Self-enforcing climate coalitions for farsighted countries: integrated analysis of heterogeneous countries

Sareh Vosooghi
University of Oxford

Maria Arvaniti
University of Bologna

Rick van der Ploeg
University of Oxford

CRETE, 2022

Introduction

- We model negotiations of countries to form climate coalitions.
- Signatories commit to maximising the joint payoffs of all coalition members when choosing their emission reduction levels.
- Our policymakers are **farsighted**: rationally predict the overall coalition structure
- We allow for **heterogeneity** across countries and a **dynamic** game.
- Bring together two strands of literature: standard IAM and Coalition Formation Theory

Contribution

- In a climate coalition formation + Integrated Assessment Model (IAM), we offer a simple algorithm to **fully characterise** the equilibrium number of climate coalitions and their number of signatories.
- Large coalitions can be stable : no small coalition paradox
- The problem of coalition formation of heterogeneous countries can be decoupled:
    - number coalitions and number of signatories
    - composition of signatories in each coalition
- Question of composition: in progress

Contribution

- The algorithm relies on Tribonacci numbers

$$\{1, 2, 4, 7, 13, 24, ...\}$$

- The **policy** message:
    - allow multiple climate coalitions!
    - large coalitions can be stable.

- Our results are robust to renegotiation and a generalised energy sector.

Literature

- Coalition formation

  Farsightedness: Chatterjee et al. (1993, REStud); Chwe(1994, JET); Bloch (1996); Ray and Vohra (1999, GEB)

  Farsightedness + public goods: Ray and Vohra (2001, JPE)

- IAMs

  Nordhaus (1993, AER); Nordhaus and Yang (1996, AER); Nordhaus (2014, JAERE)

  Closed form solution: Golosov et al. (2014, ECTA); Hassler and Krusell (2012, JEEA); Van den Bremer and Van der Ploeg (2021, AER)

- Climate coalitions + IAMs

  Cartel Stability and Numerical Approach: Lessmann et al.(2009 EM; 2015, ERE); Bosetti et al (2013, EP)

Setup

- N countries, each country is indicated by i and $I = \{1, 2, ..., N\}$
- Time is discrete and infinite, $t = 0, 1, 2, ...$
- Each country has a planner who is player in a coalition game(climate negotiations): they make proposals to coalitions and respond to proposals made to them following a **negotiation** protocol (to be defined)
- The planner can implement any desired policy in the decentralized economy.

Timing of the game

Two-stage climate coalition formation

- Beginning of period t: membership stage
- From period t onwards : **action(compliance) stage(no renegotiation)**

→ cooperative decision on emission reduction (SCC) within each coalition

→ **but cross-coalition interaction is non-cooperative**

→ country-level decisions on the implementation of the agreed policies (taxation)

- At the end of each period t emissions are observed and payoffs are realized.

The Economy (Golosov et al. (2014))
Representative household in country i

C_{it}: consumption of the final good

β: discount factor

$$\sum_{\tau=0}^{\infty} \beta^{\tau} U(C_{it+\tau})$$

where $U(C_{it}) = ln(C_{it})$

The Economy (Golosov et al. (2014))
Production sectors in country i

R_{it}: stock of fossil fuel

E_{it}: energy use

Y_{it}: final output

γ: damage coefficient

T_t: global temperature

A_i: TFP

K_{it}: aggregate capital stock

ν: output elasticity of energy

- Energy sector: $R_{it+1} = R_{it} - E_{it}$
- Final output: $Y_{it} = exp(-\gamma T_t) A_i K_{it}^{1-\nu} E_{it}^{\nu}$

→ countries are **heterogeneous** with respect to K_{i0}, R_{i0}, A_i

→ full capital depreciation
→ no trade
→ Market clearing: fossil fuel and final good

$$C_{it} + K_{it+1} = Y_{it}$$

The Economy

Climate Dynamics (Allen et al.(2009),Matthews et al. (2009))

S_t: stock of cumulative emissions of CO_2

S_0:pre-industrial stock of cum. emissions

T_0:pre-industrial temp.

ξ: transient climate response

$$S_t = S_0 + \sum_{i}^{N} \sum_{s=0}^{t} E_{it-s}$$

$$T_t = T_0 + \xi S_t$$

Climate coalition formation
Some Preliminaries

- **Coalition structure** is a partition of set I into coalitions,

$$\mathbb{M} \equiv \{M_1, M_2, \ldots, M_k\}.$$

- m is number of signatories of M.
- **Numerical coalition structure**, $\mathcal{M} \equiv \{m_1, m_2, \ldots, m_k\}$.

- The protocol of the membership stage:

Deterministic order of the initial **proposers** (P) and **respondents** (R) + unanimity rule + first rejector is the next P

- Strategy of P is a **proposal:** identity of members of M + emission reduct. plan(or SCC) + payoffs of members of M

- Strategy of R: accept or reject

Climate coalition formation
Some Preliminaries

- Open membership
- Binding
- Costless
- Irreversible (no renegotiation, to be relaxed later)
- No delay equilibria
- Farsightedness

Dynamic Game
Solution concept

- Dynamic Game between different coalitions (also singletons): coalitions act non-cooperatively against other coalitions (and cooperatively within)
- Pure strategy Markov Perfect equilibrium
  - **current state**: the formed coalitions (if any); identity (and number) of those negotiating (if any); proposal (if ongoing or signed); S_t; K_{it}; and μ_{it}.
- Strategies of country *i*: as P; as R; action stage strategies: $\{E_{it+\tau}(M, \mathbb{M}), C_{it+\tau}(M, \mathbb{M}), K_{it+\tau+1}(M, \mathbb{M}), R_{it+\tau+1}(M, \mathbb{M})\}$ from $\tau=0$ to infinity
- Solve by backwards induction
- Farsightedness (Ray and Vohra, 1999,2001)

Action stage

$\hat{\Lambda}(m) \equiv \frac{\xi\gamma m}{1-\beta}$ is per-unit SCC

μ_{it}: per-unit scarcity rent

s: saving rate

The m member of coalition M maximise,

$$\sum_{i\in M}\sum_{\tau=0}^{\infty}\beta^t\{\ln(C_{it+\tau})\}$$

subject to: resource constraint and feasibility constraint

Proposition1

- optimal emission of $i \in M$ is:

$$E_{it}(m) = \nu/[\mu_{it}(1-s) + \hat{\Lambda}(m)]$$

- emission strategies are **dominant** against what other coalitions choose
- SCC depends on the size of the coalition

F.O.Cs

Membership decision of farsighted countries

Value Function

- **Optimum Value function** of $i \in M$ is $V_i(S_t, K_{i\,t}, \mu_{it}, M, \mathbb{M})$
- **Farsightedness:** countries are required to rationally predict the entire coalition (in contrast to cartel stability and coalition-proofness)- far more realistic!
  - $\mathbb{M}^*$ is immune to unilateral and multilateral deviations by the deviating group and all the active players in the negotiation room
- The equilibrium $\mathbb{M}^*$ needs to be found **recursively**: if $N=2$, then $\mathbb{M}^* =$?. if $N = 3$, then $\mathbb{M}^* =$? ... .
- We check for which group of countries, a grand coalition forms in equilibrium
- Extra demanding with heterogeneous countries but not in our case!

Heterogeneity with respect to K_{i0} and TFP

- In a stage of the recursion, suppose j is initial P and compares $M \in \{M_1, M_2, ..., M_k\}$ versus $\{I\}$:

$$\sum_{i=1}^{m} V_i^j(S_t, K_{it}, \mu_{it}, M, \mathbb{M}) - \sum_{i=1}^{m} V_i^j(S_t, K_{it}, \mu_{it}, I)$$

- This is independent of K_{i0} and A_i and only only on **emissions** and **linearly** so the comparison reduces to

$$V_i^j(S_t, K_{it}, \mu_{it}, m, \mathcal{M}) - V_i^j(S_t, K_{it}, \mu_{it}, N)$$

so it reduces to the **symmetric** case.

Proof

Heterogeneity with respect to R_{i0} and μ_{it}

- In a stage of the recursion, suppose j is initial P and compares $M \in \{M_1, M_2, \ldots, M_k\}$ versus $\{I\}$:

$$\sum_{i=1}^{m} V_i^j(S_t, K_{it}, \mu_{it}, M, \mathbb{M}) - \sum_{i=1}^{m} V_i^j(S_t, K_{it}, \mu_{it}, I)$$

- This is independent of R_{i0} and if $\beta \to 1$, membership decisions are independent of μ_{it} as well

- This implies that the problem is once again symmetric

Proof

- In both cases, membership decisions reduce to

$$V_i^j(S_t, K_{it}, \mu_{it}, m, \mathcal{M}) - V_i^j(S_t, K_{it}, \mu_{it}, N)$$

Decoupling result

What does this imply?

- focus on equilibrium numerical coalition structure and use the solution algorithm as in Ray and Vohra, (2001)
- but the identity of members is important for questions of efficiency!

- $\mathcal{T}^*$ is the set of N for which a grand coalition forms in equilibrium.

- $D(N) = \{m_1, m_2, ..., m_k\}$ is a **decomposition** of N, such that m_k is the largest integer in $\mathcal{T}^*$ that is strictly smaller than N. Then any other element is the largest integer that is not greater than $N - \sum_{j=i+1}^{k} m_j$.

- Example: if $N = 3$, and $\mathcal{T}^* = \{1, 2\}$, $\{3\}$ or $\{2, 1\}$ or $\{1, 1, 1\}$ forms in equilibrium?
  - Because $D(3) = \{1, 2\}$, then ~~$\{1, 1, 1\}$~~
  - Why? Only the coalitions in the decomposition are farsighted stable.

Lemma 1

Let $D(N) = \{m_1, m_2, ..., m_k\}$, such that m_1 is the smallest element of $D(N)$. If $\beta \to 1$, then independent of source of heterogeneity, a grand coalition forms in equilibrium if

$$\frac{N}{m_1} < e^{(k-1)}$$

Proof

Proposition 1

If $\beta \to 1$, for any number of heterogeneous countries, a grand coalition occurs in equilibrium if N is an element of

$$\mathcal{T}^* = \{1, 2, 4, 7, 13, 24, 44, 81, 149, 274, \ldots\}$$

which is the Tribonacci sequence.

- So how do we find the equilibrium numerical coalition structure?
  - if $N \in \mathcal{T}^*$, then $\mathcal{M}^* = \{N\}$
  - if $N \notin \mathcal{T}^*$, then $\mathcal{M}^* = D(N)$
- The equilibrium number of signatory, m^*, in any coalition is always a Tribonacci number.

Proof

Example. If $N = 195$ then $\mathcal{M}^* = \{149, 44, 2\}$.

Our algorithm

$$\mathcal{T}^* = \{1, 2, 4, 7, 13, 24, 44, 81, 149, 274, ...\}$$

- There is no need for any recursion in our climate coalition + IAM.
- Relative to Cartel stability, here average SCC is 120 times larger!
- Large coalitions are stable.
- The **unique** $\mathcal{M}^*$ is independent of identity of P, identity of R and the protocol.

Efficiency

Which coalitions achieve the highest reduction in emissions?

- When the grand coalition is not stable (fully efficient outcome), equilibrium payoffs and global temperature depend on identity of P and the composition of countries across coalitions.
- For $0<\beta<1$, global Emissions are lower when the high-emitting countries are in larger coalitions
- BUT, for $\beta \to 1$, the case for which we have established the equilibrium, global emissions become asymptotically independent of the identity of the coalitions' members

Proof

Reversible agreements

- Climate negotiations in any period $t \in \{0, 1, 2, ...\}$
- Renegotiation: moving from one Markov state to another
- Binding agreement + renegotiation

  Approval committee: countries which approve the move to a new state (Hyndman and Ray, 2007 REStud)
- Fix a protocol

Proposition 2

The MPE has an absorbing Markovian membership state with the same M^* of irreversible agreements. Distribution of international transfers (if any) are renegotiation-proof too.

Proof

Conclusion

- Decoupling result: characterising $\mathcal{M}^*$ independent of composition
- Capturing various aspects of climate negotiations:

farsightedness + heterogeneity + economic growth + general equilibrium + climate dynamics

- A simple algorithm to fully characterise $\mathcal{M}^*$ in climate coalition + IAM
- Climate coalitions with Tribonacci number of signatories in equilibrium
- Suggesting a more ambitious architecture for climate treaties
- Renegotiation-proof $\mathbb{M}^*$

Green Technology: perfect substitute to E_{it}

Heterogeneous countries + renegotiation

E_{iyt}: total energy

g_{it}: green energy use

d_i: inv. cost parameter

$B(g_{it})$: cost of inv. in green tech.

$$Y_{it} = exp(-\gamma T_t) A_i K_{it}^{1-\nu} E_{iyt}^{\nu}$$

$$E_{iyt} = E_{it} + g_{it}$$

$$B(g_{it}) = \frac{d_i}{2} g_{it}^2$$

heterogeneous with respect to K_{i0}, R_{i0}, A_i, d_i

Green Technology

- A dynamic game with $3N + 1$ stocks
  $\rightarrow$ assume full depreciation of g_{it} and K_{it}
- Coalition members set $\hat{\Lambda}(M)$ jointly, then countries independently choose E_{it} and g_{it}.
- If reversible, no holdup in g_{it} investment.

Action stage

Proposition

- Unique emission level and investment in green technology in each $i \in M$ are

$$E_{it}(m) = (\nu/[\mu_{it}(1 - s_{it}) + \hat{\Lambda}(m)]) - g_{it}$$

$$g_{it}(m) = [\mu_{it}(1 - s_{it}) + \hat{\Lambda}(m)]/d_i$$

where $\hat{\Lambda}_{it}(m) \equiv (1 - s_{it}) \sum_{i \in M} \sum_{\tau=0}^{\infty} \frac{\beta^{\tau}\gamma\xi}{1 - s_{it+\tau+1}}$.

- Emissions and investments are in dominant strategies across coalitions.

- If $s_{it} = \bar{s}$, then $\hat{\Lambda}_{it}(m) \equiv \frac{\xi\gamma m}{1-\beta}$

F.O.Cs

Membership stage

Proposition

If $s_{it} = \bar{s}$, the grand coalition occurs in equilibrium if N is a member of the Tribonacchi set, i.e. $\mathcal{T}^* = \{1, 2, 4, 7, 13, 24, ...\}$.

$\mathcal{M}^*$ is characterised independent of the heterogeneity with respect to d_i, K_{i0}, A_i, R_{i0} and μ_{it}.

Proof

- Here $E_{it}(m) + g_{it}(m)$ is equal to E_{it} in the model without green technology.
- Coalition members in effect negotiate marginal productivity of E_{iyt}, regardless of its split between $E_{it}(m)$ and $g_{it}(m)$

Membership stage

Proposition

If $s_{it} = \bar{s}$, the grand coalition occurs in equilibrium if N is a member of the Tribonacchi set, i.e. $\mathcal{T}^* = \{1, 2, 4, 7, 13, 24, ...\}$.

$\mathcal{M}^*$ is characterised independent of the heterogeneity with respect to d_i, K_{i0}, A_i, R_{i0} and μ_{it}.

Proof

- Here $E_{it}(m) + g_{it}(m)$ is equal to E_{it} in the model without green technology.
- Coalition members in effect negotiate marginal productivity of E_{iyt}, regardless of its split between $E_{it}(m)$ and $g_{it}(m)$

First-order conditions of action stage

Problem of planners ithin coalition M (coalition level):

F.O.C w.r.t. E_{it}:

$$\frac{\nu Y_{it}}{E_{it}(m)} = \mu_{it} C_{it} + \hat{\Lambda}(m) Y_{it}$$

Problem of planner ithin each country: (country level)

F.O.C w.r.t. C_{it} and K_{it+1}:

$$\frac{s_{it}}{1-s_{it}} = \beta \frac{1}{1-s_{it+1}}(1-\nu)$$

$\Rightarrow s_{it} = s = \beta(1-\nu)$, for all t and all i.

F.O.C w.r.t. R_{it+1}:

$$\mu_{it} = \beta \mu_{it+1}$$

Back

Value Function

$$V_i(S_t, K_{it}, \mu_{it}, M, \mathbb{M}) = ln(C_{it}(M, \mathbb{M})) + \beta ln(C_{it+1}(M, \mathbb{M})) + ...$$
$$= \frac{(1-\nu)ln(K_{it}) + H_1 + H_2 + H_3}{1-s}$$

where

$$H_1 \equiv \frac{sln(s) - sln(1-s) + ln(A_i) - \gamma T_0}{1-\beta}$$

$$H_2 \equiv -\gamma\xi[S_t + \beta S_{t+1} + \beta^2 S_{t+2} + ...]$$

$$H_3 \equiv \nu[ln(E_{it}(m)) + \beta ln(E_{it+1}(m)) + \beta^2 ln(E_{it+2}(m)) + ...]$$

Back

Decoupling result for K_{i0} and TFP

$$
\begin{aligned}
&V_i(M,\{M_1, M_2, ..., M_k\}) - V_i(\{I\}) = \\
&\quad \frac{1}{1-\beta(1-\nu)}\Big\{\nu[ln(\frac{E_{it}(M)}{E_{it}(I)}) + \beta ln(\frac{E_{it+1}(M)}{E_{it+1}(I)}) + ...] \\
&\quad - \frac{\gamma\xi}{1-\beta}\{[\sum_{i\in M_1} E_{it}(M_1) + \sum_{i\in M_2} E_{it}(M_2) + ... + \sum_{i\in M_k} E_{it}(M_k) - \sum_{i\in I} E_{it}(I)] + \\
&\quad \beta[\sum_{i\in M_1} E_{it+1}(M_1) + \sum_{i\in M_2} E_{it+1}(M_2) + ... + \sum_{i\in M_k} E_{it+1}(M_k) - \sum_{i\in I} E_{it+1}(I)] + ...\}\Big\}
\end{aligned}
$$

This is independent of K_{i0} and A_i. Thus,

$$
\sum_{i=1}^{m} V_i^j(S_t, K_{it}, \mu_{it}, M, \mathbb{M}) - \sum_{i=1}^{m} V_i^j(S_t, K_{it}, \mu_{it}, I)
$$

only depends on m and $\mathcal{M}$ and not M and $\mathbb{M}$.

Back

Decoupling result for R_{i0} and μ_{it}

$$V_i(M,\{M_1,M_2,...,M_k\}) - V_i(\{I\}) =$$
$$\frac{1}{1-\beta(1-\nu)}\Big\{\nu[ln(\frac{E_{it}(M)}{E_{it}(I)}) + \beta ln(\frac{E_{it+1}(M)}{E_{it+1}(I)}) + ...]$$
$$-\frac{\gamma\xi}{1-\beta}\{[\sum_{i\in M_1} E_{it}(M_1) + \sum_{i\in M_2} E_{it}(M_2) + ... + \sum_{i\in M_k} E_{it}(M_k) - \sum_{i\in I} E_{it}(I)] +$$
$$\beta[\sum_{i\in M_1} E_{it+1}(M_1) + \sum_{i\in M_2} E_{it+1}(M_2) + ... + \sum_{i\in M_k} E_{it+1}(M_k) - \sum_{i\in I} E_{it+1}(I)] + ...\}\Big\}$$

This is independent of R_{i0}, and

$$\lim_{\beta\to 1} V_i(M,\{M_1,M_2,...,M_k\}) - V_i(\{I\}) = [ln(\frac{N}{m}) + ln(\frac{N}{m}) + ...] - \{[k-1] + [k-1] + ...\}$$

is independent of μ_{it}, and only depends on m and $\mathcal{M}$.

Back

Proof of Lemma

$$V_i(m_1,\{m_1,m_2,...,m_k\}) - V_i(\{N\}) =$$

$$\frac{1}{1-\beta(1-\nu)}\Big\{\nu[ln(\frac{E_{it}(m_1)}{E_{it}(N)}) + \beta ln(\frac{E_{it+1}(m_1)}{E_{it+1}(N)}) + ...]$$

$$-\frac{\gamma\xi}{1-\beta}\{[\sum_{i\in M_1} E_{it}(m_1) + \sum_{i\in M_2} E_{it}(m_2) + ... + \sum_{i\in M_k} E_{it}(m_k) - \sum_{i\in I} E_{it}(N)] +$$

$$\beta[\sum_{i\in M_1} E_{it+1}(m_1) + \sum_{i\in M_2} E_{it+1}(m_2) + ... + \sum_{i\in M_k} E_{it+1}(m_k) - \sum_{i\in I} E_{it+1}(N)] + ...\}\Big\}$$

Thus,

$$\lim_{\beta\to 1} V_i(m_1,\{m_1,m_2,...,m_k\}) - V_i(\{N\}) = \nu[ln(\frac{N}{m_1}) + ln(\frac{N}{m_1}) + ...] - \nu\{[k-1] + [k-1] + ...\} < 0$$

This is satisfied if $ln(\frac{N}{m_1}) < (k-1)$

Back

Proof of Tribonacci Proposition

$P(n) : T_n = T_{n-3} + T_{n-2} + T_{n-1}$ and $T_n \in \mathcal{T}^*$
$P(1)$ holds, as $T_{n=1} = 7$.
Strong inductive hypothesis: $P(j)$ is true for all $j = 1, 2, ...\kappa$.
We show that $P(\kappa + 1)$ is true, i.e. $\lim_{\beta \to 1} V_i(T_{\kappa-2}, \{T_{\kappa-2}, T_{\kappa-1}, T_\kappa\}) - V_i(\{T_{\kappa+1}\}) < 0$
Using Lemma, this is:

$$\frac{T_{\kappa-2} + T_{\kappa-1} + T_\kappa}{T_{\kappa-2}} < e^2$$

equivalently,

$$1 + \frac{T_{\kappa-1}}{T_{\kappa-2}} + (\frac{T_\kappa}{T_{\kappa-1}})/(\frac{T_{\kappa-2}}{T_{\kappa-1}}) < e^2$$

Using the hypothesis: L-H-S: ≈ 6.22227, while R-H-S : $e^2 \approx 7.38905$. So $P(n)$ is ture for all positive integers n.

Back

Proof of efficiency Proposition

$$V_i^i(M_{k-1},\{M_1,M_2,...,M_k\}) - V_i^i(M'_{k-1},\{M_1,M_2,...,M_{k-2},M'_{k-1},M'_k\}) =$$

$$\frac{1}{1-\beta(1-\nu)}\Big\{\nu[ln(\frac{E_{it}(M_{k-1})}{E_{it}(M'_{k-1})}) + \beta ln(\frac{E_{it+1}(M_{k-1})}{E_{it+1}(M'_{k-1})}) + ...]$$

$$-\frac{\gamma\xi}{1-\beta}\{[\sum_{i\in M_{k-1}} E_{it}(M_{k-1}) + \sum_{i\in M_k} E_{it}(M_k)] - [\sum_{i\in M'_{k-1}} E_{it}(M'_{k-1}) + \sum_{i\in M'_k} E_{it}(M'_k)] +$$

$$\beta[\sum_{i\in M_{k-1}} E_{it+1}(M_{k-1}) + \sum_{i\in M_k} E_{it+1}(M_k)] - \beta[\sum_{i\in M'_{k-1}} E_{it+1}(M'_{k-1}) + \sum_{i\in M'_k} E_{it+1}(M'_k)] + ...\}$$

The second line is zero, and

$$\lim_{\beta\to 1} V_i^i(M_{k-1},\{M_1,M_2,...,M_k\}) - V_i^i(M'_{k-1},\{M_1,M_2,...,M_{k-2},M'_{k-1},M'_k\})$$

$$= -2(1-1+1-1+...) < 0$$

Given $1/2$ as the summation of the Grandi's series. Back

Proof of reversible agreement proposition

- W.L.G. at the beginning of t, $\mathbb{M}$ is coalition structure of singletons.

  binding agreement: proposed M maximises ∞-horizon payoff of signatories

  $\Rightarrow$ dominant strategy emissions

  $\Rightarrow$ in no-delay MPE, if $\beta \to 1$, $\mathbb{M}^*$ forms.

- In period $t + 1$, the same $i \in M^* \subseteq \mathbb{M}^*$ is P.

  binding assumption $+$ approval committee $\Rightarrow$ no party has a profitable deviation.

  $\Rightarrow$ MPE has an absorbing membership state and $\mathbb{M}^*$ forms.

- Transfers are budget-balanced $\Rightarrow \nexists$ history at which R can reject proposal of i $\Rightarrow$ transfer distributions are renegotiation-proof.

Back

F.O.Cs of action stage of model with green technology

Problem of planners of coalition *M*:

F.O.C w.r.t. E_{it}:

$$\frac{\nu}{E_{iyt}} \leq (1-s_{it}) \sum_{i \in M} \sum_{\tau=0}^{\infty} \frac{\beta^{\tau}\gamma\xi}{1-s_{it+\tau+1}} + (1-s_{it})\mu_{it} \; ; \; E_{it} \geq 0 \; ; \; \text{c.s.}$$

$\Rightarrow E_{it}(m) = \frac{\nu}{\mu_{it}(1-s_{it})+\hat{\Lambda}(m)} - g_{it}(m)$, where $\hat{\Lambda}_{it} \equiv (1-s_{it}) \sum_{i \in M} \sum_{\tau=0}^{\infty} \frac{\beta^{\tau}\gamma\xi}{1-s_{it+\tau+1}}$

Problem of planner of each country:

F.O.C w.r.t. g_{it}:

$$\frac{\nu Y_{it}(M, \mathbb{M})}{E_{it}(m) + g_{it}(m)} \leq d_i g_{it}(m) \; ; \; g_{it} \geq 0 \; ; \; \text{c.s.}$$

$\Rightarrow g_{it}(m) = \frac{\mu_{it}(1-s_{it})+\hat{\Lambda}(m)}{d_i}$

Back

Proof of membership proposition of model with green technology

If $s_{it} = \bar{s} \Rightarrow \hat{\Lambda}_{it} = \frac{\gamma \xi m}{1-\beta}$

$$V_i^g(S_t, K_{it}, \mu_{it}, m, \mathcal{M}) = \frac{(1-\nu) ln(K_{it}) + H_1^g + H_2^g + H_3^g}{1-\beta(1-\nu)}$$

where H_2^g and H_3^g are the same as H_2 and H_3 in the model without green tech. just replacing E_{it} with E_{iyt}.

$$\Rightarrow E_{it}(m) + g_{it}(m) = \frac{\nu}{\hat{\Lambda}(m) + (1-\bar{s})\mu_{it}}$$

independent of heterogeneity with respect to d_i, Lemma holds here too.

Back