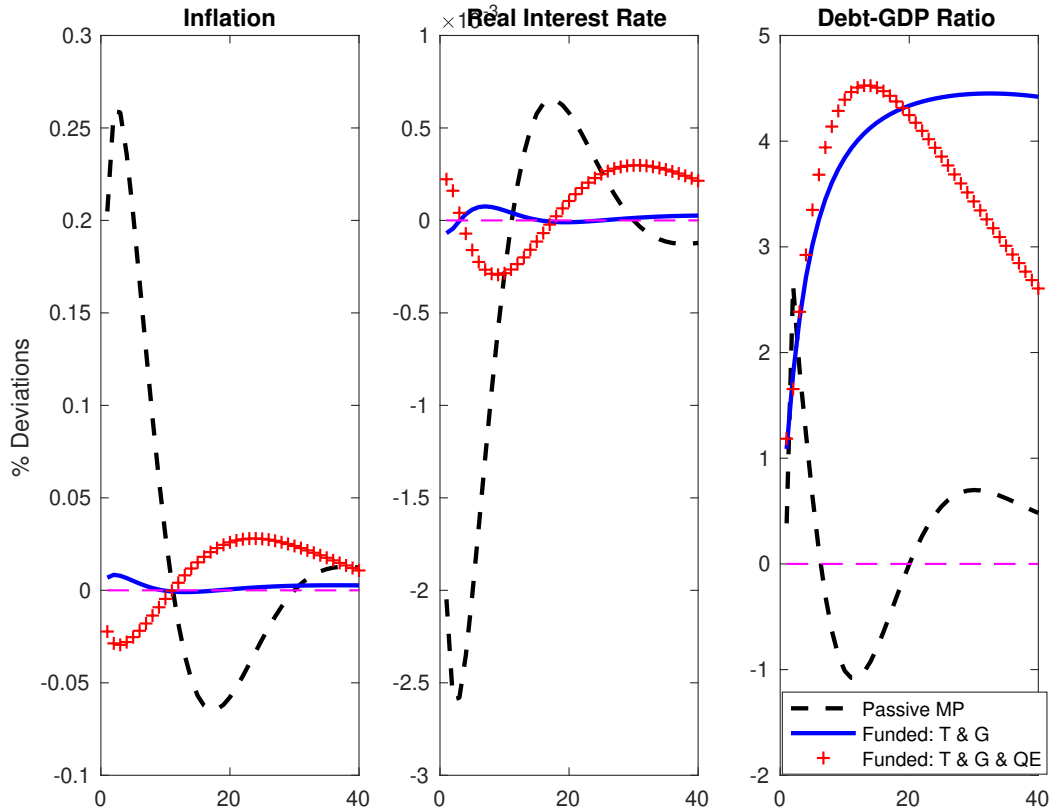


Fig. 8. A Transfer Shock under with funded and unfunded debt

## 5. Conclusion

Since COVID-19 pandemic there is an unprecedented increase in social insurance transfers both in the EA and the US. In this paper, we explore different fiscal and monetary strategies in times of large debt accumulation. We build a New Keynesian DSGE model with household heterogeneity, financial frictions, nominal rigidities, and an unconstrained central bank that can purchase bonds in exchange of reserves. We identify QE as a potential tool for debt stabilization. Profits earned from the bonds-reserves spread can be remitted from the central bank to the treasury and can be a substantial fiscal revenue. We analyse and compare QE as a debt stabilization tool versus government spending and taxation changes under an active (conventional) monetary policy and a passive monetary policy framework.

We show that QE can provide fiscal stability on top of its general equilibrium effects. This happens due to the revenues it generates for the central bank and can be remitted to the treasury to be used as fiscal revenues for debt repayments. After a transfer to the households, QE is the most efficient way of repayment relative to the classical fiscal adjustments. We also show that a passive monetary policy that increases inflation is also beneficial but hard to justify institutionally due to the inflation target mandate of the central banks.

## Appendix A Bank's Problem

This appendix describes the method used for solving the banker's problem. I solve this, with the method of undetermined coefficient in the same fashion as in [Gertler and Kiyotaki \(2010\)](#). I conjecture that a value function has the following linear form:

$$V_t(s_{j,t}, d_{j,t}, b_{j,t}^B, m_{j,t}^B) = \nu_{l,j,t}s_{j,t} + \nu_{b,j,t}b_{j,t}^B + \nu_{m^B,j,t}m_{j,t}^B - \nu_{d,j,t}d_{j,t} \quad (\text{A.1})$$

where  $\nu_{s,j,t}$  is the marginal value from credit for bank  $j$ ,  $\nu_{d,t}$  the marginal cost of deposits,  $\nu_{m^B,j,t}$  the marginal value from the central bank reserves and  $\nu_{b^B,j,t}$  the marginal value from purchasing one extra unit of sovereign bonds. The banker's decision problem is to choose  $s_{j,t}, b_{j,t}^B, m_{j,t}^B, d_{j,t}$  to maximize  $V_{j,t}$  subject to the incentive constraint (7) and the balance sheet constraint (4). Using (4) we can eliminate  $d_{j,t}$  from the value function. This yields:

$$V_{j,t} = s_{j,t}(\nu_{s,t} - \nu_{d,t}Q_t) + b_{j,t}^B(\nu_{b,j,t} - \nu_{d,j,t}q_t) + m_{j,t}^B(\nu_{m,j,t} - \nu_{d,j,t}) + \nu_{d,t}n_{j,t}^B.$$

Let  $\mathcal{L}$  be the Lagrangian of the maximization problem and  $\lambda_t$  the Lagrange multiplier.

$$\mathcal{L} = V_t + \lambda_t[V_t - \theta(Q_t s_{j,t} + \Delta q_t b_{j,t}^B + \omega m_{j,t}^B)] = (1 + \lambda_t)V_t - \lambda_t\theta(Q_t s_{j,t} + \Delta q_t b_{j,t}^B + \omega m_{j,t}^B).$$

The first order and Kuhn-Tucker conditions for the maximization problem are:

$$\frac{\theta \mathcal{L}}{\theta s_{j,t}} : (1 + \lambda_t)\left(\frac{\nu_{s,j,t}}{Q_t} - \nu_{d,j,t}\right) = \lambda_t\theta \quad (\text{A.2})$$

$$\frac{\theta \mathcal{L}}{\theta b_{j,t}^B} : (1 + \lambda_t)\left(\frac{\nu_{b^B,t}}{q_t} - \nu_{d,j,t}\right) = \Delta \lambda_t\theta \quad (\text{A.3})$$

$$\frac{\theta \mathcal{L}}{\theta m_{j,t}^B} : (1 + \lambda_t)(\nu_{m^B,t} - \nu_{d,j,t}) = \omega \lambda_t\theta \quad (\text{A.4})$$

The Kuhn-Tucker condition yields:

$$KT : \lambda_t[s_{j,t}(\nu_{s,j,t} - \nu_{d,t}Q_t) + b_{j,t}^B(\nu_{b^B,j,t} - \nu_{d,j,t}q_t) + m_{j,t}^B(\nu_{m^B,j,t} - \nu_{d,j,t}) + \nu_{d,j,t}n_{j,t}^B - \theta(Q_t s_{j,t} + \Delta q_t b_{j,t}^B + \omega m_{j,t}^B)] = 0. \quad (\text{A.5})$$

I define the excess value of bank's financial claim holdings as

$$\mu_t^s = \frac{\nu_{s,j,t}}{Q_t} - \nu_{d,j,t}. \quad (\text{A.6})$$

The excess value of bank's bond holdings relative to deposits

$$\mu_t^b = \frac{\nu_{b^B,t}}{q_t} - \nu_{d,j,t},$$

and the excess value of bank's reserve holdings relative to deposits

$$\mu_t^m = \nu_{m^B,j,t} - \nu_{d,j,t}.$$

Then from the first order conditions we have:

$$\mu_t^b = \Delta\mu_t^s. \quad (\text{A.7})$$

Setting the fraction of the absconding rate for reserves  $\omega$  to 0%, the reserves first order condition (A.4) implies that

$$\nu_{m^B,t} = \nu_{d,j,t}. \quad (\text{A.8})$$

This relationship implies that the gain from one extra unit of reserves is exactly the same with the cost of raising one extra unit of deposits. This helps us to show that when reserves is a strictly riskless asset, the bank is not taking them into account when the optimization problem is formulated. From (A.5) and (A.7) when the constraint is binding ( $\lambda_t > 0$ ) we get:

$$\begin{aligned} s_{j,t}(\nu_{s,t} - \nu_{d,t}Q_t) + b_{j,t}^B(\nu_{b^B,j,t} - \nu_{d,j,t}q_t) + m_{j,t}^B(\nu_{m^B,j,t} - \nu_{d,j,t}) + \nu_{d,t}n_{j,t} &= \theta(Q_t s_{j,t} + \Delta q_t b_{j,t}^B + \omega m_{j,t}^B) \\ s_{j,t}(\mu_t^s Q_t) + b_{j,t}^B(\mu_t^b q_t) + m_{j,t}^B(\mu_t^m) + \nu_{d,t}n_{j,t} &= \theta(Q_t s_{j,t} + \Delta q_t b_{j,t}^B + \omega m_{j,t}^B) \\ Q_t s_{j,t}(\mu_t^s - \theta) + q_t b_{j,t}^B(\Delta\mu_t^s - \Delta\theta) + m_{j,t}^B(\omega\mu_t^s - \omega\theta) + \nu_{d,t}n_{j,t} &= 0 \\ Q_t s_{j,t}(\mu_t^s - \theta) + \Delta q_t b_{j,t}^B(\mu_t^s - \theta) + \omega m_{j,t}^B(\mu_t^s - \theta) + \nu_{d,t}n_{j,t} &= 0 \end{aligned}$$

and by rearranging terms, we get equation the adjusted leverage constraint:

$$Q_t s_{j,t} + \Delta q_t b_{j,t}^B + \omega m_{j,t}^B = \frac{\nu_{d,t}n_{j,t}}{\theta - \mu_t^s} \quad (\text{A.9})$$

which gives the bank asset funding. It is given by the constraint at equality, where  $\phi_t$  is the maximum leverage allowed for the bank. The constraint limits the portfolio size to the point where the bank's required capital is exactly balanced by the fraction of the weighted measure of its assets. Hence, in times of crisis, where a deterioration of banks' net worth takes place, supply for assets will decline.

Now, in order to find the unknown coefficients I return to the guessed value function

$$V_{j,t} = Q_t s_{j,t}(\mu_t^s) + q_t b_{j,t}^B(\mu_t^b) + m_{j,t}^B(\mu_t^m) + \nu_{d,t}n_{j,t}^B. \quad (\text{A.10})$$

Substituting (A.9) into the guessed value function yields:

$$\begin{aligned} V_t &= (n_{j,t}\phi_t - \Delta q_t b_{j,t}^B - \omega m_{j,t}^B)\mu_t^s + q_t b_{j,t}^B(\mu_t^b) + m_{j,t}^B(\mu_t^m) + \nu_{d,t}n_{j,t}^B \Leftrightarrow \\ V_t &= (n_{j,t}\phi_t)\mu_t^s + q_t b_{j,t}^B(\mu_t^b - \Delta\mu_t^s) + m_{j,t}^B(\mu_t^m - \omega\mu_t^s) + \nu_{d,t}n_{j,t}^B \end{aligned} \quad (\text{A.11})$$

and by (A.7) the guessed value function (A.11) becomes:

$$V_t = \phi_t \mu_t^s n_{j,t} + \nu_{d,j,t} n_{j,t}$$

Given the linearity of the value function we get that

$$A_t^B = \phi_t \mu_t^s + \nu_{d,j,t}.$$

The Bellman equation (??) now is:

$$\begin{aligned} V_{j,t-1}(s_{j,t-1}, x_{j,t-1}, d_{j,t}, m_{j,t-1}) &= \mathbb{E}_{t-1} \Lambda_{t-1,t} \sum_{i=1}^{\infty} \{(1 - \sigma_B) n_{j,t}^B \\ &+ \sigma_B (\phi_t \mu_t^s + \nu_{d,j,t}) n_{j,t}^B\}. \end{aligned} \quad (\text{A.12})$$

By collecting terms with  $n_{j,t}$  the common factor and defining the variable  $\Omega_t$  as the marginal value of net worth:

$$\Omega_{t+1} = (1 - \sigma_B) + \sigma_B (\mu_{t+1}^s \phi_{t+1} + \nu_{d,t+1}). \quad (\text{A.13})$$

The Bellman equation becomes:

$$\begin{aligned} V_{j,t}(s_{j,t}, b_{j,t}^B, m_{j,t}^B, d_{j,t}) &= E_t \Lambda_{t,t+1} \Omega_{t+1} n_{t+1}^B = \\ &= E_t \Lambda_{t,t+1} \Omega_{t+1} [R_{k,t} Q_{t-1} s_{j,t-1} + R_{b,t} q_{t-1} b_{j,t-1}^B + R_t m_{j,t}^B - R_t d_{j,t}]. \end{aligned} \quad (\text{A.14})$$

The marginal value of net worth implies the following: Bankers who exit with probability  $(1 - \sigma_B)$  have a marginal net worth value of 1. Bankers who survive and continue with probability  $\sigma_B$ , by gaining one more unit of net worth, they can increase their assets by  $\phi_t$  and have a net profit of  $\mu_t$  per assets. By this action they acquire also the marginal cost of deposits  $\nu_{d,t}$  which is saved by the extra amount of net worth instead of an additional unit of deposits. Using the method of undetermined coefficients and comparing (A.1) with (A.14) we have the final solutions for the coefficients:

$$\begin{aligned} \nu_{s,j,t} &= E_t \Lambda_{t,t+1} \Omega_{t+1} R_{k,t+1} Q_t \\ \nu_{b^B,j,t} &= E_t \Lambda_{t,t+1} \Omega_{t+1} R_{b,t+1} q_t \\ \nu_{m^B,j,t} &= E_t \Lambda_{t,t+1} \Omega_{t+1} R_{t+1} \\ \nu_{d,j,t} &= E_t \Lambda_{t,t+1} \Omega_{t+1} R_{t+1} \\ \mu_t^s &= \frac{\nu_{s,j,t}}{Q_t} - \nu_{d,j,t} = E_t \Lambda_{t,t+1} \Omega_{t+1} [R_{k,t+1} - R_{t+1}] \\ \mu_t^b &= \frac{\nu_{b,j,t}}{Q_t} - \nu_{d,j,t} = E_t \Lambda_{t,t+1} \Omega_{t+1} [R_{b,t+1} - R_{t+1}] \\ \mu_t^m &= \nu_{m,j,t} - \nu_{d,j,t} = E_t \Lambda_{t,t+1} \Omega_{t+1} [R_{t+1} - R_{t+1}] = 0 \end{aligned} \quad (\text{A.15})$$

$$\mu_t^m = \nu_{m,j,t} - \nu_{d,j,t} = E_t \Lambda_{t,t+1} \Omega_{t+1} [R_{t+1} - R_{t+1}] = 0 \quad (\text{A.16})$$



## Appendix B Price Setting

*Final-Good Firms.*— The profit maximization problem of the retail firm is:

$$\max_{Y_t(i)} P_t \left( \int_0^1 Y_t(i)^{\frac{\zeta-1}{\zeta}} \right)^{\frac{\zeta}{\zeta-1}} - \int_0^1 P_t(i) Y_t(i) di.$$

The first order condition of the problem yields:

$$P_t \frac{\zeta}{\zeta-1} \left( \int_0^1 Y_t(i)^{\frac{\zeta-1}{\zeta}} \right)^{\frac{\zeta}{\zeta-1}-1} \frac{\zeta-1}{\zeta} Y_t(i)^{\frac{\zeta-1}{\zeta}-1} = P_t(i).$$

Combining the previous FOC with the definition of the aggregate final good we get:

$$Y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\zeta} Y_t.$$

Nominal output is the sum of prices times quantities across all retail firms  $i$ :

$$P_t Y_t = \int_0^1 P_t(i) Y_t(i) di.$$

Using the demand for each retailer we get the aggregate price level:

$$P_t = \left( \int_0^1 P_t(i)^{1-\zeta} di \right)^{\frac{1}{1-\zeta}}.$$

*Intermediate-Good Firms.*— Intermediate good firms are not freely able to change prices each period. Following the Calvo price updating specification each period there is a fixed probability  $1 - \gamma$  that a firm will be able to adjust its price.

The problem of the firm can be decomposed in two stages. Firstly, the firm hires labour and rents capital to minimize production costs subject to the technology constraint (??). Thus, it is optimal to minimize their costs which are the rental rate to capital and the wage rate for labour:

$$\min_{K_t(i), L_t(i)} P_t W_t l_t(i) + P_t Z_t K_t(i)$$

subject to

$$A_t K_t(i)^\alpha L_t(i)^{1-\alpha} \geq \left( \frac{P_t(i)}{P_t} \right)^{-\zeta} Y_t.$$

The problem's first order conditions are:

$$W_t = \frac{P_{m,t}^{nom}(i)}{P_t} (1 - \alpha) A_t \frac{Y_t(i)}{L_t(i)}, \quad (\text{B.1})$$

$$Z_t = \frac{P_{m,t}^{nom}(i)}{P_t} \alpha A_t \frac{Y_t(i)}{K_t(i)}. \quad (\text{B.2})$$

$P_{m,t}^{nom}$  is the Lagrange multiplier of the minimization problem and the marginal cost of the firms with  $P_{m,t} = \frac{P_{m,t}^{nom}(i)}{P_t}$  being the real marginal cost. Standard arguments lead to that marginal cost is equal across firms. Solving together the above equations we find an expression for the real marginal cost  $P_{m,t}$  which is independent of each specific variety:

$$P_{m,t} = \left( \frac{1}{1-\alpha} \right)^{1-\alpha} \left( \frac{1}{\alpha} \right)^\alpha W_t^{1-\alpha} Z_t^\alpha.$$

In the second stage of the firm's problem, given nominal marginal costs, the firm chooses its price to maximize profits. Firms are not freely able to change prices each period. Each period there is a fixed probability  $1-\gamma$  that a firm will adjust its price. Each firm chooses the reset price  $P_t^*$  subject to the price adjustment frequency constraint. Firms can also index their price to the lagged rate of inflation with a price indexation parameter  $\gamma_p$ . They discount profits  $s$  periods in the future by the stochastic discount factor  $\Lambda_{t,t+s}$  and the probability that a price chosen at  $t$  will remain the same for some periods  $\gamma^s$ . The second stage of the updating firm at time  $t$  us to choose  $P_t^*(i)$  to maximize discounted real profits:

$$\max_{P_t^*(i)} \mathbb{E}_t \sum_{s=0}^{\infty} \gamma^s \Lambda_{t,t+1} \left( \frac{P_t^*(i)}{P_{t+s}} - P_{m,t+s} \right) Y_{t+s}(i)$$

subject to

$$Y_{t+s}(i) = \left( \frac{P_t^*(i)}{P_{t+s}} \prod_{\kappa=1}^s (1 + \pi_{\tau+\kappa-1})^{\gamma_p} \right)^{-\zeta} Y_{t+s}.$$

where  $\pi_t$  is the rate of inflation from  $t-i$  to  $t$ . The first order condition of the problem is:

$$\mathbb{E}_t \sum_{s=0}^{\infty} \gamma^s \Lambda_{t,t+1} \left( \frac{P_t^*(i)}{P_{t+s}} \prod_{\kappa=1}^s (1 + \pi_{\tau+\kappa-1})^{\gamma_p} - P_{m,t+s} \frac{\zeta}{\zeta-1} \right) Y_{t+s}(i) = 0.$$

Using the constraint and rearranging we get:

$$P_t^*(i) = \frac{\zeta}{\zeta-1} \frac{\mathbb{E}_t \sum_{s=0}^{\infty} \gamma^s \Lambda_{t,t+1} P_{m,t+s} P_{t+s}^\zeta Y_{t+s}}{\mathbb{E}_t \sum_{s=0}^{\infty} \gamma^s \Lambda_{t,t+1} P_{t+s}^{\zeta-1} \prod_{\kappa=1}^s (1 + \pi_{\tau+\kappa-1})^{\gamma_p} Y_{t+s}}.$$

Since nothing on the right hand side depends on each firm  $i$ , all updating firms will update to the same reset price,  $P_t^*$ . By the law of large numbers the evolution of the price index is given by:

$$P_t = [(1-\gamma)(P_t^*)^{1-\zeta} + \gamma(\Pi_{t-1}^{\gamma_p} P_{t-1})^{1-\zeta}]^{\frac{1}{1-\zeta}}.$$

*Capital Goods Producers.*— Capital goods producers produce new capital and sell it

to goods producers at a price  $Q_t$ . Investment on capital ( $I_t$ ) is subject to adjustment costs. Their objective is to choose  $\{I_t\}_{t=0}^{\infty}$  to solve:

$$\max_{I_t} \mathbb{E}_t \sum_{\tau=t}^{\infty} \Lambda_{t,\tau} \left\{ Q_t I_t - \left[ 1 + \tilde{f} \left( \frac{I_\tau}{I_{\tau-1}} \right) I_\tau \right] \right\}.$$

where the adjustment cost function  $\tilde{f}$  captures the cost of investors to increase their capital stock:

$$\tilde{f} \left( \frac{I_\tau}{I_{\tau-1}} \right) = \frac{\eta}{2} \left( \frac{I_\tau}{I_{\tau-1}} - 1 \right)^2 I_\tau.$$

$\eta$  is the inverse elasticity of net investment to the price of capital. The solution to the decision problem of the investors yields the competitive price of capital:

$$Q_t = 1 + \left( \eta \frac{I_\tau}{I_{\tau-1}} \left( \frac{I_\tau}{I_{\tau-1}} - 1 \right) + \frac{\eta}{2} \left( \frac{I_\tau}{I_{\tau-1}} - 1 \right)^2 - \eta \Lambda_{t,\tau} \frac{I_{\tau+1}^2}{I_\tau^2} \left( \frac{I_\tau}{I_{\tau-1}} - 1 \right) \right).$$

## Appendix C Banking Model and Risk Weights

The aggregate bank constraint in terms of the total net worth in the economy is:

$$\omega Q_t S_t^B + \Delta q_t B_t^B + \zeta M_t^B = \phi_t N_t. \quad (\text{C.1})$$

QE, the purchase of bonds or shares by the central bank loosens the constraint leading to a reduction in spreads. In general, when the CB buys bonds, banks can leverage up and increase their loans  $S_t^B$ . The magnitude of the positive QE effects depends on the risk weights  $\omega$  and  $\Delta$  of the shares and bonds respectively. The higher the  $\omega$ , the more risky the loans and the tighter the banking constraint therefore the smaller effects from a QE. On the other hand when the CB buys bonds reducing the amount of  $B_t$ , the smaller the parameter  $\Delta$  is the smaller the effect on the banking constraint.

Figures 9 and 10 show the impulse responses after a QE shock with different weight on loans and bonds. Figure 9 shows the responses with  $\omega = 1$  and  $\Delta = 0.5$  versus the same shock with  $\omega = 2$  and  $\Delta = 0.5$  while figure 10 shows the responses with  $\omega = 1$  and  $\Delta = 0.5$  versus the same shock with  $\omega = 1$  and  $\Delta = 0.1$ . In both cases, the QE shock is expansionary by general equilibrium effects. In 9 when the loan risk weight is 200% due to the tighter bank constraint the positive effects are scaled down. In 10 when the risk weight on bonds becomes 10%, the central bank operation has much less impact than the case of  $\Delta = 0.5$ .

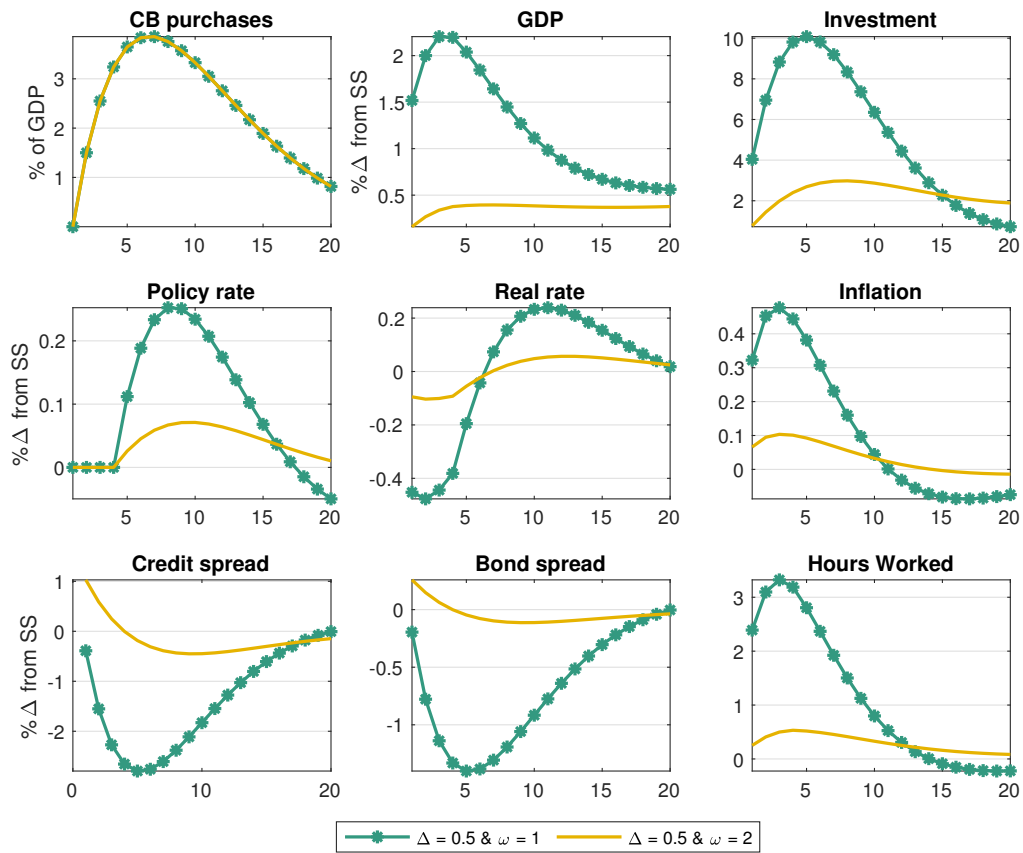


Fig. 9. QE under Different Risk Weights for Loans

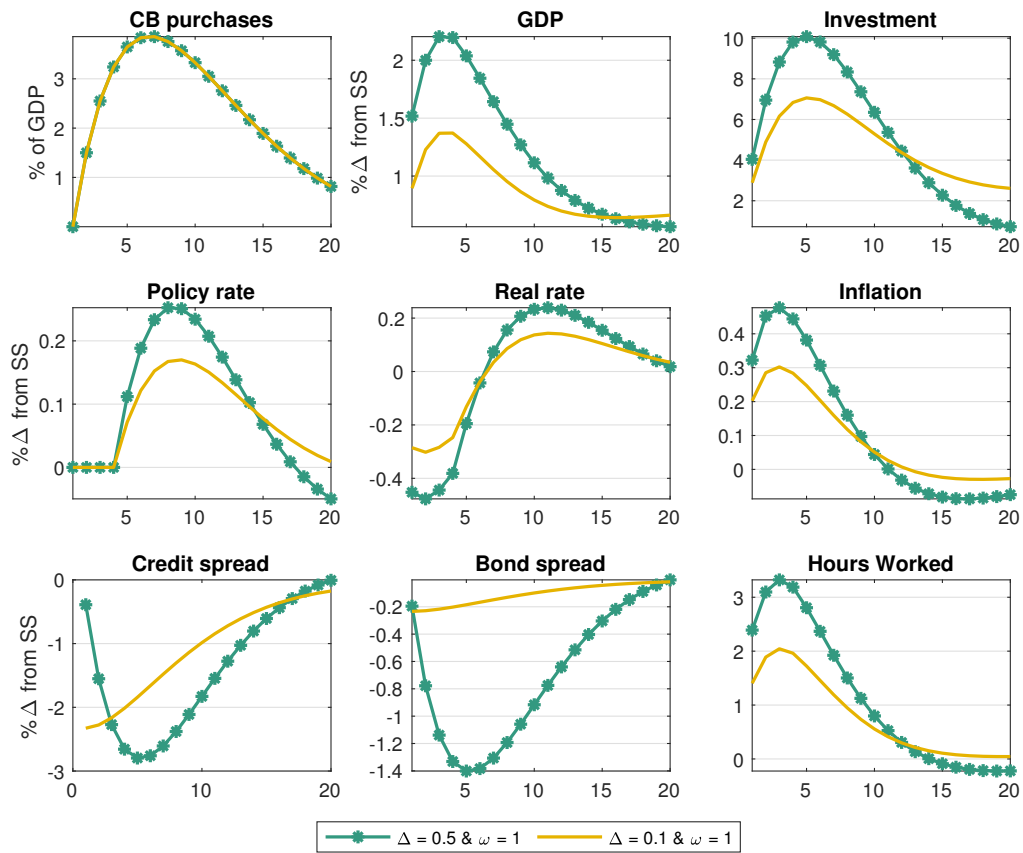


Fig. 10. QE under Different Risk Weights for Bonds

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