Binary Public Decisions, Undominated Mechanisms and Voluntary Participation

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A group of agents faces a choice between two public alternatives.
Agents’ preferences over the two alternatives differ.
Preferences constitute private information.
Utility is quasi-linear and, thus, transferable.
The planner may not rely on outside funding.
Alternative $a$ vs alternative $b$

- Enacting a bill into law
- Becoming party to an international treaty (Paris accord, arms non-proliferation, Schengen)
- Acceding a customs union, the WTO, the EU, etc
- Procurement decision regarding a binary pure public good
The literature has mostly focused on strategy-proof mechanisms that satisfy either Decision Efficiency or Budget-balance.

- Decision Efficiency + Strategy Proofness = Groves Mechanisms (Holmstrom 1979)
- Groves Mechanisms are wasteful (Green and Laffont 1979)
- The Pivotal mechanism is the least wasteful among Groves mechanisms (Guo et al. 2013)
- Budget-balance: Faltings (2005); Guo et al. (2011) etc... (randomized outcome function and welfare considerations in expected terms.)
- Inescapable tension between Decision Efficiency and Budget balance.
A new approach

- We do not require the mechanisms we consider, to satisfy a-priori neither Decision Efficiency, nor Budget-balance.
- Instead, we consider the set of **undominated mechanisms**.
- A mechanism dominates another, if each agent, at each profile of preferences, is at least as well off under the former relative to the latter, while for some agent, at some profile of preferences, the former mechanism is better.
- Athanasiou (2013), Sprumont (2013)
The two public alternatives are denoted \( a \) and \( b \)

\( N = \{1, \ldots, n\} \) constitutes the set of agents

The parameter \( \theta_i \in \mathbb{R} \) captures the utility difference between alternative \( a \) and alternative \( b \). If \( \theta_i > 0 \)...

Agents have **quasi-linear** preferences

for some element \((q, z)\) of the consumption space \( \{a, b\} \times \mathbb{R} \), agent \( i \)'s utility is denoted

\[
u_i(q, z) = \begin{cases} 
\frac{\theta_i}{2} + z & \text{if } q = a, \\
-\frac{\theta_i}{2} + z & \text{if } q = b.
\end{cases}
\]

no status quo / neutral cardinalization
For each individual $i \in \{1, 2, \ldots, n\}$, $\theta_i \in \mathbb{R}$ is $i$’s valuation for the good.
Neutral Cardinalization: No status quo.
Agents’ differ in their valuation

Pink and violet prefer $\beta$ to $\alpha$. Cyan prefers $\alpha$ to $\beta$. 
An economy is a profile $\theta_N = (\theta_i)_{i \in N} \in \mathbb{R}^n$. 
A mechanism

- A **decision criterion** is a function $f : \mathbb{R}^n \to \{a, b\}$ that associates every economy $\theta_N = (\theta_i)_{i \in N} \in \mathbb{R}^n$ with a *decision* between the two alternatives.

- A function $t_i : \mathbb{R}^n \to \mathbb{R}$ associates every economy $\theta_N = (\theta_i)_{i \in N} \in \mathbb{R}^n$ with a *transfer* for agent $i \in N$, with $t_N(\theta_N) = (t_i(\theta_N))_{i \in N}$.

- An **allocation** is a decision between the two alternatives coupled with a vector of transfers.

- A **mechanism** $\varphi = (f, t)$ is a function that assigns an allocation to each economy in the domain.
Three standard properties

A mechanism \( \varphi = (f, t) \) satisfies **Strategy-Proofness** if and only if for each \( \theta_N \in \mathbb{R}^n \), each \( \theta'_i \in \mathbb{R} \) and each \( i \in N \)

\[
    u(\varphi_i(\theta_i, \theta_{N\setminus\{i\}}); \theta_i) \geq u(\varphi_i(\theta'_i, \theta_{N\setminus\{i\}}); \theta_i).
\]

A mechanism \( \varphi = (f, t) \) satisfies **Anonymity** if and only if for each \( \theta_N \in \mathbb{R}^n \), each \( \pi \in \Pi \) and each \( i \in N \),

\[
    \varphi_i(\theta_N) = \varphi_{\pi(i)}(\theta_{\pi(N)}).
\]

(\( \theta_{\pi(N)} \) denotes the permutation of the elements of the vector \( \theta_N \) according to \( \pi \in \Pi \))

A mechanism \( \varphi = (f, t) \) satisfies **Feasibility** if and only if for each \( \theta_N \in \mathbb{R}^n \),

\[
    \sum_{i \in N} t_i(\theta_N) \leq 0.
\]
A mechanism $\varphi = (f, t)$ satisfies Decision Efficiency if and only if for each $\theta_N \in \mathbb{R}^n$

$$\sum_{i \in N} \theta_i > 0 \implies f(\theta_N) = a,$$
$$\sum_{i \in N} \theta_i < 0 \implies f(\theta_N) = b.$$
A mechanism $\varphi'$ dominates $\varphi$, if and only if,

1. for each $\theta_N \in \mathbb{R}^n$ and each $i \in N$, $u(\varphi'_i(\theta_N); \theta_i) \geq u(\varphi_i(\theta_N); \theta_i)$,
2. for some $\tilde{\theta}_N \in \mathbb{R}^n$ and some $j \in N$, $u(\varphi'_j(\tilde{\theta}_N); \tilde{\theta}_j) > u(\varphi_j(\tilde{\theta}_N); \tilde{\theta}_j)$.

A mechanism $\varphi$ is **Undominated** if and only if it is feasible, anonymous and strategy-proof and, moreover, there does not exist another feasible, anonymous and strategy-proof mechanism $\varphi'$ that dominates $\varphi$. 
What is the set of appealing *Undominated* mechanisms?
Some context

The set of appealing undominated mechanisms

- the Pivotal mechanism
  - Decision – Efficiency

Voting Rules

- Budget Balance
A rich set of possibilities
Weakening Decision Efficiency

A mechanism \( \varphi = (f, t) \) satisfies **Weak Neutrality** if and only if for each \( y \in \mathbb{R} \setminus \{0\} \),

\[
f(y, \ldots, y) \neq f(-y, \ldots, -y).
\]

A mechanism \( \varphi = (f, t) \) satisfies **Strong Unanimity** if and only if for each \( \theta_N \in \mathbb{R}^n \),

\[
\begin{align*}
\theta_N \in \mathbb{R}_+^n \setminus \{(0, \ldots, 0)\} & \implies f(\theta_N) = a, \\
\theta_N \in \mathbb{R}_-^n \setminus \{(0, \ldots, 0)\} & \implies f(\theta_N) = b.
\end{align*}
\]

\[DE \implies SU \implies WN\]

**Remark:** Weak Neutrality on its own isolates those \( P \in \mathcal{P} \) satisfying \((0, \ldots, 0) \in B(P)\)...

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Weakening No-Envy

A mechanism $\varphi = (f, t)$ satisfies No Positive Transfers to the Winners if and only if for each $\theta_N \in \mathbb{R}^n$ and each $i \in N$,

$$f(\theta_N) = a \text{ and } \theta_i > 0 \implies t_i(\theta_N) \leq 0,$$

and

$$f(\theta_N) = b \text{ and } \theta_i < 0 \implies t_i(\theta_N) \leq 0.$$ 

A mechanism $\varphi = (f, t)$ satisfies No-Envy if and only if for each $\theta_N \in \mathbb{R}^n$ and each $i, j \in N$, $u(\varphi_i(\theta_N); \theta_i) \geq u(\varphi_j(\theta_N); \theta_i)$.

$NE \implies NPTtW$
Theorem 1 (Main Result)

Suppose that $\varphi = (f, t)$, associated with the payment scheme $(P, g) \in \mathcal{P} \times \mathcal{G}$, is a strategy-proof, anonymous, feasible and undominated mechanism. Mechanism $\varphi$ satisfies Weak Neutrality and No Positive Transfers to the Winners if and only if it satisfies Conditions W and S, $(0, \ldots, 0) \in B(P)$, and for each $\theta_N \in \mathbb{R}^n$ and each $i \in N$,

$$g(\theta_N \setminus \{i\}) = \min \left\{ 0, p(\theta_N \setminus \{i\}) \right\}.$$

Once one has chosen one of the many decision criteria complying with Theorem 1... then the transfers are **uniquely** determined:

$$t_i = -p(\theta_N \setminus \{i\}) + g(\theta_N \setminus \{i\}) = -p(\theta_N \setminus \{i\}) + \min \left\{ 0, p(\theta_N \setminus \{i\}) \right\}.$$
Corollary 1 of Theorem 1 The Pivotal mechanism is the only undominated Groves mechanism satisfying No Positive Transfers to the Winners.
A mechanism $\varphi = (f, t)$ satisfies Budget-balance if and only if for each $\theta_N \in \mathbb{R}^n$, $\sum_{i \in N} t_i(\theta_N) = 0$.

**Corollary 2 of Theorem 1** A mechanism belonging to the class characterized by Theorem 1 satisfies Budget-Balance if and only if it satisfies No-Envy.
Voting Rules

For each $\theta_N \in \mathbb{R}^n$, let

$a(\theta_N) = \left| \{i \in N : \theta_i > 0\} \right|$ and $b(\theta_N) = \left| \{i \in N : \theta_i < 0\} \right|$.

A family of (strongly decisive) special majority voting rules (Fishburn 1973) that rely solely on the information conveyed by the parameters above.
Corollary 3 of Theorem 1  Suppose that $\varphi = (f, t)$, associated with the payment scheme $(P, g) \in \mathcal{P} \times \mathcal{G}$, is a strategy-proof, anonymous, feasible and undominated mechanism. Mechanism $\varphi$ satisfies Weak Neutrality and No Envy if and only if there exists a finite sequence of integers $s(0), s(1), ..., s(m - 1)$ with $m \leq n$ such that:

i. $s(k) \in \{0, 1\}$ if $k = 0$ and $s(k) \in \{1, n - k\}$ if $k = 1, ..., m - 1$;

ii. $s(0) \leq s(1) \leq ... \leq s(m - 1)$;

iii. for each $\theta_N \in \mathbb{R}^n$, $P(\theta_N) = 1$ if and only if

$$b(\theta_N) < m \text{ and } a(\theta_N) \geq s(b(\theta_N)),$$

iv. for each $\theta_N \in \mathbb{R}^n$ and each $i \in N$, $t_i(\theta_N) = 0$. 
For instance

Let $N = \{1, 2, 3\}$

$m = 1$ and $s(0) = 1$; if just one agent prefers $b$ to $a$ then $b$ is chosen.

$m = 3$ and $s(0) = 0$; $b$ is chosen only if three agents prefer $b$ to $a$.

$m = 2$, $s(0) = 1$ and $s(1) = 1$; $a$ is chosen if $1 \leq b(\theta_N) < 2$.
So far...

- A proof of concept for *Undominatedness*: A workable concept with meaningful consequences that is applicable to mechanism design in general.
- A new family of mechanisms that places the focus on the many different ways the issue of tension between decision efficiency and budget-balance may be resolved.

An embarrassment of riches: Does some mechanism emerge as a definitive proposal?
A mechanism $\varphi = (f, t)$ satisfies **Voluntary Participation** if and only if for each $\theta_N \in \mathbb{R}^n$ and each $i \in N$, $u(\varphi_i(\theta_N); \theta_i) \geq \overline{u}(\theta_i)$.

Can the instrument of transfers ensure that no-one will be coerced into participating?

Can some mechanism guarantee for all a non-negative share of the gain; if a gain is to be had relative to the status quo (say alternative $\alpha$)?

[Set for each $\theta_N \in \mathbb{R}^n$ and each $i \in N$, $\overline{u}(\theta_i) = u((a, 0); \theta_i)$.]
One of the alternatives constitutes the status quo

NATO as is vs NATO with the addition of Sweden and Finland.
Theorem 2

A mechanism \( \varphi = (f, t) \) is the **Unanimity** mechanism if and only if for each \( \theta_N \in \mathbb{R}^n \)

1. \( f(\theta_N) = a \) if and only if for each \( i \in N, \theta_i \geq 0 \), and

2. for each \( i \in N, t_i(\theta_N) = 0 \).

A mechanism \( \varphi \) that satisfies Strategy-proofness, Anonymity, Feasibility, and Voluntary Participation is Undominated if and only if it is the Unanimity Mechanism.
Concluding Remarks

- Voluntary Participation drastically restricts the amount of options.
- The possibility of transfers fails to generate possibilities.
- Unanimity or extreme status quo bias can be morally justified nearly exclusively in terms of Voluntary Participation.
- This is a novel characterization of Unanimity based on the concept of Undominatedness.
- There is good reason to expect that the result holds in a setting with an arbitrary number of public alternatives (Osheto 2000).
What is the set of *Anonymous* and *Strategy-proof* mechanisms?
A representation Theorem

Let $\mathcal{P}$ be the set of functions $P : \mathbb{R}^n \rightarrow \{-1, 1\}$ that are weakly monotonic and symmetric.

A mechanism $\varphi = (f, t)$ is strategy-proof and anonymous only if there exists $P \in \mathcal{P}$ such that

\[
\{\theta_N \in \mathbb{R}^n : P(\theta_N) = 1\} = \{\theta_N \in \mathbb{R}^n : f(\theta_N) = a\},
\]

\[
\{\theta_N \in \mathbb{R}^n : P(\theta_N) = -1\} = \{\theta_N \in \mathbb{R}^n : f(\theta_N) = b\}.
\]
A representation Theorem

Each function $P \in \mathcal{P}$ is associated with a partition of the domain of preferences into two connected sets.

Let $B(P)$ denote the **boundary** of associated with partition $P \in \mathcal{P}$. 
A representation Theorem

For each $P \in \mathcal{P}$, each $\theta_N \in \mathbb{R}^n$ and each $i \in N$, let

$$ p(\theta_N \setminus \{i\}) = \begin{cases} 
\inf\{x \in \mathbb{R} : P(x; \theta_N \setminus \{i\}) = 1\} & \text{if } P(\mathbb{R}, \theta_N \setminus \{i\}) = \{-1, 1\}, \\
0 & \text{if } P(\mathbb{R}, \theta_N \setminus \{i\}) = \{k\} \\
\text{for some } k \in \{-1, 1\}. 
\end{cases} $$
Consider the following function \( p : \mathbb{R}^{n-1} \rightarrow \mathbb{R} \):

For each \( P \in \mathcal{P} \), each \( \theta_N \in \mathbb{R}^n \) and each \( i \in \mathbb{N} \),

if \( P(x, \theta_N \setminus \{i\}) \) is constant in \( x \in \mathbb{R} \), then we set \( p(\theta_N \setminus \{i\}) = 0 \).

if \( P(x, \theta_N \setminus \{i\}) \) is non-constant in \( x \in \mathbb{R} \), then \( p(\theta_N \setminus \{i\}) \) represents the unique cut-off value such that:

\[
P(x, \theta_N \setminus \{i\}) = \begin{cases} 
1 & \text{if } x > p(\theta_N \setminus \{i\}), \\
-1 & \text{if } x < p(\theta_N \setminus \{i\}). 
\end{cases}
\]
A representation Theorem

\[ p'(\hat{\theta}_N \setminus \{i\}) \quad B(P') \]

\[ \theta_i \]

\[ \hat{\theta}_N \setminus \{i\} \]

\[ p''(\hat{\theta}_N \setminus \{i\}) = 0 \quad B(P'') \]
Let $G$ be the set of symmetric functions $g : \mathbb{R}^{n-1} \to \mathbb{R}$.

A mechanism $\varphi = (f, t)$ is strategy-proof and anonymous if and only if there exist $P \in \mathcal{P}$ and $g \in G$ such that for all $\theta_N \in \mathbb{R}^n$,

$$
\varphi(\theta_N) = \begin{cases} 
(a, (g(\theta_N \setminus \{i\}) - p(\theta_N \setminus \{i\}))_{i \in N}) & \text{if } P(\theta_N) = 1 \\
(b, (g(\theta_N \setminus \{i\}))_{i \in N}) & \text{if } P(\theta_N) = -1
\end{cases}
$$

Every strategy-proof and anonymous mechanism can be associated with a payment scheme $(P, g)$. 
A representation Theorem: Another example

\[ t_i(\theta''_i, \theta'_N\setminus\{i\}) = g(\theta'_N\setminus\{i\}) \]

\[ t_i(\theta'_i, \theta'_N\setminus\{i\}) = -p(\theta'_N\setminus\{i\}) + g(\theta'_N\setminus\{i\}) \]
A simple application of the representation Theorem:

A *Strategy-Proof* and *Anonymous* mechanism \( \varphi = (f, t) \), associated with payment scheme \((P, g)\) satisfies *Decision Efficiency* if and only if for each \( \theta_N \in \mathbb{R}^n \) and each \( i \in N \), \( p(\theta_{N\backslash\{i\}}) = -\sum_{j \in N \backslash \{i\}} \theta_j \).
Two necessary conditions

Condition $S$ : $p^1(\hat{\theta}_1) = \delta$
Condition $W$ : there exists $x \in \mathbb{R}$ s.t. $(\tilde{\theta}_1, x) \in B(P^2)$ and $p^2(\tilde{\theta}_1) = x$