A non-admitted role of central banks: Debt-dependent quantitative monetary policies

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Motivation

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- typically, this policy mix ensures stability and determinacy.
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- It seems that the politically unpleasant task of debt sustainability has fallen on Central Banks.
What we do in this paper

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- We explore the possibility that quantitative monetary policies can substitute tax-spending debt stabilization policies when it comes to stability and determinacy.
- We do so in the context of a rather standard New Keynesian general equilibrium model solved using common parameter values and data from the EZ.
Step A: We shock the initial steady state by assuming an adverse supply shock (to mimic an economic disaster) and, at the same time, an increase in government transfers (to mimic the usual fiscal stimulus that counters economic disasters) - see Hall and Sargent (2021).
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Step B: We experiment with different debt-contingent policy instruments, both fiscal and (quantitative) monetary.
Literature on fiscal-monetary policy interactions

- Leeper (1991)
- Reis (2016, 2017)
- Sims and Wu (2020)
- Bassetto and Sargent (2020)
- Hall and Sargent (2021)
Our model

- Households

Private firms: final good firms, intermediate good firms: Dixit-Stiglitz, Rotemberg nominal fixities, borrowing constraint, production-price-corporate finance decisions

Private banks: assets: loans to firms, government bonds, reserves held at the CB
liabilities: receive deposits and loans from the CB

Our model similar to Cúrdia and Woodford (2010, 2011) which means that market segmentation and costly financial intermediation give rise to asset pricing wedges and imperfect substitutability that break Wallace’ s (1981) neutrality proposition and give a real role to quantitative monetary policies (Walsh, 2017, chapter 11, for a review)
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- In simple words, the issuance of liabilities is used to finance, via loans to private banks, loans to private companies and national governments, via government bonds purchases in the secondary market.
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- The end-of-period government bonds follow residually to close the Treasury’s budget constraint.
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- The lack of “fiscal backing” in the EZ implies that the CB’s dividends to Treasury, \( n_t \), is an exogenous policy instrument (see Del Negro and Sims (2015), Hall and Reis (2017)). Also, the lack of “fiscal support” in the EZ, implies the non-negativity constraint: \( n_t \geq 0 \).
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- To the extent that currency held by the non-bank public and reserves, as well as central bank loans held by private banks are demand determined, the central bank’s budget constraint can provide an extra equation to determine the inflation rate or the price level.
Fiscal policy rules

- We adopt a rule-like approach to policy where the exogenous policy instruments, in addition to a conventional exogenous $AR(1)$ component, can also react to debt imbalances.

- The rules of the tax-spending instruments are:

  \[ s_t = \rho^s s_{t-1} + (1 - \rho^s) s - \gamma^s \left( \frac{b_t}{y_t} - \frac{b}{y} \right) \]

  \[ \tau_t = \rho^\tau \tau_t + (1 - \rho^\tau) \tau_{t-1} + \gamma^\tau \left( \frac{b_t}{y_t} - \frac{b}{y} \right) \]
Monetary policy rules

- The policy rates, \( i_t^Z \) and \( i_t^r \), follow Taylor type rules of the form:

\[
\log (1 + i_t) = (1 - \rho) \log (1 + i) + \rho \log (1 + i_{t-1}) + \gamma \pi \log \left( \frac{\pi_t}{\pi} \right)
\]

- The central bank purchases in the secondary market the fraction \((1 - \Lambda_t)\) of outstanding government bonds, at a constant price \( \Phi_t \), where \((1 - \Lambda_t)\) satisfies:

\[
\Phi_t (1 - \Lambda_t) \frac{p_{t-1}}{p_t} b_{t-1} = B_t y_t
\]

and

\[
B_t = \rho^B B_{t-1} + \gamma^B \left( \frac{b_t}{y_t} - \frac{b}{y} \right)
\]
The CB pays out to Treasury its non-negative net income every period (see e.g. Reis, 2016, 2017):

\[
  n_t = \left( m_{h,t} - \frac{p_{t-1}}{p_t} m_{t-1} \right) + i_t^z \frac{p_{t-1}}{p_t} z_{p,t-1} + \\
  + i_t^b (1 - \Lambda_t) \frac{p_{t-1}}{p_t} b_{t-1} - i_t^r \frac{p_{t-1}}{p_t} m_{p,t-1}
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Solution methodology

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- Transition dynamics driven by
  - an adverse supply shock
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  - policy reaction to debt imbalances

Compute long-run output multipliers.

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To quantify the impact of the different policies on the economy, we plot of the Impulse Response Functions (IRFs) of the main macroeconomic variables and calculate a “multiplier” (close to Uhlig’s, 2010, multipliers), $\varphi_t$, which is given by:

$$\varphi_t = \sum_{s=0}^{t} \frac{y_s - y}{(1 + i_b)^s}$$

We have experimented with other criteria, like the discounted lifetime utility in terms of consumption equivalents (see e.g. Lucas (1990)) and the welfare losses (see e.g. Schmitt-Grohé and Uribe (2007)), and the results are quantitatively close.
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Policy scenario:

- Conventional policy assignment: …scal policy instruments adjust to ensure public debt stability, while monetary policy controls inflation.
- We experiment with one instrument at a time: government investment, government consumption, consumption tax rate, income tax rate and profit tax rate (in this order).
- IRFs
- Fiscal dominance: quantitative monetary policy substitutes fiscal policy when it comes to debt stability and determinancy.
- We switch off fiscal reaction to debt and allow CB’s government bonds’ purchases in the secondary market to react to debt imbalances.

For all policy rules considered we set the persistence parameter \( \rho = 0 \), while we set the feedback parameter, \( \gamma \), to the lowest possible value that ensures dynamic stability.
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CB’s government bond purchases
Fiscal and monetary policy instruments
The impact of economic policy

\[ \phi_t \]

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(CRETE)
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Possible extensions

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- Thank you for your attention
Budget constraint:

\[(1 + \tau_t^c) c_{h,t} + j_{h,t} + m_{h,t} = (1 - \tau_t^y) w_t l_{h,t} + (1 + i_t^d) \frac{p_{t-1}}{p_t} j_{h,t-1} + \frac{p_{t-1}}{p_t} m_{h,t-1} + \pi_{h,t} + g_t^t\]

Cash-in-advance constraint:

\[m_{h,t} \geq (1 + \tau_t^c) c_{h,t}\]
Final good firms

Standard Dixit-Stiglitz technology:

\[ y_{f,t} = \left[ \sum_{i=1}^{N} \frac{1}{N} (y_{i,t})^{\theta} \right]^{\frac{1}{\theta}} \]

Real profits:

\[ \pi_{f,t} = y_{f,t} - \sum_{i=1}^{N} \frac{1}{N} \frac{p_{i,t}}{p_{t}} y_{i,t} \]
Intermediate good firms

Net profit:

\[ \pi_{i,t} = (1 - \tau_{t}^{\pi}) \left[ \frac{p_{i,t}}{p_{t}} y_{i,t} - w_{t} l_{i,t} \right] - x_{i,t} - \]

\[ - \frac{\zeta^{p}}{2} \left( \frac{p_{i,t}}{p_{i,t-1}} - 1 \right)^{2} \bar{y}_{i,t} + \left( L_{i,t}^{h} - \left[ 1 + i_{t}^{l} (1 - \tau_{t}^{\pi}) \right] \frac{p_{t-1}}{p_{t}} L_{i,t-1}^{h} \right) \]

Production function:

\[ y_{i,t} = A \left( k_{t-1}^{g} \right)^{\sigma} \left( k_{i,t-1}^{\alpha} l_{i,t}^{1-\alpha} \right)^{1-\sigma} \]
The law of motion of the firm’s capital stock:

\[ k_{i,t} = x_{i,t} + (1 - \delta) k_{i,t-1} \]

Borrowing constraint:

\[ L_{i,t} \geq \eta w_t l_{i,t} \]

Demand for product:

\[ p_{i,t} = p_t \left( \frac{y_{i,t}}{y_{f,t}} \right)^{\theta-1} \]

private firms
Private banks

The budget constraint of each bank that connects changes in its assets and liabilities is:

$$\pi_{p,t} = (1 - \tau^\pi_t) \left[ (1 + i^l_t) \frac{p_{t-1}}{p_t} L_{p,t-1} + (1 + i^r_t) \frac{p_{t-1}}{p_t} m_{p,t-1} + (1 + i^b_t) \frac{p_{t-1}}{p_t} \Lambda_t b_p + \right. $$

$$\Phi_t \frac{p_{t-1}}{p_t} (1 - \Lambda_t) b_{p,t-1} - (1 + i^d_t) \frac{p_{t-1}}{p_t} j_{p,t-1} - (1 + i^z_t) \frac{p_{t-1}}{p_t} z_{p,t-1} -$$

$$\Xi_t - L_{p,t} - b_{p,t} - m_{p,t} + j_{p,t} + z_{p,t}$$

where

$$\Xi_t = \frac{\xi^l}{2} (L_{p,t-1})^2 + \frac{\xi^b}{2} (\Lambda_t b_{p,t-1})^2 +$$

$$+ \frac{\xi^m}{2} (m_{p,t-1} + \Phi_t (1 - \Lambda_t) b_{p,t-1})^2 + \frac{\xi^z}{2} (z_{p,t-1})^2$$
The Government budget constraint

The flow budget constraint of the government written in per capita and real terms is:

\[ g^c_t + g^g_t + g^t_t + (1 + i^b_t) \frac{p_{t-1}}{p_t} b_{t-1} = b_t + \frac{T_t}{N} + n_t \]

Total tax revenues in real terms are defined as:

\[ \frac{T_t}{N} \equiv \tau^c_t c_{h,t} + \tau^y_t w_t l_{h,t} + \tau^\pi_t (y_{i,t} - w_t l_{i,t}) + \]

\[ + \tau^\pi_t (1 + i^l_t) \frac{p_{t-1}}{p_t} L_{p,t-1} + (1 + i^r_t) \frac{p_{t-1}}{p_t} m_{p,t-1} + \]

\[ + (1 + i^b_t) \frac{p_{t-1}}{p_t} \Lambda_t b_{p,t-1} + \Phi_t \frac{p_{t-1}}{p_t} (1 - \Lambda_t) b_{p,t-1} - \]

\[ - (1 + i^d_t) \frac{p_{t-1}}{p_t} j_{p,t-1} - (1 + i^z_t) \frac{p_{t-1}}{p_t} z_{p,t-1} - \frac{p^h_t}{p_t} \Xi_t ] \]
The budget constraint of the CB linking changes in assets and liabilities is (written in real and per capita terms):

$$
\Phi_t (1 - \Lambda_t) \frac{p_{t-1}}{p_t} b_{t-1} + z_{p,t} + i_t \frac{p_{t-1}}{p_t} m_{p,t-1} + n_t \equiv \\
\equiv (1 - \Lambda_t)(1 + i^b_t) \frac{p_{t-1}}{p_t} b_{t-1} + (1 + i^z_t) \frac{p_{t-1}}{p_t} z_{p,t-1} + m_t - \frac{p_{t-1}}{p_t} m_{t-1}
$$
Government investment
Government consumption
Consumption tax rate
Income tax rate
Profit tax rate