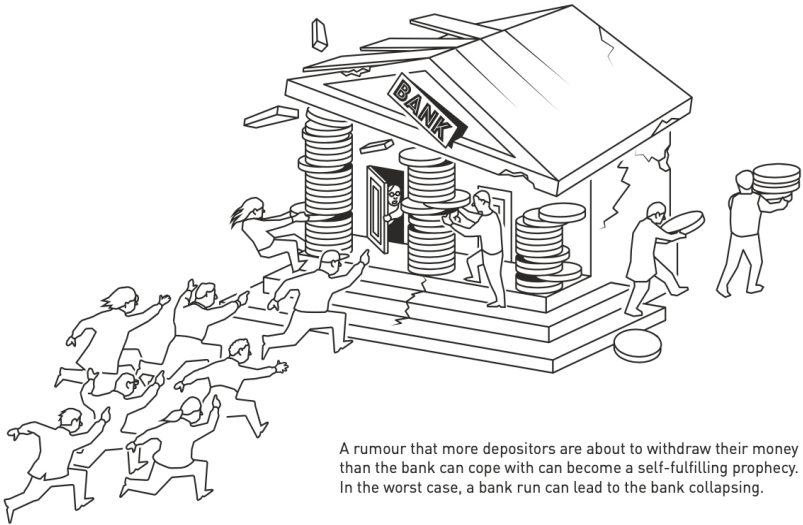
Smart Banks

Alkis Georgiadis-Harris
with Maxi Guennewig and Yuliyan Mitkov

2023



A rumour that more depositors are about to withdraw their money than the bank can cope with can become a self-fulfilling prophecy. In the worst case, a bank run can lead to the bank collapsing.

Introduction

- Diamond-Dybvig (1983) argued that maturity transformation is inherently *fragile*.
  - Efficient arrangement is prone to self-fulfilling *runs*.
  - Short-term illiquidity: liquid liabilities $>$ liquid assets.
- Runs introduce an *efficiency vs stability* trade-off.
- ...but what exactly is the *source* of fragility?
- To answer this: study the mechanism design problem.

Introduction

- Wallace (1988) identifies **sequential service** as a *necessary* constraint.
  - Depositors are serviced sequentially.
  - Bank can only condition the allocation on past interactions.
    - Cannot observe the full withdrawal demand.
  - Depositors are isolated (no trade or communication).
- However, sequential service is *not sufficient* for fragility.

Introduction

Fragility depends in subtle ways on the environment:

- Information frictions (about line-position) (Peck and Shell, 2003). → fragility
- Distributional assumptions (Green and Lin, 2003; Ennis and Keister, 2009b). → depends
- Solution concept (Huang, 2023). → depends
- **Lack of commitment power** (Ennis and Keister, 2009a). → fragility
- **Flexibility** in design of mechanisms (Andolfatto et al., 2017). → stability

Introduction

Our approach:

- Unify existing frameworks.
- Allow for **full flexibility** in the design of mechanisms.
- **Limit commitment** power of intermediary.

- We develop a Mechanism Selection *Game*:
  i. Sequential Service.
  ii. Limited Commitment.
  iii. Contractual Flexibility.

What we find:

* **Unique** equilibrium outcome is **efficient**!
* No efficiency vs stability trade-off..

Model

Environment

- N depositors and a benevolent intermediary ('Banker').

- Two periods, $t \in \{0, 1\}$. Initial resources $Y > 0$.
- Technology transforms a unit not consumed in $t = 0$, into $R > 1$ in $t = 1$.

- Each depositor has a payoff-type $\omega_i \in \{0, 1\}$ ('impatient' or 'patient').
- Preferences of Depositor i:

$$u(c + \omega_i \cdot C) \qquad u(\cdot) \text{ CRRA, coef. RRA} > 1.$$

where c is early consumption ($t = 0$), and C is late consumption ($t = 1$).

- Preferences of Banker:

$$w(\mathbf{c} \mid \omega) = \sum_{i \leq N} u(c_i + \omega_i \cdot C_i) \qquad \text{where } \mathbf{c} = (c_i, C_i)_{i \leq N}$$

where $\omega = (\omega_i)_{i \leq N}$ is the aggregate payoff-state.

Mechanism Selection Game

Stage-0

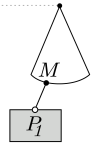
- Nature draws the state $(\omega, \lambda) \sim \mathbb{P}$ where
  - i. $\omega = (\omega_i)_{i \leq N} \in \{0,1\}^N$ is the payoff-state.
  - ii. $\lambda : \{1, \ldots, N\} \mapsto \{1, \ldots, N\}$ is a permutation.
    - $\lambda(i)$ is i's line-position in interacting with the Banker.

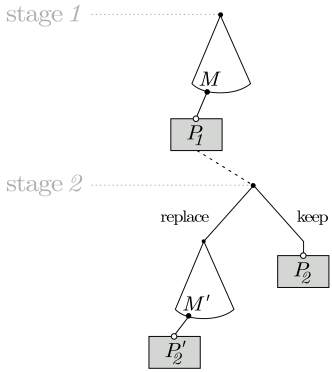
- Each Depositor-i:
  - Is privately informed of $\omega_i \in \{0,1\}$.
  - Receives a private signal κ_i about λ.

- Assumption:

$$\mathbb{P}(\omega_{k+1} = 1 \mid \omega_1, \ldots, \omega_k) = \mathbb{P}\Big(\omega_{k+1} = 1 \mid \sum_{j \leq k} \omega_j\Big)$$

stage *1*

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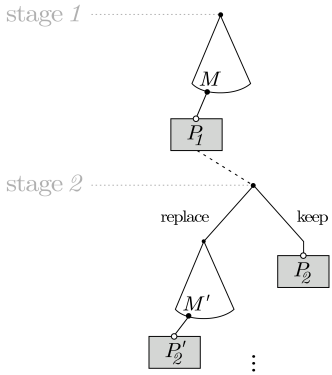
M
P_1
stage 2
replace
keep
P_2
M'
P_2'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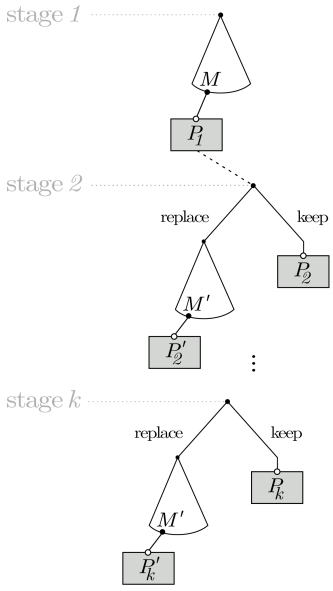
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stage k
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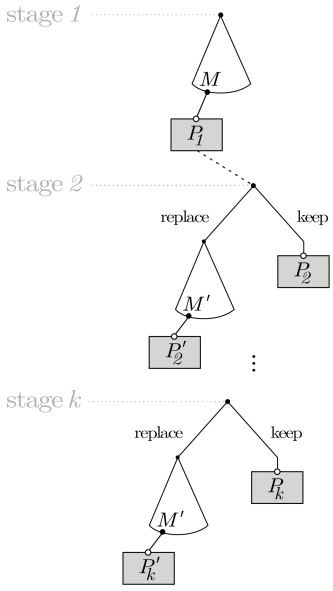
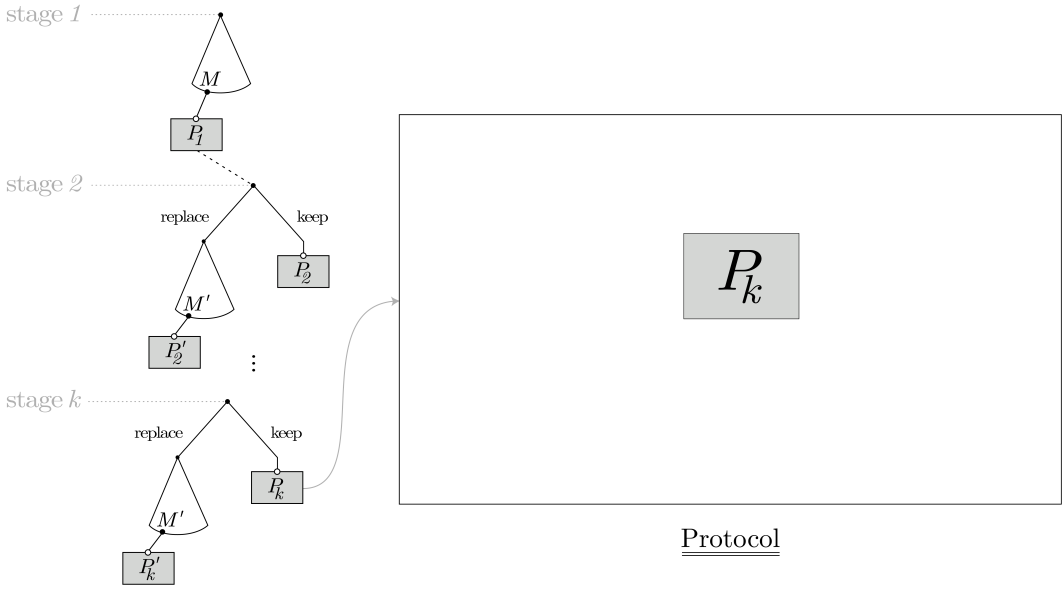


stage $N+1$*:*

- Remaning resources mature at rate $R>1$.
- Each depositor promised future consumption receives an equal share.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stage k
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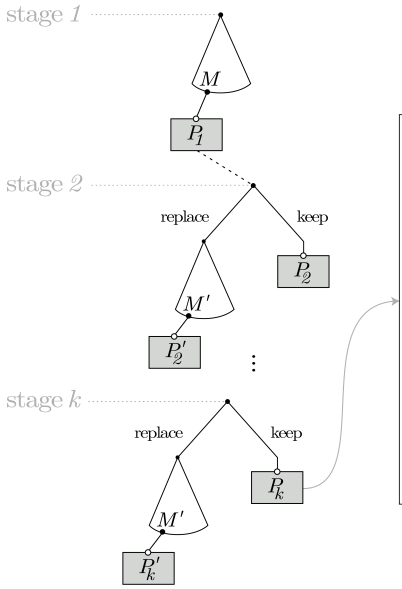
P_k
Protocol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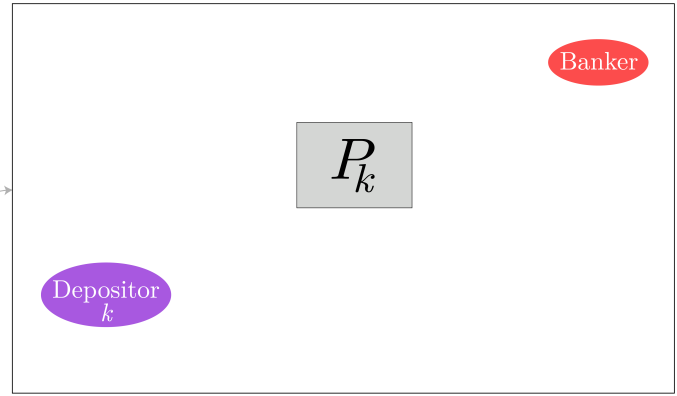
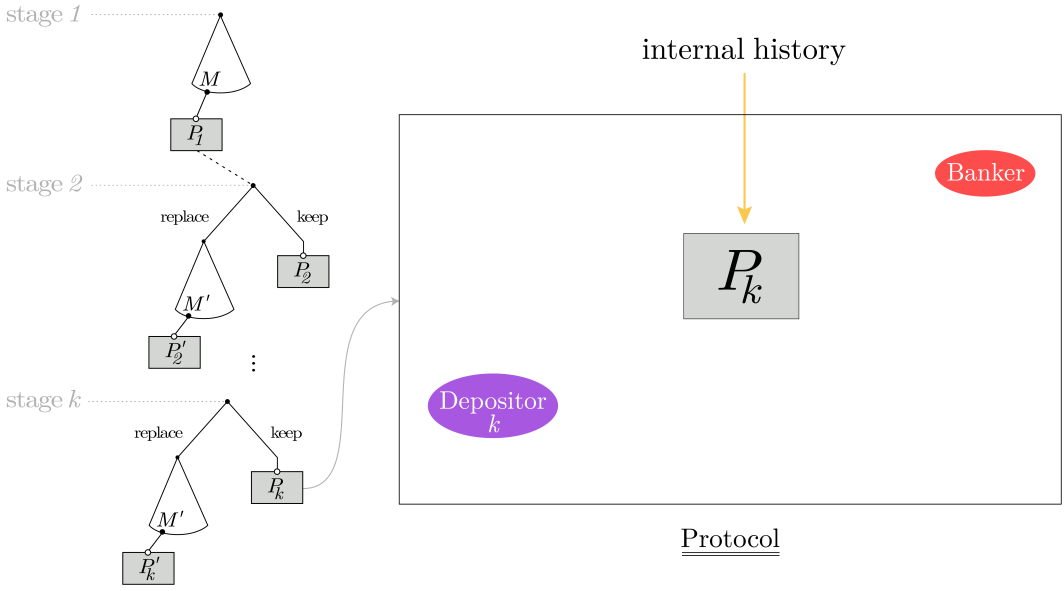

Banker
P_k
Depositor k

Protocol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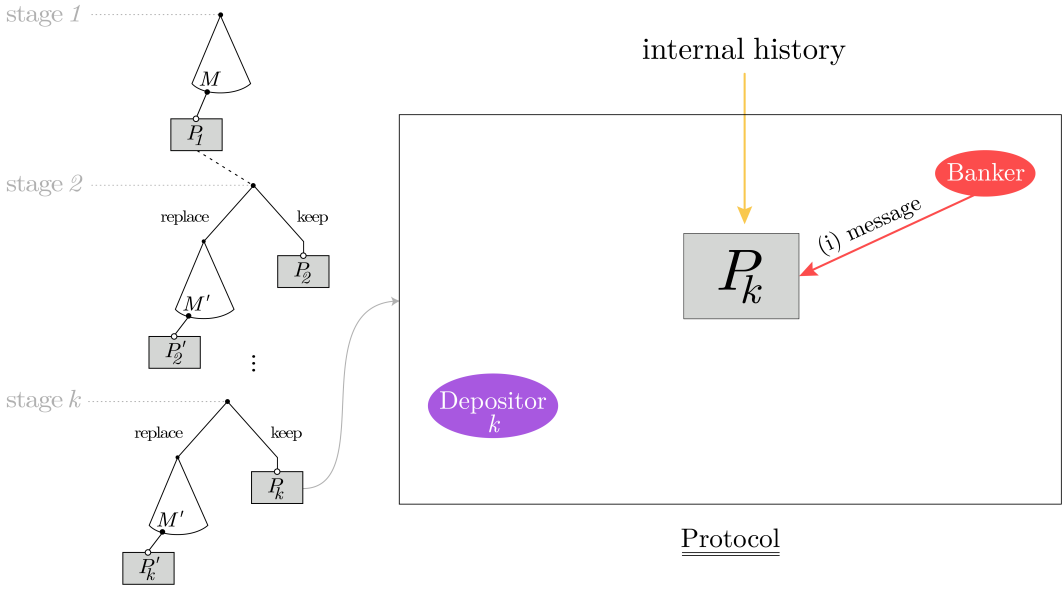
internal history
Banker
P_k
Depositor k
Protocol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stage k
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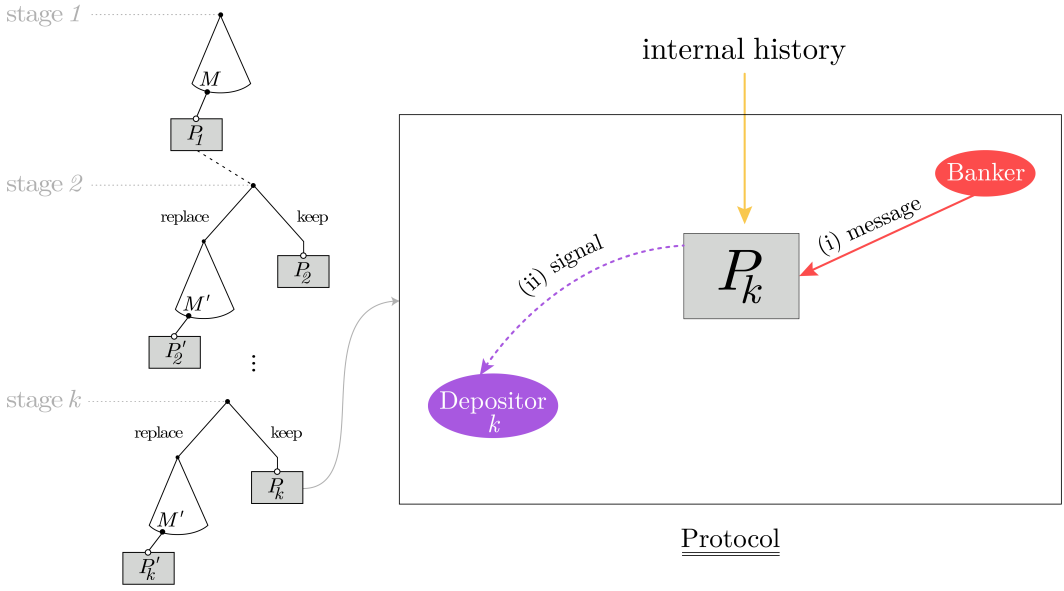
Banker
(i) message
P_k
Depositor k
Protocol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M'
P_2'
stage k
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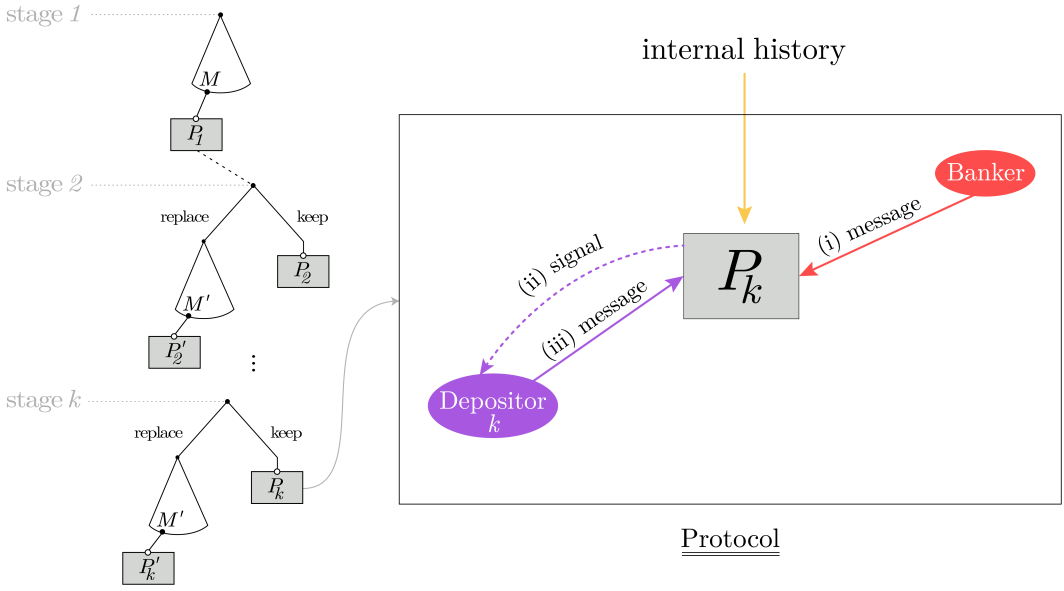
keep
P_k
M'
P_k'
internal history
Banker
(i) message
(ii) signal
P_k
Depositor k
Protocol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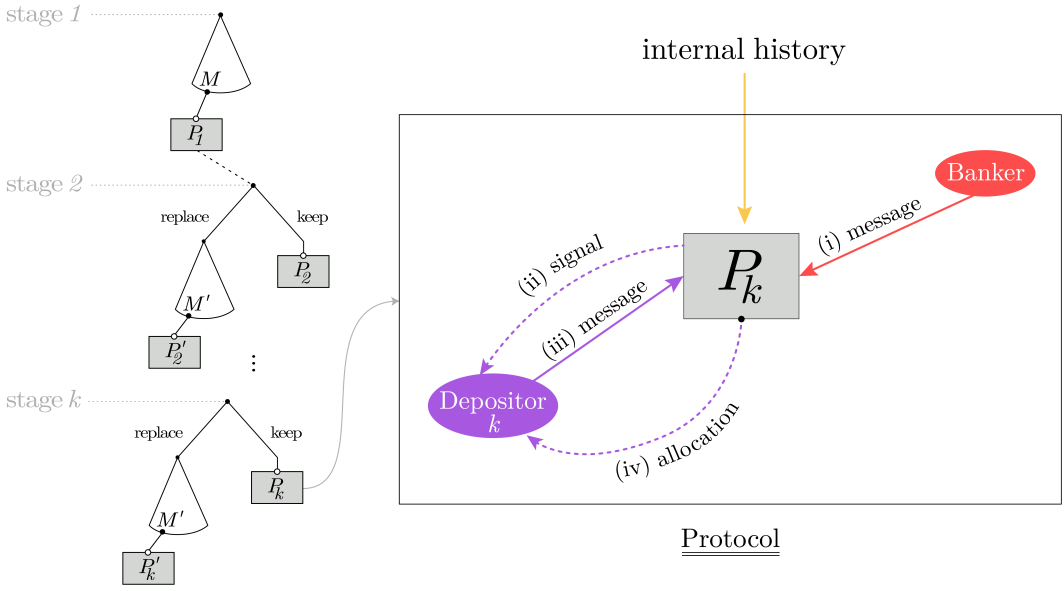
(i) message
(ii) signal
(iii) message
P_k
Depositor k
Protocol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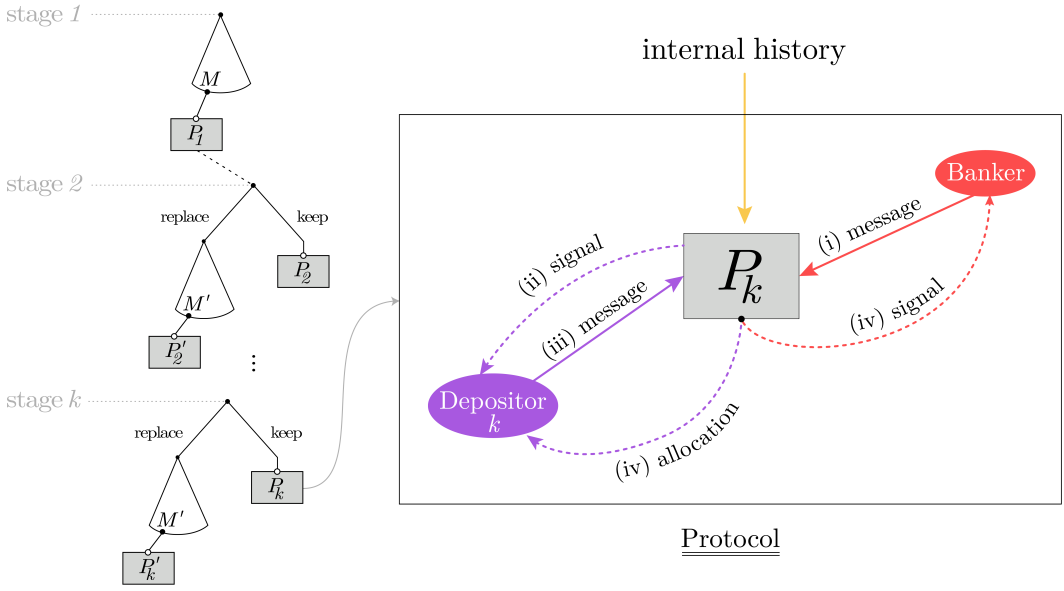
Banker
(i) message
(ii) signal
P_k
(iii) message
Depositor k
(iv) allocation
Protocol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P_k
M'
P_k'
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Banker
(i) message
(ii) signal
P_k
(iii) message
(iv) signal
Depositor k
(iv) allocation
Protocol

Banking Mechanisms

- A banking mechanism M is a sequence of **protocols**: $M = (P_k)_k$
- Banker chooses from a set $\mathcal{M}$ of banking mechanisms as above.
- **Assumption**: $\mathcal{M}$ is 'rich enough.'
  - Contains at least 3-message mechanisms.

▸ Solution Concept

Constrained Efficient Allocation

Constrained Efficient Welfare

We define the *constrained efficient* continuation welfare by:

$$W_k^* = \max_{\mathbf{c}:\ \text{feasible}} \mathbb{E}\Big[w(\mathbf{c} \mid \omega) \,\Big|\mathcal{F}_k\Big]$$

- $\mathcal{F}_k$ is the **full history** of types and their allocations:

  (Y_k, ω^k, π_k) i.e. (resources, past types, # promised future consumption)

- Corresponding allocation:

$$c_k^* = c_k^*(Y_k, \omega^k, \pi_k)$$

$$C^* = R \cdot \frac{Y_N}{\pi_N}$$

Constrained Efficient Welfare

$$W_k^* = \max_{\mathbf{c}\text{: feasible}} \mathbb{E}\Big[w(\mathbf{c} \mid \omega) \,\Big|\mathcal{F}_k\Big]$$

- This problem ignores depositors' incentives.
- ...and ignores the Banker's commitment problem. So:

- **Lemma**: W_k^* is an upper bound to any stage-k equilibrium payoff of the Banker.

Constrained Efficient Welfare

$$W_k^* = \max_{\mathbf{c}\text{: feasible}} \mathbb{E}\Big[w(\mathbf{c} \mid \omega) \,\Big|\mathcal{F}_k\Big]$$

- This allocation also satisfies *continuation IC*:
- **Lemma**: For all k,

$$\mathbb{E}\Big[u\big(c_k^*\big) \mid \omega_k = 1\Big] \leq \mathbb{E}\big[u(C^*) \mid \omega_k = 1\big]$$

- If I am patient, and all patient depositors after me consume late, I prefer consuming late!

A Special Mechanism

A Special Mechanism M^*

i. Reveal to depositor his line-position, k.

ii. Depositor-k chooses $a_k \in \{0, 1, \alpha\}$:

- $0-$ "impatient" $\xrightarrow{\textit{receive}}$ $c_k^*(\tilde{Y}_k, \tilde{\omega}^k, \tilde{\pi}_k)$
- $1-$ "patient and consume late" $\xrightarrow{\textit{promise}}$ C^*
- $\alpha-$ "patient and consume early" $\xrightarrow{\textit{receive}}$ $c_k^*(\tilde{Y}_k, \tilde{\omega}^k, \tilde{\pi}_k)$

iii. Reveal a_k to the Banker (full transparency).

Properties of M^*

Lemma: Pick an equilibrium σ^*, and a path along which M^* has been continuously deployed.

Then, at each stage k, there is common knowledge that M^* *knows* the true history:

$$(\tilde{Y}_k, \tilde{\omega}^k, \tilde{\pi}_k) = (Y_k, \omega^k, \pi_k)$$

Properties of M^*

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Moreover, if M^* is used for Depositor k:

- If Depositor k believes *all* future patient depositors will consume *late*, we must have $\sigma_k^*(1, M^*) = 1$. (continuation IC property)

Equilibrium Outcomes

Theorem: The unique equilibrium outcome is efficient.

- Fix a path where only M^* was deployed.
- We will prove by induction that the equilibrium payoff of the Banker, at any stage k, is W_k^*.
- Hence, at the initial stage the welfare from deploying M^* is W_1^*.
- Equilibrium outcome is efficient.

Proof

Fix an equilibrium profile σ^*, and consider the path where M^* has been continuously deployed.

Induction:

- Depositor N strictly prefers to consume late under M^*.
- So $\sigma_N^*(1, M^*) = 1$.
- Anticipating this the Banker can guarantee W_N^* by deploying M^*.
- This is an upper bound so W_N^* must be the equilibrium payoff at stage N.

Proof

- Suppose that for $n < N$, the equilibrium payoff along a path where only M^* has been deployed, is W_n^*.
- We will show that the equilibrium payoff at stage $(n-1)$ along that path is W_{n-1}^*.
- By the induction hypothesis, if M^* is deployed, no patient depositor after $n-1$ consumes late. (no matter the history)
- Continuation IC (lemma) $\Rightarrow \sigma_{n-1}^*(1, M^*) = 1$.
- So the Banker can guarantee W_{n-1}^* with M^*.
- This is an upper bound...

What just happened?

- We 'disassociate' the mechanism from the designer.
- This allows us to construct mechanisms which collect useful information, *even in the event of a run*!
  - No matter the history, M^*, always learns it exactly.
  - Eliminates 'belief divergence' between Banker and depositors.
- This information is used to set up the ex-post efficient allocation.
  - Patient depositors are deterred from withdrawing early.
  - Banker is deterred from replacing them.

Conclusion

Q. What is the ultimate source of fragility in Diamond-Dybvig?

A. Inability to **collect** useful information *and* **respond** to it optimally.

With enough contractual flexibility:

- Commitment is not an issue.
- Efficiency vs Stability trade-off disappears.

How much is 'enough' flexibility?

Conclusion

Smart Contracts: Protocols specifying parties' actions, and allocations given actions.

- **Automatic execution**.
  - Once in place, allocation can be implemented without need for further actions.
- **Cryptographic security**.
  - Information communicated to the protocol is encrypted.
  - Prevents parties from accessing it even if they have access to the contract's code.
- **Immutability**. promise?
  - Impossibility of ever altering or stopping the protocol.

Conclusion

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Thank you!

Strategies and Solution Concept

Strategies:

$$\sigma_k : \Big\{\text{info. set}\Big\} \longmapsto \Big\{\text{message into deployed protocol}\Big\}$$

$$\sigma_k^B : \Big\{\text{info. set}\Big\} \longmapsto \Big\{\text{mechanism to deploy}\Big\} \times \Big\{\text{message into deployed protocol}\Big\}$$

Solution Concept: Correlation + Sequential Rationality + Consistency of beliefs...

+ tie-breaking in favour of Banker.

- e.g. PBE, sunspot PBE, etc..

Assumption: $\mathcal{M}$ is such that an equilibrium exits.

- e.g. $\mathcal{M}$ discrete.

Back