

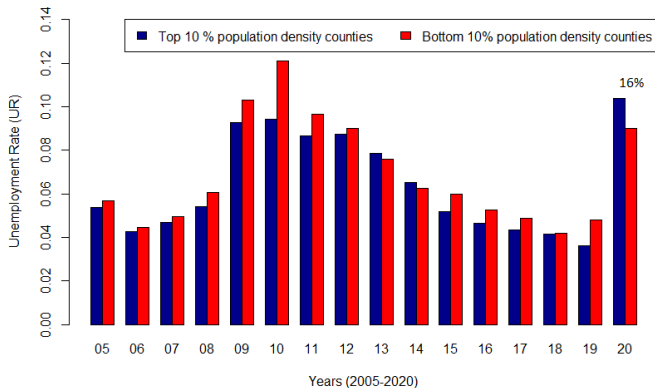
Population Density and the Local Economy pre- and post-Pandemic Breakout

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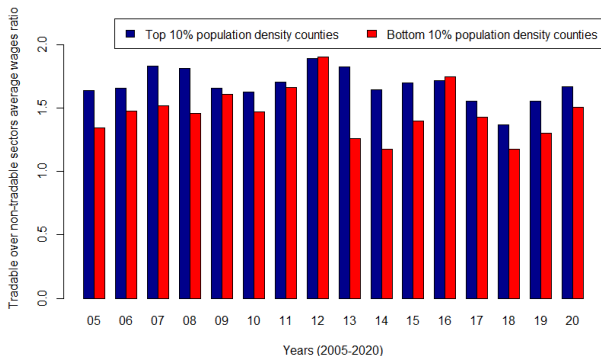
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Average unemployment rate for HD and LD counties in US



Tradable to non-tradable sector average wage ratio for HD and LD counties in US (tradable wage premium)



1. Systems-of-locations theories:

Especially those on heterogeneous agents sorting & selection (e.g., Davis & Dingel, 2019)

2. Local multiplier:

Predominantly empirical (e.g., Moretti, 2010; Moretti & Thulin, 2013)

3. Search & matching in spatial equilibrium

This paper builds upon the model of Davis and Dingel (2019) and

- **extends it:** includes labor market frictions and an endogenous mass of firms in each location
- **alters preferences:** non-homothetic, local good's expenditure share increases in income

Main mechanisms:

- Complementarity between local level of idea exchange & a worker's skill
- Skill elasticity of tradable sector workers' demand for local good non-decreasing in skill & local level of idea exchange
- Same for tradable sector workers' average wage (albeit more)

Main results:

- Higher skilled individuals collocate and are in tradable sector
- Higher local good's prices and urban costs in HD locations
- Higher employment rate in HD locations
- Higher tradable sector wage premium in HD locations

WFH and relocation of tradable sector workers:

- Boosts employment rate in destination region. Reverse in origin location
- Increases tradable sector wage premium in destination and origin location

General

- C locations, continuum of individuals of mass L per C unit areas
- 2 sectors in each location: tradable (T), non-tradable (N)
- Tradable good (TG) numéraire. Non-tradable good (NG) in location c : p_{nc}
- Individuals distributed on the interval $[zmin, zmax] \subset \mathbb{R}_+$ with pdf $\phi(z)$
- Price Independent Generalized Linearity (PIGL) type preferences (Boppart, 2014)

$$v(p_{nc}, p_{hc}, I) = \frac{1}{\epsilon} \frac{(I^\epsilon - r)}{p_{nc}^\epsilon} + \frac{r-1}{\epsilon} p_{hc}(L_c)$$

- I -income, L_c -population density in c , $p_{hc}(L_c)$ -urban cost, $\frac{\partial p_{hc}}{\partial L_c} > 0$
- Expenditures \rightarrow TG: $rI^{1-\epsilon}$, NG: $I - rI^{1-\epsilon}$, $r > 0$, $0 < \epsilon < 1$

Economy setup

Labor market frictions

- Directed search ($m(v, u) = Bv^\eta u^{1-\eta}$, $q = \frac{v}{u}$)

N sector firms

- Labor is the only factor of production
Uniform productivity normalized to 1
- k_n -vacancy cost
Maximize expected profit given reservation wage U_c

T sector firms

- $\tilde{z}(z, Z_c)$
- k_t -vacancy cost
Maximize expected profit given reservation wage $U_c(z, Z_c)$

profit maximization

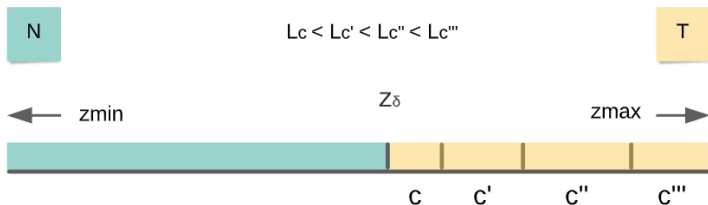
local level of idea exchange

Assumption: $\tilde{z}(z, Z_c)$ is C^2 , $\frac{\partial \tilde{z}(z, Z_c)}{\partial z}$, $\frac{\partial \tilde{z}(z, Z_c)}{\partial Z_c}$ & $\frac{\partial^2 \tilde{z}(z, Z_c)}{\partial z \partial Z_c} > 0$

Economy setup

It follows (Lemmata 1+2):

- Critical skill level, z_δ , everyone with $z > z_\delta$ looks for a job in the T sector (comparative advantage)
- If $L_c < L_{c'}$, then $p_{nc} < p_{nc'}$ and $p_{hc} < p_{hc'}$



lemma 1

lemma 2

equilibrium conditions

Two locations case (Assume $L_2 > L_1, Z_2 > Z_1$)

Sufficient conditions (Proposition 1)

- (a) \tilde{z} (log-supermodular, log-convex in z)
- (b) skill distribution (non-increasing)
- (c) r ($r \leq 1$)

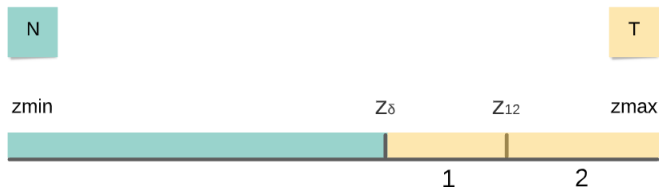
Location 2 has both:

- higher $\frac{\text{N sector employed}}{\text{Labor force}}$
- higher $\frac{\text{T sector employed}}{\text{Labor force}}$

Unemployment Rate

Proposition 1 implies

- $\frac{\% \text{change in T's demand for N}}{\% \text{change in } z}$ non-decreasing in z , Z_c
- $z_{max} - z_{12} > z_{12} - z_{\delta}$

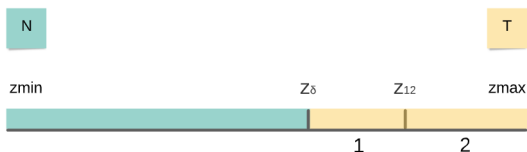


fictitious example

Wage Premium

Same conditions ensure that (Proposition 2):

$$\frac{\int_{t,1} U_1(z, Z_1) \phi(z) dz}{U_1 \int_{t,1} \phi(z) dz} < \frac{\int_{t,2} U_2(z, Z_2) \phi(z) dz}{U_2 \int_{t,2} \phi(z) dz}$$



Note: D&D (2019) and this paper's skill premia comparison

- Wage premium w/o unemployment adjustment: Beneficial effects of concentration only on **workers' income**
- Wage premium adjusted for unemployment: Beneficial effects of concentration both on **workers' income** and **matching rate**

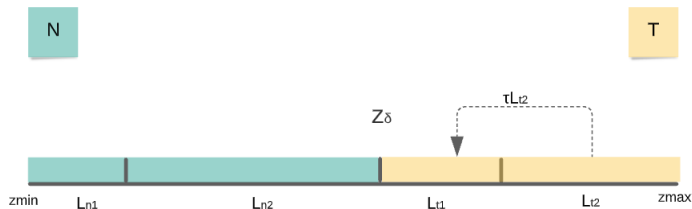
Relocation

Location 1 (LD)

- $L'_1 = L_1 + \tau L_{t2}$
- N demand: $+\tau(\text{N demand by T in 2}) \rightarrow q'_{n1} > q_{n1}^*, w'_{n1} > w_{n1} \rightarrow U'_1 > U_1$

Location 2 (HD)

- $L'_1 = L_2 - \tau L_{t2}$
- N demand: $-\tau(\text{N demand by T in 2}) \rightarrow q'_{n2} < q_{n2}^*, w'_{n2} < w_{n2} \rightarrow U'_2 < U_2$



Alternative (**B**): Individuals considered based on residence location

alternative (a)

Unemployment rate

- Unemployment rate in in 2 \uparrow
- Unemployment rate in in 1 \downarrow proposition 3

Wage premium

- Wage premium in 2 \uparrow $\left(\frac{(1-\tau) \int_{t,2} U_2(z, Z_2) \phi(z) dz}{U_2'(1-\tau) \int_{t,2} \phi(z) dz} \right)$
- Wage premium in 1 \uparrow (Proposition 4)

Data sources

- IPUMS-CPS
- US Census Bureau

Data processing

- Panel of counties
- Ranking based on average population density
- Population density weighted average of measures
- Time average for equilibrium (2016-2019, 2005-2008)

Targeted moments

- Unemployment rate in high density counties (URHD)
- Unemployment rate in low density counties (URLD)
- Tradable to non-tradable sector average wage in the high density counties (TWHD/NWHD)
- Tradable to non-tradable sector average wage in low density counties (TWLD/NWLD)
- High to low population densities ratio (HD/LD)
- High to low density average non-tradable wage ratio (NWHD/NWLD)

Distributions and forms

Table: Equilibrium

Measures	Data US	Normal, US	Uniform, US	Data PA	Normal, PA	Uniform, PA
HD/LD	3	3	3	3	3	3
URHD	4%	4%	4%	3%	3%	3%
URLD	4.3%	4.3%	4.3%	3.7%	3.7%	3.7%
TWHD/NWHD	1.59	1.59	1.59	1.51	1.55	1.51
TWLD/NWLD	1.5	1.21	1.23	1.33	1.23	1.21
NWHD/NWLD	1.13	1.13	1.15	1.24	1.24	1.24

calibrated parameter set

Table: Model relocation moments

Moments	Data US (2020)	Uniform, US	Normal, US
URLD	8.4%	8.4%	8.4%
URHD	10%	10%	10%

Table: Model relocation parameters

Parameter	Uniform, US	Normal, US
τ	6.7%	6.6%
B	1.66	1.34

Normal, US, URLD indifference curves

Normal, US, URHD indifference curves

Uniform, US, URLD indifference curves

Uniform, US, URHD indifference curves

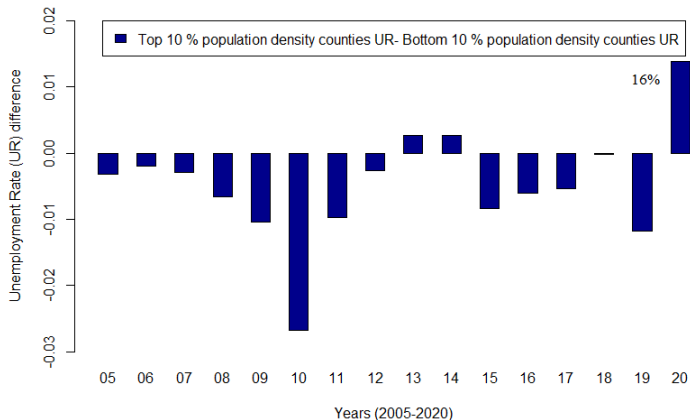
1. Facts:

- Documents higher T wage premium and lower unemployment rate (UR) in HD counties.
Atypical reversal in UR during 2020

2. Develop a framework that endogenously explains these patterns and starts from symmetric fundamentals. Some building blocks:

- Complementarity between z and Z_c
- Skill-elasticity of production non-decreasing in z , Z_c
- Non-homothetic preferences (higher income-larger N good's income share)

3. Provides a first look to potential economic rearrangements following post-covid relocation and living patterns



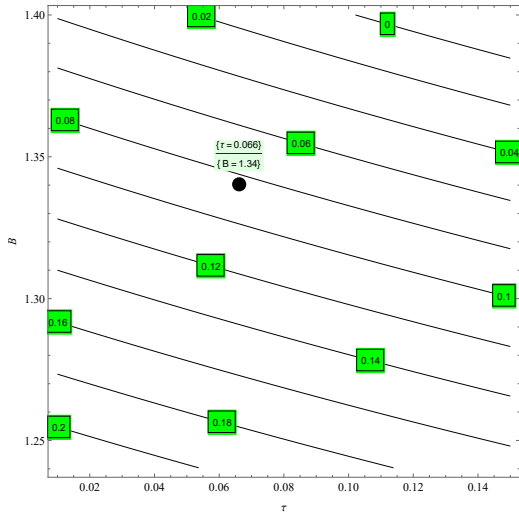
Unemployment rate difference between the most and least densely populated counties in US for the years 2005-2020

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Table: Parameters (Equilibrium)

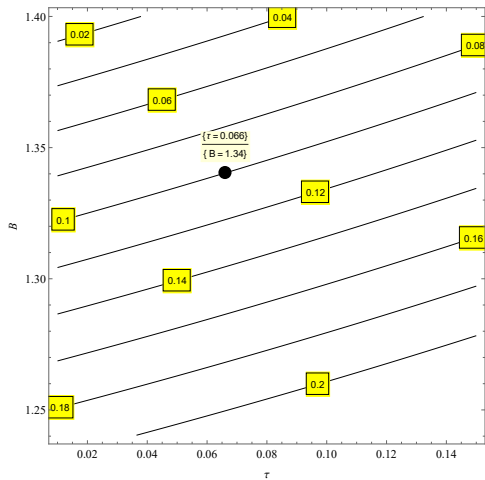
Parameters	Normal, US	Uniform, US	Normal, PA	Uniform, PA
B	1.4	1.75	1.4	1.75
η	0.38	0.35	0.38	0.35
β	0.72	0.72	0.72	0.72
ϵ	0.1	0.1	0.1	0.1
k_t	0.54	1.2	1.08	1.2
k_n	0.89	2.2	2.24	2.3
A	1.29	1.96	2.35	2
θL^γ	1.16	0.98	0.56	0.36
r	0.35	0.5	0.35	0.52
γ	0.1	0.1	0.13	0.08

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Normal, US, URLD indifference curves

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Normal, US, URHD indifference curves

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- N sector

$$\max_{q_{nc}, w_{nc}} Bq_{nc}^{\eta-1} (p_{nc} - w_{nc}) - k_n \quad \text{s.t.} \quad U_c = Bq_{nc}^\eta w_{nc}$$

Result: $q_{nc}^* = \left(\frac{U_c}{Bw_{nc}} \right)^{\frac{1}{\eta}}, w_{nc} = (1 - \eta)p_{nc}$

- T sector

$$\max_{q_{tc}, w_{tc}} Bq_{tc} (z, Z_c)^{\eta-1} (\tilde{z}(z, Z_c) - w_{tc}(z, Z_c)) - k_t$$

$$\text{s.t.} \quad U_c(z, Z_c) = Bq_{tc}^\eta(z, Z_c) w_{tc}(z, Z_c)$$

Result: $q_{tc}^*(z, Z_c) = \left(\frac{U_c(z, Z_c)}{Bw_{t,c}(z, Z_c)} \right)^{\frac{1}{\eta}}, w_{t,c}(z, Z_c) = (1 - \eta)\tilde{z}(z, Z_c)$

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$$Z_c = \left(1 - e^{\left(-\nu L \int_{t,c} (1-\beta) \phi(z) dz \right)} \right) \int_{t,c} \frac{z}{\int_{t,c} \phi(z) dz} \phi(z) dz$$

Z_c from Davis & Dingel (2019), $0 < \beta < 1$, $\nu > 0$

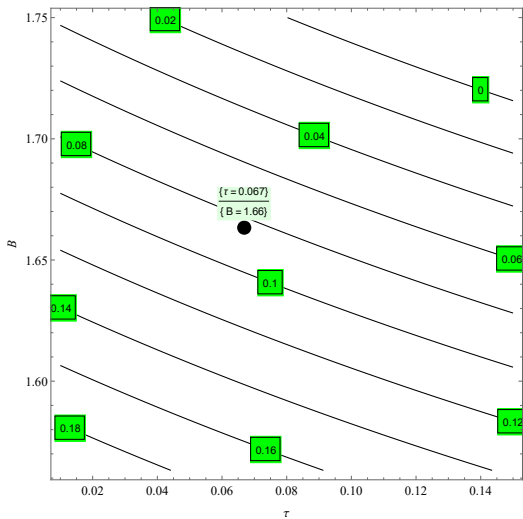
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Fictitious example:

in 2: $z = 7 \rightarrow$ demand for $N = 4$, $z = 7.7 \rightarrow$ demand for $N = 4.4$

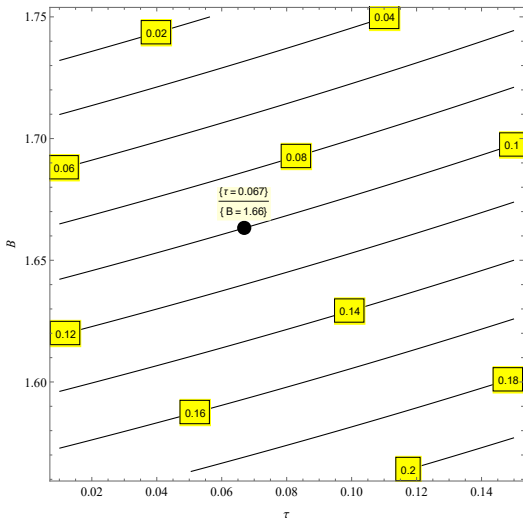
in 1: $z = 5 \rightarrow$ demand for $N = 3$, $z = 5.5 \rightarrow$ demand for $N = 3.15$

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Uniform, US, URLD indifference curves

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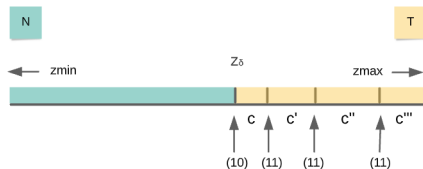
Uniform, US, URHD indifference curves

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Equilibrium

In equilibrium, firms maximize their expected profits and agents make the optimal consumption, occupational and locational choices. Moreover, the following conditions are true:

- Adding-up constraints for location populations (4), (5)
- Zero profit conditions (6), (7)
- Labor market clearing (8)
- Non-tradable goods market clearing (9)
- Skill type indifferent between the two sectors (10)
- Skill type indifferent between tradable sector in c and c' where c' is the next, population-wise, denser location than c (11)



$$L_c = L_{tc} + L_{nc} \text{ where } L_{tc} = L \int_{t,c} \phi(z) dz, L_{nc} = L \int_{n,c} \phi(z) dz \quad (4)$$

$$L = \sum_{c=1}^C L_c \quad (5)$$

$$U_c(z, Z_c) = \left(\frac{\eta}{k_t} \right)^{\frac{\eta}{1-\eta}} B^{\frac{1}{1-\eta}} (1-\eta) \tilde{z}(z, Z_c)^{\frac{1}{1-\eta}} \quad (6)$$

$$U_c = \left(\frac{\eta}{k_n} \right)^{\frac{\eta}{1-\eta}} B^{\frac{1}{1-\eta}} (1-\eta) p_{nc}^{\frac{1}{1-\eta}} \quad (7)$$

$$L \int_{t,c} q_{tc}^*(z, Z_c) \phi(z) dz = Q_{tc} \text{ and } L \int_{n,c} q_{nc}^* \phi(z) dz = Q_{nc} \quad (8)$$

$$U_c \int_{n,c} \phi(z) dz = U_c \int_{n,c} \phi(z) dz + \int_{t,c} U_c(z, Z_c) \phi(z) dz - r \left(U_c^{1-\epsilon} \int_{n,c} \phi(z) dz + \int_{t,c} U_c(z, Z_c)^{1-\epsilon} \phi(z) dz \right) \quad (9)$$

$$U_c(z_\delta, Z_c) = U_c \quad (10)$$

$$\frac{1}{\epsilon} \frac{(U_c^\epsilon(z_{cc'}, Z_c) - r)}{p_{nc}^\epsilon} - p_{hc} = \frac{1}{\epsilon} \frac{(U_{c'}^\epsilon(z_{cc'}, Z_{c'}) - r)}{p_{nc'}^\epsilon} - p_{hc'} \quad (11)$$

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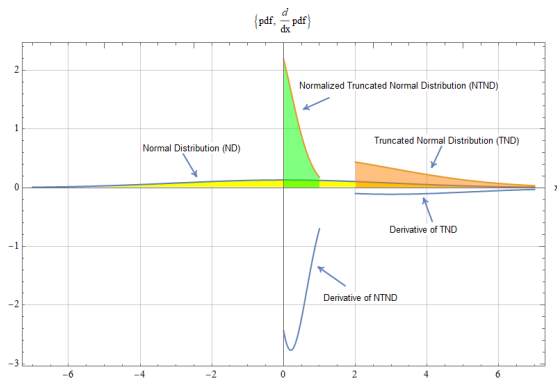
Quantitative analysis

$$Phc = \theta L_c^\gamma$$

\tilde{z} form from Davis & Dingel (2019):

$$\tilde{z}(z_n, Z_c) = \beta z_n + (1 - \beta) A z_n^2 Z_c$$

Distribution:



Unemployment rate

Alternative (A): Individuals considered based on job location

- Only N job finding rates change
- Unemployment rate in 1 ↓ and in 2 ↑

Wage premium

Alternative (A): Individuals considered based on job location

- Job finding rate and wage change only for N
- Wage premium in 1 ↓ and in 2 ↑ $\left(\frac{\int_{t,c} U_c(z, Z_c) \phi(z) dz}{U'_c \int_{t,c} \phi(z) dz} \right)$

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$z \in \Theta_{ntc}, z' \in \Theta_{tc}, z'' \in \Theta_{ntc'}, z < z' < z'',$

$$\frac{1}{\epsilon} \frac{(U_c^\epsilon - r)}{p_{nc}^\epsilon} - p_{hc} = \frac{1}{\epsilon} \frac{(U_{c'}^\epsilon - r)}{p_{nc'}^\epsilon} - p_{hc'}$$

At the same time, for z' that is looking for a job in the tradable sector in c , it holds that:

$$\frac{1}{\epsilon} \frac{(U_c^\epsilon(z', Z_c) - r)}{p_{nc}^\epsilon} - p_{hc} \geq \frac{1}{\epsilon} \frac{(U_c^\epsilon - r)}{p_{nc}^\epsilon} - p_{hc}$$

Since $z'' > z'$, given Assumption 1,

$$\frac{1}{\epsilon} \frac{(U_c^\epsilon(z'', Z_c) - r)}{p_{nc}^\epsilon} - p_{hc} > \frac{1}{\epsilon} \frac{(U_c^\epsilon(z', Z_c) - r)}{p_{nc}^\epsilon} - p_{hc}$$

and z'' would have a higher utility as tradable sector job seeker in location c than as a non-tradable sector job seeker in location c' . Hence, it cannot be the case that any non-tradable sector job seeker has higher ability than any tradable sector job seeker in the economy.

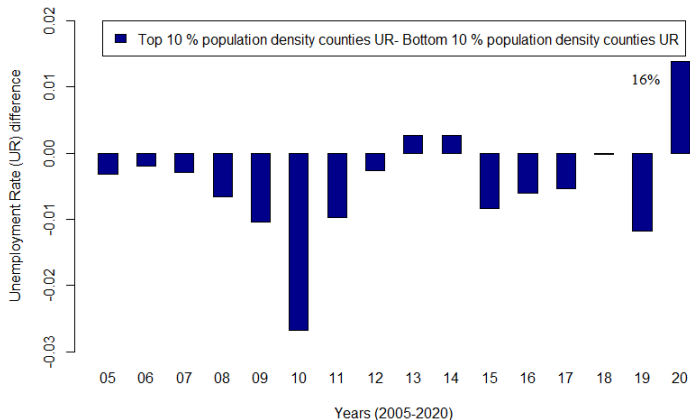
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Let us assume that $L_c < L_{c'}$, and, hence, $p_{hc} < p_{hc'}$, but that $p_{nc} > p_{nc'}$. Then, in equilibrium, from the utility equalization among non-tradable sector workers and from (8), it should hold that:

$$\begin{aligned} \frac{(U_c^\epsilon - r)}{p_{nc}^\epsilon} &< \frac{(U_{c'}^\epsilon - r)}{p_{nc'}^\epsilon} \Rightarrow \\ \frac{\left(\left(\left(\frac{\eta}{k_n} \right)^{\frac{\eta}{1-\eta}} B^{\frac{1}{1-\eta}} (1-\eta) p_{nc}^{\frac{1}{1-\eta}} \right)^\epsilon - r \right)}{p_{nc}^\epsilon} &< \frac{\left(\left(\left(\frac{\eta}{k_n} \right)^{\frac{\eta}{1-\eta}} B^{\frac{1}{1-\eta}} (1-\eta) p_{nc'}^{\frac{1}{1-\eta}} \right)^\epsilon - r \right)}{p_{nc'}^\epsilon} \Rightarrow \\ \left(\left(\frac{\eta}{k_n} \right)^{\frac{\eta}{1-\eta}} B^{\frac{1}{1-\eta}} (1-\eta) \right)^\epsilon p_{nc}^{\frac{\eta\epsilon}{1-\eta}} - \frac{r}{p_{nc}^\epsilon} &< \left(\left(\frac{\eta}{k_n} \right)^{\frac{\eta}{1-\eta}} B^{\frac{1}{1-\eta}} (1-\eta) \right)^\epsilon p_{nc'}^{\frac{\eta\epsilon}{1-\eta}} - \frac{r}{p_{nc'}^\epsilon} \Rightarrow \\ \left(\left(\frac{\eta}{k_n} \right)^{\frac{\eta}{1-\eta}} B^{\frac{1}{1-\eta}} (1-\eta) \right)^\epsilon p_{nc}^{\frac{\eta\epsilon}{1-\eta}} - \left(\left(\frac{\eta}{k_n} \right)^{\frac{\eta}{1-\eta}} B^{\frac{1}{1-\eta}} (1-\eta) \right)^\epsilon p_{nc'}^{\frac{\eta\epsilon}{1-\eta}} &< \frac{r}{p_{nc}^\epsilon} - \frac{r}{p_{nc'}^\epsilon} \end{aligned}$$

Since we assumed that $p_{nc'}^\epsilon < p_{nc}^\epsilon$, the inequality cannot hold.

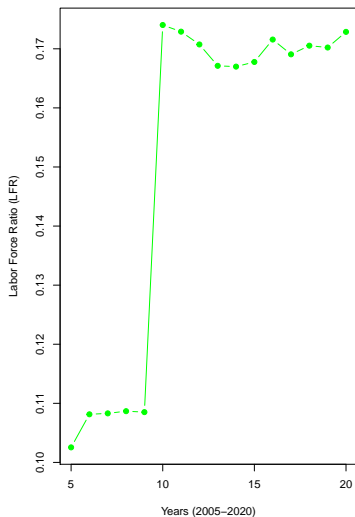
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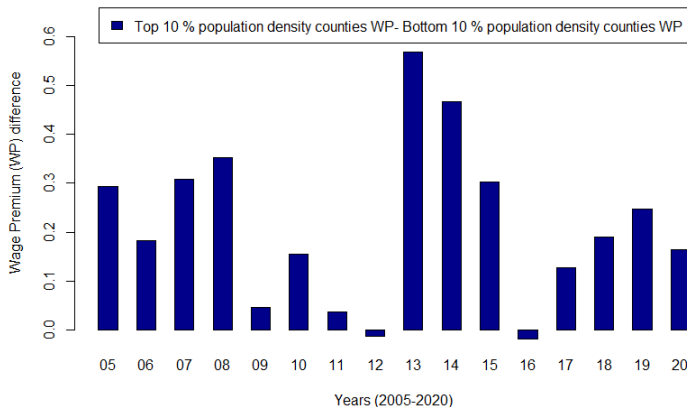


Unemployment rate difference between the most and least densely populated counties in US for the years 2005-2020

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Bottom to top counties LF ratio





Tradable sector wage premium difference between the most and least densely populated counties in US for the years 2005-2020

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Sufficient condition (Proposition 3)

(a) $\frac{\eta}{1-\epsilon} \geq 1$

- Could be problematic if very low elasticity of N's quantity produced with respect to the N expenditure of a location's T residents

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