Constrained QML Estimation for Multivariate Asymmetric MEM with Spillovers: The Practicality of Matrix Inequalities

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- To keep variables (that take only positive values, i.e. conditional variances), generated by time series models nonnegative, researchers should impose inequality constraints on the parameters of the process.
- As an example consider the GARCH(p, q) model:

$$h_t = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j}.$$

- In order to ensure that h_t is positive at all time, the (sufficient) nonnegativity constraints are

$$\omega > 0; \alpha_i, \beta_j \geq 0, \text{ for all } 1 \leq i \leq q, \, 1 \leq j \leq p.$$

(see for example Bollerslev, 1986, JE).

- Infinite ARCH Representation:

$$h_t = \omega^* + \psi_1 \varepsilon_{t-1}^2 + \psi_2 \varepsilon_{t-2}^2 + \psi_3 \varepsilon_{t-3}^2 + \cdots + \psi_k \varepsilon_{t-k}^2 + \cdots .$$

- The first researchers to relax some of these conditions for higher order models, i.e., GARCH(p, q), were Daniel Nelson and Cao (1992, JEBS)
- They show that these constraints can be substantially weakened and so should not be imposed in estimation
- Some of these parameters can be allowed to take negative values
- They provided sufficient but not necessary conditions
- They also provide empirical examples illustrating the importance of relaxing these constraints

- The Conditions in Nelson and Cao (1992, JBES):
- 1. $\omega > 0$,
- 2. ϕ_1 is real and positive
- (where ϕ_1 is the highest root from the p (distinct) inverse roots of the GARCH polynomial: $1 - \sum_{j=1}^{p} \beta_j z^j$)
- 3. $A(\phi_1^{-1}) > 0$, where $A(z)$ is the ARCH polynomial: $A(z) = \sum_{i=1}^{q} \alpha_i z^i$.
- 4. If 2 and 3 hold, then only the first ψ_i coefficients, $i = 1, \ldots, k$, should be checked (if they are positive)
- (NC, 1992, gave a simple formula for k)

- As noted by Rabemananjara and Zakoian (1993) and Knight and Satchell (2007), non-negativity constraints on the parameters maybe a source of important difficulties in running estimation procedures.
- With a shock in the past, regardless of the sign, always has a positive effect on the current volatility: the impact increases with the magnitude of the shock. Therefore, cyclical or any non-linear behavior in volatility cannot be taken into account.
- He and Teräsvirta (1999, JTSA) show that the less severe nonnegativity constraints allow more flexibility in the shape of the autocorrelation function than the constraints restricting the parameters to be nonnegative.

- The paper of Nelson and Cao (1992) generated a small literature.
- Tsai and Chan (2008, ET) show that the sufficient conditions of Nelson and Cao (1992) are indeed necessary as well.
- Conrad and Haag notice the importance of these results and they(he) applied them on the FIGARCH(p, d, q) and the HYGARCH model
- (see Conrad and Haag, JFinEco, 2006; Conrad, JE, 2010)

- The fractionaly inegrated (FI) model introduces a long memory characteristic into the GARCH process by replacing $(1 - \beta L)$ where L is the lag operator with $(1 - L)^d$ where $0 \leq d \leq 1$
- The same inequality constraints apply to the LMGARCH(p, d, q) model (see Karanasos, Psaradakis and Sola, JTSA, 2004; Conrad and Karanasos, EL, 2006)

- The problem becomes much more interesting (and important) when we move from the univariate case to the multivariate one.

Consider as an example the bivariate GARCH$(1,1)$ model:

$$\begin{aligned} h_{1t} &= \omega_1 + \alpha_{11}\varepsilon^2_{1,t-1} + \beta_{11} h_{1,t-1}, \\ h_{2t} &= \omega_2 + \alpha_{22}\varepsilon^2_{2,t-1} + \beta_{22} h_{2,t-1}, \end{aligned}$$

or in a matrix form

$$\mathbf{h}_t = \boldsymbol{\omega} + \mathbf{A}\varepsilon^2_{t-1} + \mathbf{B}\mathbf{h}_{t-1},$$

where

$$\mathbf{A} = \begin{bmatrix} \alpha_{11} & 0 \\ 0 & \alpha_{22} \end{bmatrix}, \ \mathbf{B} = \begin{bmatrix} \beta_{11} & 0 \\ 0 & \beta_{22} \end{bmatrix}$$

- One can allow for spillover effects by allowing the **A** and **B** matrices to be full:

$$\mathbf{A} = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix}, \ \mathbf{B} = \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix},$$

That is:

$$\begin{aligned} h_{1t} &= \omega_1 + \alpha_{11}\varepsilon^2_{1,t-1} + \beta_{11}h_{1,t-1} + \alpha_{12}\varepsilon^2_{2,t-1} + \beta_{12}h_{2,t-1}, \\ h_{2t} &= \omega_2 + \alpha_{22}\varepsilon^2_{2,t-1} + \beta_{22}h_{2,t-1} + \alpha_{21}\varepsilon^2_{1,t-1} + \beta_{21}h_{1,t-1}, \end{aligned}$$

- This is the so called extended bivariate GARCH$(1, 1)$ model: Jeantheau, T. (1998, ET)
- (it is extended by allowing for shock and volatility spillovers)

- If one relaxes the nonnegativity constraints and allow some of the parameters in the **A** and **B** matrix to be negative then this model is called bivariate unrestricted extended (UE) GARCH$(1, 1)$ model (see Conrad and Karanasos, ET, 2010).

- The inequality constraints are: $\mathbf{A} > \mathbf{0}$, and only two elements (of the same column) in the $\mathbf{B}$ matrix can be negative.

- We were applying this model to inflation and output growth data (see for example, Conrad, Karanasos and Zeng, EL, 2010; Karanasos and Zeng, Book, 2013)
- Why?
- Negative Volatility Spillovers: Economic and Financial theories

- The debate about the inflation-growth interaction is linked to another ongoing dispute, that of the existence or absence of a variance relationship. As Fuhrer (1997) puts it:
- ‘It is difficult to imagine a policy that embraces targets for the level of inflation or output growth without caring about their variability around their target levels. The more concerned the monetary policy is about maintaining the level of an objective at its target, the more it will care about the variability of that objective around its target’.

- There are many controversies in the theoretical literature on the relationship between the four variables (see Fountas et al., OBES, 2006; Fountas and Karanasos, JIMF, 2007 and the references therein).
- The extent to which there is an interaction of either sign between the two variances is an issue that cannot be resolved on merely theoretical grounds.
- Not only that, the models regarding the 'uncertainty link' that do exist are often ambiguous in their predictions. These considerations reinforce a widespread awareness of the need for more empirical evidence, but also make clear that a good empirical framework is lacking.

- In Conrad and Karanasos (ET, 2010) we employed the bivariate process to examine how US nominal and real uncertainties are interrelated.
- We got the following estimated results (1 is inflation and 2 is output growth):

$$\mathbf{B} = \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix} = \begin{bmatrix} 0.888 & -0.010 \\ 1.296 & 0.512 \end{bmatrix},$$

- Interestingly, there is a bidirectional feedback between the two variables.
- In particular, there is strong evidence supporting the Logue and Sweeney (1981) theory that inflation uncertainty has a positive impact on the volatility of growth.
- In sharp contrast real variability affects nominal uncertainty negatively as predicted by, among others, Fuhrer (1997).

- Clearly, the negative coefficient β would have been ruled out by the sufficient Bollerslev conditions. On the other hand, it is easy to check that in this example the conditions of Proposition 1 are satisfied for the given parameter combination. Moreover, the conditions for the existence of the unconditional second and fourth moments are satisfied as well (based on a paper by He and Terasvirta, ET, 2005).

- How we manage to obtain these inequality constraints for the multivariate case?
- That was actually relatively simply.
- We noticed that the bivariate case have the so called univariate representation

- That is, we can have the following representation:

$$
\begin{aligned}
h_{1t} &= z_1 + a_{11}^{(1)}\varepsilon_{1,t-1}^2 + a_{12}^{(1)}\varepsilon_{1,t-2}^2 + a_{11}^{(2)}\varepsilon_{2,t-1}^2 + a_{12}^{(2)}\varepsilon_{2,t-2}^2 \\
&\quad + b_{11}h_{1,t-1} + b_{12}h_{1,t-2}, \\
h_{2t} &= z_2 + a_{21}^{(1)}\varepsilon_{1,t-1}^2 + a_{22}^{(1)}\varepsilon_{1,t-2}^2 + a_{21}^{(2)}\varepsilon_{2,t-1}^2 + a_{22}^{(2)}\varepsilon_{2,t-2}^2 \\
&\quad + b_{21}h_{2,t-1} + b_{22}h_{2,t-2},
\end{aligned}
$$

  where each conditional variance can be expressed as a univariate GARCH$(2,2)$ process,
- that has two ARCH polynomials.

- In other words,
- h_{1t} depends only in its own two lags and the two lags of the two (squared) errors,
- similarly h_{2t} depends only in its own two lags $(h_{2,t-1}, h_{2,t-2})$ and the two lags of the two (squared) errors $(\varepsilon^2_{2,t-1}, \varepsilon^2_{2,t-2})$
- where the a's and the b's are functions of the α's and the β's.

- Therefore, one can directly apply the results of Nelson and Cao (1991) and Tsai and Chan (2008) to the bivariate (or multivariate case as well)
- The only difference is that in the bivariate case, you do not have only one GARCH$(2,2)$ process (kernel) to check but 4!

$$
\begin{aligned}
h_{1t} &= z_1 + a_{11}^{(2)}\varepsilon_{2,t-1}^2 + a_{12}^{(2)}\varepsilon_{2,t-2}^2 + b_{11}h_{1,t-1} + b_{12}h_{1,t-2} \\
h_{1t} &= z_1 + a_{11}^{(1)}\varepsilon_{1,t-1}^2 + a_{12}^{(1)}\varepsilon_{1,t-2}^2 + +b_{11}h_{1,t-1} + b_{12}h_{1,t-2} \\
h_{2t} &= z_2 + a_{21}^{(1)}\varepsilon_{1,t-1}^2 + a_{22}^{(1)}\varepsilon_{1,t-2}^2 + b_{21}h_{2,t-1} + b_{22}h_{2,t-2} \\
h_{2t} &= z_2 + a_{21}^{(2)}\varepsilon_{2,t-1}^2 + a_{22}^{(2)}\varepsilon_{2,t-2}^2 + b_{21}h_{2,t-1} + b_{22}h_{2,t-2}.
\end{aligned}
$$

- The research by Nelson and Cao (1992), He and Teräsvirta (1999), Gourieroux (2007, JFinEconometr), Tsai and Chan (2007, 2008, ET), Nakatani and Teräsvirta (2008, 2009, EL), underline the theoretical interest in the derivation of such necessary and sufficient conditions. This strand of the literature originated with the seminal work of Nelson (1991a,b).

- What we do in our current paper (the first part of our contribution) is to show that these non-negativity constraints are translated into simple matrix inequalities, which are easily handled
- (are dealt without difficulty)
- The matrix inequalities can be practically checked with ease and **they can even effortlessly enforced in estimatio**n.

- New theoretical results on multivariate HEAVY/MEM/GARCH models including conditions which ensure the non-negativity of the variables (conditional means/variances).
- We propose simple matrix inequalities which incorporate these constraints.
- The relevance and the importance of the proposed method is demonstrated with three empirical examples on three different real datasets.

How did we derive these matrix inequalities?

- Consider again the univariate GARCH(1, 1) model:

$$h_t = \omega + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1}.$$

- It has the following infinite ARCH representation:

$$h_t = \omega + \alpha \varepsilon_{t-1}^2 + \alpha\beta \varepsilon_{t-2}^2 + \alpha\beta^2 \varepsilon_{t-3}^2 + \alpha\beta^3 \varepsilon_{t-4}^2 + \cdots .$$

- In order for $h_t > 0 \ \forall \ t$ all the parameters in the ARCH(∞) representation must be nonnegative.
- That is ω, α and $\alpha\beta$ must be nonnegative and that's how one obtains the inequality constraints: $\omega \geq 0$, and α, $\beta > 0$.

- Thus in the univariate GARCH(1, 1) model one should check only if the first two ARCH(∞) coefficients are nonnegative:

$$h_t = \omega + \alpha\varepsilon_{t-1}^2 + \alpha\beta\varepsilon_{t-2}^2 + \alpha\beta^2\varepsilon_{t-3}^2 + \alpha\beta^3\varepsilon_{t-4}^2 + \cdots .$$

- If this is the case then all other ARCH(∞) coefficients will also be nonnegative.

- In the current contribution we notice that something similar holds for the multivariate case but instead of elements in the ARCH(∞) representation we have matrices:

$$\mathbf{h}_t = \boldsymbol{\omega}^* + \mathbf{A}\varepsilon_{t-1}^2 + \mathbf{B}\mathbf{A}\varepsilon_{t-2}^2 + \mathbf{B}^2\mathbf{A}\varepsilon_{t-3}^2 + \mathbf{B}^3\mathbf{A}\varepsilon_{t-4}^2 + \cdots .$$

- Thus, iff
- 1. $\boldsymbol{\omega}^* = adj[\mathbf{I} - \mathbf{B}]\boldsymbol{\omega} > \mathbf{0}$: this is equivalent to $\omega^* > 0$ for the univariate case
- 2. GARCH polynomial: $B(z) = \det[\mathbf{I} - \mathbf{B}z]$, ϕ_1 should be real and positive:
- this is equivalent to ϕ_1 should be real and positive for the univariate case
- 3. $adj[\mathbf{I}\phi_1 - \mathbf{B}]\mathbf{A} > \mathbf{0}$, this is equivalent to: $A(\phi_1^{-1}){>}0$ for the univariate case

- Multivariate ARCH(∞) representation

$$\mathbf{h}_t = \omega + \mathbf{A}\varepsilon_{t-1}^2 + \mathbf{BA}\varepsilon_{t-2}^2 + \mathbf{B^2A}\varepsilon_{t-3}^2 + \mathbf{B}^3\mathbf{A}\varepsilon_{t-4}^2 + \cdots .$$

- So in the bivariate case we must have $\mathbf{A}, \mathbf{BA} \geq \mathbf{0}$,
- in the trivariate case $\mathbf{A}, \mathbf{BA}, \mathbf{B^2A} \geq \mathbf{0}$ and so on...
- notice that in Conrad and Karanasos (2010) we give exact form solutions ($\mathbf{A}, \mathbf{BA} \geq \mathbf{0}$) only for the bivariate case.
- In the current paper instead of working with univariate GARCH representations
- we work with infinite ARCH representations (in the multivariate context)

- Same Model, Different Acronym: Instead of UE:
- we use the acronym: **MUFA**-GARCH:
- Multivariate Unrestricted Full Assymetric GARCH
- MUFA (written as MOUFA) in Greek means: **fake**, grotty!
- So in Greek it has a negative meaning!
- In Italian: MUFA (written as: MUFFA) means: mold, **moldy**:
- la muffa de formaggio: cheese mold
- In Spanish: MUFA means **bad luck** *(mala suerte)*, misfortune, **bad mood** *(mal humor)*! Very Negative!
- But in English MUFA means: A Monounsaturated Fatty Acid
- which is a healthy type of fat!

- M is for munificent, for you are extremely liberal in giving
- U is for upstanding, the honorable way to be
- F is for fair, always honest and true
- A is for accord, the harmony you spread

MUFA is also a Brazilian (I think) Heavy/Death metal group:

MUFA

listen to the brutality show

https://www.youtube.com/watch?v=gt4f4ZA5aic
Lyrics:
Let's start the war now
Beginning the brutality show
Troops of junkies are ready, we're gonna kill all bastards
Let's start the war now (the war now)
Begninning the brutality show
Only madness will survive
You'll see, how stupid we can be!
No hope, no fear
Today I'm gonna attack
I'm ready for destruction, I'm ready to kill
No hope, no fear
Today I'm gonna attack
I'm ready to kill, I'm ready to die
In each corner a puddle of blood
In each mother a dead son

Fuck all the cops
'Cause assholes must die
Destroy them all
By the name of the state
Then you don't even vote, cause you don't give a damn
Destroy them all
By the name of a god
Then you don't even believe, cause you don't give a fuck!
Let's start the war!
Let's start the war now!

- There are a number of papers, using multivariate MEM models with spillovers, i.e. Cipollini, Engle and Gallo (2013, JAE), Manganelli (2005, JFM), Cipollini and Galo (2010, CSDA),
- where the estimated models and parameters violate some of the inequality constraints
- We reestimate (extend) these models taking into account the inequality constraints:
- a four variate MEM model with four stock return volatilities
- a trivariate MEM with realized volatility, absolute returns and high minus low
- a trivariate model with duration, volume and stock volatility

- In what follows we graphically illustrate the necessary and sufficient parameter set for the trivariate MUFA-MEM(1,1) model. This will provide a better understanding of the results presented in the previous subsection. We discuss four examples. We allow two off-diagonal elements of B to vary from -0.5 to 0.5.
- In the fourth example, we examine if more than two off-diagonal elements in **B** matrix can be negative. To do so, we restrict b_{21} to be negative and vary b_{13} and b_{31}.
- The parameters chosen are mainly from empirical results from case FTSE in dataset 2.

The Data generation process is given by:

$$\text{DGP Ex.4}$$

$$\begin{pmatrix} 0.214 & 0.184 & 0.164 \end{pmatrix}$$

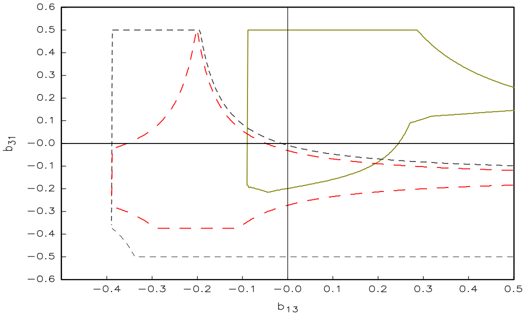
$$A$$

$$\begin{pmatrix} 0.078 & 0.012 & 0.171 \\ 0.012 & 0.005 & 0.100 \\ 0.048 & 0.029 & 0.228 \end{pmatrix}$$

$$B$$

$$\begin{pmatrix} 0.743 & 0.031 & b_{13} \\ -\mathbf{0.028} & 0.851 & 0.053 \\ b_{31} & 0.111 & 0.548 \end{pmatrix}$$

- Recall that in the trivariate case the conditions are:
- $\mathbf{A}, \mathbf{BA}, \mathbf{B}^2\mathbf{A} \geq \mathbf{0}$
- plus one more condition (that involves the adj of a matrix)
- plus a condition about the constants
- We also have the condition for the existence of the unconditional first moments



Bold solid lines represent the non-negativity restrictions (implied by our Theorem)

Dotted grey lines represent the restrictions implied by the existence of the unconditional first moment.

Dashed red lines represent the restrictions implied by the existence of the unconditional second moment.

it is interesting to see that three off-diagonal elements in **B** matrix can be negative.

- For the specification of the conditional distribution, Xu(2013) propose to use multivariate lognormal distribution (for the MEM model),
- Then the model can be estimated consistently by QMLE.
- An alternative estimation method was proposed by Cipollini et al. (2013).
- They bypass the specification the error distribution and make use of only the first two moments. An efficient GMM estimation method was proposed.
- By a simulation study, Xu(2013) has shown that both the QML and GMM are consistent
- And the efficient loss of QML compare with GMM due to misspecification of the error distribution is trivial.

- Caporin (2007) argues that highly volatile stock prices induce a decrease in trading variations, stabilizing the average volume of trades.
- While such a negative effect from stock volatility to volume variability is untestable under the Bollerslev conditions, the necessary and sufficient conditions of Proposition 1 allow for such a scenario.

Table 4. Trivariate UF MEM (1, 1) of intra day trading duration, stock volume and volatility.

	AVT			COX		
B	**0.873**	-	-	**0.873**	0.001	-
	(90.2)			(148.3)	(0.18)	
	-	**0.775**	-	**0.001**	**0.987**	**-0.004**
		(3.12)		(3.04)	(587.8)	(5.64)
	0.017	-0.223	**0.826**	0.002	0.003	**0.686**
	(2.21)	(0.75)	(81.9)	(0.45)	(0.04)	(110.1)

Notes: We use the same data set as Manganelli (2005).
1st row: duration; 2nd row: volume; 3rd row: volatility.

- Only for the COX case, does the conditional mean of volatility affect that of stock volume negatively. This result is in alignment with the work of Li and Wu (2006).
- Clearly, the negative value of β_{23} would have been ruled out by the sufficient Bollerslev conditions.
- It is easy to check that the matrix inequality constraints of our Theorem are satisfied for the given parameter combination (see Table C1 in Section C of the supplementary Appendix).

Table 4. Trivariate UF MEM (1, 1) of intra day trading duration, stock volume and volatility.

	CP			DLP		
B	**0.950**	0.001	-	**0.912**	**0.004**	0.000
	(176.3)	(0.69)		(169.1)	(3.14)	(0.72)
	-	**0.873**	-	-	**0.858**	-
		(11.97)			(20.8)	
	0.033	**-0.040**	**0.232**	**0.024**	**-0.093**	**0.826**
	(2.83)	(3.59)	(32.64)	(7.82)	(3.66)	(162.4)

- For CP and DLP, the conditional mean of stock volume has a negative impact on that of volatility (β_{32} is negative and significant), which is in line with the theory by Wang (2007).
- According to Wang foreign purchases tend to lower volatility by increasing the investor base in emerging markets, since the broadening of the investor base improves the accuracy of market information and stabilizes stock prices (see also Karanasos and Kartsaklas, 2009).
- Clearly, the negative estimated parameter β_{32} would have been ruled out by the sufficient Bollerslev conditions.

- It is important to highlight the fact that our methodology is very general, and it can be applied not only in multivariate MEM and GARCH/HEAVY processes but also to vector ARMA (VARMA)-type processes where the variables in the system should take non-negative values.
- Three indicative examples are:

- The multivariate autoregressive conditional double poison (MDACP) model of Heinen and Rengifo (2007), that is a VARMA-type system for the conditional means of counts.
- The multivariate conditional autoregressive range (MCARR) model of Fernades et al. (2005), where the conditional expectations of the range-based measures of volatility are modelled as VARMA-type processes.
- The multivariate heterogeneous autoregressive realized volatility (MHAR-RV) process of Hwang and Hong (2021) for modelling realized volatilities.
- That is, our matrix inequality constraints should be enforced on these models as well.

- If an UE model is estimated then the non-negativity conditions might be violated
- As in: Cipollini, Engle and Gallo (2013, JAE), Manganelli (2005, JFM), Cipollini and Galo (2010, CSDA)
- Therefore one should impose the non-negativity constraints (expressed as matrix inequalities) in the estimation, that is apply constrained QML estimation:
- In other words, use the MUFA Model!

- we employ the symmetric unrestricted full model of order $(1, 1)$, and by using Monte Carlo simulations, we examine the effects of ignoring the non-negativity conditions in our Theorem on:
- i) the bias of QML estimates, and ii) the out of sample forecasts. We compare three cases:
- I) imposing our matrix inequality constraints in the estimation,
- II) enforcing Bollerlsev's sufficient conditions (that is, that all parameters are allowed to take only non-negative values),
- and III) the unconstrained estimation.
- The estimates based on the matrix inequality constraints have smaller bias than the other two. The performance of the estimates without imposing any non-negativity conditions is the worst both in terms of the bias and the standard deviation.

One possible drawback is that the non-negativity constraints might be rather restrictive

- One solution to this problem is to use a multivariate exponential type of model, i.e., Multivariate EGARCH or Log-GARCH
- that is a system where we model instead of the conditional variances (h_{it}) their logarithmic transformations, $ln(h_{it})$.
- Drawbacks: EGARCH: no asymptotic theory (only for the univ. $(1,1)$ model)
- Log-GARCH: the zero returns issue
- In addition, there is no priori reason to model all variables using logs

- In another paper (work in progress) we provide an alternative strategy which includes
- the MUFA GARCH model
- as well as the multivariate exponential type of system
- as special cases.

- A new model but the same acronym: MMUFA!
- The mixture MUFA GARCH Model
- That is, some (the first $0 \leq d \leq N$) of the conditional variances in this N-dimensional mixture formulation are modelled as GARCH specifications
- (see, for example, Conrad and Karanasos, 2010, Karanasos et al. 2014, JEF, Karanasos et al. 2021, Yfanti and Karanasos, 2021, JIFMIM)
- whereas the rest are modelled as exponential processes (in the spirit of Nelson, 1991).

- But that's a story for another day
- another conference
- another presentation...
- and in case we run out of time
- (i.e., due to the Omicron/Delta coronovirus!)
- ...then perhaps for another life!

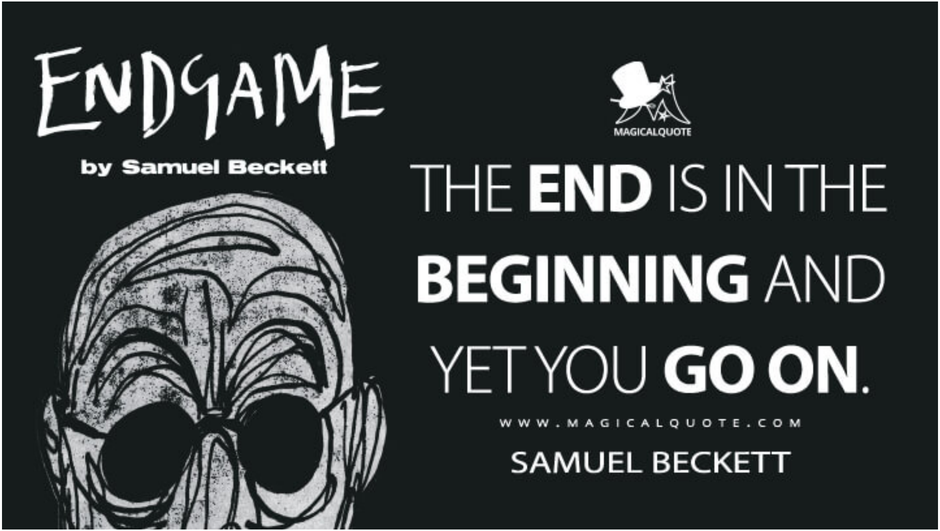
In the mean time, while you are alive

Support and Enjoy the MUFA GARCH model!

Extensions:

- Obtain exact form solution results for the MGARCH(p, q) model and not only the MGARCH$(1, q)$ process
- Tse and Terasvirta (2005, ET) obtain the autocorrelation function of the squared errors only for the MGARCH$(2, 2)$ model and even for this they used recursive solutions.
- We can now obtain exact form solutions for the MGARCH(p, q) model.
- So we can see how these autocorrelations are affected by the inequality constraints.
- Extend our results to the case of time-varying models, i.e., with abrupt breaks:
- Karanasos et al. (2022) we provide a theory for time-varying ARMA models,
- which we also have extended it (work in progress) to multivariate ARMA models.

Finish where we started



- Quoting NC, 1992:
- "Presumably, such sufficient (but not necessary) conditions should not be imposed in estimation. In practice, however, it is usually necessary to impose positivity on in-sample fitted values of h_t to keep nonlinear maximization routines from encountering overflows. For the ARCH(p), GARCH($1, q$), and GARCH($2, q$), the inequality constraints of Sections 2.1 and 2.2 should suffice. For higher-order GARCH models and multivariate GARCH, some other tactic is required. [...]This approach can easily be adapted to the multivariate case"

In Memory of Dan B. Nelson 1959-1995