Credit Crunches with Recursive Preferences

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Research Questions

2 main questions

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2. What are the implications for asset prices?

Motivation

Recent interest motivated by:

- costly debt deleveraging following the **2007-2008 financial crisis.**
- challenges of macro models to be consistent with asset-pricing phenomena.

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Theoretically, we introduce Epstein-Zin preferences to:

- 10 percentage point drop in debt-to-GDP ratio.
- deal with two fundamental challenges of most theoretical macro models.

Equity-Premium & Risk-Free Rate Puzzles

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- Empirically the average return on equity in the US has been $\simeq$ **7%**.
- Risk-free rate is $\simeq$ $\mathbf{0.9\%}$.

$\Longrightarrow$ **equity premium** $\simeq$ **6.1** % $\rightarrow$ **VERY LARGE** $\rightarrow$ PUZZLE.

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- Extremely high RA needed to generate the observed (mean) return on equity $\longrightarrow$ RA should be around 50 while micro estimates suggest this number should not be higher than 2 or 3.
- This implies a very low EIS which generates a **very high risk-free rate** $\rightarrow$ inconsistent with the data $\rightarrow$ PUZZLE.

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1. habit formation.
2. Epstein-Zin + Long-Run Risk (LLR)
   $\Longrightarrow$ extra degree of freedom.

Epstein-Zin Preferenecs

Standard expected-utility time-separable preferences:

$$\mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t U(c_t)\right] \tag{1}$$

In recursive form:

$$V_t = U(c_t) + \beta E_t V_{t+1} \tag{2}$$

EZ specification:

$$V_t = \left[c_t^{1-\rho} + \beta \left(E_t V_{t+1}^{1-\alpha}\right)^{\frac{1-\rho}{1-\alpha}}\right]^{\frac{1}{1-\rho}} \tag{3}$$

- $\frac{1}{\rho}$: EIS (between consumption today and CEq of continutation utility)
- α: coefficient of RRA

$\Longrightarrow$ **breaks** link between risk-aversion and EIS.

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3. single asset traded is a one-period risk-free bond.
4. exogenous debt limit.
5. single interest rate.

Households (HHs)

Continuum of heterogeneous HHs that supply labor and consume.

- HH i's lifetime utility is given by

$$V(b_i, \theta_i) = u(c_i, 1 - n_i) + \beta \left(EV(b_i', \theta_i')^{1-\alpha} \right)^{\frac{1}{1-\alpha}} \tag{4}$$

  - b : bond holdings.
  - θ_i : labor productivity $\longrightarrow$ markov process.

- Assume instantaneous utility is additive and iso-elastic

$$V(b_i, \theta_i) = \underbrace{\frac{c_i^{1-\gamma}}{1-\gamma} + \psi \frac{(1-n_i)^{1-\eta}}{1-\eta}}_{u(c_i, 1-n_i)} + \beta \underbrace{\left(EV(b_i', \theta_i')^{1-\alpha} \right)^{\frac{1}{1-\alpha}}}_{\mu \equiv \text{CEq}} \tag{5}$$

- $\gamma \equiv$ inverse EIS
- $\alpha = 1 - \frac{RA}{\gamma}$
- $RA = -B \cdot \frac{V_{11}(b,\theta)}{V_1(b,\theta)} + \alpha \cdot \frac{BV_1(b,\theta)}{V(b,\theta)}$

- Each HH produces consumption goods using linear technology

$$y_i = \theta_i n_i \tag{6}$$

  - $\theta^1 = 0$: unemployment $\rightarrow$ exogenous unemployment risk.

Budget Constraint:

$$qb_i' + c_i \leq b_i + y_i - \tilde{\tau}_i \tag{7}$$

  - $\tilde{\tau}_i = \tau, \;\; \theta_i > 0 \longrightarrow$ employed.
  - $\tilde{\tau}_i = \tau$ - $v, \;\; \theta_i = 0 \longrightarrow$ unemployed.

and

  - v: unemployment benefit.

- Debt is bounded below by the exogenous limit ϕ

$$b_i' \geq -\phi \tag{8}$$

Optimality Conditions

- The HHs problem is to max (1) subject to (3)

$$V(b_i,\theta_i) \equiv \max_{\{c_i,n_i,b_i'\}} \left[u\left(c_i, 1-n_i\right) + \beta \left(EV(b_i',\theta_i')^{1-\alpha} \right)^{\frac{1}{1-\alpha}} \right] \qquad (9)$$

s.t: budget constraint (7), $c_i \geq 0$, and $n_i \in [0,1]$.

- **Euler for consumption** ▸ optimality conditions

$$u_{c_i}(c_i, 1-n_i) \geq \beta(1+r)\left[EV(b_i',\theta_i')^{1-\alpha}\right]^{\frac{\alpha}{1-\alpha}} E\left[V(b_i',\theta_i')^{-\alpha} u_{c_i'}(c_i', 1-n_i')\right] \qquad (10)$$

- **Euler for labor**

$$-\frac{u_n(c_i, 1-n_i)}{u_c(c_i, 1-n_i)} \geq w \qquad (11)$$

Government

- Finances a stream of expenditures by levying taxes and issuing debt.
- The government's budget constraint is:

$$B + uv = qB' + \tau \tag{12}$$

  - $u = Pr(\theta_i = 0) \longrightarrow$ fraction of unemployed in the population.
  - tax rate (residually) adjusts to balance budget.
- Supply of bonds (B) exogenously fixed.

Recursive Competitive Equilibrium (RCE)

Definition

A RCE is a sequence of value functions $\{V\}$: $\mathbb{R}^2_+ \to \mathbb{R}$, decision rules $\{C\}$, {N}: $\mathbb{R}^2_+ \to \mathbb{R}$, interest rates $\{r\}$: $\mathbb{R} \to \mathbb{R}$, tax rates $\{\tau\}$: $\mathbb{R}_+ \to \mathbb{R}_+$, and the (joint) distribution of bond holdings and productivity levels $\{\Psi\}$: $\mathbb{R}^2 \to \mathbb{R}_+$, such that

1. given the interest rate $\{r\}$ and the tax rate $\{\tau\}$, the value function V(b,θ) solves the Bellman equation (9), and $\{C(b,\theta), N(b,\theta)\}$ are the corresponding optimal decision rules,
2. the (joint) distribution of bond holdings and productivity levels Ψ is consistent with the households' optimal consumption and labor supply decisions,
3. the tax rate equals government expenditures,
$$\tau = \nu u + \frac{rB}{1+r}$$
4. the asset (bond) market clears,
$$\int b d\Psi(b,\theta) = B.$$

Quantitative Analysis

Table: Baseline Calibration

Parameter	Value	Description	Target/Source
β	0.9749	Discount factor	r = 2.5%
γ	4	Inverse elasticity of intertemporal substitution	
RA	15	Coefficient of relative risk aversion	
η	1.5	Curvature of utility from leisure	Average Frisch elasticity = 1
ψ	15.57	Weight on utility from leisure	Nekarda and Ramey (2020)
ρ	0.967	Productivity shock persistence	Floden and Lindé (2001)
σ_ϵ	0.017	Productivity shock variance	Floden and Lindé (2001)
$\pi_{e,u}$	0.057	Transition to unemployment	Shimer (2005)
$\pi_{u,e}$	0.882	Transition to employment	Shimer (2005)
B	1.60	Bond supply	Liquid assets
ϕ	1.847	Borrowing constraint	Total gross debt
ν	0.167	Unemployment benefit	40% of average labor income

Credit Shocks

- Unexpected tightening of borrowing limit $\longrightarrow$ MIT shock.

$\Longrightarrow$ no aggregate uncertainty.

- Gradual adjustment

$$\phi_t = \max\{\phi', \phi - \Delta\phi \cdot t\}, \quad t \geq 1$$

- Shock calibrated to generate 10 p.p drop in debt-to-GDP ratio.

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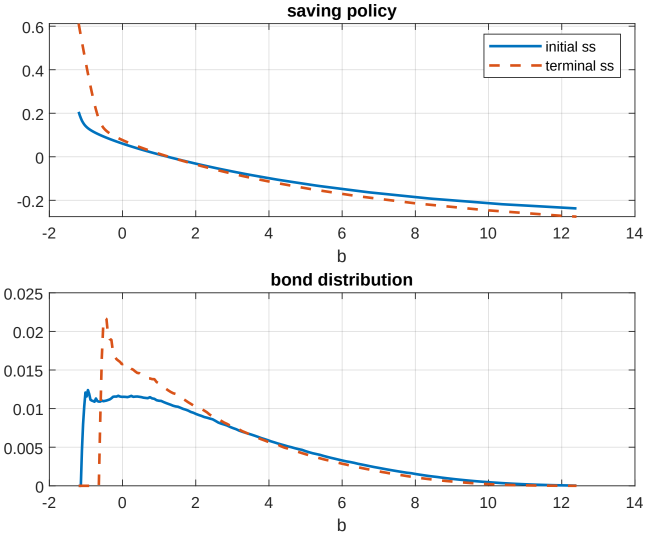
   - depends on **PE** and **GE** effects. consumption labor supply output
4. **MPC too low**
   - empirical estimates of 0.2 $\neq$ our estimate of 0.015.

$\Longrightarrow$ Overall: very decent job at matching business-cycle dynamics.

transitions

Ergodic Bond Distribution I



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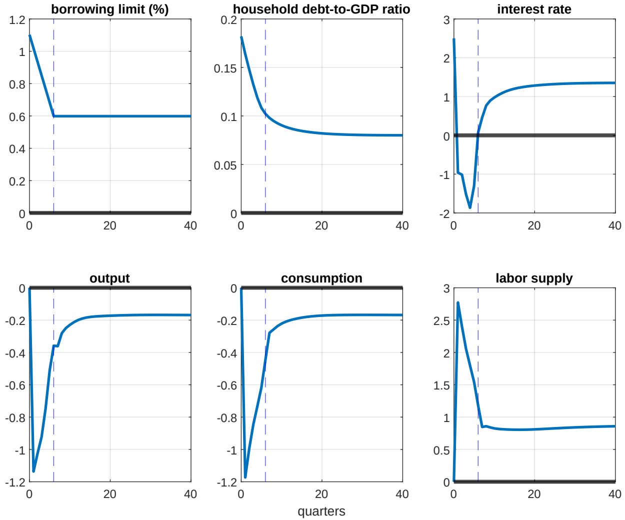
Transitional Dynamics I



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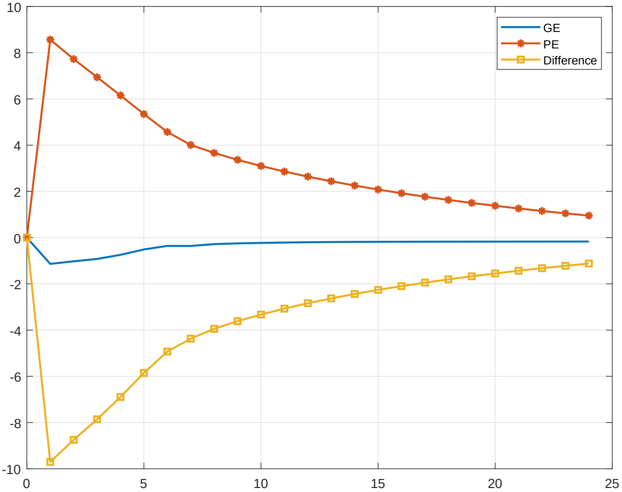
Output Response Decomposition



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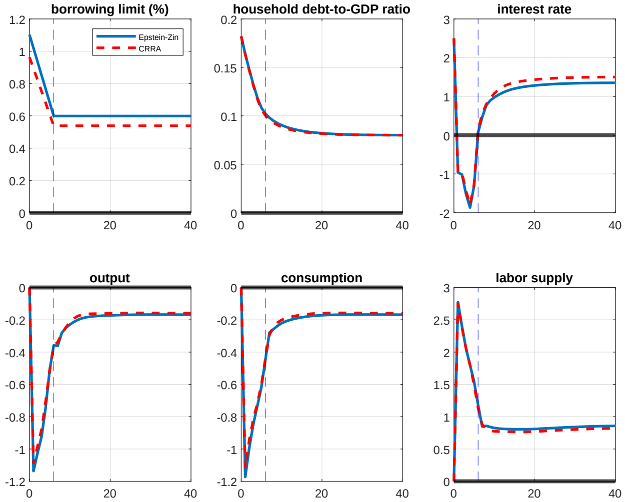
Transitional Dynamics I.1



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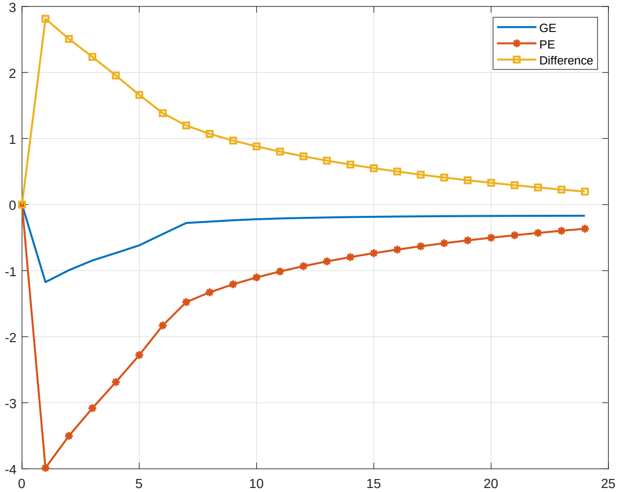
- Leaves room for the study of asset-pricing phenomena.

Future Work

- Long-run risk + asset-pricing puzzles.
- Timing and risk premia.
- Fiscal policy.

Appendix

Consumption Response Decomposition



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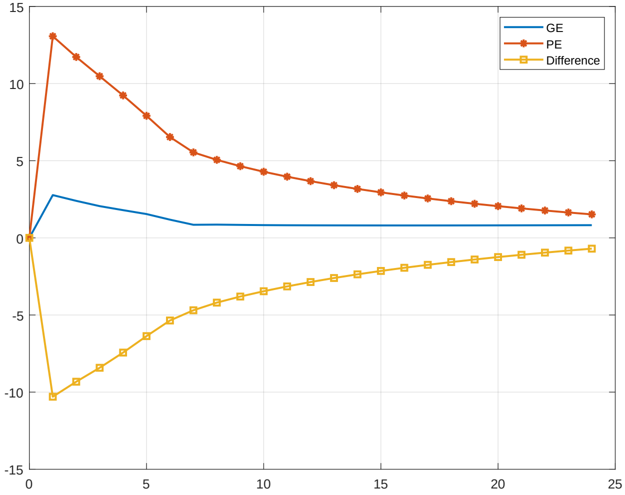
Labor Supply Response Decomposition



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Optimality Conditions

Optimality condition with respect to consumption:

$$u_c(c,n) + \beta \frac{1}{1-\alpha} \left(EV(b',\theta')^{1-\alpha} \right)^{\frac{\alpha}{1-\alpha}} (1-\alpha) E \left(V(b',\theta')^{-\alpha} V_c(b',\theta') \right) \geq 0$$

$$u_c(c,n) \geq -\beta \left(EV(b',\theta')^{1-\alpha} \right)^{\frac{\alpha}{1-\alpha}} E \left(V(b',\theta')^{-\alpha} V_{b'}(b',\theta') \left(\frac{-1}{q} \right) \right)$$

$$u_c(c,n) \geq \frac{\beta}{q} \left[EV(b',\theta')^{1-\alpha} \right]^{\frac{\alpha}{1-\alpha}} E \left[V(b',\theta')^{-\alpha} V_{b'}(b',\theta') \right].$$

Optimality condition with respect to labor supply:

$$u_n(c,n) + \beta \frac{1}{1-\alpha} \left(EV(b',\theta')^{1-\alpha} \right)^{\frac{\alpha}{1-\alpha}} (1-\alpha) E \left(V(b',\theta')^{-\alpha} V_{b'}(b',\theta') \frac{w}{q} \right) \leq 0$$

$$u_n(c,n) \leq -\frac{\beta}{q} w \left[EV(b',\theta')^{1-\alpha} \right]^{\frac{\alpha}{1-\alpha}} E \left[V(b',\theta')^{-\alpha} V_{b'}(b',\theta') \right].$$

Optimality Conditions

Derivative of value function with respect to b:

$$\frac{\partial V(b,\theta)}{\partial b} = u_c(c,n)\frac{\partial c}{\partial b} + u_n(c,n)\frac{\partial n}{\partial b} + \frac{\beta}{1-\alpha}\left[EV(b',\theta')\right]^{\frac{\alpha}{1-\alpha}}(1-\alpha)\times$$
$$\times E\left(V(b',\theta')^{-\alpha}\frac{\partial V(b',\theta')}{\partial b'}\frac{\partial b'}{\partial b}\right)$$
$$= u_c(c,n)\frac{\partial c}{\partial b} + u_n(c_t,n_t)\frac{\partial n}{\partial b} + \frac{\beta}{1-\alpha}\left(EV(b',\theta')\right)^{\frac{\alpha}{1-\alpha}}(1-\alpha)\times$$
$$\times E\left(V(b',\theta')^{-\alpha}\times\frac{\partial V(b',\theta')}{\partial b'}\left(\frac{1}{q}+\frac{w}{q}\frac{\partial n}{\partial b}-\frac{1}{q}\frac{\partial c}{\partial b}\right)\right).$$

Optimality Conditions

Combining all the above we get the Benveniste-Scheinkman **envelope condition**:

$$
\begin{aligned}
\frac{\partial V(b,\theta)}{\partial b} &= \frac{\beta}{q}\left[EV(b',\theta')^{1-\alpha}\right]^{\frac{\alpha}{1-\alpha}} E\left[V(b',\theta')^{-\alpha} V_{b'}(b',\theta')\frac{\partial c}{\partial b}\right] - \\
&- \frac{\beta}{q} w \left[EV(b',\theta')^{1-\alpha}\right]^{\frac{\alpha}{1-\alpha}} E\left[V(b',\theta')^{-\alpha} V_{b'}(b',\theta')\frac{\partial n}{\partial b}\right] + \\
&+ \frac{\beta}{q}\left[EV(b',\theta')^{1-\alpha}\right]^{\frac{\alpha}{1-\alpha}} E\left[V(b',\theta')^{-\alpha} V_{b'}(b',\theta')\right] + \\
&+ \frac{\beta}{q} w \left[EV(b',\theta')^{1-\alpha}\right]^{\frac{\alpha}{1-\alpha}} E\left[V(b',\theta')^{-\alpha} V_{b'}(b',\theta')\frac{\partial n}{\partial b}\right] - \\
&- \frac{\beta}{q}\left[EV(b',\theta')^{1-\alpha}\right]^{\frac{\alpha}{1-\alpha}} E\left[V(b',\theta')^{-\alpha} V_{b'}(b',\theta')\frac{\partial c}{\partial b}\right] \\
&= \frac{\beta}{q}\left[EV(b',\theta')^{1-\alpha}\right]^{\frac{\alpha}{1-\alpha}} E\left[V(b',\theta')^{-\alpha} V_{b'}(b',\theta')\right].
\end{aligned}
$$

Optimality Conditions

Now, combining the optimality condition with respect to consumption and the envelope condition, we get the Euler for consumption:

$$u_c(c,n) \geq \frac{\beta}{q}\left[EV(b',\theta')^{1-\alpha}\right]^{\frac{\alpha}{1-\alpha}} E\left[V(b',\theta')^{-\alpha}u_{c'}(c',n')\right]$$
$$u_c(c,n) \geq \beta(1+r)\left[EV(b',\theta')^{1-\alpha}\right]^{\frac{\alpha}{1-\alpha}} E\left[V(b',\theta')^{-\alpha}u_{c'}(c',n')\right].$$

Finally, combining equations the optimality conditions with respect to labor supply and consumption, we get the Euler for labor supply:

$$u_n(c,n) \leq -wu_c(c,n)$$
$$-\frac{u_n(c,n)}{u_c(c,n)} \geq w.$$

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