Unpacking the Birth Order Effects

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Family size is widely believed to be an important determinant of child quality.
Introduction

- Family size is widely believed to be an important determinant of child quality.
- However, researchers have documented considerable variation within families.

In a seminal paper, Black et al. (2005) find a negative effect of family size on labor market and education outcomes using exogenous variation in family size induced by twin births. BUT once birth order is introduced this effect becomes negligible. This implies that family size affects marginal children through birth order effects.
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Birth order effects

- Follow-up on birth order effects: cognitive (Booth & Kee, 2009; Conley & Glauber, 2006), non-cognitive skills (Black et al., 2018) and criminal behavior (Breining et al., 2020).
- Estimates of birth order effects vary and are sometimes positive.
- Evidence suggests that their direction/magnitude are context-specific.
  - Ecuador (De Haan et al., 2014): First-born receive less education and more likely to participate in child labor; completely driven by low-income families.
  - Philippines (Ejrnæs and Pörtner, 2004): Positive birth order effects, which are especially pronounced in low-educated families.
Sibling spillover effects

- Separate literature finds evidence of sibling spillovers for a number of educational outcomes.
- Studies on schooling achievement find a positive spillover effect (Gurantz et al., 2020; Nicoletti & Rabe, 2018; Qureshi, 2018).
- Studies on choice of college institution and major provide mixed evidence (Altmejd et al., 2021; Dahl et al., 2020).
Main findings

1. We find negative birth order effects.
2. We find a negatively signed spillover effect which implies:
   - Birth order effects are overstated.
   - The multiplier effect arising from the sibling interaction process is \( (i) \) less pronounced compared to the case where the spillover effects are positive and \( (ii) \) dissipates faster as the family size increases.

Our findings suggest that an intervention that improves the outcomes of some of the siblings will be less effective in larger families.
Econometric Model

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We first consider a \textbf{linear-in-means} model with \textbf{endogenous effects}. For each sibling $i = 1, \ldots, m_g$ in household $g = 1, \ldots, N$ of size $m_g = 2, \ldots, M$

$$y_{ig} = x_{1, ig}^t \beta_1 + x_{2, ig}^t \beta_{2,m_g} + \frac{\gamma}{m_g - 1} \sum_{j \neq i} y_{jg} + \eta_g + \varepsilon_{ig}$$

- $y_{ig}$: Education in years.

\textbf{Endogenous Effect}
We first consider a **linear-in-means** model with **endogenous effects**. For each sibling $i = 1, ..., m_g$ in household $g = 1, ..., N$ of size $m_g = 2, ..., M$

$$y_{ig} = x'_{1,ig} \beta_1 + x'_{2,ig} \beta_{2,m_g} + \frac{\gamma}{m_g - 1} \sum_{j \neq i} y_{jg} + \eta_g + \varepsilon_{ig}$$

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- $\eta_g$: Household fixed effect.
- $\varepsilon_{ig}$: Error term
Econometric Model

In matrix form

\[ Y_g = X_{1,g} \beta_1 + X_{2,g} \beta_{2,m} + \gamma W_{m_g} Y_g + 1_{m_g} \eta_g + \varepsilon_g \]

where the interaction matrix is

\[ W_{m_g} = \frac{1}{m_g - 1} \left( 1_{m_g} 1_{m_g}' - I_{m_g} \right) \]

where \( 1_{m_g} \) an \( m_g \)-vector of 1s and \( I_{m_g} \) the identity matrix.

Identification & Estimation Challenges

1. \( W_{m_g} Y_g \) is endogenous w.r.t. \( \eta_g \).
In matrix from

\[ Y_g = X_{1,g} \beta_1 + X_{2,g} \beta_{2,m_g} + \gamma W_{m_g} Y_g + 1_{m_g} \eta_g + \varepsilon_g \]

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**Identification & Estimation Challenges**

(2) Household size \( m_g \) possibly correlated with \( \eta_g \).
In matrix form

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Identification & Estimation Challenges

(3) Left-multiplying by the within operator \( Q_{m_g} : Q_{m_g} 1_{m_g} = 0 \) to eliminate
the household FE induces correlation between \( Q_{m_g} W_{m_g} Y_g \) and \( Q_{m_g} \varepsilon_g \).
Assumptions

We assume that

$$\mathbb{E}[\varepsilon_{ig} \mid X_g, W_{mg}, m_g, \eta_g] = 0 \text{ for all } i, g$$
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- Household and individual characteristics and the interaction matrix, $W_{mg}$, are exogenous relative to the outcome $y$. 
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- Not so restrictive: $(X_g, W_{mg}, m_g)$ can be related to the household FE.
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- Not so restrictive: $(X_g, W_{mg}, m_g)$ can be related to the household FE.
- Allows us to assess the total effect of a change in one of the model’s exogenous regressors.
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- Household and individual characteristics and the interaction matrix, $W_{mg}$, are exogenous relative to the outcome $y$.
- Not so restrictive: $(X_g, W_{mg}, m_g)$ can be related to the household FE.
- Allows us to assess the total effect of a change in one of the model’s exogenous regressors.
- **Note:** The identification analysis will not assume that we observe all siblings within a household. No selectivity bias is additionally assumed.
Provided that $\gamma \neq 1 - m_g$ and $\gamma \neq 1$ we get:

$$Y_g = (I_{mg} - \gamma W_{mg})^{-1}(X_{1,g} \beta_1 + X_{2,g} \beta_{2,m} + 1_{mg} \eta_g + \varepsilon_g)$$

where

$$(I_{mg} - \gamma W_{mg})^{-1} = \frac{1}{1 + \gamma / (m_g - 1)} \left[ I_{mg} + \frac{\gamma}{(1 - \gamma) (m_g - 1)} 1_{mg} 1_{mg}' \right]$$

Applying the within transformation:

$$Q_{mg} Y_g = Q_{mg} X_{1,g} \frac{\beta_1}{1 + \gamma / (m_g - 1)}$$

$$+ Q_{mg} X_{2,g} \frac{\beta_{2,m} g}{1 + \gamma / (m_g - 1)} + Q_{mg} \frac{\varepsilon_g}{1 + \gamma / (m_g - 1)}$$

Recovering $\left(\beta_1, \beta_{2,m}, \gamma\right)$ thus analogous to identifying the parameters of time-invariant covariates in panel fixed effects regression (see also Graham and Hahn, 2005).
Identification

To separately identify $\left( \beta_1, \beta_2, m_g, \gamma \right)$ we rely on group size variation (Lee 2007, Davezies et al. 2009).
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The reduced form parameter

\[
\frac{\beta_1}{1 + \gamma/(m_g - 1)}
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- As long as \(\beta_1 \neq 0\) and at least two group sizes are observed, \((\beta_1, \gamma)\) are identified.
Identification

To separately identify \((β_1, β_2, m_g, γ)\) we rely on group size variation (Lee 2007, Davezies et al. 2009).

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- Is identified under standard assumptions.
- Is an 1-to-1 function of \(m_g\).
- As long as \(β_1 \neq 0\) and at least two group sizes are observed, \((β_1, γ)\) are identified.
- Knowledge of \(γ\) allows us to recover the remaining parameters, \(β_2, m_g\).
We next consider a model with both \textit{endogenous} and \textit{contextual} effects:

\[
Y_g = X_{1a,g} \beta_1 + X_{2,g} \beta_{2,m} + X_{1b,g} \beta_3 + W_{mg} X_{1b,g} \beta_4 + \gamma W_{mg} Y_g + l_{mg} \eta_g + \varepsilon_g
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We next consider a model with both endogenous and contextual effects:

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Contextual Effects
We next consider a model with both endogenous and contextual effects:

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- **Contextual effects**: Individuals are also affected by exogenous characteristics of their reference group.
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- **Contextual effects**: Individuals are also affected by exogenous characteristics of their reference group.
- **Additional identification challenge**: The reflection problem (Manski, 1993). Hard to distinguish between endogenous and contextual effects.
Estimating Equation: Reduced Form + within-transformation

\[
Q_{mg} Y_g = Q_{mg} X_{1a,g} \frac{\beta_1}{1 + \gamma/(m_g - 1)} + Q_{mg} X_{2,g} \frac{\beta_{2,m_g}}{1 + \gamma/(m_g - 1)}
\]

\[+ Q_{mg} X_{1b,g} \frac{(m_g - 1) \beta_3 - \beta_4}{m_g - 1 + \gamma} + Q_{mg} \frac{\varepsilon_g}{1 + \gamma/(m_g - 1)} \]
Estimating Equation: Reduced Form + within-transformation

\[ Q_{m_g} Y_g = Q_{m_g} X_{1a,g} \frac{\beta_1}{1 + \gamma/(m_g - 1)} + Q_{m_g} X_{2,g} \frac{\beta_{2,m_g}}{1 + \gamma/(m_g - 1)} \]

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Reduced form parameter
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\]

- \(X_{1a,g}\) non-empty: Birth order is mutually exclusive and hence does not generate contextual effects.
Estimating Equation: Reduced Form + within-transformation

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- \( X_{1a,g} \) non-empty: Birth order is mutually exclusive and hence does **not** generate contextual effects.
- The identification analysis thus remains unaltered.
**Econometric Model**

**Estimating Equation:** Reduced Form + within-transformation

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\[ + Q_{mg} X_{1b,g} \frac{(m_g - 1)\beta_3 - \beta_4}{m_g - 1 + \gamma} + Q_{mg} \frac{\varepsilon_g}{1 + \gamma/(m_g - 1)} \]

- **X_{1a,g}** non-empty: Birth order is mutually exclusive and hence does not generate contextual effects.
- The identification analysis thus remains unaltered.
- **Note:** \((\beta_3, \beta_4)\) cannot vary by group sizes.
# Unpacking the Birth Order Effects

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Marginal Effects

We assume that $\min_g (m_g) < \gamma < 1$. Let $h, g$ be given by

$$h(\gamma, m) = 1 - \frac{\gamma^2}{(\gamma + m - 1)(\gamma - 1)}$$

$$f(\gamma, m) = -\frac{\gamma}{(\gamma - 1)(\gamma + m - 1)}$$

Then, for those regressors that generate individual effects only

$$\frac{\partial}{\partial x_{1a,ig}} \mathbb{E}[y_{ig} \mid X_g, W_{m_g}, m_g, \eta_g] = \beta_1 h(\gamma, m_g)$$
Marginal Effects

We assume that \( \min_g (m_g) < \gamma < 1 \). Let \( h, g \) be given by

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\begin{align*}
    h(\gamma, m) &= 1 - \frac{\gamma^2}{(\gamma + m - 1)(\gamma - 1)} \\
    f(\gamma, m) &= -\frac{\gamma}{(\gamma - 1)(\gamma + m - 1)}
\end{align*}
\]

Then, the marginal effect of \( x_{1a} \) is

\[
\frac{\partial}{\partial x_{1a,ig}} \mathbb{E}[y_{ig} \mid X_g, W_{mg}, m_g, \eta_g] = \beta_1 h(\gamma, m_g)
\]

- The total effect is a nonlinear function of direct (\( \beta_1 \)) and indirect (\( \gamma \)) effects.
Marginal Effects

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- The total effect is a nonlinear function of direct ($\beta_1$) and indirect ($\gamma$) effects.
- For fixed $m$, $h$ is a convex function of $\gamma$ that is minimized at $\gamma = 0$ (multiplier effect).
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- The total effect is a nonlinear function of direct ($\beta_1$) and indirect ($\gamma$) effects.
- For fixed $m$, $h$ is a convex function of $\gamma$ that is minimized at $\gamma = 0$ (multiplier effect).
- The effect dissipates as group sizes become larger.
Interpretation of Parameters

Marginal Effects

\[ y = \gamma h(\gamma) \]

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Marginal Effects

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Then, the marginal effect of $x_{1b}$ is

$$\frac{\partial}{\partial x_{1b, ig}} \mathbb{E}[y_{ig} \mid X_g, W_{mg}, m_g, \eta_g] = \beta_2 h(\gamma, m_g) + \beta_3 f(\gamma, m_g)$$
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\end{align*}
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$$

- For fixed $m$, $f > 0 (< 0)$ if $\gamma > 0 (< 0)$. 

Marginal Effects

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Then, the marginal effect of $x_{1b}$ is

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- For fixed $m$, $f > 0 (< 0)$ if $\gamma > 0 (< 0)$.
- This additional term might inflate, zero out, or even reverse the direction of the total effect depending on the values of the parameters $(\gamma, \beta_2, \beta_3)$. 
Marginal Effects
## Germany: Reduced Form & Structural parameters

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<th>Fixed-effects (by size)</th>
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</tr>
<tr>
<td><strong>N</strong></td>
<td>3157</td>
<td>3260</td>
<td>1694</td>
<td>1059</td>
</tr>
</tbody>
</table>

**Notes:** Standard errors (in parentheses) allow for correlation of errors within households. All regressions include indicators for year of birth and sex. The specification estimated by OLS additionally includes indicators for sibship size and parents education.
## Germany: Marginal Effects with Endogenous Effects

<table>
<thead>
<tr>
<th>Household Size</th>
<th>Structural Parameters</th>
<th>Marginal Effects</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\gamma$</td>
<td>$m_g = 2$</td>
</tr>
<tr>
<td></td>
<td>-0.599***</td>
<td>-0.34***</td>
</tr>
<tr>
<td></td>
<td>(0.176)</td>
<td>(0.071)</td>
</tr>
<tr>
<td>Second child</td>
<td>-0.204**</td>
<td>-0.496***</td>
</tr>
<tr>
<td></td>
<td>(0.084)</td>
<td>(0.138)</td>
</tr>
<tr>
<td>Third child</td>
<td>-0.419***</td>
<td>-0.697***</td>
</tr>
<tr>
<td></td>
<td>(0.131)</td>
<td>(0.231)</td>
</tr>
<tr>
<td>Fourth child</td>
<td>-0.657***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.219)</td>
<td></td>
</tr>
<tr>
<td>J-test</td>
<td>15.33 [0.88]</td>
<td></td>
</tr>
<tr>
<td>$N$</td>
<td>3260</td>
<td>1694</td>
</tr>
</tbody>
</table>
## Egypt: Marginal Effects with Endogenous Effects

<table>
<thead>
<tr>
<th>Household Size</th>
<th>Structural Parameters</th>
<th>Marginal Effects</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\gamma$</td>
<td>$m_g = 2$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Second child</td>
<td>-0.558**</td>
<td>-0.395***</td>
</tr>
<tr>
<td></td>
<td>(0.233)</td>
<td>(0.105)</td>
</tr>
<tr>
<td>Third child</td>
<td>-0.384**</td>
<td>-0.438**</td>
</tr>
<tr>
<td></td>
<td>(0.177)</td>
<td>(0.177)</td>
</tr>
<tr>
<td>Fourth child</td>
<td>-0.526**</td>
<td>-0.569**</td>
</tr>
<tr>
<td></td>
<td>(0.267)</td>
<td>(0.275)</td>
</tr>
<tr>
<td>Fifth child</td>
<td>-0.645</td>
<td>-0.683</td>
</tr>
<tr>
<td></td>
<td>(0.427)</td>
<td>(0.441)</td>
</tr>
<tr>
<td>J-test</td>
<td>20.81 [0.11]</td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>3196</td>
<td>536</td>
</tr>
</tbody>
</table>
## Germany: Marginal Effects with Endogenous & Contextual Effects

<table>
<thead>
<tr>
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<th>Structural Parameters</th>
<th>Marginal Effects</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$m_g = 2$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>-0.664*** (0.158)</td>
<td>-0.327*** (0.07)</td>
</tr>
<tr>
<td>Second child</td>
<td>-0.183** (0.083)</td>
<td></td>
</tr>
<tr>
<td>Third child</td>
<td>-0.377*** (0.129)</td>
<td>-0.451*** (0.131)</td>
</tr>
<tr>
<td>Fourth child</td>
<td>-0.59*** (0.210)</td>
<td></td>
</tr>
</tbody>
</table>

**Individual Effects**

| Sex            | 0.248 (0.161)       | 0.198** (0.101) | 0.236* (0.142) | 0.241 (0.15) |

**Contextual Effects**

| Sex            | 0.207 (0.169)       | 0.075 (0.134)   | 0.019 (0.036)  | 0.011 (0.021) |

| N              | 3260                | 1694             | 1059            | 507           |
Conclusions

- We employ an econometric model that allows us to study birth order effects on educational attainment accounting for sibling interactions.
- Birth order effects are found to be negative in both Germany and Egypt.
- Failing to account for sibling interactions leads to an overstatement of the birth order effects.
- The parameter characterizing sibling interactions is found to be negative implying, among other things, more pronounced family size effects.