Binary Response Dynamic Panel Data Models with Switching State Dependence

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Talk Outline

1. Motivation and Contribution
2. Literature
3. Model framework
4. Identification strategy
5. Conclusion and Work-in-progress
Heckman (1981): “The conditional probability that an individual will experience the event in the future is a function of past experience.”

- **True State Dependence**: an otherwise identical individual who did not experience the event would behave differently in the future than an individual who experienced the event.

- **Spurious State Dependence**: individuals may differ in certain unmeasured variables that influence their probability of experiencing the event but that are not influenced by the experience of the event.”
Motivation II: Switching costs

- Dubé, Hitsch, and Rossi (2010): “Researchers in both marketing and economics have documented a form of persistence in consumer choice data whereby consumers have a higher probability of choosing products that they have purchased in the past. We call this form of persistence inertia in brand choice.”
  - “There are two conceptually distinct explanations for the source of inertia in brand choice: structural state dependence and spurious state dependence”
  - “...the finding of inertia is robust to controls for unobserved consumer differences, and thus we find evidence for structural state dependence. We then explore three different economic explanations that can give rise to structural state dependence: preference changes due to past purchases (psychological switching costs) which induce a form of loyalty, search, and learning...”

- Dubé, Hitsch, and Rossi (2009): “Switching costs can come from a variety of sources, including product adoption costs, shopping/search costs, and psychological sources..... Both psychological switching costs and a monetary switching fee give rise to an observationally equivalent form of inertia in consumer purchases.”
Contributions

This paper studies identification in

- Binary response panel data models with switching state dependence

Examples:

- Consumers demand over time
  - Choice between system blades and disposable razors

- Individuals labor decisions over time
  - Choice between employment and unemployment

*Binary Response Dynamic Panel Data Models (with a twist)*

- Individuals might be reluctant to switch: Inertia in individual choices
  - Cost of switching, habits, loyalty, lock-in effects, search, learning, etc.

- Examine whether individuals condition their decisions on switching
(A very brief) Literature review

Dynamic binary response models

Dynamic ordered response models
- Muris, Raposo, and Vandoros (2020), Honoré, Muris, and Weidner (2021)

Structural dynamic panel data models

Duration models
- Lancaster (1979), Frederiksen, Honoré, and Hu (2007)

Multi-spell duration models
- Gørgens and Hyslop (2018, 2019)

IO literature: Consumer inertia, passive shoppers and switching costs
The (classical) Dynamic Binary Response Model

Individuals observed for 3 periods, \( t \in \{0, 1, 2\} \)

\[
Y_{it} = 1(X_{it} \beta + 1(Y_{i,t-1} = 1) \gamma + \alpha_i + V_{it} > 0)
\]

with

\[
1(Y_{i,t-1} = 1) = \begin{cases} 
1 & \text{if } Y_{i,t-1} = 1 \\
0 & \text{otherwise}
\end{cases}
\]

- \((\beta, \gamma) \in \Theta\) parameters of interest

**Observables**

- \(Y_i = (Y_{i1}, Y_{i2}) \in \mathcal{Y}_1 \times \mathcal{Y}_2 \equiv \{0, 1\}\)
- \(X_i = (X_{i1}, X_{i2})\) with support \(\mathcal{X}\)
- Initial condition \(Y_{i0} \in \{0, 1\}\)

**Unobservables**

- \(\alpha_i \in \mathcal{A}\)
- \(V_i = (V_{i1}, V_{i2})\) with support \(\mathcal{V}\)
The New Model

Individuals observed for 3 periods, \( t \in \{0, 1, 2\} \)

\[
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- \((\beta, \gamma) \in \Theta\) parameters of interest

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Unobservables
- \(\alpha_i \in \mathcal{A}\)
- \(V_i = (V_{i1}, V_{i2})\) with support \(\mathcal{V}\)
Econometric Challenges: (classical) DBR Model

- Describe choices between two or more discrete alternatives
  - Qualitative or Limited dependent variables models
- Distribution of time-varying unobservables
- Fixed effects: \( \alpha \) correlated with \( Y_{t-1} \) and possibly \( X_t \)
- Initial condition: Choice in period \( t = 1 \) depends on \( t = 0 \)
- Incidental parameters problem: Small T-Large N
Individual decision problem:

\[\begin{align*}
\text{When } Y_{t-1} = 0 : \quad Y_t &= \begin{cases} 
1 & \text{if } X_t \beta + \alpha + V_t > 0 \\
0 & \text{if } X_t \beta + \gamma + \alpha + V_t < 0
\end{cases} \\
\text{When } Y_{t-1} = 1 : \quad Y_t &= \begin{cases} 
1 & \text{if } X_t \beta + \gamma + \alpha + V_t > 0 \\
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\end{cases}
\end{align*}\]

Single Index Restriction is violated.
Econometric Challenges: New Model

Individual decision problem:

When $Y_{t-1} = 0$:

$$Y_t = \begin{cases} 
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When $Y_{t-1} = 1$:

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\end{cases}$$

Single Index Restriction is violated.

The choice in period $t$ depends on $1(Y_{t-1} = Y_t)$

- Current period’s choice affects current period’s utility
- The model is logically inconsistent (Maddala (1983))
- Model is incomplete and incoherent!
- In general this leads to partial identification.
Regions of Unobservables

Regions $\mathcal{R}_{(y_1, y_2)}(x, y_0; \beta, \gamma)$ partitioning $\text{supp}(V, \alpha)$ for each $(y_1, y_2)|x, y_0$

$\mathcal{R}_{(0,0)}(x, 0; \beta, \gamma) = \{(V, \alpha) \in (V, A): x_1 \beta + \gamma + \alpha + V_1 \leq 0 \& x_2 \beta + \gamma + \alpha + V_2 \leq 0\}$

$\mathcal{R}_{(0,1)}(x, 0; \beta, \gamma) = \{(V, \alpha) \in (V, A): x_2 \beta + \alpha + V_2 > 0 \geq x_1 \beta + \gamma + \alpha + V_1\}$

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Figure: \((Y_1, Y_2)\) choice when \(\gamma < 0\) and \(Y_0 = 0\)
Regions of Unobservables II

Figure: \((Y_1, Y_2)\) choice when \(\gamma > 0\) and \(Y_0 = 0\)
Regions of Unobservables III

Figure: \((Y_1, Y_2)\) choice when \(\gamma < 0\) and \(Y_0 = 1\)
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Incompleteness and Incoherence

Incompleteness:

- \( \exists (x, y_0) \in \mathcal{X} \times \mathcal{Y}, \) such that for every \((V, \alpha)\), there is not a unique solution with probability 1.

Incoherence:

- \( \exists (x, y_0) \in \mathcal{X} \times \mathcal{Y}, \) such that for every \((V, \alpha)\), there is no solution, denoted by \( \mathcal{Y}(\cdot) = \emptyset \)
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Knowing if the model is complete and coherent is important for *parameters’ estimation*

- Complete and coherent models allow for the construction of a likelihood function if the model is correctly specified.
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Knowing if the model is complete and coherent is important for parameters’ estimation

- Complete and coherent models allow for the construction of a likelihood function if the model is correctly specified.

**Incompleteness implications example:**

- when \( \gamma < 0 \): \( P(1, 0|x, 0) < F_{(V, \alpha)|x, y_0}(R_{1,0}(x, 0; \beta, \gamma)) \)
Related models and approaches proposed in the literature:

- Binomial response models with dummy endogenous regressors: Lewbel (2007)
- Truncating the limited dependent variables to lie outside the incoherency regions: Hajivassiliou and Savignac (2019)
- Selection mechanism among multiple potential equilibria: Bjorn and Vuong (1984), Bajari, Hong, and Ryan (2010)
- Imposing coherency and completeness conditions: Schmidt (1981), Blundell and Smith (1994)
Possible Identification Techniques

- Completing the model: $\gamma = 0$ - Standard static binary response model
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- Completing the model: $\gamma = 0$ - Standard static binary response model
- Inequalities characterizing the identified set when model is incoherent and incomplete: Chesher and Rosen (2012)

Table 1: A summary of the determination of outcome $Y$ for the approaches of section 3 when the model delivers no or multiple solutions.

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- None of the above approaches has been applied to dynamic panel data models with fixed effects
Possible Identification Techniques

- Completing the model: $\gamma = 0$ - Standard static binary response model

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- None of the above approaches has been applied to dynamic panel data models with fixed effects
Semiparametric Identification Strategy

Semiparametric Identification:
- No distributional assumptions on $V$ and $\alpha$
- Observable implications invariant to the fixed effect.

Initial condition:
- $Y_0$ is treated as an additional observable with no assumptions on its generation

(Partial) Identification strategy:
- Use individuals who switch in two periods conditional on their initial condition.
Aristodemou (2021): (classical) DBR model

Semiparametric Identification: conditional independence only

\[ \mathcal{B}^{DB} = \left\{ (\beta, \gamma) \in \mathcal{B} : \forall \omega \in \mathbb{R}, \right. \]

\[ \sup_{x: -\Delta x \beta - \gamma \leq \omega} P(1, 0|x, 0) \leq \inf_{x: -\Delta x \beta \geq \omega} 1 - P(0, 1|x, 0) \]

\[ \land \]

\[ \sup_{x: -\Delta x \beta \leq \omega} P(1, 0|x, 1) \leq \inf_{x: -\Delta x \beta + \gamma > \omega} 1 - P(0, 1|x, 1) \]

\[ a.e. \ x \in \mathcal{X} \]

- Can we use this approach in the incomplete and incoherent model?
Graphical explanation example: (classical) DBR model

Figure: \((Y_1, Y_2)\) choice when \(\gamma < 0\) and \(Y_0 = 0\)
Figure: \((Y_1, Y_2)\) choice when \(\gamma < 0\) and \(Y_0 = 0\)
Graphical explanation example: The New Model

\[ V_2 + \alpha \]
\[ -x_2 \beta - \gamma \]
\[ -x_1 \beta \]
\[ -x_1 \beta - \gamma \]
\[ V_1 + \alpha \]

\[ \Delta V = -\Delta x \]

Figure: Upper Bound for \((1, 0) | Y_0 = 0\)
Approach 1: \( \{ Y = \phi \} \) when \( \mathcal{Y}(\cdot) = \emptyset \)

Assume:

\[
\mathcal{Y}(\cdot) = \emptyset \implies Y = \phi \implies \mathcal{Y}^* = \{ \mathcal{Y}_1 \times \mathcal{Y}_2 \cup \phi \}
\]

Implications:

- With \( \{ Y = \phi \} \) observed,
  \[
f_{Y_1, Y_2|x, y_0}(y_1, y_2|x, y_0) = P(y_1, y_2|x, y_0)
  \]
  is identified.

- \( \{ Y = y \cap X = x \cap Y_0 = y_0 \} \implies (V, \alpha) \in \mathcal{R}_{(y_1, y_2)}(x, y_0; \beta, \gamma) \)
  - Multiple equilibria regions

- \( F_{(V, \alpha)|x, y_0}(\mathcal{R}_{(y_1, y_2)}(x, y_0; \beta, \gamma)) \geq P(y_1, y_2|x, y_0) \)
Consider $\left( Y_1 = 1, Y_2 = 0 \right) | x, Y_0 = 0$

\[ R_{(1,0)}(x, 0; \beta, \gamma) = \{(V, \alpha) \in (\mathcal{V}, A) : x_2 \beta + \alpha + V_2 < 0 \geq x_1 \beta + \alpha + V_1 \} \]

For any fixed $\alpha \in A$,

\[ \{ V \in \mathcal{V} : R_{(1,0)}(x, 0; \beta, \gamma) \} \subseteq \{ V \in \mathcal{V} : \Delta V < -\Delta x \beta \} \]

\[ \Rightarrow \{ P(1, 0 | x, 0) < F_{(V, \alpha)} | x, Y_0 (R_{(1,0)}(x, 0; \beta, \gamma)) < P_{\Delta V | Y_0} [\Delta V < -\Delta x \beta | Y_0 = 0] \} \]

Recall: $V \perp X | Y_0$
Example: Semiparametric Identification Bounds

Consider \((Y_1 = 1, Y_2 = 0)|x, Y_0 = 0\)

\[\mathcal{R}_{(1,0)}(x, 0; \beta, \gamma) = \{(V, \alpha) \in (\mathcal{V}, \mathcal{A}) : x_2\beta + \alpha + V_2 < 0 \geq x_1\beta + \alpha + V_1\}\]

For any fixed \(\alpha \in \mathcal{A}\),

\[\{V \in \mathcal{V} : \mathcal{R}_{(1,0)}(x, 0; \beta, \gamma)\} \subseteq \{V \in \mathcal{V} : \Delta V < -\Delta x\beta\}\]

\[\Rightarrow \{P(1, 0|x, 0) < F_{(V, \alpha)}|x, Y_0(R_{(1,0)}(x, 0; \beta, \gamma)) < P_{\Delta V|Y_0}[\Delta V < -\Delta x\beta|Y_0 = 0]\}\]

Recall: \(V \perp X|Y_0\)

**Theorem**

*For any fixed pair \(x\) and \(y_0\), \((\beta, \gamma)\) satisfies*

\[P(1, 0|x, 0) \leq P_{\Delta V|Y_0}[\Delta V < -\Delta x\beta|Y_0 = 0]\]

\[1 - P(0, 1|x, 0) \geq P_{\Delta V|Y_0}[\Delta V < -\Delta x\beta + \gamma|Y_0 = 0]\]

\[P(1, 0|x, 1) \leq P_{\Delta V|Y_0}[\Delta V < -\Delta x\beta + \gamma|Y_0 = 1]\]

\[1 - P(0, 1|x, 1) \geq P_{\Delta V|Y_0}[\Delta V < -\Delta x\beta|Y_0 = 1]\]

*where \(\Delta X = X_2 - X_1\) and \(\Delta V = V_2 - V_1\).*
Example: Semiparametric Identification Bounds

**Theorem**

Bounds (outer regions) for $\beta, \gamma$ are given by the set:

$$
\mathcal{B} = \left\{ (\beta, \gamma) \in \mathcal{B} : \forall \omega \in \mathbb{R}, \right. \\
\left. \quad \sup_{x : -\Delta x \beta \leq \omega} P(1, 0|x, 0) \leq \inf_{x : -\Delta x \beta + \gamma \geq \omega} \quad 1 - P(0, 1|x, 0) \right. \\
\left. \quad \wedge \right. \\
\left. \quad \sup_{x : -\Delta x \beta + \gamma \leq \omega} P(1, 0|x, 1) \leq \inf_{x : -\Delta x \beta > \omega} \quad 1 - P(0, 1|x, 1) \right\}
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$a.e. \, x \in \mathcal{X}$
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\left. \text{a.e. } x \in \mathcal{X} \right\}
$$

Outline: Conditioning on $Y_0 = 0$ for any constant $\omega \in \mathbb{R}$

- $(Y_1, Y_2) = (1, 0) \cap -\Delta X \beta \leq \omega \Rightarrow \Delta V < \omega$
- If $-\Delta X \beta \leq \omega : P[(Y_1, Y_2) = (1, 0) \cap -\Delta X \beta \leq \omega|x, 0] = P(1, 0|x, 0)$
- $\forall x \in \mathcal{X}$ s.t. $-\Delta X \beta \leq \omega : P(1, 0|x, 0) \leq P(\Delta V < \omega|Y_0)$
- $P(\Delta V < \omega|Y_0)$ independent of $x$: $\sup_{x : -\Delta x \beta \leq \omega} P(1, 0|x, 0) \leq F_{\Delta V|Y_0}(\omega)$
Approach 2: Nothing observed if \( \mathcal{Y}() = \emptyset \)

Assume: \( \mathcal{Y}() = \emptyset \Rightarrow \) null outcomes never observed

Implications:

- Null outcomes never observed,
  \[
f_{y_1,y_2|x,y_0}(y_1, y_2|x, y_0) = P(y_1, y_2|x, y_0, \mathcal{Y}() = \emptyset)
\]
  is identified.

- \( \{ Y = y \cap X = x \cap Y_0 = y_0 \} \Rightarrow (V, \alpha) \in \mathcal{R}_{(y_1,y_2)}(x, y_0; \beta, \gamma) \)
  - Multiple equilibria regions

- \( F(V, \alpha)|x, y_0(\mathcal{R}_{(y_1,y_2)}(x, y_0; \beta, \gamma)) \geq P[(y_1, y_2)|x, y_0] \)

- Bayes Rule:
  \[
  F(V, \alpha)|x, y_0(\mathcal{R}_{(y_1,y_2)}(x, y_0; \beta, \gamma)) \\
  \geq [1 - F(V, \alpha)|x, y_0(\mathcal{Y}() = \emptyset)]f_{y_1,y_2|x,y_0}(y_1, y_2|x, y_0)
  \]
Example: Semiparametric Identification Bounds

Consider $(Y_1 = 1, Y_2 = 0)|x, Y_0 = 0$

\[ R_{(1,0)}(x, 0; \beta, \gamma) = \{(V, \alpha) \in (V, A) : x_2 \beta + \alpha + V_2 < 0 \geq x_1 \beta + \alpha + V_1\} \]

For any fixed $\alpha \in A$,

\[ \{V \in V : R_{(1,0)}(x, 0; \beta, \gamma)\} \subseteq \{V \in V : \Delta V < -\Delta x \beta\} \]

\[ [1 - F_{(V, \alpha)}|x, Y_0(Y() = \emptyset)]f_{y_1, y_2|x, y_0}(y_1, y_2|x, y_0) \leq F_{(V, \alpha)}|x, Y_0(R_{(1,0)}(x, 0; \beta, \gamma)) \leq P_{\Delta V|Y_0}[\Delta V < -\Delta x \beta|Y_0 = 0] \]

Recall: $V \perp X|Y_0$

\[ f_{y_1, y_2|x, y_0}(y_1, y_2|x, y_0) \leq P_{\Delta V|Y_0}[\Delta V < -\Delta x \beta|Y_0 = 0, Y \neq \emptyset] \]
Example: Semiparametric Identification Bounds

Consider \((Y_1 = 1, Y_2 = 0 | x, Y_0 = 0)\)

\[
\mathcal{R}_{(1,0)}(x, 0; \beta, \gamma) = \{(V, \alpha) \in (\mathcal{V}, \mathcal{A}) : x_2 \beta + \alpha + V_2 < 0 \geq x_1 \beta + \alpha + V_1\}
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For any fixed \(\alpha \in \mathcal{A}\),

\[
\{V \in \mathcal{V} : \mathcal{R}_{(1,0)}(x, 0; \beta, \gamma)\} \subseteq \{V \in \mathcal{V} : \Delta V < -\Delta x \beta\}
\]

\[
[1 - F(\mathcal{V}, \alpha)|x, y_0(\mathcal{Y}() = \emptyset)]f_{y_1, y_2|x, y_0}(y_1, y_2|x, y_0) \leq F(\mathcal{V}, \alpha)|x, y_0(\mathcal{R}_{(1,0)}(x, 0; \beta, \gamma)) \\
\leq P_{\Delta V|Y_0}[\Delta V < -\Delta x \beta| Y_0 = 0]
\]

Recall: \(V \perp X|Y_0\)

\[
f_{y_1, y_2|x, y_0}(y_1, y_2|x, y_0) \leq P_{\Delta V|Y_0}[\Delta V < -\Delta x \beta| Y_0 = 0, \mathcal{Y} \neq \emptyset]
\]

Theorem

For any fixed pair \(x\) and \(y_0\), \((\beta, \gamma)\) satisfies

\[
P(1,0|x, 0) \leq P_{\Delta V|Y_0}[\Delta V < -\Delta x \beta| Y_0 = 0, \mathcal{Y} \neq \emptyset]
\]

\[
1 - P(0,1|x, 0) \geq P_{\Delta V|Y_0}[\Delta V < -\Delta x \beta + \gamma| Y_0 = 0, \mathcal{Y} \neq \emptyset]
\]

\[
P(1,0|x, 1) \leq P_{\Delta V|Y_0}[\Delta V < -\Delta x \beta + \gamma| Y_0 = 1, \mathcal{Y} \neq \emptyset]
\]

\[
1 - P(0,1|x, 1) \geq P_{\Delta V|Y_0}[\Delta V < -\Delta x \beta| Y_0 = 1, \mathcal{Y} \neq \emptyset]
\]

where \(\Delta X = X_2 - X_1\) and \(\Delta V = V_2 - V_1\).
Conclusion and Work-in-progress

- New binary response model with fixed effects and switching dependence

- Model is incomplete and incoherent which leads to impose econometric and identification challenges

- Identification Bounds using different techniques still in progress

Further questions:

- How informative are these bounds?
- How to estimate the model?

- Extension: Application to consumer demand using the Nielsen dataset.
Thank you for your attention!

Any questions or comments?


Hajivassiliou, V., and F. Savignac (2019): “Novel approaches to coherency conditions in dynamic LDV models: quantifying financing constraints and a firm’s decision and ability to innovate,”.


