Asset pricing with and without garbage: The overlooked triple-hypothesis problem

Stefanos Delikouras, University of Miami
George M. Korniotis, University of Miami and CEPR

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Research contribution

- Empirical performance of Consumption-based Capital Asset Pricing Models (C-CAPM) is problematic
- Alternative measures of consumption have been proposed to replace the standard Bureau of Economic Analysis (BEA) aggregate U.S. consumption:
  1. ultimate consumption. Q4-Q4, garbage, unfiltered consumption, ...
  2. alternative measures imply relatively low values for the structural risk aversion parameter (e.g., equity premium puzzle in Mehra and Prescott (1985))
  3. alternative consumption measures seem to do a decent job at pricing the cross-section of Fama-French portfolios
- Testing consumption-based asset pricing models is a triple hypothesis, which, in addition to selecting consumption data, requires examining the assumptions for investor preferences and consumption dynamics
- We formalize the triple-hypothesis within a GMM framework that
  1. relaxes the CRRA assumption
  2. jointly estimates consumption dynamics with Euler equations
  3. highlights the importance of the variance of the risk-free rate as a target moment

Research contribution

This paper is directly related to

- Savov (2011, JF): Asset pricing with garbage
- Kroncke (2017, JF): Asset pricing without garbage

And is a continuation of the agenda in consumption-based asset pricing

- Delikouras (2017, RFS): Where's the kink? disappointment events in consumption growth and equilibrium asset prices
- Delikouras and Kostakis (2019, JFQA): A single factor consumption-based asset pricing model

Methodology

- Choosing the “correct" consumption measure is only one aspect of testing C-CAPMs
- Assessing the cross-sectional fit of C-CAPM is a triple hypothesis:

  Investor Preferences × Consumption Dynamics × Consumption Measures

Methodology

Investor Preferences $\times$ Consumption Dynamics $\times$ Consumption Measures

- Relax the CRRA assumption by estimating the Epstein and Zin (1989) model
  - disentangle time preferences from risk aversion
  - risk aversion and elasticity of intertemporal substitution determined by two different parameters in the utility function
- Useful dichotomy to jointly fit the cross-section of risk premia and the variance of the risk-fee rate
- Esptein-Zin: Least controversial alternative preference specification
  - we do not take a stance in favor of Epstein-Zin
  - showcase the effect of alternative preferences on fit of various consumption measures.

Methodology

Investor Preferences × Consumption Dynamics × Consumption Measures

- Alternative assumptions for consumption dynamics (e.g., i.i.d., AR, ARMA) affect the fit of C-CAPM for the cross-section of risk premia and the moments of the risk-free rate
- Our proposed empirical methodology jointly estimates consumption dynamics with Euler equations for risk premia and the risk-free rate
- Persistence vs. volatility of consumption drive the wedge between matching the risk-free rate volatility and the cross-section of risk premia
- We do not take a stance in favor of a specific time-series assumption
  - showcase the effect of alternative assumptions for consumption dynamics
  - different consumption measures are characterized by different dynamics (e.g., BEA is most likely AR(1) or ARMA(1,1), garbage and unfiltered consumption are i.i.d., etc.)

Methodology

Investor Preferences × Consumption Dynamics × Consumption Measures

- Focus on the cross-section:
  - estimate C-CAPMs with alternative consumption measures, preference specifications, and consumption dynamics using annual size, value, investment, and profitability portfolios
  - correspond to return-generated factors in Fama and French (1993, 2015) and Hou et al. (2016)
- Include the mean and volatility of risk-free rate in the set of target moment conditions
  - average risk-free rate: fixes the mean of the discount factor
  - variance of the risk-free rate: identifies the elasticity of intertemporal substitution (EIS) and the Epstein-Zin pricing kernel
- Proposed empirical methodology is conducted withing the standard GMM framework commonly used to test C-CAPMs

Roadmap

1. Theoretical background
2. Data and estimation
3. Empirical results
4. Conclusion

Theoretical Background

CRRA preferences

- Endowment economy with complete markets

- CRRA stochastic discount factor for representative investor

$$M_t^{CRRA} = \beta e^{-\gamma \Delta c_t} \tag{1}$$

  Δc_t: aggregate log-consumption growth, γ: coefficient of risk aversion, β: rate of time preference

- Unconditional Euler equation for the excess return on the stock market

$$\mathbb{E}\left[\beta e^{-\gamma \Delta c_t}\left(R_{mt} - R_{ft}\right)\right] = 0$$

  R_{mt}: return on the stock market, R_{ft}: risk-free rate

CRRA preferences

- Expression for the equity risk premium (Cochrane (2001))
  1. solve Euler equation for risk premia
  2. linearize CRRA discount factor
  3. replace covariance with correlation

$$\mathbb{E}\left[R_{mt} - R_{ft}\right] \approx \gamma \rho_{m,c} \sigma_m \sigma_c \tag{2}$$

  $\rho_{m,c}$: correlation of market excess returns to consumption growth, σ_m: volatility of market excess returns, σ_c: consumption growth volatility

- Solve for implied risk aversion coefficient

$$\gamma \approx \frac{\mathbb{E}\left[R_{mt} - R_{ft}\right]}{\rho_{m,c} \sigma_m \sigma_c} = \frac{11.68\% - 5.16\%}{0.36 * 18.09\% * 1.35\%} = 75 \tag{3}$$

This is the equity premium puzzle of Mehra and Prescott (1985)

CRRA: Equity risk premium and alternative consumption

$$\gamma \approx \frac{\mathbb{E}\left[R_{mt} - R_{ft}\right]}{\rho_{m,c}\sigma_m \sigma_c}$$

- Find an alternative consumption measure with similar correlation to market returns and higher volatility
- Volatility of standard BEA consumption: 1.35%
- Volatility of garbage measure: 3%
- Risk aversion coefficient decreases from 75 to 33
- Equity premium puzzle (partially) resolved!
  - 33 is still a large number for risk aversion (e.g., Rabin (2000))

Alternative preference specifications: Epstein-Zin

- Contrary to implications of the CRRA model, risk aversion is not necessarily the inverse of the elasticity of intertemporal substitution
  - lifetime utility is not necessarily the sum of per period utilities (Epstein and Zin (1989))
- Persistence in consumption growth amplifies consumption risk

Esptein-Zin model

Epstein-Zin stochastic discount factor with ARMA(1,1) consumption dynamics:

$$M_{t+1} = \underbrace{\tilde{\beta} e^{(\rho-1)\Delta c_{t+1}}}_{\text{time preferences}}$$

Time preferences:

- Δc_{t+1}: consumption growth
- $\tilde{\beta}$: rate of time preference
- ρ: EIS, sensitivity of consumption growth to risk-free rate
  - EIS = $1/(1-\rho)$

Esptein-Zin model

Epstein-Zin stochastic discount factor with ARMA(1,1) consumption dynamics:

$$M_{t+1} = \tilde{\beta} e^{(\rho-1)\Delta c_{t+1}} \times \underbrace{e^{-\frac{\rho-1+\gamma}{1-\kappa_1\phi_c}\Delta c_{t+1} + \frac{\rho-1+\gamma}{1-\kappa_1\phi_c}\phi_c\Delta c_t - \frac{\rho-1+\gamma}{1-\kappa_1\phi_c}\theta_c\kappa_1\epsilon_{t+1} + \frac{(\rho-1+\gamma)}{1-\kappa_1\phi_c}\theta_c\epsilon_t}}_{\text{risk preferences}}$$

Risk preferences:

- γ: risk aversion coefficient
- ϕ_c: autocorrelation of consumption growth
- θ_c: moving average of consumption growth
- κ_1: log-linearization constant
- risk aversion and EIS are determined by two distinct parameters (γ, ρ)
- for $\gamma = 1 - \rho \Rightarrow$ CRRA model (EIS is inverse of risk aversion)
- for $\phi_c = \theta_c = 0 \Rightarrow$ CRRA model (i.i.d. consumption growth)
- Functional form of Epstein-Zin kernel depends on consumption dynamics and consumption parameters

Consumption growth dynamics

- In deriving the Epstein-Zin discount factor, we assume ARMA(1,1) consumption growth (Δc_t) with constant volatility and N(0,σ_c^2) shocks ϵ_t

$$\Delta c_t = \mu_c + \phi_c \Delta c_{t-1} + \theta_c \epsilon_{t-1} + \epsilon_t.$$

  - μ_c: unconditional mean of consumption growth
  - ϕ_c: autocorrelation of consumption growth
  - θ_c: moving average of consumption growth
  - σ_c: unconditional volatility of shocks to consumption growth

- Alternative assumptions about consumption dynamics imply different functional forms for Esptein-Zin kernel

- The ARMA(1,1) nests the AR(1) and i.i.d. cases

Consumption growth dynamics

- There is an important debate in consumption-based asset pricing whether consumption growth is predictable (e.g., Bansal and Yaron (2004)) or i.i.d. (e.g., Cochrane (2001))

- In estimating the various consumption models with alternative consumption measures, we jointly estimate consumption growth dynamics (μ_c, ϕ_c, θ_c, σ_c) to verify the correct assumption for each alternative consumption process

- Joint estimation of consumption growth dynamics with Euler equations is key for identifying Epstein-Zin kernel and fitting the variance of the risk-free rate

Variance of the risk-free rate

- C-CAPMs imply explicit solutions for the risk-free rate since

$$R_{f,t+1} = \mathbb{E}_t[M_{t+1}]^{-1}$$

  - the risk-free rate is the conditional expectation of the pricing kernel
  - this restriction is important when testing C-CAPMs with alternative consumption measures

- Based on the ARMA(1,1) assumption for consumption growth, the variance of the log risk-free rate for all models (CRRA, Epstein-Zin) is

$$var(r_{f,t+1}) = (1-\rho)^2\Big(\frac{1+\theta_c^2+2\phi_c\theta_c}{1-\phi_c^2}\phi_c^2\sigma_c^2 + \theta_c^2\sigma_c^2 + 2\phi_c\theta_c\sigma_c^2\Big)$$

  - variance of risk-free rate depends on inverse of EIS ($(1-\rho)^2$), consumption growth persistence (ϕ_c^2, θ_c^2), and consumption growth variance (σ_c^2)

- The variance of the risk-free rate is an important moment condition for estimating C-CAPMs
  - allows estimation of the Epstein-Zin model with distinct EIS and risk aversion parameters
  - largely overlooked by the literature in cross-sectional tests of alternative consumption measures

Variance of the risk-free rate

- Variance of the risk-free rate depends hugely on assumption about consumption dynamics

- For AR(1) consumption growth, the variance of the risk-free rate is

$$var(r_{f,t+1}) = (1-\rho)^2 \frac{\phi_c^2}{1-\phi_c^2} \sigma_c^2$$

- For i.i.d. consumption growth, the variance of the risk-free rate is zero

Variance of the risk-free rate

- Variance of the risk-free rate causes tension in the CRRA model

- For the CRRA model, the variance of the risk-free rate is

$$var(r_{f,t+1}) = \gamma^2 \left(\frac{1+\theta_c^2+2\phi_c\theta_c}{1-\phi_c^2} \phi_c^2\sigma_c^2 + \theta_c^2\sigma_c^2 + 2\phi_c\theta_c\sigma_c^2 \right)$$

  - inverse of EIS is risk aversion ($1-\rho=\gamma$)

- Risk aversion (inverse of EIS) implied by the variance of the risk-free rate with BEA consumption

$$\gamma = \sqrt{\frac{var(r_{f,t+1})}{\frac{1+\theta_c^2+2\phi_c\theta_c}{1-\phi_c^2}\phi_c^2\sigma_c^2 + \theta_c^2\sigma_c^2 + 2\phi_c\theta_c\sigma_c^2}} = 3.25$$

- Risk aversion implied by equity risk-premium with BEA consumption

$$\gamma \approx \frac{\mathbb{E}\left[R_{mt} - R_{ft}\right]}{\rho_{m,c}\sigma_m\sigma_c} = 75$$

CRRA results

Data and Estimation

Benchmark BEA aggregate consumption

- Standard consumption process (SNonD):
  - obtained from BEA
  - consumption of services and non-durables
  - normalized by population and price index
  - separate consumption measures for services (S-K) and non-durables (NonD-K)

Alternative aggregate consumptions

- Parker and Julliard (2003): consumption measure over longer time intervals, e.g., three-year periods
  - investors slowly adjust their consumption in response to financial shocks
  - ultimate consumption (Ult)
- Jagannathan and Wang (2007): consumption measure at the end of each period instead of flow over the entire period
  - investors do not pay attention to financial decisions all year around
  - fourth quarter to fourth quarter annual consumption growth (Q4)
- Savov (2011): proxy consumption with the amount of municipal garbage produced every year
  - garbage output may be a more accurate measure of economic activity than BEA consumption
  - asset pricing with garbage (Gbg)

Alternative aggregate consumptions

- Kroencke (2017): consumption observed by BEA econometrician is measured with error
  - Kalman filter approach to obtain the true aggregate consumption process from the observed one
  - unfiltered consumption processes for i) services and non-durables, ii) non-durables, and iii) fourth quarter non-durables (SNonD-U, NonD-U, Q4NonD-U)

- Breeden (1979): in a no-trade endowment economy consumption is entirely financed by dividends
  - real per capita aggregate dividend growth (Div) from Robert Shiller's website

Test assets

- Nine equity portfolios: Stock market + 8 extreme portfolios (4 low and 4 high) from 40 decile portfolios independently sorted on
  1. size
  2. book-to-market
  3. profitability
  4. investment

- Basis portfolios for the most commonly used asset pricing factors: RmRf, HML, SMB, CMA, RMW

- Mean and variance of the risk-free rate

- Baseline sample consists of annual returns for 1964-2016

- Robustness: Annual returns for 1930-2016
  1. includes (part of) Great Depression
  2. drop alternative consumption measures (Q4, garbage) and test assets (investment, profitability)

Estimation

Goals of empirical analysis:

- Estimate consumption growth moments
- Estimate structural parameters (e.g., risk aversion, EIS, rate of time prefence)
- Test cross-sectional fit of alternative C-CAPM specifications and consumption measures

Estimation

- Methodology: First-stage GMM (Hansen and Singleton (1982))
- Moment conditions include
  1. score vector for consumption growth moments

$$\begin{bmatrix} \mathbb{E}[\partial l(\mu_c, \phi_c, \theta_c, \sigma_c^2; \Delta c_t, \Delta c_{t-1}, \epsilon_{t-1})/\partial \mu_c] \\ \mathbb{E}[\partial l(\mu_c, \phi_c, \theta_c, \sigma_c^2; \Delta c_t, \Delta c_{t-1}, \epsilon_{t-1})/\partial \phi_c] \\ \mathbb{E}[\partial l(\mu_c, \phi_c, \theta_c, \sigma_c^2; \Delta c_t, \Delta c_{t-1}, \epsilon_{t-1})/\partial \theta_c] \\ \mathbb{E}[\partial l(\mu_c, \phi_c, \theta_c, \sigma_c^2; \Delta c_t, \Delta c_{t-1}, \epsilon_{t-1})/\partial \sigma_c^2] \end{bmatrix}$$

- Function $l()$ is the conditional log-likelihood for consumption dynamics
- For ARMA(1,1) consumption growth, score vector is calculated numerically
- For AR(1) and i.i.d. assumption, score vector is calculated analytically
- For AR(1) consumption growth, $\partial l / \partial \theta_c = 0$
- For i.i.d. consumption growth, $\partial l / \partial \phi_c = \partial l / \partial \theta_c = 0$

Estimation

- Methodology: First-stage GMM (Hansen and Singleton (1982))
- Moment conditions include
  1. score vector for consumption growth moments
  2. mean and variance of risk-free rate

$$\begin{bmatrix} \mathbb{E}[\partial l(\mu_c, \phi_c, \theta_c, \sigma_c^2; \Delta c_t, \Delta c_{t-1}, \epsilon_{t-1})/\partial \mu_c] \\ \mathbb{E}[\partial l(\mu_c, \phi_c, \theta_c, \sigma_c^2; \Delta c_t, \Delta c_{t-1}, \epsilon_{t-1})/\partial \phi_c] \\ \mathbb{E}[\partial l(\mu_c, \phi_c, \theta_c, \sigma_c^2; \Delta c_t, \Delta c_{t-1}, \epsilon_{t-1})/\partial \theta_c] \\ \mathbb{E}[\partial l(\mu_c, \phi_c, \theta_c, \sigma_c^2; \Delta c_t, \Delta c_{t-1}, \epsilon_{t-1})/\partial \sigma_c^2] \\ \mathbb{E}[(logR_{ft})^2 - \mathbb{E}[logR_{ft}]^2] - (1-\rho)^2\left(\frac{\phi_c^2(1+\theta_c^2+2\phi_c\theta_c)\sigma_c^2}{1-\phi_c^2} + \theta_c^2\sigma_c^2 + 2\theta_c\phi_c\sigma_c^2\right) \\ \mathbb{E}\left[R_{ft}M_t\right] - 1 \end{bmatrix}$$

- For i.i.d. consumption growth, the model-implied risk-free rate is constant, and the variance of the risk-free rate is zero

Estimation

- Methodology: First-stage GMM (Hansen and Singleton (1982))
- Moment conditions include
  1. score vector for consumption growth moments
  2. mean and variance of risk-free rate
  3. unconditional Euler equations for the stock market and the cross-section of risk premia

$$\begin{bmatrix} \mathbb{E}[\partial l(\mu_c, \phi_c, \theta_c, \sigma_c^2; \Delta c_t, \Delta c_{t-1}, \epsilon_{t-1})/\partial \mu_c] \\ \mathbb{E}[\partial l(\mu_c, \phi_c, \theta_c, \sigma_c^2; \Delta c_t, \Delta c_{t-1}, \epsilon_{t-1})/\partial \phi_c] \\ \mathbb{E}[\partial l(\mu_c, \phi_c, \theta_c, \sigma_c^2; \Delta c_t, \Delta c_{t-1}, \epsilon_{t-1})/\partial \theta_c] \\ \mathbb{E}[\partial l(\mu_c, \phi_c, \theta_c, \sigma_c^2; \Delta c_t, \Delta c_{t-1}, \epsilon_{t-1})/\partial \sigma_c^2] \\ \mathbb{E}[(logR_{ft})^2 - \mathbb{E}[logR_{ft}]^2] - (1-\rho)^2\left(\frac{\phi_c^2(1+\theta_c^2+2\phi_c\theta_c)\sigma_c^2}{1-\phi_c^2} + \theta_c^2\sigma_c^2 + 2\theta_c\phi_c\sigma_c^2\right) \\ \mathbb{E}\Big[R_{ft}M_t\Big] - 1 \\ \color{red}{\mathbb{E}\Big[\Big(R_{mt} - R_{ft}\Big)M_t\Big]} \\ \color{red}{\mathbb{E}\Big[\Big(R_{it} - R_{ft}\Big)M_t\Big] \quad \text{for} \quad i = 1, 2, \ldots, n} \end{bmatrix}$$

Estimation

- Methodology: First-stage GMM (Hansen and Singleton (1982))
- Moment conditions include
  1. score vector for consumption growth moments
  2. mean and variance of risk-free rate
  3. unconditional Euler equations for the stock market and the cross-section of risk premia
  4. log-linearization constant κ_1 for Epstein-Zin utility from aggregate price-dividend growth ($\Delta d_{m,t}$) identity in Campbell and Shiller (1988)

$$\begin{bmatrix} \mathbb{E}[\partial l(\mu_c, \phi_c, \theta_c, \sigma_c^2; \Delta c_t, \Delta c_{t-1}, \epsilon_{t-1})/\partial \mu_c] \\ \mathbb{E}[\partial l(\mu_c, \phi_c, \theta_c, \sigma_c^2; \Delta c_t, \Delta c_{t-1}, \epsilon_{t-1})/\partial \phi_c] \\ \mathbb{E}[\partial l(\mu_c, \phi_c, \theta_c, \sigma_c^2; \Delta c_t, \Delta c_{t-1}, \epsilon_{t-1})/\partial \theta_c] \\ \mathbb{E}[\partial l(\mu_c, \phi_c, \theta_c, \sigma_c^2; \Delta c_t, \Delta c_{t-1}, \epsilon_{t-1})/\partial \sigma_c^2] \\ \mathbb{E}[(logR_{ft})^2 - \mathbb{E}[logR_{ft}]^2] - (1-\rho)^2\left(\frac{\phi_c^2(1+\theta_c^2+2\phi_c\theta_c)\sigma_c^2}{1-\phi_c^2} + \theta_c^2\sigma_c^2 + 2\theta_c\phi_c\sigma_c^2\right) \\ \mathbb{E}\left[R_{ft}M_t\right] - 1 \\ \mathbb{E}\left[\left(R_{mt} - R_{ft}\right)M_t\right] \\ \mathbb{E}\left[\left(R_{it} - R_{ft}\right)M_t\right] \quad \text{for} \quad i = 1, 2, \ldots, n \\ \mathbb{E}[-logR_{mt} + \Delta d_{mt}] - log\kappa_1 \end{bmatrix} = \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{z_t})$$

$\boldsymbol{x}$: vector of parameters, $\boldsymbol{z_t}$: vector of variables

Estimation

- Estimate model parameters and assess cross-sectional fit by minimizing the first-stage GMM objective function

$$\min_{\boldsymbol{x}} \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{z_t})' \boldsymbol{W} \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{z_t})$$

- Pre-specified diagonal GMM weighting matrix $\boldsymbol{W}$ overweighs moment conditions for consumption and the risk-free rate
  - consumption parameters and risk-premia have different scales
  - importance of fitting the risk-free rate
- GMM is cross-sectional regression of risk premia on covariances
  - covariances are estimated regressors
  - GMM estimates covariances and runs the regression simultaneously adjusting standard errors for EIV

Estimation

- Assess cross-sectional accuracy of the different C-CAPMs with alternative consumption measures based on
  1. the cross-sectional R-square (R^2)
  2. the root-mean-square-prediction error (*rmspe*)

$$rmspe = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\mathbb{E}[R_{it}-R_{ft}]_{\text{sample}}-\mathbb{E}[R_{it}-R_{ft}]_{\text{fitted}}\right)^2}$$

with

$$\mathbb{E}[R_{it}-R_{ft}]_{\text{fitted}} = -Cov\left(R_{it}-R_{ft}, M_t\right)/\mathbb{E}[M_t]$$

Results

GMM results for the CRRA model (1964-2016)

AR(1) consumption growth, $\Delta c_t = \mu_c + \phi_c \Delta c_{t-1} + \epsilon_t$; GMM includes the variance of the log risk-free rate

	SNonD	S-K	NonD-K	Ult	Q4	Q4NonD	SNonD-U	NonD-U	Q4-U	Q4NonD-U	Gbg	Div
γ	4.19	3.50	3.27	1.07	4.17	5.63	26.68	23.19	33.95	20.75	18.09	0.70
	(2.65)	(2.60)	(3.26)	(3.51)	(2.20)	(1.20)	(2.06)	(1.88)	(1.72)	(2.21)	(2.02)	(2.13)
β	1.08	1.03	1.05	1.04	1.06	1.05	1.26	1.06	1.37	1.08	1.03	0.99
	(27.31)	(48.43)	(45.38)	(55.92)	(28.05)	(16.46)	(8.70)	(10.10)	(7.89)	(11.36)	(11.63)	(130.32)
μ_c	1.32%	0.84%	1.02%	1.29%	1.18%	1.18%	1.69%	1.34%	1.88%	1.56%	1.19%	0.61%
	(3.81)	(3.11)	(3.47)	(2.58)	(3.63)	(3.19)	(4.01)	(2.25)	(3.20)	(1.61)	(1.22)	(0.77)
σ_c	1.05%	1.41%	1.00%	1.61%	1.26%	1.95%	2.43%	2.61%	2.39%	2.88%	2.86%	5.39%
	(4.66)	(3.85)	(5.26)	(5.77)	(5.47)	(4.57)	(4.73)	(3.85)	(5.74)	(4.29)	(3.82)	(2.02)
ϕ_c	0.41	0.37	0.51	0.76	0.36	0.18	0.03	0.03	0.02	-0.03	-0.03	0.47
	(3.38)	(3.39)	(5.01)	(10.82)	(2.69)	(1.33)	(2.22)	(2.09)	(1.72)	(-2.21)	(-1.96)	(3.12)
χ^2_1	42.77	43.19	44.86	44.94	45.64	31.36	15.88	16.25	15.28	22.79	17.50	42.23
dof_1	9	9	9	9	9	9	9	9	9	9	9	9
p_1	0	0	0	0	0	0	0.06	0.06	0.08	0	0.04	0
χ^2_2	42.77	43.19	44.86	44.94	45.64	31.36	15.88	16.25	15.28	22.79	17.50	42.23
dof_2	9	9	9	9	9	9	9	9	9	9	9	9
p_2	0	0	0	0	0	0	0.06	0.06	0.08	0	0.04	0
R^2	-615.07%	-590.80%	-651.81%	-671.67%	-617.42%	-514.90%	13.05%	42.78%	41.30%	48.44%	5.19%	-621.55%
rmspe	7.68%	7.55%	7.88%	7.99%	7.70%	7.12%	2.58%	2.09%	2.12%	1.99%	2.69%	7.72%

- estimate γ from variance of risk-free rate: $\gamma = \sqrt{var(r_{ft})(1-\phi_c^2)/(\phi_c^2\sigma_c^2)}$
- high persistence (high ϕ_c) implies low γ estimates
- fit of the model in the cross-section according to Euler equation: $\mathbb{E}[R_{it} - R_{ft}] \approx \gamma \rho_{c,i} \sigma_c \sigma_i$
- low γ estimates lead to poor fit

Volatility wedge

GMM results for the Epstein-Zin model (1964-2016)

AR(1) consumption growth, $\Delta c_t = \mu_c + \phi_c \Delta c_{t-1} + \epsilon_t$; GMM includes the variance of the log risk-free rate

	SNonD	S-K	NonD-K	Ult	Q4	Q4NonD	SNonD-U	NonD-U	Q4-U	Q4NonD-U	Gbg	Div
γ	34.40	27.27	37.87	23.15	61.17	24.97	26.56	23.53	33.91	19.43	16.97	4.29
	(1.94)	(2.06)	(1.82)	(1.79)	(1.68)	(1.82)	(2.13)	(2.15)	(1.75)	(1.79)	(1.84)	(1.88)
ρ	-2.57	-2.35	-2.10	-0.26	-2.90	-3.96	-40.77	-15.84	-20.59	-59.99	-28.24	0.30
	(-2.10)	(-1.88)	(-2.29)	(-0.81)	(-1.72)	(-1.03)	(-0.16)	(-0.36)	(-0.23)	(-0.06)	(-0.14)	(0.92)
$\tilde{\beta}$	1.63	1.12	1.56	1.58	1.52	1.14	1.27	1.05	1.36	1.08	1.03	0.92
	(3.73)	(8.74)	(3.97)	(3.30)	(3.48)	(9.03)	(7.10)	(8.79)	(6.34)	(10.91)	(14.32)	(21.76)
κ_1	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96
	(48.99)	(48.99)	(48.99)	(48.99)	(48.99)	(48.99)	(48.99)	(48.99)	(48.99)	(48.99)	(48.99)	(48.99)
μ_c	1.16%	0.80%	0.97%	1.74%	1.12%	1.09%	1.75%	1.26%	1.80%	1.39%	1.07%	0.60%
	(3.58)	(3.07)	(3.51)	(3.16)	(3.64)	(2.99)	(3.72)	(2.96)	(3.98)	(2.78)	(2.22)	(0.75)
σ_c	1.05%	1.41%	1.05%	1.64%	1.26%	1.95%	2.43%	2.61%	2.39%	2.88%	2.86%	5.40%
	(4.64)	(3.87)	(5.27)	(5.32)	(5.48)	(4.59)	(4.72)	(3.88)	(5.74)	(4.80)	(4.06)	(2.03)
ϕ_c	0.47	0.39	0.53	0.70	0.38	0.20	0.02	0.04	0.03	-0.01	-0.02	0.48
	(3.96)	(3.54)	(5.35)	(8.48)	(2.90)	(1.46)	(0.16)	(0.39)	(0.24)	(-0.06)	(-0.15)	(3.12)
χ_1^2	226.37	16.48	14.19	14.50	7.64	20.37	15.41	15.97	12.57	19.74	17.43	23.78
dof_1	8	8	8	8	8	8	8	8	8	8	8	8
p_1	0	0.03	0.07	0.06	0.46	0	0.05	0.04	0.12	0.01	0.02	0
χ_2^2	184.38	16.48	14.19	14.50	7.64	20.37	15.41	15.97	12.57	19.74	17.43	23.78
dof_2	8	8	8	8	8	8	8	8	8	8	8	8
p_2	0	0.03	0.07	0.06	0.46	0	0.05	0.04	0.12	0.01	0.02	0
R^2	53.25%	47.05%	-2.18%	-121.74%	20.63%	52.89%	15.14%	44.26%	41.83%	50.49%	8.41%	30.28%
rmspe	1.90%	2.02%	2.81%	4.15%	2.48%	1.91%	2.56%	2.08%	2.12%	1.96%	2.66%	2.32%

- estimate ρ from variance of risk-free rate: $\rho = 1 - \sqrt{var(r_{ft})(1-\phi_c^2)/(\phi_c^2\sigma_c^2)}$
- low persistence (low ϕ_c) implies large $|\rho|$ estimates and near-zero EIS (EIS = $1/(1-\rho)$)

GMM results for the Epstein-Zin model (1964-2016)

AR(1) consumption growth, $\Delta c_t = \mu_c + \phi_c \Delta c_{t-1} + \epsilon_t$***; GMM includes the variance of the log risk-free rate***

	SNonD	S-K	NonD-K	Ult	Q4	Q4NonD	SNonD-U	NonD-U	Q4-U	Q4NonD-U	Gbg	Div
γ	34.40	27.27	37.87	23.15	61.17	24.97	26.56	23.53	33.91	19.43	16.97	4.29
	(1.94)	(2.06)	(1.82)	(1.79)	(1.68)	(1.82)	(2.13)	(2.15)	(1.75)	(1.79)	(1.84)	(1.88)
ρ	-2.57	-2.35	-2.10	-0.26	-2.90	-3.96	-40.77	-15.84	-20.59	-59.99	-28.24	0.30
	(-2.10)	(-1.88)	(-2.29)	(-0.81)	(-1.72)	(-1.03)	(-0.16)	(-0.36)	(-0.23)	(-0.06)	(-0.14)	(0.92)
$\tilde{\beta}$	1.63	1.12	1.56	1.58	1.52	1.14	1.27	1.05	1.36	1.08	1.03	0.92
	(3.73)	(8.74)	(3.97)	(3.30)	(3.48)	(9.03)	(7.10)	(8.79)	(6.34)	(10.91)	(14.32)	(21.76)
κ_1	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96
	(48.99)	(48.99)	(48.99)	(48.99)	(48.99)	(48.99)	(48.99)	(48.99)	(48.99)	(48.99)	(48.99)	(48.99)
μ_c	1.16%	0.80%	0.97%	1.74%	1.12%	1.09%	1.75%	1.26%	1.80%	1.39%	1.07%	0.60%
	(3.58)	(3.07)	(3.51)	(3.16)	(3.64)	(2.99)	(3.72)	(2.96)	(3.98)	(2.78)	(2.22)	(0.75)
σ_c	1.05%	1.41%	1.05%	1.64%	1.26%	1.95%	2.43%	2.61%	2.39%	2.88%	2.86%	5.40%
	(4.64)	(3.87)	(5.27)	(5.32)	(5.48)	(4.59)	(4.72)	(3.88)	(5.74)	(4.80)	(4.06)	(2.03)
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	(3.96)	(3.54)	(5.35)	(8.48)	(2.90)	(1.46)	(0.16)	(0.39)	(0.24)	(-0.06)	(-0.15)	(3.12)
χ^2_1	226.37	16.48	14.19	14.50	7.64	20.37	15.41	15.97	12.57	19.74	17.43	23.78
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dof_2	8	8	8	8	8	8	8	8	8	8	8	8
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R^2	53.25%	47.05%	-2.18%	-121.74%	20.63%	52.89%	15.14%	44.26%	41.83%	50.49%	8.41%	30.28%
rmspe	1.90%	2.02%	2.81%	4.15%	2.48%	1.91%	2.56%	2.08%	2.12%	1.96%	2.66%	2.32%

- risk aversion γ is distinct from intertemporal substitution ρ
- fit of the model in the cross-section: $\mathbb{E}[R_{it} - R_{ft}] \approx \gamma \rho_{c,i} \sigma_c \sigma_i$
- fit of the model does not depend on EIS as in the CRRA case
- fit of the model depends on alignment of risk premia ($\mathbb{E}[R_{it} - R_{ft}]$) with consumption growth correlations ($\rho_{c,i}$)
- Similar results hold Epstein-Zin preferences and ARMA(1,1) assumption

More results

- All possible combinations between two preference specifications (CRRA, Epstein-Zin) and three types of consumption dynamics (i.i.d., AR(1), ARMA(1,1)) across several alternative consumption measures
- Analysis for rate of time preference β
- Robustness: 1930 - 2016 sample with Great Depression

Conclusion

Conclusion

- We propose a novel triple-hypothesis framework for evaluating the cross-sectional fit of C-CAPMs with alternative proxies for aggregate consumption:

  Investor Preferences × Consumption Dynamics × Consumption Measures

- Within the CRRA framework, alternative consumption measures:
  - imply lower risk aversion parameters than standard BEA consumption but yield an almost zero EIS
  - better fit than standard BEA consumption
  - fit is quite poor with low cross-sectional R^2s and large pricing errors
  - replicate results of existing literature

- Epstein-Zin model with persistence assumption (AR(1) or ARMA(1,1)) for consumption growth
  - BEA consumption outperforms alternative consumption measures in terms of cross-sectional accuracy
  - most alternative consumption measures perform quite poorly within the Epstein-Zin model due to low autocorrelation of these measures
  - BEA consumption still implies large risk aversion parameters but alternative consumption processes yield almost zero EIS

Conclusion

- Results of alternative consumption literature rely heavily on
  - assumption of CRRA preferences
  - the fact that moments of the risk-free rate are usually ignored

- Ability of alternative consumptions to generate plausible prices of risk and improve the cross-sectional fit diminishes when stochastic discount factor disentangles time preferences from risk aversion (e.g., Epstein-Zin)

- More realistically plausible preferences (e.g., Epstein-Zin) are likely more important in improving the cross-sectional fit of consumption models than alternative consumption measures