Information Aggregation under Ambiguity: Theory and Experimental Evidence

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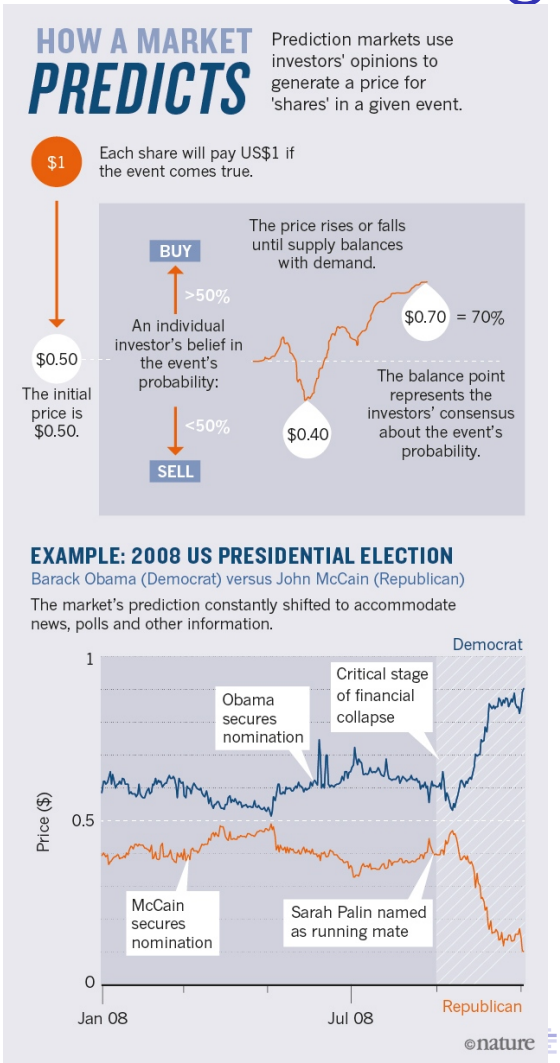
- Do financial markets aggregate and reveal the private information held by individual traders?
- The mechanism, through prices, is intuitive
- If the price is low and some traders have private information that the real value of the security is high, they will increase their demand and the price, and vice versa
- Price movements reveal information that others have about the true value of the security
- Starting from [Hayek, 1945], there is extensive literature on information aggregation under various settings

- Prediction markets are specifically designed to aggregate information by trading a security
- Google, Microsoft, Ford, General Electric, HP run internal prediction markets as a corporate governance tool
- Cultivate Labs, Inkling Markets, Consensus Point, Crowdcast, Iowa Electronic Markets run public markets on sports, political events, Hollywood box office success

- Are they better than other forecasting methods (e.g. polls, average opinion)?
- Results are mixed

Case 1: prediction market is better than average opinion

- Iowa Electronic Markets outperformed 964 polls 74% of the time during five presidential elections (1988 - 2004), 100% for forecasts 100 days in advance



Case 2: prediction market is similar to average opinion

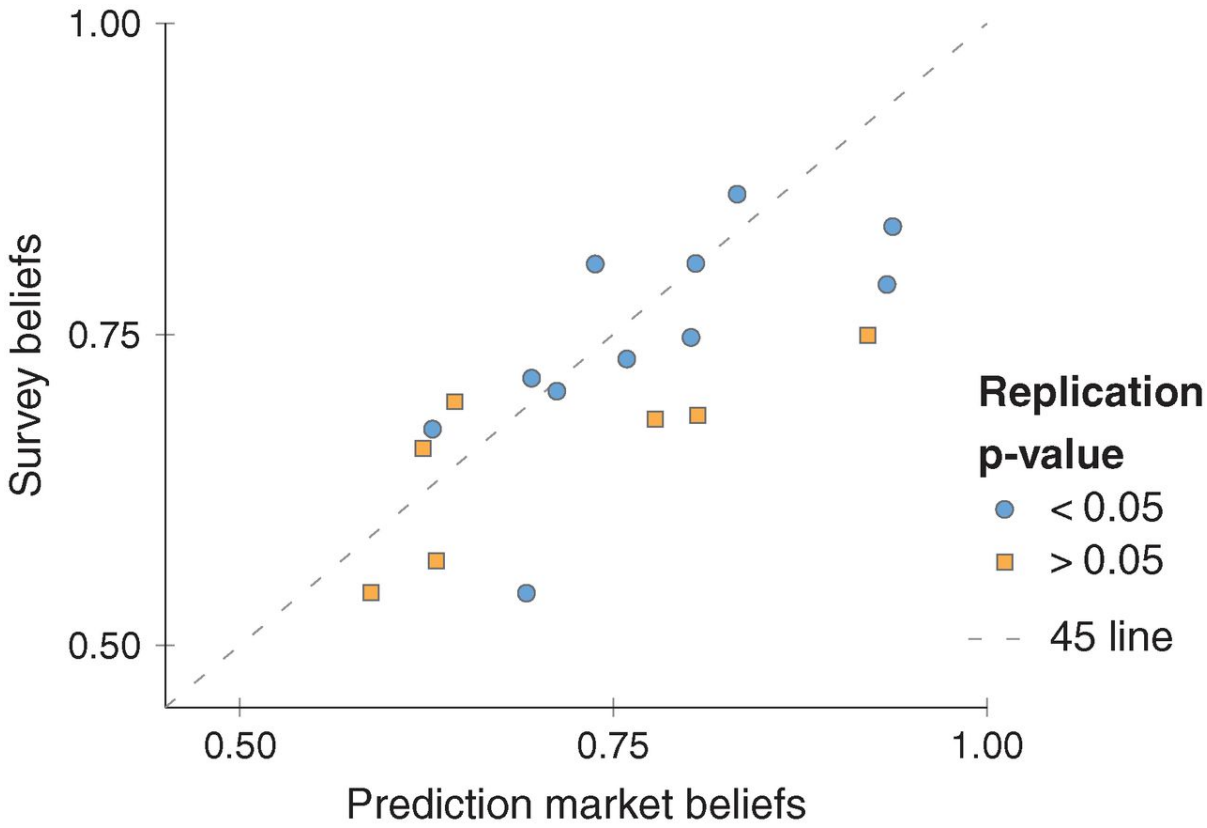


Figure: Camerer et al. "Evaluating replicability of laboratory experiments in economics." Science 351.6280 (2016)

Case 3: prediction market is worse than average opinion

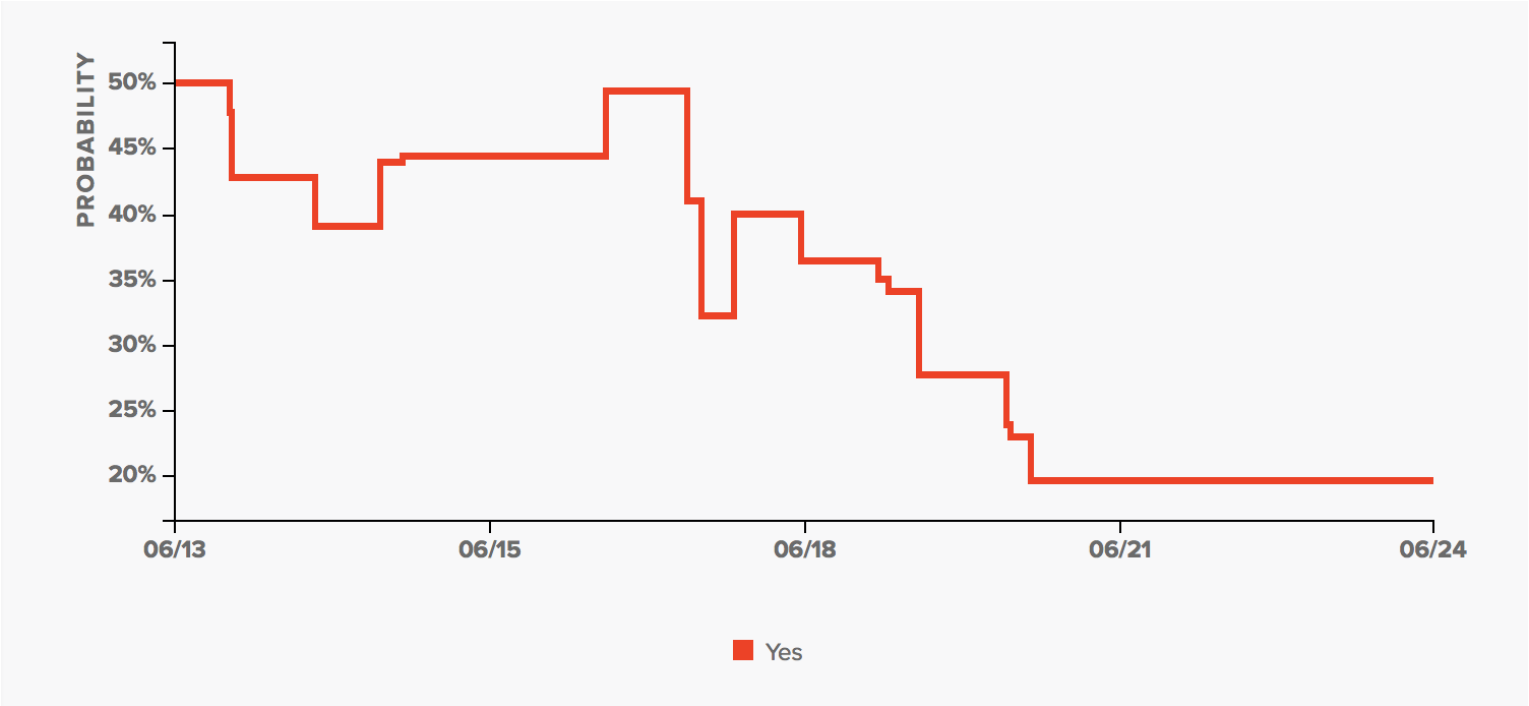


Figure: Will the UK vote to leave the EU in the June 2016 referendum? A Financial Times poll of polls found 48% remain, 46% leave.

Motivation

- When do prediction markets (or more generally financial markets) fail to aggregate information relative to other methods?
- Most theoretical results on information aggregation assume precise beliefs and expected utility
- But for uncommon or "once in a generation" events, for which there is no experience in forecasting, maybe beliefs are imprecise
- We examine this question by introducing imprecise beliefs and ambiguity

Separability and expected utility

- [Ostrovsky, 2012]: With expected utility, "separable" securities aggregate information in both strategic and non-strategic settings
- A security X is separable if, whenever its expected value is common knowledge, there is no uncertainty about is value
- Immune to manipulation
- An Arrow-Debreu security is always separable

Separability and ambiguity

- Separable securities may fail to aggregate information even if belief imprecision is vanishingly small
- Outcome can be manipulated by an uninformed market maker who sets the initial value
- Traders may eventually agree on the wrong value of the security

Main results I

- Show theoretically and experimentally that separable securities
  - may fail to aggregate information under ambiguity
  - the initial price can influence the result, allowing for manipulation
- Define a new class of securities, called **strongly separable**
- Show theoretically and experimentally that they
  - aggregate information under ambiguity in both strategic and non-strategic settings
  - They are immune to manipulation

Main results II

- There is no security that is strongly separable for all information structures
  - unlike with EU, where Arrow-Debreu are always separable
  - With ambiguity, it is impossible to design a market that aggregates information if we don't know the information structure
- We use the MEU model of [Gilboa and Schmeidler, 1989]
  - Results generalise to variational preferences ([Maccheroni et al., 2006])
  - Surprisingly, strongly separable securities stay the same with variational preferences

Related literature

- No trade theorems: [Aumann, 1976], [Milgrom and Stokey, 1982], [Geanakoplos and Polemarchakis, 1982], [Sebenius and Geanakoplos, 1983]
- Market Scoring Rules: [McKelvey and Page, 1990] and [Hanson, 2003, Hanson, 2007]
- Separability and information aggregation: [DeMarzo and Skiadas, 1998], [Ostrovsky, 2012], [Chen et al., 2012], [Dimitrov and Sami, 2008]

Preliminaries

- Finite state space Ω and $i = 1, ..., n$ traders
- Security $X : \Omega \to \mathbb{R}$
- Asymmetric information, where $\Pi = \{\Pi_1, ..., \Pi_n\}$ is the information structure and their join is the finest partition

Ambiguity

- MEU preferences [Gilboa and Schmeidler, 1989]
- Common set of priors $\mathcal{P}$
- Prior-by-prior updating
- Traders are risk neutral

Sequence of play

- Market maker's initial announcement is y_0
- In period t_1, player 1 announces y_1, in t_2 player 2 announces y_2 ...
- There are infinitely many periods

Payoffs

- Let $x^* = X(\omega)$ be the true value of the security, if the state is ω
- Trader i's payoff from announcing y_t at time t is

$$s(y_t, x^*) - s(y_{t-1}, x^*)$$

- $s(y_t, x^*)$ is a *strictly proper scoring rule*
- For example, quadratic scoring rule: $s(y, x^*) = -(y - x^*)^2$
- For every prior p and security X, the expectation of $s(y, x)$ is maximised, uniquely, for $y = E_p[X]$

Two settings

- Non-strategic $\Gamma^M(\Omega, I, \Pi, X, \mathcal{P}, y_0, Y, s)$, where traders only care about the current payoff
- Strategic $\Gamma^S(\Omega, I, \Pi, X, \mathcal{P}, y_0, Y, s, \beta)$ where traders discount their future payoffs using β

Information aggregation

Under a profile of strategies in Γ^M or Γ^S, information gets aggregated if sequence $\{y_k\}_{k=1}^{\infty}$ converges in probability to random variable $X(\omega)$, for all $\omega \in \Omega$

Separability

A security X is non-separable under partition structure Π if there exists probability p and value $v \in \mathbb{R}$ such that:

(*i*) $X(\omega) \neq v$ for some $\omega \in Supp(p)$

(*ii*) $E_p[X|\Pi_i(\omega)] = v$ for all $i = 1, ..., n$ and $\omega \in Supp(p)$

Otherwise, it is separable

A "generalization" of expectation

- With MEU preferences, myopic announcement is

$$d_{\mathcal{P}}(E, v) = \underset{y \in Y}{argmax} \min_{p \in \mathcal{P}_E} E_p\left[s(y, X) - s(v, X)\right]$$

- Unique by Lemma 1
- Equal to $E_p[X|F]$ for some $p \in \mathcal{P}$
- Depends on previous announcement v if $\mathcal{P}$ is not a singleton

Strong separability

A security X is not strongly separable under partition structure Π and proper scoring rule s if there exist a regular $\mathcal{P} \subseteq \Delta(\Omega)$ with respect to each Π_i, $i = 1, ..., n$, and $v \in \mathbb{R}$ such that:

(*i*) $X(\omega) \neq v$ for some $\omega \in \bigcup_{p \in \mathcal{P}} Supp(p)$

(*ii*) $d_{\mathcal{P}}(\Pi_i(\omega), v) = v$ for all $i = 1, ..., n$ and $\omega \in \bigcup_{p \in \mathcal{P}} Supp(p)$

Otherwise, it is strongly separable

Information aggregation with myopic traders

Theorem

Fix security X, information structure Π and continuous strictly proper scoring rule s. Information gets aggregated for any regular $\Gamma^M(\Omega, \Pi, I, X, \mathcal{P}, y_0, Y, s)$ if and only if X is strongly separable

Dynamic Consistency (DC)

- Prior-by-prior updating violates DC, unless beliefs are rectangular
- DC is problematic with common priors and ambiguity aversion ([Ellis, 2018])
- We do not impose DC but suggest the Revision-proof equilibrium, which is immune to any 'collective' deviations by a trader and her future selves

Revision-proof equilibrium

- Two main differences from the Revision-proof equilibrium of [Asheim, 1997] and [Ales and Sleet, 2014]
- Their notion is only for complete information games, so we generalise
- We check for deviations for all future selfs of a specific player, instead of all future players

Theorem

Fix information structure Π *and bounds* Y

(*i*) *If security* X *is strongly separable under* Π*, then for any* Γ^S *and any Revision-proof equilibrium, information gets aggregated*

(*ii*) *If security* X *is not strongly separable under* Π*, then there exist game* Γ^S *and a Revision-proof equilibrium such that information does not get aggregated*

Testable implications

- Strongly separable securities aggregate information in both SEU and Ambiguity environments
- The initial value does not matter, so an uninformed market maker cannot manipulate the market
- Separable securities may not perform as well

Design

- Our experimental design focuses on three dimensions
  1. Subjective Expected Utility (SEU) vs. Maxmin Expected Utilty (MEU)
  2. Separable vs. Strongly Separable securities
  3. The uninformed market maker's initial value of 0 vs. 50
- $2 \times 2 \times 2$ experimental between-subjects' design

Conclusions

- Separable securities may not aggregate information if traders have imprecise beliefs
- Prices might be manipulated
- We introduce a new class of securities, strongly separable, which aggregate information in both strategic and non-strategic environments and are immune to manipulation
- These results are confirmed experimentally
- There is no security that can aggregate information for all information structures

Thank you!

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