Credit Crunches with Recursive Preferences

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\textbf{CRETE}

\textit{July 14, 2022}
2 main questions

1. What is the effect of a credit crunch on households' decisions?
2. What are the implications for asset prices?
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2. What are the implications for asset prices?
Recent interest motivated by:

- costly debt deleveraging following the 2007-2008 financial crisis.
- challenges of macro models to be consistent with asset-pricing phenomena.
Our contribution

We build a unified theoretical framework to:

▶ replicate the observed downturn of the U.S economy.
▶ account for the dynamics of asset prices.
▶ are the first to MIT shocks in a setup featuring HA and recursive utility.
▶ credit crunches and the 2008 financial crisis with Epstein-Zin preferences.
▶ asset pricing puzzles in HA economy.

Theoretically, we introduce Epstein-Zin preferences to:

▶ 10 percentage point drop in debt-to-GDP ratio.
▶ deal with two fundamental challenges of most theoretical macro models.
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Risk-free rate is \( \approx 0.9\% \);

\[ \Rightarrow \text{equity premium} \approx 6.1\% \rightarrow \text{VERY LARGE PUZZLE.} \]

Q: Can existing macro models generate such a high equity premium?

\[ \rightarrow \text{time-separable preferences is the fundamental cause of this puzzle.} \]

Why:

Extremely high RA needed to generate the observed (mean) return on equity \( \rightarrow \) RA should be around 50 while micro estimates suggest this number should not be higher than 2 or 3.

This implies a very low EIS which generates a very high risk-free rate \( \rightarrow \) inconsistent with the data \( \rightarrow \) PUZZLE.
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Advantages

▶ mathematical convenience.
▶ dynamic consistency.
▶ scale invariance.
▶ first-order conditions log-linear in consumption.

Shortcomings

▶ restriction that risk attitudes be tied to timing attitudes.
▶ timing of resolution of uncertainty does not matter.

⇒ fundamental cause of equity-premium / risk-free rate puzzles.

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Epstein-Zin Preferences

Standard expected-utility time-separable preferences:

\[
E \left[ \sum_{t=0}^{\infty} \beta^t U(c_t) \right]
\]  

(1)

In recursive form:

\[
V_t = U(c_t) + \beta E_t V_{t+1}
\]  

(2)

EZ specification:

\[
V_t = \left[ c_t^{1-\rho} + \beta \left( E_t V_{t+1}^{1-\alpha} \right)^{\frac{1-\rho}{1-\alpha}} \right]^{\frac{1}{1-\rho}}
\]  

(3)

- $\frac{1}{\rho}$: EIS (between consumption today and CEq of continuation utility)
- $\alpha$: coefficient of RRA

--- breaks link between risk-aversion and EIS.
Model

We build upon Guerrieri and Lorenzoni (2017).
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- government
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3. single asset traded is a one-period risk-free bond.
4. exogenous debt limit.
5. single interest rate.
Households (HHs)

Continuum of heterogeneous HHs that supply labor and consume.

- HH $i$’s lifetime utility is given by

$$V(b_i, \theta_i) = u(c_i, 1 - n_i) + \beta \left( EV(b'_i, \theta'_i)^{1-\alpha} \right)^{\frac{1}{1-\alpha}} \tag{4}$$

- $b$: bond holdings.
- $\theta_i$: labor productivity $\rightarrow$ markov process.

- Assume instantaneous utility is additive and iso-elastic

$$V(b_i, \theta_i) = \underbrace{\frac{c_i^{1-\gamma}}{1-\gamma}}_{u(c_i, 1 - n_i)} + \psi \frac{(1 - n_i)^{1-\eta}}{1 - \eta} + \beta \left( EV(b'_i, \theta'_i)^{1-\alpha} \right)^{\frac{1}{1-\alpha}} \mu \equiv CEq \tag{5}$$

- $\gamma \equiv$ inverse EIS
- $\alpha = 1 - \frac{RA}{\gamma}$
- $RA = -B \cdot \frac{V_{11}(b,\theta)}{V_1(b,\theta)} + \alpha \cdot \frac{BV_1(b,\theta)}{V(b,\theta)}$
Each HH produces consumption goods using linear technology

\[ y_i = \theta_i n_i \]  

- \( \theta^1 = 0 \): unemployment \( \rightarrow \) **exogenous unemployment risk**.

**Budget Constraint:**

\[ q b'_i + c_i \leq b_i + y_i - \tilde{\tau}_i \]  

- \( \tilde{\tau}_i = \tau, \quad \theta_i > 0 \) \( \rightarrow \) employed.
- \( \tilde{\tau}_i = \tau - v, \quad \theta_i = 0 \) \( \rightarrow \) unemployed.

and

- \( v \): unemployment benefit.

Debt is bounded below by the exogenous limit \( \phi \)

\[ b'_i \geq -\phi \]
Optimality Conditions

▶ The HHs problem is to max (1) subject to (3)

\[ V(b_i, \theta_i) \equiv \max_{\{c_i, n_i, b'_i\}} \left[ u(c_i, 1 - n_i) + \beta \left( EV(b'_i, \theta'_i)^{1-\alpha} \right)^{\frac{1}{1-\alpha}} \right] \]  

(9)

s.t: budget constraint (7), \( c_i \geq 0 \), and \( n_i \in [0, 1] \).

▶ Euler for consumption

\[ u_{c_i}(c_i, 1 - n_i) \geq \beta(1 + r) \left[ EV(b'_i, \theta'_i)^{1-\alpha} \right]^{\frac{\alpha}{1-\alpha}} E \left[ V(b'_i, \theta'_i)^{-\alpha} u_{c'_i}(c'_i, 1 - n'_i) \right] \]  

(10)

▶ Euler for labor

\[ \frac{-u_n(c_i, 1 - n_i)}{u_c(c_i, 1 - n_i)} \geq w \]  

(11)
Government

- Finances a stream of expenditures by levying taxes and issuing debt.
- The government’s budget constraint is:
  \[ B + uv = qB' + \tau \]  
  \[ (12) \]
  - \( u = Pr(\theta_i = 0) \) → fraction of unemployed in the population.
  - tax rate (residually) adjusts to balance budget.
- Supply of bonds (\( B \)) exogenously fixed.
A RCE is a sequence of value functions \( \{V\}: \mathbb{R}_+^2 \to \mathbb{R} \), decision rules \( \{C\}, \{N\}: \mathbb{R}_+^2 \to \mathbb{R} \), interest rates \( \{r\}: \mathbb{R} \to \mathbb{R} \), tax rates \( \{\tau\}: \mathbb{R}_+ \to \mathbb{R}_+ \), and the (joint) distribution of bond holdings and productivity levels \( \{\Psi\}: \mathbb{R}^2 \to \mathbb{R}_+ \), such that

1. given the interest rate \( \{r\} \) and the tax rate \( \{\tau\} \), the value function \( V(b, \theta) \) solves the Bellman equation (9), and \( \{C(b, \theta), N(b, \theta)\} \) are the corresponding optimal decision rules,

2. the (joint) distribution of bond holdings and productivity levels \( \Psi \) is consistent with the households’ optimal consumption and labor supply decisions,

3. the tax rate equals government expenditures,

\[
\tau = \nu u + \frac{rB}{1 + r}
\]

4. the asset (bond) market clears,

\[
\int bd\Psi(b, \theta) = B.
\]
Table: Baseline Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
<th>Target/Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.9749</td>
<td>Discount factor</td>
<td>$r = 2.5%$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>4</td>
<td>Inverse elasticity of intertemporal substitution</td>
<td></td>
</tr>
<tr>
<td>$RA$</td>
<td>15</td>
<td>Coefficient of relative risk aversion</td>
<td></td>
</tr>
<tr>
<td>$\eta$</td>
<td>1.5</td>
<td>Curvature of utility from leisure</td>
<td>Average Frisch elasticity = 1</td>
</tr>
<tr>
<td>$\psi$</td>
<td>15.57</td>
<td>Weight on utility from leisure</td>
<td>Nekarda and Ramey (2020)</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.967</td>
<td>Productivity shock persistence</td>
<td>Floden and Lindé (2001)</td>
</tr>
<tr>
<td>$\sigma_\varepsilon$</td>
<td>0.017</td>
<td>Productivity shock variance</td>
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<tr>
<td>$\pi_{e,u}$</td>
<td>0.057</td>
<td>Transition to unemployment</td>
<td>Shimer (2005)</td>
</tr>
<tr>
<td>$\pi_{u,e}$</td>
<td>0.882</td>
<td>Transition to employment</td>
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</tr>
<tr>
<td>$B$</td>
<td>1.60</td>
<td>Bond supply</td>
<td>Liquid assets</td>
</tr>
<tr>
<td>$\phi$</td>
<td>1.847</td>
<td>Borrowing constraint</td>
<td>Total gross debt</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.167</td>
<td>Unemployment benefit</td>
<td>40% of average labor income</td>
</tr>
</tbody>
</table>
Credit Shocks

- Unexpected tightening of borrowing limit $\rightarrow$ MIT shock.

$\rightarrow$ no aggregate uncertainty.

- Gradual adjustment

$$\phi_t = \max\{\phi', \phi - \Delta \phi \cdot t\}, \quad t \geq 1$$

- Shock calibrated to generate 10 p.p drop in debt-to-GDP ratio.
Key Takeaways I

Credit crunch leads to:

1. More concentrated asset distribution
2. Interest rate (negative) overshooting
3. Transition dynamics
4. MPC too low

Empirical estimates of 0.2 ≠ our estimate of 0.015.

⇒ Overall: very decent job at matching business-cycle dynamics.

Transitions de Groot and Veneris (U of Liverpool)
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   - ergodic distribution

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   - depends on PE and GE effects.
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Ergodic Bond Distribution I

![Graph showing saving policy and bond distribution](image)

- **Saving Policy**
  - Blue line: initial ss
  - Dashed red line: terminal ss

- **Bond Distribution**
  - X-axis: b
  - Y-axis: probability density

- Parameters:
  - b values range from -2 to 14
  - Probability density values range from 0 to 0.025

- Graph indicates the distribution of bond savings for different values of b.
Transitional Dynamics I

- **borrowing limit (%)**
- **household debt-to-GDP ratio**
- **interest rate**
- **output**
- **consumption**
- **labor supply**

Graphs showing changes over time in various economic indicators.
Transitional Dynamics I.1

- Borrowing limit (%)
  - Epstein-Zin
  - CRRA

- Household debt-to-GDP ratio

- Interest rate

- Output

- Consumption

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Conclusion

Overall, baseline model does a very decent job at explaining the drop of the U.S economic activity following the 2008 crisis, for plausible parameter values.

Leaves room for the study of asset-pricing phenomena.

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Future Work

- Long-run risk + asset-pricing puzzles.
- Timing and risk premia.
- Fiscal policy.
Appendix
Optimality Conditions

Optimality condition with respect to consumption:

\[ u_c(c, n) + \beta \frac{1}{1 - \alpha} \left( EV(b', \theta')^{1-\alpha} \right)^{\frac{\alpha}{1-\alpha}} (1 - \alpha)E \left( V(b', \theta')^{-\alpha} V_c(b', \theta') \right) \geq 0 \]

\[ u_c(c, n) \geq -\beta \left( EV(b', \theta')^{1-\alpha} \right)^{\frac{\alpha}{1-\alpha}} E \left( V(b', \theta')^{-\alpha} V_b'(b', \theta') \left( \frac{-1}{q} \right) \right) \]

\[ u_c(c, n) \geq \frac{\beta}{q} \left[ EV(b', \theta')^{1-\alpha} \right]^{\frac{\alpha}{1-\alpha}} E \left[ V(b', \theta')^{-\alpha} V_b'(b', \theta') \right]. \]

Optimality condition with respect to labor supply:

\[ u_n(c, n) + \beta \frac{1}{1 - \alpha} \left( EV(b', \theta')^{1-\alpha} \right)^{\frac{\alpha}{1-\alpha}} (1 - \alpha)E \left( V(b', \theta')^{-\alpha} V_b'(b', \theta') \frac{w}{q} \right) \leq 0 \]

\[ u_n(c, n) \leq -\frac{\beta}{q} w \left[ EV(b', \theta')^{1-\alpha} \right]^{\frac{\alpha}{1-\alpha}} E \left[ V(b', \theta')^{-\alpha} V_b'(b', \theta') \right]. \]
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Derivative of value function with respect to $b$:

\[
\frac{\partial V(b, \theta)}{\partial b} = u_c(c, n) \frac{\partial c}{\partial b} + u_n(c, n) \frac{\partial n}{\partial b} + \frac{\beta}{1 - \alpha} [EV(b', \theta')]^{\frac{\alpha}{1 - \alpha}} (1 - \alpha) \times \\
\times E \left( V(b', \theta')^{-\alpha} \frac{\partial V(b', \theta')}{\partial b'} \frac{\partial b'}{\partial b} \right) \\
= u_c(c, n) \frac{\partial c}{\partial b} + u_n(c_t, n_t) \frac{\partial n}{\partial b} + \frac{\beta}{1 - \alpha} (EV(b', \theta'))^{\frac{\alpha}{1 - \alpha}} (1 - \alpha) \times \\
\times E \left( V(b', \theta')^{-\alpha} \times \frac{\partial V(b', \theta')}{\partial b'} \left( \frac{1}{q} + \frac{w}{q} \frac{\partial n}{\partial b} - \frac{1}{q} \frac{\partial c}{\partial b} \right) \right) .
\]
Optimality Conditions

Combining all the above we get the Benveniste-Scheinkman envelope condition:

\[
\frac{\partial V(b, \theta)}{\partial b} = \frac{\beta}{q} \left[ EV(b', \theta')^{1-\alpha} \right]^{\frac{\alpha}{1-\alpha}} E \left[ V(b', \theta')^{-\alpha} V_b'(b', \theta') \frac{\partial c}{\partial b} \right] - \\
- \frac{\beta}{q} w \left[ EV(b', \theta')^{1-\alpha} \right]^{\frac{\alpha}{1-\alpha}} E \left[ V(b', \theta')^{-\alpha} V_b'(b', \theta') \frac{\partial n}{\partial b} \right] + \\
+ \frac{\beta}{q} \left[ EV(b', \theta')^{1-\alpha} \right]^{\frac{\alpha}{1-\alpha}} E \left[ V(b', \theta')^{-\alpha} V_{bb'}(b', \theta') \frac{\partial n}{\partial b} \right] - \\
- \frac{\beta}{q} w \left[ EV(b', \theta')^{1-\alpha} \right]^{\frac{\alpha}{1-\alpha}} E \left[ V(b', \theta')^{-\alpha} V_{bb'}(b', \theta') \frac{\partial c}{\partial b} \right] + \\
= \frac{\beta}{q} \left[ EV(b', \theta')^{1-\alpha} \right]^{\frac{\alpha}{1-\alpha}} E \left[ V(b', \theta')^{-\alpha} V_b'(b', \theta') \right].
\]
Now, combining the optimality condition with respect to consumption and the envelope condition, we get the Euler for consumption:

\[
u_c(c, n) \geq \frac{\beta}{q} \left[ EV(b', \theta')^{1-\alpha} \right]^{\frac{\alpha}{1-\alpha}} E \left[ V(b', \theta')^{-\alpha} u_c'(c', n') \right] \]

\[
u_c(c, n) \geq \beta(1 + r) \left[ EV(b', \theta')^{1-\alpha} \right]^{\frac{\alpha}{1-\alpha}} E \left[ V(b', \theta')^{-\alpha} u_c'(c', n') \right].
\]

Finally, combining equations the optimality conditions with respect to labor supply and consumption, we get the Euler for labor supply:

\[
u_n(c, n) \leq -w u_c(c, n)
\]

\[-\frac{u_n(c, n)}{u_c(c, n)} \geq w.
\]