# Capacity Constraints on Renewable Energy Resources: Modelling through a Green Ramsey Model

PhD Thesis (Working Draft Paper)

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# Introduction

- If renewable energy sources exist forever, then why we all cannot be 100% dependent on this energy and abandon exhaustible resources (e.g. oil) (saving them for a 'rainy day')?
- Although renewable energy sources exist forever, factors such as nature constraints and limited equipment, land, human/market resources and other barriers prevent businesses, industries and countries in general to achieve the desired renewable energy output (Amigues et al., 2015; Wang & Zhao, 2018).

All these restrictions are examples of the physical/capacity constraints.

# Introduction

**Research Aim:** Study the capacity constraints in the renewable energy context.
**Contribution:** The literature in the specific topic is quite a little, seek to expand the knowledge over the implications of capacity constraints in the optimal climate policy framework.

# Literature Review

**Macias & Matilla-Garcia (2012)** investigate the role of energy inputs in a Ramsey-Hotelling-EROI (Energy Return on Energy Invested) model with CES production function. They concluded that when energy is scarce, the energy sector size slightly decreases when capital-energy substitutability in the long-run is below zero, it is constant if the latter is zero, and increases if it is higher than zero.

**Amigues et al. (2015)** investigate the transition from non-renewable to renewable energy under capacity constraints. They concluded that the aforementioned transition is smooth and investing in renewable energy may begin either before the production of renewables or be delayed till the energy price reaches a sufficient point above the renewables cost.

# Literature Review

**Wang & Zhao (2018)** examine the impacts of renewable energy support policies on energy prices, exhaustible energy supply and thus carbon emissions from fossil fuels and climate change. They concluded that these impacts are dependent mostly on renewable energies' production capacity, as well as market power in the fossil fuel sector.

# Literature Review

**van der Ploeg & Withagen (2014)**: By using a Green Ramsey Model, investigate the optimal energy use where four energy regimes occur:
*Regime I*: Only-oil phase – only-renewable phase
(carbon tax rises till oil is phased out / Green Paradox: no carbon tax but renewables subsidy -> more oil is pumped up)
*Regime II*: Only-oil phase – oil-renewable phase
(carbon tax may decline gradually, more oil is left in situ)
*Regime III*: Only-renewable phase – oil-renewable phase
*Regime IV*: Only-renewable phase forever
(Regimes III and IV are unlikely to occur till there is a breakthrough in the technology of renewables)

# The Green Ramsey Model

ASSUMPTIONS

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  $$\dot{S}(t) = -O(t),\ S(0) = S_0 > 0$$
- The stock of atmospheric carbon $E(t)$, for all $t \geq 0$, is proportional to oil depletion:
  $$\dot{E}(t) = O(t) - gE(t),\ E(0) = E_0 > 0$$
  where $0 < g < 1$ is the atmospheric carbon decay and $E_0$ the initial stock of atmospheric carbon.

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  2. $\lim_{S \to 0} G(S(t)) = \infty$, $\lim_{S \to \infty} G(S(t)) = 0$

- Intertemporal Social Welfare:

$$\max W = \int_0^{\infty} e^{-\rho t}(U(C(t)) - D(E(t))dt$$

where $\rho > 0$ the constant rate of time preference, U(.) is an instantaneous, increasing and concave utility function, which ensures positive consumption throughout, D(.) an increasing and convex function of atmospheric carbon.

- Renewable capital accumulation-depreciation equation:

$$\dot{K}(t)_R = R(t) - \delta K(t)_R,\ K(0)_R = K_{0R}$$

where $K(t)_R$ the capital needed to produce renewable energy capacity, $R(t)$ the annual investment in the stock of renewable energy at time t and $0 < \delta < 1$.

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- Renewable capital constraint: $0 < K(t)_R < \bar{K}_R$
  where $\bar{K}_R < +\infty$ is the maximum productive capital stock limit land can support, which will generate a renewable energy at its maximum point; more capital stock that exceeds the limit will be useless and may cause negative impacts to the natural and human environment.

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  3. non-increasing returns to scale function
  4. $0 \leq h(K(t)_R) \leq h(\bar{K}_R)$
  5. $\lim_{K^R \to 0} h'(K(t)_R) = \infty$, $\lim_{K^R \to \bar{K}^R} h'(K(t)_R) = 0$, for all $K(t)_R > 0$

- Material Balance Equation:

$$\dot{K}(t) = F(K(t), O(t) + h(K(t)_R)) - R(t) - G(S(t))O(t) - C(t) - \delta K(t),$$

$K(0) = K_0$, $0 < \delta < 1$

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  F is a non-increasing returns to scale function, strictly increasing and concave, zero for at least one zero input (capital and/or both energy sources) and satisfies the Inada Conditions.
- Useful notation: $\sigma(C) = -U''(C)/U'(C)C > 0$ (elasticity of intertemporal substitution), $p = F_O(K(t), O(t) + h(K(t)_R))$ (marginal product of oil energy),
  $r \equiv F_K(K(t), O(t) + h(K(t)_R)) - \delta$ (net rate of return on capital)

Social Planner Program:

$$\max W = \int_0^\infty e^{-\rho t}(U(C(t)) - D(E(t))dt \text{ (1)}$$

s.t.

$\dot{S}(t) = -O(t)$ (2), $S(0) = S_0 > 0$

$\dot{E}(t) = O(t) - gE(t)$ (3), $E(0) = E_0 > 0$, $0 < g < 1$

$\dot{K}(t)_R = R(t) - \delta K(t)_R$ (4), $0 < \delta < 1$

$0 \leq K(t)_R \leq \bar{K}_R$ (5)

$\dot{K}(t) =$
$F(K(t), O(t) + h(K(t)_R)) - R(t) - G(S(t))O(t) - C(t) - \delta K(t)$ (6),
$K(0) = K_0 > 0$

social cost of carbon: $\tau = -\mu_E/\mu_K = \frac{\int_t^\infty e^{-\rho(t-s)} D'(E(s))ds}{U'(C(t))}$(7)
control variables: $\{C(t), O(t), R(t)\}$ state variables:
$\{S(t), E(t), K(t), K(t)_R\}$.

**PROPOSITION 1**: The planning program satisfies equations (2) - (6) and the optimality conditions indicate the following facts:

1) $\mu_K = \mu_{K_R}$

2) consumers consume up to the point where the net rate of return of capital $r$ requals the return on consumption $\rho$, i.e. $r = \rho$ (two Euler equations for $K$ and $K_R$)

3) The return on leaving a marginal barrel in the earth, $\dot{p}$, equals the net rate on capital return on the net revenues of depleting a marginal barrel, $r(p - G(S))$, plus the social cost of carbon reduced due to the natural decays of CO2 in the atmosphere, $g\tau$ minus the marginal global warming damages $\frac{D'(E)}{U'(C)}$. So: $\dot{p} = r(p - G(S)) + g\tau - \frac{D'(E)}{U'(C)}$

4) The rate of change of social cost of carbon over time $\dot{\tau}$, equals the net rate of return on capital $r$ plus the natural degradation of $CO_2$ on the social cost of carbon, $\tau(r + g)$ minus the marginal global warming damages $\frac{D'(E)}{U'(C)}$. So: $\dot{\tau} = \tau(r + g) - \frac{D'(E)}{U'(C)}$

**PROPOSITION 2**: The planning program, as mentioned in Proposition 1, assuming that optimality conditions (A) - (E) and the production properties of $F$ and $h$ provide a steady state, they do so only when $K_R = \bar{K}_R$ (This maximum level is maintained over time as long as $R = \delta \bar{K}_R$). Then:

$\frac{\dot{C}}{C} = \sigma(F_K(K, O + \overline{h}) - (\rho + \delta))$ (A')

$\mu_\lambda = U'(C)(F_{\bar{K}_R}(K, O + \overline{h})h'(\bar{K}_R) - (\rho + \delta))$ (B'), $\mu_\lambda > 0$, $\xi = 0$

$\dot{p} = r(p - G(S)) + g\tau - \frac{D'(E)}{U'(C)}$ (C')

$\dot{\tau} = \tau(r + g) - \frac{D'(E)}{U'(C)}$ (D')

$\lim_{t\to\infty}[\mu_K K + \mu_S O + \mu_E E]e^{-\rho t} = 0$ (E')

# Phases of Optimal Energy

1. Renewables - Only Phase $(O(t) = 0)$

$\dot{K} = F(K, \overline{h}) - R - C - \delta K$ (1R)

$\frac{\dot{C}}{C} = \sigma(F_K(K, +\overline{h}) - (\rho + \delta))$ (2R)

The dynamical system (1R) - (2R) has a solution which is a saddle-path.

## Lemma

*Assume that the initial capital and oil stock (under a "pivotal" economy case) satisfy the following conditions where the marginal cost of oil extraction exceeds its marginal benefit:*

$$p(0) - \frac{g}{\rho}\tau(0) < G(S^*) + \frac{D'(E_0)}{\rho U'(C^*)},\ K_0 < K^*$$
$$p(0) - \frac{g}{\rho}\tau(0) < G(S^*) + \frac{D'(E_0)}{\rho U'(C^*)},\ K_0 > K^*$$

If these conditions hold, then the optimal program uses only renewables.

2 Oil - Only Phase ($h = 0$)

Ruled out. If we had $K_R = 0$, then the economy would not have installed any renewable capacity, it would never be optimal to use only oil forever as the exhaustible energy would gradually be phased out and ultimately, $F(.) = 0$.

3 Oil-Renewables Phase

$\dot{S} = -O$ (1OR)
$\dot{E} = O - gE$ (2OR)
$\dot{K} = F(K, O + \overline{h}) - G(S)O - C - \delta(K + \bar{K}_R)$ (3OR)
$\frac{\dot{C}}{C} = \sigma(F_K(K, O + \overline{h}) - (\rho + \delta))$ (4OR)
$\mu_\lambda = U'(C)(F_{\bar{K}_R}(K, O + \overline{h})h'(\bar{K}_R) - (\rho + \delta))$ (5OR)
$\dot{p} = r(p - G(S)) - \frac{D'(E) + g\mu_E}{U'(C)}$ (6OR)
$\dot{\mu}_E = D'(E) + (\rho + g)\mu_E$ (7OR)

The solution of the dynamical system (1OR) - (7OR) has a saddle-path point.

### Lemma

*The planning program can use oil and renewables simultaneously only if $K > K^*$.*

# Climate Policy

"laissez-faire" economy: $D \equiv 0$

Therefore, it holds that at a critical time T: $p(T) = G(S(T))$, where is indifferent to use the available oil stock or left it in situ.

So: , $G(S(T))_{sc} < G(S(T))_{lf}$, so the social optimum is more cost efficient than in the "laissez-faire" economy.

# First Best Outcome: Optimal Carbon Tax

**PROPOSITION 3**: In an optimal regulation, the government could lead to the socially optimal outcome if it levies carbon tax $\tau$ that is equal to the social cost of carbon evaluated in the first-best.

$\tau = -\mu_E / \mu_K = \frac{\int_t^\infty e^{-\rho(t-s)} D'(E(s))ds}{U'(C(t))}$

where $\mu_E < 0$ as it is expressed in terms of social loss of welfare. Then, optimal carbon tax will rise over time as long as $r > \frac{D'(E)}{\tau U'(C)} - g$.

If the economy starts to develop, then the optimal carbon tax will rise; should be forced during the oil-renewable energy use and be maintained after the end of this period, so only renewables are used.

# Second Best Outcome: Renewable Subsidy

A constant backstop subsidy $\upsilon$ for renewables which is financed by a lump-sum tax.
The subsidy of renewables may lead the economy to extract and use oil more rapidly, so it will be phased out more quickly and cause more global warming damages (Green Paradox).
Therefore, it holds that at a critical time $T$: $p(T) - \upsilon = G(S(T))$, so this reduces the amount of $CO_2$ emissions in the atmosphere and more oil is left in situ than the first-best 'laissez-faire' outcome.

But the results may be the opposite if the economy is at a low level of development; this is because such subsidy implies a large loss in utility of consumption so private well owners will try to offset this loss by pumping oil more rapidly than without a green subsidy.

# Using Carbon Revenues

- improve environmental awareness by informing consumers and firms about the natural environment's protection (Bollino & Micheli, 2014).
- finance R&D activities in the renewable sector (Bollino & Micheli, 2014).
- the government could use carbon tax revenues for public spending $G(t)$ unrelated to the climate change: (Barro, 1990; Futagami et al., 1993).

# Concluding Remarks of the Benchmark Model

- the economy uses either only renewable capacity or renewables alongside oil.
- under the renewable capacity constraint framework, the social planning economy will have the maximum renewable capacity installed forever as long as $R^* = \delta \bar{K}_R$.
- the constraint renders renewable energy a fundamental resource to the economy and a swift towards more environmentally friendly technologies.
- If carbon tax is not maintained during the only-renewables phase, then the economy would have the incentive to re-use oil energy alongside renewables.
- the optimal subsidy for the second-best outcome should be low enough, especially for low growth levels of the economy, so as not to pump more oil.

# Green Ramsey Model with Public Spending (Barro, 1990)

Social Planner Program:

$$\max W = \int_0^\infty e^{-\rho t}(U(C(t)) - D(E(t))dt \text{ (1)'}$$

s.t.

$\dot{S}(t) = -O(t)$ (2)', $S(0) = S_0 > 0$

$\dot{E}(t) = O(t) - gE(t)$ (3)', $E(0) = E_0 > 0$, $0 < g < 1$

$\dot{K}(t)_R = R(t) - \delta K(t)_R$ (4)', $0 < \delta < 1$

$0 \leq K(t)_R \leq \bar{K}_R$ (5)'

$\dot{K}(t) = (1 - \tau_{tax})F(K(t), G_{sp}, O(t) + h(K(t)_R)) - R(t) - G(S(t))O(t) - C(t) - \delta K(t)$ (6)', $K(0) = K_0 > 0$

$G_{sp} = \tau_{tax} F(K(t), G_{sp}, O(t) + h(K(t)_R))$ (7)' (Government budget constraint)

# Green Ramsey Model with Knowledge Production Function and Clean Environment Dynamic (Bolino & Micheli, 2014)

Social Planner Program:

$$\max W = \int_0^\infty e^{-\rho t}(U(C(t), M(t)) - D(E(t))dt \text{ (1)''}$$

s.t.

$\dot{S}(t) = -O(t)$ (2)'', $S(0) = S_0 > 0$

$\dot{E}(t) = O(t) - gE(t)$ (3)'', $E(0) = E_0 > 0$, $0 < g < 1$

$\dot{K}(t)_R = R(t) - \delta K(t)_R$ (4)'', $0 < \delta < 1$

$0 \leq K(t)_R \leq \bar{K}_R$ (5)''

$\dot{K}(t) =$
$F(K(t), O(t) + h(K(t)_R)) - R(t) - G(S(t))O(t) - C(t) - \delta K(t)$ (6)'',
$K(0) = K_0 > 0$

$\dot{H} = H(R, H)$ (7)''

$\dot{M} = Z + bM - O$ (8)''

# Future Work

- Dynamics of the augmented Green Ramsey Model
- Calibrations!!!
- Climate change's additive or multiplicative effect on production function (e.g. destroying a natural ecosystem (additive), causing health problems and low productivity (multiplicative)) (Rezai et al., 2012)
- Green Ramsey Model with investment cost function instead of a damage one & upper limits of atmospheric carbon imposed by climate policy (Pommeret & Schubert, 2022).

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