Information Aggregation by Council

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The Question

General Motivation:

Can deliberation improve collective-decision making?

Some Answers so far:

When allowing truth-seeking agents to communicate, outcomes typically improve

→ For instance: *Levit and Malenko (2011)*; *Battaglini (2017)*; *Ekmekci and Lauermann (2019)*; *Goeree and Yariv (2011)*.

But to derive these results:

- we compare groups of equal size

  → efficiency of the same group when allowed or not to deliberate.
- we often assume no aggregate uncertainty and/or specific families of info structures.

The trade-off

Councils can deliberate but have access to few pieces of information; while electorates do not deliberate but have access to all existing information.

Councils have better tools but less resources compared to electorates.

Is there a way to compare these two institutions meaningfully?

Which features of the environment matter?

Our Contribution

Questions:

Does a (small) deliberative body fare better or worse than a (big) electorate?

What is the role of aggregate uncertainty in this respect?

Quick Answers:

It depends! ☹

Not, really. Council is better! ☺

Preview of Results:

- Council better when substantial aggregate uncertainty.
- When electorate does better, it does so only slightly.
- When council does better, difference can be prodigious.
- Rationale on why collective entities use representative democracy institutions, delegating decisions to small deliberative bodies rather than large electorates.

Literature - Information Aggregation by Electorates

- *Condorcet*: In a society of independent voters who vote sincerely, majority voting leads to the correct outcome.
- *Austen–Smith and Banks (1996)*: Sincere voting is rarely an equilibrium.
- *McLennan (1998)*: Even if sincere voting is not an equilibrium, there is always an equilibrium that efficiently aggregates information.
- *Barelli et al. (2020)*: This is true also for environments with multiple alternatives and general information structures.

→ Information aggregation is not perfect
→ Strong assumptions are required to be achieved
→ Absence of other considerations like state-dependent participation (Ekmekci and Lauermann 2022), or reputation concerns (Bos et al. 2022)

Literature - Information Aggregation (II)

- *Bouton, et al. (2022)*: "Flexibility" is good for information aggregation.
- *Dhillon, et al. (2022)*, *Casella et al. (2022)*: Vote delegation/Liquid democracy allows agents also to aggregate information better.

→ Information structure still plays an important role though.

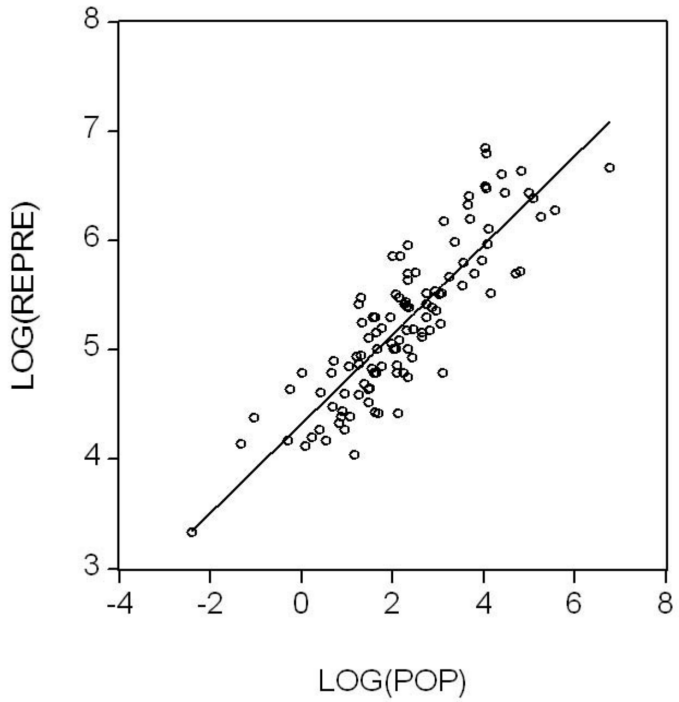
→ Substantial aggregate uncertainty can prevent information aggregation.

→ What can we do about that then?

Let a "small" council decide. Small?

Its absolute size increases as the population increases, but it becomes negligible as a share of the population (e.g. $\sqrt{n}$). Taagepera (1972).

Auriol and Gary-Bobo (2012): Empirical evidence that as population n increases, parliament size increases at $n^{0.4}$.



The Problem

Society faces a binary decision to be made collectively

- All individuals are truth-seeking (common preferences)
- Individuals are imperfectly and privately informed.

Compare relative informational efficiency of:

- Council: a small deliberative body with the power to decide
- Electorate: a large non-deliberative body deciding by majority

Tradeoff:

- Electorate: more information, but less elegant tools to aggregate them
- Council: less information, but can process it more efficiently via deliberation.

Failure of plurality voting to aggregate info - I

Two alternatives: $Hire$ or $Not\ Hire$.

Three states (equally likely): $Soft$, $Aggressive$, and $Balanced$.

- $Not\ Hire$ is optimal if candidate is $Soft$ or $Aggressive$.
- $Hire$ is optimal if candidate is $Balanced$.

Each individual observes a private signal, $Risky$ or $Chicken$.

- If $Soft$ signal is $Risky$ with prob. 0%.
- If $Aggressive$ signal is $Risky$ with prob. 100%.
- If $Balanced$ signal is $Risky$ with prob. 50%.

Failure of plurality voting to aggregate info - II

Strategy can only depend on signal. For instance: Sincere Voting

[Sincere Voting: vote the alternative most likely to be correct given private info.]

- Sincere Voting $\Rightarrow$ vote $Not\ Hire$ after either signal.
- Definitely fails to aggregate information.
- But is just sincere voting the problem? No!

What about another common strategy:

$P(Hire|Risky) := \mu_r$, $P(Hire|Chicken) := \mu_c$

[Symmetry is used for exposition. Result is general.]

Failure of plurality voting to aggregate info - III

Then, the ex-ante probability of voting for to Hire in each state is:

- in $Soft$ $\rightarrow 0 \cdot \mu_r + 1 \cdot \mu_c = \mu_c$
- in $Aggressive$ $\rightarrow 1 \cdot \mu_r + 0 \cdot \mu_c = \mu_r$
- in $Balanced$ $\rightarrow \frac{1}{2} \cdot \mu_r + \frac{1}{2} \cdot \mu_c = \frac{\mu_r + \mu_c}{2}$

Information aggregation would require $\mu_c < 1/2$, $\mu_r < 1/2$ and $\frac{\mu_r + \mu_c}{2} > 1/2$

$\rightarrow$ Impossible [$\pi\mu_r + (1-\pi)\mu_c$ is weakly monotonic on π].

$\rightarrow$ Even an arbitrarily large society would sometimes fail to make the correct decision in some state.

$\rightarrow$ A deliberative council that could communicate signals would succeed.

Basic Setup: Preferences

Society of n individuals $N = \{1, \ldots, n\}$ has to select among alternatives a_1, a_2.

Selection through a collective decision-making process. *[to be described in detail]*

Individuals have common preferences.

Namely, nature chooses a state x from a finite set $X = \{x_1, \ldots, x_r\}$.

States determine desirability each alternative.

There is a partition $\{\Pi_1, \Pi_2\}$ of X such that for each $a_j \in A = \{a_1, a_2\}$,

$$u(a_j, x_l) = \begin{cases} 1 & \text{if } x_l \in \Pi_j \\ 0 & \text{if } x_l \notin \Pi_j \end{cases}$$

i.e. alternative a_j is preferred than a_{-j} if the state belongs to Π_j.

Basic Setup: Information

But, states are unobservable.

Individuals receive potentially different information about the underlying state.

Information affects beliefs about which alternative should be selected.

Individuals observe privately i.i.d. signals from a finite set $S = \{s_1, \ldots, s_m\}$.

- Probability of each signal are state dependent.
- Distributions are commonly known.

In what follows, we will consider *[will elaborate more on that in a bit]*:

- Multiple states, $|X| = r \geq 3$.
- Binary signals, $S = \{s_1, s_2\}$
- Probability of observing s_1 when the state is x_l denoted by p_l.
- $p_l \neq p_{l'}$ for all $x_l \neq x_{l'}$.

Basic Setup: Full-Information Equivalence

Determine the probability that society manages to choose the correct alternative.

Can large societies eliminate discrepancies arising from information asymmetries?

Full-information equivalence (FIE) achieved through a collective-decsion process if the signal profile eventually reveals the state in large societies.

More formally, in the current setup:

Definition (Full Information Equivalence)

A collective decision-making process achieves full-information equivalence for a set of admissible parametrizations if for each admissible parametrization in the set there exists a sequence of equilibria such that the ex-ante probability of the correct alternative being chosen converges to one as the society becomes arbitrarily large.

Decision-Making Process I: Plurality Rule Election

The society chooses an alternative through an election that follows *plurality rule*.

- Each voter votes for one of the two alternatives.
- For simplicity, abstention is not allowed.
- The alternative that receives most votes is the winner of the election.

Common action space $A = \{a_1, a_2\}$ for all individuals.

- For each i, a mixed (ex-ante) strategy is a vector function $\mu_i : S \to \Delta(A)$.
- $\mu_i(s_k) = (\mu_i(a_1|s_k), \mu_i(a_2|s_k))$ prob. dist. over A conditional on s_k.
- $\mu = (\mu_i)_{i \in N}$ denotes a strategy profile and M is the strategy space.

Decision-Making Process II: Council Ruling

Society delegates decision to a council consisting of fraction g_n of population.

- g_n increases in n, $g_n \to \infty$, but $\frac{g_n}{n} \to 0$.

Council members are able to share their private information.

- Each council member $i \in \{2, \ldots, g_n\}$ sends a (non-necessarily truthful) report of the observed signal.
- Messages are sent to $i = 1$, call her *the decision maker*
- Decision maker collects messages and chooses one of the two alternatives.

Strategies:

- For each council member, mixed (ex-ante) strategy: $m_i : S \to \Delta(S)$.
- DM's strategy function of messages vector and own signal, $v : S^{g(n)} \to \Delta(A)$
- Strategy profile $\mu = (v, m_1, \ldots, m_{g_n})$ contains messages and DM's decision.

Full-Information Equivalence

For either of the above processes, we can determine:

→ $W_n(\mu)$: ex-ante prob. the correct alternative is selected for a profile μ.

→ Also describes the ex-ante expected utility of each i, i.e. $EU_i(\mu) = W_n(\mu)$.

→ μ^* an equilibrium if, for each i, μ_i^* maximizes EU_i when others follow μ^*.

→ μ_n^* denotes an equilibrium strategy profile in a society of n individuals.

Definition (Full-Information Equivalence - Formal)

A collective decision-making process achieves full-information equivalence for a set of admissible parametrizations if for each admissible parametrization there exists a sequence of equilibria $\{\mu_n^*\}_{n=2}^{\infty}$ such that $W_n(\mu_n^*) \to 1$ as $n \to \infty$.

Full-Information Equivalence in Plurality Elections

By Barelli et al. (2020) FIE is achieved generically in plurality elections when:

- Number of alternatives equal to the number of states.
- Number of signals exceeds number of states.

FIE tougher when state space is rich compared to signal space.

- This is the reason we focus on binary signals and multiple states.

Proposition (FIE in Plurality Elections)

In a plurality-rule election, full-information equivalence is achieved if and only if the set of admissible parametrizations satisfies:

$$\max_{x_l \in \Pi_1} p_l < \min_{x_{l'} \in \Pi_2} p_{l'} \text{ or } \min_{x_l \in \Pi_1} p_l > \max_{x_{l'} \in \Pi_2} p_{l'}$$

→ Requires some sort of monotonicity on the likelihood of observing each signal.

→ Observing s_1 more/less likely in any state of Π_1 than in any of Π_2.

→ Each signal associated with one element of the partition.

Full-Information Equivalence in Council Ruling

Could information sharing within a council solve this problem?

- YES!
- As the society grows, DM eventually learns the state.
- Only requirement states to be distinguishable ($p_l \neq p_{l'}$ for each $x_l \neq x_{l'}$).

Proposition (FIE under Council Ruling)

In a council ruling, full-information equivalence is achieved for any set of admissible parametrizations.

→ Sincere messages + Thershold strategy by DM lead to FIE.

→ Found sequence of strategy profiles that "does the job".

→ Given common preferences, will also exist a sequence of equilibria that does it.

Comparison of Processes - I

Corollary

When the signal structure is "non-monotonic", a council can yield superior results in large societies compared to an electorate.

→ Follows directly from the fact that in such cases a council achieves FIE, whereas the election does not.

But what about cases in which both processes achieve FIE?

Proposition

When the signal structure is "monotonic", there exists n^*, such that, for every $n > n^*$ the electorate performs better than the council.

→ For a given n, the condition is $g_n \leq \frac{n-1}{2}$ (and by assumption $\frac{g_n}{n} \to 0$)

Why?

Comparison of Processes - II

Intuition:

- Council of similar size to society: Clear advantage (info sharing).
- But, a sufficiently large society can imitate any strategy profile a small council can achieve and improve upon that.

Important Points:

- Typical advantage of councils is their small size.
- Cutoff council size sufficiently large to ensure a meaningful comparison.
- Efficiency difference shrinks as the population increases

Conclusion

We compare information efficiency of:

- Council: a small deliberative body with the power to decide
- Electorate: a large non-deliberative body deciding by majority.

Findings:

- Either can aggregate info better. Depending on info structure.
- When electorate does better, it does so only slightly.
- When council does better, difference can be prodigious.

A novel rationale as to why most collective entities delegate decisions to councils rather than electorates.