Binary Response Dynamic Panel Data Models with Switching State Dependence

Eleni Aristodemou
University of Cyprus

20th Conference on Research on Economic Theory and Econometrics,
Tinos
July 14, 2022

Talk Outline

1. Motivation and Contribution
2. Literature
3. Model framework
4. Identification strategy
5. Conclusion and Work-in-progress

Motivation I: Intertemporal correlation

Heckman (1981): "The conditional probability that an individual will experience the event in the future is a function of past experience."

- "**True State Dependence**: an otherwise identical individual who did not experience the event would behave differently in the future than an individual who experienced the event."
- "**Spurious State Dependence**: individuals may differ in certain unmeasured variables that influence their probability of experiencing the event but that are not influenced by the experience of the event"

Motivation II: Switching costs

- Dubé, Hitsch, and Rossi (2010): "Researchers in both marketing and economics have documented a form of **persistence** in consumer choice data whereby consumers have a higher probability of choosing products that they have purchased in the past. We call this form of persistence **inertia** in brand choice."
  - "There are two conceptually distinct explanations for the source of inertia in brand choice: structural state dependence and spurious state dependence"
  - "...the finding of inertia is robust to controls for unobserved consumer differences, and thus we find **evidence for structural state dependence**. We then explore three different economic explanations that can give rise to structural state dependence: preference changes due to past purchases (**psychological switching costs**) which induce a form of loyalty, search, and learning..."
- Dubé, Hitsch, and Rossi (2009): "**Switching costs** can come from a variety of sources, including product adoption costs, shopping/search costs, and psychological sources..... Both psychological switching costs and a monetary switching fee give rise to an **observationally equivalent form of inertia** in consumer purchases."

Contribution

This paper studies identification in

- Binary response panel data models with switching state dependence

Examples:

- Consumers demand over time
  - Choice between system blades and disposable razors
- Individuals labor decisions over time
  - Choice between employment and unemployment

Binary Response Dynamic Panel Data Models (with a twist)

- Individuals might be reluctant to switch: Inertia in individual choices
  - Cost of switching, habits, loyalty, lock-in effects, search, learning....
- Examine whether individuals condition their decisions on **switching**

(A very brief) Literature review

Dynamic binary response models

- Honoré and Kyriazidou (2000), Honoré and Tamer (2006), Aristodemou (2021), Khan, Ponomareva, and Tamer (2019), Honoré and Weidner (2020)

Dynamic ordered response models

- Muris, Raposo, and Vandoros (2020), Honoré, Muris, and Weidner (2021)

Structural dynamic panel data models

- Aguirregabiria, Gu, and Luo (2021), Pakes, Porter, Shepard, and Calder-Wang (2021)

Duration models

- Lancaster (1979), Frederiksen, Honoré, and Hu (2007)

Multi-spell duration models

- Gørgens and Hyslop (2018, 2019)

IO literature: Consumer inertia, passive shoppers and switching costs

- Dubé, Hitsch, and Rossi (2009, 2010), Clerides and Courty (2017)

The (classical) Dynamic Binary Response Model

Individuals observed for 3 periods, $t \in \{0, 1, 2\}$

$$Y_{it} \quad = \quad 1(X_{it}\beta + 1(Y_{i,t-1} = 1)\gamma + \alpha_i + V_{it} > 0)$$

with

$$1(Y_{i,t-1} = 1) = \begin{cases} 1 & \text{if } Y_{i,t-1} = 1 \\ 0 & \text{otherwise} \end{cases}$$

- $(\beta, \gamma) \in \Theta$ parameters of interest

Observables

- $Y_i = (Y_{i1}, Y_{i2}) \in \mathcal{Y}_1 \times \mathcal{Y}_2 \equiv \{0, 1\}$
- $X_i = (X_{i1}, X_{i2})$ with support $\mathcal{X}$
- Initial condition $Y_{i0} \in \{0, 1\}$

Unobservables

- $\alpha_i \in \mathcal{A}$
- $V_i = (V_{i1}, V_{i2})$ with support $\mathcal{V}$

The New Model

Individuals observed for 3 periods, $t \in \{0, 1, 2\}$

$$Y_{it} = 1(X_{it}\beta + 1(Y_{i,t-1} = Y_{it})\gamma + \alpha_i + V_{it} > 0)$$

with

$$1(Y_{i,t-1} = Y_{it}) = \begin{cases} 1 & \textit{if } Y_{i,t-1} = Y_{it} \\ 0 & \textit{otherwise} \end{cases}$$

- $(\beta, \gamma) \in \Theta$ parameters of interest

Observables

- $Y_i = (Y_{i1}, Y_{i2}) \in \mathcal{Y}_1 \times \mathcal{Y}_2 \equiv \{0, 1\}$
- $X_i = (X_{i1}, X_{i2})$ with support $\mathcal{X}$
- Initial condition $Y_{i0} \in \{0, 1\}$

Unobservables

- $\alpha_i \in \mathcal{A}$
- $V_i = (V_{i1}, V_{i2})$ with support $\mathcal{V}$

Econometric Challenges: (classical) DBR Model

- Describe choices between two or more discrete alternatives
  - Qualitative or Limited dependent variables models
- Distribution of time-varying unobservables
- Fixed effects: α correlated with Y_{t-1} and possibly X_t
- Initial condition: Choice in period $t = 1$ depends on $t = 0$
- Incidental parameters problem: Small T-Large N

Econometric Challenges: New Model

Individual decision problem:

$$When\ Y_{t-1} = 0: \ \ Y_t = \begin{cases} 1 & if\ X_t\beta + \alpha + V_t > 0 \\ 0 & X_t\beta + \gamma + \alpha + V_t < 0 \end{cases}$$

$$When\ Y_{t-1} = 1: \ \ Y_t = \begin{cases} 1 & if\ X_t\beta + \gamma + \alpha + V_t > 0 \\ 0 & X_t\beta + \alpha + V_t < 0 \end{cases}$$

Single Index Restriction is violated.

Econometric Challenges: New Model

Individual decision problem:

$$When\ Y_{t-1} = 0: \ Y_t = \begin{cases} 1 & if\ X_t\beta + \alpha + V_t > 0 \\ 0 & X_t\beta + \gamma + \alpha + V_t < 0 \end{cases}$$

$$When\ Y_{t-1} = 1: \ Y_t = \begin{cases} 1 & if\ X_t\beta + \gamma + \alpha + V_t > 0 \\ 0 & X_t\beta + \alpha + V_t < 0 \end{cases}$$

Single Index Restriction is violated.

The choice in period t depends on $1(Y_{t-1} = Y_t)$

- Current period's choice affects current period's utility
- The model is *logically inconsistent* (Maddala (1983))
- Model is incomplete and incoherent!
- In general this leads to partial identification.

Regions of Unobservables

Regions $\mathcal{R}_{(y_1,y_2)}(x, y_0; \beta, \gamma)$ partitioning *supp*(V, α) for each $(y_1, y_2)|x, y_0$

$$
\begin{aligned}
\mathcal{R}_{(0,0)}(x,0;\beta,\gamma) &= \{(V,\alpha)\in(\mathcal{V},\mathcal{A}) : x_1\beta+\gamma+\alpha+V_1\le 0 \ \&\ x_2\beta+\gamma+\alpha+V_2\le 0\}\\
\mathcal{R}_{(0,1)}(x,0;\beta,\gamma) &= \{(V,\alpha)\in(\mathcal{V},\mathcal{A}) : x_2\beta+\alpha+V_2> 0\ge x_1\beta+\gamma+\alpha+V_1\}\\
\mathcal{R}_{(1,0)}(x,0;\beta,\gamma) &= \{(V,\alpha)\in(\mathcal{V},\mathcal{A}) : x_2\beta+\alpha+V_2\le 0< x_1\beta+\alpha+V_1\}\\
\mathcal{R}_{(1,1)}(x,0;\beta,\gamma) &= \{(V,\alpha)\in(\mathcal{V},\mathcal{A}) : x_1\beta+\alpha+V_1\ge 0 \ \&\ x_2\beta+\gamma+\alpha+V_2\ge 0\}\\
\mathcal{R}_{(0,0)}(x,1;\beta,\gamma) &= \{(V,\alpha)\in(\mathcal{V},\mathcal{A}) : x_1\beta+\alpha+V_1\le 0 \ \&\ x_2\beta+\gamma+\alpha+V_2\le 0\}\\
\mathcal{R}_{(0,1)}(x,1;\beta,\gamma) &= \{(V,\alpha)\in(\mathcal{V},\mathcal{A}) : x_2\beta+\alpha+V_2> 0\ge x_1\beta+\alpha+V_1\}\\
\mathcal{R}_{(1,0)}(x,1;\beta,\gamma) &= \{(V,\alpha)\in(\mathcal{V},\mathcal{A}) : x_2\beta+\alpha+V_2\le 0< x_1\beta+\gamma+\alpha+V_1\}\\
\mathcal{R}_{(1,1)}(x,1;\beta,\gamma) &= \{(V,\alpha)\in(\mathcal{V},\mathcal{A}) : x_1\beta+\gamma+\alpha+V_1> 0 \ \&\ x_2\beta+\gamma+\alpha+V_2> 0\}
\end{aligned}
$$

Regions of Unobservables I

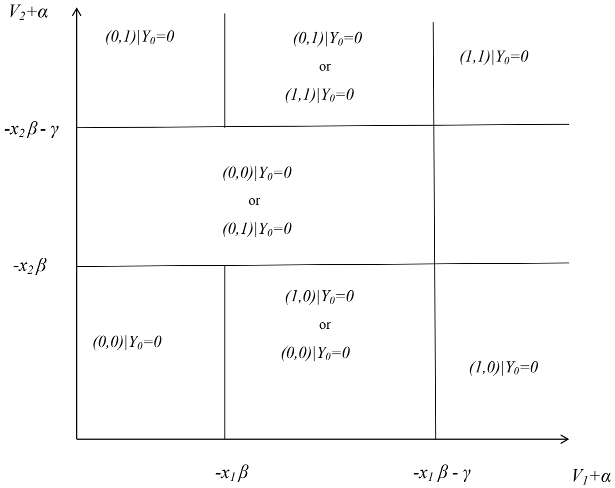

Figure: (Y_1, Y_2) choice when $\gamma < 0$ and $Y_0 = 0$

Regions of Unobservables II

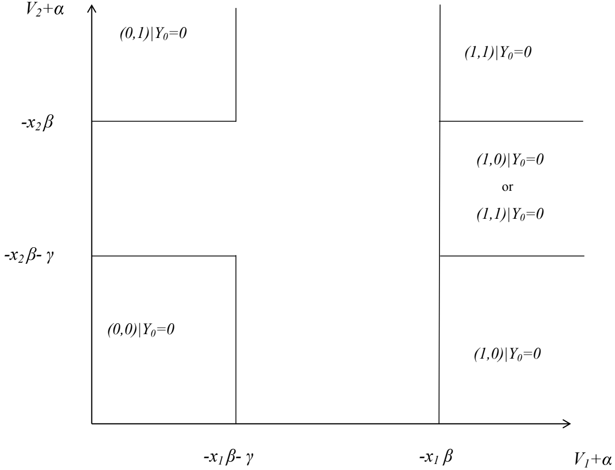

Figure: (Y_1, Y_2) choice when $\gamma > 0$ and $Y_0 = 0$

Regions of Unobservables III

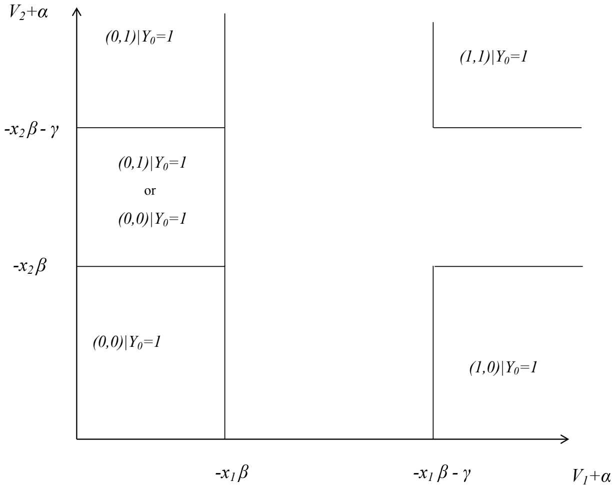

Figure: (Y_1, Y_2) choice when $\gamma < 0$ and $Y_0 = 1$

Regions of Unobservables IV

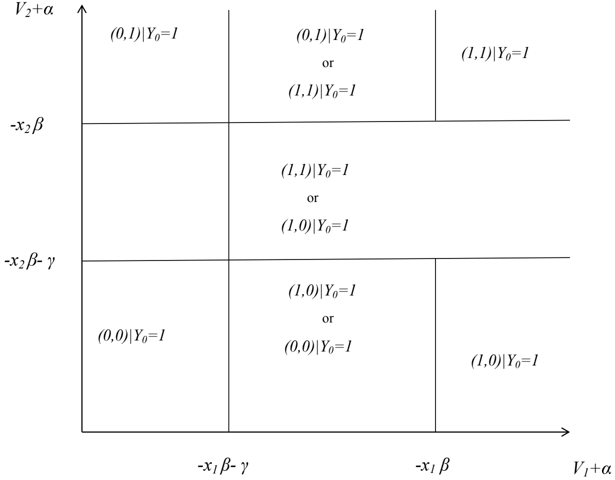

Figure: (Y_1, Y_2) choice when $\gamma > 0$ and $Y_0 = 1$

Incompleteness and Incoherence

Incompleteness:

- $\exists(x, y_0) \in \mathcal{X} \times \mathcal{Y}_I$, such that for every (V, α), there is not a unique solution with probability 1.

Incoherence:

- $\exists(x, y_0) \in \mathcal{X} \times \mathcal{Y}_I$, such that for every (V, α), there is no solution, denoted by $\mathcal{Y}() = \emptyset$

Incompleteness and Incoherence

Incompleteness:

- $\exists(x, y_0) \in \mathcal{X} \times \mathcal{Y}_I$, such that for every (V, α), there is not a unique solution with probability 1.

Incoherence:

- $\exists(x, y_0) \in \mathcal{X} \times \mathcal{Y}_I$, such that for every (V, α), there is no solution, denoted by $\mathcal{Y}() = \emptyset$

Knowing if the model is complete and coherent is important for *parameters' estimation*

- Complete and coherent models allow for the construction of a likelihood function if the model is correctly specified.

Incompleteness and Incoherence

Incompleteness:

- $\exists(x, y_0) \in \mathcal{X} \times \mathcal{Y}_I$, such that for every (V, α), there is not a unique solution with probability 1.

Incoherence:

- $\exists(x, y_0) \in \mathcal{X} \times \mathcal{Y}_I$, such that for every (V, α), there is no solution, denoted by $\mathcal{Y}() = \emptyset$

Knowing if the model is complete and coherent is important for *parameters' estimation*

- Complete and coherent models allow for the construction of a likelihood function if the model is correctly specified.

Incompleteness implications example:

- when $\gamma < 0$: $P(1, 0|x, 0) < F_{(V,\alpha)|X,Y_0}(R_{(1,0)}(x, 0; \beta, \gamma))$

Approaches in Literature

Related models and approaches proposed in the literature:

- Games with simultaneous responses + identification at infinity: Tamer (2003)
- Binomial response models with dummy endogenous regressors: Lewbel (2007)
- Characterization of identified sets using random set theory: Beresteanu, Molchanov, and Molinari (2011), Chesher and Rosen (2012)
- Truncating the limited dependent variables to lie outside the incoherency regions: Hajivassiliou and Savignac (2019)
- Selection mechanism among multiple potential equilibria: Bjorn and Vuong (1984), Bajari, Hong, and Ryan (2010)
- Imposing coherency and completeness conditions: Schmidt (1981), Blundell and Smith (1994)

Possible Identification Techniques

- Completing the model: $\gamma = 0$ - Standard static binary response model
  - Chamberlain (1984, 2010) and Manski (1987)

Possible Identification Techniques

- Completing the model: $\gamma = 0$ - Standard static binary response model
  - Chamberlain (1984, 2010) and Manski (1987)
- Inequalities characterizing the identified set when model is incoherent and incomplete: Chesher and Rosen (2012)

Table 1: A summary of the determination of outcome Y for the approaches of section 3 when the model delivers no or multiple solutions.

	Restrictions for the determination of Y when:	
	$\mathcal{Y}(Z,U;h) = \emptyset$ (no solution)	$\#\mathcal{Y}(Z,U;h) \geq 2$ (multiple solutions)
Approach 1:	$Y = \phi$	$Y \in \mathcal{Y}(Z,U;h)$
Approach 2:	not observed	$Y \in \mathcal{Y}(Z,U;h)$
Approach 3:	$Y \in \mathcal{Y}$	$Y \in \mathcal{Y}(Z,U;h)$
Approach 4:	not observed	not observed

- None of the above approaches has been applied to dynamic panel data models with fixed effects

Possible Identification Techniques

- Completing the model: $\gamma = 0$ - Standard static binary response model
  - Chamberlain (1984, 2010) and Manski (1987)
- Inequalities characterizing the identified set when model is incoherent and incomplete: Chesher and Rosen (2012)

Table 1: A summary of the determination of outcome Y for the approaches of section 3 when the model delivers no or multiple solutions.

	Restrictions for the determination of Y when:	
	$\mathcal{Y}(Z,U;h) = \emptyset$ (no solution)	$\#\mathcal{Y}(Z,U;h) \geq 2$ (multiple solutions)
Approach 1:	$Y = \phi$	$Y \in \mathcal{Y}(Z,U;h)$
Approach 2:	not observed	$Y \in \mathcal{Y}(Z,U;h)$
Approach 3:	$Y \in \mathcal{Y}$	$Y \in \mathcal{Y}(Z,U;h)$
Approach 4:	not observed	not observed

- None of the above approaches has been applied to dynamic panel data models with fixed effects

Semiparametric Identification Strategy

Semiparametric Identification:

- No distributional assumptions on V and α
- Observable implications invariant to the fixed effect.

Initial condition:

- Y_0 is treated as an additional observable with no assumptions on its generation

(Partial) Identification strategy:

- Use individuals who switch in two periods conditional on their initial condition.

Aristodemou (2021):(classical) DBR model

Semiparametric Identification: conditional independence only

$$\mathcal{B}^{DB} = \left\{ \begin{array}{c} (\beta,\gamma) \in \mathcal{B}: \ \forall \omega \in \mathbb{R}, \\ \\ \sup_{x: -\Delta x \beta - \gamma \leq \omega} P(1,0|x,0) \leq \inf_{x: -\Delta x \beta \geq \omega} 1 - P(0,1|x,0) \\ \wedge \\ \sup_{x: -\Delta x \beta \leq \omega} P(1,0|x,1) \leq \inf_{x: -\Delta x \beta + \gamma > \omega} 1 - P(0,1|x,1) \\ a.e.\ x \in \mathcal{X} \end{array} \right\}$$

- Can we use this approach in the incomplete and incoherent model?

Graphical explanation example: (classical) DBR model

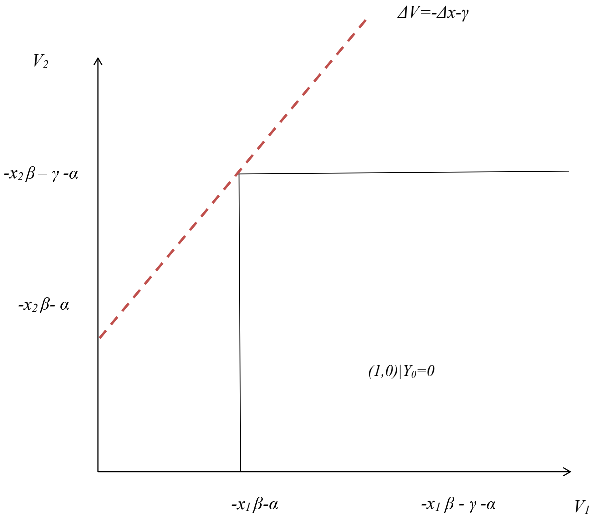

Figure: (Y_1, Y_2) choice when $\gamma < 0$ and $Y_0 = 0$

Regions of Unobservables I: Revisited

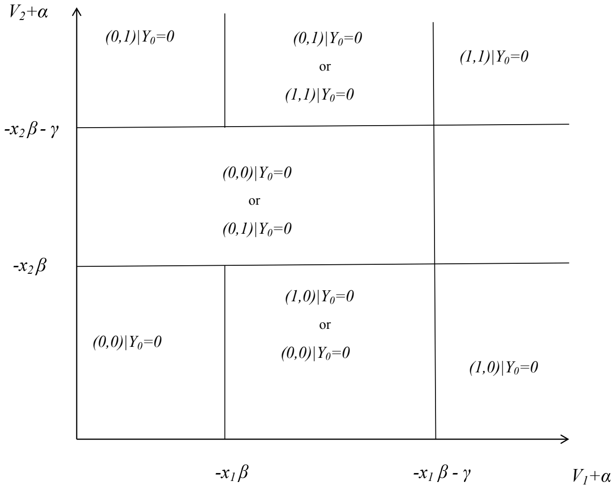

Figure: (Y_1, Y_2) choice when $\gamma < 0$ and $Y_0 = 0$

Graphical explanation example: The New Model

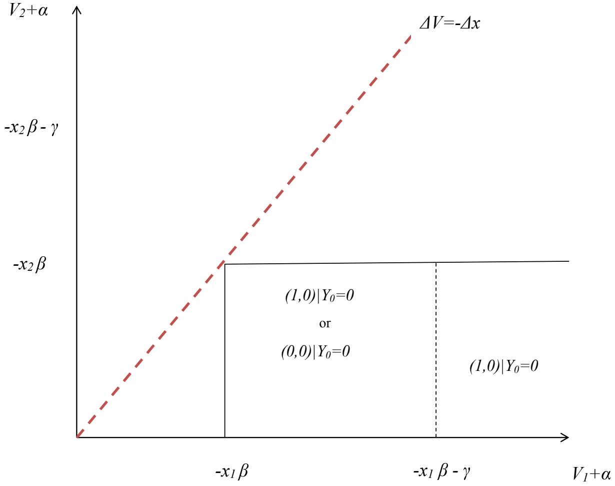

Figure: Upper Bound for $(1,0)|Y_0 = 0$

Approach 1: $\{Y = \phi\}$ when $\mathcal{Y}() = \emptyset$

Assume:

$$\mathcal{Y}() = \emptyset \Rightarrow Y = \phi \to \mathcal{Y}^* = \{\mathcal{Y}_1 \times \mathcal{Y}_2 \cup \phi\}$$

Implications:

- With $\{Y = \phi\}$ observed, $f_{y_1,y_2|x,y_0}(y_1, y_2|x, y_0) = P(y_1, y_2|x, y_0)$ is identified.
- $\{Y = y \cap X = x \cap Y_0 = y_0\} \Rightarrow (V, \alpha) \in \mathcal{R}_{(y_1,y_2)}(x, y_0; \beta, \gamma)$
  - Multiple equilibria regions
- $\mathbf{F}_{(\mathbf{V},\alpha)|\mathbf{X},\mathbf{Y_0}}(\mathcal{R}_{(\mathbf{y_1},\mathbf{y_2})}(\mathbf{x}, \mathbf{y_0}; \beta, \gamma)) \geq \mathbf{P}(\mathbf{y_1}, \mathbf{y_2}|\mathbf{x}, \mathbf{y_0})$

Example: Semiparametric Identification Bounds

Consider $(Y_1 = 1, Y_2 = 0)|x, Y_0 = 0$

$\mathcal{R}_{(1,0)}(x, 0; \beta, \gamma) = \{(V, \alpha) \in (\mathcal{V}, \mathcal{A}) : x_2\beta + \alpha + V_2 < 0 \geq x_1\beta + \alpha + V_1\}$

For any fixed $\alpha \in \mathcal{A}$,

$$\{V \in \mathcal{V} : \mathcal{R}_{(1,0)}(x, 0; \beta, \gamma)\} \subseteq \{V \in \mathcal{V} : \Delta V < -\Delta x\beta\}$$

$\Rightarrow \{P(1, 0|x, 0) < F_{(V,\alpha)|X,Y_0}(R_{(1,0)}(x, 0; \beta, \gamma)) < P_{\Delta V|Y_0}[\Delta V < -\Delta x\beta|Y_0 = 0]\}$

Recall: $V \perp X|Y_0$

Example: Semiparametric Identification Bounds

Consider $(Y_1 = 1, Y_2 = 0)|x, Y_0 = 0$

$\mathcal{R}_{(1,0)}(x, 0; \beta, \gamma) = \{(V, \alpha) \in (\mathcal{V}, \mathcal{A}) : x_2\beta + \alpha + V_2 < 0 \geq x_1\beta + \alpha + V_1\}$

For any fixed $\alpha \in \mathcal{A}$,

$$\{V \in \mathcal{V} : \mathcal{R}_{(1,0)}(x, 0; \beta, \gamma)\} \subseteq \{V \in \mathcal{V} : \Delta V < -\Delta x\beta\}$$

$\Rightarrow \{P(1, 0|x, 0) < F_{(V,\alpha)|X,Y_0}(R_{(1,0)}(x, 0; \beta, \gamma)) < P_{\Delta V|Y_0}[\Delta V < -\Delta x\beta|Y_0 = 0]\}$

Recall: $V \perp X|Y_0$

Theorem

For any fixed pair x and y_0, (β, γ) satisfies

$$\begin{aligned}
P(1, 0|x, 0) &\leq P_{\Delta V|Y_0}[\Delta V < -\Delta x\beta|Y_0 = 0] \\
1 - P(0, 1|x, 0) &\geq P_{\Delta V|Y_0}[\Delta V < -\Delta x\beta + \gamma|Y_0 = 0] \\
P(1, 0|x, 1) &\leq P_{\Delta V|Y_0}[\Delta V < -\Delta x\beta + \gamma|Y_0 = 1] \\
1 - P(0, 1|x, 1) &\geq P_{\Delta V|Y_0}[\Delta V < -\Delta x\beta|Y_0 = 1]
\end{aligned}$$

where $\Delta X = X_2 - X_1$ and $\Delta V = V_2 - V_1$.

Example: Semiparametric Identification Bounds

Theorem

Bounds (outer regions) for β, γ are given by the set:

$$\mathcal{B} = \left\{ \begin{array}{c} (\beta,\gamma) \in \mathcal{B}: \ \forall \omega \in \mathbb{R}, \\ \\ \sup_{x: -\Delta x\beta \leq \omega} P(1,0|x,0) \leq \inf_{x: -\Delta x\beta + \gamma \geq \omega} 1 - P(0,1|x,0) \\ \wedge \\ \sup_{x: -\Delta x\beta + \gamma \leq \omega} P(1,0|x,1) \leq \inf_{x: -\Delta x\beta > \omega} 1 - P(0,1|x,1) \\ a.e.\ x \in \mathcal{X} \end{array} \right\}$$

Example: Semiparametric Identification Bounds

Theorem

Bounds (outer regions) for β, γ are given by the set:

$$\mathcal{B} = \left\{ \begin{array}{c} (\beta,\gamma) \in \mathcal{B} : \ \forall \omega \in \mathbb{R}, \\ \\ \sup_{x: -\Delta x\beta \leq \omega} P(1,0|x,0) \leq \inf_{x: -\Delta x\beta + \gamma \geq \omega} 1 - P(0,1|x,0) \\ \wedge \\ \sup_{x: -\Delta x\beta + \gamma \leq \omega} P(1,0|x,1) \leq \inf_{x: -\Delta x\beta > \omega} 1 - P(0,1|x,1) \\ a.e.\ x \in \mathcal{X} \end{array} \right\}$$

Outline: Conditioning on $Y_0 = 0$ for any constant $\omega \in \mathbb{R}$

- $(Y_1, Y_2) = (1,0) \cap -\Delta X\beta \leq \omega \Rightarrow \Delta V < \omega$
- If $-\Delta X\beta \leq \omega : P[(Y_1, Y_2) = (1,0) \cap -\Delta X\beta \leq \omega | x, 0] = P(1,0|x,0)$
- $\forall x \in \mathcal{X}$ s.t. $-\Delta X\beta \leq \omega : P(1,0|x,0) \leq P(\Delta V < \omega | Y_0)$
- $P(\Delta V < \omega | Y_0)$ independent of x: $\sup_{x: -\Delta x\beta \leq \omega} P(1,0|x,0) \leq F_{\Delta V|Y_0}(\omega)$

Approach 2: Nothing observed if $\mathcal{Y}() = \emptyset$

Assume: $\mathcal{Y}() = \emptyset \Rightarrow$ null outcomes never observed

Implications:

- Null outcomes never observed,
  $f_{y_1,y_2|x,y_0}(y_1, y_2|x, y_0) = P(y_1, y_2|x, y_0, \mathcal{Y}() = \emptyset)$ is identified.
- $\{Y = y \cap X = x \cap Y_0 = y_0\} \Rightarrow (V, \alpha) \in \mathcal{R}_{(y_1,y_2)}(x, y_0; \beta, \gamma)$
    - Multiple equilibria regions
- $F_{(V,\alpha)|X,Y_0}(\mathcal{R}_{(y_1,y_2)}(x, y_0; \beta, \gamma) \geq P[(y_1, y_2)|x, y_0]$
- Bayes Rule:
  $F_{(V,\alpha)|X,Y_0}(\mathcal{R}_{(y_1,y_2)}(x, y_0; \beta, \gamma)$
  $\geq [1 - F_{(V,\alpha)|X,Y_0}(\mathcal{Y}() = \emptyset))]f_{y_1,y_2|x,y_0}(y_1, y_2|x, y_0)$

Example: Semiparametric Identification Bounds

Consider $(Y_1 = 1, Y_2 = 0)|x, Y_0 = 0$

$$\mathcal{R}_{(1,0)}(x, 0; \beta, \gamma) = \{(V, \alpha) \in (\mathcal{V}, \mathcal{A}) : x_2\beta + \alpha + V_2 < 0 \geq x_1\beta + \alpha + V_1\}$$

For any fixed $\alpha \in \mathcal{A}$,

$$\{V \in \mathcal{V} : \mathcal{R}_{(1,0)}(x, 0; \beta, \gamma)\} \subseteq \{V \in \mathcal{V} : \Delta V < -\Delta x\beta\}$$

$$\begin{aligned}[1 - F_{(V,\alpha)|X,Y_0}(\mathcal{Y}() = \emptyset)] f_{y_1,y_2|x,y_0}(y_1, y_2|x, y_0) &\leq F_{(V,\alpha)|X,Y_0}(R_{(1,0)}(x, 0; \beta, \gamma)) \\ &\leq P_{\Delta V|Y_0}[\Delta V < -\Delta x\beta | Y_0 = 0]\end{aligned}$$

Recall: $V \perp X | Y_0$

$$f_{y_1,y_2|x,y_0}(y_1, y_2|x, y_0) \leq P_{\Delta V|Y_0}[\Delta V < -\Delta x\beta | Y_0 = 0, \mathcal{Y} \neq \emptyset]$$

Example: Semiparametric Identification Bounds

Consider $(Y_1 = 1, Y_2 = 0)|x, Y_0 = 0$

$$\mathcal{R}_{(1,0)}(x, 0; \beta, \gamma) = \{(V, \alpha) \in (\mathcal{V}, \mathcal{A}) : x_2\beta + \alpha + V_2 < 0 \geq x_1\beta + \alpha + V_1\}$$

For any fixed $\alpha \in \mathcal{A}$,

$$\{V \in \mathcal{V} : \mathcal{R}_{(1,0)}(x, 0; \beta, \gamma)\} \subseteq \{V \in \mathcal{V} : \Delta V < -\Delta x\beta\}$$

$$\begin{aligned}
[1 - F_{(V,\alpha)|X,Y_0}(\mathcal{Y}() = \emptyset)] f_{y_1,y_2|x,y_0}(y_1, y_2|x, y_0) &\leq F_{(V,\alpha)|X,Y_0}(R_{(1,0)}(x, 0; \beta, \gamma)) \\
&\leq P_{\Delta V|Y_0}[\Delta V < -\Delta x\beta | Y_0 = 0]
\end{aligned}$$

Recall: $V \perp X | Y_0$

$$f_{y_1,y_2|x,y_0}(y_1, y_2|x, y_0) \leq P_{\Delta V|Y_0}[\Delta V < -\Delta x\beta | Y_0 = 0, \mathcal{Y} \neq \emptyset]$$

Theorem

For any fixed pair x and y_0, (β, γ) satisfies

$$\begin{aligned}
P(1, 0|x, 0) &\leq P_{\Delta V|Y_0}[\Delta V < -\Delta x\beta | Y_0 = 0, \mathcal{Y} \neq \emptyset] \\
1 - P(0, 1|x, 0) &\geq P_{\Delta V|Y_0}[\Delta V < -\Delta x\beta + \gamma | Y_0 = 0, \mathcal{Y} \neq \emptyset] \\
P(1, 0|x, 1) &\leq P_{\Delta V|Y_0}[\Delta V < -\Delta x\beta + \gamma | Y_0 = 1, \mathcal{Y} \neq \emptyset] \\
1 - P(0, 1|x, 1) &\geq P_{\Delta V|Y_0}[\Delta V < -\Delta x\beta | Y_0 = 1, \mathcal{Y} \neq \emptyset]
\end{aligned}$$

where $\Delta X = X_2 - X_1$ and $\Delta V = V_2 - V_1$.

Conclusion and Work-in-progress

- New binary response model with fixed effects and switching dependence
- Model is incomplete and incoherent which leads to impose econometric and identification challenges
- Identification Bounds using different techniques still in progress

Further questions:

- How informative are these bounds?
- How to estimate the model?
- Extension: Application to consumer demand using the Nielsen dataset.

Thank you for your attention!

Any questions or comments?

Aguirregabiria, V., J. Gu, and Y. Luo (2021): "Sufficient statistics for unobserved heterogeneity in structural dynamic logit models," *Journal of Econometrics*, 223(2), 280–311.

Aristodemou, E. (2021): "Semiparametric identification in panel data discrete response models," *Journal of Econometrics*, 220(2), 253–271.

Bajari, P., H. Hong, and S. P. Ryan (2010): "Identification and estimation of a discrete game of complete information," *Econometrica*, 78(5), 1529–1568.

Beresteanu, A., I. Molchanov, and F. Molinari (2011): "Sharp identification regions in models with convex moment predictions," *Econometrica*, 79(6), 1785–1821.

Bjorn, P. A., and Q. H. Vuong (1984): "Simultaneous equations models for dummy endogenous variables: a game theoretic formulation with an application to labor force participation," .

Blundell, R., and R. J. Smith (1994): "Coherency and estimation in simultaneous models with censored or qualitative dependent variables," *Journal of Econometrics*, 64(1-2), 355–373.

Chamberlain, G. (1984): "Panel data," *Handbook of econometrics*, 2, 1247–1318.

——— (2010): "Binary response models for panel data: Identification and information," *Econometrica*, 78(1), 159–168.

CHESHER, A., AND A. M. ROSEN (2012): "Simultaneous Equations Models for Discrete Outcomes: Completeness, Coherence, and Identification," Discussion paper, cemmap working paper, Centre for Microdata Methods and Practice.

CLERIDES, S., AND P. COURTY (2017): "Sales, quantity surcharge, and consumer inattention," *Review of Economics and Statistics*, 99(2), 357–370.

DUBÉ, J.-P., G. J. HITSCH, AND P. E. ROSSI (2009): "Do switching costs make markets less competitive?," *Journal of Marketing Research*, 46(4), 435–445.

——— (2010): "State dependence and alternative explanations for consumer inertia," *The RAND Journal of Economics*, 41(3), 417–445.

FREDERIKSEN, A., B. E. HONORÉ, AND L. HU (2007): "Discrete time duration models with group-level heterogeneity," *Journal of Econometrics*, 141(2), 1014–1043.

Gørgens, T., and D. Hyslop (2018): "Equivalent representations of discrete-time two-state panel data models," *Economics Letters*, 163, 65–67.

——— (2019): "The specification of dynamic discrete-time two-state panel data models," *Econometrics*, 7(1), 1.

Hajivassiliou, V., and F. Savignac (2019): "Novel approaches to coherency conditions in dynamic LDV models: quantifying financing constraints and a firm's decision and ability to innovate," .

Heckman, J. J. (1981): "Heterogeneity and state dependence," in *Studies in labor markets*. University of Chicago Press.

Honoré, B. E., and E. Kyriazidou (2000): "Panel data discrete choice models with lagged dependent variables," *Econometrica*, 68(4), 839–874.

Honoré, B. E., C. Muris, and M. Weidner (2021): "Dynamic Ordered Panel Logit Models," *arXiv preprint arXiv:2107.03253*.

Honoré, B. E., and E. Tamer (2006): "Bounds on parameters in panel dynamic discrete choice models," *Econometrica*, 74(3), 611–629.

Honoré, B. E., and M. Weidner (2020): "Moment Conditions for Dynamic Panel Logit Models with Fixed Effects," .

Khan, S., M. Ponomareva, and E. T. Tamer (2019): *Identification of dynamic panel binary response models*. Boston College.

Lancaster, T. (1979): "Econometric methods for the duration of unemployment," *Econometrica: Journal of the Econometric Society*, pp. 939–956.

Lewbel, A. (2007): "Coherency And Completeness Of Structural Models Containing A Dummy Endogenous Variable," *International Economic Review*, 48(4), 1379–1392.

Maddala, G. S. (1983): *Limited-dependent and qualitative variables in econometrics*. Cambridge University Press. Cambridge, UK.

Manski, C. F. (1987): "Semiparametric analysis of random effects linear models from binary panel data," *Econometrica*, 55(2), 357–362.

Muris, C., P. Raposo, and S. Vandoros (2020): "A dynamic ordered logit model with fixed effects," *arXiv preprint arXiv:2008.05517*.

Pakes, A., J. R. Porter, M. Shepard, and S. Calder-Wang (2021): "Unobserved Heterogeneity, State Dependence, and Health Plan Choices," Discussion paper, National Bureau of Economic Research.

Schmidt, P. (1981): "Constraints on the parameters in simultaneous tobit and probit models," *Structural analysis of discrete data with econometric applications*, 1, 423–434.

Tamer, E. (2003): "Incomplete simultaneous discrete response model with multiple equilibria," *The Review of Economic Studies*, 70(1), 147–165.