Credit Crashes with Recursive Preferences

Oliver de Groot † Panagiotis Veneris ‡

Abstract

Recursive preferences have substantial effects on household behavior. In an otherwise standard model of incomplete markets and heterogeneous agents we establish two general results. First, when properly calibrated, our baseline model with recursive preferences can fit U.S business cycle moments and be in line with the predictions of the von Neumann-Morgenstern (VNM) expected-utility framework. Second, a shock of identical size is shown to dampen households’ response compared to the model in which leisure and consumption enter the utility in an additive separable fashion. This is because risk aversion and intertemporal substitution are distinct structural parameters, and allowing the former to take on large values we are able to generate substantial swings in consumer behavior. Our model also lends itself to the intersection of macroeconomics and finance, as it facilitates the investigation of asset returns and the potential resolution of the equity premium and risk free rate puzzles.

Keywords: Heterogeneous Agents; Incomplete Markets; Recursive Utility; Precautionary Savings

JEL Codes: E32, E44, D52, D80, G52

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I INTRODUCTION

In this paper we study the effects of recursive utility on the response of the economy to an exogenous contraction in borrowing capacity, and the implications for asset pricing. Extending the Guerrieri and Lorenzoni (2017) incomplete markets model to incorporate Epstein and Zin (1989) preferences, we evaluate households’ optimal policies and their propagation to aggregate economic activity when credit gets tighter.

The reason for choosing this setup is twofold. First, market incompleteness is essential to capture the distinctive effects of heterogeneity, as it makes the distributional impact of shocks matter at the aggregate level. The reason is that, with incomplete markets aggregation theorems cease to hold, and aggregate demand is no longer a linear function of aggregate wealth, but depends on the distribution of wealth across agents. Agents’ behavior becomes state-contingent, and their response to aggregate shocks asymmetric. This allows us to study different adjustment mechanisms to the tighter debt limit, depending on the quantile in the distribution the agents belong. On the one hand, households up against the constraint deleverage in an attempt to meet the new stricter borrowing conditions. On the other hand, unconstrained households engage in buffer-stock savings to shield themselves from bearing losses due to adverse future shocks. The precautionary savings channel, absent in models with complete markets, is pivotal to account for the sharp decline in economic activity following a credit shock, for it constitutes an important amplification mechanism of the consumption response to aggregate shocks.

Second, our model provides a fertile ground to study the behavior of asset returns, and in particular the observed difference between the short-term interest rate and the expected return on equity. The reason is that, in contrast to expected utility time-separable preferences in which risk aversion and intertemporal substitution are directly related, with Epstein-Zin preferences these two objects are distinct structural parameters. Disentangling risk attitudes from timing attitudes provides an extra degree of freedom, as it permits to raise risk aversion while keeping the elasticity of intertemporal substitution constant, a feature

\[\text{Heathcote et al. (2009) offer an extensive review of the literature on the importance of heterogeneity and market incompleteness.}\]
that helps generate a higher mean return on equity.

The tension in an expected-utility setting is the implied restriction that agents’ willingness to substitute across dates be related to their willingness to substitute across states. In such a setting, a high degree of risk aversion is needed to match the observed return on equity, which entails a low degree of intertemporal substitution that calls for a high risk-free rate. This is inconsistent with empirical evidence, in which the return to safe assets appears to be in the 1%-3% range (Jordà et al., 2019; Van Binsbergen et al., 2022). In contrast, in our setup we can assess the economy’s reaction to a credit crunch while remaining free from the pervasive model failures arising from the use of expected utility preferences, such as the Mehra and Prescott (1985) equity premium and Weil (1989) risk-free rate puzzles.

Recursive preferences are particularly relevant for studies on the intersection of macroeconomics and finance, for their ability to predict realistic asset prices and equity risk premia. The most representative study that popularized the practicality of recursive preferences is the seminal work of Bansal and Yaron (2004). Using a model with Epstein-Zin preferences and uncertainty about long-run consumption growth prospects, they match empirical estimates of the equity premium and risk-free rate, resolving the corresponding puzzles. Rudebusch and Swanson (2012) fit stylized business cycle and asset pricing moments augmenting Epstein-Zin preferences and long-run inflation risk in a vanilla New Keynesian setup. Pidkuyko et al. (2019) add durable consumption in a standard Lucas (1978) tree economy with recursive utility to address the puzzle of excessive timing and risk premia implied by existing long-run risk models. Nezafat and Slavík (2021) build a heterogeneous-agent, production-based asset pricing model with productivity and liquidity shocks to reconcile business cycle fluctuations and the levels of equity premium and risk-free rate, and de Groot et al. (2022) estimate a sequence of endowment asset pricing models with long-run cash-flow risk and stochastic volatility to highlight the importance of the proper incorporation of time-preference shocks in the Epstein-Zin utility for the success of valuation risk in predicting asset prices. Most of these papers, with a notable exception of Nezafat and Slavík (2021), rely on the representative agent paradigm with complete markets to bridge the gap between asset pricing predictions and business cycle frequencies. Our paper extends this literature, and complements Nezafat and Slavík (2021), by examining the dynamics of a credit crunch and the
implications for asset returns in a unified framework featuring household heterogeneity and uninsurable idiosyncratic risk. The main deviation from Nezafat and Slavík (2021) is that our model does not admit explicit aggregation, which, under certain assumption, holds in their model and greatly simplifies the analysis.

Besides their ability to be consistent with key asset pricing moments and business cycle statistics, recursive preferences can also have important policy implications. Karantounias (2018) in a representative agent model with perfect insurance, Epstein-Zin preferences, and exogenous shocks to government spending, shows that the long-standing tax-smoothing prescription of Lucas and Stokey (1983) breaks, and optimal fiscal policy calls for cyclical fiscal surpluses and deficits.

Our model’s key characteristic, and important factor for the aggregate behavior of the economy, is an occasionally binding borrowing constraint that sets an upper bound on the amount of debt that households can hold at any point in time. This constraint and incomplete insurance drive the internal dynamics of households’ decision-making. Since households do not have access to state-contingent securities, imperfect insurance impairs their ability to absorb negative shocks and retain a smooth consumption path. The extent to which they adjust to an adverse shock and the adjustment mechanism entirely depend on their initial asset position. Households at the left tail of the distribution adjust by reducing their debt to meet the new tighter borrowing conditions. At the same time, households away from the limit respond by increasing their savings. This is due to strong precautionary motives. In this sense, precautionary savings is the main amplification channel in our framework. The aggregate effect of these adjustment mechanisms is an increase in the net demand for assets. Under the assumption that the supply of assets in our model is exogenously fixed, for the asset market to clear the interest rate has to decline in equilibrium.

The importance of individual heterogeneity in quantitative incomplete markets models has been a long-lasting topic in the macroeconomics research agenda, yet received little attention until the wake of the financial crisis. Early papers that embrace heterogeneity are the seminal works of Bewley (1977), Imrohoroğlu (1989), Huggett (1993) and Aiyagari (1994). Imrohoroğlu (1989) examines whether incomplete markets magnify the welfare cost of business cycles. Exploiting different sources of imperfect insurance against income risk, she finds
that the welfare cost of business cycles is six times larger compared to the complete markets benchmark, when a no-borrowing constraint is imposed. After accounting for borrowing, the cost falls below the complete markets estimate. In an endowment economy, Huggett (1993) explores the effect of ex-post heterogeneity coupled with imperfect insurance markets on the risk-free rate, and finds that this combination generates a more realistic equilibrium real interest rate, lower than what representative-agent complete markets models could predict. Using a similar setup, Aiyagari (1994) studies the quantitative implications of individual uncertainty for aggregate savings and the distributions of income and wealth in a production economy. His findings point towards a marginal effect of idiosyncratic risk on savings, at least for plausible values of risk aversion. In a recent paper, Bayer et al. (2019) analyze the ramifications of heightened income risk on aggregate activity, and its distributional implications when markets are incomplete. Their analysis marks income risk as a crucial piece to better understand the intensity of the Great Recession. Complete markets models that mute the role of idiosyncratic uncertainty cannot capture the distributional impact of shocks, delivering implications much different from what empirical studies suggest. For this reason, we follow the incomplete markets paradigm by assuming that households can only partially insure against idiosyncratic risk, in a consumption-savings model with household heterogeneity and within-household production.

The nature of the (aggregate) shock itself has very important implications for the qualitative and quantitative predictions of quantitative macroeconomics models, and its proper modelling relies on the corresponding phenomenon to be explored. Prominent examples in this direction include Fornaro (2018), Justiniano et al. (2015), and Eggertsson and Krugman (2012). Fornaro (2018) investigates the implications of a permanent tightening in the borrowing limit in financially integrated countries in an open economy setting. Justiniano et al. (2015) explore the causes and consequences of a leveraging and deleveraging cycle in a heterogeneous household setup featuring a collateral constraint that periodically relaxes and tightens. Eggertsson and Krugman (2012) analyze the emergence of a liquidity trap resulting from a one-time, sufficiently aggressive tightening of the borrowing constraint, in a representative borrower - representative lender closed economy setup. Here, in line with Fornaro (2018), we choose to focus on a one-time permanent shock in the access to credit,
to capture the adjustment path that households experienced following the 2007 collapse of the financial system.

Central in our model is the role of precautionary savings in intensifying the response of the household sector to exogenous disturbances. There has been a growing body of work that stresses the importance of precautionary savings in business cycle models. Challe and Ragot (2016) build a heterogeneous agent economy in which precautionary savings arise as a cushion against counter-cyclical unemployment risk. They show that the response of aggregate consumption is amplified through the precautionary savings channel, propagating business cycle fluctuations. In a similar manner, Bayer et al. (2019) gauge the contribution of precautionary savings and portfolio rebalancing in aggregate activity. In our model, and in contrast to Challe and Ragot (2016) which makes simplifying assumptions to get exact aggregation, the cross-sectional wealth distribution matters for buffer stock savings as it induces unconstrained households to save more in response to a tightening in borrowing capacity, leading to a sharper decline in aggregate consumption, and hence on the equilibrium interest rate.

The remainder of the paper proceeds as follows. Section II lays out the model and discusses the preference specification. Section III describes the calibration, and the key takeaways from the credit crunch experiment. Section IV contrasts the findings from the expected-utility case with those under the Epstein-Zin specification. Section V concludes.

## II MODEL

In modelling the household sector, we closely follow the approach in Guerrieri and Lorenzoni (2017). Time is discrete and the horizon is infinite. Uncertainty in this economy comes from exogenous idiosyncratic shocks to labor productivity. The economy is populated by a continuum of households who face independent uninsurable productivity draws, consume, supply labor, pay labor income taxes, and trade on a one-period risk-less bond to smooth the effects of idiosyncratic risk on consumption. Households differ each period with respect to income and their initial asset position. The absence of a full set of state-contingent Arrow securities implies that the only avenue of insurance against negative income surprises is
through bond accumulation. For each household $i$, $c_i \geq 0$, and $n_i \in [0,1]$ are consumption and labor supply, respectively, and $\tilde{\tau}_i$ are taxes on labor income.

Every household produces consumption goods using a linear technology of the form

$$y_i = \theta_i n_i,$$  \hspace{1cm} (1)

where $\theta_i$ denotes the idiosyncratic labor productivity shock that household $i$ faces in period $t$. When positive, this shock follows an AR(1) process in logs, and is approximated by a Markov chain on the space \{\theta^2, ..., \theta^S\}. We also allow for unemployment by assuming that households hit by an income shock $\theta^1 = 0$ to be unemployed. Hence, income realizations span the interval \{\theta^1, ..., \theta^S\}. Household $i$’s budget constraint is given by:

$$qb'_i + c_i = b_i + y_i - \tilde{\tau}_i,$$  \hspace{1cm} (2)

where $b_i$ are bond holdings traded at price $q$, and $\tilde{\tau}_i$ are tax payments. The tax structure is the following - while every household pays a lump-sum tax $\tau$, the unemployed receive a transfer $\nu$ in a lump-sum fashion, which can be interpreted as an unemployment benefit. Thus, $\tilde{\tau}_i = \tau$ for employed ($\theta_i > 0$), and $\tilde{\tau}_i = \tau - \nu$ for unemployed ($\theta_i = 0$). Households are constrained to borrow up to an exogenous limit $\phi$, and hence bond holdings each period must satisfy a borrowing constraint of the form:

$$b'_i \geq -\phi.$$  \hspace{1cm} (3)

The government levies lump-sum taxes on households and issues non-contingent debt to finance an exogenous stream of government expenditures, consisting of transfers $\nu$ given to a fraction $u = Pr(\theta_i = \theta^1)$ of unemployed in the population, and outstanding debt obligations. More formally, the budget constraint takes the form:

$$B + u\nu = qB' + \tau.$$  \hspace{1cm} (4)

Under the assumption of a fixed supply of government bonds $B$ and unemployment benefits $\nu$, the government can always balance its budget by adjusting the tax schedule $\tau$.

We close the model by making two simplifying assumptions. First, we assume that in this economy the only supply of liquid assets $B$, comes either from the household sector

\footnote{The borrowing constraint is assumed to be always tighter than the natural borrowing limit.}
or from the government. These assets play a crucial role in households’ behavior, since they act as shock absorbers and the only avenue of insurance against current and future low-endowment realizations. Finally, we assume that the rate at which households borrow is the same as the rate at which they lend, that is we abstract from interest rate deviations.

II.A Epstein-Zin Preferences

Households’ preferences follow the recursive structure of Kreps and Porteus (1978). We focus on the isoelastic preferences of Epstein and Zin (1989) and Weil (1990), which generalize expected utility time-separable preferences, and can be written as:

$$\tilde{V}(b_i, \theta_i) = \left[ \rho_i^\alpha + \beta \left( EV(b'_i, \theta'_i)^\alpha \right) \right]^{\frac{1}{\rho}}.$$  \hspace{1cm} (5)

When labor supply is endogenous, (5) is reformulated as:

$$\tilde{V}(b_i, \theta_i) = \left[ u(c_i, 1 - n_i) + \beta \left( EV(b'_i, \theta'_i)^\alpha \right) \right]^{\frac{1}{\rho}},$$  \hspace{1cm} (6)

that is, households derive utility from consumption and leisure, and a certainty equivalent of continuation utility. If we let $V = \tilde{V}^\rho$ and $\alpha = 1 - \tilde{\alpha}/\rho$, then equation (6) corresponds to:

$$V(b_i, \theta_i) = u(c_i, 1 - n_i) + \beta \left( EV(b'_i, \theta'_i)^{1-\alpha} \right)^{\frac{1}{1-\alpha}},$$  \hspace{1cm} (7)

where $u(c_i, 1 - n_i)$ is period utility stemming from consumption and leisure streams, $\beta \in (0,1)$ is the discount factor, $E(.)$ is the expectations operator conditional on period $t$ information, $\mu = (EV(b'_i, \theta'_i)^{1-\alpha})^{\frac{1}{1-\alpha}}$ is a certainty equivalent of next period’s continuation utility, and $\alpha$ is the Epstein-Zin parameter. The Epstein-Zin parameter is a linear combination of the risk aversion parameter $\tilde{\alpha}$ and the elasticity of intertemporal substitution $1/\rho$. The certainty equivalent operator translates random future continuation utility into units of the composite good comprising consumption and leisure.

Notice that the original Epstein-Zin specification in (6) imposes a restriction on the weights in the time aggregator. In particular, the weights do not sum to one ($1 + \beta \neq 1$). de Groot et al. (2022) show that this restriction is not always innocuous. In particular, they

3We implicitly normalize all assets held by the household sector to equal $B$.
4We relax this assumption in a following section.
prove that for models which embed valuation risk in an Epstein-Zin utility, this restriction creates a discontinuity, in that the behavior of asset pricing moments is strikingly different along the domain of the EIS. Instead, they recommend the use of the alternative specification:

\[
\tilde{V}(b_i, \theta_i) = \left( (1 - \beta)u(c_i, 1 - n_i) + \beta E\tilde{V}(b'_i, \theta'_i)^{\tilde{\alpha}} \right)^{\frac{1}{\tilde{\beta}}},
\]

which lifts that restriction. So, researchers interested in the asset-pricing implications of recursive utility should be very cautious regarding the choice of functional specification. However, since we abstract from these considerations, we proceed with equation (7).

For the period utility we assume that it is additive and isoelastic in consumption and leisure, so that

\[
u(c_i, 1 - n_i) = \frac{c_i^{1-\gamma}}{1-\gamma} + \psi \frac{(1 - n_i)^{1-\eta}}{1-\eta},
\]

where \(u(c_i, 1 - n_i)\) is twice-differentiable, strictly increasing and strictly concave in consumption, and strictly decreasing and strictly convex in labor supply. The usual non-negativity constraint on consumption, \(c_i \geq 0\), and the feasibility constraint on labor, \(n_i \in [0,1]\), apply. If we let \(u \geq 0\) everywhere, then there exists a value function \(V \geq 0\) that solves the Bellman equation (7), and the utility recursion becomes

\[
 V(b_i, \theta_i) = \frac{c_i^{1-\gamma}}{1-\gamma} + \psi \frac{(1 - n_i)^{1-\eta}}{1-\eta} + \beta E V(b'_i, \theta'_i)^{1-\alpha}.
\]

Parameter \(\psi\) measures how much households value leisure, and \(\eta\) stands for the inverse Frisch elasticity of labor supply to changes in the wage rate. Parameter \(\alpha\) is the Epstein-Zin composite parameter, and is defined as \(\alpha = 1 - RA/\gamma\). Since labor supply is endogenous and households can absorb shocks varying both consumption and labor supply, we follow Swanson (2012) and define the coefficient of relative risk aversion over static gambles as:

\[
RA = -B \cdot \frac{V_{11}(b, \theta)}{V_1(b, \theta)} + \alpha \cdot \frac{BV_1(b, \theta)}{V(b, \theta)}.
\]

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5The implications of valuation risk in the current framework is left for future research.

6Theorem 3.1 in Epstein and Zin (1989).

7There is an equivalence between the definition of the Epstein-Zin parameter in equations (10) and (7), with \(RA \equiv \tilde{\alpha}\), and \(1/\gamma \equiv 1/\rho\).
where $V_1$ and $V_{11}$ are the respective first and second derivatives of the value function with respect to its first argument, and $B$ stands for households’ positive wealth measured by the units of bond holdings. The parameter $1/\gamma$ governs the constant elasticity of intertemporal substitution between the composite good and the certainty equivalent of continuation utilities. In the special case where the Epstein-Zin composite parameter $\alpha$ equals zero, our preference specification in (7) is equivalent to the recursive representation of standard time-additive expected utility:

$$V(b_i, \theta_i) = u(c_i, 1 - n_i) + \beta EV(b'_i, \theta'_i), \quad (12)$$

in which the elasticity of intertemporal substitution remains unaltered (and equal to $1/\gamma$), and the coefficient of relative risk aversion takes the form:

$$RA = -B \cdot \frac{V_{11}(b, \theta)}{V_1(b, \theta)}. \quad (13)$$

The definition in equation (13) is in contrast to the definition prevailing in the literature, which mutes the role of the labor margin when interpreting households’ attitudes towards risk, and according to which the parameter of risk aversion is given by:

$$RA = -\frac{u_{11}(c, 1 - n)}{u_1(c, 1 - n)} \cdot c. \quad (14)$$

This highlights the weakness of modern macroeconomic models to properly specify the right measure of risk aversion when labor supply becomes an endogenous choice variable.

The variation between the Epstein-Zin specification in (7) and the time-separable specification in (12) is that in the former, the value function is twisted and untwisted by $1 - \alpha$, and the coefficient of risk aversion is amplified or attenuated by the curvature parameter $\alpha$. However, the main feature of Epstein-Zin utility (10) is that it separates risk aversion from the elasticity of intertemporal substitution (EIS). These two parameters are now allowed to vary independently of each other. While EIS in (10) is identical to the expected-utility case (12), risk aversion is linear in the EIS and the Epstein-Zin parameter $\alpha$. This is in contrast to the coefficient of relative risk aversion in (12), given by $(1/\gamma + 1/\eta)^{-1}$ (Swanson, 2012), and

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8Expanding the expression, and given the utility function is additively separable, the second argument drops out.
which under the assumption of a constant $\eta$ gets tied to the EIS. Therefore, our Epstein-Zin setup provides an extra degree of freedom as we can vary risk aversion without altering the EIS, a property that is critical for the resolution of some long-standing asset puzzles, and the proper study of asset returns.

Finally, if $u \leq 0$ everywhere, then to avoid complex numbers we let $V \leq 0$ and rewrite the recursion (10) as

$$V(b_i, \theta_i) = c_i^{1-\gamma} + \frac{1}{1-\gamma} - \frac{(1 - n_i)^{1-\eta}}{1 - \eta} - \beta (E(-V(b'_i, \theta'_i))^{1-\alpha})^{\frac{1}{1-\alpha}}$$

(15)

Recall that a household with recursive utility as (7) is averse to volatility in future utility, i.e prefers early resolution of uncertainty, if and only if $RA > \gamma$. In contrast, it loves volatility in future consumption, i.e prefers late resolution of uncertainty, if and only if $\gamma > RA$. Here we make the assumption that $RA > \gamma$, which implies that recursive utility increases the curvature with regard to future risks, making agents strictly prefer early resolution of uncertainty. This is an essential characteristic to account for many asset pricing facts.

II.B Equilibrium

II.B.1 Households’ Problem. Households choose a stream of consumption and labor supply, $\{c, n\}_{t=0}^{t=\infty}$, to maximize their lifetime utility, $V(c, n)$, subject to the budget constraint (2), the non-negativity constraint on consumption, $c \geq 0$, and the feasibility constraint for labor, $n \in [0, 1]$. Using the budget constraint, we can express the choice variables as functions of the state variables, that is, $C(b, \theta)$ and $N(b, \theta)$.

II.B.2 Bellman Equation. We assume that household $i$’s value function exists and satisfies the Bellman equation:

$$V(b_i, \theta_i) = \max_{\{c_i, n_i, b'_i\}} \left[ u(c_i, 1 - n_i) + \beta \left( EV(b'_i, \theta'_i)^{1-\alpha} \right)^{\frac{1}{1-\alpha}} \right]$$

(16)

subject to

$$c_i \geq 0, \quad n_i \in [0, 1],$$

(17)
and the household’s budget constraint (2). The optimal choices of consumption and labor supply are summarized in two equations.\(^9\) The intertemporal optimality condition (Euler for consumption):

\[
u_i(c_i, 1 - n_i) \geq \beta(1 + r) \left[ EV(b'_i, \theta'_i)^{1-\alpha} \right]^{\frac{\alpha}{1-\alpha}} E \left[ V(b'_i, \theta'_i)^{-\alpha} u'_i(c'_i, 1 - n'_i) \right], \tag{18}
\]
captures the trade-off households face between consumption today and in the future. Kuhn-Tucker conditions ensure that as long as the debt constraint \(b'_i \geq -\phi\) does not bind, this equation holds with equality. The intratemporal optimality condition (Euler for labor supply) is given by:

\[
-\frac{u_n(c_i, 1 - n_i)}{u_c(c_i, 1 - n_i)} \geq w, \tag{19}
\]
and equates the marginal rate of substitution between consumption and leisure with the real wage. It holds with equality when \(n_i \geq 0\).

**Definition 1.** A recursive competitive equilibrium in this economy is a sequence of value functions \(\{V\} : \mathbb{R}^2_+ \rightarrow \mathbb{R}\), decision rules \(\{C\}, \{N\} : \mathbb{R}^2_+ \rightarrow \mathbb{R}\), interest rates \(\{r\} : \mathbb{R} \rightarrow \mathbb{R}\), tax rates \(\{\tau\} : \mathbb{R}_+ \rightarrow \mathbb{R}_+\), and the (joint) distribution of bond holdings and productivity levels \(\{\Psi\} : \mathbb{R}^2 \rightarrow \mathbb{R}_+\), such that

1. **given the interest rate** \(\{r\}\) and the tax rate \(\{\tau\}\), the value function \(V(b, \theta)\) solves the Bellman equation (16), and \(\{C(b, \theta), N(b, \theta)\}\) are the corresponding optimal decision rules,

2. **the (joint) distribution of bond holdings and productivity levels** \(\Psi\) is consistent with the households’ optimal consumption and labor supply decisions,

3. **the tax rate equals government expenditures,**

\[
\tau = \nu u + \frac{rB}{1 + r}
\]

4. **the asset (bond) market clears,**

\[
\int bd\Psi(b, \theta) = B.
\]

\(^9\)See APPENDIX for analytical derivations.
III QUANTITATIVE ANALYSIS

In this section we quantitatively analyze the response of the economy to an exogenous tightening of the borrowing constraint.

III.A Calibration

Table 1: Baseline Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
<th>Target/Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.9749</td>
<td>Discount factor</td>
<td>$r = 2.5%$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>4</td>
<td>Inverse elasticity of intertemporal substitution</td>
<td></td>
</tr>
<tr>
<td>$RA$</td>
<td>15</td>
<td>Coefficient of relative risk aversion</td>
<td></td>
</tr>
<tr>
<td>$\eta$</td>
<td>1.5</td>
<td>Curvature of utility from leisure</td>
<td>Average Frisch elasticity = 1</td>
</tr>
<tr>
<td>$\psi$</td>
<td>15.57</td>
<td>Weight on utility from leisure</td>
<td>[Nekarda and Ramey (2020)]</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.967</td>
<td>Productivity shock persistence</td>
<td>[Floden and Lindé (2001)]</td>
</tr>
<tr>
<td>$\sigma_e$</td>
<td>0.017</td>
<td>Productivity shock variance</td>
<td></td>
</tr>
<tr>
<td>$\pi_{e,u}$</td>
<td>0.057</td>
<td>Transition to unemployment</td>
<td>[Shimer (2005)]</td>
</tr>
<tr>
<td>$\pi_{u,e}$</td>
<td>0.882</td>
<td>Transition to employment</td>
<td>[Shimer (2005)]</td>
</tr>
<tr>
<td>$B$</td>
<td>1.60</td>
<td>Bond supply</td>
<td>Liquid assets</td>
</tr>
<tr>
<td>$\phi$</td>
<td>1.847</td>
<td>Borrowing constraint</td>
<td>Total gross debt</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.167</td>
<td>Unemployment benefit</td>
<td>40% of average labor income</td>
</tr>
</tbody>
</table>

The calibration of the model is based on [Guerrieri and Lorenzoni (2017)](https://doi.org/10.1017/CBO9781139729230), and utilizes quarterly frequency data to match some salient moments in the US economy. The baseline parameters are shown in Table 1. The discount factor $\beta$ is set to give an annual interest rate of 2.5%. Note that here we follow an unconventional approach. Instead of fixing the value of the discount factor and search for the steady state interest rate consistent with our $\beta$, we set the interest rate and search for the $\beta$ that pins down that interest rate. For the elasticity of intertemporal substitution $\gamma$, typical values in the literature vary widely, mostly
ranging from 0 to 1, and in some instances to 2.\(^{10}\) In our baseline calibration we set the EIS to 1/4, a value upon which we experiment later. The parameter that governs the inverse Frisch elasticity \(\eta\), is chosen to yield a unitary elasticity of labor supply that is within the range of micro estimates (Chetty et al., 2011), and widely used in macroeconomic studies (Justiniano et al., 2015; Bigio and Schneider, 2017; Perri and Quadrini, 2018; Karantounias, 2018; Boppart et al., 2018; Kaplan et al., 2018; Bayer et al., 2019; Cui and Sterk, 2021; Debortoli and Galí, 2022). Parameter \(\psi\) is fixed to target an average of hours worked of 40% of the time endowment of employed workers, as empirically estimated in Nekarda and Ramey (2020). Floden and Lindé (2001), using data on wages and hours worked at the micro level, estimate the stochastic wage process for Sweden and the US, and show that the persistent component for the US displays a degree of autocorrelation equal to 0.91, and a conditional variance of 0.0426. Therefore, the degree of persistence and the variance of our logarithmic AR(1) process, \(\rho\) and \(\sigma_\epsilon\), are set to be in line with that evidence. Shimer (2005), using unemployment-duration data adjusted by the BLS, estimates the job finding and separation rates at business cycle frequencies. The transition probabilities from employment to unemployment, \(\pi_{e,u}\), and from unemployment to employment, \(\pi_{u,e}\), are set to match those rates. \(B\) reflects the sum of bond holdings held by the household sector, and is chosen to match the (liquid) assets to GDP ratio of 1.78 in the US in 2006. The borrowing limit \(\phi\) is such that it yields a 18% consumer debt-to-GDP ratio in the initial steady state and the unemployment benefit \(\nu\) is chosen to be 40% of average labor income, as in Shimer (2005).

Finally, when modeling risk attitudes, there is little consensus regarding the range of plausible values for the coefficient of relative risk aversion, with its choice being mainly dependent on the model economy.\(^{11}\) For instance, representative agent DSGE models with model-consistent expectations, and hence low uncertainty about the economic environment...

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\(^{10}\)Clark and Summers (1982) estimate the EIS to be close to 1, and Hall (1988) finds that EIS ranges from 0 to 0.2. Attanasio and Weber (1993) and Attanasio and Weber (1995) find an elasticity of substitution equal to 0.8. Recent studies, such as Bansal et al. (2016), Best et al. (2020), and Martinez et al. (2021) report estimates of 2.18, 0.1, and 0.5, respectively.

\(^{11}\)In their long-run risk model, Bansal and Yaron (2004) set the coefficient of relative risk aversion equal to 7.5. Piazzesi et al. (2006) fix it to 57, Rudebusch and Swanson (2012) and Van Binsbergen et al. (2012) use values of 75 and 80, respectively, while Karantounias (2018) uses a value of 10 in his benchmark calibration.
(rational expectations hypothesis), need relatively higher values of risk aversion to match the moments in the data.\textsuperscript{12} In contrast, long-run risk models largely respect the upper bound of ten (Epstein et al., 2014), as rationalized by Mehra and Prescott (1985). In our Epstein-Zin setup, idiosyncratic income uncertainty entails sufficient consumption risk, and we do not need to resort to very high values of risk aversion to confront with the data. Therefore, we set the coefficient of risk aversion equal to 15.

III.B Steady State

Figure 1 sketches the optimal decisions of households, with respect to consumption and labor supply, as a function of the initial level of bond holdings and all levels of productivity, from the lowest productivity level $\theta^1$ (unemployment state) to the highest productivity level $\theta^{13}$. This is to uncover the different dynamics at play, at different regions of the wealth distribution.

Two things are worth mentioning. First, there is much heterogeneity in the consumption and labor supply responses between low-wealth and wealthy agents, that is agents at the left tail and the right tail of the wealth distribution, respectively. With regard to consumption, for low levels of bond holdings, the consumption function displays high concavity, while for high levels of bond holdings consumption is almost linear in wealth. The concavity of the consumption function came into the spotlight in the seminal work of Zeldes (1989) who argued that contrary to the case without income uncertainty in which the consumption rule is linear, the presence of uncertainty makes the consumption function concave in wealth. This is also summarized in Carroll and Kimball (1996) who prove that when income risk is introduced, the consumption of low-wealth households tends to be highly sensitive to changes in wealth or transitory income, while for wealthy households the opposite is true.\textsuperscript{13,14} When it comes to labor supply, our benchmark calibration yields a convex labor supply function,

\textsuperscript{12}For an extensive discussion see Rudebusch and Swanson (2012).
\textsuperscript{13}Its origin dates back to Keynes and his book the ‘General Theory of Employment, Interest, and Money’.
\textsuperscript{14}This is in contrast to Kimball (1990) who shows that the MPC alternates for given levels of consumption but not of wealth.
that is an inverse relationship between asset holdings and labor supply. This is intuitive, as households at the lower end of the distribution work harder to increase their labor income and better insure against negative future shocks, indicative of strong income effects that prevail at low levels of wealth. The concavity of the consumption function along with the convexity
of labor supply are key ingredients for the determination of households’ consumption and savings decisions, and hence for the behavior of the wealth distribution.

Second, in a recent paper Wang et al. (2016) show that the concavity of consumption function is directly related to households’ efficacy to self-insure against idiosyncratic risk, with better-insured households experiencing lower consumption volatility. The intuition is that at high levels of asset holdings, the materialization of the shock does not significantly alter the consumption habits of households, given these households have the capacity to utilize their existing savings to hold on to a smooth consumption path. Conversely, the savings of households with low wealth are inadequate to keep their consumption levels unchanged, and hence the shock is partially absorbed by a downward adjustment in consumption. In that sense, consumption at low levels of wealth becomes more volatile and the marginal propensity to consume (MPC) increases. This is in line with our findings, and highlights the importance of the interplay between idiosyncratic risk and the occasionally binding borrowing constraint in understanding the behavior of the MPC, and the subsequent impact of the credit crunch. Quantitatively, our model produces an aggregate marginal propensity to consume of 0.019 in the initial steady state.

III.C Credit Crunch

In this section we describe our baseline experiment. The economy is in its steady state in period $t = 0$, and the maximum amount of debt that households can take on is limited to $\phi = 1.847$. In period $t = 1$ the economy is hit by an exogenous and unexpected one-time shock to the borrowing limit, which is assumed to be permanent in nature and to gradually reduce households’ borrowing capacity to $\phi' = 1.003$. That is, households are now required to adjust their debt schedules downward up until the new constraint. The choice of $\phi'$ is not arbitrary, but rather reflects the 10 percentage point reduction in the debt-to-GDP ratio that the U.S economy experienced in the apex of the 2008 financial crisis. Hence, when we calibrate the model, we choose $\phi'$ to generate a 10 percentage point reduction terminal steady state debt-to-GDP ratio.

One innovation is that we allow the transition path from $\phi$ to $\phi'$ to be smooth, by letting $\phi$ periodically tighten by an amount equal to $\Delta \phi$. The value of $\Delta \phi$ is chosen such that
households meet the new constraint after six periods. Thus, the debt position of households takes the general form:\footnote{This is similar in fashion to Guerrieri and Lorenzoni (2017).}

$$\phi_t = \max\{\phi', \phi - \Delta \phi \cdot t\}, \quad t \geq 1$$

The reason for relying on a slow adjustment of the borrowing constraint is twofold. First, we model bonds as quarterly-traded assets. In reality, debt maturities are much longer than a single quarter. Second, if we assume that households should meet the constraint in such a short time, this would require a sufficiently aggressive degree of adjustment, especially on the part of households closer to the constraint, which could induce some of them to default. To avoid all these unintended consequences, we let households gradually shift to the new lower borrowing limit. An important side effect of the assumption of a gradual transition to the tighter limit is the nontrivial implications for the adjustment path that households choose to follow, with the extend of households’ reaction to be entirely dependent on their initial asset position.

\textit{III.C.1 Implications for savings and the distribution of assets}

Figure 2 contrasts the demand for savings and the distribution of assets in the initial steady state with their counterparts in the terminal steady state.

We start with the first panel of Figure 2 and the policy functions for savings. Our initial calibration delivers convex savings functions. The convexity of the savings functions suggests that households at the left end of the distribution (\textit{low-wealth}) want to increase their savings to move away from the borrowing limit. Comparing the behavior before and after the shock, we observe that following a credit tightening, \textit{low-wealth households} increase their savings whereas those with high initial asset-holdings (\textit{wealthy}) act in the opposite direction, cutting back on their savings in favor of a higher consumption path. The implications of the savings behavior manifest in the second panel, and the ergodic distribution of bonds. In particular, following a credit tightening the ergodic bond distribution concentrates to the right. However, since the aggregate supply of bonds is exogenously fixed, this shift
Figure 2: Savings and Bond Distribution

Notes: Solid line: Initial steady state. Dashed line: Terminal steady state.

should not be interpreted as an increase in the aggregate amount of savings, but rather a rebalancing of households’ balance-sheets. More precisely households up against or close to the constraint increase asset demand, and those at the right tail of the asset distribution decumulate assets. In that sense, the aggregate level of savings is preserved across steady states, and both distributions display the same averages.

III.C.2 Transitional Dynamics

The aggregate behavior of households can be explained in terms of the consumption and labor supply functions, and the occasionally binding borrowing constraint.

When the aggregate shock hits, households who are now debt constrained, respond by
Figure 3: Impulse Response Functions

Notes: The interest rate is annualized. Consumption and labor supply are in levels, whereas output is in percent deviation from its initial steady state value. The dashed blue line is the period in which the borrowing constraint reaches its new steady state level.

... sharply decreasing their consumption \((MPC \cdot \Delta \phi)\), and increasing their labor supply in order to meet the new, lower borrowing limit. This also holds for unconstrained households due to strong precautionary saving motives, and is the outcome of the partial equilibrium response to the credit tightening.

However, since our analysis is in a general equilibrium context, the endogenous response of prices, and in particular that of the interest rate, is pivotal for the precise inference about households’ behavior, and the observed dynamics of savings and the bond distribution. Figure 3 shows the full transition path following the exogenous shock to the credit limit.

\footnote{For a formal proof see the appendix in Guerrieri and Lorenzoni (2017).}
**Interest Rate Response.** The interest rate experiences a sharp decline, going far below zero and reaching its peak of 1.9% after four periods. To get an intuition, it is useful to look at Figure 2. As the bottom panel of Figure 2 suggests, a very high fraction of households starts with very low assets in the initial steady state. After the credit tightening, part of these households will find themselves against the constraint, and will be forced to deleverage. The rest will strive to stay away from the new tighter constraint, building a stock of savings. The net effect is a strong increase in the demand for assets for low-wealth households. Given that assets are in zero net supply in equilibrium, the change in the portfolio composition of low-wealth households has to be compensated by an increase in the supply of assets from wealthy households. Wealthy households decumulate savings due to the convexity of the savings function. However, the size of the wealthy population is not sufficient to compensate for the excess asset demand from households at the left end of the distribution. For the asset market to clear, the interest rate drops in equilibrium.

**Output Response.** Output contracts by 1.1% on-impact, and then it starts converging to the new lower steady state. To better understand the response of output, we should look at the partial and general equilibrium effects in both consumption and labor supply margins. Treating each margin separately and then aggregating is essential as households choose to absorb the credit shock by altering both their consumption and labor supply decisions. Therefore, beyond the demand side effect of consumption on output, output also relies on the supply side and the extend to which households are willing to work more to self-insure against future shocks.

As we analyzed in the beginning of the subsection, the partial equilibrium effect induces both constrained and unconstrained agents to lower consumption and increase savings. Figure 4 gives a snapshot of the consumption response decomposition into partial equilibrium and general equilibrium. Our simulations point towards a partial equilibrium effect that reduces consumption by 4% on-impact. The same intuition applies when we look at Figure 5 and the labor margin. Initially, constrained and unconstrained households want to absorb the adverse shock partly by increasing their labor supply, which translates into a higher labor income and, consequently, higher bond holdings. We document an increase in labor supply
Figure 4: Consumption Response Decomposition

Notes: Solid line: General equilibrium effect. Asterisks: Partial equilibrium effect. Squares: Difference between general equilibrium and partial equilibrium effects. Consumption in percent deviations from its initial steady state value.

of 13.07% on impact, in partial equilibrium.

Both effects lead to an increase in the net demand for assets. Since the supply of assets is (exogenously) fixed, the interest rate has to drop to induce equilibrium in the market for assets. As the interest rate transitions to its new lower equilibrium value, the intertemporal price of consumption increases. Moreover, leisure starts to become more attractive as its cost in terms of consumption units decreases. Therefore, both constrained and unconstrained households adjust by increasing consumption today due to intertemporal substitution, and decreasing labor supply due to the lower cost of leisure. This is the general equilibrium effect. Quantitatively, the endogenous drop in the interest rate of 1.9% in the short-run, leads to
Figure 5: Labor Supply Response Decomposition

Notes: Solid line: General equilibrium effect. Asterisks: Partial equilibrium effect. Squares: Difference between general equilibrium and partial equilibrium effects. Labor supply in percent deviations from its initial steady state value.

an increase in aggregate consumption of 2.8\% on impact, considerably mitigating the partial equilibrium effect. The reverse intuition applies when looking at labor supply. In particular, the general equilibrium effect contracts aggregate hours worked by 10\% on impact, partially offsetting the strong initial increase absent equilibrating forces.

Given our baseline calibration, the partial equilibrium effect dominates along the supply and demand sides of the economy. This points toward a reduction in aggregate consumption and an increase in aggregate labor supply. One would be tempted to infer that these two offsetting forces would have opposing effects on output. In our model this is not the case. In particular, the increased aggregate labor supply does not translate into an increase in
Figure 6: Output Response Decomposition

Notes: Solid line: General equilibrium effect. Asterisks: Partial equilibrium effect. Squares: Difference between general equilibrium and partial equilibrium effects. Labor supply in percent deviations from its initial steady state value.

output. This is due to compositional effects that determine the extend to which workers with different productivity levels adjust their labor supply. Since in our model labor income is tied to productivity, low-productivity households with low asset holdings are the ones to increase labor supply aggressively since they are interest-insensitive and the partial equilibrium effect dominates, whereas high-productivity households at the right end of the asset distribution work less hours due to a dominant general equilibrium effect. Therefore, even though the partial equilibrium effect dominates in equilibrium and hours worked increase on aggregate, the contractionary effect coming from the declining labor supply of productive, wealthy households more than offsets the increase in output due to higher hours worked of under-
productive, low-wealth households. In other words, the aggregate labor productivity declines. Hence, contrary to the conventional wisdom, both effects lead to a decline in output. As Figure 6 suggests, although the partial equilibrium effect calls for a strong increase in output of 8.5% on impact, this increase is balanced by the endogenous drop in the interest rate. Overall, taking into account the demand and supply side effects, output drops by 1.1% on-impact.

IV INSPECTING THE MECHANISM

In this section we present alternative calibrations which differ with respect to the coefficient of relative risk aversion, RA, and the borrowing limits, $\phi$, that are consistent with a debt-to-GDP ratio of 18% and 8% in the initial and terminal steady states, respectively.

IV.A CRRA vs Epstein-Zin

We begin by analyzing an economy with CRRA preferences. To this end, we set RA = 4, which implies an elasticity of intertemporal substitution of 1/4. This calibration replicates the expected-utility case, as the coefficient of relative risk aversion and the elasticity of intertemporal substitution are reciprocal of one another. The main motive behind this exercise is to use the expected-utility case as a natural benchmark against which we compare our findings under Epstein-Zin preferences. This helps to uncover the extend to which the preference specification tilts the response of the economy to an unexpected one-time shock, and the implications for the dynamics of the business-cycle.

Figure 7 reports the policy functions for consumption and labor supply when the utility function displays constant relative risk aversion (CRRA). What we observe is that the consumption function becomes more concave as the coefficient of relative risk aversion reduces to the value that replicates the CRRA case. This is at odds with the expected-utility benchmark, in which the concavity of consumption is a positive function of the risk aversion parameter (Guerrieri and Lorenzoni (2017)). To understand the intuition behind this result, it is useful to look at the first panel of Figure 8. With CRRA preferences, the demand for savings for low-wealth households is lower than when households have recursive utility. This,
in turn, implies that those households have less power to self-insure against idiosyncratic income risk, facing greater consumption volatility. Greater consumption volatility is the key element to account for the higher curvature of the consumption function at that region of the bond distribution.
Beyond its impact on the shape of the consumption function, the extend to which households adjust their demand for savings has strong implications for the distribution of wealth. The panels in Figure 9 contrast the density of bond holdings in the initial (top panel) and terminal (bottom panel) steady states, respectively. It is straightforward that the asset distribution in the CRRA case displays a higher mass of agents closer to the borrowing limit, relative to the distribution under Epstein-Zin preferences, which is more concentrated to higher values of asset holdings. The intuition is simple. Even absent the aggregate shock, as the degree of risk aversion decreases, households, irrespective of their initial asset position, become less prudent and start to decumulate savings. Thus, the distribution displays more households near the borrowing constraint. However, it is important to mention that the aggregate amount of savings is identical across preference specifications.

A common feature of the bond distribution across preference specifications is that it shifts to the right in the terminal steady state. The main idea behind the more concentrated
The terminal-steady-state distribution is that after the shock is realized, constrained agents are forced to adjust their consumption downwards, thus increasing their savings, while unconstrained agents engage in bond accumulation to preclude the possibility of hitting the constraint in a later period. Hence, all agents end up with a higher level of assets, and the terminal steady state shifts to the right.

The macroeconomic implications of households’ decision making in response to a credit tightening are reflected in the panels of Figure 10. The figure depicts the time-path of macroeconomic aggregates, and makes a quantitative comparison between the transition dynamics for the two main preference specifications. We observe that our variables of interest follow a qualitative pattern characterized by an aggressive downward adjustment of output, the debt-to-GDP ratio, and the interest rate, on impact. This is the outcome of the tighter borrowing limit, which depresses the aggregate economic activity. The intuition behind these results carries over from the Epstein-Zin case. When comparing the quantitative behavior
Figure 10: Epstein-Zin & CRRA

Notes: The interest rate is annualized. Dashed red line: CRRA preferences. Solid blue line: Epstein-Zin preferences. Dashed blue line: period in which the borrowing constraint reaches its new steady state level. Output, consumption, and labor supply in percent deviation from their initial steady state values.

of macroeconomic aggregates, the key takeaway is an overall similar degree of adjustment that persists for almost the entire time-horizon, with negligible differences in the terminal steady state. This suggests that our model with Epstein-Zin preferences does a fairly decent job at explaining the dynamics of the U.S business cycle in the apex of the 2008 financial crisis, producing moments that match those of the expected-utility counterpart. This is particularly encouraging, especially for models with recursive utility, as our baseline model, that rest on plausible parameter values, that is, parameter values within the well-respected range that the macroeconomics and finance literature dictates.
IV.B  Fixed Borrowing Limits

This section studies the quantitative implications of fixing the borrowing limits across the main preference specifications. To this end, we take our benchmark model with Epstein-Zin preferences and fix values for the borrowing limits in initial and terminal steady states that coincide with the calibrated values under the expected-utility framework. In particular, we set the borrowing constraints to $\phi_1 = 1.6079$ and $\phi_2 = 0.8988$, respectively. The implication

Figure 11: Epstein-Zin & CRRA

Notes: The interest rate is annualized. Dashed red line: CRRA preferences. Solid blue line: Epstein-Zin preferences. Dashed blue line: period in which the borrowing constraint reaches its new steady state level. Output, consumption, and labor supply in percent deviation from their initial steady state values.

of this tweak is that we no longer target a specific debt-to-GDP ratio before and after the shock; rather we let it adjust to levels consistent with $\phi_1$ and $\phi_2$. The bottom line is that we
can study the extend to which the responses of macro aggregates in both economies differ when hit by the same shock.

**Figure 12:** Bond distribution

![Bond distribution](image)

**Figure 11** shows the full transition path of the main macro aggregates following a credit tightening. The key takeaway from the impulse response analysis is an overall smoother adjustment to the shock in the Epstein-Zin framework. The reason behind this result is straightforward. With Epstein-Zin preferences - and the borrowing limits fixed to the values prevailing in the CRRA case - the required degree of downward adjustment in the debt-to-GDP ratio is lower. In particular, the debt-to-GDP ratio falls by roughly 8 percentage points in the terminal steady state, which is much lower than the 10 percentage point decline that we impose when households have expected-utility preferences. The reasoning follows from the structure of Epstein-Zin preferences, and in particular the common practice of endowing
households with greater degrees of risk aversion.\textsuperscript{17} Higher risk aversion in turn makes the marginal utility more convex, and households more prudent. This implies that households choose take on less debt in the initial steady state.

Figure 12 shows the ergodic distribution of bond holdings in the initial and terminal steady states across preference specifications. It is clear from the top panel that when preferences follow the Epstein-Zin recursive structure, a lower fraction of households is concentrated around the borrowing limit before the shock. Less excessive (initial) steady state leveraging calls for a less costly adjustment to the tighter limit. Therefore, consumption and output fall by less, and labor supply increases by less. Consistent with the smoother downward adjustment of output, the response of the real rate is far less pronounced.

V CONCLUSION

In this paper we add Epstein-Zin preferences in an otherwise standard incomplete markets model with idiosyncratic and aggregate shocks. The reason for choosing this setup is twofold. First, it serves as an experiment to understand whether altering the preference specification has any meaningful quantitative impact on households’ decision-making and, in turn, on the macro aggregates, when the borrowing constraint tightens unexpectedly. Second, it lays the foundations to study the behavior of asset returns and some of the long-standing puzzles prevalent in model with expected utility preferences, such as the equity premium and the risk-free rate puzzles.

Quantitatively, our model produces a less concave consumption function, higher aggregate savings that shift the wealth distribution to the right, and a sharper interest rate and output decline than its expected-utility counterpart. These results are attributed to the stronger precautionary savings motive that the higher coefficient of risk aversion induces.

The model can be extended in many different directions if one wants to get richer dynamics and capture other interesting dimensions of the economy. First, adding risky assets is a natural extension of our Epstein-Zin specification, as it allows to study both the behavior of asset prices and the macroeconomic impact of shocks in a unified framework.

\textsuperscript{17}Check earlier footnote for a discussion on values of risk-aversion in models with Epstein-Zin preferences.
Second, one can give rise to endogenous business cycles, by using a framework with capital accumulation in which long-run growth is subject to firms’ investment in innovation, as in Benigno and Fornaro (2018). This is useful if one wants to study whether alternative preference specifications can have real effects in long-run growth prospects. Finally, studying a mix of fiscal and monetary policies in a novel setup is always of interest.
References


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APPENDIX

A. Optimality Conditions

To get the Euler equations for consumption and leisure we take the following steps. First, we get the first order condition with respect to consumption, which states as:

\[
 u_c(c, n) + \beta \frac{1}{1 - \alpha} (EV(b', \theta')^{1-\alpha})^{\frac{\alpha}{1 - \alpha}} (1 - \alpha)E \left( (V(b', \theta')^{-\alpha}V_c(b', \theta')) \right) \geq 0
\]

\[
 u_c(c, n) \geq -\beta (EV(b', \theta')^{1-\alpha})^{\frac{\alpha}{1 - \alpha}} E \left( (V(b', \theta')^{-\alpha}V_c(b', \theta')) \left( \frac{-1}{q} \right) \right)
\]

\[
 u_c(c, n) \geq \frac{\beta}{q} [EV(b', \theta')^{1-\alpha}]^{\frac{\alpha}{1 - \alpha}} E \left( (V(b', \theta')^{-\alpha}V_c(b', \theta')) \right), \quad (1)
\]

and the optimality condition with respect to labor supply:

\[
 u_n(c, n) + \beta \frac{1}{1 - \alpha} (EV(b', \theta')^{1-\alpha})^{\frac{\alpha}{1 - \alpha}} (1 - \alpha)E \left( (V(b', \theta')^{-\alpha}V_c(b', \theta')) \left( \frac{w}{q} \right) \right) \leq 0
\]

\[
 u_n(c, n) \leq -\beta w [EV(b', \theta')^{1-\alpha}]^{\frac{\alpha}{1 - \alpha}} E \left( (V(b', \theta')^{-\alpha}V_c(b', \theta')) \right). \quad (2)
\]

The problem that pops up is that we do not know how to find the derivative of the value function \( V(b', \theta') \) with respect to \( b' \). However, if we assume that \( b' \) has been chosen optimally, we can take the derivative of the value function with respect to \( b \), given by:

\[
 \frac{\partial V(b, \theta)}{\partial b} = u_c(c, n) \frac{\partial c}{\partial b} + u_n(c, n) \frac{\partial n}{\partial b} + \frac{\beta}{1 - \alpha} [EV(b', \theta')]^{\frac{\alpha}{1 - \alpha}} (1 - \alpha)E \left( (V(b', \theta')^{-\alpha} \frac{\partial V(b', \theta')}{\partial b'}) \frac{\partial b'}{\partial b} \right)
\]

\[
 = u_c(c, n) \frac{\partial c}{\partial b} + u_n(c, n) \frac{\partial n}{\partial b} + \frac{\beta}{1 - \alpha} (EV(b', \theta'))^{\frac{\alpha}{1 - \alpha}} (1 - \alpha) \times
\]

\[
 \times E \left( (V(b', \theta')^{-\alpha} \frac{\partial V(b', \theta')}{\partial b'}) \left( \frac{1}{q} + \frac{w \frac{\partial n}{\partial b}}{q} - \frac{1}{q} \frac{\partial c}{\partial b} \right) \right) \quad (3)
\]
Inserting (1) and (2) into (3), we get the Benveniste-Scheinkman envelope condition:

\[
\frac{\partial V(b, \theta)}{\partial b} = \frac{\beta}{q} \left[ EV(b', \theta')^{1-\alpha} \right]^{\frac{\alpha}{1-\alpha}} E \left[ V(b', \theta')^{-\alpha} V(b', \theta') \frac{\partial c}{\partial b} \right] - \frac{\beta}{q} w \left[ EV(b', \theta')^{1-\alpha} \right]^{\frac{\alpha}{1-\alpha}} \times E \left[ V(b', \theta')^{-\alpha} V(b', \theta') \frac{\partial n}{\partial b} \right] + \frac{\beta}{q} w \left[ EV(b', \theta')^{1-\alpha} \right]^{\frac{\alpha}{1-\alpha}} E \left[ V(b', \theta')^{-\alpha} V(b', \theta') \frac{\partial n}{\partial b} \right] - \frac{\beta}{q} w \left[ EV(b', \theta')^{1-\alpha} \right]^{\frac{\alpha}{1-\alpha}} \times E \left[ V(b', \theta')^{-\alpha} V(b', \theta') \frac{\partial c}{\partial b} \right] + \frac{\beta}{q} \left[ EV(b', \theta')^{1-\alpha} \right]^{\frac{\alpha}{1-\alpha}} E \left[ V(b', \theta')^{-\alpha} V(b', \theta') \right]
\]

Now, combining (1) and the envelope condition (4) we get the Euler for consumption:

\[
u_c(c, n) \geq \frac{\beta}{q} \left[ EV(b', \theta')^{1-\alpha} \right]^{\frac{\alpha}{1-\alpha}} E \left[ V(b', \theta')^{-\alpha} u_c(c', n') \right]
\]

This is the intertemporal optimality condition, and holds with equality when the exogenous debt limit \( b' \geq 0 \) is not binding. Finally, combining equations (1) and (2) we get the Euler for labor supply:

\[
u_n(c, n) \leq -w u_c(c, n)
\]

an intratemporal optimality condition that equates the marginal rate of substitution between consumption and leisure with the real wage. It holds with equality when \( n \geq 0 \).

**B. Solution Algorithm**

To compute the optimal decision rules for consumption and labor supply, we proceed by discretizing the state space. We setup a non-linear grid for the endogenous state (assets) comprising 300 values. The grid is denser for lower values of bonds, because the policy functions at that region display higher curvature, and hence we more gridpoints are needed to get a better approximation. We fix the upper bound of the asset grid at 50, and the
lower bound at -3. Since the decision rules are functions of the asset choices and the income realizations, they are allowed to be nonlinear between gridpoints.

The employment status follows an AR(1) stochastic process which is discretized using 12 equally spaced nodes, delivering a probability transition matrix of size 12x12. For the discretization we use the method of Tauchen (1986).

To solve for the equilibrium, we employ the method of Endogenous Gridpoints originally proposed by Carroll (2006). The main advantage of this method is that it skips the optimization step necessary to build the Euler equation, the most time-consuming step in the traditional Value Function Iteration approach.

The complexity of the model increases when we introduce an aggregate shock in borrowing capacity, on top of the idiosyncratic income shocks that hit households each period. This is because, with aggregate shocks there is an extra state variable that agents need to take into account when making optimal decisions about consumption and labor supply, the wealth distribution. This can be a high dimensional, or, sometimes, an infinite dimensional object, making the analysis very challenging. Most of the literature on incomplete markets models with aggregate shocks relies on computationally expensive techniques, such as the one developed in Krusell and Smith (1998), which approximates the wealth distribution with a small number of moments that incorporate all the necessary information about that distribution. In our model, we assume away aggregate uncertainty by focusing on an MIT shock, after which agents have perfect foresight. Hence, the distribution of assets does not enter into households’ state space, making the analysis less demanding.

To compute the ergodic set of bonds, we start from a uniform distribution that reflects the percentage (fraction) of agents at each employment-asset node, and iterate on the probability transition matrix until convergence. In this way, we keep track of the fraction of agents at each node across time, and not of individual agents. In principle, we could have proceeded by finding the eigenvector of the probability transition matrix consistent with the unitary eigenvalue, but since the matrix is big this would be more costly in terms of computational time.

Finally, to compute the transitional dynamics we proceed by backwards induction. In particular, we start by giving a guess about the full path of interest rates. Then, we assume
that the economy is in the terminal steady state in period $t = T$ (we set $T = 100$) with $r_t = r_T$, and iterate on the Euler equation backwards, using the endogenous gridpoints method, to get the individual policy rules from $t = T - 1$ to $t = 0$. Next, we iterate on the bond distribution forward from $t = 0$ to $t = T$, using the computed policy rules, to calculate the aggregate demand for bonds at each period. Finally, we check for market clearing; if the asset market does not clear, we update the interest rate using the simple linear rule $r_t^\kappa = r_t^{(\kappa - 1)} - w \cdot (B_s - B^d)$, where $\kappa$ denotes the $\kappa$-th iteration, and start over again; otherwise we stop.

C Elasticity of Intertemporal Substitution

Figure C.1 displays the policy functions when we increase the elasticity of intertemporal substitution to 0.5 (hence, $\gamma$ equals 2 in Table 1), keeping the parameter of relative risk aversion fixed at 25. The effect of a higher EIS on consumption is conditional on agents’ wealth position in the distribution. For *wealthy* households, the increase in the EIS makes current consumption more attractive, raising their demand for goods at the cost of lower consumption in the future. The opposite is true for households with levels of wealth below a threshold, with higher EIS be associated with intertemporal substitution in consumption, hence higher consumption in subsequent periods. At the aggregate level, the increased demand for consumption of wealthy households dominates the subdued demand of households with asset levels below the threshold, and consumption increases.
Figure C.1: Policy Functions

Consumption

Labor Supply
**Figure C.2: Bond Distribution & Savings**

![Graphs showing Initial and Terminal steady states for different values of \( \gamma \)](image)

**Notes:** Solid lines: Initial steady state. Dashed lines: Terminal steady state.

Besides the effect on consumption, raising the elasticity of intertemporal substitution has a direct effect on the steady-state level of savings. In particular, when we move from \( \gamma = 4 \) to \( \gamma = 2 \), the steady-state level of savings jumps only by 4.9%, indicating that the elasticity of intertemporal substitution has a minor contribution to savings decisions. These results accord with the findings in Wang et al. (2016).