A general theory of tax-smoothing

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The basic idea

• **Question**: should we tax today or should we *postpone* taxes and issue debt ($\equiv$ *future* taxes)?
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• What matters for this *trade-off?* the price of government debt.

• **This paper:** Build a *general* theory of optimal fiscal policy around the following “*tax-smoothing*” *principle*:

\[
\begin{align*}
\text{Future taxes} & = \Phi \times \text{Marginal Revenue} \\
\text{MC of debt} & \quad \text{MB of debt}
\end{align*}
\]

• **Optimality condition wrt to (some measure of) debt.**
  • *LHS:* MC of issuing more debt: *costly* due to *more* taxes tomorrow.
  • *RHS:* Marginal revenue of new *debt issuance* × *social value* of relaxing the government budget.
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- **This paper:** Build a *general* theory of optimal fiscal policy around the following "tax-smoothing" principle:

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\text{Future taxes} \propto \Phi \times \left[p + \left(\frac{\partial p}{\partial b'} \cdot b'\right)\right]
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  - **RHS**: Marginal revenue of new debt issuance × social value of relaxing the government budget.

- **Principle**: Levy more taxes on states/dates if MR of debt is high

⇒ *Tax more tomorrow vs today if it is cheaper to issue debt!*
Asset prices matter!

- **Market value** of the government debt portfolio depends on:
  1. Stochastic Discount Factor (e.g. time-additive or recursive utility).
  2. Market structure (complete or incomplete markets).
  3. Timing protocol (commitment versus discretion).

- What I do: Take asset prices seriously.

- Use a plausible model of asset returns ⇒ (Generalized) recursive utility.

- Market structure: consider complete or incomplete markets.

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- The MR is activated with recursive utility.

- The same principle Taxes = \Phi \times MR emerges in each environment \implies tax-smoothing!
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- Related literature
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- ← Related literature
Preview of results

Complete markets

• Time-additive utility: Lucas and Stokey (1983)
• Keep labor tax essentially constant ⇒ tax-smoothing.
• No drifts.
• No endogenous persistence.
• Recursive utility.
• Taxes are not constant ⇒ tax more in good times and less in bad times.
• Back-loading of distortions.
• High endogenous persistence.

Incomplete markets

• Time-additive utility: Barro (1979) and Aiyagari et al. (2002)
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- Economy without capital and exogenous and stochastic $g_t$ (TFP shocks can be easily incorporated)

$$c_t(g^t) + g_t = h_t(g^t)$$
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- Two market structures:

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b_t(g^t) = \tau_t(g^t)w_t(g^t)h_t(g^t) - g_t + \sum_{g_{t+1}} p_t(g_{t+1}, g^t)b_{t+1}(g^{t+1})
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  - **primary surplus**
  - **portfolio of new debt**
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 2. Non-contingent debt as in Aiyagari et al. (2002):

$$b_{t-1}(g^{t-1}) = \tau_t(g^t)w_t(g^t)h_t(g^t) - g_t + q_t(g^t)b_t(g^t)$$
Preferences

- *General* form of recursive utility (Kreps and Porteus (1978)):

\[ V_t = u(c_t, 1-h_t) + \beta H^{-1}(E_t H(V_{t+1})) \]

Certainty equivalent \( \mu_t \)

- \( H \) increasing and *concave* \( \Rightarrow \) aversion towards risks in \( V_{t+1} \).

- \( A(x) \equiv -H''/H' \) coefficient of *absolute* risk aversion.

- Time-additive utility: \( H(x) = x \).
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• Three *parametric* examples:
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Stochastic Discount Factor

- Two components: Consumption (*short-run*) risk vs Continuation value risk (*long-run*):

\[ S_{t+1} = \beta \frac{u_{c,t+1}}{u_{ct}} \frac{H'(V_{t+1})}{H'(\mu_t)} \equiv m_{t+1} \]
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- Agent dislikes volatility in utility ⇒ $V_{t+1} \downarrow \Rightarrow$ SDF $\uparrow$. 
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\[ m_{t+1} = \frac{\exp(-AV_{t+1})}{E_t \exp(-AV_{t+1})}, E_t m_{t+1} = 1 \]
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- **Logarithmic CE:**

\[
m_{t+1} = \exp\left(-\left(v_{t+1} - E_t v_{t+1}\right)\right), \quad v_{t+1} \equiv \ln V_{t+1}, E_t \ln m_{t+1} = 1.
\]
Optimal policy under commitment

- Distortionary taxation:

\[
\frac{u_{lt}}{u_{ct}} = (1 - \tau_t)w_t
\]

- **Optimal policy problem**: choose \( \tau \) to maximize the utility of the representative household at \( t = 0 \).

- Formulate commitment problem recursively as in Kydland and Prescott (1980).

- **Important**: \( V_t \) shows up in the implementability constraints due to recursive utility.

- State variables
  - **Complete markets**: \( z_t \equiv u_{ct}b_t \), debt in MU units.
  - **Incomplete markets**: \( B_t \equiv E_t m_{t+1} u_{c,t+1} \cdot b_t \), debt in average MU units.

- Value function with **complete** markets:  

  - Value function with **incomplete** markets:  

- \( \Phi_t \) : excess burden (multiplier on implementability constraint) \( \Rightarrow \) Captures taxes.
Recursive utility: price effect of continuation values

• Let $g_L < g_H$. Planner *insures* ex-ante:
  
  • *sells* debt against $g_L \Rightarrow$ to be paid with a *surplus* when $g' = g_L$
  
  • *buys* assets against $g_H \Rightarrow$ finances a *deficit* when $g' = g_H$. 

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  • *buys* assets against $g_H \Rightarrow$ finances a *deficit* when $g' = g_H$.

• *How much debt/assets?*
  • Expected utility: Make tax rate *constant* across $g_i, i = L, H$. 
Recursive utility: price effect of continuation values

• Let $g_L < g_H$. Planner *insures* ex-ante:
  • *sells* debt against $g_L \Rightarrow$ to be paid with a *surplus* when $g' = g_L$
  • *buys* assets against $g_H \Rightarrow$ finances a *deficit* when $g' = g_H$.

• *How much debt/assets?*
  • *Expected utility*: Make tax rate *constant* across $g_i, i = L, H$.
  • *Recursive utility*: Planner *over-insures*:
    • Sells *more* debt against $g_L$ and *increase* taxes when $g' = g_L$
    • Buys *more* assets against $g_H$ and *decrease* taxes when $g' = g_H$. 
Recursive utility: price effect of continuation values

• Let \( g_L < g_H \). Planner insures ex-ante:
  • sells debt against \( g_L \) ⇒ to be paid with a surplus when \( g' = g_L \)
  • buys assets against \( g_H \) ⇒ finances a deficit when \( g' = g_H \).

• How much debt/assets?
  • Expected utility: Make tax rate constant across \( g_i, i = L, H \).
  • Recursive utility: Planner over-insures:
    • Sells more debt against \( g_L \) and increase taxes when \( g' = g_L \)
    • Buys more assets against \( g_H \) and decrease taxes when \( g' = g_H \).

• Why? \( Debt_L \uparrow \Rightarrow V_L \downarrow \Rightarrow SDF_L \uparrow \): price of claims sold \( \uparrow \).
  • Tax more at \( g_L \) since it becomes cheaper to issue debt against \( g_L \).
  • Tax less at \( g_H \) because assets against \( g_H \) become more profitable \((SDF_H \downarrow)\).
Excess burden with complete markets I

- Optimality condition wrt $z_{t+1} \equiv u_{c,t+1}b_{t+1}$. 

$\eta_{t+1} \equiv A(V_{t+1}z_{t+1})$ adjusted by $A(x) \equiv -A''/A'$.

$\eta_{t+1} \equiv 0$ for time-additive utility (or for the deterministic case).
Excess burden with complete markets I

• **Time-additive utility**: Lucas and Stokey (1983)

\[
\Phi_{t+1} = \Phi_t \cdot 1, \forall t, s^t
\]

• MR part **trivial** ⇒ keep distortions **constant** over states and dates ("tax-smoothing").

\[
\Phi_t = \Phi_{t+1} = \Phi_{t+1} \cdot 1, \forall t, s^t
\]
Excess burden with complete markets I

- Recursive utility:

\[ \frac{1}{\Phi_{t+1}} = \frac{1}{\Phi_t} - \eta_{t+1} \]
Excess burden with complete markets I

- Recursive utility:

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\[
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\]

“debt” value of portfolio
Excess burden with complete markets I

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\[ \frac{1}{\Phi_{t+1}} = \frac{1}{\Phi_t} - \eta_{t+1} \]

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\[ \eta_{t+1} \equiv A(V_{t+1})z_{t+1} - A(\mu_t) \begin{cases} 
E_{tm_{t+1}}z_{t+1} \text{ \text{value of portfolio}} \end{cases} \]


• \( \eta_{t+1} \equiv 0 \) for time-additive utility (or for the deterministic case).
Excess burden with complete markets II

- **LoM** in terms of inverse excess burden of taxation

\[
\frac{1}{\Phi_{t+1}} = \frac{1}{\Phi_t} - \eta_{t+1}
\]

- Tax *more* tomorrow vs today \((\Phi_{t+1}(g') > \Phi_t)\) when issue relatively more debt \((\eta_{t+1}(g') > 0)\).
- Tax *less* tomorrow vs today \((\Phi_{t+1}(g') < \Phi_t)\) when issue relatively less debt \((\eta_{t+1}(g') < 0)\).

• parametric examples, persistence and drifts
• optimal tax rate
• numerical exercises
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- Expect \(\eta_{t+1}(g_L) > 0 > \eta_{t+1}(g_H)\) due to fiscal hedging (issue more debt against good times) \(\Rightarrow \Phi_{t+1}(g_L) > \Phi_t > \Phi_{t+1}(g_H)\).
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- \(\Rightarrow\) Tax more in good times and less in bad times \(\Rightarrow\) amplify Lucas and Stokey (1983).
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\(\Rightarrow\) *Tax more in good times and less in bad times ⇒ amplify Lucas and Stokey (1983).*

\(\Rightarrow\) run larger surpluses in good times and larger deficits in bad times.
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- **optimal tax rate** **numerical exercises**
What happens with incomplete markets?

- Debt *non-contingent* $\Rightarrow$ less room for manipulation of SDF?

  - Let $g_L < g_H$. Planner issues non-contingent debt:
    - In good times, $g'_L = g_L$, the planner will tax less to repay debt and finance $g_L$.
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  - How much non-contingent debt does the planner issue?
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      - Is there a counter-acting force (tax more in good times and less in bad times)? NO.
      - Price manipulation: Use continuation values to make the average SDF (inverse of interest rate) large.
    - Result: put more tax distortions on events with high $u_c \Rightarrow$ tax even more bad times with high $u_c$. 
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- **How much non-contingent debt does the planner issue?**
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Excess burden with incomplete markets II

- Optimality condition wrt to $B_t \equiv E_t m_{t+1} u_{c, t+1} b_t$:
Excess burden with incomplete markets II

- **Time-additive utility:** (AMSS 2002)

\[ E_t x_{t+1} \Phi_{t+1} = \Phi_t, \quad x_{t+1} \equiv \frac{u_{c,t+1}}{E_t u_{c,t+1}} \]

⇒ keep distortions on “average” constant (Barro (1979)).
Excess burden with incomplete markets II

• Recursive utility: LoM for the inverse average excess burden

\[
\frac{1}{E_t n_{t+1} \Phi_{t+1}} = \frac{1}{\Phi_t} - \frac{E_{t-1} n_t \Phi_t}{\Phi_t} \cdot \xi_t \cdot b_{t-1}, \quad n_{t+1} \equiv m_{t+1} \cdot \frac{u_{c,t+1}}{E_t m_{t+1} u_{c,t+1}}
\]

where \( \xi_t \) is the relative marginal utility adjusted by \( A(x) \)

\[
\xi_t \equiv A(V_t)u_{ct} - A(\mu_{t-1})E_{t-1} m_t u_{ct}
\]

• MR: depends on \( \xi_t \) times non-contingent debt \( b_{t-1} \).
Excess burden with incomplete markets II

- **Recursive utility**: LoM for the inverse *average* excess burden

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Excess burden with incomplete markets II

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- \( u_c \) high in bad times \( \Rightarrow \) expect \( \xi_t(g_H) > 0 > \xi_t(g_L) \). Thus, if \( b_{t-1} > 0 \)
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Excess burden with incomplete markets II

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- **Tax more in bad times and less in good times** \( \Rightarrow \) larger weight on \( u_c \) in bad times ( \( q_t \uparrow \) \( \Rightarrow \) amplify Aiyagari et al. (2002)).

- **optimal tax rate**
Concluding remarks

- Minimization of welfare distortions ⇒ tax more events against which it is *cheap* to issue debt (Taxes=Φ × MR).

- This insight holds in *all* environments ⇒ provides a *general principle* of taxation.

- This does *not* mean taxes are (on average or not) *smooth*!

- Taking asset prices seriously ⇒ *amplification* of standard taxation and debt issuance motives.
THANK YOU!
### Table: Optimal fiscal policy with time-additive utility.

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<thead>
<tr>
<th></th>
<th>Commitment</th>
<th>Discretion</th>
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<td><strong>Markets</strong></td>
<td>Chari et al. (1994), Zhu (1992)</td>
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Related literature

Table: Optimal fiscal policy with \textit{time-additive} utility.

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<td>Incomplete Markets</td>
<td>Aiyagari et al. (2002), Farhi (2010)</td>
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<td>Bhandari et al. (2017), Martin (2009)</td>
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<td>Karantounias (2017)</td>
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Table: Optimal fiscal policy with \textit{recursive} utility.

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<td>This paper: more \textit{general} utility</td>
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</table>
Value function with complete markets under commitment

- $z \equiv u_c \cdot b.$

$$V(z, g) = \max_{c \geq 0, h \in [0,1], z'_g \in Z(g')} u(c, 1 - h) + \beta H^{-1} \left( \sum_{g'} \pi(g'|g) H(V(z'_g, g')) \right)$$

subject to

$$z = u_c c - u_l h + \beta \sum_{g'} \pi(g'|g) m'_g z'_g$$

surplus

$$c + g = h$$

price x debt
Value function with complete markets under commitment

• \( z \equiv u_c \cdot b \).

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V(z, g) = \max_{c \geq 0, h \in [0,1], z_{g'}, \in Z(g')} u(c, 1 - h) + \beta H^{-1} \left( \sum_{g'} \pi(g' | g) H \left( V(z_{g'}, g') \right) \right)
\]

subject to

\[
z = u_c c - u_l h + \beta \sum_{g'} \pi(g' | g) m'_{g'} z'_{g'}
\]

\[
\text{surplus}
\]

\[
c + g = h
\]

• Value functions in the constraint due to the SDF:

\[
m'_{g'} \equiv \frac{H'(V(z'_{g'}, g'))}{H'(\mu)}, \quad \text{and} \quad \mu \equiv H^{-1} \left( \sum_{g'} \pi(g' | g) H \left( V(z'_{g'}, g') \right) \right).
\]

• \( m'_{g'} \equiv 1 \) for time-additive utility.
Value function with incomplete markets under commitment

- State variable $B_t \equiv E_t m_{t+1} u_{c,t+1} \cdot b_t$. 

subject to 

\[
\sum_g \pi(g | g - h_g) m_g u_c(c_g, 1 - h_g) = u_c c_g - u_l h_g \quad \forall g
\]

\[
B_g \leq \bar{B}_g \quad \forall g.
\]
Value function with incomplete markets under commitment

- State variable $B_t \equiv E_t m_{t+1} u_{c,t+1} \cdot b_t$.

- Commit to *average* marginal utility: promises *across* states $g$.

$$W(B_-, g-) = \max_{c_g \geq 0, h_g \in [0,1], B_g} H^{-1} \left( \sum_g \pi(g|g-) H \left( u(c_g, 1 - h_g) + \beta W(B_g, g) \right) \right)$$

subject to

$$\frac{u_c(c_g, 1 - h_g)}{\sum_g \pi(g|g-) m_g u_c(c_g, 1 - h_g)} B_- = u_c c_g - u_l h_g + \beta B_g \quad \text{surplus} \quad \text{new debt} , \forall g$$

$$c_g + g = h_g, \forall g$$

$$B_g \leq B_g \leq \bar{B}_g , \forall g.$$
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subject to

$$\frac{u_c(c_g, 1 - h_g)}{\sum_g \pi(g|g_-) m_g u_c(c_g, 1 - h_g)} B_- = u_c c_g - u_l h_g + \beta B_g, \forall g$$

surplus new debt

$$c_g + g = h_g, \forall g$$

$$B_g \leq B_g \leq \bar{B}_g, \forall g.$$ 

• Value functions $W$ in $m_g$:

$$m_g = \frac{H'(u(c_g, 1 - h_g) + \beta W(B_g, g))}{H'(H^{-1}\left(\sum_g \pi(g|g_-) H(u(c_g, 1 - h_g) + \beta W(B_g, g))\right))}, \forall g.$$
Parametric examples and drifts

- **Exponential CE:**

  \[ \eta_{t+1} = A \cdot [z_{t+1} - E_t m_{t+1} z_{t+1}] \Rightarrow E_t m_{t+1} \eta_{t+1} = 0 \]

- \( \Rightarrow \frac{1}{\Phi_t} \text{ martingale wrt } \pi_t \cdot M_t \)

- **Drift wrt physical measure?**

  \[ E_t \Phi_t \eta_{t+1} \geq \Phi_t - \text{Cov} \left( m_{t+1}, \Phi_{t+1} \right) \Rightarrow \text{if } \text{Cov} < 0 \Rightarrow \text{positive drift wrt } \pi_t \cdot M_t \]

- **Power CE, } \alpha \neq 1 \)

  \[ \eta_{t+1} = \alpha \cdot [V - 1 \cdot z_{t+1} - E_t \kappa_{t+1} V - 1 \cdot z_{t+1}] \Rightarrow E_t \kappa_{t+1} \eta_{t+1} = 0 \]

  \( \Rightarrow \frac{1}{\Phi_t} \text{ martingale wrt } \pi_t \cdot K_t \Rightarrow \text{positive drift wrt } \pi_t \cdot K_t \)

- **Logarithmic CE:**

  \[ \eta_{t+1} = V - 1 \cdot z_{t+1} - E_t V - 1 \cdot z_{t+1} \]

  \( \Rightarrow \frac{1}{\Phi_t} \text{ martingale wrt } \pi_t \Rightarrow \text{positive drift wrt } \pi_t \)
Parametric examples and drifts

• Exponential CE:

\[ \eta_{t+1} = A \cdot [z_{t+1} - E_t m_{t+1} z_{t+1}] \Rightarrow E_t m_{t+1} \eta_{t+1} = 0 \]

• \( \Rightarrow 1/\Phi_t \) martingale wrt \( \pi_t \cdot M_t \) \( \Rightarrow \Phi_t \) submartingale wrt \( \pi_t \cdot M_t \),

\[ E_t m_{t+1} \Phi_{t+1} \geq \Phi_t. \]
Parametric examples and drifts

- **Exponential CE:**

  \[ \eta_{t+1} = A \cdot [z_{t+1} - E_t m_{t+1} z_{t+1}] \Rightarrow E_t m_{t+1} \eta_{t+1} = 0 \]

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  \( E_t m_{t+1} \Phi_{t+1} \geq \Phi_t. \)

- Drift wrt *physical* measure?

  \[ E_t \Phi_{t+1} \geq \Phi_t - \text{Cov}_t(m_{t+1}, \Phi_{t+1}) \]

  \( \Rightarrow \text{if Cov}_t < 0 \Rightarrow \text{positive drift wrt } \pi_t. \)
Parametric examples and drifts

- **Exponential CE:**
  \[
  \eta_{t+1} = A \cdot [z_{t+1} - E_t m_{t+1} z_{t+1}] \Rightarrow E_t m_{t+1} \eta_{t+1} = 0
  \]

- \(1/\Phi_t\) martingale wrt \(\pi_t \cdot M_t \Rightarrow \Phi_t\) submartingale wrt \(\pi_t \cdot M_t\),
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  \[
  E_t \Phi_{t+1} \geq \Phi_t - Cov_t(m_{t+1}, \Phi_{t+1})
  \]

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  \[
  \eta_{t+1} = \alpha \cdot [V_{t+1}^{-1} z_{t+1} - E_t \kappa_{t+1} V_{t+1}^{-1} z_{t+1}] \Rightarrow E_t \kappa_{t+1} \eta_{t+1} = 0
  \]

- \(1/\Phi_t\) martingale wrt \(\pi_t \cdot K_t \Rightarrow\) positive drift wrt \(\pi_t \cdot K_t\).
Parametric examples and drifts

• Exponential CE:

\[ \eta_{t+1} = A \cdot [z_{t+1} - E_t m_{t+1} z_{t+1}] \Rightarrow E_t m_{t+1} \eta_{t+1} = 0 \]

\[ \Rightarrow 1/\Phi_t \text{ martingale wrt } \pi_t \cdot M_t \Rightarrow \Phi_t \text{ submartingale wrt } \pi_t \cdot M_t, \]

\[ E_t m_{t+1} \Phi_{t+1} \geq \Phi_t. \]

• Drift wrt *physical* measure?

\[ E_t \Phi_{t+1} \geq \Phi_t - \text{Cov}_t(m_{t+1}, \Phi_{t+1}) \]

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\[ \eta_{t+1} = \alpha \cdot [V_{t+1}^{-1} z_{t+1} - E_t \kappa_{t+1} V_{t+1}^{-1} z_{t+1}] \Rightarrow E_t \kappa_{t+1} \eta_{t+1} = 0 \]

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• Logarithmic CE:

\[ \eta_{t+1} = V_{t+1}^{-1} z_{t+1} - E_t V_{t+1}^{-1} z_{t+1} \]

\[ \Rightarrow 1/\Phi_t \text{ martingale wrt } \pi_t \Rightarrow \text{positive drift wrt } \pi_t. \]
Optimal tax rate I

- Complete markets and commitment, \( t \geq 1 \)

\[
\tau_t = \frac{\Phi_t (\epsilon_{cc,t} + \epsilon_{ch,t} + \epsilon_{hh,t} + \epsilon_{hc,t})}{1 + \Phi_t (1 + \epsilon_{hh,t} + \epsilon_{hc,t})}
\]

where \( \epsilon_{cc} \equiv -\frac{u_{cc}}{u_c} \), \( \epsilon_{ch} \equiv \frac{u_{cl}}{u_c} \) and \( \epsilon_{hh} \equiv -\frac{u_{ll}}{u_l} \), \( \epsilon_{hc} \equiv \frac{u_{cl}}{u_l} \), the respective own and cross elasticities.
Optimal tax rate I

• Complete markets and commitment, \( t \geq 1 \)

\[
\tau_t = \frac{\Phi_t (\epsilon_{cc,t} + \epsilon_{ch,t} + \epsilon_{hh,t} + \epsilon_{hc,t})}{1 + \Phi_t (1 + \epsilon_{hh,t} + \epsilon_{hc,t})}
\]

where \( \epsilon_{cc} \equiv -u_{cc} c / u_c, \epsilon_{ch} \equiv u_{cl} h / u_c \) and \( \epsilon_{hh} \equiv -u_{ll} h / u_l, \epsilon_{hc} \equiv u_{cl} c / u_l \), the respective own and cross elasticities.

• Assume a utility function with constant elasticities

\[
U(c, 1-h) = \frac{c^{1-\rho} - 1}{1 - \rho} - a_h \frac{h^{1+\phi_h}}{1 + \phi_h}
\]

• \( \Rightarrow \) \( \tau_t \) moves 1-1 with \( \Phi_t \), with law of motion

\[
\frac{1}{\tau_{t+1}} = \frac{1}{\tau_t} - \frac{1}{\rho + \phi_h} \eta_{t+1}
\]
Optimal tax rate II: Incomplete markets and commitment

- Power in \( c \) and \( h \) (constant Frisch): The optimal tax rate with recursive utility is

\[
\tau_t = \frac{\Phi_t (\rho + \phi_h) - \rho [\Phi_t - E_{t-1} n_t \Phi_t] \frac{b_{t-1}}{c_t}}{1 - (E_{t-1} n_t \Phi_t) \xi_t b_{t-1} + \Phi_t (1 + \phi_h)}
\]

- The respective tax rate for the time-additive case of Aiyagari et al. (2002) is

\[
\tau_t = \frac{\Phi_t (\rho + \phi_h) - \rho [\Phi_t - \Phi_{t-1}] \frac{b_{t-1}}{c_t}}{1 + \Phi_t (1 + \phi_h)}
\]

- if \( \xi_t > 0 \) (marginal utility relatively high) \( \Rightarrow \) tax rate ↑.
Excess burden without commitment and complete markets

- **Value functions:**

\[
\begin{align*}
\Phi_{t+1} &= \Phi_t \cdot \left[1 + \frac{\partial C_{t+1}}{\partial B_{t+1}} \cdot B_{t+1}\right] \\
&\propto \Phi_t \cdot MR_t.
\end{align*}
\]

- **Excess burden with recursive utility:**

\[
\begin{align*}
\Phi_{t+1} &= \left[1 + \frac{\partial C_{t+1}}{\partial b_{t+1}} \cdot b_{t+1}\right] - \frac{1}{\Phi_t - \nu_{t+1}} \\
&\equiv A\left(V_{t+1}\right) u_{c,t+1} b_{t+1} - A\left(\mu_t\right) \cdot E_t m_{t+1} u_{c,t+1} b_{t+1}.
\end{align*}
\]

- **Relative “debt” position:**

\[
\nu_{t+1} = \frac{A\left(V_{t+1}\right) u_{c,t+1} b_{t+1}}{A\left(\mu_t\right) \cdot E_t m_{t+1} u_{c,t+1} b_{t+1}}.
\]

- **\( u'_{c,\text{channel}} \):** tax more tomorrow vs today if you issue debt.

- **\( V_{t+1} \):** tax more (less) if debt is relatively high (low).

\[\Rightarrow\] the two incentives may oppose each other.
Excess burden without commitment and complete markets

- **Value functions:**

  
  - **Complete markets- MPE**

- **Excess burden with time-additive utility:**

  \[ \Phi_{t+1} = \Phi_t \cdot \left[ 1 + \frac{u_{cc,t+1} - u_{cl,t+1}}{u_{c,t+1}} \frac{\partial C}{\partial B_{t+1}} \cdot B_{t+1} \right] \text{ or } \Phi \times MR \]

- **Excess burden with recursive utility:**

  \[ \Phi_{t+1} = \Phi_t \cdot \left[ 1 + \frac{u_{cc,t+1} - u_{cl,t+1}}{u_{c,t+1}} \frac{\partial C}{\partial b_{t+1}} \cdot b_{t+1} \right] - \frac{1}{\Phi_t} \left[ 1 + \frac{u_{cc,t+1} - u_{cl,t+1}}{u_{c,t+1}} \frac{\partial C}{\partial b_{t+1}} \cdot b_{t+1} \right] \]

- **Relative “debt” position:**

  \[ \nu_{t+1} \equiv A(\nu_{t+1}) \frac{u_{c,t+1} b_{t+1}}{u_{c,t+1} b_{t+1} - A(\mu_{t}) \cdot E_{t} \mu_{t+1}} \]

- **Return channel:**

  - Tax more tomorrow vs today if you issue debt.
  - Tax more (less) if debt is relatively high (low).

- \Rightarrow the two incentives may oppose each other.
Excess burden without commitment and complete markets

- **Value functions:** Complete markets- MPE

- **Excess burden with time-additive utility:**
  \[ \Phi_{t+1} = \Phi_t \cdot \left[ 1 + \frac{u_{cc,t+1} - u_{cl,t+1}}{u_{c,t+1}} \frac{\partial C}{\partial B_{t+1}} \cdot B_{t+1} \right]. \]

- **Excess burden with recursive utility:**
  \[ \frac{1}{\Phi_{t+1}} = \left[ 1 + \frac{u_{cc,t+1} - u_{cl,t+1}}{u_{c,t+1}} \frac{\partial C}{\partial b_{t+1}} \cdot b_{t+1} \right]^{-1} \left[ \frac{1}{\Phi_t} - \nu_{t+1} \right] \]

- **Relative “debt” position:**
  \[ \nu_{t+1} \equiv A(V_{t+1})u_{c,t+1}b_{t+1} - A(\mu_t) \cdot E_t m_{t+1} u_{c,t+1}b_{t+1}. \]
Excess burden without commitment and complete markets

- Value functions: Complete markets - MPE

- Excess burden with time-additive utility:
  \[ \Phi_{t+1} = \Phi_t \cdot \left[ 1 + \frac{u_{cc,t+1} - u_{cl,t+1}}{u_{c,t+1}} \frac{\partial C}{\partial B_{t+1}} \cdot B_{t+1} \right]. \]
  \[ \propto \Phi \times MR \]

- Excess burden with recursive utility:
  \[ \frac{1}{\Phi_{t+1}} = \left[ 1 + \frac{u_{cc,t+1} - u_{cl,t+1}}{u_{c,t+1}} \frac{\partial C}{\partial b_{t+1}} \cdot b_{t+1} \right]^{-1}\left[ \frac{1}{\Phi_t} - \nu_{t+1} \right] \]

- Relative “debt” position:
  \[ \nu_{t+1} \equiv A(V_{t+1})u_{c,t+1}b_{t+1} - A(\mu_t) \cdot E_{t}m_{t+1}u_{c,t+1}b_{t+1}. \]

- \( u_{c}' \) channel: tax more tomorrow vs today if you issue debt.

- \( V_{t+1} \): tax more (less) if debt is relatively high (low).

\[ \Rightarrow \text{the two incentives may oppose each other.} \]
Excess burden without commitment and incomplete markets

- **Value function:**
  Incomplete markets -MPE

- **Excess burden with time-additive utility:**
  \[
  E_t x_{t+1} \Phi_{t+1} = \Phi_t \cdot \left[ 1 + E_t x_{t+1} \frac{u_{cc,t+1} - u_{cl,t+1}}{u_{c,t+1}} \frac{\partial C}{\partial B_{t+1}} \cdot B_{t+1} \right]
  \]
  where \( x_{t+1} \equiv u_{c,t+1}/E_t u_{c,t+1} \)
Excess burden without commitment and incomplete markets

- Value function: Incomplete markets - MPE

- **Excess burden with time-additive utility:**

  \[
  E_t x_{t+1} \Phi_{t+1} = \Phi_t \cdot \left[ 1 + E_t x_{t+1} \frac{u_{cc,t+1} - u_{cl,t+1}}{u_{c,t+1}} \frac{\partial C}{\partial B_{t+1}} \cdot B_{t+1} \right]
  \]

  where \(x_{t+1} \equiv \frac{u_{c,t+1}}{E_t u_{c,t+1}}\)

- **Excess burden with recursive utility:**

  \[
  E_t n_{t+1} \Phi_{t+1} (1 - \xi_{t+1} b_t \Phi_t) = \Phi_t \left[ 1 + E_t n_{t+1} \frac{u_{cc,t+1} - u_{cl,t+1}}{u_{c,t+1}} C_{b,t+1} \cdot b_t \right]
  \]

  with \(\xi_{t+1} \equiv A(V_{t+1}) u_{c,t+1} - A(\mu_t) E_t m_{t+1} u_{c,t+1}\).

  “Averaging” with respect to \(n_{t+1}\) measure.

  Continuation value channel depends on relative marginal utility \(\xi_{t+1}\).
Value function with complete markets and no commitment

- Markov-perfect equilibrium: state variable \((b, g)\).

\[
V(b, g) = \max_{c, h, b'} u(c, 1 - h) + \beta H^{-1} \left( \sum_{g'} \pi(g'|g) H(V(b'_g, g')) \right)
\]

subject to

\[
u_c b = u_c c - u_l h + \beta \sum_{g'} \pi(g'|g) m'_g u_c(C(b'_g, g'), 1 - H(b'_g, g')) b'_g
\]

\[
c + g = h
\]

- where \(m'_g \equiv \frac{H'(V(b'_g, g'))}{H'(\mu)}\)

- MPE: \(c = C, h = H\).
Value function with incomplete markets and no commitment

- State variable is non-contingent debt: \((b_-, g)\).

\[
V(b_-, g) = \max_{c \geq 0, h \in [0,1], b \in \mathcal{B}} u(c, 1-h) + \beta H^{-1}(\sum_{g'} \pi(g'|g) H(V(b, g'))) \\
\text{subject to} \\
u_c(c, 1-h)b_- = u_c c - u_l h + \beta \left( \sum_{g'} \pi(g'|g)m'_{g'} u_c(C(b, g'), 1 - \mathcal{H}(b, g')) \right) \cdot b \\
c + g = h \\
\text{where } m'_{g'} \equiv \frac{H'(V(b,g'))}{H'(H^{-1}(\sum_{g'} \pi(g'|g) H(V(b, g'))))}. \\
\text{MPE: } c = C, h = \mathcal{H}.
Numerical exercises

**Calibration:**

- Utility function: \( \rho = 1 < \gamma \)

\[
v_t = \ln c_t - a_h \frac{h_t^{1+\phi_h}}{1 + \phi_h} + \frac{\beta}{(1 - \beta)(1 - \gamma)} \ln E_t \exp((1 - \beta)(1 - \gamma)v_{t+1})
\]

- Parameters: \((\beta, \phi_h, \gamma) = (0.96, 1, 10)\)

- Shocks
  - i.i.d. shocks: mean 20% and std 2%.
  - Chari et al. (1994) shocks.

**Computational issues:**

- Endogenous state space.

- Lack of the *contraction* property due to the value function in the constraint.

- Non-convexities.
Instructive sample path

- **g**
- **Debt in marginal utility units z**
- **Tax rate in %**
- **Consumption**
- **Surplus-output ratio in %**
- **Debt-output ratio in %**

Graphs and data points are shown for different time periods (t) indicating trends and values for each category.
Random sample paths

![Graphs of tax rate and debt-to-output ratio over time.](image-url)
Volatility and back-loading of distortions

- Positive drift.
- Increasing volatility over time, “fanning-out” of the distribution.
Stationary moments

<table>
<thead>
<tr>
<th>Tax rate in %</th>
<th>i.i.d.</th>
<th>CCK shocks</th>
<th>$2 \times \text{std}(g)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>30.86</td>
<td>30.49</td>
<td>31.26</td>
</tr>
<tr>
<td>St. Dev</td>
<td>4.94</td>
<td>5.52</td>
<td>7.76</td>
</tr>
<tr>
<td>St. Dev of $\Delta$</td>
<td>0.17</td>
<td>0.41</td>
<td>0.90</td>
</tr>
<tr>
<td>Autocorrelation</td>
<td>0.9994</td>
<td>0.9972</td>
<td>0.9932</td>
</tr>
</tbody>
</table>

- Enormous volatility of the tax rate and therefore of debt.
- Chari et al. (1994): volatility of tax rate of 5-15 basis points.
Stationary distribution: debt

<table>
<thead>
<tr>
<th>debt/output in %</th>
<th>i.i.d.</th>
<th>CCK shocks</th>
<th>$2 \times \text{std}(g)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>181.97</td>
<td>172.15</td>
<td>180.34</td>
</tr>
<tr>
<td>St. Dev</td>
<td>104.28</td>
<td>117.05</td>
<td>163.22</td>
</tr>
<tr>
<td>St. Dev of $\Delta$</td>
<td>12.72</td>
<td>12.48</td>
<td>26.07</td>
</tr>
<tr>
<td>Autocorrelation</td>
<td>0.9926</td>
<td>0.9972</td>
<td>0.9877</td>
</tr>
</tbody>
</table>


