Forecasting Realized Volatility with wavelet decomposition

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In a nutshell...

- The implications of realized volatility (RV) in the fields of risk management, asset allocation and asset pricing has attracted academic interest during recent decades.
- Schwert (1989) argued that there is little connection between RV and economic variables.
- Academic literature has made significant progress both in terms of methods, mainly focusing of the countercyclical movements.
- The RV forecasting literature has provided evidence in favor of macroeconomic variables (see among others Paye, 2012; Christiansen et al., 2012; Conrad and Loch, 2015; Wang et al., 2018).
- Technical Indicators are gaining ground in the academic literature (e.g. Gehrig and Menkhoff, 2006; Cheung and Chinn, 2001; Taylor and Allen, 1992), but mainly in forecasting experiments related to returns.
- We highlight the importance of frequency decomposition in order to provide insightful information regarding the performance of the predictors at hand, as well as a useful tool in favor of modelling RV.

Objectives...

Our objective/contribution is to...

- investigrate the predictive ability of some common macroeconomic and financial predictors.
- investigrate technical indicators as a new candidate group of predictors.
- explore a new explanation regarding the reasons that different groups of predictors could be complementary, namely forecast differing frequency components of RV.
- identify the sources of the forecasting ability which could lead to enhanced RV modelling, both statistically and economically motivated.
- investigate timing effects in the forecasting performance.
- evaluate the economic performance of the forecasts.

Findings

Theoretical backround - Macroeconomic Predictors

The relationship is mainly established on the uncertainty regarding the existing state of the economy and/or the expected stock returns which generate volatility in the ex-post return. The economic drivers of volatility can be summarized and labelled as i) the business cycle, ii) liquidity effects, iii) learning effects iv) market sentiment/uncertainty about fundamentals and v) country characteristics.

- Mele (2007) finds that risk premia change asymmetrically when the **economic state changes**; specifically, when the economy enters a negative state then risk premia increase and thus generates volatility.
- **Shortage of liquidity** in the market creates forces in the market that increase trading volumes (Schwert, 1989). Leveraged asset holders face pressure to short sale, in order to maintain liquidity (Brunnermeier and Pedersen, 2009).
- Timmermann (1993) argues that agents follow a **learning curve** over time. The investor adjusts the parameters of their subjective dividend growth model, since they tend to overreact to news.
- Officer (1973) argues that **volatility derived from macroeconomic variables** and market volatility are associated.
- Aggarwal et al. (1999) find that large increases in volatility in emerging **markets** are generally local events such as currency crises, hyperinflation or political issues.

Theoretical backround - Technical Indicators

The relationship between RV and technical indicators is sparse. This group of predictors is mainly a short-run decision toolkit, used by less "informed investors“. As a results, it can create "noise“ in the market and deviations from the fundamental value. We identify three transmission channels: i) investors are not always rational, ii) negative signals create more volatility and iii) sentiment periods.

- Early studies, such as Daniel et al. (1998), have documented that traders are **not always rational** when applying decision criteria. Brown and Cliff (2004) claim that the acquisition of information is costly.

- Negative signals are adjusted to more quickly, leading to larger absolute price changes and thus higher volatility (Chen and Ghysels, 2011). This **asymmetric effect** is further strengthened a) by short-sales constraints, and b) volatility feedback effects can lead to positive signals being more muted whereas negative signals can be amplified (Campbell and Hentschel, 1991).

- Smith et al. (2016) show that the superiority of technical rules during **high-sentiment seasons** can be attributed to the fact that markets exhibit trends (e.g. Lee et al., 2002). Shu and Chang (2015) demonstrate that higher sentiment can be associated with lower risk aversion and becoming more patient which leads to lower volatility.

Data-Candidate Predictors

Data

- We use $S\&P500$ as the world leading equity market index.
- We calculate RV based on the standard framework, such as

$$RV_t = ln\sqrt{\sum_{i=1}^{n} r_i^2}, \quad t = 1, 2, .., T$$

- Our sample covers the period extending from January 1950 to December 2018 ($T = 828$ observations).
- The in-sample period ends in December 1965 (as in Neely et al., 2014). The remaining is used as an out-of-sample to evaluate our model.

Candidate Predictors

- We use 14 common technical indicators: MA(1,9) , MA(2,9) , MA(3,9) , MA(1,12) , MA(2,12) , MA(3,12) , MOM(9) , MOM(12) , RSI(7), RSI(14) , EMA(3,9), EMA(5,9) , EMA(3,12), EMA(5,12).
- We use the standard Goyal and Welch dataset: DP , DY , EP , DE , RVOL, BM , NTIS , TBL , LTY , LTR , TMS , DFY , DFR , INFL.

Methodology

The information set to generate the forecast at time $t+1$ utilizes solely information available up to time t. The information set is updated recursively.

Individual Predictors

- The benchmark:

$$RV_t = a_j + \sum_{i=1}^{p} b_{j,i} RV_{t-i} + u_t$$

- Simple linear bivariate model forecasts:

$$RV_t = a_j + \sum_{i=1}^{p} b_{j,i} RV_{t-i} + \beta_1 X + u_t$$

- The forecasts are evaluated based on the R^2_{OS} and the Clark and West test (2007).
- To aggregate information from all available predictors we use a number of commonly applied methods: POOL, MEDIAN, TRIM3, PCA.

Table 1: Out-of-Sample Forecasts

Variables	$R^2_{OOS,rec}$	$R^2_{OOS,roll}$	Variables	$R^2_{OOS,rec}$	$R^2_{OOS,roll}$
MA(1,9)	2.52***	1.09***	DP	0.34*	-0.52***
MA(2,9)	1.99***	0.58***	DY	0.74**	-0.67**
MA(3,9)	1.66***	0.50***	EP	0.09*	0.74***
MA(1,12)	1.68***	0.38**	DE	0.26***	-0.02***
MA(2,12)	1.04***	-0.18**	RVOL	0.02**	0.91***
MA(3,12)	1.07***	-0.02**	BM	0.17***	0.09***
MOM(9)	1.54***	0.28**	NTIS	-0.80	-1.81
MOM(12)	0.68**	-0.24*	TBL	-0.11**	-1.29**
RSI(7)	1.06***	-0.41*	LTY	0.34***	0.33***
RSI(14)	0.19	-1.00	LTR	-0.68*	-0.50*
EMA(3,9)	-0.43	-0.90	TMS	-0.48	-0.78
EMA(5,9)	-0.67	-0.89	DFY	1.05***	0.31***
EMA(3,12)	-0.79	-1.07	DFR	0.79*	-0.15*
EMA(5,12)	-1.09	-1.31	INFL	0.56**	0.72***
POOL	1.13***	0.25**	POOL	1.51***	2.56***
MEDIAN	1.17***	0.23**	MEDIAN	1.28***	1.87***
TRIM3	1.13***	0.00**	TRIM3	1.72***	2.45***
PCA	1.94***	0.43***	PCA	0.75***	-1.12***

Shed further light...

Wavelet analysis is a powerful tool to decompose time-series data into orthogonal components with different frequencies. Following Risse (2019), Faria and Verona (2018) and Caraiani (2017), we employ wavelet decomposition analysis with the use of maximal overlap discrete wavelet transform (MODWT) and Haar wavelet filter with reflecting boundary conditions. We also apply 6 levels of Multiresolution Analysis (MRA) . Hence, we break the initial series into 6 wavelet details, plus the smooth component . The new series capture gradually more and more abrupt changes, while the smooth detail works more as a long trend/memory series, such as:

$$RV_t = RV_t^{L1} + RV_t^{L2} + RV_t^{L3} + RV_t^{L4} + RV_t^{L5} + RV_t^{L6} + RV_t^{S} \quad (1)$$

The short is signal is calculated as $RV_t^{SS} = RV_t^{L1} + RV_t^{L2}$,
the medium as $RV_t^{MS} = RV_t^{L3} + RV_t^{L4} + RV_t^{L5}$, and finally,
the long signal as $RV_t^{LS} = RV_t^{L6} + RV_t^{S}$.

The forecasting process for SS, MS and LS is given by:

$$\hat{R}V_t^f = \hat{a}_j + \sum_{i=1}^{p} \hat{b}_{j,i} RV_{t-i} + \hat{\beta}_j X_{j,t-1} \quad (2)$$

Faria and Verona (2018a) build on the work of Ferreira and Santa-Clara (2011) by proposing the sum of some of the forecasted decomposed parts. We follow a similar approach, such as:

$$\hat{R}V_{t,j} = \hat{R}V_{t,j}^{SS} + \hat{R}V_{t,j}^{MS} + \hat{R}V_{t,j}^{LS} \quad (3)$$

Table 2: Out-of-Sample Forecasts for Decomposed RV

Panel A: Technical Indicators					Panel B: Macroeconomic Predictors				
Var.	RV^{SS}	RV^{MS}	RV^{LS}	$\sum_{f=1}^{3} \hat{R}V_f$	Var	RV^{SS}	RV^{MS}	RV^{LS}	$\sum_{f=1}^{3} \hat{R}V_f$
MA(1,9)	5.04***	1.65***	0.00	2.74***	DP	0.00	6.24***	13.62***	0.58**
MA(2,9)	4.42***	1.35***	0.00	2.21***	DY	0.00	5.93***	13.93***	0.63**
MA(3,9)	3.95***	1.23***	0.00	1.90***	EP	0.00	2.57***	7.32***	0.50**
MA(1,12)	4.09***	1.19***	0.00	1.99***	DE	0.00	1.64***	-2.06***	0.52***
MA(2,12)	3.35***	0.95***	0.00	1.30***	RVOL	0.00	0.83**	10.09***	-0.05**
MA(3,12)	3.15***	0.92***	0.00	1.33***	BM	0.00	4.40***	12.15***	0.75***
MOM(9)	3.92***	1.09***	0.00	1.91***	NTIS	0.00	-0.48	-5.21***	-0.60
MOM(12)	2.80***	0.84***	0.00	0.85***	TBL	0.00	4.45***	2.87***	-0.21**
RSI(7)	3.47***	1.19***	0.00	1.46***	LTY	0.00	6.13***	12.66***	0.14***
RSI(14)	1.58***	2.32***	0.00	0.03*	LTR	0.00	-0.30	-1.23	0.06
EMA(3,9)	0.87***	0.88***	0.00	-0.23	TMS	0.00	13.38***	15.31***	-0.43
EMA(5,9)	0.31**	0.84***	0.00	-0.27	DFY	0.00	5.78***	19.61***	0.96***
EMA(3,12)	0.32*	1.02***	0.00	-0.19	DFR	0.00	0.84**	-0.86	0.61**
EMA(5,12)	-0.57	0.64***	0.00	-0.41	INFL	0.00	0.69***	0.55***	1.03***
POOL	3.26***	1.22***	0.00	1.40***	POOL	0.00	5.78***	15.32***	1.42***
MEDIAN	3.69***	1.17***	0.00	1.66***	MEDIAN	0.00	1.97***	8.17***	1.08***
TRIM3	3.45***	1.25***	0.00	1.40***	TRIM3	0.00	4.52***	14.64***	1.57***
PCA	4.08***	1.42***	0.00	2.30***	PCA	0.00	15.84***	35.77***	1.76***

Aggregating Information

We provide further evidence in favor of RV forecasting:

Statistical modelling approaches

A) By aggregating information from the entire dataset , i.e. including both technical indicators and macroeconomic predictors; denoted as ALL. Intuitively, technical indicators and macroeconomic predictors should be able to provide complementary information regarding the frequency components of RV.
B) We are including all the predictors to forecast each frequency using all of the data for each of the combination techniques. In the next step, we sum together the forecasts for each of the frequency components.

Economically motivated approach

C) We impose economically motivated restriction in the spirit of Campbell and Thompson (2008). Intuitively, technical indicators and macroeconomic predictors should be able to provide complementary information regarding the frequency components of RV. Here we impose that the short signal is based on technical indicators and the long signal is based on macro-financial predictors with the medium component based on both series.

Table 3: Aggregating Information

Method	**ALL** (A)	$\hat{R}V_{SS}^{SA}$	$\hat{R}V_{MS}^{SA}$	$\hat{R}V_{LS}^{SA}$	$\sum_{f=1}^{3} \hat{\boldsymbol{R}}\boldsymbol{V}_f^{SA}$ (B)	$\sum_{f=1}^{3} \hat{\boldsymbol{R}}\boldsymbol{V}_f^{EA}$ (C)
POOL	1.56***	2.31***	3.66***	9.60***	1.56***	2.88***
MEDIAN	0.93***	1.08***	1.68***	3.31***	0.07	2.83***
TRIM3	1.36***	2.00***	3.13***	9.61***	1.36***	2.99***
PCA	0.08***	2.78***	13.02***	35.64***	2.57***	2.96***

Timing and other Effects

Do Financial Conditions Matter?

- Performance during Recessions and Expansions

$$R_c^2 = 1 - \frac{\sum_1^P (RV_t - \hat{R}V_t)^2 I_t^c}{\sum_1^P (RV_t - \hat{R}V_{b,t}) I_t^c}^2 \quad , c = expansion, recession \tag{4}$$

- Performance during High and Low VIX periods

$$VIX - period_t = \left\{ \begin{array}{l} high \;\; if \;\; VIX_t \succeq median(VIX_{1:t}) \\ low \;\; if \;\; VIX_t \prec median(VIX_{1:t}) \end{array} \right\} \tag{5}$$

- Performance during Alternative OOS Periods, 1983:1-2018:12 and 2000:1-2018:12.

h-month-ahead forecasts

$h = [3, 6, 12]$

Leverage Effect

The literature has established a negative relationship between market returns and volatility. Our objective is to identify whether the forecasts of the proposed methods contain distinct information from market movements, such as: $\hat{R}V_t^f = \hat{a}_j + \sum_{i=1}^{p} \hat{b}_{j,i} RV_{t-i} + \hat{\beta}_j X_{j,t-1} + \hat{\gamma} EP_{t-1}$

Results; Timing Effects

Table 4: Forecasts during different timing in Financial Conditions

	Exp.	Rec.	H-VIX	L-VIX	OOS1983	OOS2000
Panel A: Technical Indicators						
POOL	0.83**	2.60***	1.92**	2.12**	1.29***	2.01***
MEDIAN	0.87**	2.69***	2.21**	2.21**	1.39***	2.20***
TRIM3	0.78**	2.93***	2.07**	2.15**	1.30***	2.18***
PCA	1.60***	3.66***	1.24*	4.50***	2.29***	3.84***
Panel B: Macroeconomic Predictors						
POOL	0.64**	5.86***	0.36	-0.25	0.15	0.36**
MEDIAN	0.68***	4.26***	0.36	-0.32	0.21	0.17
TRIM3	0.62**	7.22***	0.52	-1.34	-0.17	-0.37
PCA	-1.76**	13.34***	-1.89	-3.50	-1.35	0.69*
Panel C: All predictors Taken Together						
POOL	0.97***	4.51***	1.28**	1.18**	0.89***	1.36***
MEDIAN	0.62***	2.51***	1.09**	0.47	0.72***	1.26***
TRIM3	0.76***	4.35***	1.07**	0.71	0.64**	1.08***
PCA	-1.88*	9.90***	-0.38	-0.21	-0.39	1.58**
Panel D: Statistically Motivated Approach						
POOL	0.97***	4.51***	1.28**	1.18**	0.89***	1.36***
MEDIAN	-0.10	0.94**	0.87**	0.38	0.38**	0.61**
TRIM3	0.76***	4.35***	1.07**	0.71	0.64**	1.08***
PCA	-0.35***	17.21***	4.62***	3.40***	2.71***	3.78***
Panel E: Economically Motivated Approach						
POOL	1.64***	9.12***	2.37**	1.64*	1.45***	2.48***
MEDIAN	1.96***	7.22***	2.48**	2.08**	1.58***	2.59***
TRIM3	1.53***	10.31***	2.56**	1.55**	1.34***	2.04***
PCA	-0.15***	18.52***	4.14**	3.02***	3.24***	4.92***

Results; h-month-ahead and Leverage Effect

Table 5: h-month-ahead

	TECH	MACRO	ALL	SE	EA
Panel A: $h = 3$					
POOL	-0.91	0.95***	0.18	0.18	0.11*
MEDIAN	-1.15	0.80***	-0.18	-0.39	-0.36
TRIM3	-1.26	1.07***	0.23	0.23	0.00**
PCA	-1.83	0.82***	-1.53	-0.75***	0.26***
Panel B: $h = 6$					
POOL	-0.31	1.02***	0.49**	0.49**	1.02***
MEDIAN	-0.37	0.88***	0.19	-0.18	0.86***
TRIM3	-0.39	1.39***	0.52***	0.52***	1.42***
PCA	-1.18	0.76***	-0.44**	-1.03***	1.25***
Panel C: $h = 12$					
POOL	-0.63	0.85***	0.22*	0.22*	0.16*
MEDIAN	-0.76	0.47***	0.02	-0.17	-0.39
TRIM3	-0.82	1.31***	0.35**	0.35**	0.81***
PCA	-1.30	0.61***	-0.24**	-1.29**	0.59***

Table 6: Leverage Effect

POOL	5.39***	2.81***	5.63***	6.19***	6.92***
MEDIAN	5.20***	2.45***	5.58***	5.51***	6.40***
TRIM3	5.15***	3.01***	5.59***	6.31***	6.97***
PCA	5.40***	3.72***	3.75***	6.30***	6.60***

The applicability of the proposed RV forecasting setup has economic interest. For this purpose, we isolate the volatility effects and formulate the portfolios based on the volatility managed portfolios framework. Our setup is inspired by Moreira and Muir (2017). We consider a US based investor that rebalances her portfolio at the end of each month t between the risky asset and the risk free. We measure the performance of the portfolio based on the Certainty Equivalent Return (CER) and Sharpe Ratio (SR), as well as, the end-of-period portfolio return; we assume that $\gamma = 3$.

The investor allocates weights such as:

$$w_{t+1} = \frac{\bar{R}V_{1:t}}{\hat{R}V_{t+1}} \tag{6}$$

where $\hat{R}V_{t+1}$ is the forecast of each specification or the RV historical average, namely $\bar{R}V_{1:t}$, when employing the buy-and-hold strategy. Thus, in periods of high forecasted RV, the investor reduces the exposure to the risky asset, and vice versa.

Results; Economic Performance

Table 7: Volatility-Managed Portfolio

	CER	cumRet	SR(ann.)	R_p(ann.)	σ_p(ann.)	CER	cumRet	SR(ann.)	R_p(ann.)	σ_p(ann.)
$\hat{R}V_b$=Buy & Hold	4.89	137.23	0.38	5.69	14.93	4.89	137.23	0.38	5.69	14.93
$\hat{R}V_b$=60m roll.	5.15	47.36	0.30	3.11	10.41	5.06	45.39	0.29	3.03	10.42
$\hat{R}V_b$=AR(p)	6.60	132.44	0.45	5.17	11.54	5.81	87.71	0.38	4.39	11.57
Panel A: Technical Indicators										
POOL	6.63	136.06	0.45	5.22	11.59	5.83	89.77	0.38	4.44	11.61
MEDIAN	6.64	136.42	0.45	5.23	11.59	5.83	89.85	0.38	4.44	11.62
TRIM3	6.64	136.98	0.45	5.24	11.60	5.83	90.00	0.38	4.44	11.63
PCA	6.75	144.06	0.46	5.33	11.57	5.89	92.04	0.39	4.48	11.59
Panel B: Macroeconomic Predictors										
POOL	6.66	132.70	0.45	5.16	11.42	5.89	88.66	0.38	4.39	11.44
MEDIAN	6.63	132.67	0.45	5.16	11.48	5.85	88.40	0.38	4.40	11.50
TRIM3	6.69	133.35	0.45	5.16	11.38	5.92	89.36	0.39	4.41	11.40
PCA	6.69	130.16	0.45	5.11	11.28	5.94	88.41	0.39	4.38	11.30
Panel C: All Predictors Taken Together										
POOL	6.65	134.11	0.45	5.19	11.49	5.86	89.26	0.38	4.42	11.52
MEDIAN	6.63	134.41	0.45	5.20	11.54	5.84	89.17	0.38	4.42	11.56
TRIM3	6.64	133.28	0.45	5.17	11.49	5.85	88.77	0.38	4.41	11.52
PCA	6.76	133.77	0.46	5.16	11.25	5.97	89.25	0.39	4.39	11.28
Panel D: Statistical Approach, Frequency Aggregates										
POOL	6.65	134.11	0.45	5.19	11.49	5.86	89.26	0.38	4.42	11.52
MEDIAN	6.59	132.60	0.45	5.17	11.58	5.79	87.79	0.38	4.39	11.60
TRIM3	6.64	133.28	0.45	5.17	11.49	5.85	88.77	0.38	4.41	11.52
PCA	6.53	110.43	0.43	4.76	10.94	5.78	74.74	0.37	4.02	10.97
Panel E: Economically Motivated Approach, Frequency Aggregates										
POOL	6.65	131.82	0.45	5.15	11.41	5.86	87.55	0.38	4.37	11.44
MEDIAN	6.68	136.58	0.45	5.22	11.50	5.88	90.14	0.38	4.44	11.53
TRIM3	6.67	132.93	0.45	5.16	11.41	5.88	88.11	0.38	4.38	11.44
PCA	6.77	117.52	0.45	4.84	10.65	6.04	80.33	0.39	4.13	10.68

Conclusions

- Our first contribution to this dialogue is to provide further insight and show the role that technical indicators play in the RV forecasting context.
- Our second contribution is to shed light on the complementarity of technical indicators and macro predictors at different time frequencies.
- Our third contribution is to demonstrate that economically motivated constraints can substantially enhances forecasting performance of RV by explicitly recognizing the short-term nature of technical indicators and the longer-term nature of macroeconomic variables.
- In terms of timing, the positive performance is stronger during crisis periods. In addition, our results show that technical indicators do better during recessions, as suggested by the literature, and low sentiment periods, whereas the proposed amalgamation model demonstrates a smoother performance and is less affected by sporadic turbulence in sentiment or the economy.
- The results provide strong evidence that our framework is able to deliver significantly superior economic gains against a simple buy-and-hold strategy. Moreover, the portfolios generate higher annual returns and are less volatile