CHILE*

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Abstract

We introduce a tractable asset-pricing framework with general and heterogeneous utilities, heterogeneous information sets, Bayesian inferences, and no restrictions on payoff distributions, called "Continuous Heterogeneous Information Large Economy" (CHILE). We then derive a log-linear equilibrium with log-normal payoffs in closed form. With non-CARA utility, initial wealth gets factored into equilibrium, determining price efficiency, which in turn determines the distribution of post-trade wealth. The fixed point of this mapping is a "trade-invariant" wealth distribution: even though the wealth of individual agents changes as they trade, its distribution across different agents does not. With CRRA utility this distribution is a power law.

Keywords: inefficient markets, information aggregation, rational expectations with non-CARA preferences, wealth effects

JEL Codes: D01, D53, D82, E19, G12, G14.

1 Introduction

How do risk preferences affect market inefficiency? This straightforward question, fundamental as it may be, is also deceptively difficult to answer. As it is widely acknowledged, the crux

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of the matter is that standard approaches lack tractability, making the treatment of market inefficiency with general preferences look like an unattainable goal. While models of financial markets without frictions—as in, for example, the popular textbook "Asset Pricing" (Cochrane, 2005)—are able to capture many empirically relevant sources of risk and match a number of interesting quantities, they are notoriously unable to handle asymmetric information, a force broadly believed to be a key component of market inefficiency.

What about those asymmetric-information models collectively known as Rational Expectations Equilibria (REE)—do they fare any better? At least with respect to non-revealing prices, REE appear to be stuck with the same-old CARA-normal framework we see at least as far back as Grossman (1976). Their Constant Absolute Risk Aversion (CARA) preferences, essentially a version of the mean-variance criterion, offer us linearity and combine well with normally distributed payoffs, so they have understandably become a favorite among information economists (authors included). Nevertheless, the CARA-Normal framework is not suitable for achieving at least two modest research goals: incorporating wealth effects, and developing models beyond linear—that is, occasionally negative(!)—prices.[1]

Aiming to address these issues, we introduce a tractable framework with general preferences and fully heterogeneous traders. In particular, we model a large economy made up of a continuum of traders, indexed by $a \in [0, 1]$. We then parameterize trader characteristics (utility, risk aversion, signal precision, and so on) by arbitrary functions of a. Assuming log-normally distributed assets, we obtain a log-linear equilibrium with all equilibrium quantities characterized in closed-form.[2] Such tractability is remarkable given rich heterogeneity of our model and general, non-CARA utilities.

What enables us to derive these results is the way we model information. Originating in Hellwig (1980), the traditional approach assumes that each trader has a signal of finite precision,

[1]Notable papers that go beyond the CARA-Normal framework, while still requiring other assumptions for tractability, include Peress (2004), Peress (2014), Breon-Drish (2015), Malamud (2015) and Chabakauri, Yuan, and Zachariadis (2022). While allowing for non-Gaussian distribution, both Breon-Drish (2015) and Chabakauri et al. (2022) require CARA utility and thus do not incorporate wealth effects. Malamud (2015) requires complete markets. Peress (2004) and Peress (2014) achieve tractability by requiring the risk of assets to be small. See Section 6 for more details.

[2]Our framework is flexible enough to handle more general probability distributions, a subject of ongoing work.

implying that as the number of traders becomes large, so does the total amount of information.[3] What is more, as traders base their demands on signals with finite precision, they make finite speculative trades, with the unfortunate consequence that aggregate demand explodes for large numbers of traders.

In our model, however, neither the total amount of information nor the aggregate demand explode, even as the number of traders becomes large. To obtain this property, we adapt an assumption of Kyle (1989, Section 9), whereby a finite total amount of information is distributed among all traders. By definition, then, the total amount of information is finite. Consequently, in our large economy, traders base their demands on signals with small precision, making small speculative trades that aggregate to finite aggregate demand.

A key contribution of our paper is the introduction of a formalism that can represent a large economy made up of traders with small signals. As it turns out, the right formalism uses a type of stochastic calculus, where the dimension that typically represents time is "transposed" to represent agents.[4]

To fix ideas, we offer the following example. Suppose we represent a continuum of agents as a unit interval. Using a to denote one of the agents, let us imagine that a lives on a segment of size da, and that he observes the signal

$$ds(a) = v\,da + dB(a),$$

where v is the fundamental value of a traded asset, and where $dB(a)$ is an increment of a Brownian Motion over the agent interval. The aggregate sum of all signals—a sufficient statistic for all private signals—is an Itô integral, a key property that gives us access to the full arsenal of stochastic calculus. But perhaps even more importantly, this type of aggregation also allows us to think of the noise component of the sufficient statistic as consisting of a large number of small idiosyncratic shocks, resembling what Black (1986) calls "noise in the sense of a large number of small events."

[3]Here "finite precision" means that precision is neither infinite nor infinitesimal (infinitely small). As the total amount of information is the precision of the sufficient statistic combining all the private signals of an economy, it can be shown to equal the sum of precisions of all trader signals.

[4]See Gârleanu, Panageas, and Yu (2015) for a similar trick applied to firms rather than traders.

Our equilibrium has the following properties. First, traders with higher risk tolerance provide more liquidity, while traders with higher precision-weighted risk tolerance trade more aggressively. Although these properties hold in CARA-Normal models, the non-CARA utility of our model links the absolute risk tolerance to wealth. Assuming, for example, decreasing absolute risk aversion, we can say that wealthier traders provide more liquidity (see section 3.2 for more implications).

We also demonstrate that the distribution of initial wealth affects important equilibrium quantities, such as price efficiency. We show, for example, that economies with wealthier traders have more informative prices, but economies with greater dispersion in trader wealth have less informative prices. Conversely, as price efficiency affects speculative profits, it also affects the post-trade distribution of wealth. We therefore think about a "trade-invariant" wealth distribution where, even though the wealth of individuals changes as they trade, the cross-sectional distribution of wealth post-trade is the same as ex-ante. Leveraging the stochastic-calculus toolbox, we show how to derive invariant wealth distributions by a Kolmogorov Forward Equation (KFE). With CRRA utility, the KFE becomes one familiar from continuous-time random-growth models (see Gabaix, 2009, for a review), implying that the invariant wealth distribution obeys a power law.

2 Setup

Our economy unfolds over two time periods, $t = \{1, 2\}$, and it is made of a continuum of traders of total mass one, described in more detail below. There are two assets: one risky, and one risk-free, with the returns of both assets realized at $t = 2$. The supply of the risk-free asset is perfectly elastic and its gross return is R_f, while the liquidation value of the risky asset is

$$V(v),\, v \sim N\left(0, \tau_v^{-1}\right).$$

Our framework is tractable for a general function $V(\cdot)$.[5] As, however, our goal here is to focus on the effects arising from non-CARA utility (and not the general distribution), we assume a

[5]We are exploring general $V(\cdot)$ functions in ongoing work.

log-normal payoff,

$$V(v) = \exp(v).$$

Trade happens at $t = 1$, determining the price P.[6] We denote

$$p = \log P \text{ and } R = V/P.$$

There are two groups of traders (*agents*), indexed by a, living on the interval $[0, 1)$. *Rational traders* live on the segment $[\nu, 1)$, whereas *noise traders* live on the segment $a \in [0, \nu)$.

Agent a trades a *dollar amount* of x (price per share times number of shares) at time $t = 1$, so his realized wealth at time $t = 2$ is

$$W = W_0(a)R_f + x(R - R_f),$$

and his realized utility is

$$u(W, a).$$

Here we see the first two dimensions of heterogeneity of our model: initial wealth $W_0(a)$, and preferences $u(\cdot, a)$.[7] Our only restriction is that expected utility $E[u(W, a)]$ is thrice differentiable with respect to x in a small open neighbourhood around $x = 0$.

The last primitive of our model is a standard Brownian Motion $B(\cdot)$ on the interval $[0, 1)$, running through the cross-section of traders, who are located on the same interval. We will make heavy use of $B(\cdot)$, as it will provide the price noise in our economy.

In what follows, we micro-found the key quantities of our continuous equilibrium as limits of corresponding quantities of a discrete economy built on traditional economic primitives. We begin by describing our limit economy in loose terms in our next section, providing our readers with "user-friendly" heuristics involving Brownian differentials, as if our economy was setup directly at the limit. We tighten up our exposition in the sequel (section 2.2), describing our

[6] Adding information acquisition at $t = 0$ to our framework is straightforward, and it is the subject of ongoing work.

[7] Our third and final dimension of heterogeneity is signal precision, discussed in the next section.

discrete economy and showing how we take the limit.

2.1 A Heuristic Presentation of the Continuous Economy

A given agent $a \in [0, 1)$ has initial wealth $W_0(a)$ and utility function over final wealth $u(\cdot, a)$. He lives in segment $[a, a + da)$, with his location determining his type as discussed above.

A rational trader a observes the signal

$$ds(a) = v\, da + \frac{1}{\sqrt{t(a)}} dB(a),\ a \in [\nu, 1), \tag{1}$$

where $B(\cdot)$ is the Brownian motion introduced earlier.

Conditional on v, trader a's signal is normally distributed with precision

$$t(a)da.$$

To gain some intuition for why, we can (loosely) argue that $ds(a)$ has the same informational content as a signal written in the "truth plus noise" form $v + dB(a)/\left(\sqrt{t(a)}da\right)$. As the precision of a signal is the reciprocal of the variance of the noise, we can think of the precision of (1) as $\text{Var}\left(dB(a)/\left(\sqrt{t(a)}da\right)\right)^{-1} = [da/(t(a)da^2)]^{-1} = t(a)da$. Abusing terminology, we refer to $t(a)$ as the *precision of trader* a.

Agents in $[0, \nu)$ are Fisher-Black noise traders (see Black, 1986). Although they believe they are observing the process in (1), their actual signal is pure noise. We model their "signal" as

$$ds(a) = \frac{1}{\sqrt{t(a)}} dB(a),\ a \in [0, \nu). \tag{2}$$

We refer to $t(a)$ for a noise trader a as a *perceived precision* or just precision, for brevity. Noise trader a is only wrong about his signal and has correct beliefs about the equilibrium price function.[8]

[8] As a modeling choice, our assumption in (2) imposes discipline on the degree of irrationality of the noise traders. It can be relaxed with little loss of general tractability, albeit at the expense of losing our closed form.

Finally, we note that conditional on v, the aggregate signal process $s(a) = \int_0^a ds(b)$ is a continuous martingale. We call an economy with heterogeneous signals such that the aggregate signal is a continuous martingale a *continuous heterogeneous information* economy.

2.2 The Continuous Economy as a Limiting Discrete Economy

Consider an economy with n traders. Trader i, $i = 1, 2, \ldots, n$ lives in segment $[a^i, a^i + m)$, where $a^i = m \cdot (i - 1)$. The length of the agent's segment is the same as his size (or *mass*), $m = 1/n$.[9] Trader i's wealth is $W(a^i)$ and his utility is $u(\cdot, a^i)$.

The agents with $a^i \in [0, \nu)$ are noise traders. They observe the signal

$$\Delta s^i = \frac{1}{\sqrt{t(a^i)}} \left(B(a^i + m) - B(a^i) \right), \tag{3}$$

but act as if their signal is

$$\Delta s^i = v \cdot m + \frac{1}{\sqrt{t(a^i)}} \left(B(a^i + m) - B(a^i) \right). \tag{4}$$

The agents with $a^i \in [\nu, 1)$ are rational traders, and they observe the signal in (4).

Let

$$X(p, \Delta s; m, a)$$

denote the demand of the trader who lives in segment $[a, a + m)$, and has signal Δs. As X is a demand function, it depends on the price p, the signal Δs, the size of the agent m, and on the agent's characteristics (wealth, utility, precision), captured by the argument a.

2.2.1 An Aggregation Lemma

As we highlight with the following lemma, the key reason for the tractability of our analysis is that as trader size becomes small, the aggregate demand in our discrete economy above

[9] We assume that all agents have the same size as a matter of expositional clarity. Once can instead consider any partition of the unit interval, but only at the expense of more cumbersome notation.

converges to a stochastic integral, allowing us to think in terms of functions and their derivatives, rather than pointwise sequences of discrete random variables.

Lemma 1. ***(Aggregation lemma)*** *Consider a continuous function $X(p,s;m,a):\mathbb{R}\times\mathbb{R}\times[0,1]\times[0,1]\to\mathbb{R}$ with continuous partial derivatives X_s, X_{ss} and X_m. Take a partition $[0,y)=\cup_{i=1}^{n}[a^i,a^i+m^i)$, and let $m=\min_i m_i$, $\Delta s^i=s(a^{i+1})-s(a^i)$, and $\Delta X^i=X(p,\Delta s^i,m,a^i)-X(p,0,0,a^i)$. Finally, suppose that there exists an $M>0$ such that $|\sum_{i=1}^{n}X^i|<M$ for any n. Then, for any partition such that $m\to 0$ as $n\to\infty$, we have that*

$$\sum_{i=0}^{n-1}\Delta X^i \to \int_0^y X_s(p,0,0,a)ds(a)+\int_0^y\left(\frac{1}{2t_a}X_{ss}(p,0,0,a)+X_m(p,0,0,a)\right)da. \tag{5}$$

In what follows we will write that

$$\Delta X \xrightarrow{agg} dX = X_s(p,0,0,a)ds(a)+\left(\frac{1}{2t(a)}X_{ss}(p,0,0,a)+X_m(p,0,0,a)\right)da$$

whenever (5) holds. The idea behind the notation is that whenever $\Delta X \xrightarrow{agg} dX$ the *aggregation* of ΔX and dX is the same in the limit.

The above lemma underlies the tractability of our analysis. Assuming that the demand of each trader becomes 0 as his size goes to 0 (otherwise aggregate demand explodes), we have $\Delta X(a)=X(a)$. Denoting $\beta(p,a)=X_s(p,0,0,a)$ and $\aleph(p,a)=\frac{1}{2t(a)}X_{ss}(p,0,0,a)+X_m(p,0,0,a)$ the aggregation lemma tells us that

$$X(a)\xrightarrow{agg}\beta(p,a)ds(a)+\aleph(p,a)da.$$

This further implies that for any payoff function $V(\cdot)$, our economy would feature a generalized linear equilibrium: given the price, one can always compute the sufficient statistic $\int_0^1\beta(p,a)ds(a)$, which, as a linear combination of signals of rational traders and noise traders, is conditionally normally distributed. This conditional normality simplifies inferences significantly, overcoming a common obstacle to extending asymmetric information models beyond the

CARA-normal framework. Conveniently, our log-normal setup comes with an additional simplification, stemming from that $\beta(p,a)$ does not depend on p, and that $\aleph(p,a) = \alpha(a) - \gamma(a)p$ is affine in p.

Remark 1. The condition $|\sum_{i=1}^{n} X^i| < M$ is not restrictive, since any the aggregate demand is bounded by market-clearing.

2.3 Definition of equilibrium

Our equilibrium definition proceeds in several steps.

1. We are looking for log-linear equilibria. Denoting $p = \log P$, the dollar demand of traders in the interval $[a, a+da)$ is

$$dx(a) = \alpha(a) + \beta(a)ds_a - \gamma(a) \cdot pda. \tag{6}$$

   The rigorous meaning of the above is that the aggregate demand of traders in some segment $\mathcal{A}$ is $\int_{\mathcal{A}} dx_a$.

2. We need to define dx_a. The idea is that: (i) we can rigorously define $x(\Delta s_i, p, m, a^i)$, the demand of a trader i located in $[a^i; a^i + m]$ in the discrete economy, and (ii) require that $x(\cdot, a) \xrightarrow{agg} dx(a)$, where $dx(a)$ is given by (6). More formally:

   (a) The demand $x(\Delta s_i, p, m, a^i)$ solves

$$x(p, \Delta s^i; m, a^i) = \arg\max_x \left\{ E\left[u\left(W_0(a^i)R_f + x\left(R - R_f\right), a\right) | p, \Delta s^i \right] \right\}$$

   Where all traders act as if they get Δs^i given by (4) but noise traders ($a^i \in [0, \nu)$) actually get (3). The p must clear the market

$$\int_0^1 (\hat{\alpha}(a;m)da + \hat{\beta}(a;m)ds_a - \hat{\gamma}(a,m) \cdot pda) = 0.$$

   That, is each trader in the discrete economy assumes that the price function is generated by a continuous economy, albeit with different coefficients $\hat{\alpha}(a;m)$, $\hat{\beta}(a;m)$

and $\hat{\gamma}(a,m)$. We require that in the limit, such beliefs about price function are correct

$$\hat{\alpha}(a;0)=\alpha(a),\ \hat{\beta}(a;0)=\beta(a),\ \text{and}\ \hat{\gamma}(a,0)=\gamma(a).$$

(b) We also require that limiting demands are consistent with optimisation problems described above. That is, $x(p,\Delta s^i;m,a^i)\xrightarrow{agg} dx_a=\alpha(a)+\beta(a)ds(a)-\gamma(a)\cdot pda$, which means that for a partition with $m\to 0$ as $n\to\infty$

$$\sum_{i=0}^{n-1} x(p,\Delta s^i;m^i,a^i)\xrightarrow[n\to\infty]{}\alpha(a)+\beta(a)ds_a-\gamma(a)\cdot pda.$$

Remark 2. A remark on notation. We will often use "hat" to denote equilibrium an equilibrium quantity X in the discrete economy with the size of the agent m by $\hat{X}(m)$. Its limit in the continuous economy is denoted without hat, $X=\hat{X}(0)$. An example is $\hat{\alpha}(a;0)=\alpha(a)$ above.

3 Equilibrium characterisation

3.1 Inference

From the perspective of each agent in the discrete economy, the price function satisfies

$$\int_0^1 (\hat{\alpha}(a;m)da+\hat{\beta}(a;m)ds(a)-\hat{\gamma}(a,m)\cdot pda)=0. \tag{7}$$

Substituting (1) and (2) into (7), integrating, and rearranging gives

$$s_p\equiv v+\frac{\int_0^1\hat{\beta}(a;m)/\sqrt{t(a)}dB(a)}{\int_\nu^1\hat{\beta}(a;m)da}=\frac{\int_0^1\hat{\gamma}(a;m)da}{\int_\nu^1\hat{\beta}(a;m)da}p-\frac{\int_0^1\hat{\alpha}(a;m)da}{\int_\nu^1\hat{\beta}(a;m)da}. \tag{8}$$

We note that the form "truth plus noise" shows up again, this time for the signal s_p defined as the expression on the left-hand side of (8), with truth being v and the noise being $\left(\int_0^1\hat{\beta}(a;m)/\sqrt{t(a)}dB(a)\right)/\left(\int_\nu^1\hat{\beta}(a;m)da\right)$. This noise, in particular, is normally distributed with mean zero and variance

$$\frac{\int_0^1 \hat{\beta}(a;m)^2/t(a)da}{\left(\int_\nu^1 \hat{\beta}(a;m)da\right)^2},$$

a result that follows from standard properties of stochastic integration.[10] We summarize the information content of prices in the following lemma.

Lemma 2. *The information content of prices is the unbiased signal s_p defined in (8). Given the price p, the realization of this signal can be computed as shown on the right-hand side of (8), while its precision can be written as*

$$\hat{\tau}_p(m) = \frac{\left(\int_\nu^1 \hat{\beta}(a;m)da\right)^2}{\int_0^1 \frac{\hat{\beta}(a;m)^2}{t(a)}da}.$$

Conditional on v, the covariance of the price signal with agent i's private signal is

$$\operatorname{cov}(s_p, \Delta s^i|v) = \frac{\hat{\beta}(a^i;m)}{\sqrt{t^i}} \int_{a^i}^{a^i+m} \frac{\hat{\beta}(a;m)}{t(a)}da.$$

We now turn to deriving the first-order condition (FOC), by writing the choice problem of trader i as

$$\max_x E\left[u\left(W_0(a^i)R_f + x\left(R - R_f\right), a^i\right)|\Delta s^i, s_p\right]. \tag{9}$$

Lemma 3. *There exists a unique best response for trader i. It solves (9) and satisfies the FOC*

$$E\left[u'\left(W_0(a^i)R_f + x\left(R - R_f\right), a^i\right)(R - R_f)|\Delta s^i, s_p\right] = 0,$$

which has a unique solution.

The conditional distribution of v given $\mathcal{F}^i = \{\Delta s^i, s_p\}$ is $N\left(E^i[v](\Delta s^i, m), 1/\hat{\tau}^i(m)\right)$, where $\hat{\tau}^i(m)$ can be computed in closed form for any m, but the solution is particularly simple for

[10] For a deterministic function f, the stochastic integral $\int_0^1 f(a)dB(a)$ is normally distributed with mean zero and variance $\int_0^1 f(a)^2 da$.

$m = 0$ (continuous economy limit), which is the only thing we need going forward.

$$\tau = \hat{\tau}^i(0) = \text{Var}[v|p]^{-1} = \tau_v + \tau_p.$$

where we have denoted $\tau_p = \hat{\tau}_p(0)$. Note that $\tau(0)$ is the same for all agents, hence we do not use superscript i. For a variable $\hat{x}(m)$ we'll use the notation x to denote $\hat{x}(0)$.

Similarly,

$$E[v|\Delta s^i, p](\Delta s^i, m) = \hat{c}_{\bar{v}}(m)\bar{v} + \hat{c}_s(m)\Delta s^i + \hat{c}_p(m)s_p,$$

where s_p is given by the right-hand side of (8). Again, even though they can be calculated in closed form for any m, we will only need the value of the coefficients $\hat{c}_{\bar{v}}(m)$, $\hat{c}_s(m)$ and $\hat{c}_p(m)$ at 0 :

$$c_{\bar{v}} = \frac{\tau_v}{\tau},\ c_s = \frac{t(a^i)}{\tau},\ \text{and}\ c_p = \frac{\tau_p}{\tau}.$$

Given the information $\mathcal{F}_i = \{\Delta s^i, p\}$, the random variable $z = \frac{v - E[v|\Delta s^i, p]}{\sqrt{\text{Var}[v|\Delta s^i, p]}}$ is standard normal. Then, we can write

$$v \equiv v^i(p, \Delta s^i, m, z) = E[v|\Delta s^i, p] + z\sqrt{\text{Var}[v|\Delta s^i, p]},\ \text{where}\ z|\mathcal{F}_i \sim N(0,1).$$

Substituting v^i instead of v to (3) yields

$$\int \left(\begin{array}{c} u'\left(W_0(a^i)R_f + (v^i(p, \Delta s^i, m, z) - p) - R_f, a^i\right) x \\ \cdot \left(\exp\left(v^i(p, \Delta s^i, m, z) - p\right) - R_f\right) \end{array} \right) d\mathcal{N}(z) = 0, \tag{10}$$

where $\mathcal{N}(z)$ denotes standard normal CDF. Substitution of v^i instead of v is convenient as it makes explicit the dependence of conditional distribution of v on Δs^i and m. Eq. (10) defines implicitly the demand

$$X(p, \Delta s^i, m, a^i).$$

In order to proceed we make several observations summarized in the following lemma.

Lemma 4. *The equilibrium exists only if $X(p, 0, 0, a^i)$ (the limit of $X(p, \Delta s^i, m, a^i)$, defined by*

(10) *as* $m \to 0$*) is zero. This implies that*

$$E[R|P] = R_f, \tag{11}$$

$$\frac{\tau_p}{\tau} \frac{\int_0^1 \gamma(a)da}{\int_\nu^1 \beta(a)da} = 1, \tag{12}$$

and

$$\exp\left(\frac{\tau_v}{\tau}\bar{v} - \frac{\tau_p}{\tau}\frac{\int_0^1 \alpha(a)da}{\int_0^1 \gamma(a)da} + \frac{1}{2\tau}\right) = R_f.$$

We comment here on the condition (11). In our limiting continuous economy the market is efficient. This makes sense: if it is not, then each trader would make a finite speculative trade and the aggregate demand would explode. What's more, in our economy the risk premium is zero. This is because we assume that the asset is in zero supply. One can introduce risk premium by endowing the agents not only with cash $W_0(a)$ but also with risky asset endowments.[11]

3.2 The three Greeks of CHILE

According to the Aggregation Lemma, we can compute the demand coefficients $\alpha(a)$, $\beta(a)$ and $\gamma(a)$ (the three Greeks) as follows. First, consider the demand function $X(p, \Delta s, m, a)$: the demand of agent sitting in the interval $[a, a+m)$. This demand is defined implicitly by (10). Then, demand coefficients can be found by first computing the following two functions of partials of $X(\cdot)$:

$$\mathfrak{f}^1(p,a) \equiv X_s(p,0,0,a), \text{ and } \mathfrak{f}^2(p,a) \equiv \left(\frac{1}{2t(a)}X_{ss}(p,0,0,a) + X_m(p,0,0,a)\right).$$

According to the Aggregation Lemma, the coefficient $\beta(a)$ is simply

$$\beta(a) = \mathfrak{f}^1(p,a).$$

[11]We explore such generalization in our ongoing work.

Moreover, for our guess of log-linear equilibrium to be correct, $\mathfrak{f}^1(p, a)$ shall not depend on p. Similarly, coefficients $\alpha(a)$ and $\gamma(a)$ can be found from

$$\alpha(a) - \gamma(a)p = \mathfrak{f}^2(p, a).$$

If our guess is correct, then $\mathfrak{f}^2(p, a)$ is indeed an affine function of p. Then $\gamma(a)$ (resp., $\alpha(a)$) can be identified as a slope (resp., intercept) of that function. Proceeding as we just described is straightforward and we do so in the Appendix. We summarize equilibrium derivation in the theorem below.

Theorem 1. *There exists a unique equilibrium. The demand coefficients are*

$$\beta(a) = \frac{R_f t(a)/\tau}{\rho(a)\mathrm{Var}[R|p]}, \gamma(a) = \frac{R_f h_\gamma}{\rho(a)\mathrm{Var}[R|p]}, \text{ and } \alpha(a) = \frac{R_f h_\alpha}{\rho(a)Var[R|p]} + \kappa(a), \tag{13}$$

with

$$\mathrm{Var}[R|p] = R_f^2(\exp(\tau^{-1}) - 1)$$

and $\tau = \tau_v + \tau_p$*, where* τ_p *is the equilibrium price informativeness, given as*

$$\tau_p = \frac{\left(\int_\nu^1 \frac{t(a)}{\rho(a)} da\right)^2}{\int_0^1 \frac{t(a)}{\rho(a)^2} da}. \tag{14}$$

The coefficient h_γ *is given by*

$$h_\gamma = \frac{\int_0^1 t(a)/\rho(a)^2 da}{\int_\nu^1 t(a)/\rho(a)da \int_0^1 1/\rho(a)da}$$

and the coefficients h_α *and* $\kappa(a)$ *are given in closed-form in* (29) *and* (30) *of the Appendix. Moreover, the equilibrium demand is*

$$dx(a) = \frac{E[R - R_f|ds(a), p]}{\rho(a)Var[R|p]} + \kappa(a)da.$$

where $E[R - R_f|ds(a), p]$ *is the infinitesimal conditional excess return from the perspective of* *trader a.*[12]

Theorem 1 above illustrates the power of our framework. It can deliver a closed-form equilibrium under assumptions that are usually quite difficult to incorporate in REE, such as rich heterogeneity, non-CARA utility, and non-normally distributed payoffs.

Directly from (13) we obtain the following corollary that describes the cross-section of $\beta(a)$ and $\gamma(a)$.

Corollary 1. *Traders with higher risk tolerance provide more liquidity, while traders with higher precision-weighted risk tolerance trade more aggressively. More formally, $\gamma(a) > \gamma(b)$ iff $1/\rho(a) > 1/\rho(b)$ and $\beta(a) > \beta(b))$ iff $t(a)/\rho(a) > t(b)/\rho(b)$.*

The above corollary implies that the following is true. Suppose all trader precisions are homogeneous, $t(a) = \bar{t}$, and that they all have the same utility with decreasing absolute risk aversion. Then wealthier traders will trade more aggressively and will provide more liquidity. The next corollary examines the role of wealth distribution for equilibrium price informativeness.

Corollary 2. *Suppose that $t(a) = \bar{t}$ and that all traders have CRRA utility function with relative risk aversion RRA. Then price informativeness is given by*

$$\tau_p = \bar{t}\frac{\left(\int_\nu^1 W_0(a)da\right)^2}{\int_0^1 W_0(a)^2 da}. \tag{15}$$

The price informativeness increases in $\bar{t}$. A mean-preserving spread[13] *in cross-section of wealth $W_0(a)$ decreases price informativeness.*

[12] The conditional return $E[R - R_f|ds(a), p]$ is defined as follows. Let

$$ER_e^i(\Delta s^i, m) = E[R - R_f|\Delta s^i, p] - R_f.$$

be the expected excess return process, and denote its stochastic differential at $m = 0$ as

$$E[R - R_f|ds(a^i), p].$$

Such notation makes sense, since $dER_e^i = E[R - R_f|ds^i, p] - E[R - R_f|p]$ and $E[R - R_f|p]$ is zero.

[13] Preserving the average wealth of both rational and noise traders. By definition, the change in the cross-section of wealth from $W_0(a)$ to $\check{W}(a)$ is mean preserving if both $\int_0^\nu \left(W_0(a) - \check{W}(a)\right) da = 0$ and $\int_\nu^1 \left(W_0(a) - \check{W}(a)\right) da = 0$ and, $\check{W}(a)$ is mean-independent of $W(a)$ and $\check{W}(a)$ is different from $W_0(a)$ on a set of points with positive Lebesgue measure.

4 Traders' characteristics and their distribution

Our model allows for the following three dimensions of heterogeneity: initial wealth $W_0(a)$, precisions $t(a)$ and utilities $u(\cdot, a)$. We will proceed further by assuming that the heterogeneity in utilities is captured by a vector of parameters $\mathfrak{p}(a)$, where each element in $\mathfrak{p}(a)$ is a real number. As it is clear from our equilibrium characterization, in our setup parameters of the utility entering equilibrium quantities are absolute risk aversion $\rho(a)$ and prudence $\pi(a)$ evaluated at $W_0(a)$. Thus, there is no loss of generality by going from $u(\cdot, a)$ to $\mathfrak{p}(a)$.

Once the choice of information precision $t(a)$ is endogenized, it becomes a function of trader's wealth and utility, so $t(a) = t(W_0(a), \mathfrak{p}(a))$ and is no longer an independent characterstic. We refer to the tuple $(W_0(a), \mathfrak{p}(a))$ as *characteristics* of agent a and denote it

$$\mathcal{C}(a) = (W_0(a), \mathfrak{p}(a)).$$

For now we assume some exogenous function $t(a) = t(W_0(a), \mathfrak{p}(a))$. In fact, for most of the analysis that follows we simply assume that precisions are homogeneous, $t(a) = \bar{t}$.[14]

We assume that characteristics $\mathcal{C}(a)$ live in some finite measure space and that the mapping between the set of characteristics and the agents is given by a strictly increasing and continuous function $a = \Phi(\mathcal{C})$.[15] This means that the functions describing the characteristics of each trader a are given by $(W_0(a), \mathfrak{p}(a)) = \Phi^{-1}(a)$. The function $\Phi(\mathcal{C})$ is a CDF of a cross-sectional distribution of characteristics.

In the following section we endogenize the distribution $\Phi(\cdot)$ by requiring it to be *trade-invariant.*

[14] With endogenous information acquisition and decreasing absolute risk aversion, $\tau(W)$ will be increasing in W. Then, assumption that $t(W) = \bar{t}$ for large enough W amounts to assuming that there is an upper bound on the information that can be acquired.

[15] In fact, for many of our results it is enough to assume that $\Phi(\cdot)$ has a Lebesgue measure zero of discontinuity points.

5 Kolmogorov in CHILE

In this section we consider the following exercise. Suppose, for simplicity, that all traders in our economy have the same utility.[16] The only dimension of heterogeneity is their initial wealth $W_0(a)$. Consider a discrete economy of size m and suppose that the initial wealth of each trader is drawn form the distribution $\hat{\phi}(w;m)$. After trade the distribution of wealth will be different and will depend on realized v and p. Denote it $\hat{\phi}_+(w;v,p,m)$. We say that the distribution is *trade-invariant* if ex-ante distribution of post-trade wealth coincides with $\hat{\phi}(w;m)$, i.e.,

$$E[\hat{\phi}_+(w;v,p,m)] = \hat{\phi}(w,m).$$

The trade-invariant density in large economy $\phi(w)$ is simply the limit, as $m \to 0$, of the invariant density $\hat{\phi}(w,m)$, i.e., $\phi(w) = \hat{\phi}(w,0)$.

We proceed below with a heuristic derivation and we leave the rigorous derivation for the Appendix. First, write the the change in wealth due to risky asset trade, $dW = W_2 - R_f W_0$ as follows

$$\begin{aligned} dW &= (R - R_f)dx \\ &= (R - R_f)\left((\alpha - \gamma p)\, da + \beta ds_a\right) \\ &= (R - R_f)\left((\alpha - \gamma p + \beta v)\, da + \beta/\sqrt{t_a} dB_a\right) \\ &= \mu_W da + \sigma_W dB_a. \end{aligned}$$

Here we denoted

$$\mu_W = (R - R_f)\left(\alpha(W) - \gamma(W)p + \beta(W)v\right) \text{ and } \sigma_W = \beta(W)/\sqrt{t(W)}.$$

We emphasize that the coefficients α, β and γ as well as precision t depend on W.

Recall that $\hat{\phi}_+(w;m) = Pr(W \in [w, w+dw])/dw$ is the PDF of time-2 wealth in the discrete economy with agent size m. Recall that $\hat{\phi}(w;m) = Pr(W_0 \in [w, w + dw])/dw$ is the PDF of

[16] It's easy to extend to heterogeneous preferences. With heterogeneous preferences, this section characterizes $\phi(W, \mathfrak{p})$, the joint density, for a specific preferences, characterized by a particular $\mathfrak{p}$.

the initial wealth in the discrete economy of with agent size m. Now note that in the limit as the size of each agent becomes zero ($m \to 0$), agents do not trade and pre- and post-trade distributions are the same. Thus, we have $\hat{\phi}_+(w,0) = \hat{\phi}(w,0)$. Then $\hat{\phi}_+(w,m) - \hat{\phi}(x,m) = \hat{\phi}_+(x,m) - \hat{\phi}_+(x,0)$. The Kolmogorov Forward Equation (KFE), written at $m = 0$ tells us that

$$\frac{\partial \hat{\phi}_+}{\partial m} = -\frac{\partial}{\partial W}(\mu_W \hat{\phi}_+(W,0)) + \frac{1}{2}\frac{\partial^2}{\partial W^2}(\sigma_W^2 \hat{\phi}_+(W,0)).$$

The trade-invariant density is such that $E[\hat{\phi}_+(w,m) - \hat{\phi}(w,m)] = E[\hat{\phi}_+(w,m) - \hat{\phi}_+(w,0)] = 0$ which implies that $\frac{\partial E[\hat{\phi}_+(w,0)]}{\partial m} = 0$. Taking expectation over v and p, we obtain the following ODE on trade-invariant density

$$0 = -\frac{\partial}{\partial W}(E[\mu_W]\phi(W)) + \frac{1}{2}\frac{\partial^2}{\partial W^2}(E[\sigma_W^2]\phi(W)). \tag{16}$$

We can compute in close-form

$$E[\mu_W] = (1-\nu)\beta(W)\tau^{-1}R_f$$

and

$$E\left[\sigma_W^2\right] = \beta^2(W)/t(W)Var[R|p]$$

(see Appendix for the derivation).

It is instructive to consider a special case of CRRA utility, so that $\rho(W) = W/RRA$. We also assume that $t(W)$ is constant, $t(W) = \bar{t}$. Then $E[\mu_W] \propto W$, and $E\left[\sigma_W^2\right] \propto W^2$, and the KFE coincides with the KFE describing the stationary density of a Geometric Brownian Motion. It is well-known that such density does not exist (the GBM density is log-normal with variance increasing over time). This is well-understood in the literature (e.g. Gabaix (1999)) and so the papers additionally introduce a "stabilizing force": a reflecting lower barrier, exit and reinjection or death (see Gabaix (1999) and Gabaix, Lasry, Lions, and Moll (2016, Appendix D)).

For our context, we use death as a natural stabilizing force. More specifically, we assume that a "death shock," modeled as a Poisson arrival of intensity δ, lands somewhere in the cross-section of agents. If this shock hits agent i, this agent dies before $t = 2$ and his wealth is replaced with a draw from the distribution $\psi(\cdot)$, satisfying certain regularity conditions . In that case the KFE becomes (see the Proof)

$$0 = -\frac{\partial}{\partial W}(E[\mu_W]\phi(W)) + \frac{1}{2}\frac{\partial^2}{\partial W^2}(E[\sigma_W^2]\phi(W)) - \delta \cdot \phi(W) + \delta \cdot \psi(W). \tag{17}$$

The intuition for the transition from (16) to (17) is as follows. With intensity δ the agent dies and his wealth is taken out, which contributes to "outflow" $-\delta\phi(\cdot)$. His wealth is then replaced with a draw from $\psi(\cdot)$ which contributes to an "inflow" $\delta\psi(\cdot)$.

For the case of CRRA utility and $t(W) = \bar{t}$ (for large enough W), the ODE above can be translated to the ODE for the log-wealth, which would have the form identical to the ODE (5) in Gabaix et al. (2016). The results in Gabaix et al. (2016) then imply that $\phi(W)$ has a power-law tail

$$Pr(W > x) \sim Cx^{-\zeta},$$

where

$$\zeta = -\frac{E[\mu_W]/W}{E[\sigma_W^2]/W^2} + \frac{1}{2} + \sqrt{\left(\frac{E[\mu_W]/W}{E[\sigma_W^2]/W^2} - \frac{1}{2}\right)^2 + \frac{2\delta}{E[\sigma_W^2]/W^2}}.$$

Substituting closed-form solutions for $E[\mu_W]$ and $E[\sigma_W^2]$ and simplifying we obtain

$$\zeta = -(1-\nu)RRA + \frac{1}{2} + \sqrt{\left((1-\nu)RRA - \frac{1}{2}\right)^2 + RRA^2\tau^2\left(\exp\left(\tau^{-1}\right) - 1\right)\frac{2\delta}{\bar{t}}}. \tag{18}$$

We summarize in the proposition below.

Proposition 1. *The stationary density solves ODE (17). Suppose that (i) agents have CRRA utility with relative risk aversion RRA, (ii) $t(W) = \bar{t}$ for W large enough and (iii) $\psi \sim x^{-\zeta_\psi}$, where ζ_ψ is the greater than the right-hand side of (18). Then, stationary distribution has a Pareto tail with exponent ζ given by (18).*

6 Literature review

There are several branches of literature that our paper is related to. First, there is a literature on REE models that go beyond the CARA-Normal framework. Breon-Drish (2015) extends the CARA-Normal framework beyond normality in a single asset setup. Albagli, Hellwig, and Tsyvinski (2021) consider a setup with general distribution and risk-neutral traders subject to position limits. Chabakauri et al. (2022) further extends Breon-Drish (2015) by allowing for multiple assets. All of these papers assume CARA utility and so abstract away from wealth effects that are central to our paper.[17]

Malamud (2015) considers an REE model with a continuum of assets. Central to the tractability of his framework is the assumption of market completeness.[18] In contrast, we have one asset and continuum of states of the world. Hence the market is incomplete in our setup. One of the central results in Malamud (2015, Theorem 2.1) is that with non-CARA utility the equilibrium is fully revealing. In contrast, in our setup, there is no full revelation for any utility function.

Peress (2004) was the first (to our knowledge) to study wealth effects in a noisy REE model. Similarly to our paper, his model features log-normally distributed payoffs and non-CARA utilities.[19] The key difference is that Peress (2004) relies on "small risk" approximation, where the riskiness of the risky asset is small. In the limit, the variance of risky asset return is zero, making such approach not suitable for quantitative work (it will be hard to match variance). Our approximation is essentially "small information". In contrast to Peress (2004), in our model the asset stays risky even in the limit. Additionally: (i) in our model the equilibrium quantities are affected by absolute risk aversion and absolute prudence, whereas in Peress (2004) only risk-aversion plays a role and (ii) conditional skewness plays a role in our model, but not in Peress (2004). Peress (2014) embeds the REE model of Peress (2004) into a growth model. An interesting direction for future research is to embed our model of financial market into a

[17]Here we consider risk-neutral preferences as a special case of CARA with risk-aversion equal to zero.

[18]Related, DeMarzo and Skiadas (1998) and DeMarzo and Skiadas (1999) analyze REE models, where the market is *quasi-complete.*

[19]The wealth effect can arise even in CARA model, because the wealth may affect the tightness of financial constraints, as in Glebkin, Gondhi, and Kuong (2021).

growth model.

Second, our paper is related to the literature on power laws in economics and finance. See Gabaix (1999), Gabaix et al. (2016) and, for a review (Gabaix (2009) and Gabaix et al. (2016)). As in most of the papers in that literature, the power law emerges via a random growth mechanism. To the best of our knowledge we are the first to make the heterogeneity of information a part of the random growth mechanism and thus to relate the tail exponent in the economy to information efficiency.

Third, our paper is also related to the literature on mean-field games. See Lasry and Lions (2007), Achdou, Han, Lasry, Lions, and Moll (2022) and, for a review, Guéant, Lasry, and Lions (2011). A typical mean-field game is described by HJB equation determining the dynamics of optimal policy of each agent type and KFE describing the dynamics of the distribution of agent types over time. Our model is static. Yet the tools we use are similar. In our model the KFE describes the trade-invariant distribution that is analogous to a stationary distribution in a dynamic game. Moreover, as Achdou et al. (2022) note: *" The name (Mean Field Games) comes from an analogy to the continuum limit taken in "Mean Field theory" which approximates large systems of interacting particles by assuming that these interact only with the statistical mean of other particles."* This analogy holds in our model. The effect of other traders on a trader of interest in our economy is summarized by several statistics of cross-sectional distribution of traders' characteristics. These statistics can be viewed as a "mean field" that influences each trader's equilibrium behaviour. As in Mean Field theory other traders do not affect a trader of interest directly, but only through their (infinitesimal) contribution to the mean field.

Fourth, on a technical side, our paper is also related to literature that uses stochastic calculus tools outside the domain of continuous time finance and economics. Examples include Malamud (2015) who models the noise in a continuum of assets as a cross-sectional stochastic process; Gârleanu et al. (2015) who use Brownian bridge to represent the dividends for firms located on a circle; and Glebkin et al. (2021) who use stochastic calculus techniques to derive a marginal value of information in a static model.[20] The most closely related paper is Avdis (2018), that introduces a model with continuous heterogeneous information, albeit with CARA preferences

[20]There is also a related econometric literature on the unit roots. A good example is Phillips (1987).

and, as a result, without wealth effects.

7 Conclusion

Thank you for reaching this part of the paper. At some point we will add a conclusion here.

Appendices

A Summary of notation

Notation	Explanation
	General mathematical notation
$dB(a)$	An increment of the Brownian motion
a, b	Index of agents in the continuous limit
i, j	Index of agents in the discrete economy
n	Number of agents in the discrete economy
$m = 1/n$	Mass of an agent in the discrete economy
$E^i[\cdot]$	Conditional expectation, given agent i's information set
$E_y[\cdot]$	Expectation taken with respect to r.v. y
$\mathcal{N}(\cdot)$	Standard normal CDF
	Model variables
General note. Subscripts indicate partial derivatives.	
$dx(a) = \alpha(a)da + \beta(a)ds_a - \gamma(a)pda$	Demand of trader in $[a, a + da]$
$\rho(a) = -u''(a)/u'(a)$	Absolute risk aversion of trader a
$\pi(a) = -u'''(a)/u''(a)$	Absolute prudence of trader a
Π	Profit

B Derivation of demand coefficients

The Lemma 5 in the appendix implies that $\mathfrak{f}^1(p,a)$ and $\mathfrak{f}^2(p,a)$ can be computed as follows. Denote $F(\Delta s^i(m), m)$ the left-hand side of the first order condition (10), where one substitutes the demand of an agent $X(\cdot)$ instead of x.

$$F(\Delta s^i(m), m) = \int \left(\begin{array}{c} U'\left(W_0^i R_f + \left(\exp\left(v^i(p, \Delta s^i, m, z) - p\right) - R_f\right) X(\cdot)\right) \\ \cdot \left(\exp\left(v^i(p, \Delta s^i, m, z) - p\right) - R_f\right) \end{array} \right) d\mathcal{N}(z)$$

Note, that $F(\Delta s^i(m), m)$ is an Ito process, driven by the process

$$ds^i = v da + \frac{1}{\sqrt{t(a)}} dB_a.$$

We then apply Ito's lemma to the process F at $m = 0$ and equalize both drift and diffusion to zero (the FOC should hold for any $\tilde{m}$, so F is zero).

To proceed we note that $X(p, \Delta s^i, m, a)$ and $v^i(p, \Delta s^i, m, z)$ are also Ito processes and by Ito's lemma their differentials at $m = 0$ can be written as

$$dX = \mathcal{D}X da + \sigma_X ds(a) \text{ and } dv^i = \mathcal{D}v^i \, da + \sigma_{v,i} ds(a),$$

where

$$\begin{aligned} \mathcal{D}X &= X_{ss}(p, 0, 0, a) + X_m(p, 0, 0, a), \\ \mathcal{D}v^i &= v^i_{ss}(p, 0, 0, z) + v^i_m(p, 0, 0, z), \\ \sigma_X &= X_s(p, 0, 0, a), \\ \sigma_{v,i} &= v^i_s(p, 0, 0, z). \end{aligned}$$

The functions $\mathfrak{f}^1(p,a)$ and $\mathfrak{f}^2(p,a)$ are identified as

$$\mathfrak{f}^1 = \sigma_X \text{ and } \mathfrak{f}^2 = \mathcal{D}X.$$

The computation of equilibrium can be done in several steps. First, we determine the equilibrium coefficients $\beta(a)$. Once these are known, we can compute all other coefficients in closed-form.

B.1 Finding $\beta(a)$

Computing the diffusion term of $F(\cdot)$ at $m = 0$ and equalizing it to zero yields

$$\sigma_{v,i} u'(R_f W_0(a)) \int \left(\exp\left(v^i(p,0,0,z) - p\right)\right) d\mathcal{N}(z) + \tag{19}$$
$$\sigma_X u''(R_f W_0(a)) \int \left(\exp\left(v^i(p,0,0,z) - p\right) - R_f\right)^2 d\mathcal{N}(z) = 0$$

Now we note that

$$\int \exp\left(v^i(p,0,0,z) - p\right) d\mathcal{N}(z) = E[R|p] = R_f,$$

where the last equality follows from (11). It then follows that

$$\int \left(\exp\left(v^i(p,0,0,z) - p\right) - R_f\right)^2 d\mathcal{N}(z) = Var[R|p] = R_f^2(\exp\left(\tau(0)^{-1}\right) - 1),$$

where the last equality computes $Var[R|p]$ using the fact that $R|p$ is lognormally distributed.[21]

Denoting absolute risk aversion of agent a

$$\rho(a) \equiv -\frac{u''(W_0 R_f)}{u'(W_0 R_f)},$$

and noting that $\sigma_{v,i} = c_s(0) = \frac{t(a)}{\tau}$ we express $\beta(a) = \sigma_X$ from (19) as follows

$$\beta(a) = \sigma_X = \frac{R_f t(a)/\tau}{\rho(a)\mathrm{Var}[R|p]} = \frac{t(a)}{R_f \rho(a) \tau (\exp\left(\tau^{-1}\right) - 1)}.$$

[21] The calculation is as follows. We know that $v|p \sim N(\mu_{v|p}, \sigma_{v|p})$. Moreover, $E[R|p] = E[\exp(v-p)|p] = \exp(\mu_{v|p} + \sigma^2_{v|p}/2) = R_f$. Here the second transition uses the standard formula for mean of lognormal distribution and third transition uses (11). Then, $\mathrm{Var}[R|P] = (\exp(\sigma^2_{v|p}) - 1)\exp(2\mu_{v|p} + \sigma^2_{v|p}) = (\exp(\sigma^2_{v|p}) - 1)R_f^2$.

In accordance with our guess σ_X does not depend on p.

The following proposition then summarizes how the solution $\beta(a)$ is found.

Proposition 2. *The coefficients $\beta(a)$ are given by*

$$\beta(a) = \frac{R_f t(a)/\tau}{\rho(a)\mathrm{Var}[R|p]}.$$

where

$$\mathrm{Var}[R|p] = R_f^2(\exp(\tau^{-1}) - 1)$$

and $\tau = \tau_v + \tau_p$. The price informativeness τ_p is given by

$$\tau_p = \frac{\left(\int_\nu^1 \frac{t(a)}{\rho(a)} da\right)^2}{\int_0^1 \frac{t(a)}{\rho(a)^2} da}. \tag{20}$$

B.2 Finding $\gamma(a)$

Computing the drift term of $dF(\cdot)$ at $m = 0$ and equalizing it to zero yields:

$$\int \mathcal{D}v^i \exp\left(v^i(p,0,0,z) - p\right) d\mathcal{N}(z) - \tag{21}$$

$$\mathcal{D}X\rho(a) \int \left(\exp\left(v^i(p,0,0,z) - p\right) - R_f\right)^2 d\mathcal{N}(z) + \tag{22}$$

$$\frac{\pi(a)\rho(a)\sigma_X^2}{2t(a)} \int (\exp\left(v^i(p,0,0,z) - p\right) - R_f)^3 d\mathcal{N}(z) - \tag{23}$$

$$\frac{2\rho(a)\sigma_{v,i}\sigma_X}{t(a)} \int \exp\left(v^i(p,0,0,z) - p\right)\left(\exp\left(v^i(p,0,0,z) - p\right) - R_f\right) d\mathcal{N}(z) + \tag{24}$$

$$\frac{\sigma_{v,i}^2 \int \exp\left(v^i(p,0,0,z) - p\right) d\mathcal{N}(z)}{2t(a)} = 0, \tag{25}$$

where we have denoted

$$\pi(a) = -\frac{u'''(R_f W_0(a))}{u''(R_f W_0(a))}$$

The absolute prudence of agent a. While the above equation is big, only the first two lines depend on p. Denoting

$$\hat{h}(m) \equiv c_p(m)\frac{\int_0^1 \gamma(a,m)da}{\int_\nu^1 \hat{\beta}(a,m)da},$$

and letting "$\cdots$" represent the terms that do not depend on p we write

$$\mathcal{D}v^i = \hat{h}(m)'p + \cdots$$

Noting that $\mathcal{D}X = -\gamma(a)p + \cdots$ equation (21)-(25) then becomes

$$R_f\hat{h}(0)'p + \gamma(a)\rho(a)Var[R|p]p + \cdots = 0$$

From which we find

$$\gamma(a) = -\frac{R_f\hat{h}(0)'}{\rho(a)\mathrm{Var}[R|p]}. \tag{26}$$

Given that our equilibrium concept only requires the functions $\hat{\alpha}(a,m)$, $\hat{\beta}(a,m)$ and $\hat{\gamma}(a,m)$ to be equal to $\alpha(a)$, $\beta(a)$ and $\gamma(a)$ and does not put explicit restrictions on the derivatives of these functions with respect to m, one might be concerned about equilibrium indeterminacy, as the aforementioned conditions put no restrictions on $\hat{h}(m)'$. However, the observations of Lemma 4 allow to pin down $\hat{h}(m)'$ as follows. The key is the condition (12).

Note that τ_p, τ and $\int_\nu^1 \beta(a)da$ are determined once $\beta(a)$ is solved for. Then

$$\hat{h}(0)' \equiv -h_\gamma = -\frac{\tau\int_\nu^1 \beta(a)da\mathrm{Var}[R|p]}{R_f\tau_p\int_0^1 1/\rho(a)da} = -\frac{\int_0^1 t(a)/\rho(a)^2da}{\int_\nu^1 t(a)/\rho(a)da\int_0^1 1/\rho(a)da}. \tag{27}$$

We summarize in the proposition below.

Proposition 3. *The coefficients $\gamma(a)$ are given by (26), where $\hat{h}(0)'$ is given by (27).*

B.3 Finding $\alpha(a)$

We turn to finding $\alpha(a)$. We work our way through (21)-(25) line by line. We start with (21). Denoting "$\cdots$" the terms proportional to p (which are already accounted for) we write

$$\mathcal{D}v^i = c_{\bar{v}}(0)'\bar{v} + (c_p(m)\bar{\alpha}(m)\bar{\beta}(m))' - \frac{1}{2}\tau^{-3/2}\hat{\tau}(0)'z + \cdots$$

While the next term can be computed in closed-form, it is only important for us to note that it does not depend on a and denote it by h_α.

$$\int \mathcal{D}v^i \exp\left(v^i(p,0,0,z) - p\right) d\mathcal{N}(z) = h_\alpha.$$

We turn to (22). Sice $\mathcal{D}X = \alpha(a) + \cdots$, we have

$$\mathcal{D}X\rho(a)\int \left(\exp\left(v^i(p,0,0,z) - p\right) - R_f\right)^2 d\mathcal{N}(z) = \alpha(a)\rho(a)Var[R|p] + \cdots$$

We turn to (23). Substituting $\sigma_X = \beta(z)$, and $\int(\exp\left(v^i(p,0,0,z) - p\right) - R_f)^3 d\mathcal{N}(z) = \mathrm{Sk}[v|p]\mathrm{Var}[R|p]^{3/2}$ we get

$$\frac{\pi(a)\rho(a)\sigma_X^2}{2t(a)}\int(\exp\left(v^i(p,0,0,z) - p\right) - R_f)^3 d\mathcal{N}(z) = \frac{\pi(a)\rho(a)\beta(a)^2}{2t(a)}\mathrm{Var}[R|p]^{3/2}\mathrm{Sk}[R|p],$$

where the skewness can be computed in closed-form

$$\mathrm{Sk}[R|p] = \left(\exp\left(\tau^{-1}\right) + 2\right)\sqrt{\exp\left(\tau^{-1}\right) - 1}.$$

We proceed to (24). Here we substitute $\sigma_{v,i} = t(a)/\tau$ and

$$\int \exp\left(v^i(p,0,0,z) - p\right)\left(\exp\left(v^i(p,0,0,z) - p\right) - R_f\right) d\mathcal{N}(z) = \mathrm{Var}[R|p]$$

and get

$$\frac{2\rho(a)\sigma_{v,i}\sigma_X}{t(a)}\int \exp\left(v^i(p,0,0,z)-p\right)\left(\exp\left(v^i(p,0,0,z)-p\right)-R_f\right)d\mathcal{N}(z)=\frac{2\rho(a)\beta(a)}{\tau}Var[R|p]$$

Finally, for (25) we get

$$\frac{\sigma_{v,i}^2\int \exp\left(v^i(p,0,0,z)-p\right)d\mathcal{N}(z)}{2t(a)}=\frac{R_f t(a)}{2\tau^2}$$

Combining it all together we get

$$h_\alpha-\alpha(a)\rho(a)Var[R|p]$$
$$+\frac{\pi(a)\rho(a)\beta(a)^2}{2t(a)}\mathrm{Var}[R|p]^{3/2}\mathrm{Sk}[R|p]-\frac{2\rho(a)\beta(a)}{\tau}Var[R|p]+\frac{R_f t(a)}{2\tau^2}=0$$

After mild simplification we get

$$\alpha(a)=\frac{h_\alpha}{\rho(a)Var[R|p]}+\frac{\pi(a)\beta(a)^2}{2t(a)}\mathrm{Var}[R|p]^{1/2}\mathrm{Sk}[R|p]-\frac{3\beta(a)}{2\tau}. \tag{28}$$

As before, the discipline on h_α comes from the Lemma 4 and it is found as a unique solution to

$$\int_0^1\alpha(a)da=h_\alpha\int_0^1\frac{da}{\rho(a)Var[R|p]}+\mathrm{Var}[R|p]^{1/2}\mathrm{Sk}[R|p]\int_0^1\frac{\pi(a)\beta(a)^2}{2t(a)}da-\frac{3}{2\tau}\int_0^1\beta(a)da. \tag{29}$$

We summarize.

Proposition 4. *The coefficients $\alpha(a)$ are given by (28), where h_α is the unique solution to (29).*

B.4 An alternative representation of demand

Define the expected excess return process

$$ER_e^i(\Delta s^i, m) = E[R - R_f|\Delta s^i, p].$$

Denote its stochastic differential at $m = 0$

$$E[R - R_f|ds^i, p].$$

Such notation makes sense, since $dER_e^i = E[R - R_f|ds^i, p] - E[R - R_f|p]$ and the term $E[R - R_f|p]$ is zero. Applying Ito's lemma it's easy to compute

$$E[R - R_f|ds^i, p] = R_f c_s(0) ds^i + R_f \hat{h}(0)' p dm + h_\alpha da.$$

Then the following proposition follows immediately.

Proposition 5. *The equilibrium demand can be written as*

$$dX_a = \frac{E[R - R_f|ds^i, p]}{\rho(a) Var[R|p]} + \kappa(a) da,$$

where

$$\kappa(a) = \frac{\pi(a)\beta(a)^2}{2t(a)} \mathrm{Var}[R|p]^{1/2} \mathrm{Sk}[R|p] - \frac{3\beta(a)}{2\tau}. \tag{30}$$

C Proofs

C.1 Proof of Lemma 1

Proof of Lemma 1. Fix p. We drop the argument p in what follows. That is, $X(\Delta s^i; m, a^i)$ denotes $X(p, \Delta s^i; m, a^i)$.

By Ito's lemma we have

$$
\begin{aligned}
&X(\Delta s^i, m, a^i) - X(0,0,a^i) = \\
&= \int_{a^i}^{a^{i+1}} X_s(s(a) - s(a^i); a - a^i, a^i) ds_a + \\
&+ \int_{a^i}^{a^{i+1}} \left(\frac{1}{2t_a} X_{ss}(p, s(a) - s(a^i); a - a^i, a^i) + X_m(p, s(a) - s(a^i); a - a^i, a^i) \right) da.
\end{aligned}
$$

Denote the i-th segement $\mathcal{A}^i = [a^i, a^{i+1})$ and $\hat{a}^i = \sum_{i=1}^{n-1} a^i 1(a \in \mathcal{A}^i)$ ($\hat{a}^i$ gives a^i whenever a falls in the i-th segment $\mathcal{A}^i$). With such notation we can write

$$
\begin{aligned}
&\sum_{i=0}^{n-1} \left(X(\Delta s^i; m, a^i) - X(0; 0, a^i) \right) = \\
&= \int_0^y X_s(s(a) - s(\hat{a}^i); a - \hat{a}^i, \hat{a}^i) ds_a + \\
&+ \int_0^y \left(\frac{1}{2t_a} X_{ss}(s(a) - s(\hat{a}^i); a - \hat{a}^i, \hat{a}^i) + X_d(s(a) - s(\hat{a}^i); a - \hat{a}^i, \hat{a}^i) \right) da.
\end{aligned}
$$

Denote $LI = \int_0^y \left(\frac{1}{2t_a} X_{ss}(s(a) - s(\hat{a}^i); a - \hat{a}^i, \hat{a}^i) + X_d(s(a) - s(\hat{a}^i); a - \hat{a}^i, \hat{a}^i) \right) da$ and $SI = \int_0^y X_s(s(a) - s(\hat{a}^i); a - \hat{a}^i, \hat{a}^i) ds_a$. That $LI \to \int \left(\frac{1}{2t_a} X_{ss}(p, 0; 0, a) + X_d(p, 0; 0, a) \right) da$ follows from the Continuous Mapping Theorem.[22] We can also apply Continuous Mapping Theorem to the stochastic integral, since in our case it is bounded (since $\sum_{i=1}^n X^i$ is bounded). Thus, we get

$$
\begin{aligned}
&\sum_{i=0}^{n-1} \left(X(p, \Delta s^i; m, a^i) - X(p, 0; 0, a^i) \right) \xrightarrow[n \to \infty]{} \\
&\int X_s(p, 0; 0, a) ds_a + \int \left(\frac{1}{2t_a} X_{ss}(p, 0; 0, a) + X_d(p, 0; 0, a) \right) da.
\end{aligned}
$$

■

[22] Here we use the fact that Lebesgue integral is a continuous mapping from $C[0,1]$ to $C[0,1]$, the space of functions continuous on $[0,1]$.

C.2 Proof of Lemma 3

Proof of Lemma 3. We start with FOC (we use shortcut $E^i[\cdot] = E[\cdot|\Delta s^i, s_p]$)

$$E^i\left[U'\left(W_0 R_f + (R - R_f)\,x\right)((R - R_f))\right] = 0.$$

The second derivative of the objective function is

$$E^i\left[U''(\cdot)(R - R_f)^2\right] < 0$$

Hence, the objective function is strictly concave. Hence, the FOC is both necessary and sufficient. ■

C.3 Proof of Lemma 4

Proof of Lemma 4. Denote, for this proof only $\bar{\gamma}(m) = \int_0^1 \gamma(a,m)da$, $\bar{\beta}(m) = \int_\nu^1 \beta(a,m)da$ and $\bar{\alpha}(m) = \int_0^1 \alpha(a,m)da$.

Indeed, if the limit is not zero, then the aggregate demand is infinite. Eq. (11) follows by substituting $x = 0$ to (10) and noting that $\mathcal{F}_i = p$ in the limit when $m = 0$. Note that

$$E[R|p] = \exp\left(E[v|p] - p + \frac{1}{2\tau}\right) = R_f \text{ as } m \to 0.$$

We must have that $\partial/\partial p$ of LHS is zero. It's only possible if

$$\partial/\partial p E^i[v] = 1 \implies \frac{\tau_p^i(0)}{\tau^i(0)}\frac{\bar{\gamma}(0)}{\bar{\beta}(0)} = 1.$$

It then follows that $E[v|p] - p + \frac{1}{2\tau} = \frac{\tau_v}{\tau}\bar{v} - \frac{\tau_p}{\tau}\frac{\bar{\alpha}}{\bar{\gamma}} + \frac{1}{2\tau}$ and the last statement follows.

■

C.4 Proof of Corollary 2

Proof of Corollary 2. Eq.(15) follows from (20) by substituting $\rho(a) = RRA/W(a)$ and $t(a) = \bar{\tau}$. From there, the only part of the corollary that requires a proof is a statement about mean preserving spread.

By definition, the change in the cross-section of wealth from $W_0(a)$ to $\check{W}(a)$ is mean preserving if

$$\int_0^{\nu} \left(W_0(a) - \check{W}(a)\right) da = 0 \text{ and } \int_{\nu}^{1} \left(W_0(a) - \check{W}(a)\right) da = 0$$

and $\check{W}(a)$ is different from $W_0(a)$ on a set of points with positive Lebesgue measure. Then, the only "affected" term is $\int_0^1 W_0(a)^2 da$. We have

$$\begin{aligned}
\int_0^1 \check{W}(a)^2 da &= \int_0^1 \left(W_0(a) + \check{W}_0(a) - W_0(a)\right)^2 da \\
&= \int_0^1 W_0(a)^2 da + \int_0^1 \left(\check{W}_0(a) - W_0(a)\right)^2 da + 2\int_0^1 W_0(a) da \int_0^1 \left(\check{W}_0(a) - W_0(a)\right) da \\
&= \int_0^1 W_0(a)^2 da + \int_0^1 \left(\check{W}_0(a) - W_0(a)\right)^2 da \\
&> \int_0^1 W_0(a)^2 da
\end{aligned}$$

■

C.5 Proof of Proposition 1

Proof of Proposition 1 . Fix v and p such that $\exp(v - p) \neq R_f$. For an optimal wealth of an agent of size l with signal $\Delta\tilde{s}^i = s^i(a^i + l) - s^i(a^i)$ in the discrete economy we can write

$$dW = \mu_W(\Delta\tilde{s}^i, l; W, v, p)da + \sigma_W(\Delta\tilde{s}^i, l; W, v, p)dB_a.$$

In what follows we'll only write $W$and l as arguments of μ_W and σ_W.

The drift and the diffision μ_W and σ_W can be computed by applying Ito's lemma to the

FOC

$$\int u'(W)\left(\exp(v^i - p) - R_f\right) d\mathcal{N}(z) = 0.$$

Where,

$$v^i(p, \Delta\tilde{s}^i, l, z) = E[v|\Delta\tilde{s}^i, p] + z\sqrt{\text{Var}[v|\Delta\tilde{s}^i, p]}.$$

While we can compute $\mu_W(l, W)$ and $\sigma_W(l, W)$ for any l, we will only need them at $l = 0$. There it is easily calculated from $dW = (R - R_f)dx$, recalling that $dx = \alpha da + \beta ds_a - \gamma pda$

$$\mu_W(0, W) = (R - R_f)\left(\alpha(W) - \gamma(W)p + \beta(w)v\right),$$

for rational traders and

$$\mu_W(0, W) = (R - R_f)\left(\alpha(W) - \gamma(W)p\right),$$

for noise traders. We also have

$$\sigma_W(0, W) = (R - R_f)\,\beta/\sqrt{t(a)}.$$

By Kolmogorov Forward Equation, the PDF of optimal wealth for the size of an agent equal to l , $f(w, l) = Pr(W(l) \in [w, w + dw])/dw$ of the process given by SDE above satisfies

$$f(w,l) = \left(f(w,0) + \int_0^m \left(-\frac{\partial}{\partial w}(\mu_W(l,w)f(w,l)) + \frac{1}{2}\frac{\partial^2}{\partial W^2}(\sigma_W^2(w,l)f(w,l))\right) dl\right)(1 - Pr(\text{death})) +$$
$$+ Pr(\text{death})\psi(w)$$

The death probability is $\delta m + O(m^2)$. The stationary density is such that $E[f(w, l)] = f(w, 0)$. Taking expectations, dividing by m and taking the limit as $m \to 0$ ... Plus accounting for the fact that $f(w, 0)$ does not depend on v or p and can be taken out of expectaion we obtain the

ODE

$$0 = -\frac{\partial}{\partial w}(E\left[\mu_W(0,w)\right] f(w,0)) + \frac{1}{2}\frac{\partial^2}{\partial W^2}(E\left[\sigma_W^2(w,l)\right] f(w,0)) + \delta f(w,0) - \delta\psi(w).$$

Now we compute

$$\begin{aligned}
E\left[\mu_W(0,w)\right] &= E\left[E\left[(R - R_f)\,|p\right](\alpha(W) - \gamma(W)p)\right] + (1-\nu)\beta(w)E\left[v\left(R - R_f\right)\right] \\
&= (1-\nu)\beta(w)E\left[v\left(R - R_f\right)\right] \\
&= (1-\nu)\beta(w)\tau^{-1}R_f
\end{aligned}$$

Where the last transition is thanks to Lemma 6.

$$E\left[\sigma_W^2(w,l)\right] = \beta^2(w)/t(a)Var[R|p].$$

$$\frac{E\left[\mu_W(w,0)\right]}{E\left[\sigma_W^2(w,0)\right]} = (1-\nu)\frac{t(a)\tau^{-1}R_f}{\beta(w)Var[R|p]} = (1-\nu)\rho(W).$$

■

C.6 Proof of Proposition 2

Proof of Proposition 2. To get (20) note that

$$\tau_p = \frac{\left(\int_\nu^1 \beta(a)da\right)^2}{\int_0^1 \frac{\beta(a)^2}{t(a)}da}.$$

Then (20) follows by substituting $\beta(a) = \frac{t(a)}{R_f\rho(a)\tau(\exp(\tau^{-1})-1)}$ to integrals $\int_0^1 \beta(a)da$ and $\int_0^1 \frac{\beta(a)^2}{t(a)}da$ and rearranging.

■

C.7 Proof of Lemma 5.

Lemma 5. *Consider an Ito process* $dz(a) = \mu(z,a)da + \sigma(z,a)dB(a)$. *Consider a function* $y = \phi(z(a), a)$ *defined implicitly by* $F(y,z,a) = 0$. *Then,* $\sigma_y = \frac{\partial\phi}{\partial z}$ *and* $\mu_y = \frac{\partial\phi}{\partial a} + \frac{1}{2}\sigma^2\frac{\partial^2\phi}{\partial z^2}$ *are given by*

$$\sigma_y = -\frac{F_z}{F_y},$$

$$\mu_y = -(F_a + F_z\mu + \frac{1}{2}F_{yy}\sigma_y^2\sigma^2 + F_{yz}\sigma_y\sigma + \frac{1}{2}F_{zz}\sigma^2)/F_y,$$

where μ, σ *and partials of* ϕ *are evaluated at* $(z(0), 0)$ *and partials of* F *are evaluated at* $(\phi(z(0), 0), 0, 0)$. *These expressions can be obtained by applying Ito's lemma to* $F(y,z,a)$ *at* $a = 0$, $dF = \mu_F da + \sigma_F dz$ *and equalizing both* μ_F *and* σ_F *to zero.*

Proof. We apply Ito's lemma at $a = 0$ to the identity

$$F(\phi(z(a), a), z, a) = 0.$$

We have

$$dF = F_y d\phi + F_z dz + F_a da + \frac{1}{2}F_{yy}d\phi^2 + F_{yz}d\phi dz + \frac{1}{2}F_{zz}dz^2.$$

Since $d\phi = \phi_z dz + \phi_a da + \frac{1}{2}\phi_{zz}dz^2 = \sigma_y dz + \mu_y da$ we have

$$\mu_F = F_y\mu_y + F_z\mu + F_a + \frac{1}{2}F_{yy}\sigma_y^2\sigma^2 + F_{yz}\sigma_y\sigma^2 + \frac{1}{2}F_{zz}\sigma^2 = 0$$

$$\sigma_F = F_y\sigma_y + F_z = 0.$$

Solving for μ_y and σ_y from the two equations above yields the stated result. ■

Lemma 6. $E[v\,(R - R_f)] = \tau^{-1}R_f$.

Proof. Note that by law of iterated expectations we have

$$\begin{aligned} E[v\left(R-R_f\right)] &= E\left[E\left[v\left(R-R_f\right)|p\right]\right] \\ &= E\left[E\left[v\left(R-R_f\right)|p\right]\right]. \end{aligned}$$

Denoting $\mu_{v|p} = E[v|p]$ and $\tau^{-1} = Var[v|p]$ we have

$$\begin{aligned} E\left[v\left(R-R_f\right)|p\right] &= \frac{\partial}{\partial t} \left. E\left[\exp\left(tv-p\right)\right]\right|_{t=1} - \mu_{v|p}R_f \\ &= \frac{\partial}{\partial t} \exp\left. \left(t\mu_{v|p} + \frac{1}{2}t^2\tau^{-1} - p\right)\right|_{t=1} - \mu_{v|p}R_f \\ &= \left(\mu_{v|p} + \tau^{-1}\right) \underbrace{\exp\left(\mu_{v|p} + \frac{1}{2}\tau^{-1} - p\right)}_{=E[R|p]=R_f} - \mu_{v|p}R_f \\ &= \tau^{-1}R_f \end{aligned}$$

■

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