Population Density and the Local Economy pre- and post-Pandemic Breakout

Despoina Balouktsi

20th Conference on Research on Economic Theory and Econometrics

11 July 2022
Motivation

Average unemployment rate for HD and LD counties in US

![Graph showing unemployment rate from 2005 to 2020 for Top 10% and Bottom 10% population density counties.](image-url)
Motivation

 Tradable to non-tradable sector average wage ratio for HD and LD counties in US (tradable wage premium)

![Bar chart showing the tradable to non-tradable sector average wage ratio for HD and LD counties in the US over the years 2005 to 2020. The chart compares the ratio for counties in the top 10% and bottom 10% of population density.](chart.png)
1. **Systems-of-locations theories:**
Especially those on heterogeneous agents sorting & selection (e.g., Davis & Dingel, 2019)

2. **Local multiplier:**
Predominantly empirical (e.g., Moretti, 2010; Moretti & Thulin, 2013)

3. **Search & matching in spatial equilibrium**

This paper builds upon the model of Davis and Dingel (2019) and

- **extends it:** includes labor market frictions and an endogenous mass of firms in each location

- **alters preferences:** non-homothetic, local good’s expenditure share increases in income
Mechanisms & Results

Main mechanisms:

- Complementarity between local level of idea exchange & a worker’s skill
- Skill elasticity of tradable sector workers’ demand for local good non-decreasing in skill & local level of idea exchange
- Same for tradable sector workers’ average wage (albeit more)

Main results:

- Higher skilled individuals collocate and are in tradable sector
- Higher local good’s prices and urban costs in HD locations
- Higher employment rate in HD locations
- Higher tradable sector wage premium in HD locations

WFH and relocation of tradable sector workers:

- Boosts employment rate in destination region. Reverse in origin location
- Increases tradable sector wage premium in destination and origin location
Economy setup

General

- $C$ locations, continuum of individuals of mass $L$ per $C$ unit areas
- 2 sectors in each location: tradable (T), non-tradable (N)
- Tradable good (TG) numéraire. Non-tradable good (NG) in location $c$: $p_{nc}$
- Individuals distributed on the interval $[z_{min}, z_{max}] \subset \mathbb{R}_+$ with pdf $\phi(z)$
- Price Independent Generalized Linearity (PIGL) type preferences (Boppart, 2014)

$$v(p_{nc}, p_{hc}, I) = \frac{1}{\epsilon} \left( \frac{I^{\epsilon} - r}{p_{nc}^{\epsilon}} \right) + \frac{r-1}{\epsilon} - p_{hc}(L_c)$$

- $I$-income, $L_c$-population density in $c$, $p_{hc}(L_c)$-urban cost, $\frac{\partial p_{hc}}{\partial L_c} > 0$
- Expenditures $\rightarrow$ TG: $rI^{1-\epsilon}$, NG: $I - rI^{1-\epsilon}$, $r > 0$, $0 < \epsilon < 1$
Economy setup

Labor market frictions

- Directed search \( m(v, u) = Bv^\eta u^{1-\eta}, \ q = \frac{v}{u} \)

N sector firms

- Labor is the only factor of production
  - Uniform productivity normalized to 1
- \( k_n \)-vacancy cost
  - Maximize expected profit given reservation wage \( U_c \)

T sector firms

- \( \tilde{z}(z, Z_c) \)
- \( k_t \)-vacancy cost
  - Maximize expected profit given reservation wage \( U_c(z, Z_c) \)

Assumption: \( \tilde{z}(z, Z_c) \) is \( C^2 \), \( \frac{\partial \tilde{z}(z, Z_c)}{\partial z} \), \( \frac{\partial \tilde{z}(z, Z_c)}{\partial Z_c} \) & \( \frac{\partial^2 \tilde{z}(z, Z_c)}{\partial z \partial Z_c} > 0 \)
It follows (Lemmata 1+2):

- Critical skill level, $z_\delta$, everyone with $z > z_\delta$ looks for a job in the T sector (comparative advantage)

- If $L_c < L_c'$, then $p_{nc} < p_{nc'}$ and $p_{hc} < p_{hc'}$
Two locations case (Assume $L_2 > L_1, Z_2 > Z_1$)

Sufficient conditions (Proposition 1)

(a) $\tilde{z}$ (log-supermodular, log-convex in $z$)
(b) skill distribution (non-increasing)
(c) $r$ ($r \leq 1$)

Location 2 has both:

- higher $\frac{N \text{ sector employed}}{\text{Labor force}}$
- higher $\frac{T \text{ sector employed}}{\text{Labor force}}$
Proposition 1 implies

\[
\frac{\text{% change in T's demand for } N}{\text{% change in } z} \quad \text{non-decreasing in } z, \ Z_c
\]

\[z_{max} - z_{12} > z_{12} - z_\delta\]
Wage Premium

Same conditions ensure that (Proposition 2):

\[
\frac{\int_{t_1} U_1(z, Z_1) \phi(z) \, dz}{U_1 \int_{t_1} \phi(z) \, dz} < \frac{\int_{t_2} U_2(z, Z_2) \phi(z) \, dz}{U_2 \int_{t_2} \phi(z) \, dz}
\]

Note: D&D (2019) and this paper’s skill premia comparison

- Wage premium w/o unemployment adjustment: Beneficial effects of concentration only on workers’ income
- Wage premium adjusted for unemployment: Beneficial effects of concentration both on workers’ income and matching rate
Relocation

Location 1 (LD)

- \( L'_1 = L_1 + \tau L_{t2} \)

- N demand: \( +\tau (N \text{ demand by } T \text{ in } 2) \rightarrow q'_{n1} > q^*_n, w'_{n1} > w_{n1} \rightarrow U'_1 > U'_1 \)

Location 2 (HD)

- \( L'_1 = L_2 - \tau L_{t2} \)

- N demand: \( -\tau (N \text{ demand by } T \text{ in } 2) \rightarrow q'_{n2} < q^*_n, w'_{n2} < w_{n1} \rightarrow U'_2 < U'_2 \)
Relocation: Unemployment Rate & Wage Premium

Alternative (\(B\)): Individuals considered based on residence location

Unemployment rate

- Unemployment rate in in 2 \(\uparrow\)
- Unemployment rate in in 1 \(\downarrow\) (Proposition 3)

Wage premium

- Wage premium in 2 \(\uparrow\) \(\left(\frac{(1-\tau) \int_{t,2} U_2(z,Z_2) \phi(z) \, dz}{U_2'(1-\tau) \int_{t,2} \phi(z) \, dz}\right)\)
- Wage premium in 1 \(\uparrow\) (Proposition 4)
Quantitative analysis

Data sources

- IPUMS-CPS
- US Census Bureau

Data processing

- Panel of counties
- Ranking based on average population density
- Population density weighted average of measures
Quantitative analysis

Targeted moments

- Unemployment rate in high density counties (URHD)
- Unemployment rate in low density counties (URLD)
- Tradable to non-tradable sector average wage in the high density counties (TWHD/NWHD)
- Tradable to non-tradable sector average wage in low density counties (TWLD/NWLD)
- High to low population densities ratio (HD/LD)
- High to low density average non-tradable wage ratio (NWHD/NWLD)
## Quantitative analysis

### Table: Equilibrium

<table>
<thead>
<tr>
<th>Measures</th>
<th>Data US</th>
<th>Normal, US</th>
<th>Uniform, US</th>
<th>Data PA</th>
<th>Normal, PA</th>
<th>Uniform, PA</th>
</tr>
</thead>
<tbody>
<tr>
<td>HD/LD</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>URHD</td>
<td>4%</td>
<td>4%</td>
<td>4%</td>
<td>3%</td>
<td>3%</td>
<td>3%</td>
</tr>
<tr>
<td>URLD</td>
<td>4.3%</td>
<td>4.3%</td>
<td>4.3%</td>
<td>3.7%</td>
<td>3.7%</td>
<td>3.7%</td>
</tr>
<tr>
<td>TWHD/NWHD</td>
<td>1.59</td>
<td>1.59</td>
<td>1.59</td>
<td>1.51</td>
<td>1.55</td>
<td>1.51</td>
</tr>
<tr>
<td>TWLD/NWLD</td>
<td>1.5</td>
<td>1.21</td>
<td>1.23</td>
<td>1.33</td>
<td>1.23</td>
<td>1.21</td>
</tr>
<tr>
<td>NWHD/NWLD</td>
<td>1.13</td>
<td>1.13</td>
<td>1.15</td>
<td>1.24</td>
<td>1.24</td>
<td>1.24</td>
</tr>
</tbody>
</table>

*calibrated parameter set*
Quantitative analysis

Table: Model relocation moments

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>URLD</td>
<td>8.4%</td>
<td>8.4%</td>
<td>8.4%</td>
</tr>
<tr>
<td>URHD</td>
<td>10%</td>
<td>10%</td>
<td>10%</td>
</tr>
</tbody>
</table>

Table: Model relocation parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Uniform, US</th>
<th>Normal, US</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau$</td>
<td>6.7%</td>
<td>6.6%</td>
</tr>
<tr>
<td>$B$</td>
<td>1.66</td>
<td>1.34</td>
</tr>
</tbody>
</table>
Conclusions

1. Facts:
   - Documents higher T wage premium and lower unemployment rate (UR) in HD counties. Atypical reversal in UR during 2020

2. Develop a framework that endogenously explains these patterns and starts from symmetric fundamentals. Some building blocks:
   - Complementarity between $z$ and $Z_c$
   - Skill-elasticity of production non-decreasing in $z$, $Z_c$
   - Non-homothetic preferences (higher income-larger N good’s income share)

3. Provides a first look to potential economic rearrangements following post-covid relocation and living patterns
Unemployment rate difference between the most and least densely populated counties in US for the years 2005-2020.
Table: Parameters (Equilibrium)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Normal, US</th>
<th>Uniform, US</th>
<th>Normal, PA</th>
<th>Uniform, PA</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>1.4</td>
<td>1.75</td>
<td>1.4</td>
<td>1.75</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.38</td>
<td>0.35</td>
<td>0.38</td>
<td>0.35</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.72</td>
<td>0.72</td>
<td>0.72</td>
<td>0.72</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>$k_t$</td>
<td>0.54</td>
<td>1.2</td>
<td>1.08</td>
<td>1.2</td>
</tr>
<tr>
<td>$k_n$</td>
<td>0.89</td>
<td>2.2</td>
<td>2.24</td>
<td>2.3</td>
</tr>
<tr>
<td>$A$</td>
<td>1.29</td>
<td>1.96</td>
<td>2.35</td>
<td>2</td>
</tr>
<tr>
<td>$\theta L\gamma$</td>
<td>1.16</td>
<td>0.98</td>
<td>0.56</td>
<td>0.36</td>
</tr>
<tr>
<td>$r$</td>
<td>0.35</td>
<td>0.5</td>
<td>0.35</td>
<td>0.52</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.1</td>
<td>0.1</td>
<td>0.13</td>
<td>0.08</td>
</tr>
</tbody>
</table>
Normal, US, URLD indifference curves

Back to Quantitative analysis
Normal, US, URHD indifference curves
N sector

\[
\max_{q_{nc}, w_{nc}} B q_{nc}^{\eta - 1} (p_{nc} - w_{nc}) - k_n \quad \text{s.t.} \quad U_c = B q_{nc}^\eta w_{nc}
\]

Result: \( q_{nc}^* = \left( \frac{U_c}{B w_{nc}} \right)^{\frac{1}{\eta}}, w_{nc} = (1 - \eta) p_{nc} \)

T sector

\[
\max_{q_{tc}, w_{tc}} B q_{tc} (z, Z_c)^{\eta - 1} (\tilde{z} (z, Z_c) - w_{tc} (z, Z_c)) - k_t
\]

s.t. \( U_c (z, Z_c) = B q_{tc}^\eta (z, Z_c) w_{tc} (z, Z_c) \)

Result: \( q_{tc}^* (z, Z_c) = \left( \frac{U_c(z,Z_c)}{B w_{tc}(z,Z_c)} \right)^{\frac{1}{\eta}}, w_{t,c} (z, Z_c) = (1 - \eta) \tilde{z}(z, Z_c) \)
\[ Z_c = \left(1 - e^{-\nu L \int_{t,c} (1-\beta) \phi(z) \, dz}\right) \int_{t,c} \frac{z}{\int_{t,c} \phi(z) \, dz} \phi(z) \, dz \]

\(Z_c\) from Davis & Dingel (2019), 0 < \(\beta\) < 1, \(\nu > 0\)

Back to Setup
Fictitious example:

in 2: \( z = 7 \rightarrow \text{demand for } N = 4, \ z = 7.7 \rightarrow \text{demand for } N = 4.4 \)
in 1: \( z = 5 \rightarrow \text{demand for } N = 3, \ z = 5.5 \rightarrow \text{demand for } N = 3.15 \)

Back to Proposition 1
Uniform, US, URLD indifference curves
Uniform, US, URHD indifference curves
In equilibrium, firms maximize their expected profits and agents make the optimal consumption, occupational and locational choices. Moreover, the following conditions are true:

- Adding-up constraints for location populations (4), (5)
- Zero profit conditions (6), (7)
- Labor market clearing (8)
- Non-tradable goods market clearing (9)
- Skill type indifferent between the two sectors (10)
- Skill type indifferent between tradable sector in \( c \) and \( c' \) where \( c' \) is the next, population-wise, denser location than \( c \) (11)
\[ L_c = L_{tc} + L_{nc} \text{ where } L_{tc} = L \int_{t,c} \phi(z) \, dz , \, L_{nc} = L \int_{n,c} \phi(z) \, dz \] (4)

\[ L = \sum_{c=1}^{C} L_c \] (5)

\[ U_c(z, Z_c) = \left( \frac{\eta}{k_t} \right)^{\frac{n}{1-\eta}} B^{\frac{1}{1-\eta}} (1 - \eta) \tilde{z}(z, Z_c) \] (6)

\[ U_c = \left( \frac{\eta}{k_n} \right)^{\frac{n}{1-\eta}} B^{\frac{1}{1-\eta}} (1 - \eta) p_{nc}^{\frac{1}{1-\eta}} \] (7)

\[ L \int_{t,c} q_{tc}^*(z, Z_c) \phi(z) \, dz = Q_{tc} \text{ and } L \int_{n,c} q_{nc}^* \phi(z) \, dz = Q_{nc} \] (8)

\[ U_c \int_{n,c} \phi(z) \, dz = U_c \int_{n,c} \phi(z) \, dz + \int_{t,c} U_c(z, Z_c) \phi(z) \, dz - r \left( U_c^{1-\epsilon} \int_{n,c} \phi(z) \, dz + \int_{t,c} U_c(z, Z_c)^{1-\epsilon} \phi(z) \, dz \right) \] (9)

\[ U_c(z, \delta, Z_c) = U_c \] (10)

\[ \frac{1}{\epsilon} \frac{(U_c^e(z_{cc'}, Z_c) - r)}{p_{nc}^e} - p_{hc} = \frac{1}{\epsilon} \frac{(U_{c'}^e(z_{cc'}, Z_{c'}) - r)}{p_{nc'}^e} - p_{hc'} \] (11)

Back to Equilibrium
Quantitative analysis

\[ p_{hc} = \theta L_c^\gamma \]

\( \tilde{z} \) form from Davis & Dingel (2019):

\[ \tilde{z}(z_n, Z_c) = \beta z_n + (1 - \beta) A z_n^2 Z_c \]

Distribution:
Unemployment rate

Alternative (A): Individuals considered based on job location

- Only N job finding rates change
- Unemployment rate in 1 ↓ and in 2 ↑

Wage premium

Alternative (A): Individuals considered based on job location

- Job finding rate and wage change only for N
- Wage premium in 1 ↓ and in 2 ↑ \[ \left( \frac{\int_t^c U_c(z, Z_c) \phi(z) \, dz}{U'_c \int_t^c \phi(z) \, dz} \right) \]
$z \in \Theta_{ntc}, z' \in \Theta_{tc}, z'' \in \Theta_{ntc}', z < z' < z''$, 

$$
\frac{1}{\epsilon} \frac{(U_c^e - r)}{p_{nc}^e} - p_{hc} = \frac{1}{\epsilon} \frac{(U_{c'}^e - r)}{p_{nc'}^e} - p_{hc'}
$$

At the same time, for $z'$ that is looking for a job in the tradable sector in $c$, it holds that:

$$
\frac{1}{\epsilon} \frac{(U_c^e (z', Z_c) - r)}{p_{nc}^e} - p_{hc} \geq \frac{1}{\epsilon} \frac{(U_c^e - r)}{p_{nc}^e} - p_{hc}
$$

Since $z'' > z'$, given Assumption 1,

$$
\frac{1}{\epsilon} \frac{(U_c^e (z'', Z_c) - r)}{p_{nc}^e} - p_{hc} > \frac{1}{\epsilon} \frac{(U_c^e (z', Z_c) - r)}{p_{nc}^e} - p_{hc}
$$

and $z''$ would have a higher utility as tradable sector job seeker in location $c$ than as a non-tradable sector job seeker in location $c'$. Hence, it cannot be the case that any non-tradable sector job seeker has higher ability than any tradable sector job seeker in the economy.

Back to [Equilibrium]
Let us assume that $L_c < L_c'$, and, hence, $p_{hc} < p_{hc}'$, but that $p_{nc} > p_{nc}'$. Then, in equilibrium, from the utility equalization among non-tradable sector workers and from (8), it should hold that:

$$\frac{(U_c^\varepsilon - r)}{p_{nc}^\varepsilon} < \frac{(U_{c'}^\varepsilon - r)}{p_{nc'}^\varepsilon} \Rightarrow$$

$$\left(\left(\frac{\eta}{k_n}\right)^{\frac{\eta}{1-\eta}} B^{\frac{1}{1-\eta}} (1 - \eta)p_{nc}^{\frac{1}{1-\eta}}\right)^\varepsilon - r \left(\left(\frac{\eta}{k_n}\right)^{\frac{\eta}{1-\eta}} B^{\frac{1}{1-\eta}} (1 - \eta)p_{nc'}^{\frac{1}{1-\eta}}\right)^\varepsilon - r \Rightarrow$$

$$\left(\left(\frac{\eta}{k_n}\right)^{\frac{\eta}{1-\eta}} B^{\frac{1}{1-\eta}} (1 - \eta)\right)^\varepsilon p_{nc}^{\varepsilon} - \frac{r}{p_{nc}^\varepsilon} < \left(\left(\frac{\eta}{k_n}\right)^{\frac{\eta}{1-\eta}} B^{\frac{1}{1-\eta}} (1 - \eta)\right)^\varepsilon p_{nc'}^{\varepsilon} - \frac{r}{p_{nc'}^\varepsilon} \Rightarrow$$

$$\left(\left(\frac{\eta}{k_n}\right)^{\frac{\eta}{1-\eta}} B^{\frac{1}{1-\eta}} (1 - \eta)\right)^\varepsilon p_{nc}^{\varepsilon} - \left(\left(\frac{\eta}{k_n}\right)^{\frac{\eta}{1-\eta}} B^{\frac{1}{1-\eta}} (1 - \eta)\right)^\varepsilon p_{nc'}^{\varepsilon} < \frac{r}{p_{nc}^\varepsilon} - \frac{r}{p_{nc'}^\varepsilon}$$

Since we assumed that $p_{nc'}^{\varepsilon} < p_{nc}^{\varepsilon}$, the inequality cannot hold.

Back to Equilibrium
Unemployment rate difference between the most and least densely populated counties in US for the years 2005-2020

Back to Motivation
Tradable sector wage premium difference between the most and least densely populated counties in US for the years 2005-2020
**Sufficient condition** (Proposition 3)

(a) \( \frac{n}{1-\epsilon} \geq 1 \)

- Could be problematic if very low elasticity of N’s quantity produced with respect to the N expenditure of a location’s T residents

Back to Relocation unemployment rate