Optimal Tariffs with Downstream Uncertainty and Fixed Development Cost

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❖ Introduction

- ➢ There are instances where upstream firms support financially their downstream clients. For example:

  - ▪ a franchising company may provide its franchisee with part of the fixed equipment necessary for the downstream operation.
    - This support may take the form of providing part of the installation cost, local advertising, or other necessary costs for the downstream activity

  - ▪ a multinational company may offer free technical assistance to its independent retailers.

  - ▪ The upstream firm's assistance to its downstream partner is often justified on the basis of better knowledge or economies of scale

- ➢ Uncertainty provides a new argument for upstream cost – assistance:
  - ▪ While the expected profitability of a given market may be positive, risk-aversion may prevent the downstream firm from undertaking the job
  - ▪ By reducing the burden of downstream investment the upstream firm may induce downstream participation and reap net profit

- *The risk-sharing incentive for subsidizing part of the downstream cost may be present even if the upstream firm is itself risk-averse and even more risk-averse than its downstream partner.*

- *Prima facie* the upstream assistance seems to be welfare improving since it allows the survival of markets and provision of products that would otherwise be extinct.

  - ***Is it really so?*** **Our work investigates the validity of the above argument.**

- Subsidization
  - We show that when the fixed cost is sufficiently high, the optimal TPT contains a **negative payment** (subsidy) that covers only part (***never the entirety***) of downstream fixed cost.
  - In exchange, *U* sells the input with positive price-cost margins

- Thus, our question boils down to comparing the efficiency of a Two-part tariff (TPT)—i.e., a pricing system with subsidization—to that of a Linear tariff (no subsidization) under uncertainty.

❖ **Main Results:**

- When the fixed cost is very high, subsidization saves the market (the argument is correct).

However,

- ***For intermediate (high but not so high) levels of the fixed cost, subsidization conceals an effort to extract higher profit at the expense of consumer surplus, and results to lower total welfare.***

- **Short look at previous works:**
  - Rey & Tirole (1986) points out that a higher *w* reduces final downstream profit variance, thus explaining why setting the input price *w* above the input’s marginal production cost may represent a form of insurance that *U* offers to *D*. Even the input price under TPT contains some price-cost margin.
  - Lomo (2020) also shows that under uncertainty, the optimal two-part tariff (TPT) includes an input price that is above the input’s marginal production cost.
  - None of the above deals explicitly with fixed costs.

❖ The setting

- ***Bilateral monopoly*** in a vertical chain: *U* sells an input to *D* who sells the final product to consumers

- ***Uncertainty***. The demand for the final product is uncertain: with probability *z* there is a positive demand sufficient to generate "satisfactory" returns to both *U, D,* whereas with probability $(1-z)$ quantity demanded is zero at any price.
  - By "satisfactory" we mean beneficial despite potential risk-aversion of one or both parties.

Risk aversion. Both *U, D,* can be risk-averse receiving **a mean-variance utility** from the project, where $\hat{R}_i$ represents the expectation and $\sigma^2_{R_i}$ the variance of the uncertain part in each party's proceeds.

$$u_i = \hat{R}_i - \lambda_i \sigma^2_{R_i}, \quad i = U, D,$$

Letting $q, \pi,$ represent the optimal quantity and the resulting profit, respectively, we have

$$u_D = z\pi - \lambda_D \sigma^2_{\pi} - F - T,$$
$$u_U = zq - \lambda_U \sigma^2_q + T$$

- for notational simplicity we assume:
  - $\lambda_D = \lambda$ and $\lambda_U = r\lambda$, with $r > 0$
    - ♦ $r = 0$ implies that *U* is risk-neutral
    - ♦ Values of $r < (>) 1$ imply that *U* is less (more) risk-averse than *D.*

- **Production**
  - The input is produced at zero cost and sold at price w to D
  - The production of the final product requires
    - A fixed cost F that must be paid before the state of demand is known
    - No other variable cost, except for the input price. Each unit of output requires a unit of input, hence the variable cost for D is w.

- **Input Pricing**
  - U makes take-it-or-leave-it offers that can be either
    - A two-part-tariff contract (TPT) specifying a fixed transfer T and an input price w
    - A linear contract (L) specifying an input price w.

- **Timing**
  - U makes a take-or-leave-it offer, determining w and T in case of TPT
  - If D accepts, pays the fixed cost F and the fixed transfer T in case of TPT
  - The state of demand is revealed
  - D decides the quantity to be sold and buys the necessary input at the predetermined price w
  - Sales are made and the proceeds go to D.

❖ TPT Equilibrium

➢ Downstream Equilibrium

- The downstream firm chooses q to maximize gross operating profits:

$$\max_{q} \pi = [p(q) - w]q \tag{1}$$

- The first- and second-order conditions are

$$w = p + qp' \quad \text{and} \quad 2p' + qp'' \tag{2}$$

  - For future reference, define the derived demand elasticity as

$$\varepsilon^d = \frac{dq(w)}{dw}\frac{w}{q(w)} < 0 \tag{3}$$

- Letting $p(q(w)) \equiv p(w)$, firm D's operating profit is $\pi_D(w) = (p(w) - w)q(w)$. Expected profit is thus

$$\hat{\pi}_D(w) = z\pi_D(w) = z(p(w) - w)q(w) \tag{4}$$

  and **downstream profit variance** is

$$\sigma_D^2(w) = z(1 - z)[(p(w) - w)q(w)]^2. \tag{5}$$

- **Downstream utility** is therefore

$$v_D(w) = (\hat{\pi}_D(w) - F) - T - \lambda\sigma_D^2(w) \tag{6}$$

- Hence, **downstream participation** requires that

$$T \leq z(p(w) - w)q(w)[1 - \lambda(1 - z)(p(w) - w)q(w)] - F \qquad (6a)$$

- Before closing this section, notice that downstream returns are decreasing in w and their variance is also decreasing in w, which may make the overall impact of w on u to be either positive or negative. We assume that firm D is always worse-off by a *ceteris paribus* increase in w, *i.e.*

$$\frac{du_D}{dw} < 0$$

  which can be shown to hold iff

$$\lambda < \bar{\lambda}(w; z) \equiv \frac{1}{2(1 - z)(p(w) - w)q(w)} > 0 \qquad (dPer\text{ condition})$$

- **Upstream equilibrium**
  - U's target is to maximize:

$$\max_{w,T} v_U(w) = \hat{\pi}_U(w) + T - r\lambda\sigma_U^2(w), \qquad s.t.\ v_D(w) \geq 0.$$

    where $\pi_U(w)$ is the risky part in *U*'s profit, with expectation

$$\hat{\pi}_U(w) = zwq(w) \tag{7}$$

    and variance

$$\sigma_U^2(w) = z(1-z)[wq(w)]^2. \tag{8}$$

  - Treating the downstream firm's participation constraint as an equality, i.e., $v_D(w) = 0$, we obtain

$$\max_{w} v_U(w) = \hat{\pi}_U(w) + \hat{\pi}_D(w) - F - \lambda\sigma_D^2(w) - r\lambda\sigma_U^2(w) \tag{9}$$

- The first-order condition of the above maximization problem is:

$$\frac{dq(w)}{dw}w + 2\lambda(1-z)q(w)\left[(p(w)-w)q(w) - wr\left(\frac{dq(w)}{dw}w + q(w)\right)\right] = 0 \qquad (10)$$

- Note that

  - for $z = 1$ and/or $\lambda = 0$, $w = 0$, *i.e.,* if success is certain and/or U and D are risk-neutral, the optimal policy for U is marginal-cost pricing.
  - if $z < 1$ and $\lambda > 0$, the second term in the LHS of (10) is no longer zero, but rather positive and the optimal upstream price is set above marginal cost.

❖ Equilibrium in the TPT case

- **Proposition 1:** *For all the admissible values of* $(\boldsymbol{\lambda}, \boldsymbol{r})$ *there exists a value of* $\boldsymbol{F} = \widetilde{\mathbf{F}}$ *such that,* $\forall \boldsymbol{F} \geq \widetilde{\boldsymbol{F}}$ *the optimal TPT implies* $\boldsymbol{T} < \mathbf{0}$*, i.e., a fixed upfront payment from U to D. The condition for* $\boldsymbol{T} < \mathbf{0}$ *is that*

$$\frac{z}{1-z}(4\lambda)^{-1}[-\varepsilon^{d}(2+\varepsilon^{d})+A] < F \tag{12}$$

with

$$A = 4\lambda(1-z)wq(w)r(1+\varepsilon^{d})^{2}[1-\lambda(1-z)wq(w)r]$$

and $\boldsymbol{\varepsilon}^{\mathbf{d}}\big(\boldsymbol{w}(\boldsymbol{\lambda}, \boldsymbol{r}, \boldsymbol{z})\big) < \mathbf{1}$ *is the elasticity of the derived demand function. When* $\mathbf{T} < \mathbf{0}$*, it is always in absolute value less than F.*

Corollary: When U is risk neutral, i.e., $r = 0$, then $A = 0$, and the condition in (12) reduces to

$$\frac{z}{1-z}(4\lambda)^{-1}[-\varepsilon^{d}(2+\varepsilon^{d})] < F \tag{12a}$$

- Illustration of Proposition 1
  - Linear demand $q = 1 - p$, and $z = 1/2$.

<table>
<tr><td>Horizontal axis: F</td><td>Light Blue area: T > 0 in all cases</td><td rowspan="2">Blue line: T = 0 when r = 0 (U is risk-neutral)

Green line: T = 0 when r = 1 (U is as risk-averse as D)

Red line: T = 0 when r = 2 (U is more risk-averse than D)</td></tr>
<tr><td>Vertical axis: λ</td><td>Light Green area: T < 0 in all cases</td></tr>
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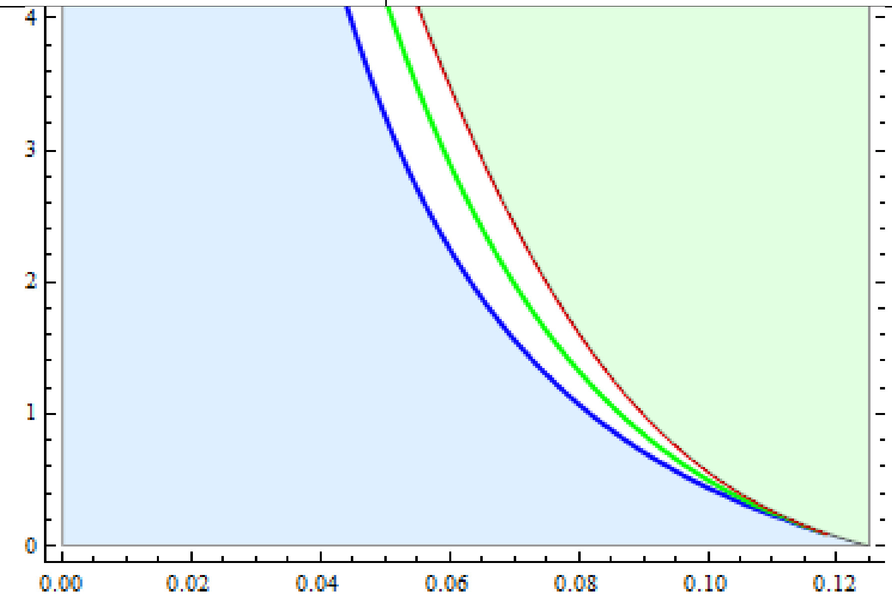


❖ Equilibrium in the L-case

➢ *Downstream Equilibrium*

- The downstream equilibrium is represented by expressions (1) – (5) and (6) for T = 0,

$$v_D(w) = (\hat{\pi}_D(w) - F) - \lambda\sigma_D^2(w)$$

➢ *Upstream Equilibrium*

- U's target is to maximize:

$$\max_{w,T} v_U(w) = \hat{\pi}_U(w) - r\lambda\sigma_U^2(w), \tag{13}$$

$$s.t.\ v_D(w) \geq 0.$$

- If the above admits an interior solution, the optimal input price is determined by

$$\frac{dv_U(w)}{dw} = \left[\frac{dq(w)}{dw}w + q(w)\right] - \frac{2\lambda(1-z)q(w)[(p(w)-w)q(w)]}{1-2r\lambda(1-z)wq(w)} = 0$$

  - If assume no upstream risk-aversion (assumed hereafter) the above becomes

$$\frac{dq(w)}{dw}w + q(w) = 0,$$

- ♦ Which implies that when the problem admits interior solution, the optimal w, w_{un}^L, is determined by:

$$\varepsilon^d(w) + 1 = 0. \qquad .$$

  - ▪ Obviously, $w_{un}^L > 0$ and $dw_{un}^L/dF = 0$.

- ▪ On the other hand, if

$$F > \breve{F} = \hat{\pi}_D(w_{un}^L) - \lambda\sigma_D^2(w_{un}^L) > 0, \tag{14}$$

then D's constraint is not satisfied at the interior solution, and the optimal w, w_{con}^L, must satisfy:

$$\hat{\pi}_D(w_{con}^L) - \lambda\sigma_D^2(w_{con}^L) = F. \tag{15}$$

- The *dPer* condition implies that the LHS of (15) is decreasing in w, thus w_{con}^L falls when F rises, $dw_{con}^L/dF < 0$.
  - ♦ Setting $w_{con}^L = 0$ imposes an upper bound on F:

$$\bar{F} \equiv zp(0)q(0) - \lambda\sigma_D^2(0). \tag{16}$$

- For $F > \bar{F}$, the market cannot be served under L. Since $\hat{\pi}_D(w) - \lambda\sigma_D^2(w)$ is decreasing in w and $w_{un}^L > 0$, $\breve{F} < \bar{F}$.
- The above are summarized in the following Proposition.

➢ **Proposition 2.** ***Under a linear tariff, the optimal input price satisfies:***

$$\begin{cases} \varepsilon^d(w_{un}^L) = -1 & if \quad 0 < F \leq \breve{F} \\ \hat{\pi}_D(w_{con}^L) - \lambda\sigma_D^2(w_{con}^L) = K & if \quad \breve{F} \leq F < \bar{F} \end{cases}$$

with $\breve{\mathbf{F}}$ and $\bar{\mathbf{F}}$ given in (14) and (16) respectively.

- When derived demand is inelastic, $\varepsilon^d(w) > -1$, upstream expected profit $zwq(w)$ increases with w, reaching its maximum at $\varepsilon^d(w) = -1$. Thus, since $w_{con}^L < w_{un}^L$, w_{con}^L belongs to the inelastic part of the derived demand.

❖ Comparison

- Comparing the 1st order conditions of the Linear and TPT problems (expressions (10) and (15) yields:

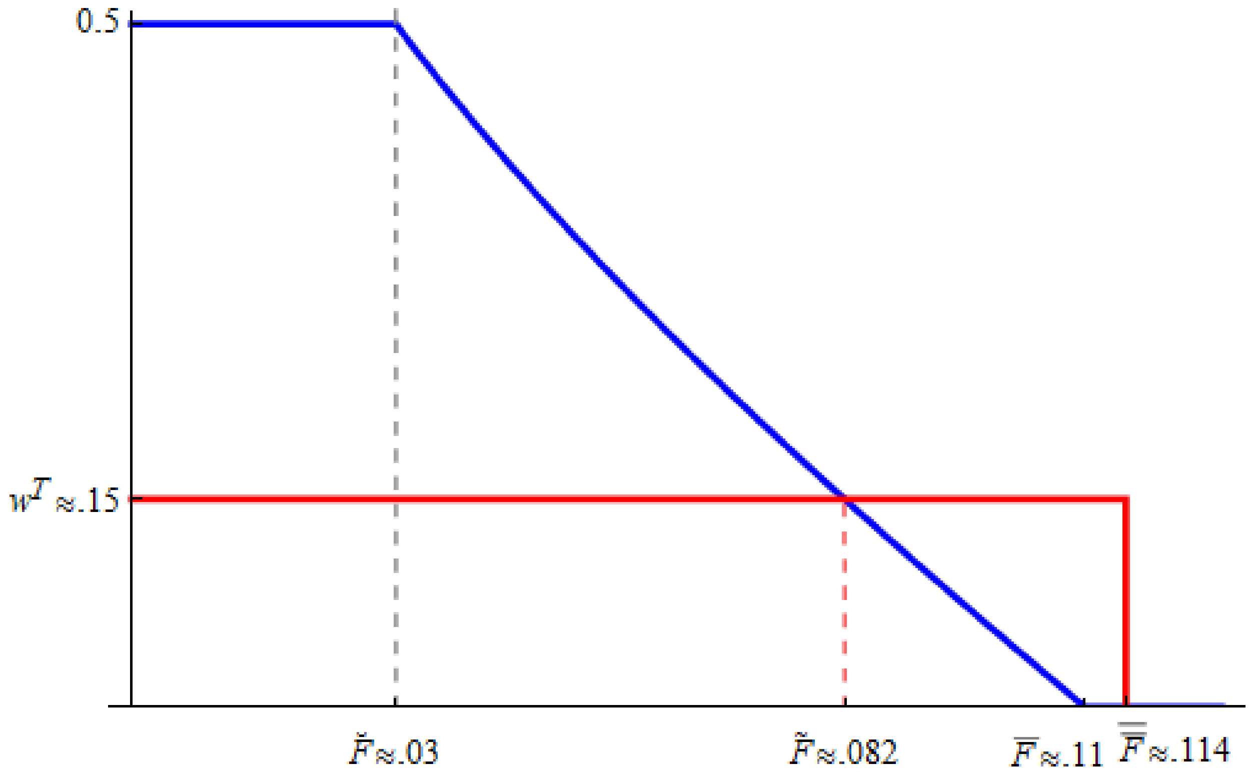
➢ **Proposition 3**: ***When*** $\boldsymbol{T < (\geq) 0}$***, then*** $\boldsymbol{w_L < (\geq) w_{TPT}}$***; the TPT always produces higher profit for U, independently of the sign of T.***

- *When the upstream firm is risk-averse, higher profit does not necessarily mean higher upstream utility. Nevertheless, we find values of* (λ, r) *with that* $\lambda < \bar{\lambda}$ *and* $r \geq 1$ *such that the upstream utility* u_D *produced from a TPT with T < 0 is higher than the corresponding utility under L.*

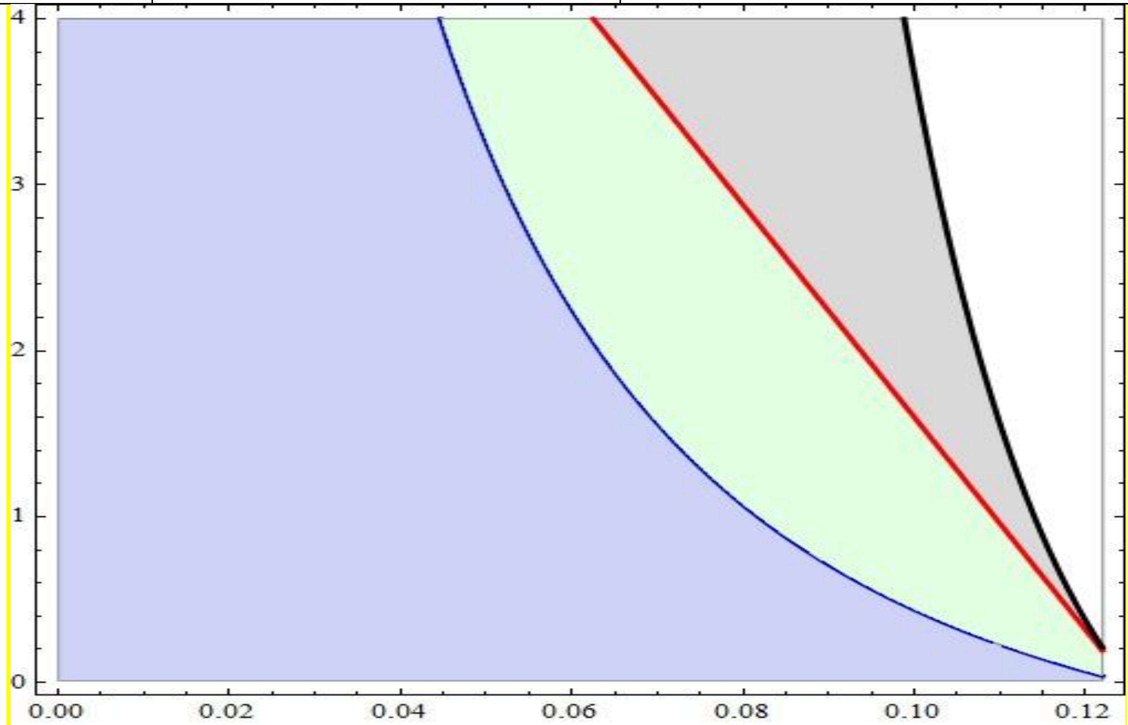
➢ **Input-price Comparison between TPT and Linear**

- Linear demand $q = 1 - p$, and $z = 1/2$.
- Risk-aversion coefficient: $\lambda = 1$

➢ **Blue line: Input price under Linear tariff** **Red line: Input price under Two-Part tariff**



Horizontal axis: *F*	**Light Blue Area:** T > 0, π, CS, and W are all higher under TPT	**Light Green Area:** T < 0, π is higher under TPT CS and W are lower under TPT
Vertical axis: *λ*	**White Area:** no product is offered	**Gray Area:** T < 0, The good is provided only if subsidization (TPT)



❖ Conclusions

- When *F* is sufficiently high, it is optimal for *U* to subsidize part of *D's* fixed cost.
  - This subsidization is part of the optimal TPT, which in this case implies **negative transfer**.
    - When negative, the transfer is never so high as to cover the entire downstream fixed cost *F*
- The benefit from offering some insurance to *D* is that *U* can keep a higher input price compared to the no-subsidization scheme (*L tariff*). Thus compared to a Linear tariff
  - a TPT creates a ***stronger*** double-marginalization distortion
  - consumers pay an *ex-post* higher price
  - profits are higher
  - total welfare is lower
- However
  - Due to subsidization the TPT scheme may still be profitable at very high levels of the fixed cost where the good could not have been provided under a Linear price (no subsidization)
- Testing numerically, the results are robust to the use of a quadratic utility function

THANKS FOR YOUR ATTENTION!