CHILE

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Ask *not* what you can do for CHILE, ask what CHILE can do for *you*

Why should you care about CHILE?

- Glebkin is coming up for tenure

- CHILE answers a common critique we call the Dumas Dichotomy: “one topic, two fields”:

  “asset pricing” and “rational expectations/microstructure”

- Wealth×information effects!
  - Heterogeneity galore
  - Non CARA-Normal-Linear “quasi” **rational expectations**
  - Room for new models (and fresh faces!)

But first, what is a CHILE? (alt. "χLE")

That's NOT a CHILE

That's NOT a CHILE

Continuous and
Heterogenous
Information in a
Large
Economy

THAT's a CHILE

How to make CHILE: ingredients & cookware

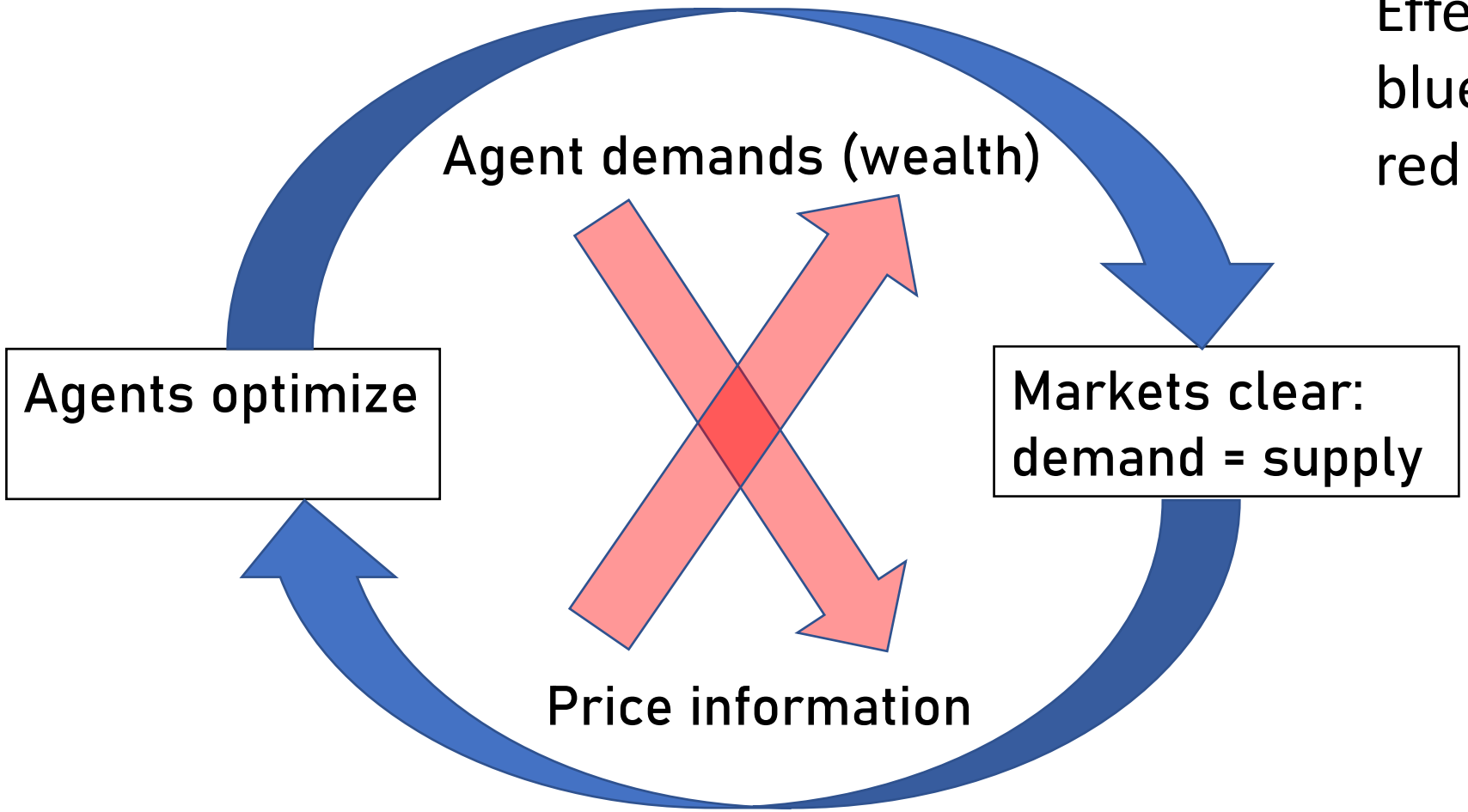
Primitives

- A Brownian Motion $\{B_a\}_{0 \le a < 1}$
- A continuum of agents, with characteristics represented by continuous functions of the agent index a in [0,1)
- An asset of value unknown to the agents
- Signals as “truth + noise”, with noise coming from the increments of B

Mechanisms

- Market clearing for price P
- Bayesian learning for agent expectations
- Aggregation à la Itô, not Kolmogorov
- “Holophrenic” price taking (alternatives possible)

The “market chemistry” we study in microstructure/REE



Effects:
blue typically studied
red typically ignored

Ingredients in detail

The asset value is $V(v)$, v is Normal, $\mathrm{V}(\cdot)$ is (weakly) monotone

An agent $a \in [0,1)$ has

- size da
- utility $\mathrm{du}(a,\cdot)$, with discrete setting equivalent (d.s.e.) $\mathrm{u}_i, i = 1,2, \ldots$
- wealth endowment $\mathrm{W}_0(a)$
- absolute-risk-aversion coefficient $\rho(a,\cdot)$
- a signal $\mathrm{d}s(a)$ about v , with precision $\mathrm{t}(a)da$

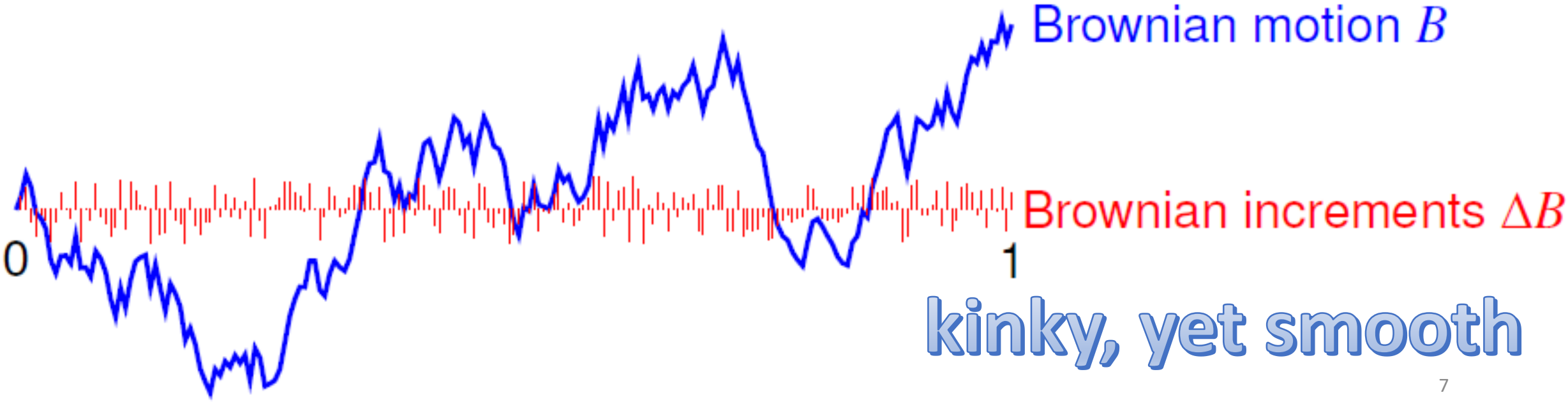
i.i.d. over a

$$ds(a) = v\, da + \frac{1}{\sqrt{t(a)}}\, dB_a$$

On the nature of noise: on a grid or in a continuum?

noise$_n$

standard, yet clunky



kinky, yet smooth

Why? Grids & sequences vs *intervals & functions*

typical REE demand
requires CARA-Normal-linearity

$$\alpha P + \beta s_i$$

standard, yet clunky

typical CHILE demand
allows any utility and nonlinearity*

$$\text{א}(P, a)da + \text{ב}(P, a)ds(a)$$

kinky, yet smooth

REE cookware: agents' attitudes towards how they affect prices

Revelation of own information

Market impact	Account	Ignore
Account	<u>Imperfect competition</u> (Kyle '89 and descendants): "rational" agents *CHI treatment possible*	
Ignore	<u>Perfect competition</u> (Grossman '76 and descendants): "schizophrenic" agents	<u>CHILE</u> "holophrenic," or "snowflake" agents: they think that because they are small, they are the only ones learning from the price

So, what is it that can CHILE do for you?

The demand of agent a with signal $ds(a)$, for price P

$$\text{א}(P,a)da + \text{ב}(P,a)ds(a)$$

Just one example

$V(v) = e^v$

$P = e^p$

$u(a)$ is CRRA, so $\rho(a)$ depends on $W_0(a)$ and a

$$[\alpha(a) + \gamma(a)\, p\,]da + \beta(a)ds(a)$$

From the distribution of wealth to price efficiency

price informativeness = $\dfrac{\left(\int_{\nu}^{1} \frac{t(a)}{\rho(a)}\right)^2}{\int_0^1 \frac{t(a)}{\rho^2(a)} da}$

Just one example

homogeneous precisions, $t(a) = t$ (working on generalizations)
ν is fraction of Fisher-Black noise traders

A Matthew effect on information:
A mean-preserving spread of wealth decreases price informativeness

$$\mathrm{t}\frac{\left(\int_{\nu}^{1} W_0(a)da\right)^2}{\int_0^1 W_0^2(a)da} = \mathrm{t}\frac{(1-\nu)^2}{CV}$$

Coefficient of variation of the wealth distribution

From price efficiency to the distribution of wealth

Homage to Kolmogorov: we can discipline the distribution of wealth ϕ

- **Le père de la probabilité dit:**

$$\delta\,\phi(w) = \delta\,\psi(w) - \frac{\partial}{\partial w}(\phi(w)E[\text{drift of wealth change}]) + \frac{\partial^2}{\partial w^2}(\phi(w)\sigma[\text{drift of wealth change}])$$

- **The solution of this ODE can be a power law!**

Green stuff: exogenous—thanks Siméon-Denis!

δ death rate

ψ replacement density

Blue stuff: endogenous—thanks Kiyoshi!

Red stuff: disciplined—thanks Andrey!

Common **misconceptions**

- You cannot do that! You are using a “time-oriented” filtration!
  - Yes we can
  - We are not using a time filtration, only stochastic integration
- Fisher-Black (1986) noise traders à la Itô? What?
  - Well, are we not already using them *implicitly*?
- How do we think of continuous agents? Are they “conjoined”?
  - A model of a large market as a “healthy” limit of finite markets
  - Working on posing it directly in the small-agent limit
  - Can incorporate market impact and remove “snowflakyness,” at the cost wealth power laws

Conclusion: resolving the Dumas Dichotomy

That's NOT a Dumas Dichotomy

THAT's a Dumas Dichotomy

Old workhorse + young(ish) theory donkeys = a new mule

We have a young model, we need fresh faces!