Sequential choice and self-reinforcing rankings

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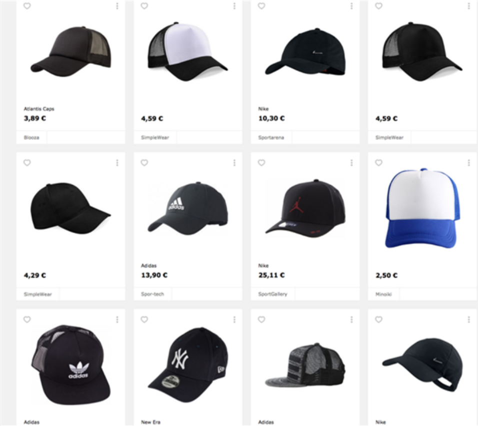
with Pantelis P. Analytis, Francesco Cerigioni & Hrvoje Stojic

Conference on Research on Economic Theory and Econometrics (CRETE)
Tinos, July 2022

Rankings are widespread



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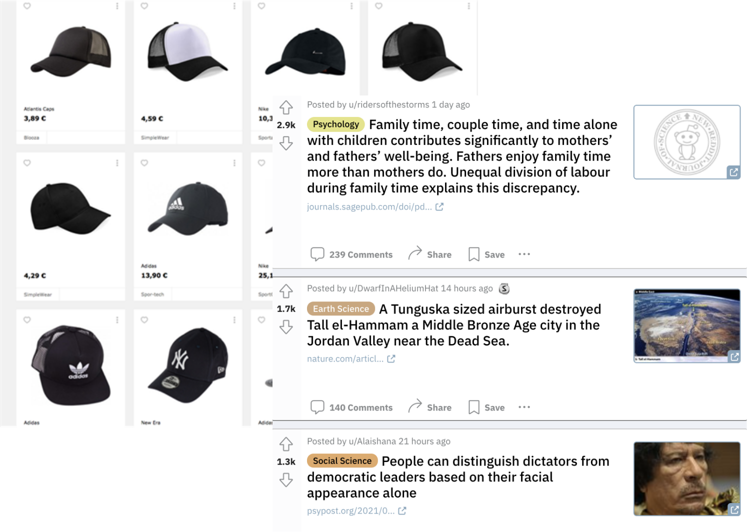


Posted by u/ridersofthestorms 1 day ago

Psychology Family time, couple time, and time alone with children contributes significantly to mothers' and fathers' well-being. Fathers enjoy family time more than mothers do. Unequal division of labour during family time explains this discrepancy.

journals.sagepub.com/doi/pd...

Posted by u/DwarfInAHeliumHat 14 hours ago

Earth Science A Tunguska sized airburst destroyed Tall el-Hammam a Middle Bronze Age city in the Jordan Valley near the Dead Sea.

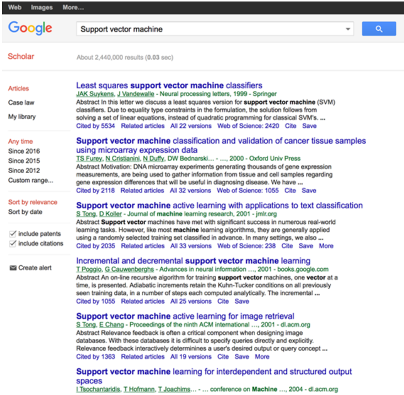
nature.com/articl...

Posted by u/Alaishana 21 hours ago

Social Science People can distinguish dictators from democratic leaders based on their facial appearance alone

psypost.org/2021/0...

Rankings influence choice probabilities



Support vector machine

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Least squares **support vector machine** classifiers
JAK Suykens, J Vandewalle - Neural processing letters, 1999 - Springer
Abstract In this letter we discuss a least squares version for **support vector machine** (SVM) classifiers. Due to equality type constraints in the formulation, the solution follows from solving a set of linear equations, instead of quadratic programming for classical SVM's. ...
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Support vector machine classification and validation of cancer tissue samples using microarray expression data
TS Furey, N Cristianini, N Duffy, DW Bednarski... - ..., 2000 - Oxford Univ Press
Abstract Motivation: DNA microarray experiments generating thousands of gene expression measurements, are being used to gather information from tissue and cell samples regarding gene expression differences that will be useful in diagnosing disease. We have ...
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Support vector machine active learning with applications to text classification
S Tong, D Koller - Journal of **machine** learning research, 2001 - jmlr.org
Abstract **Support vector** machines have met with significant success in numerous real-world learning tasks. However, like most **machine** learning algorithms, they are generally applied using a randomly selected training set classified in advance. In many settings, we also ...
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Incremental and decremental **support vector machine** learning
T Poggio, G Cauwenberghs - Advances in neural information ..., 2001 - books.google.com
Abstract An on-line recursive algorithm for training **support vector** machines, one **vector** at a time, is presented. Adiabatic increments retain the Kuhn-Tucker conditions on all previously seen training data, in a number of steps each computed analytically. The incremental ...
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Support vector machine active learning for image retrieval
S Tong, E Chang - Proceedings of the ninth ACM international ..., 2001 - dl.acm.org
Abstract Relevance feedback is often a critical component when designing image databases. With these databases it is difficult to specify queries directly and explicitly. Relevance feedback interactively determinines a user's desired output or query concept ...
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Support vector machine learning for interdependent and structured output spaces
I Tsochantaridis, T Hofmann, T Joachims... - ... conference on **Machine** ..., 2004 - dl.acm.org

[PDF] wustl.edu

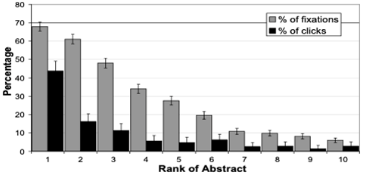

Figure 1: Percentage of times an abstract was viewed/clicked depending on the rank of the result.

Observation: The probabilities of choosing options are directly influenced by rank-orders (e.g. see Joachims et al. 2005)

Research question and previous work

Self-reinforcement of popularity creates non-trivial dynamics

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Previous work:

- Generalized Pólya urns (Arthur et al., 1986)
- Specific algorithms & user assumptions with some theoretical results (Germano et al., 2019; Gaeta et al., 2022; Ciampaglia et al., 2018)

Overview

What we'll see:

- A unified framework to talk about sequential choice under the effect of rankings

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- A unified framework to talk about sequential choice under the effect of rankings
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- Examples of models from the literature covered by our framework
- Extensions and consequences for utility (if time allows)

The model in simple terms

- Fixed set of items

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- Large number of agents, choosing sequentially
- Keeping track of items' popularity (i.e. number of agents that have chosen each item)
- Choice probabilities of each agent determined by the ranking by popularity at the time.

General formulation

- X: set of items to choose from

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- **Popularity** of $x \in X$ at time $n \in \mathbb{N}$:

$$Z_n(x) = card\{k \leq n : A_k = x\}$$

A_k: Item chosen by agent k

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Choice probabilities for n-th agent:

$$\mathrm{P}\left(A_n = x \mid R_{n-1} = r\right) = q(x; r)$$

where

$$q : X \times R \to (0; 1]; \qquad \sum_{x \in X} q(x; r) = 1; \quad \forall r \in R$$

Two items

Choice probability	Item 1 first
Choose item 1	0.7
Choose item 2	0.3

Two items

Choice probability	Item 1 first	Item 2 first
Choose item 1	0:7	0:4
Choose item 2	0:3	0:6

Two items

Choice probability	Item 1 first	Item 2 first	Tie
Choose item 1	0:7	0:4	0:55
Choose item 2	0:3	0:6	0:45

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Case 2 occurs with probability 32%.

Example - No convergence with 3 items

r	$q(a; r)$	$q(b; r)$	$q(c; r)$
$(a; b; c)$	0:6	0:15	0:25
$(a; c; b)$	0:4	0:1	0:5
$(b; a; c)$	0:5	0:4	0:1
$(c; a; b)$	0:15	0:25	0:6
$(b; c; a)$	0:25	0:6	0:15
$(c; b; a)$	0:1	0:5	0:4

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(b; c; a)	0:25	0:6	0:15
(c; b; a)	0:1	0:5	0:4

Choice probabilities satisfy the following:

Whenever an item goes up one position, its choice probability increases .

Main results 1 - Convergence

Stochastic preference any total order P on X

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Definition (Stochastic preference compatibility - SPC)

A ranking-based stochastic choice function q is stochastic preference compatible if there exists some total order P on X such that for any $a, b \in X$ and any ranking $r \in R$,

$$\text{if} \quad aPb \quad \text{and} \quad a \;_{r}\, b, \quad \text{then} \quad q(a; r) > q(b; r).$$

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Note: The limit of R_n does not necessarily agree with P.

Main results 1b - Convergence

Definition (Stochastic preference compatibility - SPC)

A ranking-based stochastic choice function q is stochastic preference compatible if there exists some strict partial order P on X such that for any $a, b \in X$ and any ranking $r \in R$,

$$\text{if} \quad b \not P a \quad \text{and} \quad a \succ_r b, \quad \text{then} \quad q(a; r) > q(b; r).$$

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Main results 1c - Convergence

Definition (Stochastic preference compatibility - SPC)

A ranking-based stochastic choice function q is stochastic preference compatible if there exists some asymmetric binary relation P on X such that for any $a; b \in X$ and any ranking $r \in R$,

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Main results 2 - Possible limits

R_n and $\frac{Z_n}{n}$ converge, but to what limit?

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For any $r \in \mathcal{R}$, we have $P(R_n \to r) > 0$ (for any initial condition) if and only if r contains no ties and it ranks the items by descending choice probability, that is

$$q\left(r^{-1}(1); r\right) > q\left(r^{-1}(2); r\right) > \dots > q\left(r^{-1}(N); r\right).$$

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Item	Choice probability
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4	0.1

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Item	Choice probability	Item	Choice probability	Item	Choice probability
3	0.4	3	0.4	2	0.32
1	0.3	2	0.27	3	0.35
2	0.2	1	0.23	1	0.23
4	0.1	4	0.1	4	0.1

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Not a possible limit

Behavioral models - 1

- Search and satisficing [Simon, 1955], with search order induced by ranking:

$$q(a;r) = \frac{\prod_{x \succ_r a}(1 - p_x)\, p_a}{\sum_{y \in X} \prod_{x \succ_r y}(1 - p_x)\, p_y};$$

p_x: probability x accepted when searched.

SPC satisfied with aPb, $p_a > p_b$.

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- Multiplicative attention modulation (Germano et al., 2019):

$$q(a;r) = \frac{{}^{r(a)}\, v_a}{\sum_{x \in X} {}^{r(x)}\, v_x}$$

v_x: intrinsic quality of item x, $\in (0;1)$

SPC satisfied with aPb, $v_a > v_b$.

See also (Ciampaglia et al., 2018; Gaeta et al., 2022).

Behavioral models - 2

- **Consideration sets model** (Manzini & Mariotti, 2014; Masatlioglu, Nakajima, & Ozbay, 2012), with top- K ranked items in the consideration set, then use Luce rule:

$$q(a;r) = \sum_{k} P(K=k) \frac{v(a)}{\sum_{x:r(x)\le k} v(x)} 1_{r(a)\le k}.$$

- **Random utility model** with additive ranking-dependent term (Pope, 2009):

$$q(a;r) = P\left(a = \arg\max_{x\in X} u(x;r)\right);$$

where

$$u(x;r) = v(x) + b(r(x)) + \epsilon(x)$$

b: decreasing function

ϵ: random error term

- **Herding model** - Rational agents with unreliable private information (Banerjee, 1992).

Heterogeneous agents

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$$\bar{q}(x,r) = \int_{w \in W} q(x,r,w)\, dQ$$

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Proposition

If $q(\cdot;\cdot;w)$ is compatible with a (fixed) stochastic preference relation P for each $w \in W$, then $\bar{q}$ is also compatible with P.

Utility and optimal ranking

Each agent experiences a utility form their choice:

$$u : X \quad R \quad W \,! \qquad R$$

(e.g. search and satisficing model, random utility model)

Utility and optimal ranking

Each agent experiences a utility form their choice:

$$u : \mathcal{X} \times \mathcal{R} \times \mathcal{W} \to \mathbb{R}$$

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Average utility experienced by first n agents:

$$\bar{u}_n = \frac{1}{n} \sum_{k=1}^{n} u(A_k, R_{k-1}, W_k),$$

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If $R_n \to r$, then $\overline{u}_n \to V(r)$ with probability 1, where

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Proposition

If $R_n \to r^*$, then $\bar{u}_n \to V(r^*)$ with probability 1, where

$$V(r) = \sum_{x \in \mathcal{X}} \int_{w \in \mathcal{W}} q(x, r, w) \cdot u(x, r, w) \, dQ,$$

Optimal ranking: $r^* = \arg\max_{r \in \mathcal{R}} V(r)$

Utility and optimal ranking

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Optimal ranking: $r^* = \arg\max_{r \in R} V(r)$

$\Rightarrow$ R_n may in general converge to a suboptimal ranking

Summary and outlook

- A new theoretical framework for sequential choice systems and popularity dynamics
- Results about long-term behavior (possible outcomes, market shares), easy to check conditions
- Covers a wide variety of behavioral models
- Theory can cover other types of quantities with rich-get-richer dynamics (instead of popularity)
- Important caveat: It is restricted to ranking-based dynamics

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